(R)EVOLUTION OF ASSET ALLOCATION

Asset allocation – the identification of a mix of assets that in the long run optimally balances risk constraints with return expectations – is at the heart of every portfolio construction process and crucial to its success. Though as diverse as they are innovative, the approaches used to pinpoint the optimal mix of assets mostly have common roots. In the following paper, we address this commonality in depth. First, we outline the portfolio construction process and highlight empirically the importance of asset allocation with respect to a portfolio's return. Second, the evolution of portfolio theory is put into a historical perspective. Third, we present a unified optimisation framework for asset allocation and show that most well-known asset allocation techniques fit exactly in that framework. Finally, an illustrative example brings to light the similiarities and differences of three prevalent approaches and highlights implications for practitioners.

ASSET ALLOCATION – THE CORNERSTONE OF EVERY PORTFOLIO

The investment process of any investor – be it a private person saving for retirement, a life insurer hedging annuity obligations, or a hedge fund manager trying to achieve the highest risk-adjusted return – generally follows a standard sequence of basic steps. Based on a thorough analysis of the individual investment objective, all investors must first define the level of risk they are able and willing to bear, and what return expectations can reasonably be associated with that risk budget. Then, these objectives and constraints need to be mapped into a portfolio that has the highest probability of meeting the expectations. This process has two dimensions. First, the allocation policy among general asset classes must be defined and then, the security selection within those classes. Having created an asset mix that – at least ex ante – meets the expectations, the relevant trades have to be executed and the portfolio monitored on a continuous basis.

While each of the aforementioned steps are vital in themselves, this paper focuses on the second stage – the asset allocation. Commonly, asset allocation strategies are distinguished as either *strategic* or *tactical*. The *strategic asset allocation (SAA)* forms the cornerstone of every portfolio strategy by identifying a mix of assets that in the long run optimally balances the risk constraints with the return expectations. Needless to say, markets may

By Fabian Dori, Frank Häusler and David Stefanovits, July 2011.

and will at least temporarily deviate from their on average expected risk/return attributes. The theory of rational beliefs that is propagated by Mordecai Kurz (1994a, 1994b) from Stanford University, for example, assumes that while investors may have differing beliefs and that they are prone to forecasting errors, their expectations are also correlated. This mutual manipulation may result in time-varying degrees of optimism and pessimism, which in turn considerably affect market prices. Actively exploiting such shorter lived market movements is the aim of the *tactical asset allocation (TAA)*.

In a seminal paper, Brinson et al. (1986) concluded that the strategic asset allocation accounts for the majority of a portfolio's total return and volatility over time. Despite criticism of the interpretation and universality of these early results (e.g. Jahnke [1997], Statman [2002]), further studies (e.g. Brinson et al. [1991], Ibbotson and Kaplan [2000]) confirmed that with respect to both the historical and cross-sectional dimension, setting the strategic weights among asset classes is much more important than selecting the security weights within the individual classes.

Percentage of return variation explained

	Average	Minimum	Maximum
SAA	91.5%	67.7%	98.2%
SAA and TAA	93.3%	69.4%	98.3%
SAA and security selection	96.1%	76.2%	99.8%

Source: Brinson et al. (1991), Database: 82 large US pension plans 1977–1987.

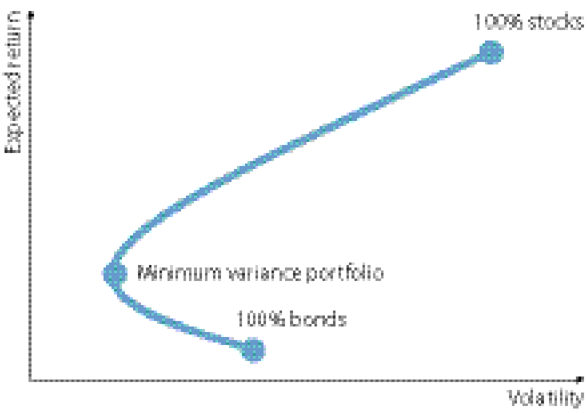
In light of these findings, it is not surprising that several important academic contributions evolved that not only formalised a stringent asset allocation theory, but were also rewarded with the Nobel Prize. So let's turn to the revolutionary evolution of asset allocation theory.

HISTORY OF ASSET ALLOCATION

A natural starting point to discuss the evolution of asset allocation would of course be the landmark 1952 paper of Harry Markowitz on portfolio selection. Yet, as Markowitz (1999) states in his overview of the early history of portfolio theory, he was hardly the first to consider the advantages of diversification. Many authors before him grasped its risk-mitigating effect, at least intuitively. For example, as early as 1738, Daniel Bernoulli argued that *"… it is advisable to divide goods which are exposed to some small danger into several portions rather than to risk them all together"*. John Burr Williams (1938) had an even stronger view on the merits of diversification by claiming that investing in a sufficiently large number of securities would virtually eliminate risk. Leavens (1945) subsequently qualified this view with the condition that the risks be sufficiently independent of each other. Overall, it is fair to say that early in the last century the benefit of diversification as well as an intuitive model of covariance was understood and deemed to be important for investment decisions. However, a formal analysis of these tools, and consequently, a stringent portfolio theory had yet to be developed.

This situation changed considerably in the 1950s with Markowitz's (1952, 1959) work on portfolio selection. Probably the most important contribution centred on his formal proof that it is not a security's idiosyncratic risk that is the most relevant, but rather its contribution to the risk of the entire portfolio. Markowitz was primarily concerned about risk in the sense of the portfolio's variance, and consequently, about a security's covariance with all other instruments within the portfolio. Based on this insight, Markowitz proposed that mean returns, variances, and covariances of securities may be estimated in order to derive efficient mean-variance combinations, from which an investor can choose the portfolio matching his risk

Figure 1: Mean-variance frontier



appetite. Efficient in this context means that a portfolio either yields the highest expected return given a particular level of risk, or the lowest risk given a specific expected level of return (see figure 1).

Virtually at the same time, Roy (1952) also proposed defining portfolio weights on the basis of the covariance of returns, as this determines the expected mean and variance of the entire portfolio. However, unlike Markowitz, who suggested choosing the desired portfolio from the efficient mean-variance combinations, Roy recommended the choice of a specific portfolio. According to his understanding of "safety first", Roy considered the optimal portfolio as that which maximises the risk-adjusted expected excess return, *E[r]*, over a fixed threshold return, r_L: $(E[r] - r_L)/\sigma$.

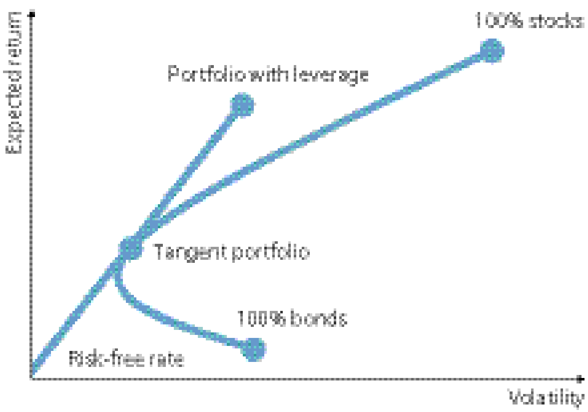
Whilst the contribution of Roy may be classified as an alternative to Markowitz's work, the next significant contribution to the growing literature on portfolio theory stemmed from Tobin (1958). Assuming that besides a number of risky assets the investor has access to a risk-free asset, Tobin showed that an optimal portfolio made up of risky assets only is independent of the investor's attitude towards risk and expected return. Accordingly, all investors should invest in exactly the same mix of risky assets, however, with differing degrees of leverage. This so called *separation theorem* has two important implications. First, in the mean-standard deviation world of Markowitz, it reduced the portfolio problem to finding – on the efficient frontier – the tangency portfolio to a ray of lines that spread out of the riskless asset (see figure 2). The tangency portfolio is the portfolio that maximises the ratio of the mean excess return over the risk free asset to the standard deviation (the Sharpe ratio). Additionally, if all investors hold exactly the same mix of risky assets, the weights of the individual assets in the tangency portfolio must be relative to their market capitalisation. It also paved the way for the *mutual fund theorem*. If the composition of the risky asset portfolio is fixed in any case, all investors can obtain their desired risk/return profile by simply mixing two assets, namely the risk-free asset and the tangency portfolio. For example, a very risk-tolerant investor may borrow at the risk-free rate and leverage his portfolio up in order to increase expected returns, whereas a risk-averse investor probably lends at the risk-free rate and tempers his portfolio risk by investing only a fraction of his wealth in the risky asset mix.

With the portfolio theory advancing step-by-step, new problems arose as well. In particular, the efficient estimation of a vast number of inputs, together with the fact that even small changes in these input factors may lead to considerably different allocation decisions makes the practical implementation of mean-variance analysis a challenging task. In order to solve these issues, two different schools of thought emerged. Whereas one focused on improving the quality of and sensitivity to input estimates, the other aimed at eliminating the portfolio problem's dependency on (unreliable) expected return estimates.

IMPROVING THE QUALITY OF INPUT ESTIMATES

The simplest and earliest models developed for better estimating input factors were single-index models, in particular the market model popularised by Sharpe (1967). By uniquely linking the expected return of any security to its sensitivity with respect to the overall market return, Sharpe's *Capital Asset Pricing Model (CAPM)* not only allows a considerable reduction in the number of estimates

Figure 2: The tangency portfolio



required, but also improves the accuracy of portfolio optimisation. However, the CAPM fails on many dimensions. Above all, it does not capture all risk factors affecting a security's return.

Accord-ingly, soon after the advent of the market model, multi-index models were explored. While both the nature and number of such indices are not restricted from a theoretical point of view, the parsimonious *three-factor asset pricing model of Fama and French* (1992) has attained particular attention. Besides the market return, Fama and French identified size and value as major driving forces explaining an individual security's return. To further reduce the uncertainty of sample estimates, Fischer Black and Robert Litterman (1992) sug-gested the *Black-Litterman model*. In the spirit of the separation theorem, this strategy assumes that the optimal asset allocation is proportional to the market values of the available assets. Accordingly, equilibrium expected returns can be derived from observable security prices, and modified to represent the optimiser's specific opinion about that assets future perspective. Similarly, in an attempt to reduce the sensitivity of the final allocation to the accuracy of the input estimates, Michaud (1998) proposed the *resampled efficient frontier* approach. By resampling the available data, the number of efficient portfolio sets can be increased, and an aver-age over these sets can be taken. The resulting portfolio is not optimal with respect to any of the underlying resampled optimisation problems (in general), but is considered more stable with respect to input parameters.

REDUCING DEPENDENCY ON EXPECTED RETURNS

While the above-mentioned methods significantly improved both the accuracy and robustness of the mean-variance portfolio theory, they still depend on expected returns. Popular techniques developed with the aim of eliminating that dependency are the minimum variance, the equally-weighted portfolio, and the equal contribution to risk approaches. The *minimum variance strategy* is not only a specific, but also unique portfolio on the efficient frontier (see figure 1). As the name suggests, it selects the portfolio with the lowest variance, irrespective of the return associated with it. Thus, the minimum variance portfolio has the advantage of not relying on expected return estimates. However, the clear drawback of this approach is that it generally lacks diversification. Accordingly, despite its low risk profile from a variance point of view, it may suffer considerably from single extreme events due to its often high cluster risk. The *equally-weighted portfolio* (DeMiguel et al. [2007]) invests the same amount of money in all available assets. Accordingly, it is well diversified in nominal terms. Nevertheless, it does not take the differing risk contributions of the individual securities or liquidity into account. Only when no single asset's risk dominates can diversification of risks be achieved. A novel approach somewhere between minimum variance and equally-weighted portfolios is the *equal contribution to risk* or *risk parity portfolio* (Maillard et al. [2010]). This strategy equalises the risk contributions – for example in terms of the assets' volatilities – of the various portfolio components. As a consequence, no security contributes more than its peers to the total risk of the portfolio. In this sense, the equal contribution to risk portfolio is an approach that truly diversifies the risk dimension. An investor can still pick a portfolio matching his individual risk-tolerance, but may be required to lever positions up or down in order for each asset class or instrument to contribute the same amount of risk and attain the desired portfolio risk-target.

	Since 80s	**Since 90s**	**Today**
Model	Traditional	Yale	Risk parity
Asset classes	- Stocks - Bonds	- Stocks - Bonds - Commodities - Hedge funds - Private equity	- Stocks - Bonds - Commodities - Alpha strategies
Geographical focus	Home bias	Global	Global
Liquidity:			
- Instruments	High	Party low	**High**
- Investment vehicle	Variable	Low	**High**
Allocation	Static	Static	**Dynamic**
Risk measure	Volatility	Value-at-Risk	Tail-risk
Weighting according to	**Capital**	**Capital**	**Risk**

These academic achievements in portfolio theory of course considerably affected the implementation of asset allocation techniques in practice. For example, up to the 1990s, the asset allocation was predominantly driven by the traditional Markowitz approach. Accordingly, risk was measured in terms of the portfolio volatility. Additionally, the separation and mutual fund theorem led both institutional and private investors to statically invest in stocks and bonds only. The weighting scheme was determined according to capital. Often, focusing on what they knew best, portfolio managers heavily focused on securities from domestic firms, resulting in a home bias. Whilst such portfolios were inspired by academic insights, they suffered severe drawbacks. First, for volatility to be an adequate risk measure, returns need to be normally distributed. Unfortunately, history demonstrates that this is clearly not the case and even if it were, it is unclear why returns and losses should be treated symmetrically. Moreover, the lack of diversification with respect to both asset classes and geographical considerations made such portfolios vulnerable to falling stock and bond prices.

Leading US university endowment funds, therefore, started improving their asset allocation in the early 1990s. By adding lowly or uncorrelated assets such as commodities, hedge funds, and private equity vehicles to the portfolio, and focusing on a global perspective, these endowment funds aimed to achieve more robust portfolio returns. Moreover, a new risk concept called Value-at-Risk was expected to more precisely indicate potential losses. And the crisis in the aftermath of the IT bubble proved such endowment funds – at least temporarily – to be right. However, during the financial crisis, the evaporating liquidity of many of their assets coupled with an unexpected increase in the correlation between the portfolio components caused heavy losses.

Accordingly, the focus started to shift from static, capital-weighted investing to dynamic, risk-based investing. By eliminating the dependency on expected return estimates and instead relying on more robust portfolio construction techniques, approaches such as the minimum variance, the equally-weighted portfolio, and the equal contribution to risk methodologies gained momentum.

A UNIFIED FRAMEWORK

In the previous section, we examined the prominent asset allocation methodologies. We now try to generalise these methodologies into a unified framework for the portfolio problem. Accordingly, the above-mentioned methods such as mean-variance or indexing methodologies (equal-weighted, capital-weighted, etc) should be included in the general framework.

We propose to rewrite the asset allocation in the form of a general optimisation problem:

$$\begin{aligned} &\textbf{minimize} && f(x) \\ &\textbf{subject to} && g(x) \le 0 \end{aligned}$$

where x is the vector of the portfolio weights, f is the objective function, and g represents constraints which may be defined as a vector valued function[1]. The optimisation problem can then be solved by choosing the optimal weights such that the objective function is minimised and at the same time all constraints are fulfilled. Having stated the asset allocation problem in its most general form, we will now discuss objective functions and then constraints[2], in order to provide intuitively understandable examples.

Whenever an asset allocation decision has to be taken, the objective has to be crystal clear beforehand. Otherwise, it is impossible to derive a suitable objective function. Furthermore, constraints have to be taken into consideration simultaneously as they directly influence the allocation. Practical experience supports this statement and thus, it is no surprise that the above model can cope with most asset allocation problems.

GENERAL RISK MEASURES (FOR EXAMPLE MEAN-VARIANCE PORTFOLIOS)

Several asset allocation problems are stated as risk minimisation. The best known example would be Markowitz's minimum-variance portfolio, where the investor has the objective of minimising the variance of his portfolio for a given level of expected return, subject to, for example, the restriction that only long positions may be taken. Therefore, we can write the asset allocation problem in

[1] We neither discuss issues concerned with well-posedness of the above problem, nor do we treat any existence of solution issues. We always assume that a solution to the problem exists. Further issues with major impact on the optimisation problem, such as estimation of statistical number, etc are not discussed.

[2] Although objective functions and constraints are not always separable, we do not further address this issue here.

general and for the minimum variance investor, respectively, in the following forms:

	General	Minimum variance
minimize	RiskMeasure(x)	$\sigma(x)^2$
subject to	$g(x) \leq 0$	$-x \leq 0$

In a similar spirit, it is often not necessary to minimise the absolute level of risk, but to bring it as close as possible to a specific target. Let us take the above example of the minimum variance investor, and let us assume that she is aware of the fact that a higher expected return can only be achieved by a higher risk tolerance (in terms of the portfolio variance). Given her specific risk tolerance, the investor can then select the portfolio that matches her risk attitude, while at the same time, again only allowing for long investments. In this case, the general and specific risk targeting problem reads as follows:

	General	Target variance
minimize	(RiskMeasures(x) - RiskTarget)^2	$(\sigma(x)^2 - \sigma^2_{Target})^2$
subject to	$g(x) \leq 0$	$-x \leq 0$

In the purest risk minimisation or risk targeting form, only risk measures have to be estimated and no return (forecasts) are required. As we have argued in the history section, this makes such optimisation results relatively robust. Another robust estimation procedure is risk parity. This approach has the additional advantage that it not only considers the risk of the portfolio as a whole, but also accounts for the specific risk of every individual portfolio component.

EQUAL CONTRIBUTION TO RISK (RISK PARITY)

Equal contribution to risk (and not capital) divides risk equally among investments. It has been shown that a *risk measure (RM)* can be decomposed into the contributions stemming from each instrument according to:

$$RM(x) = sum_i(x_i \, dRM(x)/dx_i)$$

In order for the formula to be valid, certain conditions have to be fulfilled[3]. For example, risk measures such as variance, value at risk and conditional value at risk are decomposable according to this formula. The total contribution of instrument *i* is then equal to:

$$x_i \, dRM(x)/dx_i$$

Intuitively, the term $dRM(x)/dx_i$ represents the marginal contribution to risk, that is, the amount by which the risk of the total portfolio changes if we incrementally increase or lower the absolute holding x_i of instrument *i*.

Usually, in risk budgeting considerations, a certain risk budget has to be divided equally among investments. Let us therefore assume that we have a target risk of TR. Then, each instrument has to contribute the same amount of risk:

$$TR/n = x_i \, dRM(x)/dx_i \text{ for all } i$$

where n is the number of instruments. Generally, there is no closed form solution to this problem and it is solved with the help of least square optimisation

minimize	$sum_i(x_i \, dRM(x)/dx_i - TR/n)^2$
subject to	$g(x) \leq 0$

fitting exactly in the general asset allocation minimisation problem.

Instead of deriving the optimal portfolio weights indirectly by minimising an absolute or relative risk measure, other methods that directly optimise the problem with respect to the weights also exist. Examples of such methods are capital- and equal-weighted portfolios.

CAPITAL- AND EQUAL-WEIGHTED PORTFOLIOS

Although not the standard approach, it is possible to restate equal-weighted and capital-weighted portfolios in such a way that they fit into the general asset allocation minimisation framework. The following examples illustrate the optimisation problem if we further restrict the portfolio to having positive holdings only:

	General	Equal weighted	Capital weighted
minimize	$sum(x_i - x_{i,target})^2$	$sum_i(x_i - 1/n)^2$	$sum_i(x_i - cap_i)^2$
subject to	$g(x) \leq 0$	$-x \leq 0$	$-x \leq 0$

[3] We chose not to discuss them in this paper, but refer to the comprehensive study in Stefanovits (2010)

where n is the number of instruments, cap_i is the capital weight of instrument i, and x_i is the effective weight of instrument i.

ANALYSIS OF DIFFERENT METHODOLOGIES

Having described the evolution of portfolio theory from a historical point of view and shown that we can basically generalise the different approaches to asset allocation into a unified framework, we now turn to a more detailed empirical analysis. Given the prominence of the minimum variance (MV) approach, the intuitive appeal of the equal weighted portfolio (EW), and the novelty of the equal contribution to risk methodology (ERC), these three asset allocation strategies are compared below.

	S&P 500	USA bonds	SPI	CH bonds
Return p.a.	11.62%	5.85%	38.77%	4.54%
Volatility p.a.	19.17%	6.88%	16.43%	3.15%
Max. annual return	42.86%	14.35%	33.35%	9.62%
Min. annual return	-29.45%	-9.22%	-24.62%	-3.57%
Sharpe	0.61	0.85	0.53	1.44

We base the analysis on monthly data for US and Swiss equities and bonds[4] over the time period between January 1931 and December 2008. Each strategy is rebalanced monthly. The descriptive statistics for this sample show that US stocks on average performed better than Swiss equities, and Swiss bonds beat US bonds in terms of risk-adjusted returns. Not surprisingly, equities exhibited much higher volatility than bonds.

In order to test and compare the three asset allocation strategies, standard deviation is used as the risk measure when constructing minimum variance and equal contribution to risk strategies. Each month we estimate the covariance matrix using a constant correlation coefficient shrinkage procedure (see Muirhead [1987] and Ledoit and Wolf [2003 1 & 2]) based on the previous 24 monthly observations. We then solve numerically the corresponding optimisation problems under long-only and fully-invested constraints. Accordingly, no leverage is applied. Table 3 presents the portfolio statistics for the three different strategies.

	MV	ERC	EW
Return p.a.	4.46%	5.47%	7.55%
Volatility p.a.	2.92%	3.79%	8.11%
Value at risk p.a.	5.75%	8.13%	23.12%
Conditional VaR p.a.	9.16%	11.02%	28.71%
Max draw down	10.28%	11.94%	44.26%
Return/volatility	1.53	1.44	0.93
Return/value at risk	0.78	0.67	0.33
Return/CVaR	0.49	0.50	0.26
Return/Max DD	0.43	0.46	0.17
Turnover p.a.	61%	41%	0%

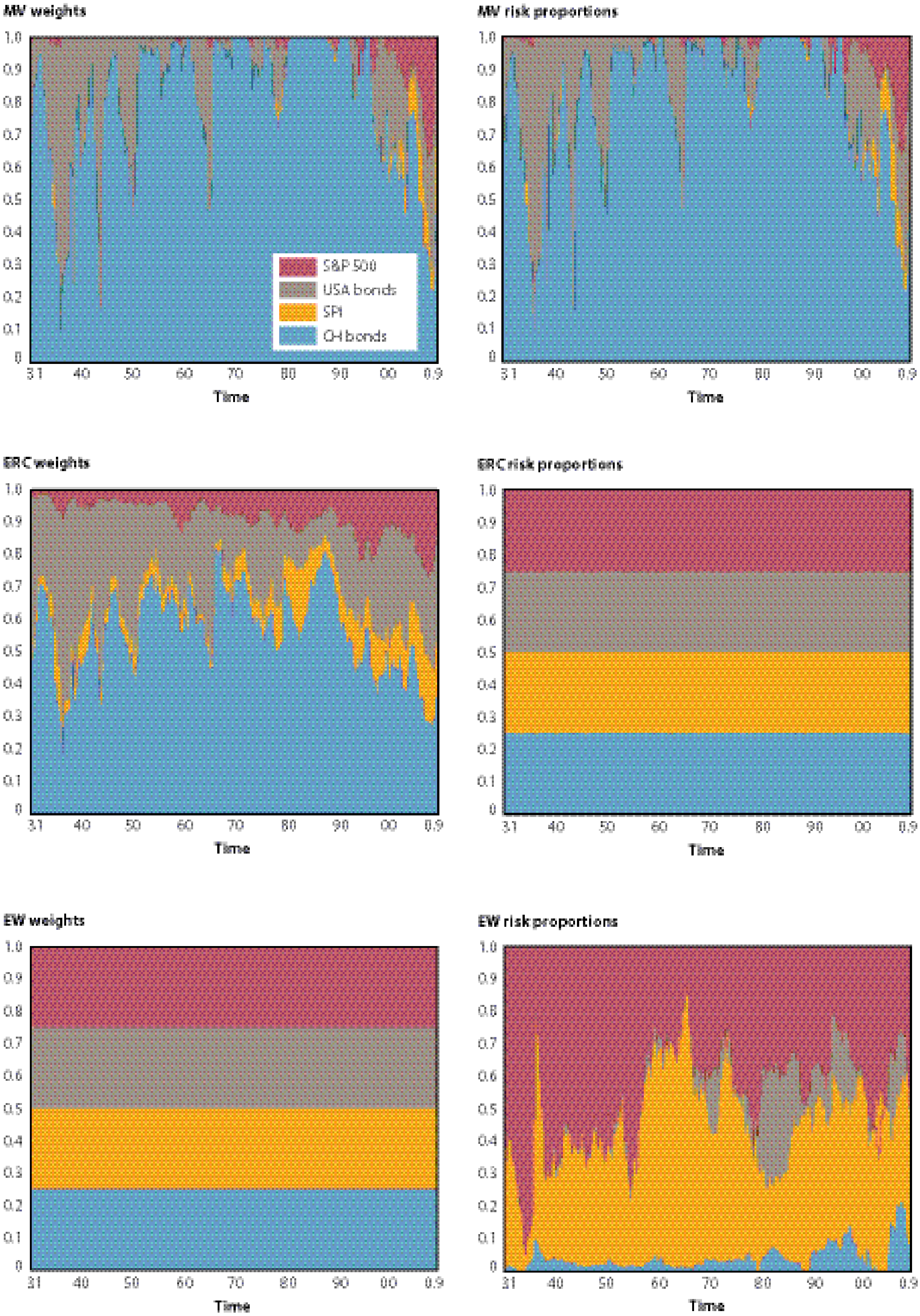
We can observe from the portfolio statistics that the ERC strategy is located between the MV and EW strategies in terms of volatility. ERC and MV perform much better than the EW portfolio in terms of risk-adjusted returns, irrespective of the performance measure considered (return per risk). Noteworthy is also the fact that the Sharpe ratio of the EW strategy is much lower than that of Swiss bonds over the entire period. This clearly indicates poor risk diversification for the EW portfolio. We also note that the EW strategy suffers from a deeper drawdown compared to the MV and ERC portfolios (even after adjusting to the same volatility level). The EW portfolio exhibits the best cumulative return, however, only at the cost of much higher volatility. If all positions are leveraged up to the same level of volatility, MV and ERC considerably outperform the EW strategy.

Comparing the optimal portfolio weights for the three different strategies in figure 3 (left hand side), we see quite clearly that the minimum variance portfolio is – not surprisingly – highly dominated by bonds, especially Swiss bonds. The equity exposure is most of the time minimal. The risk parity approach also results in an allocation biased to bonds, but relies on a fair amount of equity exposure for most of the time. Naturally, the weights of the equally weighted portfolio are constant.

On the right hand side of figure 3, the risk contribution (in terms of standard deviation) is represented. For the MV allocation, it is striking to observe that almost all the risk is allocated in bonds. Accordingly, the MV portfolio is extremely sensitive to shocks in the bond instruments, since

[4] Swiss bonds: Bloomberg, SZG4TR Index (EFFAS Gov Bond Total Return Index 7-10 Year); Swiss equities: Datastream, SITOTMK Total Return (SPI); US bonds: Bloomberg, USG4TR Index (EFFAS Gov Bond Total Return Index 7-10 Year); US equities: Datastream, S&PCOMP Total Return (S&P 500)

Figure 3: Empirical comparison



equities only play a minor role. It is important to note that portfolio weights and risk proportions look identical for the MV strategy. In fact, it is mathematically possible to show that this is always the case, see for example Stefanovits (2010). The chart for the ERC portfolio indicates that the optimisation has been successfully performed, given that all the risk contributions are equalised. As a result, this portfolio is much more robust with respect to the pricing factors of the individual instruments. Regarding risk contributions, the equal-weighted portfolio is basically the counterpart to the MV portfolio, as stocks contributed heavily to the entire portfolio risk, while bonds only contributed a very limited amount of risk.

To summarise, the MV and EW portfolios are direct counterparts in relation to return and risk characteristics. The MV portfolio per construction minimises the portfolio variance, and thereby, also limits its drawdown. However, this risk restraint comes at the cost of a low average return. The EW portfolio on the other hand exhibits relatively pronounced volatility and drawdown, but also benefits from a high average return. Both portfolio strategies concentrate their risk in only one asset class, namely bonds for the MV, and equities for the EW portfolio. The ERC approach represents a middle way between these two extremes. Not only does it spread the risk proportions equally among all instruments, but it also provides a more balanced set of risk/return attributes. The optimal portfolio type for an individual investor of course depends on the specific objectives and constraints.

CONCLUSION

Asset allocation methodologies have historically evolved and matured considerably, particularly over the last 60 years. Starting from an intuitive grasp of the merits of diversification, Markowitz was the first to formally prove the importance of the covariance between individual securities for reducing a portfolio's total risk. Subsequent approaches first focused on improving the reliability of the inputs necessary for the mean-variance technique. Newer methods then aimed at eliminating the entire portfolio problem from dependency on unreliable expected return estimates. While these efforts produced a multiplicity of methods, this paper shows that they can essentially be mapped into a unified portfolio optimisation framework which highlights the importance of knowing the objective of asset allocation. Without a clear objective and constraints, the optimisation problem can not be formulated and therefore not solved. Hence, at the very heart of asset allocation is the fundamental question: What do you want to achieve with your asset allocation?

Using this general setting to compare robust portfolio construction methodologies, it is evident that the equal contribution to risk approach represents a middle way between the minimum variance and equal-weighted portfolio techniques. Additionally, it has the advantages of not only avoiding the heavy concentration risk of the minimum variance portfolio, but also enhancing the equal-weighted portfolio approach by simultaneously considering individual and joint risk contributions of the individual portfolio components.

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