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# UNDERSTANDING RISK AND RETURN

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# UNDERSTANDING RISK AND RETURN

## **ABSTRACT**

This paper uses an intertemporal equilibrium asset pricing model to interpret the cross-sectional pattern of stock and bond returns. The model relates assets' mean returns to their covariances with the contemporaneous return and news about future returns on the market portfolio. In a departure from standard practice, the market portfolio return is measured using data on both the aggregate stock market and aggregate labor income. The paper finds that aggregate stock market risk is the main factor determining excess stock and bond returns, but that the price of stock market risk does not equal the coefficient of relative risk aversion as would be implied by the static Capital Asset Pricing Model.

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#### 1. Introduction

Much recent research has described the cross-sectional and time-series behavior of asset returns. Asset pricing models with multiple observable or unobservable factors appear to summarize the cross-sectional pattern of returns on common stocks, including some phenomena which are anomalies for the single-factor Capital Asset Pricing Model (CAPM). In the time-series, several variables have emerged as reliable forecasters of returns on stocks and other long-term assets. Fama (1991) provides a comprehensive recent survey of this work.<sup>1</sup>

Fama's survey emphasizes the extent to which cross-sectional research has proceeded independently of time-series research. Yet the time-series behavior of returns can provide vital additional evidence needed to interpret and validate cross-sectional models. As Fama puts it, existing multi-factor models "leave one hungry for economic insights about how the factors relate to uncertainties about consumption and portfolio opportunities that are of concern to investors, that is, the hedging arguments for multifactor models of Fama (1970) and Merton (1973)" (Fama 1991 p. 1594). In addition, "since multifactor models offer at best vague predictions about the variables that are important in returns and expected returns, there is the danger that measured relations between returns and economic factors are spurious, the result of special features of a particular sample (factor dredging)" (p. 1595). Fama calls for "a coherent story that relates the variation through time in expected returns to models for the cross-section of expected returns" (p. 1610).

In this paper I explore the empirical implications of the story developed in Campbell (1993). There I showed that if asset returns are conditionally lognormal and homoskedastic, and if there is a representative agent with the objective function proposed by Epstein and Zin (1989, 1991) and Weil (1989) (a generalization of power utility), then a loglinear approximation to the budget constraint gives a closed-form solution for consumption. One can then derive an asset pricing formula that makes no reference to consumption, instead relating assets' returns to their covariances with the market return and news about future market returns. In this framework the factors in a multi-factor model are innovations in variables that forecast the market return, and

<sup>&</sup>lt;sup>1</sup> Both the cross-sectional and time-series literatures are far too large to cite adequately. A partial list of cross-sectional references might include Chan, Chen, and Hsieh (1985), Chen, Roll, and Ross (1986), Fama and French (1992, 1993), Jagannathan and Wang (1993), Roll and Ross (1880), and Shanken and Weinstein (1990). A partial list of time-series references might include Campbell (1987), Campbell and Shiller (1988), Fama and French (1988a,b, 1989), Fama and Schwert (1977b), Keim and Stambaugh (1986), and Poterba and Summers (1988). Ferson and Harvey (1991) is one of the few papers that bridges these two literatures.

the prices of the factors are determined by the coefficient of relative risk aversion and the time-series behavior of the market return. This result generalizes straightforwardly to the case where asset returns are conditionally heteroskedastic, provided that the elasticity of intertemporal substitution is sufficiently close to one.

In applying this approach, an obvious problem is the "Roll critique" that the market portfolio return may not be easily observable (Roll 1977). It is standard in the finance literature to use the return on a value-weighted index of common stocks as a measure of the market portfolio return, but this excludes many other components of wealth. Perhaps most important, it omits human capital. Here I depart from standard practice by using a time-series model for aggregate labor income to proxy the return on human capital. The priced factors in the multi-factor asset pricing model then include innovations in variables that forecast labor income.<sup>2</sup>

The organization of the paper is as follows. Section 2 reviews the theory in Campbell (1993), and section 3 discusses strategies for measuring the return on the market portfolio. Section 4 discusses econometric methodology. Section 5 describes the data and presents empirical results, while section 6 concludes.

<sup>&</sup>lt;sup>2</sup> Hardouvelis, Kim, and Wizman (1992) and Li (1991) have used the intertemporal asset pricing model of Campbell (1993) but have treated the stock index return as the market portfolio return. Fams and Schwert (1977a) and Jagannathan and Wang (1993) have used labor income growth to test Mayers' (1972) version of the CAPM allowing for human capital. Both these papers assume that labor income growth is unforecastable. Shiller (1993) is closer to the present paper in that it uses a time-series model to construct innovations in the present value of aggregate income forecasts.

# 2. Intertemporal Asset Pricing Without Consumption Data: A Summary

## Approximating the budget constraint

The model considered by Campbell (1993) is a representative agent economy in which human capital is tradable along with other assets. Define  $W_t$  and  $C_t$  as wealth and consumption at the beginning of time t, and  $R_{m,t+1}$  as the gross simple return on invested wealth (the "market portfolio"). The representative agent's dynamic budget constraint can then be written as

$$W_{t+1} = R_{m,t+1}(W_t - C_t). (2.1)$$

Labor income does not appear explicitly in this budget constraint because the market value of tradable human capital is included in wealth.

The budget constraint in (2.1) is nonlinear because of the interaction between subtraction (of consumption from wealth) and multiplication (of market return and invested wealth). Campbell (1993) suggests dividing (2.1) by  $W_t$ , taking logs, and then using a first-order Taylor approximation around the mean log consumption-wealth ratio c-w. If one defines a parameter  $\rho \equiv 1 - \exp(c-w)$ , the approximation can be written as

$$\Delta w_{t+1} \approx r_{m,t+1} + k_w + \left(1 - \frac{1}{\rho}\right)(c_t - w_t),$$
 (2.2)

where lower-case letters are used for logs and  $k_w$  is a constant that need not concern us here. Campbell shows that if the log consumption-wealth ratio is stationary, this approximation implies

$$c_{t+1} - E_t c_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r_{m,t+1+j}$$
$$- (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j}. \tag{2.3}$$

Equation (2.3) says that an upward surprise in consumption today must correspond to an unexpected return on wealth today (the first term in the first sum on the right hand side of the equation), or to news that future returns will be higher (the remaining terms in the first sum), or to a downward revision in expected future consumption growth (the second sum on the right hand side).

The consumer's objective function

The next step is to use a loglinear Euler equation to eliminate expected future consumption growth from the right hand side of (2.3), leaving only current and expected future asset returns. Campbell (1993) uses the non-expected utility model proposed by Epstein and Zin (1989, 1991) and Weil (1989) in order to distinguish the coefficient of relative risk aversion and the elasticity of intertemporal substitution. In the standard model of time-separable power utility, relative risk aversion is the reciprocal of the elasticity of intertemporal substitution, but these concepts play quite different roles in the asset pricing theory. The non-expected utility model also allows intertemporal considerations to affect asset prices even when the consumption-wealth ratio is constant, something which is not possible with time-separable power utility.

The objective function for a simple non-expected utility model is defined recursively by

$$U_{t} = \left\{ (1-\beta)C_{t}^{1-\frac{1}{\sigma}} + \beta \left( E_{t} U_{t+1}^{1-\gamma} \right)^{\frac{1-\frac{1}{\sigma}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\sigma}}}$$

$$= \left\{ (1-\beta)C_{t}^{\frac{1-\gamma}{\theta}} + \beta \left( E_{t} U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}. \tag{2.4}$$

Here  $\gamma$  is the coefficient of relative risk aversion,  $\sigma$  is the elasticity of intertemporal substitution, and  $\theta$  is defined, following Giovannini and Weil (1989), as  $\theta = (1 - \gamma)/(1 - \frac{1}{\sigma})$ . Note that in general the coefficient  $\theta$  can have either sign. Important special cases of the model include the case where the coefficient of relative risk aversion  $\gamma$  approaches one, so that  $\theta$  approaches zero; the case where the elasticity of intertemporal substitution  $\sigma$  approaches one, so that  $\theta$  approaches infinity; and the case where  $\gamma = 1/\sigma$ , so that  $\theta = 1$ . Inspection of (2.4) shows that this last case gives the standard time-separable

power utility function with relative risk aversion  $\gamma$ . When both  $\gamma$  and  $\sigma$  equal one, the objective function is the time-separable log utility function.

Epstein and Zin (1989, 1991) have solved for the Euler equations corresponding to this objective function when the budget constraint is described by (2.1). Assume for the present that asset prices and consumption are conditionally homoskedastic. Also, either assume that asset prices and consumption are jointly lognormal or use a second-order Taylor approximation to the Euler equations. Then these equations can be written in loglinear form as

$$E_t \Delta c_{t+1} = \mu_m + \sigma E_t r_{m,t+1}, \qquad (2.5)$$

and

$$E_{t} r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \theta \frac{V_{ic}}{\sigma} + (1 - \theta) V_{im}. \tag{2.6}$$

In (2.5)  $\mu_m$  is an intercept term related to the second moments of consumption and the market return, which have been assumed constant. In (2.6)  $r_{f,t+1}$  is a riskless real interest rate, while  $V_{ii}$  denotes  $\text{Var}(r_{i,t+1} - E_t r_{i,t+1})$  and other expressions of the form  $V_{xy}$  are defined in analogous fashion. The assumption of homoskedasticity ensures that the unconditional variances and covariances of innovations are the same as the constant conditional variances and covariances of these innovations.

Equation (2.5) is the familiar time-series Euler equation that one obtains also with power utility. It says that expected consumption growth is a constant, plus the elasticity of intertemporal substitution times the expected return on the market portfolio.

Equation (2.6) is the implication of the model emphasized by Giovannini and Weil (1989). In this expression all risk premia are constant over time because of the assumption that asset returns and consumption are homoskedastic. (2.6) says that the expected excess log return on an asset is determined by its own variance (a Jensen's Inequality effect) and by a weighted average of two covariances. The first covariance is with consumption growth divided by the intertemporal elasticity of substitution; this gets a weight of  $\theta$ . The second covariance is with the return on the market portfolio; this gets a weight of  $1 - \theta$ .

Three special cases are worth noting. When the objective function is a time-separable power utility function, the coefficient  $\theta=1$  and the model collapses to the loglinear consumption CAPM of Hansen and Singleton (1983). When the coefficient of relative risk aversion  $\gamma=1$ ,  $\theta=0$  and a logarithmic version of the static CAPM pricing formula holds. Most important for the present paper, as the elasticity of intertemporal substitution  $\sigma$  approaches one the coefficient  $\theta$  goes to infinity. At the same time the variability of the consumption-wealth ratio decreases so that the covariance  $V_{ic}$  approaches  $V_{im}$ . It does not follow, however, that the risk premium is determined only by  $V_{im}$  in this case. Giovannini and Weil (1989) show that the convergence rates are such that asset pricing is not myopic when  $\sigma=1$  unless also  $\gamma=1$  (the log utility case).

#### Substituting out consumption

These loglinear Euler equations can now be combined with the approximate loglinear budget constraint. Substituting (2.5) into (2.3), one obtains

$$c_{t+1} - E_t c_{t+1} = r_{m,t+1} - E_t r_{m,t+1}$$
  
  $+ (1 - \sigma)(E_{t+1} - E_t) \sum_{i=1}^{\infty} \rho^j r_{m,t+1+j}.$  (2.7)

The intuition here is that an unexpected return on invested wealth has a one-for-one effect on consumption, no matter what the parameters of the utility function. (This follows from the scale independence of the objective function (2.4)). An increase in expected future returns raises or lowers consumption depending on whether  $\sigma$ , the elasticity of intertemporal substitution, is greater or less than one.

Equation (2.7) implies that the covariance of any asset return with consumption growth can be rewritten in terms of covariances with the return on the market and revisions in expectations of future returns on the market. The covariance satisfies

$$Cov(r_{i,t+1} - E_t r_{i,t+1}, \Delta c_{t+1} - E_t \Delta c_{t+1}) \equiv V_{ic} = V_{im} + (1 - \sigma)V_{ih}, \quad (2.8)$$

where  $V_{ih} \equiv \text{Cov}\left(r_{i,t+1} - E_t r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{m,t+1+j}\right)$ .  $V_{ih}$  is the covariance of the unexpected return on asset i with good news about future returns on the market, i.e. upward revisions in expected future returns.

Substituting (2.8) into (2.6) and using the definition of  $\theta$  in terms of the underlying parameters  $\sigma$  and  $\gamma$ , one obtains a cross-sectional asset pricing formula that makes no reference to consumption:

$$E_t r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \gamma V_{im} + (\gamma - 1)V_{ih}. \tag{2.9}$$

Equation (2.9) is the starting point for the empirical work of this paper. It says that assets can be priced without direct reference to their covariance with consumption growth, using instead their covariances with the return on invested wealth and with news about future returns on invested wealth in the manner of Merton (1973). Moreover, the only parameter of the utility function that enters (2.9) is the coefficient of relative risk aversion  $\gamma$ . The elasticity of intertemporal substitution  $\sigma$  does not appear once consumption has been substituted out of the model.

Equation (2.9) expresses the risk premium (net of the Jensen's inequality effect) as a weighted sum of two terms. The first term, with a weight of  $\gamma$ , is the asset's covariance with the market portfolio. The second term, with a weight of  $\gamma-1$ , is the asset's covariance with news about future returns on the market. When  $\gamma<1$ , assets that do well when there is good news about future returns on the market have lower mean returns, but when  $\gamma>1$ , such assets have higher mean returns. The intuitive explanation is that such assets are desirable because they enable the consumer to profit from improved investment opportunities, but undesirable because they reduce the consumer's ability to hedge against a deterioration in investment opportunities. When  $\gamma<1$  the former effect dominates, and consumers are willing to accept a lower return in order to hold assets that pay off when wealth is most productive. When  $\gamma>1$  the latter effect dominates, and consumers require a higher return to hold such assets.

Finally, equation (2.9) implies that a logarithmic version of the static CAPM holds if  $\gamma = 1$ , if  $V_{ih} = 0$  for all assets, or (less restrictively) if  $V_{ih}$  is proportional to  $V_{im}$  for all assets. The empirical work of this paper suggests that the first two conditions do not hold even approximately, but that the third condition describes the data surprisingly well.

### Heteroskedasticity

So far I have assumed that asset returns and consumption (or equivalently, asset returns and news about future asset returns) are jointly homoskedastic. This assumption simplifies the analysis but is unrealistic. Campbell (1993) discusses various ways to allow for heteroskedasticity. The simplest approach is to assume that the elasticity of intertemporal substitution  $\sigma=1$ , in which case the asset pricing formula (2.9) holds exactly when asset returns are homoskedastic. With heteroskedastic returns, the formula still holds exactly if one uses conditional expected excess returns and conditional variances and covariances:  $V_{ii}$  becomes  $V_{ii,t}$  and so forth.<sup>3</sup> One can then take unconditional expectations of the conditional version of (2.9) to put it back in unconditional form. (This is valid because all variances and covariances are of innovations with respect to a conditional information set.) In this paper I assume that  $\sigma=1$  or is close enough to 1 that (2.9) is a good approximate asset pricing model even in the presence of heteroskedasticity, and I test the unconditional implications of this model.

<sup>&</sup>lt;sup>3</sup> Nieuwand (1991) and Restoy (1992) discuss other ways to extend the framework of Campbell (1993) to handle heteroskedasticity.

## 3. Measuring the Return on the Market

The asset pricing model developed in the previous section is empirically testable only if one can measure the return on the market portfolio. Financial economists commonly proxy the market portfolio by a value-weighted index of common stocks, but this practice is questionable. Even if the stock index return captures the return on financial wealth, it is unlikely to capture the return on human wealth. Approximately two-thirds of the GNP goes to labor and only one-third to capital so human wealth is likely to be about twice financial wealth, suggesting that this is a serious omission.

Here I propose a simple way to bring human wealth into the analysis. I start with the relationship

$$R_{m,t+1} = (1 - \nu_t)R_{a,t+1} + \nu_t R_{\nu,t+1} \tag{3.1}$$

where  $\nu_t$  is the ratio of human wealth to total wealth,  $R_{a,t+1}$  is the return on financial wealth (a refers to financial assets), and  $R_{y,t+1}$  is the return on human wealth (y refers to the stream of labor income). The next step is to take logs of (3.1) and linearize around the means of  $\nu_t$ ,  $r_{a,t+1}$  and  $r_{y,t+1}$ , assuming that the means of the latter two variables are the same, that is, that the average log return on financial wealth equals the average log return on human wealth. The result is

$$r_{m,t+1} \approx k_m + (1-\nu)r_{a,t+1} + \nu r_{y,t+1},$$
 (3.2)

where  $k_m$  is a constant that plays no role in what follows, and  $\nu$  is the mean of  $\nu_t$ . This approximation can also be obtained by noting that  $r_{a,t+1} \approx R_{a,t+1} - 1$  and  $r_{y,t+1} \approx R_{y,t+1} - 1$ , and linearizing around the mean of  $\nu_t$ .

Of course, the return on human wealth is not directly observable. What is observable is aggregate labor income  $y_t$ , which can be thought of as the dividend on human wealth. If I assume that the conditional expected return on financial wealth equals the conditional expected return on human wealth (a slightly stronger assumption than the

<sup>&</sup>lt;sup>4</sup>This statement abstracts from variations in work effort that might affect marginal utility in a fully-specified model with endogenous labor supply.

one used to derive (3.2)), then the log-linear approximation of Campbell and Shiller (1988) and Campbell (1991) implies that

$$r_{y,t+1} - E_t r_{y,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta y_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{a,t+1+j}.$$
(3.3)

Increases in expected future labor income cause a positive return on human capital, but increases in expected future asset returns cause a negative return on human capital because the labor income stream is now discounted at a higher rate and is therefore worth less today. Equation (3.3) is similar to the formula used in Shiller (1993), except that Shiller discounts aggregate income at a constant rate and therefore includes only the first summation in (3.3). Fama and Schwert (1977a) and Jagannathan and Wang (1993) also discount income at a constant rate, but in addition they assume that labor income growth is unforecastable so that they include only the first term in the first summation in (3.3):  $r_{y,t+1} - E_t r_{y,t+1} = \Delta y_{t+1}$ .

Substituting (3.3) into (3.2) gives

$$r_{m,t+1} - E_t r_{m,t+1} = (1 - \nu)(r_{a,t+1} - E_t r_{a,t+1})$$

$$+ \nu (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta y_{t+1+j}$$

$$- \nu (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{a,t+1+j}. \tag{3.4}$$

Because I am now using forecasts of future labor income to calculate the human capital component of the market return, forecasts of future stock returns also appear in the formula as the discount rates applied to labor income.

Substituting (3.4) into (2.7), I obtain

$$c_{t+1} - E_t c_{t+1} = (1 - \nu)(r_{a,t+1} - E_t r_{a,t+1})$$

$$+ \nu (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta y_{t+1+j}$$

$$+ (1 - \sigma - \nu)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{a,t+1+j}. \tag{3.5}$$

Changes in interest rates (expected future stock returns) affect consumption directly through their effect on the value of human wealth, as well as indirectly through intertemporal substitution. The former effect depends only on  $\nu$ , while the latter depends on  $\sigma$ . When  $\nu=0$ , (3.5) gives the conventional result that increases in interest rates drive down consumption only if  $\sigma>1$ . As human capital becomes more important, however, interest rate increases drive down consumption even with lower values of  $\sigma$ . In the extreme case where  $\nu=1$ , so that there is only human wealth and no financial wealth, interest rate increases drive down consumption for any value of  $\sigma$ . Summers (1982) has also emphasized the importance of human capital in determining the response of consumption to interest rates.

The focus of this paper is on the risk premium formula (2.9), which becomes

$$E_{t} r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \gamma (1-\nu)V_{ia} + \gamma \nu V_{iy} + (\gamma (1-\nu) - 1)V_{ih}, \quad (3.6)$$

where  $V_{iy} \equiv \operatorname{Cov}\left(r_{i,t+1} - E_t r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta y_{t+1+j}\right)$ .  $V_{iy}$  is the covariance of the return on asset i with good news about future labor income. This appears in (3.6) with a weight of  $\gamma \nu$ , while the covariance with the stock market return  $V_{ia}$  has a weight of  $\gamma(1-\nu)$ . Since  $\nu$  is likely to be on the order of two-thirds, this formula shows that labor income risk can be important in pricing assets.

# 4. Econometric Methodology

# A vector autoregressive factor model

To derive testable implications of the asset pricing formula (3.6), I adapt the vector autoregressive (VAR) approach of Campbell (1991). I write the real stock index return as the first element of a K-element state vector  $z_t$ , and real labor income growth as the second element. The other elements of  $z_t$  are variables that are known to the market by the end of period t and are relevant for forecasting future stock returns and labor income growth. For simplicity, I assume that all the variables in  $z_t$  have zero means or have been demeaned before the analysis begins, and I assume that the vector  $z_t$  follows a first-order VAR:

$$z_{t+1} = Az_t + \epsilon_{t+1}. \tag{4.1}$$

The assumption that the VAR is first-order is not restrictive, since a higher-order VAR can always be stacked into first-order (companion) form in the manner discussed by Campbell and Shiller (1988a). The matrix A is known as the companion matrix of the VAR.<sup>5</sup> The advantage of working with a first-order VAR is that it generates simple multi-period forecasts of future returns:

$$E_t z_{t+1+j} = A^{j+1} z_t. (4.2)$$

Next I define a K-element vector e1, whose first element is one and whose other elements are all zero. This vector picks out the stock return  $r_{at}$  from the vector  $z_t$ :  $r_{at} = e1'z_t$ , and  $r_{a,t+1} - E_t r_{a,t+1} = e1'\epsilon_{t+1}$ . Similarly, I define a K-element vector e2, whose second element is one and whose other elements are all zero, which picks out labor income growth  $\Delta y_t$  from the vector  $z_t$ .

It follows that the discounted sum of revisions in forecast stock returns can be written as

<sup>&</sup>lt;sup>5</sup> As is well known, VAR systems can be normalized in different ways. For example, the variables in the state vector can be orthogonalized so that the variance-covariance matrix of the error vector  $\epsilon$  is diagonal. The results given below hold for any observationally equivalent normalization of the VAR system.

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{a,t+1+j} = e^{1} \sum_{j=1}^{\infty} \rho^j A^j \epsilon_{t+1}$$

$$= e^{1} \rho A (I - \rho A)^{-1} \epsilon_{t+1} = \lambda'_h \epsilon_{t+1}, \qquad (4.3)$$

where  $\lambda_h'$  is defined to equal  $e1'\rho A(I-\rho A)^{-1}$ , a nonlinear function of the VAR coefficients. The elements of the vector  $\lambda_h$  measure the importance of each state variable in forecasting future returns on the market. If a particular element  $\lambda_{h,k}$  is large and positive, then a shock to variable k is an important piece of good news about future investment opportunities.

Similarly, the discounted sum of revisions in expected future labor income growth is

$$(E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \, \Delta y_{t+1+j} = e2' \sum_{j=0}^{\infty} \rho^j \, A^j \epsilon_{t+1}$$

$$= e2' (I - \rho A)^{-1} \epsilon_{t+1} = \lambda'_{t} \epsilon_{t+1}, \tag{4.4}$$

where  $\lambda'_y$  is defined to equal  $e2'(I-\rho A)^{-1}$ . The elements of  $\lambda_y$  measure the importance of each state variable in forecasting future labor income.

I now define  $V_{ik} \equiv \operatorname{Cov}_t(r_{i,t+1}, \epsilon_{k,t+1})$ , where  $\epsilon_{k,t+1}$  is the k'th element of  $\epsilon_{t+1}$ . Since the first element of the state vector is the stock index return,  $V_{i1} = V_{ia}$ . Then equation (3.6) implies that

$$E_{t} r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \gamma (1-\nu) V_{i1} + \sum_{k=1}^{K} (\gamma \nu \lambda_{y,k} + (\gamma (1-\nu) - 1) \lambda_{h,k}) V_{ik}. \tag{4.5}$$

This is a standard K-factor asset pricing model of the type implied by the Arbitrage Pricing Theory of Ross (1976) and many other financial models. The contribution of

the intertemporal optimization problem is a set of restrictions on the risk prices of the factors. Factors have large risk prices if they are good forecasters of labor income growth or expected future stock returns. The intertemporal model thus implies that priced factors should be found not by running a factor analysis on the covariance matrix of returns (Roll and Ross 1980), nor by selecting important macroeconomic variables (Chen, Roll, and Ross 1986). Instead, variables that have been shown to forecast stock returns and labor income should be used in cross-sectional asset pricing studies. This is the strategy adopted in the empirical work of this paper.

#### Estimating the model

A natural approach for estimating and testing asset pricing models is the Generalized Method of Moments (GMM) of Hansen (1982). To use GMM, one must define a set of orthogonality conditions that identify the parameters of the model. If there are more orthogonality conditions than parameters, then the model is overidentified and can be tested following Hansen (1982). Hall (1992) and Ogaki (1992) are useful recent surveys of GMM methodology. This section discusses the orthogonality conditions implied by the VAR factor asset pricing model. For simplicity, I first describe the case where asset returns are conditionally homoskedastic.

It is useful to think of the orthogonality conditions in three blocks. First, there are the orthogonality conditions that identify the VAR system (4.1). We have

$$z_{t+1} - Az_t \equiv \epsilon_{t+1} \perp z_t, \tag{4.6}$$

where the symbol  $\perp$  indicates orthogonality. Since there are K variables in the state vector  $z_t$ , there are  $K^2$  parameters in the matrix A and equation (4.6) defines  $K^2$  orthogonality conditions. Taken in isolation, the VAR system is just-identified.<sup>6</sup>

Second, there are orthogonality conditions that identify the vector of mean excess returns on assets. If we are working with I excess returns  $e_{i,t+1} = r_{i,t+1} - r_{f,t+1}$ , i = 1...I, then the vector of excess returns  $e_{t+1}$  and the vector of unconditional means  $\mu$  are both  $I \times 1$ . We have

<sup>&</sup>lt;sup>6</sup>Recall that for simplicity, the variables in the state vector are assumed to have been demeaned beforehand. Estimation of means can be incorporated into (4.6) without difficulty; the number of orthogonality conditions and the number of parameters to be estimated both increase by K. The VAR system remains just-identified.

$$e_{t+1} - \mu \equiv \eta_{t+1} \perp \{1, z_t\},$$
 (4.7)

which gives I(K+1) orthogonality conditions to identify I parameters. There are IK overidentifying restrictions in this part of the model arising from the restriction that expected excess returns are constant through time.

An unrestricted factor asset pricing model can be written as

$$\mu_{i} = -\frac{V_{ii}}{2} + \sum_{k=1}^{K} p_{k} V_{ik}, \tag{4.8}$$

where  $p_k$  is the price of risk for the k'th factor. To estimate this, we note that  $V_{ii} = E[\eta_{i,t+1}^2]$  and  $V_{ik} = E[\eta_{i,t+1}\epsilon_{k,t+1}]$ . Thus we can define an expost version of (4.8):

$$u_{i,t+1} \equiv \mu_i + \left(\frac{1}{2}\right) \eta_{i,t+1}^2 - \sum_{k=1}^K p_k \, \eta_{i,t+1} \epsilon_{k,t+1} \perp \{1, z_t\}. \tag{4.9}$$

This gives I(K+1) orthogonality conditions to identify only K new parameters  $p_k$ . Hence there are IK+I-K overidentifying restrictions arising from this part of the system.

Adding up across the three parts of the model, the homoskedastic model with free factor risk prices has  $K^2 + I + K$  parameters,  $K^2 + 2I(K+1)$  orthogonality conditions, and 2IK + I - K restrictions. The intertemporal model (4.5) says that the K factor risk prices  $p_k$  are functions of the VAR parameters and the coefficient of relative risk aversion  $\gamma$ ; this reduces the number of parameters and increases the number of restrictions by K - 1, giving  $K^2 + I + 1$  parameters and 2IK + I - 1 restrictions.

As stated so far, the asset pricing model is very unlikely to describe the data because homoskedasticity requires that squared innovations in factors and returns are orthogonal to the instrument vector  $z_t$ . If asset returns are heteroskedastic, but one uses the unconditional moments of return innovations to estimate the model, then one can drop  $z_t$  from the instrument list in equations (4.7) and (4.9). The model with free factor risk prices has only  $K^2 + I + K$  parameters,  $K^2 + 2I$  orthogonality conditions,

and I - K overidentifying restrictions, while the intertemporal model has  $K^2 + I + 1$  parameters and I - 1 overidentifying restrictions. This is the main approach used in the empirical work below.

Alternatively, one can apply the heteroskedastic asset pricing model to conditional moments of returns. The VAR block of the model remains unchanged, but the block of the model defining mean returns becomes

$$e_{t+1} - \mu - Mz_t \equiv \eta_{t+1} \perp \{1, z_t\},$$
 (4.10)

which gives I(K+1) orthogonality conditions to identify I(K+1) parameters. There are no overidentifying restrictions in this part of the model, because mean returns now vary with the instruments as described by the matrix of parameters M. The factor asset pricing block of the model becomes

$$u_{i,t+1} \equiv \mu_i + M_i z_t + \left(\frac{1}{2}\right) \eta_{i,t+1}^2 - \sum_{k=1}^K p_k \, \eta_{i,t+1} \epsilon_{k,t+1} \perp \{1, z_t\}. \tag{4.11}$$

This gives I(K+1) orthogonality conditions to identify only K new parameters  $p_k$ . As in the homoskedastic model, there are IK + I - K overidentifying restrictions arising from this part of the system. Adding up across the three parts of the conditional model, there are in total (I+K)(K+1) parameters,  $K^2 + 2I(K+1)$  orthogonality conditions, and I(K+1) - K restrictions in the model with free factor risk prices. The intertemporal model subtracts K-1 parameters and adds K-1 restrictions for a total of I(K+1) - 1 restrictions.

# 5. Data and Empirical Results

In this paper I study asset pricing in two separate data sets. The first data set is monthly and runs from January 1952 through December 1990, giving 468 observations. The second data set, based on that used in Campbell and Shiller (1988), is annual and runs from 1871 through 1990, giving 120 observations. All the tables report monthly results in panel A and annual results in panel B.

The first step in implementing the VAR factor model is to define the variables which enter the state vector  $z_t$ . These variables play a double role in the empirical work. First, they are forecasting variables which should be chosen for their ability to predict market returns and labor income growth. Second, innovations in these variables are factors in a cross-sectional asset pricing model so they should be chosen for their ability to explain the cross-sectional pattern of asset returns. If the intertemporal asset pricing model (4.5) is correct, of course, then these two criteria for choosing state variables coincide.

The top part of Table 1, panel A, lists the state variables used in the monthly model. The analysis requires the first two variables to be a real stock index return RVW (the value-weighted index return from the CRSP tape) and real labor income growth LBR (obtained from CITIBASE). Both series are deflated using the Consumer Price Index, adjusted before 1983 to reflect the improved treatment of housing costs that is used in the official index only after 1983.

The remaining variables in the system are the dividend yield on the CRSP value-weighted index DIV (measured in standard fashion as a one-year backward moving average of dividends divided by the most recent stock price); the "relative bill rate" RTB (the difference between the one-month Treasury bill rate from the CRSP Fama file and its one-year backward moving average); and the yield spread between long-and short-term government bonds TRM (obtained from the Federal Reserve Bulletin). All three of these variables have been found to forecast asset returns. The variable RTB, which has been used by Campbell (1991) and Hodrick (1992), can be thought of as a stochastically detrended short-term interest rate. It is equivalent to a triangular weighted average of changes in the short rate, so it is stationary even if there is a unit root in the short rate. Innovations in RTB are effectively innovations in the short rate, so by including RTB and TRM I allow short and long rate innovations to be priced

<sup>7</sup> Starting in 1952 avoids the period of interest rate pegging before the Fed-Treasury Accord of 1951. The dynamics of interest rates were quite different during that period.

factors in the cross-sectional model.

The variables used here include many of the forecasting variables used in the time-series work of Campbell (1987, 1991), Chen (1991), Ferson and Harvey (1991), Hodrick (1992), and Li (1991). Innovations in these variables are similar to factors used in the cross-sectional work of Chan, Chen, and Hsieh (1985), Chen, Roll, and Ross (1986), Ferson and Harvey (1991), Li (1991), and Shanken and Weinstein (1990). However, parsimony is particularly important in the VAR system because the number of parameters to be estimated increases with the square of the number of variables. For this reason some variables used in previous work are omitted here. The default spread, for example, is omitted because it has no marginal explanatory power for stock returns or income when the dividend yield is in the system (Fama and French 1989, Chen 1991). The return on a small stock portfolio is omitted because it is highly correlated with the value-weighted index return and does not forecast that return or labor income. The inflation rate and the industrial production growth rate are also omitted, since the nominal short rate and labor income should be reasonable proxies for these variables.8

The remainder of Table 1, panel A, lists the portfolios used to measure the crosssectional pattern of returns. Following Ferson and Harvey (1991), I use 10 valueweighted size portfolios which are rebalanced annually, 12 value-weighted industry portfolios grouped by 2-digit SIC codes, and 3 bond portfolios. This gives 25 portfolios with a fairly wide range of average returns.

Panel B of Table 1 summarizes the state variables and portfolios used in the annual model. The state variables are as similar as possible to those in the monthly model, but there are two main changes. First, labor income data are not available over the period 1871-1990 so I use GNP data instead. There is some controversy about the measurement of GNP before 1929; I use Romer's (1989) series. Second, in annual data the "relative bill rate" is not well-defined. Since the behavior of short-term nominal interest rates has changed several times during the last century, and since parsimony is important given the smaller number of observations in the annual model, I drop the short-term nominal interest rate and work with 4 rather than 5 factors.

Far fewer portfolio returns are available over the period 1871-1990 than over the

<sup>9</sup> These portfolios were constructed from the raw CRSP data and then checked them against the Ferson-Harvey data, kindly provided by Wayne Ferson. - 18 -

<sup>8</sup> In preliminary work I estimated a 7-variable system including the 5 variables reported in the paper plus the default spread and the small stock return. Most results were similar to those reported here, but I encountered numerical difficulties in estimating the restricted intertemporal asset pricing model.

period 1952-1990. Besides the return on the stock index itself, I use returns on long-term government bonds and on gold, taken from Siegel (1992).

### Dynamics of the state variables

Table 2, panel A, summarizes the dynamic behavior of the state variables. The table reports the coefficients in a 1-lag VAR, estimated monthly in panel A and annually in panel B. The matrix of coefficients is the matrix A in equation (4.1). All variables are measured in percentage points, at a monthly rate in panel A and at an annual rate in panel B. Table 2 also reports the  $R^2$  statistic and standard error of estimate for each equation in the VAR, and a matrix giving the variances, covariances, and correlations of innovations to the system. (Correlations are reported in bold face above the diagonal of the matrix.)

The first row of Table 2, panel A, shows the monthly forecasting equation for the real value-weighted stock index RVW. This equation is similar to many that have been estimated in the literature, and the pattern of coefficients is quite familiar. There is minimal serial correlation in monthly stock returns; hence the coefficient on lagged RVW is small and statistically insignificant. Past labor income growth LBR also has little effect on stock returns. However the dividend yield DIV has a significant positive coefficient, and the interest rate variables RTB and TRM enter with negative and positive signs respectively. These variables are jointly although not individually significant. The equation has a modest  $R^2$  of 0.07, and the standard deviation of stock return innovations is about 4% per month. The annual forecasting equation, reported in panel B, also has a strongly significant dividend yield coefficient but the other variables play little role in forecasting stock returns. The  $R^2$  is again about 0.07, and the standard deviation of stock return innovations is about 17% per year.

The second row of Table 2, panel A, shows the monthly forecasting equation for real labor income growth LBR. Lagged labor income growth has a marginally significant positive coefficient, while the term spread and relative bill rate have strongly significant positive coefficients and the dividend yield has a significant negative coefficient. The  $R^2$  is 0.10 and the standard deviation of monthly innovations to labor income is about

<sup>&</sup>lt;sup>10</sup> A 1-lag VAR seems to capture the dynamics of the data fairly well. Parsimony dictates that the VAR lag length be kept fairly short, but as a robustness check I have also estimated 3-lag monthly and 2-lag annual VARs. The results, summarized in the text below, are generally very similar to those reported in the tables. In an earlier version of this paper I also aggregated the postwar monthly data to a quarterly frequency, estimated a 1-lag quarterly VAR, and again found similar results.

0.5% per month. 11 The annual model estimated in panel B has strong positive effects from lagged GNP growth and the term spread. The annual  $R^2$  is 0.24 and the standard deviation of annual innovations to labor income is about 4.5%. These results strongly reject the assumption of Fama and Schwert (1977a) and Jagannathan and Wang (1993) that labor income growth is unforecastable.

The remaining rows of Table 2, panel A, give the monthly dynamics of the forecasting variables. To a first approximation the variables DIV, RTB, and TRM all behave like persistent AR(1) processes with coefficients of 0.98, 0.81, and 0.93 respectively, although some other variables do enter. In particular, the relative bill rate helps to forecast the dividend yield. The panel B results for annual data are comparable, but of course the own lag coefficients tend to be smaller with lower frequency data.

Finally, Table 2 reports the variances, covariances, and correlations of innovations to the VAR system. Innovations to income are much less volatile than stock returns, suggesting that market risk will be overstated by the traditional procedure which uses stock returns alone. Innovations to the other state variables are much less volatile again. Also there are large negative correlations between innovations to RVW and DIV, and (in monthly data) between innovations to RTB and TRM.

These correlations and differences in volatility make it hard to interpret estimation results for a VAR factor model unless the factors are orthogonalized and scaled in some way. I proceed in the manner of Sims (1980), triangularizing the system so that the innovation in RVW is unaffected, the orthogonalized innovation in LBR is that component of the original LBR innovation orthogonal to RVW, the orthogonalized innovation in DIV is that component of the original DIV innovation orthogonal to RVW and LBR, and so on. I also scale all innovations to have the same variance as the innovation in RVW. The variables in the system are ordered so that the resulting factors are easy to interpret. The orthogonalized innovation to DIV is a shock to the dividend (a change in the dividend-price ratio with no change in the stock return). Thus LBR and DIV measure shocks to labor income and capital income respectively. Similarly, RTB and TRM measure shocks to short rates and long rates that are orthogonal to stock returns and income.

<sup>&</sup>lt;sup>11</sup> Chen (1991) and Estrella and Hardouvelis (1991) estimate a simple quarterly postwar regression of GNP growth on the term spread and find a significant positive coefficient. Chen also runs simple quarterly regressions of GNP growth on other variables in the VAR system, finding significant negative coefficients on lagged RVW and DIV, and a negative coefficient on RTB. It is of course inappropriate to directly compare the multiple monthly regression coefficients reported here with these simple quarterly regression coefficients.

## News about future stock returns and labor income

The VAR systems estimated in Table 2 give long-run forecasts of future stock returns and future labor income growth. Revisions in these forecasts are linear combinations of shocks to the state variables, combinations which are defined by the vectors  $\lambda_h$  and  $\lambda_y$  in equations (4.3) and (4.4). Table 3 reports these vectors for both raw shocks and orthogonalized shocks. As before, monthly results are reported in panel A and annual results in panel B.

Table 3 shows that monthly shocks to LBR, DIV, RTB, and TRM all have positive effects on long-run stock return forecasts that are significant at the 10% level. In annual data shocks to RVW and DIV have a significant effect on long-run stock return forecasts. In both monthly and annual data the main shock driving long-run forecasts of income growth is the current innovation to income, but innovations to TRM also have some positive effect. If labor income growth were unforecastable, current labor income growth would have a weight of one in the vector  $\lambda_y$  and all other variables would have weights of zero; this hypothesis can be rejected at conventional significance levels.

When the VAR innovations are orthogonalized and scaled, a somewhat different pattern emerges. The first element of  $\lambda_h$  now becomes -0.92 in the monthly system, indicating that 92% of a stock return innovation is reversed in the long run. In the raw system this mean reversion is obscured because it operates through the negative correlation of the dividend yield and the contemporaneous market return. 12 The standard error for the first element of  $\lambda_h$  is 0.11, so one can reject at the 5% level the hypothesis that less than 70% of a stock return innovation is reversed in the long run. The importance of other shocks can now be judged by the size of their  $\lambda_h$  coefficients, since all shocks have been scaled to have the same variance. Positive shocks to monthly labor income have a statistically significant but very small positive effect on long-run stock return forecasts. Short rate innovations have a small and insignificant negative effect, long rate innovations have a somewhat larger and marginally significant positive effect, and dividend innovations have a positive effect that is larger again and strongly statistically significant. Thus positive shocks to labor income, capital income, and the long-term interest rate are associated with increased expected stock returns.

In annual data, the estimate of stock market mean reversion is much smaller; the

<sup>12</sup> Campbell (1991) discusses the mean reversion implied by a dividend yield forecasting equation.

first element of the orthogonalized  $\lambda_h$  vector is only -0.21 with a standard error of 0.12. The effect of dividend income on stock returns, however, is stronger than in monthly data. These results reflect the fact that annual variation in dividend yields is more strongly affected by movements in dividends, and less strongly affected by market returns, than is monthly variation in dividend yields.

In the orthogonalized system, the coefficients defining  $\lambda_y$  tend to be smaller than the coefficients defining  $\lambda_h$ . This reflects the fact that long-run labor income forecasts are less volatile than long-run market return forecasts. (Since the orthogonalized shocks have the same variances and zero covariances, the variance of the forecast revisions is just the sum of squared coefficients in  $\lambda_h$  and  $\lambda_y$ .) Shocks to RVW, LBR, RTB, and TRM are about equally important but RTB has a negative sign while the other variables have positive signs. In annual data GNP and TRM are the important variables.

Table 4 reports the variances, covariances, and correlations of four variables: the current stock index return  $e1'\epsilon_{t+1}$ , news about future stock returns  $\lambda'_h\epsilon_{t+1}$ , news about current and future labor income growth  $\lambda'_y\epsilon_{t+1}$ , and current labor income growth  $e2'\epsilon_{t+1}$ . Variances and covariances are reported on and below the diagonal of each matrix, while correlations are reported in boldface above the diagonal. These numbers of course do not depend on whether the original VAR system has been orthogonalized. Several points are striking.

First, news about future stock returns is extremely volatile. In monthly data the variance of news about future stock returns is even slightly larger than the variance of the current return itself. (This is made possible by the fact that the VAR has multiple shocks; it would be impossible in a univariate system.) Second, news about future stock returns is highly negatively correlated with the current return, indicating that return forecasts fall when the stock market rises. This phenomenon is particularly strong in monthly data, reflecting the fact that the dividend yield forecasts future stock returns and its short-run movements are largely driven by current stock returns. Third, news about current and future labor income is positively correlated with the current stock return, particularly in monthly data. This correlation arises from the fact that when the market falls the dividend yield rises, increasing forecasts of future stock returns and lowering forecasts of future labor income growth. Fourth, news about current and future labor income is substantially more volatile than current labor income growth, reflecting the forecastability of the income growth process. Finally, news about current and future labor income has a positive correlation with current labor income growth;

this correlation is 0.54 in monthly data but 0.92 in annual data. Current income is an important signal of long-run prospects, but other information is also relevant in at least the monthly data.

Most of the results described in this section are robust to the choice of VAR lag length. In a 3-lag monthly VAR estimated over the postwar period, the estimate of long-run mean reversion in stock returns is -0.87 with a standard error of 0.09, very little different from the 1-lag estimate of -0.92 with a standard error of 0.11. The main effects of increasing the lag length are that LBR no longer enters as a significant forecaster of long-run stock returns, while TRM no longer enters as a significant forecaster of long-run labor income growth. The relative volatilities and correlations of the news variables in Table 4 are insensitive to lag length. The annual results hold up in a similar manner when a 2-lag VAR is estimated.

#### A first look at the cross-section

I now turn to the cross-sectional aspects of the data. The first column of Table 5 reports the mean excess log return on each stock portfolio over the 1-month Treasury bill rate. The second column reports the mean excess log return adjusted for Jensen's Inequality by adding one-half the own variance of the log return. As before, panel A gives monthly and panel B gives annual results. The units in the table are percentage points per month (panel A) or per year (panel B).

The table shows several well-known facts about average asset returns. First, the average excess return on the value-weighted stock index has been large, over 0.5% at a monthly rate in the monthly data and over 5% at an annual rate in the annual data. <sup>13</sup> Second, small stocks have had a higher average return than large stocks, as shown both by the higher return of the equal-weighted index and by the pattern of returns on size decile portfolios. Third, bonds have had much lower returns than stocks; none of the monthly Ibbotson bond portfolios have an average excess return over 1% at an annual rate, and long-term bonds have a negative average excess return in the annual data set. Finally, the annual data show that average excess returns on gold have been even lower than those on bonds.

The right-hand part of Table 5 shows the covariances of each portfolio with underlying sources of risk: the return on the value-weighted stock index  $(V_{ia})$ , news about

<sup>&</sup>lt;sup>13</sup> The annual equity premium reported here is slightly smaller than that in Mehra and Prescott (1985) because I use the commercial paper rate throughout the annual data set while Mehra and Prescott splice together a commercial paper rate and a Treasury bill rate.

future labor income  $(V_{iy})$ , and news about future stock index returns  $(V_{ih})$ . The traditional CAPM prices assets using only the  $V_{ia}$  column, while a CAPM that takes account of aggregate labor income risk also uses the  $V_{iy}$  column and the intertemporal model also uses the  $V_{ih}$  column. These covariances are all reported in units that match the excess return units in the left hand part of the table. That is, the covariances of natural variables are multiplied by 100 since the mean excess returns have been multiplied by 100 to express them in percentage points. <sup>14</sup>

The equity premium and the coefficient of relative risk aversion

Friend and Blume (1975) used the traditional CAPM and numbers like those in Table 5 to estimate the coefficient of relative risk aversion  $\gamma$ . Recall that the model of this paper is

$$\overline{er}_i + \frac{V_{ii}}{2} = \gamma(1-\nu)V_{ia} + \gamma\nu V_{iy} + (\gamma(1-\nu)-1)V_{ih},$$
 (5.1)

where  $\overline{er}_i \equiv E[r_{i,t+1} - r_{f,t+1}]$ . The traditional CAPM is the special case where human capital is ignored by setting  $\nu = 0$ , and where changing expected returns are ignored by setting  $V_{ih} = 0$ . In this case (5.1) becomes  $\overline{er}_i + V_{ii}/2 = \gamma V_{ia}$ , so  $\gamma$  should equal the average log excess return adjusted for Jensen's Inequality in the second column of Table 5, divided by the stock market covariance in the third column of Table 5. If one uses the value-weighted index row in Table 5, panel A, this gives an estimate of  $\gamma$  of 0.541/0.168 = 3.2. The same row in panel B gives a fairly similar estimate of  $\gamma = 2.1$ .

More recently, Mehra and Prescott (1985) and others have emphasized that a consumption-based approach implies much larger values of  $\gamma$ . Yet it is unclear why the consumption-based approach and the Friend and Blume approach give such different answers. Table 6 presents some back-of-the-envelope calculations to address this issue. The table takes the point estimates from the value-weighted index row of Table 5, and uses them to calculate the value of  $\gamma$  implied by different assumptions about  $\nu$  (the share of human capital) and  $V_{ih}$  (the mean-reversion of stock returns).

The two rows in each panel of Table 6 correspond to two assumptions about the predictability of market returns. The first row sets  $V_{ih} = 0$ , which corresponds to the traditional assumption that the market return is unforecastable (or at least that

<sup>&</sup>lt;sup>14</sup> The variances and covariances here thus are 100 times smaller than those reported in Tables 2 and 4.

any revisions in return forecasts are uncorrelated with the contemporaneous market return). The second row sets  $V_{ih}$  equal to the estimated value from Table 5, -0.156 monthly or -0.817 annually. The four columns in Table 6 correspond to values of  $\nu$  ranging from zero (the traditional approach that ignores human capital) to one (the opposite extreme that ignores the stock market). A reasonable value of  $\nu$  is two-thirds, since this is roughly the share of labor in national output.

Table 6 uses the formula

$$\gamma = \frac{\overline{er}_i + V_{ii}/2 + V_{ih}}{(1 - \nu)(V_{ia} + V_{ih}) + \nu V_{iv}}, \tag{5.2}$$

which follows directly from (5.1). The table shows that estimates of  $\gamma$  as low as Friend and Blume's can be obtained only by ignoring both human capital and stock market mean reversion. In monthly data  $\gamma$  changes most dramatically when one allows for mean reversion; for any value of  $\nu$ ,  $\gamma$  exceeds 20 when mean reversion is estimated rather than assumed to be zero. Changing  $\nu$  has a smaller effect except when  $\nu$  is very close to one. In annual data there is less estimated mean reversion, and the main effects on  $\gamma$  come from taking account of human capital.<sup>15</sup>

To understand these findings, it is helpful to think about the two extreme cases where  $\nu=0$  and  $\nu=1$ . When  $\nu=0$ , (5.1) implies that  $\overline{er}_i+V_{ii}/2=\gamma(V_{ia}+V_{ih})-V_{ih}$ . Since  $V_{ih}$  is negative and (in monthly data) is almost as large in absolute value as  $V_{ia}$ , the sum  $V_{ia}+V_{ih}$  is small, requiring a large  $\gamma$  to fit the equity premium. The mean reversion of the stock market makes its long-run risk much smaller than its short-run risk, so a large coefficient of risk aversion is required to explain a large equity premium. When  $\nu=1$ , (5.1) implies that  $\overline{er}_i+V_{ii}/2=\gamma V_{iy}-V_{ih}$ . But  $V_{iy}$  is small, so again a large  $\gamma$  is required to fit the equity premium. In this case the explanation is that human capital risk is much smaller than short-run stock market risk.

Fama and Schwert (1977a) have argued that Mayers' (1972) version of the CAPM, which allows for human capital, does not differ greatly from the standard CAPM in its empirical predictions. Their argument is based on the empirical claim that  $V_{iy} \approx 0$  for all assets. If one sets  $V_{iy} = 0$  and also  $V_{ih} = 0$  (since Fama and Schwert do not allow

<sup>16</sup> Black (1990) also emphasizes the fact that mean reversion can dramatically alter the relationship between risk aversion and the equity premium.

<sup>15</sup> Very similar results are obtained from 3-lag monthly and 2-lag annual VAR systems. The 3-lag monthly VAR gives somewhat larger coefficients of risk aversion for all specifications, but this does not change the relative magnitudes of the risk aversion coefficients across specifications.

for stock market mean reversion), then (5.2) becomes  $\gamma = (\overline{er}_i + V_{ii}/2)/((1 - \nu)V_{ia})$ . In this case estimates of risk aversion depend on mean returns and covariances with the aggregate stock market, but they also depend on  $\nu$ , the share of human capital in aggregate wealth. Fama and Schwert miss this point because they work with the beta representation of the model and do not consider the relation between the expected excess return on the aggregate stock market and the underlying risk aversion of investors.

System estimation and the pattern of risk prices

The parameter  $\gamma$  and the vectors  $\lambda_h$  and  $\lambda_y$  determine the risk prices associated with the factors in the intertemporal model. Equation (4.5) can be rewritten as

$$\overline{er}_{i} + V_{ii}/2 = \left[ \gamma(1-\nu) + \gamma\nu\lambda_{y1} + (\gamma(1-\nu)-1)\lambda_{h1} \right] V_{i1} + \sum_{k=2}^{K} \left( \gamma\nu\lambda_{yk} + (\gamma(1-\nu)-1)\lambda_{hk} \right) V_{ik}.$$
(5.3)

It is easiest to understand this formula by considering the special cases  $\nu=0$  and  $\nu=1$ . When  $\nu=0$ , (5.3) becomes

$$\overline{\epsilon r}_i + V_{ii}/2 = \left[\gamma + (\gamma - 1)\lambda_{h1}\right]V_{i1} + \sum_{k=2}^K (\gamma - 1)\lambda_{hk}V_{ik}$$

$$\approx \gamma(1 + \lambda_{h1})V_{i1} + \sum_{k=2}^K \gamma\lambda_{hk}V_{ik}, \qquad (5.4)$$

where the approximate equality holds for large  $\gamma$ . Equation (5.4) implies that the risk price for the first factor (the stock market return) is approximately  $\gamma(1+\lambda_{h1})$ . This is much less than  $\gamma$  when  $\lambda_{h1}$  is negative, as it is in the estimates of Table 3. This is another way to see the difficulty with the Friend and Blume calculation of risk aversion. The risk prices for the other factors are approximately  $\gamma\lambda_{hk}$ . Given large  $\gamma$  and the orthogonalized  $\lambda_h$  estimates of Table 3, this means that several different orthogonalized factors have risk prices that are of the same order of magnitude as the stock market risk price.

When  $\nu = 1$ , the stock market factor loses its unique role and equation (5.3) becomes

$$\overline{er}_i + V_{ii}/2 = \sum_{k=1}^K (\gamma \lambda_{yk} - \lambda_{hk}) V_{ik}. \tag{5.5}$$

For large  $\gamma$  the risk prices become approximately proportional to the elements of the vector  $\lambda_y$  that forecasts labor income. The Table 3 estimates of  $\lambda_y$  imply that in this case too, several different orthogonalized factors have risk prices of comparable magnitude.

Table 7 reports estimates of the full unconditional asset pricing model, using all 25 portfolios monthly and 3 portfolios annually. The model is estimated with and without intertemporal restrictions. In the intertemporally restricted specification the parameter  $\nu$  is fixed a priori at 0, 2/3, or 1, while the risk aversion coefficient  $\gamma$  is estimated. In the monthly data the 25 mean portfolio returns then identify one parameter and there are 24 overidentifying restrictions. In the unrestricted 5-factor specification the factor risk prices are freely estimated, so with 25 portfolios there are 20 overidentifying restrictions. In the annual data there are only 3 portfolios so the intertemporal models have 2 overidentifying restrictions and the unrestricted 4-factor model is unidentified.

In the monthly data of panel A, the estimated coefficient of risk aversion ranges from 16 when  $\nu=0$  to 21 when  $\nu=2/3$ . These estimates have very large standard errors, so only the  $\nu=0$  estimate is significantly different from zero at the 5% level. Of the factor risk prices, only the price of stock market risk is precisely estimated. The estimates lie between 2.8 and 4.0, with standard errors around 1. Presumably this reflects the fact that most of the portfolios have much larger covariances with the stock market than with the other factors, enabling this risk price to be pinned down more precisely. The price of stock market risk is Friend and Blume's estimate of the coefficient of risk aversion, but the intertemporal model implies greater risk aversion for the reasons already discussed.

In the intertemporal model, the other factors have risk prices that are related to

<sup>&</sup>lt;sup>17</sup> The reported results are based on 3 iterations of GMM. Starting values for the parameters are as follows: OLS estimates are used for the VAR coefficients, sample mean asset returns are used for the vector  $\mu$ , cross-sectional regression estimates are used for unrestricted factor risk prices, and the  $\gamma$  estimates from Table 6 are used for the restricted models. The initial weighting matrix is optimal conditional on the starting values of the parameters.

their forecasting role. These risk prices are similar in magnitude to the price of stock market risk, although much less precisely estimated. Labor income has a positive risk price that increases with the importance of human capital, since labor income is a strong forecaster of future labor income growth. The dividend yield has a positive risk price when  $\nu$  is small, since a high dividend yield forecasts high future stock returns. This risk price turns negative when  $\nu$  is large, since a high dividend yield forecasts slow future labor income growth. The short rate has a negative risk price for all values of  $\nu$  since a high short rate forecasts low future stock returns and slow future labor income growth. Finally, the long rate has a positive risk price for all values of  $\nu$  since a high long rate forecasts high future stock returns and rapid future labor income growth.

In the unrestricted factor model the risk prices for RVW, RTB, and TRM are similar to those in the intertemporal models. The risk price for DIV is negative, contrary to the intertemporal models with low  $\nu$ , but this risk price is very imprecisely estimated. More seriously, the risk price for LBR is significantly negative in the unrestricted factor model whereas it is always positive in the intertemporal models, particularly when  $\nu$  is high. Chi-squared tests of the overidentifying restrictions reject the unrestricted factor model at the 5.9% level, and reject the three intertemporal models at the 4.9% level when  $\nu=0$ , the 4.3% level when  $\nu=2/3$ , and the 2.9% level when  $\nu=1$ . This pattern of results reflects the mispricing of labor income risk.

In the annual data set there are not enough portfolios to estimate an unrestricted 4-factor model. Accordingly only restricted intertemporal models are estimated in Table 7; the pattern of results is quite similar to that in the monthly data, except that estimated risk aversion coefficients are smaller. None of the models' overidentifying restrictions are rejected at even the 30% level.

The contribution of orthogonalized factors to the cross-sectional pattern of mean returns depends not only on factor risk prices but also on the cross-sectional pattern of assets' covariances with factors. Since the factors have been orthogonalized, the covariance of the real value-weighted index return with the other factors is zero. The covariance of the excess value-weighted index return is not exactly zero since the commercial paper rate is not included in the VAR information set, but it is always very close to zero. By construction, then, the other factors play little role in explaining the excess return on the stock market as a whole. The individual stock portfolios move closely with the aggregate stock market index and thus for these portfolios too the other factors play a secondary role. Bond portfolios behave somewhat differently, covarying

more strongly with the interest rate factors than with the stock market factor. As one would expect, the two long-term bond portfolios covary more strongly with the long rate factor, while the short-term bond portfolio covaries more strongly with the short rate factor.

Table 8 combines the portfolio-factor covariances with a representative set of factor risk prices shown in Table 7. (The risk prices from the intertemporally restricted model with  $\nu=2/3$  are used.) Thus Table 8 shows the contribution of each factor to the expected excess return on each asset. It is immediately obvious that the stock market factor is far more important than any of the other factors in determining expected returns on stock portfolios. Labor income risk increases small stock returns and reduces large stock returns, but this effect is not important enough to explain the size effect in sample average returns. For bond portfolios the important factors are the stock market, the short rate, and the long rate. The first two of these raise bond returns while the last reduces bond returns; since long-term bonds have greater covariances with long rates, this helps to explain the low or negative average term premia over the sample period. Overall the table shows that much of the cross-sectional variation in asset returns can be explained by the stock market covariance emphasized in the traditional CAPM. Figure 1 gives a visual impression of this by plotting sample average returns in the monthly data set against covariances with the value-weighted stock index.

#### Implied behavior of consumption

In the intertemporal model the behavior of risk premia is determined only by the coefficient of relative risk aversion  $\gamma$ ; the intertemporal elasticity of substitution  $\sigma$  plays no role. Conversely, (3.5) shows that the behavior of consumption is determined only by  $\sigma$  and not by  $\gamma$ . An interesting exercise is to calculate the consumption behavior implied by the estimated VAR model for different values of  $\nu$  and  $\sigma$ .

Combining (3.5) with the VAR model, one can calculate the implied consumption innovation as

$$c_{t+1} - E_t c_{t+1} = \left[ (1 - \nu)e^{1'} + \nu \lambda'_y + (1 - \sigma - \nu)\lambda'_h \right] \epsilon_{t+1}.$$
 (5.6)

Table 9, panel A, shows the implied standard deviation of quarterly consumption innovations, and their correlations with quarterly RVW and LBR, for a range of  $\sigma$  and  $\nu$  values. For comparison the table also reports the sample moments for per capita real consumption of nondurables and services, the series most often used in empirical work on consumption-based asset pricing. Quarterly rather than monthly postwar data are used here to try to reduce the effect of measurement error on the consumption data.

As one would expect, theoretical consumption becomes smoother when  $\sigma$  is small, for then stock market mean-reversion smooths consumption. Even when  $\sigma=0$ , however, theoretical consumption is still at least four times as volatile as measured consumption. Looking at the correlations, low values of  $\sigma$  are necessary to get the correlation with stock returns much below one and to get a high correlation between consumption growth and labor income growth. Annual results over the period 1890-1990, reported in panel B, are qualitatively similar. These admittedly rough calculations suggest that low intertemporal substitution is necessary to match consumption behavior with asset return behavior. Kandel and Stambaugh (1991) make the same point by numerically solving a discrete-state asset pricing model calibrated to aggregate U.S. consumption data.

#### 6. Conclusion

The traditional Capital Asset Pricing Model still has an important place in most economists' thinking about asset returns. Does the CAPM give a good first approximation to the cross-sectional pattern of returns, or should the model be abandoned? In this paper I have used a more general intertemporal asset pricing model to try to answer this question.

I have argued that the traditional CAPM is seriously flawed because it ignores time-variation in expected stock returns and the fact that human capital is an important component of wealth. These omitted considerations can have a major impact on asset pricing. Specifically, the CAPM implies that the coefficient of relative risk aversion equals the price of stock market risk (about 3), but the intertemporal model gives a much larger estimate of risk aversion. This is because the intertemporal model allows for long-run mean reversion in the stock market and for a less volatile human capital component of market risk; both these factors reduce the estimated risk of stock market investment and thus increase the risk aversion coefficient needed to fit the equity premium. Also, in a 5-factor version of the intertemporal model the other factors have risk prices of comparable magnitude to the stock market risk price when the factors are orthogonalized and scaled to have the same variance.

Despite these flaws, the CAPM does capture most of the variation in expected excess returns across the assets studied here. At a mechanical level, this result may not be surprising since the market is the first factor in all the multi-factor models studied here. At a more fundamental level, this result comes from the fact that all the stock portfolios studied here have high average excess returns and large covariances with the stock market, while the bond portfolios have low average excess returns and small covariances with the stock market. In the annual data set, gold has a negative average excess return and a negative covariance with the stock market. This cross-sectional variation in covariances with the stock market dwarfs the cross-sectional variation in covariances with any of the other factors, and in this limited sense the CAPM is a good approximate model of stock and bond pricing.

Fama and French (1992) have recently argued that the CAPM is entirely inadequate as a description of cross-sectional asset pricing. They reach this conclusion by showing that stocks' betas are almost cross-sectionally uncorrelated with their expected returns once one controls for their market and book values. Fama and French (1993) argue that a model with 5 factors can account for the cross-sectional pattern of returns. The factors include portfolios which capture common variation in returns on small stocks and on stocks with high ratios of book to market value. As Fama and French admit, these factors "have no special standing in asset pricing theory" and their risk prices are freely estimated to fit the data.

This paper does not directly address Fama and French's findings because it uses traditional size and industry portfolios rather than portfolios grouped by size, beta, and book-to-market ratio. An interesting extension of the research in this paper will be to relate Fama and French's results to the intertemporal model used here. The intertemporal model imposes additional structure on the investigation, and this has several virtues.

First, it reduces the freedom of the researcher to search across specifications. Factors can only appear in the cross-sectional asset pricing model if they are innovations in variables that help to forecast market returns and labor income. A small-stock return, for example, can only be a priced factor if some variable such as the small-stock dividend yield has forecasting power for returns and income. This discipline reduces "data-snooping" bias (Lo and MacKinlay 1990) since variables that spuriously explain the cross-section are unlikely to be the same as variables that spuriously forecast the time-series.

Second, the intertemporal model derives factor risk prices from underlying characteristics of the economy rather than estimating them freely. MacKinlay (1993) argues that multi-factor models can account for observed deviations from the CAPM only if they have unreasonably high risk prices for the extra factors. MacKinlay uses intuition to judge what is a reasonable factor risk price; this paper offers a more formal approach. Ultimately, a satisfying model of risk and return must explain the magnitudes of the rewards that investors receive for bearing different kinds of risk. This paper explores one simple framework in which these rewards can be understood.

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#### TABLE 1

#### Variable Definitions

#### A: Monthly, 1952-1990

Code	Variable	Source
RVW	Real value-weighted stock index return	CRSP
LBR	Real labor income growth rate	CITIBASE
DIV	Dividend yield on value-weighted index	CRSP
RTB	Relative bill rate (bill rate less 1-year moving average)	CRSP Fama file
TRM	Long-short government bond yield spread	Federal Reserve Bulletin
SIZE 1-10	Return on annually rebalanced size deciles (1 small, 10 large)	CRSP
PET	Petroleum industry return (SIC 13, 29)	CRSP
FRE	Finance/real estate industry return (SIC 60~69)	CRSP
CDR	Consumer durables industry return (SIC 25, 30, 36-37, 50, 55, 57)	CRSP
BAS	Basic industry return (SIC 10, 12, 14, 24, 26, 28, 33)	CRSP
FTB	Food/tobacco industry return (SIC 1, 20, 21, 54)	CRSP
CNS	Construction industry return (SIC 15-17, 32, 52)	CRSP
CAP	Capital goods industry return (SIC 34-35, 38)	CRSP
TRN	Transportation industry return (SIC 40-42, 44, 45, 47)	CRSP
UTI	Utilities industry return (SIC 46, 48, 49)	CRSP
TEX	Textiles/trade industry return (SIC 22-23, 31, 51, 53, 56, 59)	CRSP
SVS	Services industry return (SIC 72-73, 75, 80, 82, 89)	CRSP
LSR	Leisure industry return (SIC 27, 58, 70, 78-79)	CRSP
LTG	Long-term government bond return	Ibbotson Associates
STG	Short-term government bond return	Ibbotson Associates
CRP	Corporate bond return	Ibbotson Associates

Notes: Real series are deflated using the CPI adjusted for housing costs before 1983 in the manner adopted for the official CPI after 1983. Stock portfolio returns are value-weighted. Stock and bond returns are measured as an excess over the 1-month Treasury bill rate unless otherwise stated.

## TABLE 1 (CONTINUED)

#### Variable Definitions

## B: Annual, 1871-1990

Code	Variable	Source
RVW	Real return on Cowles/S&P index	Campbell-Shiller (1988), updated from S&P
GNP	Real GNP growth rate	1871-1929: Romer (1989) 1930-1990: NIPA
DIV	Dividend yield on Cowles/S&P index	Campbell-Shiller (1988), updated from S&P
TRM	Long-short government bond yield spread	Long rate: Siegel (1992) Short rate: Campbell-Shiller (1988), updated from S&P
LTG	Long-term government bond return	Siegel (1992)
GLD	Return on gold	Siegel (1992)

Notes: Real series are deflated using the GNP deflator.

TABLE 2

VAR Summary: Dynamics of Risk Factors

## Regression Coefficients

Dependent Regressors						
variable	RVW	LBR	DIV	RTB	TRM	$R^2$
RVW	0.029 (0.058)	0.270 (0.363)	6.903 (2.864)	-6.284 (3.793)	4.099 (2.854)	0.067 (4.15)
LBR	-0.008 (0.006)	$0.132 \\ (0.066)$	-1.441 $(0.371)$	$1.012 \\ (0.326)$	$0.999 \\ (0.233)$	0.101 (0.519)
DIV	-0.000 (0.000)	-0.000 (0.001)	$0.979 \\ (0.011)$	$0.031 \\ (0.014)$	-0.010 (0.010)	0.957 (0.015)
RTB	0.001 (0.001)	$0.007 \\ (0.005)$	-0.081 $(0.049)$	0.806 (0.065)	$0.054 \\ (0.037)$	0.595 (0.056)
TRM	-0.001 (0.001)	-0.004 (0.005)	$0.069 \ (0.047)$	0.097 (0.061)	0.926 (0.038)	0.768 (0.056)

Innovation Variances, Covariances, and Correlations

	Shocks to				
Shocks to	RVW	LBR	DIV	RTB	TRM
RVW	17.04	0.169	-0.942	-0.063	-0.019
LBR	0.360	0.266	-0.129	0.048	-0.051
DIV	-0.057	-0.001	0.000	0.078	0.006
RTB	-0.015	0.001	0.000	0.003	-0.938
TRM	-0.004	-0.001	0.000	-0.003	0.003

## TABLE 2 (CONTINUED)

VAR Summary: Dynamics of Risk Factors

B: Annual, 1871-1990

## Regression Coefficients

Dependent	1				
variable	RVW	GNP	DIV	TRM	$R^2$
RVW	0.171 (0.120)	0.189 (0.331)	2.938 (1.067)	-1.214 (1.139)	0.065 (16.8)
GNP	0.064 (0.029)	$0.319 \\ (0.111)$	$0.129 \\ (0.326)$	$0.752 \\ (0.346)$	0.236 (4.5)
DIV	0.00 <b>3</b> (0.00 <b>7</b> )	$0.007 \\ (0.029)$	$0.686 \\ (0.071)$	$0.012 \\ (0.070)$	0.443 (1.1)
TRM	0.002 (0.007)	-0.030 (0.017)	$0.111 \\ (0.075)$	$0.609 \\ (0.093)$	0.362 (1.2)

## Innovation Variances, Covariances, and Correlations

	Shocks to				
Shocks to	RVW	GNP	DIV	TRM	
RVW	273.2	0.130	-0.751	0.263	
GNP	9.595	19.94	-0.052	0.044	
DIV	-13.13	-0.245	1.120	-0.259	
TRM	4.953	0.223	-0.312	1.295	
	[				

TABLE 3

News About Future Market Returns and Labor Income

A: Monthly, 1952-1990

	Orthog-			Shocks to		
Vector	onalized?	RVW	LBR	DIV	RTB	TRM
$\lambda_h'$	No	0.001 (0.026)	0.349 (0.170)	267.2 (32.38)	33.51 (17.36)	46.01 (23.22)
$\lambda_{m{y}}'$	No	-0.021 (0.017)	1.140 (0.118)	-39.26 (22.04)	13.98 (12.03)	27.79 (15.10)
$\lambda_h'$	Yes	-0.922 (0.109)	$0.068 \ (0.024)$	$0.320 \\ (0.041)$	-0.126 (0.105)	0.207 (0.105)
$\lambda_{m{y}}'$	Yes	0.114 (0.076)	0.129 (0.016)	-0.049 (0.028)	-0.161 (0.075)	$0.125 \\ (0.068)$

B: Annual, 1871-1990

	Orthog-		Shocks to			
Vector	onalized?	RVW	GNP	DIV	TRM	
$\lambda_h'$	No	0.258 (0.131)	0.497 (0.393)	9.311 (3.546)	-2.293 (2.204)	
$\lambda_{m{y}}'$	No	0.112 (0.060)	1.391 (0.244)	1.961 (1.665)	2.063 (1.161)	
$\lambda_h'$	Yes	-0.214 (0.124)	0.159 (0.106)	0.408 (0.149)	-0.152 (0.146)	
$\lambda_{m{y}}'$	Yes	0.104 (0.077)	0. <b>3</b> 80 (0.066)	0.069 (0.067)	0.1 <b>36</b> (0.077)	

TABLE 4

## Financial and Human Capital Risk: Covariances and Correlations of News Variables

A: Monthly, 1952-1990

	Current RVW	Future RVW	Current and future LBR	Current LBR
	$e1'\epsilon_{t+1}$	$\lambda_h' \epsilon_{t+1}$	$\lambda_y'\epsilon_{t+1}$	$\epsilon 2' \epsilon_{t+1}$
$e1'\epsilon_{t+1}$	17.04	-0.915	0.422	0.169
$\lambda_h' \epsilon_{t+1}$	-15.71	17.29	-0.242	-0.088
$\lambda_y' \epsilon_{t+1}$	1.950	-1.128	1.256	0.541
$e2'\epsilon_{t+1}$	0.360	-0.188	0.313	0.266

B: Annual, 1871-1990

	Current RVW	Future RVW	Current and future GNP	Current GNP
	$e1'\epsilon_{t+1}$	$\lambda_h' \epsilon_{t+1}$	$\lambda_y' \epsilon_{t+1}$	$e2'\epsilon_{t+1}$
$e^{1'}\epsilon_{t+1}$	273.2	-0.420	0.246	0.130
$\lambda_h' \epsilon_{t+1}$	-58.48	71.10	0.213	0.255
$\lambda_y' \epsilon_{t+1}$	28.38	12.52	48.74	0.923
$e2'\epsilon_{t+1}$	9.595	9.591	28.78	19.94

TABLE 5
Unconditional Risk and Return: A Summary

Portfolio	ēr;	$\overline{er}_i + V_{ii}/2$	$V_{ia}$	$V_{iy}$	$V_{ih}$
EW	0.562	0.690	0.184	0.022	-0.170
vw	0.451	0.541	0.169	0.019	-0.156
Size 1	0.641	0.850	0.190	0.022	-0.178
Size 2	0.644	0.812	0.188	0.023	-0.176
Size 3	0.595	0.757	0.191	0.023	-0.176
Size 4	0.594	0.739	0.189	0.023	-0.174
Size 5	0.533	0.671	0.187	0.022	-0.172
Size 6	0.554	0.681	0.183	0.021	-0.168
Size 7	0.547	0.669	0.185	0.021	-0.169
Size 8	0.542	0.656	0.182	0.021	-0.168
Size 9	0.491	0.592	0.173	0.020	-0.161
Size 10	0.387	0.472	0.158	0.019	-0.145
PET	0.548	0.683	0.152	0.017	-0.145
FRE	0.420	0.535	0.174	0.018	-0.166
CDR	0.408	0.546	0.182	0.022	-0.164
BAS	0.421	0.534	0.180	0.021	-0.166
FTB	0.645	0.730	0.144	0.016	-0.134
CNS	0.350	0.513	0.196	0.020	-0.183
CAP	0.459	0.588	0.173	0.023	-0.155
TRN	0.309	0.481	0.198	0.023	-0.183
UTI	0.450	0.511	0.109	0.012	-0.102
TEX	0.435	0.580	0.180	0.019	-0.162
SVS	0.445	0.624	0.191	0.022	-0.174
LSR	0.590	0.776	0.208	0.023	-0.188
LTG	-0.009	0.026	0.028	0.002	-0.034
STG	0.065	0.076	0.012	0.002	-0.014
CRP	0.018	0.048	0.030	0.003	-0.035

# TABLE 5 (CONTINUED)

Unconditional Risk and Return: A Summary

B: Annual, 1871-1990

Portfolio	er <sub>i</sub>	$\overline{er}_i + V_{ii}/2$	$V_{ia}$	$V_{iy}$	$V_{ih}$
RVW	3.501	5.197	2.525	0.342	-0.817
LTG	-0.322	-0.142	0.231	0.065	-0.133
GLD	-2.349	-1.429	-0.314	-0.046	-0.079

Notes: All variables are in percentage points.

TABLE 6

# Fitting the Equity Premium: A Back-of-the-Envelope Calculation

A: Monthly, 1952-1990

	ν =				
$V_{ih} =$	0	1/3	2/3	1	
0	3.2	4.6	7.8 23	28	
Estimated	31	26	23	20	

B: Annual, 1871-1990

	$\nu =$				
$V_{ih} =$	0	1/3	2/3	1	
0 Estimated	2.1	2.9	4.9	15	
Estimated	2.6	3.5	5.5	-13	

TABLE 7

Estimates of Risk Aversion and Risk Prices

Portfolios			İ	Ri	sk prices	for	
(Restricted?)	ν	γ	RVW	LBR	DIV	RTB	TRM
22 stocks, 3 bonds (Yes)	0	15.61 (7.368)	3.903 (1.153)	0.758 (0.615)	3.893 (2.444)	-4.858 (2.126)	$4.401 \ (2.054)$
22 stocks, 3 bonds (Yes)	2/3	21.21 (11.25)	3.406 (1.090)	$2.202 \\ (1.324)$	1.083 $(1.164)$	$-4.256 \\ (1.969)$	$\frac{3.694}{(2.035)}$
22 stocks, 3 bonds (Yes)	1	17.66 (11.64)	2.840 (0.986)	2.185 (1.604)	-1.228 (0.404)	-3.885 (1.980)	$\frac{2.893}{(2.004)}$
22 stocks, 3 bonds (No)	N/A	N/A	3.075 (1.315)	-10.35 (4.339)	-1.227 (4.307)	-4.046 $(2.612)$	5.097 $(2.634)$

B: Annual, 1871-1990

Portfolios			1	Ri	sk prices	for	
(Restricted?)	ν	γ	RVW	GNP	DIV	TRM	
Stocks, bonds, gold (Yes)	0	2.683 (1.102)	2.355 (0.860)	$0.291 \\ (0.235)$	$0.661 \\ (0.503)$	-0.307 (0.285)	
Stocks, bonds, gold (Yes)	2/3	5.305 (2.442)	2.107 (0.806)	$     \begin{array}{r}       1.542 \\       (0.783)     \end{array} $	$0.494 \\ (0.530)$	$0.229 \\ (0.298)$	
Stocks, bonds, gold (Yes)	1	9.657 (5.943)	1.608 (0.720)	$3.721 \\ (2.408)$	$0.076 \\ (0.704)$	$0.965 \\ (0.755)$	

TABLE 8
Factor Contributions to Expected Returns

Portfolio	ēr;	$\overline{er}_i + V_{ii}/2$	RVW	LBR	DIV	RTB	TRM
EW	0.562	0.690	0.615	-0.001	0.001	0.003	0.004
VW	0.451	0.541	0.565	-0.010	0.001	-0.010	0.014
Size 1	0.641	0.850	0.632	0.010	-0.005	-0.026	0.009
Size 2	0.644	0.812	0.626	0.011	-0.004	-0.004	0.002
Size 3	0.595	0.757	0.636	0.013	0.001	-0.004	0.013
Size 4	0.594	0.739	0.632	0.001	0.005	0.015	0.003
Size 5	0.533	0.671	0.623	0.001	0.003	0.013	-0.000
Size 6	0.554	0.681	0.609	-0.004	0.005	0.008	0.004
Size 7	0.547	0.669	0.616	-0.010	0.008	0.012	0.007
Size 8	0.542	0.656	0.608	-0.013	0.005	0.017	-0.006
Size 9	0.491	0.592	0.580	-0.009	0.002	0.013	-0.012
Size 10	0.387	0.472	0.528	-0.009	-0.002	-0.006	0.036
PET	0.548	0.683	0.507	-0.016	-0.018	-0.065	0.061
FRE	0.420	0.535	0.584	-0.026	-0.004	0.033	-0.053
CDR	0.408	0.546	0.607	0.004	0.009	-0.017	0.054
BAS	0.421	0.534	0.604	-0.010	-0.003	-0.010	0.038
FTB	0.645	0.730	0.484	-0.017	0.003	0.030	-0.029
CNS	0.350	0.513	0.651	-0.019	0.006	-0.020	-0.027
CAP	0.459	0.588	0.577	-0.013	0.003	0.014	0.101
TRN	0.309	0.481	0.660	0.009	0.000	-0.021	0.021
UTI	0.450	0.511	0.366	-0.010	0.003	0.063	-0.057
TEX	0.435	0.580	0.600	-0.026	0.018	0.010	-0.006
SVS	0.445	0.624	0.636	-0.004	0.006	-0.002	0.023
LSR	0.590	0.776	0.693	-0.022	0.016	-0.008	0.022
LTG	-0.009	0.026	0.095	-0.022	-0.007	0.154	-0.178
STG	0.065	0.076	0.040	-0.011	-0.005	0.124	-0.093
CRP	0.018	0.048	0.101	-0.012	-0.006	0.143	-0.166

Notes: All variables are in percentage points at a monthly rate.

## TABLE 8 (CONTINUED)

## Factor Contributions to Expected Returns

B: Annual, 1871-1990

Portfolio	ēī;	$\overline{\epsilon r}_i + V_{ii}/2$	RVW	GNP	DIV	TRM
RVW	3.501	5.197	5.385	0.370	-0.320	0.069
LTG	-0.322	-0.142	0.495	-0.058	-0.013	0.102
GLD	-2.349	-1.429	-0.659	-0.000	-0.160	0.028

Notes: All variables are in percentage points at an annual rate

TABLE 9

Actual and Theoretical Consumption Behavior

A: Quarterly, 1952-1990

Standard deviation of consumption innovation (actual = 0.43)

	ν =					
σ =	0	1/3	2/3	1		
0	2.6	2.1	1.7	1.8		
0.5	4.5	4.4	4.6	4.8		
1	7.6	7.8	8.0	8.3		
2	14.5	14.7	15.0	15.3		

Correlation of consumption and RVW innovations (actual = 0.26)

	$\nu =$					
σ =	0	1/3	2/3	1		
0	0.38	0.52	0.67	0.69		
0.5	0.96	0.99	0.98	0.94		
1	1.00	0.99	0.98	0.95		
2	0 00	0.08	0.96	0.95		

Correlation of consumption and LBR innovations (actual = 0.47)

	$\nu =$					
$\sigma =$	0	1/3	2/3	1		
0	0.18	0.34	0.53	0.64		
0.5	0.20	0.26	0.30	0.33		
1	0.18	0.21	0.23	0.25		
2	0.16	0.17	0.18	0.19		

## TABLE 9 (CONTINUED)

Actual and Theoretical Consumption Behavior

B: Annual, 1890-1990

Standard deviation of consumption innovation (actual = 2.9)

	$\nu =$					
$\sigma =$	0	1/3	2/3	_ 1		
0	15.2	11.3	8.4	7.5		
0.5	15.6	11.8	9.0	8.0		
1	17.3	13.9	11.4	10.6		
2	22.9	20.4	18.8	18.2		

Correlation of consumption and RVW innovations (actual = 0.07)

	$\nu =$					
$\sigma =$	0	1/3	2/3	_ 1		
0	0.86	0.82	0.67	0.25		
0.5	0.97	0.97	0.86	0.50		
1	1.00	0.98	0.86	0.58		
2	0.94	0.87	0.75	0.57		

Correlation of consumption and GNP innovations (actual = 0.37)

	$\nu =$					
$\sigma =$	0	1/3	2/3	1		
0	0.27	0.45	0.72	0.93		
0.5	0.20	0.35	0.56	0.75		
1	0.13	0.23	0.36	0.47		
2	0.01	0.06	ი 11	0.17		

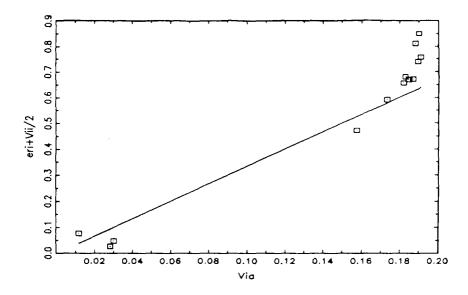


Figure 1
The Fit of the CAPM