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Intraday information from S&P 500 Index futures options[☆]

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ABSTRACT

In this paper, we employ the intraday transaction prices of liquid E-mini S&P 500 index futures options to form 10-min ahead risk-neutral skewness forecasts and show profitable options trading strategy net of transaction costs. We do not find profitable trading based on 10-min ahead risk-neutral volatility and only very marginal cases of profitable trading using kurtosis forecasts. The skewness profitability anomaly may be an indication of informational inefficiency in intraday S&P 500 futures options trading, which is contrary to findings using longer-span daily and weekly moments. Our results lend credence to the persistence of intraday trading activities in the markets.

1. Introduction

We study the intraday dynamics of risk-neutral moments of S&P 500 Index futures prices, and test intraday informational market efficiency in the index futures options market. There have been many tests of daily, weekly, and monthly options prices and the general deduction of market efficiency. However, there are relatively few studies on the efficiency of intraday information derived from the traded prices of index or index futures options. In this paper, we employ information of risk-neutral moments on S&P 500 Index futures returns extracted from liquid E-mini Index futures options to form 10-min ahead forecasts and develop profitable option trading strategies. We find that strategies capturing risk-neutral skewness information are profitable net of transaction costs. We do not find profitable trading based on 10-min ahead risk-neutral volatility and only very marginal cases of profitable trading using kurtosis forecasts.

Neumann and Skiadopoulos (2013) use daily S&P 500 Index options over January 1996 to October 2010 to extract risk-neutral moments for forecasting and for testing trading strategies over one-day, one-week, and one-month horizons. They find that all the risk-neutral moments can generally be predicted better out-of-sample relative to the random walk benchmark. Using one-day ahead forecasts of the risk-neutral moments to pre-determine option trades, they find that except for the one-day ahead

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skewness forecast, the other moment forecasts did not support profitable trading. After considering transaction cost in the form of the bid-ask spread, they report that skewness forecasts also did not deliver positive profitability. The results were similar for forecasts involving longer horizons of a week or longer. They thus conclude that the hypothesis of the efficiency of the S&P 500 Index options market cannot be rejected.

Earlier studies had also investigated if a one-day ahead volatility forecast can lead to profitable option trading net of transaction costs. Using S&P 500 Index options and forecasts based on the history of implied volatility from the Black-Scholes model, Gonçalves and Guidolin (2006) find positive profit using OLS regression for forecast and applying a \$0.25 round-trip transaction cost (see their Table 8), but this is reduced to a loss when a higher round-trip transaction cost of \$0.50 is applied. Similarly Harvey and Whaley (1992) identify implied volatility from short-term, nearest at-the-money S&P 100 options, and conclude that one-day ahead forecasts of volatility are accurate, but such forecasts did not provide for profitable option trading after transaction cost was accounted for. However, Noh et al. (1994) find that a GARCH forecast of volatility using S&P 500 Index returns applied to straddle trading produced significant profit of up to 0.89% average daily return based on a \$0.50 filter and a round-trip transaction cost of \$0.25. They also find that a forecast using implied volatilities did not produce profitable results.

The cited studies basically find historical prices to be useful in predicting the future volatility of the stock indices, and they then check for profitability via options trading based on the forecast volatility. Profitability net of transaction cost would indicate market inefficiency with respect to the historical price information. Non-profitability would support the maintained hypothesis of market efficiency. Predictability on volatility is possible with both market efficiency and market inefficiency. Neumann and Skiadopoulos (2013) open up a new line of enquiry into efficiency on the options market by testing daily trading strategies based also on risk-neutral skewness and risk-neutral kurtosis forecasts. The risk-neutral probability distribution of the underlying asset to an option embodies a large amount of information on market expectations, as well as its risk preferences. In a 2013 NYU Stern–Federal Reserve Conference on Risk Neutral Probability Density, Figlewski (2013) suggested that searching for profitable trading strategies is a good question for research. This is indeed a clever insight as trading profitability not only has obvious attractions for the finance industry, but it also has deep implications on theory.

Risk-neutral probability information via risk-neutral volatility, skewness, and kurtosis are forward-looking with a particular maturity associated with the options from which the moments are extracted. Most studies in recent years use the extraction method by Bakshi et al. (2003) that is known to be model-free. The idea behind forecasting and trading is to try to beat the market by taking a long or short position in the portfolio of options after the forecast is made and before the market forms the value of the portfolio next period. The risk-neutral moments also serve as ex-ante proxy measures of the underlying return volatility, skewness, and kurtosis. Other areas of studies such as in equilibrium asset pricing make use of these ex-ante measures to explain cross-sectional stock returns.

Chang et al. (2013) estimate risk-neutral moments from daily S&P 500 Index option data. They find that risk-neutral market-wide or aggregate skewness is an important risk factor in explaining the cross-section of stock returns, and yields a robustly negative risk premium. Bali and Murray (2013) use monthly stock options data from 1996 to 2010 to extract individual stock risk-neutral skewness with a one-month forward horizon. They form monthly portfolios of skewness assets sorted based on deciles of the extracted skewness, and find a strong negative relation between the skewness of asset returns and risk-neutral skewness, indicating investor preference for idiosyncratic risk-neutral skewness. Conrad et al. (2013) use daily individual option prices from 1996 to 2005 to infer their underlying stock return risk-neutral moments over horizons from one-month to one-year. They find that the individual stock's moments were strongly related to future returns after controlling for other firm characteristics, and provide evidence of risk premiums in idiosyncratic moments.

Clearly empirical results in asset pricing have indicated highly relevant forward-looking and dynamic information contained in the risk-neutral moments of stocks or of an aggregated index from traded options. In equilibrium, assuming market investors hold optimal portfolios aligned with their risk preferences, they also receive the risk premiums for undertaking positions with risks, such as for example, receiving on average over time a higher return for portfolios with higher volatility, negative skewness, and larger kurtosis. Such forward-looking and dynamic information in the risk-neutral moments or the conditional moments naturally provide history of the moments as fodder for forecasting what the next period risk-neutral distribution and therefore what the prices of the portfolio of options may be. As prices change dynamically throughout each trading day, there should be a huge interest to explore the dynamics or conditional moments within a trading day. These intraday trading forecasts and forecast-based intraday trading receives time-averaged profits, if any, due to informational superiority over the market, but the informational advantages would not correlate with market factors and thus are not compensation for risk premium.

Our paper makes a number of contributions to the literature in the area of intraday futures options trading profitability and intraday equity futures index return moments predictability. As far as we know, our study is one of the first on intraday implied moments of S&P 500 Index futures returns using intraday or high-frequency futures option prices. Firstly, we use intraday E-mini S&P 500 European-style futures options data and improve on existing techniques to extract the first four moments of the risk-neutral return distribution. Secondly, we perform intraday out-of-sample forecasting or prediction, and also document the intraday dynamics of the index futures return risk-neutral moments. We also introduce a novel local autoregression method that allows variable window in fitting the autoregressive parameters. This is particularly useful in situations when there may be intraday news that cause structural changes in the returns or price distributions. It also distinguishes itself from the conventional autoregressive model with predetermined sample lengths. Thirdly, we show profitability in the trading strategies involving the various risk-neutral moment forecasts, particularly those involving skewness. The positive profitability after transaction costs in skewness trading indicates an anomaly that may be an indication of market inefficiency in intraday options trading – it could be due to information inefficiency within short spans of time as in 10-min intervals. We also explain why persistence is not necessarily the only condition for accurate out-of-sample forecasts and the resulting profitability.

In Section 2, we discuss the method for extracting the risk-neutral moments. The data and implementation procedures are then explained. Section 3 provides a discussion of the forecasting models used in the forecast of intraday 10-min ahead risk-neutral moments. The local autoregressive model is also explained. Section 4 contains the empirical results showing the forecasting performances of the various models. Section 5 provides the results based on different option trading strategies involving the risk-neutral moment forecasts. A sub-section provides a discussion of the persistence of the risk-neutral moments and the relationship with our forecasting and trading performances. We report results trying to forecast intraday 10-min ahead futures returns based on information of ex-ante risk-neutral moments in Section 6. Section 7 contains our conclusions.

2. Implied risk-neutral moments

Predictions of returns moments for the purposes of financial trading, hedging, and asset pricing, is prevalent in finance. The most common forecast is that of predictive means usually obtained from a regression model. More general frameworks for setting up predictability ranges from the modeling of stochastic models to time series modeling such as GARCH. For forecasting realized volatility, Andersen et al. (2003) is a fundamental paper on the modeling and forecasting of high-frequency intraday data. Ghysels et al. (1996) and Barndorff-Nielsen et al. (2002) provide reviews on stochastic volatility models. Poon and Granger (2003) provide a comprehensive review of forecasting volatility in financial markets including detailed discussion of the pioneering dynamic time series models of Engle (1982) and Bollerslev (1986). Harvey and Siddique (1999, 2000) are some pioneering studies of skewness in financial markets. Brooks et al. (2005) study autoregressive conditional kurtosis. There is a vast literature on the empirical measures of asset returns moments, particularly on volatility and skewness, but a relative scarcity of studies on the risk-neutral measures of similar moments.

The seminal paper by Breeden and Litzenberger (1978) connected option prices with no-arbitrage state prices, the equivalence of discounted risk-neutral densities. Later papers of the same genre include Rubinstein (1996) and Jackwerth and Rubinstein (1996). Practical approaches of extracting or implying the risk-neutral moments using option prices were developed in Bakshi et al. (2003) and Jiang and Tian (2005). Due to the more general framework and higher moments inferable via the Bakshi et al. (2003) method (hereafter BKM), there has been a surge of interest in using the BKM method to study risk-neutral moments. Unless otherwise stated, the research discussed below employs risk-neutral higher moments that are computed using the BKM method.

As for the option prices and their extracted risk-neutral moments, Bakshi et al. (2003) and Taylor et al. (2009) find that risk-neutral skewness implied from individual stock options are less negative than that implied from stock index options. Gârleanu et al. (2009) find that recent option-pricing puzzles can be explained by the fact that there is a significant difference between index option prices and the prices of single-stock options due to differences in end-user demands. Figlewski (2008) suggests that the estimation of individual stock risk-neutral density is especially hampered by two serious problems, as stock options trade a relatively small number of strikes, and also face significant microstructural noise. Due to the above, we consider the study of S&P 500 Index futures options to be appropriate for the purpose of examining predictability and trading profitability. The S&P 500 Index futures options and Index futures, as well as Index options, are not only more liquid, but also possess more negative skewness moments on the underlying whereby skewness trading strategy could be effected.

We elaborate on the difference between S&P 500 Index futures options and S&P 500 Index options as follows. S&P 500 (SPX) options traded at the Chicago Board Options Exchange (CBOE) are European-style and cash-settled with the underlying as a non-traded index. SPX options are index options and these data are used in studies such as Chang et al. (2013) and Neumann and Skiadopoulos (2013). Most S&P 500 futures options traded at the Chicago Mercantile Exchange (CME) have the underlying as the S&P 500 E-mini Index futures ($50 multiplier). CME options (ES) that traded with quarterly maturities are American-style, whereas the more frequent CME options (EW) that traded with weekly maturities are European-style. There is a smaller number of CME futures options (EV) based on the standard S&P 500 futures ($250 multiplier). CME index futures options EV are used in studies such as Cremers et al. (2015). On page 587 of their paper, they state: “We focus on S&P 500 futures options rather than S&P 500 Index options, because the former are more liquid and have historical data available over a longer sample period … sample period for our analysis begins in January 1988 when the CME started trading … and ends in December 2011.”

In the 2009 Derivatives Market Survey published by the World Federation of Exchanges in May 2010, the table on page 39 shows that of the 12 most actively traded index and index futures options in the world, the CBOE S&P 500 options traded 155 million contracts in 2009 and the CME Group S&P 500 and E-mini S&P 500 futures options traded 28 million contracts. The WFE/IOMA Derivatives Market Survey 2012 published by the World Federation of Exchanges in May 2013, shows on page 20 that of the top 12 most actively traded index and index futures options in the world, the CBOE S&P 500 Index options traded 174 million contracts in 2012 while the CME Group E-mini S&P 500 options traded 39.5 million contracts. Other CME futures index options traded an additional 12.1 million contracts. Furthermore, “The S&P 500 Index is one of the world’s most widely followed financial indicators, and the E-Mini S&P 500 futures contracts represent a significant part of the stock index futures activity … The E-Mini S&P 500 futures was by far the most actively traded stock index futures contract in the world, with almost twice the activity of the second most active … For the E-Mini S&P 500 futures and options contract, … the smaller size of the E-Mini S&P contract relative to the S&P 500 futures contract has facilitated an active and liquid trade in both the futures and options markets. This provides an opportunity to trade with lower margining costs."[1] The above sources show that E-Mini futures and options are liquid and are actively traded. For our study, we use the S&P 500 E-mini EW Index futures and its corresponding futures options.

2.1. BKM method

The BKM method can be utilized for the case of S&P 500 options on futures. We define the τ-period random log return (or rate of change) of the underlying futures contract at time t as $R(t,\tau) \equiv \ln\left(\frac{F_{t+\tau}}{F_t}\right)$, where F_t is the S&P 500 E-mini Index futures price at t. To obtain the risk-neutral variance, skewness, and kurtosis of $R(t,\tau)$, it is sufficient to obtain the first four risk-neutral moments of $\mathbb{E}^Q[R(t,\tau)]$, $\mathbb{E}^Q[R(t,\tau)^2]$, $\mathbb{E}^Q[R(t,\tau)^3]$, $\mathbb{E}^Q[R(t,\tau)^4]$ under the risk-neutral probability measure Q.

Each of the moments above can be viewed as an expected payoff at maturity $T \equiv t + \tau$ and is a function of some underlying portfolio. Here we rely on a well-known result in Carr and Madan (2001) that any expected payoff of underlying futures can be spanned and priced using a traded set of options across different strike prices. For example, a forward contract can be decomposed as a long call and a short put with the same strike. A call spread can be decomposed as a long call with a lower strike and a short call with a higher strike. For a more complicated payoff, we need more options with different strikes to replicate the payoff. This can be done assuming that the payoff function is twice continuously differentiable in the underlying futures price. We can write $H(j)[F_T] = R(t,\tau)^j$, for $j = 2,3,4$, where $T = t + \tau$. Then consider the expansion of $H(j)[F_T]$:

$$H(j)[F_T] = H(j)[F_t] + \left(F_T - F_t\right) H_F(j)[F_t] + \int_{F_t}^{\infty} H_{FF}[K](F_T - K)^+ dK + \int_0^{F_t} H_{FF}[K](K - F_T)^+ dK\,,$$

where $H_F(j)$ refers to the derivative of function $H(j)$ with respect to F_T.

Letting $V_t(\tau) \equiv E_t^Q\left(e^{-r\tau}H(2)[F_T]\right)$, where Q is the risk-neutral measure and r is the risk-free rate, we have:

$$V_t(\tau) = \int_{F_t}^{\infty} \frac{2(1-\ln(K/F_t))}{K^2} C_t(\tau;K)dK + \int_0^{F_t} \frac{2(1+\ln(F_t/K))}{K^2} P_t(\tau;K)dK\,,$$

where $C_t(\tau;K)$ and $P_t(\tau;K)$ are respectively the European call and put on the underlying stock index futures price F_t, with maturity τ and strike price K.

The no-arbitrage prices of discounted payoffs, $E_t^Q\left(e^{-r\tau}H(3)[F_T]\right)$ and $E_t^Q\left(e^{-r\tau}H(4)[F_T]\right)$, can be written as $W_t(\tau)$ and $X_t(\tau)$, and are similarly found as:

$$W_t(\tau) = \int_{F_t}^{\infty} \frac{6\ln(K/F_t) - 3(\ln(K/F_t))^2}{K^2} C_t(\tau;K)dK - \int_0^{F_t} \frac{6\ln(F_t/K) + 3(\ln(F_t/K))^2}{K^2} P_t(\tau;K)dK$$

$$X_t(\tau) = \int_{F_t}^{\infty} \frac{12(\ln(K/F_t))^2 - 4(\ln(K/F_t))^3}{K^2} C_t(\tau;K)dK + \int_0^{F_t} \frac{12(\ln(F_t/K))^2 + 4(\ln(F_t/K))^3}{K^2} P_t(\tau;K)dK\,.$$

These contract prices $V_t(\tau)$, $W_t(\tau)$, and $X_t(\tau)$ are also the respective discounted risk-neutral moments of the underlying futures return or rate of change over period interval τ. The equations involve weighted sums of out-of-the-money options across varying strike prices. Using these prices, the risk-neutral (central) moments RNMs of variance, skewness, and kurtosis can be calculated as:

$$VAR_t^Q(\tau) = e^{r\tau}V_t(\tau) - \mu_t(\tau)^2$$

$$SKEW_t^Q(\tau) = \frac{e^{r\tau}W_t(\tau) - 3\mu_t(\tau)e^{r\tau}V_t(\tau) + 2\mu_t(\tau)^3}{[e^{r\tau}V_t(\tau) - \mu_t(\tau)^2]^{3/2}}$$

$$KURT_t^Q(\tau) = \frac{e^{r\tau}X_t(\tau) - 4\mu_t(\tau)e^{r\tau}W_t(\tau) + 6\mu_t(\tau)^2e^{r\tau}V_t(\tau) - 3\mu_t(\tau)^4}{[e^{r\tau}V_t(\tau) - \mu_t(\tau)^2]^2}\,,$$

where $\mu_t(\tau) \equiv \mathbb{E}^Q[R(t,\tau)] \approx e^{r\tau}\left[1 - \frac{1}{2}V_t(\tau) - \frac{1}{6}W_t(\tau) - \frac{1}{24}X_t(\tau)\right] - 1$. The computed RNMs are those of the underlying futures return $\ln\left(\frac{F_T}{F_t}\right)$.

2.2. Data and implementations

The widely disseminated intraday volatility index (VIX) and its traded futures price is an indication of changes in the market perception of volatility and sensitivity or fear of market return losses. The level of the VIX was high from 2009 to mid-2012, and then returned to a calmer level ranging between 10 and 20 points up until the end of 2014. There is therefore a priori a

[1] CME Institute, "E-mini S&P's - The Ultimate Index?", 5 pp. White Paper 2017.

higher chance of observing market anomalies during the more turbulent period of 2009–2012. In common with other cited studies extracting risk-neutral moments on or before 2013, e.g. Bali and Murray (2013), Conrad et al. (2013), and Neumann and Skiadopoulos (2013), we use intra-day E-mini S&P 500 European-style options time-stamped traded price data on weekly index futures series (EW1, EW2, and EW4) from August 2009 to December 2012 purchased from the CME. Prior to August 2009, cleaned time-stamped data on the stated contracts were not available. The stated options typically trade actively two weeks before their expiration dates. The price data are actual transactions data that are time-stamped to the second. They are not mid-points of bid and ask prices as used in other studies. The price data reflects a transaction occurring when a buyer decides to move a spread above to transact at an ask price, or else a transaction occurring when a seller decides to move a spread below to transact at a bid price. However, the data available do not provide information on the bid-ask prices nor the volume transacted. Transactions data were similarly obtained on the E-mini futures prices.

We only consider E-mini options (futures options) traded between 0830 h and 1500 h that correspond to regular trading hours, and ignore options traded on the expiration date itself. This is because options are generally illiquid during irregular trading hours, and abnormal option prices tend to occur more frequently on the day of expiration. We incorporate all options, including in-the-money (ITM) options, to capture the information contained in these options. We also obtain the underlying E-mini S&P 500 Futures intra-day price data from CME. The options data are cleaned by removing a small percentage of prices that violated the no-arbitrage bounds discussed in Merton (1973). The risk-free rate used in our computation is the yield on the secondary market 4-weeks Treasury bills reported in the Federal Reserve Report H.15. Since the risk-free rates in our sample period are close to zero, we can safely assume the risk-free rate is constant throughout any given trading day without incurring non-trivial approximation error.

In order to calculate the risk-neutral moments via the BKM method, we need to compute $V(t,\tau)$, $W(t,\tau)$, and $X(t,\tau)$, which require in theory an infinite number of OTM call and put option prices across a continuum of strike prices. In reality, however, market option prices are available for only a finite number of discrete strike prices even in highly liquid markets. See Jiang and Tian (2005) for a discussion of how to estimate the RNM by joining the call and put price points in a smooth manner. In general, the more call and put option prices are available, the less is any bias due to the discrete number of option prices. As all call and put prices in $V(t,\tau)$, $W(t,\tau)$, and $X(t,\tau)$ are no-arbitrage prices, any call or put prices with the same maturity formed out of other no-arbitrage call and put prices at the same time t must also be part of $V(t,\tau)$, $W(t,\tau)$, and $X(t,\tau)$. To increase the number of OTM options for the purpose of computing $V(t,\tau)$, $W(t,\tau)$, and $X(t,\tau)$, we consider in-the-money (ITM) call and put options as well to augment the use of OTM options in standard practice. These ITM options do carry information about their underlying asset returns and should not be unnecessarily ignored.

To incorporate these ITM options, once we apply the data filters, we use the put-call parity equation to transform ITM call option prices into OTM put option prices, and ITM put option prices into OTM call option prices. In this way we are able to increase the number of option price observations in our sample space by including these synthetic OTM option prices. Since the put-call parity equation is a model-free no-arbitrage formula, by using this transformation, we are able to utilize the information content in these ITM options and at the same time use it to calculate the risk-neutral moments without imposing any model bias. We should mention that in our empirical results we did try the smaller data sample without the above augmentation, and the results are approximately similar although weaker.

Besides theoretical justification, we also provide empirical reasons why the use of ITM futures options are valid, even though in index options, ITM index calls may be relatively less liquid. We obtain from Datastream the E-Mini EW series of call and put option daily contract volumes. The data were available starting only May 2, 2011. We also obtain the SPX call and put option daily contract volumes from CBOE, and match the dates of the two series of option volumes. The average daily trading volume of EW options over this period is 139,248 contracts while that of SPX options is 763,856. If we gross these up by about 250 days to annual contract volumes, the numbers are consistent with the numbers reported in WFE Derivatives Market Surveys of 2012, and also 2009. The EW numbers are consistent with the CME Group Monthly Options Review in July 2013 that stated for year-on-year, weekly EW options average daily trading volume was 113,466. The call/put volume ratio of the E-Mini EW futures options at 68.7% is higher than that of the SPX options at 59.6%. Thus call trading volumes are relatively higher in the CME futures options. Based on our own CME data, the average ITM calls to OTM call proportion during this period is about 40%.

In CBOE Market Statistics 2012, the table on page 66 give the average number of index option contracts per trade as 46 in 2012. Thus, the estimated average number of trades per day during 2012 for SPX options is 16,605 (or 763,856/46). From our CME sample intraday trading records, over the 2011–2012 period, the average number of trades per regular-hours day for CME EW Index futures options is 10,768, which is comparable in magnitude to the 16,605 of the SPX. This works out to an average of about 300 trades per 10-min period divided into calls and puts and of different strikes. The average number of contracts per trade for EW options is about 11. Thus, in general, ITM calls and puts are reasonably liquid.

More pertinently, regarding retrieving information from futures ITM calls by converting to OTM puts using put-call parity, this no-arbitrage price is inherently more feasible than a conversion using index ITM calls. This is because the underlying E-mini S&P 500 futures are easy to buy or sell in obtaining a put price. In the index option case, converting to OTM puts is not so straightforward and would require buying or selling a large portfolio of 500 stocks. Use of index funds to accomplish the latter would incur a tracking error. In Alex and Gallagher (2001), of the 42 S&P 500 Index mutual funds reported in Morningstar, 0.04% to 0.10% tracking errors per month were found. The tracking errors were attributed to transaction costs, dividend forecast errors, execution delays due to the large number of stocks in the index, corporate news impacting share trading, and changes in index composition. Use of synthetic constructions to trade the index would also involve a number of separate trades, incurring more costs and tracking errors than a single futures trade. In any case, the point of our method is to extract risk-neutral moment information from no-arbitrage option prices, so our approach is reasonably in line.

For each 10-min period within the regular trading hours, we use as many strike prices as are possible where options were traded on those strike prices. In order to study the market's ability to absorb information within a very short time interval, we prefer 10-min to 30-min and so on; we find that choosing an interval shorter than 10-min is not feasible as there would be some intervals with an insufficient number of traded option prices to extract the risk-neutral moments. Also, we find that in many cases, 15-min intervals do not yield any significant change in the reported results. For traded calls (puts) having the same strike price within each 10-min band, we select only the call (put) price that was transacted closest to the end of the 10-min interval. These call and put prices are then used for computing the risk-neutral moments at every intraday 10-min intervals starting at 8:40 a.m., 8:50 a.m., …, 2:40 p.m., up to 2:50 p.m.

For moment extraction in the intraday intervals, we use at least two OTM calls and two OTM puts. After that, we employ the numerical method of piece-wise cubic Hermite interpolation to evaluate the integrals for finding the risk-neutral moments. Piece-wise cubic hermite interpolation has a local smoothing property, and therefore produces more stable estimates as compared to cubic splines. The extracted one-day to ten-day constant maturity risk-neutral moments are used in our analysis. We do not use options with maturities longer than ten days because trading for longer E-mini options are less liquid and the price data are inadequate for the purpose of constructing the risk-neutral moments.

Table 1 reports the descriptive statistics of the extracted S&P 500 risk-neutral moments including the risk-neutral volatility, risk-neutral skewness, and risk-neutral kurtosis. E-mini S&P 500 options with different time-to-maturity of one day to ten days are used to produce the risk-neutral moments corresponding to the different time-to-maturity of n days. Altogether 19,859 moments, one for each 10-min interval, are estimated. For the risk-neutral volatility, each n-day volatility is scaled by $\sqrt{252/n}$ so that they are easily compared on an annual basis. The volatilities are reported in percents. The risk-neutral skewness and kurtosis, however, are reported without any scaling as these quantities do not have simple distribution-free aggregation properties. Except for volatility, which is reported in percent, the other moments are reported in decimals.

The averages of moments across all 10-min intervals on all dates where their maturities are the same n-day are reflected as the mean for the n-day. Table 1 shows that the mean volatility is quite stable across all maturities. The mean and median skewness are all negative for all maturities. This is consistent with similar results reported earlier in the literature. Kurtosis generally decreases as maturities increase. Most of the kurtosis measures are in excess of 3, indicating large deviations from

Table 1

Descriptive Statistics of S&P 500 Risk-Neutral Moments. This table reports the descriptive statistics of the extracted S&P 500 risk-neutral moments including the risk-neutral volatility, risk-neutral skewness, and risk-neutral kurtosis in Panels A, B, and C, respectively. E-mini S&P 500 futures options with different time-to-maturities of 1 day–10 days are used to produce the risk-neutral moments corresponding to the different time-to-maturities of n days. The options data are collected over the sample period of August 24, 2009 to December 31, 2012. Altogether 19,859 10-min interval estimates of risk-neutral moments are obtained. For the risk-neutral volatility, each n-day volatility is scaled by $\sqrt{252/n}$ so that they are easily compared on an annual basis. The risk-neutral volatility numbers are computed in %. The risk-neutral skewness and kurtosis, however, are reported without any scaling as these quantities do not have simple distribution-free aggregation properties.

Time-to-Maturity n	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
Panel A: Risk-Neutral Volatility (%)										
No. of Obs.	2628	2769	2594	2277	1920	1987	1784	1471	1205	1224
Mean	21.05	19.85	18.97	19.07	20.49	20.31	20.35	20.89	20.43	21.41
Median	18.47	17.30	17.02	17.10	17.64	17.60	18.29	18.14	17.60	17.73
Max	67.44	85.90	54.26	84.68	81.59	84.20	72.56	79.24	83.25	77.49
Min	9.70	9.35	8.84	8.83	8.61	8.92	9.42	9.61	9.65	7.93
Std Dev.(%)	8.21	7.68	6.90	7.32	7.91	8.18	7.34	8.25	8.43	8.97
Skewness	1.85	2.14	1.62	2.33	1.98	1.83	1.41	1.38	1.61	1.32
Kurtosis	7.29	10.19	5.78	12.27	10.12	8.51	6.31	6.86	6.10	4.62
Panel B: Risk-Neutral Skewness										
No. of Obs.	2628	2769	2594	2277	1920	1987	1784	1471	1205	1224
Mean	−1.01	−1.03	−1.07	−1.17	−1.26	−1.12	−1.08	−1.05	−1.03	−1.02
Median	−0.95	−0.97	−1.00	−1.10	−1.21	−1.08	−1.05	−1.04	−0.97	−1.02
Max	2.34	2.07	1.03	1.80	2.26	0.87	0.88	1.02	0.88	1.18
Min	−3.65	−3.81	−3.53	−3.17	−3.45	−3.14	−3.00	−3.64	−3.68	−3.27
Std Dev.(%)	0.62	0.58	0.61	0.61	0.62	0.52	0.48	0.51	0.52	0.56
Skewness	−0.41	−0.59	−0.60	−0.46	−0.31	−0.23	−0.22	−0.04	−0.61	0.15
Kurtosis	3.89	4.79	4.00	3.92	4.36	4.01	4.21	4.16	5.19	4.61
Panel C: Risk-Neutral Kurtosis										
No. of Obs	2628	2769	2594	2277	1920	1987	1784	1471	1205	1224
Mean	6.28	5.72	5.76	6.28	6.53	5.16	4.68	4.49	4.37	4.14
Median	4.90	4.56	4.48	4.98	5.38	4.34	4.06	3.81	3.58	3.59
Max	30.11	29.60	28.97	30.11	29.70	24.70	19.47	21.25	28.91	26.40
Min	0.88	0.01	0.03	0.02	0.02	0.02	0.03	0.03	0.03	0.03
Std Dev.(%)	4.41	3.94	4.00	4.20	4.30	3.09	2.60	2.62	2.97	2.64
Skewness	2.11	2.32	2.09	1.77	1.89	1.90	1.87	1.83	2.89	2.50
Kurtosis	8.73	10.32	8.54	6.69	7.41	8.30	8.09	7.95	16.21	14.17

the normal distribution. While the skewness measures are left-skewed, the volatility and kurtosis measures are right-skewed in their frequency distributions over time.

Our results indicate that the risk-neutral distributions on average have more negative skewness and much higher kurtosis than that of normal distributions. For each horizon n, the risk-neutral moments computed for each 10-min intervals are highly variable, as can be seen by their standard deviations and maximum-minimum ranges. This may reflect the large amounts of new information impacting the market at 10-min intervals.

The annualized standard deviations of the various 10-min interval risk-neutral moments show that the risk-neutral volatility (Table 1 Panel A) is the most variable and the intraday time series process is not smooth. On the other hand, the risk-neutral skewness (Table 1 Panel B) shows a smoother process as its risk-neutral moments evolve slowly through any day. In our sample, 97.9% of all intraday risk-neutral skewness measures are negative. The risk-neutral kurtosis (Table 1 Panel C) is also relatively smooth given its annualized standard deviations are also smaller than those of the risk-neutral volatility. The smoother process may imply higher chances of successful forecasting.

When we compare the descriptive statistics with the one-month, two-month, and three-month risk-neutral moments reported in Neumann and Skiadopoulos (2013, Table 1), it can be seen that annualized volatilities for different horizons remain rather constant for even up to three months. Similarly, risk-neutral skewness remains on average negative and does not appear to change much. However, it is clearly the case that our one to ten day risk-neutral kurtosis measures are larger than the longer horizon thirty-day to ninety-day kurtosis measures.

3. Forecasting models

In this section, we describe the regression models that we use to fit the time series of each of the risk-neutral moments. For each of the three different risk-neutral moments, the regressions are also performed on different time series belonging to the different constant maturities. For ease of exposition, we use the notation $RNM_t(\tau)$ to represent any of the three risk-neutral moments at time t, or the tth interval, with maturity τ. More specifically $RNV_t(\tau)$, $RNS_t(\tau)$, and $RNK_t(\tau)$ denote risk-neutral volatility, risk-neutral skewness, and risk-neutral kurtosis, respectively. Each intraday time series $RNM_t(\tau)$, for t taking 10-min intervals within the day, is associated with a particular trading day. For each trading day, each τ, and each risk-neutral moment, we run a forecasting model on the intraday time series.

Firstly, we perform a test of the stationarity of each of the risk-neutral moment time series. We report that for all cases of maturities, each day and each RNM, the series are stationary based on the augmented Dickey-Fuller tests (results are available from the authors). In all cases, unit root is rejected at less than the 1% significance level. This is different from the results in Neumann and Skiadopoulos (2013) where daily risk-neutral moments can display unit roots, so that they require use of ARIMA and ARIMAX processes for forecasting. We do not need to use forecasting models on integrated processes.

We consider eight competing models: a benchmark Random Walk Model (RW), an Autoregressive (AR) lag-one Model, an Autoregressive Moving Average Model (ARMA(1,1)), an Autoregressive (AR(1)) Model with GARCH (generalized autoregressive conditional heteroskedastic) error – AR(G), Vector Autoregressive (VAR) lag-one Model where all three lagged RNMs enter as regressors, Vector Autoregressive (VAR) lag-one Model with GARCH errors for each of the three vector elements – VAR(G), Vector Error Correction Model (VECM), and the Local Autogressive (LAR) lag-one Model. Experimenting with higher lag-orders generally does not yield any clearer results or improvement in analyses. As the lag-order is understood, we do not clutter the notation and leave out the lag-one notation. In what follows, each interval $[t, t+1)$ is 10-min within a trading day.

For the RW Model, for each RNM:

$$RNM_{t+1}(\tau) = RNM_t(\tau) + \epsilon_{t+1} ,$$

where ϵ_{t+1} is an i.i.d. noise.

For the AR Model, for each RNM:

$$RNM_{t+1}(\tau) = b_0 + b_1 RNM_t(\tau) + \epsilon_{t+1} ,$$

where b_0 and $b_1 < 1$ are constants and ϵ_{t+1} is i.i.d.

For the ARMA Model, for each RNM:

$$RNM_{t+1}(\tau) = b_0 + b_1 RNM_t(\tau) + \epsilon_{t+1} ,$$

where b_0 and $b_1 < 1$ are constants and ϵ_{t+1} is MA(1), with $\epsilon_{t+1} = \alpha\epsilon_t + \varepsilon_{t+1}$, $\alpha < 1$, and ε_t i.i.d.

For the AR(G) Model, for each RNM:

$$RNM_{t+1}(\tau) = b_0 + b_1 RNM_t(\tau) + \epsilon_{t+1} ,$$

where b_0 and $b_1 < 1$ are constants and $\mathrm{var}(\epsilon_{t+1}) = \alpha_0 + \alpha_1 \mathrm{var}(\epsilon_t) + \alpha_2 \epsilon_{t+1}^2$ with constants $\alpha_0 > 0$, and $\alpha_1 + \alpha_2 < 1$.

For the VAR Model:

$$\begin{pmatrix} RNV_{t+1}(\tau) \\ RNS_{t+1}(\tau) \\ RNK_{t+1}(\tau) \end{pmatrix} = B_0 + B_1 \begin{pmatrix} RNV_t(\tau) \\ RNS_t(\tau) \\ RNK_t(\tau) \end{pmatrix} + e_{t+1} ,$$

where B_0 is a 3 × 1 vector of constants, B_1 is a 3 × 3 matrix of constants, and e_{t+1} is a 3 × 1 vector of i.i.d. disturbance terms.

For the VAR(G) Model: the above VAR Model is used except that each element of the vector error e_{t+1} is modelled as GARCH (1,1).

For the VECM Model:

$$\begin{pmatrix} \triangle RNV_{t+1}(\tau) \\ \triangle RNS_{t+1}(\tau) \\ \triangle RNK_{t+1}(\tau) \end{pmatrix} = \Gamma_0 + \Gamma_1 \begin{pmatrix} RNV_t(\tau) \\ RNS_t(\tau) \\ RNK_t(\tau) \end{pmatrix} + \Gamma_2 \begin{pmatrix} \triangle RNV_t(\tau) \\ \triangle RNS_t(\tau) \\ \triangle RNK_t(\tau) \end{pmatrix} + e_{t+1} ,$$

where Γ_0 is a 3 × 1 vector of constants, Γ_1 and Γ_2 are 3 × 3 constant matrices, and e_{t+1} is a 3 × 1 vector of i.i.d. disturbance terms.

For the LAR Model:

$$RNM_{t+1}(\tau) = b_{0,I_d} + b_{1,I_d} RNM_t(\tau) + \epsilon_{t+1} ,$$

where I_d denotes a subset of the sample points on day d. In the current context, this subset consists of sample data from the latest time point before forecasting, i.e., 12:00 p.m., to a lagged time point not earlier than 8:40 a.m. The statistical procedure in which this subset I_d is selected is explained in the next subsection. LAR basically selects an optimal local window to perform the regression fitting where structural breaks do not occur. While it has the advantage of providing a better fit and possibly better forecast in time series that are not smooth and that may have breaks, the disadvantage is that if the time series is not smooth, the shorter sampling window may yield forecasts and estimates with larger standard errors.

The maximum likelihood regression method, equivalent to least squares in cases of normal random errors, is utilized, except that in the LAR case, the selection of window adds to the regression procedures. We next explore some intraday dynamics of the risk-neutral moments of the S&P 500 Index futures returns or rate of change. The in-sample dynamics are examined using autoregressive regressions. Table 2 reports the summary results of the intraday regressions involving the AR(1) model. It reports in-sample regression results of the various risk-neutral moments. For each maturity of one to ten days, the extracted 10-min interval RNMs of E-mini S&P 500 futures returns or rate of change over August 24, 2009 to December 31, 2012 sample period are used for a regression each trading day. On day d for maturity τ, the following autoregressive AR(1) regression is performed for each of the risk-neutral moments:

$$RNM_{t+1}(\tau) = b_0 + b_1 RNM_t(\tau) + \epsilon_{t+1} ,$$

where maturity is τ, t denotes the particular 10-min interval during the trading day d, and ϵ_{t+1} is assumed to be i.i.d.

The estimated $\widehat{b}_0$ and $\widehat{b}_1$ are recorded for every trading day and across all days in the sample period. Their averages are reported as Const and Slope respectively. Note that the averages Const and Slope are reported for each of risk-neutral volatility, skewness, kurtosis, and each maturity from one day to ten days. Next, in-sample, for each day d, for each t within d, a next 10-min interval forecast of the $RNM_{t+1}(\tau)$ is made using $E_t(RNM_{t+1}(\tau)) = \widehat{b}_0 + \widehat{b}_1 RNM_t(\tau)$. The sign of the expected change is noted and compared with actual $RNM_t(\tau)$ change at t, $RNM_{t+1}(\tau) - RNM_t(\tau)$.

If both signs are the same, i.e., correct directional in-sample prediction, then correct prediction count is increased by one. Over all such t's in the sample period, for each τ and each RNM, the % of correct prediction counts is denoted as the mean correct prediction MCP (in %) and reported in Table 2. In each RNM, each τ, for each d, each t, if the directional in-sample prediction is correct, conditional on this, the actual magnitude of $RNM_t(\tau)$ change at t, Abs $\left(RNM_{t+1}(\tau)/RNM_t(\tau) - 1\right)$ is noted. This is averaged across all its occurrences and reported as M$\triangle$ in the table.

For risk-neutral volatility and all maturities, especially the shorter ones, the estimated constants and slopes are all positive. The percentages of the slope estimates that are significantly different from zero at the 10% significance level are also shown. For risk-neutral skewness and kurtosis, about 25% of the estimates are significantly different from zero. This percentage of significant estimates is higher for the slope estimates of risk-neutral volatility. RNV regressions have smaller constants of regressions relative to absolute values of the skewness and kurtosis regression constants. They also have higher slopes. From Table 2, it is seen that the RNV regressions generally increase in constants and reduce in slopes as maturity increases. For skewness and kurtosis, the constants and slopes remain about the same with different maturities, although there is a slight decrease in constants for kurtosis. The risk-neutral skewness regressions mostly show a negative constant.

Clearly the autoregressive or slope coefficients are estimated to be largely positive for all RNMs. The lag effects do not appear to diminish with maturity for skewness and kurtosis. However, for volatility, there is a reduction in the magnitude of the lag effect as maturity lengthens. There is higher persistence in RNV. The significance of the regression coefficient estimates in these in-sample regressions indicates the plausibility of using lagged moments to forecast next 10-min moments within the day. The out-of-sample forecasting is done and reported in Section 4 while the use of the latter forecasts for options trading strategies and profitability is discussed in Section 5.

Besides understanding the characteristics of the intraday RNM dynamics, Table 2 also provides an indication of how often an autoregressive model could correctly predict the next interval increase or decrease in the actual RNM. Thus the in-sample percentage of correct prediction counts MCP is reported in the table. It is seen that within-sample, despite the higher persistence of RNV, RNV does not produce higher, but in fact lower MCPs than RNS and RNK autoregressions. The constants, slope of regressions as well as how closely the data bunched up and in fact how closely they follow the regression lines, contribute to the MCP counts. Conditional on a correct prediction, the actual magnitude of $RNM_t(\tau)$ change at t, Abs $\left(RNM_{t+1}(\tau)/RNM_t(\tau) - 1\right)$,

Table 2

Statistics of In-Sample Regressions. This table reports in-sample regressions of the risk-neutral moments. For the risk-neutral volatility, each n-day volatility is scaled by $\sqrt{252/n}$ so that they are easily compared on an annual basis. The risk-neutral volatility numbers are in decimals. The risk-neutral skewness and kurtosis are reported without any scaling. For each maturity, the extracted 10-min interval risk-neutral moments (RNMs) of E-mini S&P 500 futures returns or rate of change over the sample period August 24, 2009 to December 31, 2012 are used for a regression each trading day. On day d for maturity τ, the following autoregressive AR(1) regression is performed for each of the risk-neutral moments:

$$RNM_{t+1}(\tau) = b_0 + b_1 RNM_t(\tau) + \epsilon_{t+1} \,,$$

where maturity is τ, t denotes the particular 10-min interval during the trading day d, and ϵ_{t+1} is assumed to be i.i.d. The estimated $\widehat{b}_0$ and $\widehat{b}_1$ are recorded for every day. Their averages are reported as Const and Slope respectively. The averages Const and Slope are reported for each of risk-neutral volatility, skewness, kurtosis, and each maturity from 1 day to 10 days. For each τ, % significance reports the percentage of estimated $\widehat{b}_1$'s that are significantly different from zero at the 10% significance level. Next, in-sample, for each day d, for each t within d, a next 10-min interval forecast of the $RNM_{t+1}(\tau)$ is made using $E_t(RNM_{t+1}(\tau)) = \widehat{b}_0 + \widehat{b}_1 RNM_t(\tau)$. The sign of the expected change is noted and compared with actual $RNM_t(\tau)$ change at t, $RNM_{t+1}(\tau) - RNM_t(\tau)$. If both signs are the same, i.e. correct directional in-sample 'prediction', then correct prediction count is increased by one. Over all such t's, for each τ and each RNM, the % of correct prediction counts is denoted as MCP (in %). In each RNM, each τ, for each d, each t, if the directional in-sample prediction is correct, conditional on this, the actual magnitude of $RNM_t(\tau)$ change at t, Abs $\left(RNM_{t+1}(\tau)/RNM_t(\tau) - 1\right)$ is noted. This is averaged across all its occurrences and reported as M$\triangle$.

Days	Risk-Neutral Volatility					Risk-Neutral Skewness					Risk-Neutral Kurtosis				
τ	Const	Slope	%Sig	MCP	M$\triangle$	Const	Slope	%Sig	MCP	M$\triangle$	Const	Slope	%Sig	MCP	M$\triangle$
1	0.077	0.60	81.60	63.7	0.03	−0.922	0.13	20.49	72.0	1.13	5.388	0.16	24.05	70.0	0.54
2	0.082	0.52	73.71	62.9	0.04	−0.932	0.11	17.22	72.7	1.22	4.972	0.16	26.12	70.9	0.70
3	0.093	0.45	66.14	65.4	0.04	−0.919	0.08	18.90	72.5	1.14	4.786	0.14	26.51	71.5	0.62
4	0.106	0.42	61.13	64.2	0.05	−0.997	0.16	23.97	69.9	1.88	5.324	0.15	27.68	70.7	1.49
5	0.124	0.35	52.83	63.3	0.06	−1.115	0.12	26.90	73.5	1.11	5.616	0.16	31.57	71.1	2.00
6	0.127	0.32	45.38	64.3	0.05	−0.971	0.06	10.07	72.8	1.01	4.263	0.11	17.77	71.1	1.21
7	0.137	0.27	38.21	65.5	0.05	−0.969	0.05	15.94	72.6	0.73	3.989	0.12	18.18	72.6	0.86
8	0.152	0.27	36.57	67.5	0.06	−0.924	0.09	18.12	71.8	2.74	3.635	0.19	25.44	72.0	0.95
9	0.153	0.24	32.52	67.5	0.07	−0.890	0.15	17.27	70.7	0.73	3.440	0.23	27.91	68.9	0.73
10	0.169	0.18	25.57	69.0	0.06	−0.929	0.12	23.70	70.8	1.28	3.643	0.15	27.23	70.1	0.65

is also larger for RNS and RNK. Thus we have reasons to believe that if out-of-sample processes are similar to within-sample processes, then RNS particularly with higher MCP and M$\triangle$ could outperform RNV and RNK in terms of forecasts and trading profits.

As shown in Table 1, the time series of risk-neutral volatility is less smooth with indication of plausible breaks in the stationarity at some points within the trading day. One method to address this issue with intraday study is to employ a local autoregression (LAR) model, a technique that is gaining popularity in statistical analyses but not familiar in financial applications. We employ the LAR model as an alternative model in our forecasts.

3.1. Local autoregressive model

For each one-dimensional risk-neutral moment RNM_t representing any of the three risk-neutral moments (risk-neutral volatility RNV_t, risk-neutral skewness RNS_t, and risk-neutral kurtosis RNK_t), the idea is to fit an AR(1) model for out-of-sample forecasting, but by using a sample set of 10-min RNMs on day d, starting at time $t \geq$ 8:40 a.m. and ending at 12:00 p.m.

The optimal sample is selected from competing sample sets in the time interval (t,12:00 p.m.) such that maximum $t = t'$ provides for 10 observation points in the sample. The question is of course how to identify the sub-sample of local homogeneity. In any day of forecast, we seek the longest sub-sample, beyond which there is a high possibility of structural change occurring. A sequential testing procedure is proposed to detect the optimal sub-sample among a number of candidates with increasing sample lengths starting from (t', 12:00 p.m.). The time-dependent estimation windows distinguish the LAR model from the conventional AR model with predetermined sample length. See Chen et al. (2010) for justification of this local adaptive method over traditional time series modeling.

The LAR model is particularly useful when the series of RNMs for estimation and forecasting may not be stationary and contains structural breaks at some time points during intraday trading as is common with breaking news or disturbances due to swift changes in high-frequency trading volumes and directions, as well as the switches between dark pools and exchange trading orders. The RNM process is specified as follows.

$$RNM_{t+1} = b_{0,I_n} + b_{1,I_n} RNM_t + \varepsilon_{t+1,I_n}, \quad \varepsilon_{t+1} \sim (0, \sigma^2_{I_n}),$$

where I_n denotes a time interval of the local sub-sample (t_n, 12:00 p.m.). Increasing sub-samples are denoted by $I_K \supset I_{K-1} \supset \ldots \supset I_1$, where I_1 corresponds to (t', 12:00 p.m.) and $t_K < t_{K-1}$.

The local maximum likelihood estimators (MLE) $\widetilde{\theta}_n = (\widetilde{b}_{0,I_n}, \widetilde{b}_{1,I_n}, \widetilde{\sigma}_{I_n})^\top$ are obtained via:

$$\begin{aligned}
\widetilde{\theta}_n &= \arg\max_{\theta\in\Theta} L(RNM; I_n, \theta) \\
&= \arg\max_{\theta\in\Theta} \left\{ -(m_n+1)\log\sigma_{I_n} - \frac{1}{2\sigma^2_{I_n}} \sum_{j\in I_n} (RNM_{j+1} - b_{0,I_n} - b_{1,I_n} RNM_j)^2 \right\},
\end{aligned}$$

where m_n is number of data points in I_n, Θ is the parameter space, and $L(RNM; I_n, \theta)$ is the local log-likelihood function.

Starting from the shortest sub-sample I_1, let an adaptive estimator be $\widehat{\theta}_1 = \widetilde{\theta}_1$. The selection procedure then iteratively extends the sub-sample with more 10-min sample points and sequentially tests for possible structural breaks in the next longer sub-sample. The significance of structural breaks is measured by a sequence of log-likelihood ratio tests. Once a break is detected before reaching I_K, say at n', then $\widetilde{\theta}_{n'}$ is taken as the optimator LAR estimator for day d. The test statistic is defined in each iterative step following sub-sample I_n as:

$$T_{n+1} = \left| L(I_{n+1}, \widetilde{\theta}_{n+1}) - L(I_n, \widetilde{\theta}_n) \right|^{1/2}, \quad n = 1, \ldots, K-1, \tag{1}$$

where $L(I_n, \widetilde{\theta}_n) = \max_{\theta\in\Theta} L(RNM; I_n, \theta)$ denotes fitted log-likelihood. If there does not exist a significant structural break in the time within $I_{n+1} - I_n$, the MLE $\widetilde{\theta}_{n+1}$ is not far from the estimate $\widetilde{\theta}_n$, and the test-statistic T_{n+1} is small. In this case, the extended sub-sample with more information I_{n+1} is accepted and the corresponding MLE $\widetilde{\theta}_{n+1}$ replaces $\widetilde{\theta}_n$ for improved estimation accuracy. On the other hand, suppose the test statistic is significantly large, then the selection procedure terminates and the latest accepted sub-sample I_n yields the optimal MLE $\widetilde{\theta}_n$. A set of critical values $\zeta_1, \ldots, \zeta_K$ is calibrated in Monte Carlo experiments and used to determine the significance level of T_n for each n. The details of this calibration are available from the authors.

4. Forecasting performance

After the regression models are estimated, the estimated coefficients are used to provide a fitted model for the purpose of predicting the next period or future RNMs. Parameters are estimated in the window on the same day from 8:40 a.m. to 12:00 p.m., after which the fitted model is used for forecasting during 12:10 p.m. to 2:50 p.m. Unlike daily or weekly methods, we do not use rolling windows over the 10-min intervals within a trading day. This helps in focusing on days with highly liquid transactions at start of day trading to fix the parameters for forecast and trading for the rest of the day. As mentioned before, some days whereby there are insufficient risk-neutral moments for estimation during 8:40 a.m. to 12:00 p.m. are excluded from the sample.

Forecasts are made for RNMs pertaining to different horizons τ of one up to ten days. The various models are as follows.

For the RW Model:

$$E_t\left(RNM_{t+1}(\tau)\right)=RNM_t(\tau)\,,$$

where the subscript to the expectation operator denotes a condition on the information at t.

For the AR Model, for each RNM:

$$E_t\left(RNM_{t+1}(\tau)\right)=\widehat{b}_0+\widehat{b}_1 RNM_t(\tau)\,,$$

where $\widehat{b}_0$ and $\widehat{b}_1$ are estimated parameters.

For the ARMA Model, for each RNM:

$$E_t\left(RNM_{t+1}(\tau)\right)=\widehat{b}_0+\widehat{b}_1 RNM_t(\tau)+\widehat{\alpha}\widehat{\epsilon}_t\,,$$

where $\widehat{\epsilon}_t=RNM_t(\tau)-\widehat{b}_0-\widehat{b}_1 RNM_{t-1}(\tau)$.

For the AR(G) Model, for each RNM:

$$E_t\left(RNM_{t+1}(\tau)\right)=\widehat{b}_0^G+\widehat{b}_1^G RNM_t(\tau)\,,$$

where $\widehat{b}_0^G$ and $\widehat{b}_1^G$ are estimated parameters based on maximum likelihood procedures recognizing the GARCH variance in the residuals.

For the VAR Model, for all three RNMs at once:

$$E_t\begin{pmatrix}RNV_{t+1}(\tau)\\ RNS_{t+1}(\tau)\\ RNK_{t+1}(\tau)\end{pmatrix}=\widehat{B}_0+\widehat{B}_1\begin{pmatrix}RNV_t(\tau)\\ RNS_t(\tau)\\ RNK_t(\tau)\end{pmatrix}\,,$$

where $\widehat{B}_0$ and $\widehat{B}_1$ are the estimated parameters.

For the VAR(G) Model, for all three RNMs at once:

$$E_t\begin{pmatrix}RNV_{t+1}(\tau)\\ RNS_{t+1}(\tau)\\ RNK_{t+1}(\tau)\end{pmatrix}=\widehat{B}_0^G+\widehat{B}_1^G\begin{pmatrix}RNV_t(\tau)\\ RNS_t(\tau)\\ RNK_t(\tau)\end{pmatrix}\,,$$

where $\widehat{B}_0^G$ and $\widehat{B}_1^G$ are the estimated parameters based on GARCH(1,1) errors e_{t+1}.

For the VECM Model, for all three RNMs at once:

$$E_t\begin{pmatrix}RNV_{t+1}(\tau)\\ RNS_{t+1}(\tau)\\ RNK_{t+1}(\tau)\end{pmatrix}=\widehat{\Gamma}_0+\left(\widehat{\Gamma}_1+I\right)\begin{pmatrix}RNV_t(\tau)\\ RNS_t(\tau)\\ RNK_t(\tau)\end{pmatrix}+\widehat{\Gamma}_2\begin{pmatrix}\triangle RNV_t(\tau)\\ \triangle RNS_t(\tau)\\ \triangle RNK_t(\tau)\end{pmatrix}\,,$$

where $\widehat{\Gamma}_0$, $\widehat{\Gamma}_1$, and $\widehat{\Gamma}_2$ are the estimated parameters.

For the LAR Model:

$$E_t\left(RNM_{t+1}(\tau)\right)=\widehat{b}_{0,I_n}+\widehat{b}_{1,I_n} RNM_t(\tau),$$

where $\widehat{b}_{0,I_n}$ and $\widehat{b}_{1,I_n}$ are the estimated parameters in I_n.

4.1. Error metrics

To measure the forecasting performances of these models, we employ three error metrics or loss functions.

The root mean square error (RMSE) is defined as:

$$RMSE=\sqrt{\frac{1}{T-1}\sum_{t=1}^{T-1}\left(RNM_{t+1}(\tau)-E_t\left(RNM_{t+1}(\tau)\right)\right)^2}\quad,$$

where T is the number of periods of forecasts, each period being a 10-min interval.

The mean absolute deviation (MAD) is defined as:

$$MAD=\frac{1}{T-1}\sum_{t=1}^{T-1}\left|RNM_{t+1}(\tau)-E_t\left(RNM_{t+1}(\tau)\right)\right|\,.$$

Table 3
Out-of-Sample Error Metrics for Forecasting Models. This table reports the out-of-sample performances of the Random Walk (RW) model, the Autoregressive (AR) model, the Autoregressive Moving Average (ARMA) model, the Autogressive with Garch error (AR(G)) model, the Vector Autogressive (VAR) model, the Vector Autoregressive with Garch errors (VAR(G)) model, the Vector Error Correction model (VECM), and the Local Autoregressive (LAR) model. The respective autoregressive models are lag-one models. Results are reported for each of the risk-neutral moments of volatility, skewness, and kurtosis. For each risk-neutral moment (RNM) category, the out-of-sample regression results of all maturities are pooled. There are 8534 observations for each RNM regression. Every trade day from August 24, 2009 to December 31, 2012, the risk-neutral moments computed in each 10-min intervals from 8:40 a.m. to 12:00 p.m. are used to estimate the parameters of each model. The estimated or fitted model is then used to forecast the risk-neutral moments for each 10-min interval from 12:10 p.m. to 2:50 p.m. The error metrics or loss functions of root mean square error (RMSE), mean absolute deviation (MAD), and mean correct prediction (MCP) are shown in the table. The MCP is the % of times that the forecast of directional change in the risk-neutral moment is correct.

	Risk-Neutral Volatility			Risk-Neutral Skewness			Risk-Neutral Kurtosis		
	RMSE	MAD	MCP %	RMSE	MAD	MCP %	RMSE	MAD	MCP %
RW	0.662	0.174	50.00	0.647	0.463	50.00	4.001	2.490	50.00
AR	0.580	0.175	58.15	0.511	0.374	70.75	3.252	2.174	68.98
ARMA	0.598	0.175	59.66	0.581	0.424	69.44	3.619	2.373	68.10
AR(G)	0.498	0.162	61.18	0.521	0.378	70.27	3.230	2.131	69.63
VAR	0.654	0.210	58.35	0.541	0.387	69.87	3.465	2.255	68.40
VAR(G)	0.569	0.182	63.04	0.625	0.433	67.83	3.387	2.209	68.27
VECM	3.619	0.396	60.36	1.358	0.544	63.20	7.995	3.092	61.99
LAR	0.526	0.159	68.67	0.535	0.374	71.01	3.391	2.110	70.23

The mean correct prediction (MCP) percentage is defined as:

$$MCP = \frac{1}{T-1}\sum_{t=1}^{T-1} J_{t+1} \times 100,$$

where indicator $J_{t+1} = 1$ if $\left(RNM_{t+1}(\tau) - RNM_t(\tau)\right)\left(E_t\left(RNM_{t+1}(\tau)\right) - RNM_t(\tau)\right) > 0$, and $J_{t+1} = 0$ otherwise.

Table 3 reports the out-of-sample statistical performances of all the models. The autoregressive models are lag-one models. Results are reported for each of the RNMs of volatility, skewness, and kurtosis. For each RNM category, the regression results of all maturities are pooled. There is a total of 8534 observations for each RNM regression. Every trade day from August 24, 2009 to December 31, 2012, the RNMs computed in each 10-min intervals from 8:40 a.m. to 12:00 p.m. are used to estimate the parameters of each model. The estimated or fitted model is then used to forecast the RNMs for each 10-min interval from 12:10 p.m. to 2:50 p.m. The error metrics or loss functions of RMSE, MAD, and MCP are shown in the table. The MCP is the percentage of times that the forecast of directional change in the RNM is correct.

The results in Table 3 provide clear indications of the following. Firstly, the VECM performs the worst; this shows that more complicated models with more than one lag could yield less accurate forecasts. Similarly, the VAR Model does not perform well as there may be multi-correlation of the RNMs in a finite sample setting. Secondly, across all RNMs, the AR(G) Model, the LAR Model, and the AR Model perform better than the rest in terms of lower RMSE, lower MAD, and higher MCP. For the more volatile RNV processes, the LAR appears to perform slightly better in MAD and MCP. These latter models are all autoregressive in nature. This may not be surprising since the results in Table 2 show the pervasion of autoregression. Remarkably, all methods perform better than the RW in terms of MCP higher than 50%. In summary, there is statistical evidence of a lot of intraday information that can be utilized to successfully make rather accurate predictions of next period RNMs over short intervals of 10-min.

Table 4 reports the statistical performances of the models in out-of-sample forecasting. The autoregressive models are lag-one models. A separate regression is performed for the RNMs data corresponding to each time-to-maturity. Every trade day from August 24, 2009 to December 31, 2012, the RNMs computed in each 10-min intervals from 8:40 a.m. to 12:00 p.m. are used to estimate the parameters in each regression. The estimated or fitted model is then used to forecast the risk-neutral moments for each 10-min interval from 12:10 p.m. to 2:50 p.m. The error metrics or loss functions of RMSE, MAD, and MCP are shown in Panels A, B, and C, of Table 4 respectively. The MCP is the percentage of times that the forecast of the directional change in the RNM is correct. Note that the risk-neutral volatility in our study is expressed as a percentage. If expressed as decimals, the RMSE and MAD error metrics would be multiplied by 10^{-2}.

In Table 4, the VECM, VAR, and VAR(G) are mostly underperforming with larger RMSE and MAD relative to the other forecasting models. VECM and VAR also do not perform as well in MCP, although the VAR(G) does reasonably well. As observed in Table 3, AR(G), LAR, and AR or ARMA are overall better performers in the out-of-sample forecasting. All models outperform RW in terms of MCP, though not necessarily in RMSE and MAD. For RMSE and MAD, forecasting performances for all models generally deteriorate with longer maturities. The MCPs, however, stay approximately the same for all maturities. For the RNV, LAR yields the best MCP amongst all models except for the longest maturity of 10 days.

In Table 5, for skewness, forecasting with AR outperforms with AR(G) and LAR coming in as close seconds. This is the case for all three error metrics. Except for VECM and VAR(G), all models beat the RW model in skewness forecasting on metrics RMSE and MAD. However, all models outperform the RW in terms of MCP. Distinctly, unlike the RNV, there is no apparent deterioration in forecasting for skewness with longer maturities. In most cases, the forecasting errors reduce for longer maturities. This suggests that the RNMs equations for volatility contain a much larger residual noise, so that the apparently higher first order correlations in volatility shown in Table 2 do not imply better forecasts over longer horizons. All the models beat the RW in terms of MCP.

Table 4

Out-of-Sample Error Metrics for Forecasting Risk-Neutral Volatility. This table reports the statistical performances of the Random Walk (RW), the Autoregressive (AR) model, the Autoregressive Moving Average (ARMA) model, the Autogressive with Garch error (AR(G)) model, the Vector Autogressive (VAR) model, the Vector Autoregressive with Garch errors (VAR(G)) model, the Vector Error Correction model (VECM), and the Local Autoregressive (LAR) model. The respective autoregressive models are lag-one models. A separate regression is performed for moments data corresponding to each time-to-maturity. Every trade day from August 24, 2009 to December 31, 2012, the risk-neutral moments computed in each 10-min intervals from 8:40 a.m. to 12:00 p.m. are used to estimate the parameters in each regression. The estimated or fitted model is then used to forecast the risk-neutral moments for each 10-min interval from 12:10 p.m. to 2:50 p.m. The error metrics or loss functions of root mean square error (RMSE), mean absolute deviation (MAD), and mean correct prediction (MCP) % are shown in Panels A, B, and C respectively. The MCP is the % of times that the forecast of directional change in the risk-neutral moment is correct. Note: The risk-neutral volatility in our study is expressed in %. If expressed as decimals, the magnitudes of the RMSE and MAD error metrics would be reduced by a multiple of 10^{-2}.

Time-to-Maturity *n*	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
Panel A: Root-Mean-Square Error										
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	0.079	0.296	0.258	0.827	0.996	0.499	0.779	0.904	0.900	0.998
AR	0.078	0.286	0.193	0.679	0.911	0.454	0.680	0.771	0.920	0.747
ARMA	0.086	0.263	0.195	0.750	0.923	0.520	0.689	0.735	0.924	0.806
AR(G)	0.079	0.226	0.208	0.639	0.743	0.448	0.563	0.590	0.691	0.757
VAR	0.138	0.294	0.213	0.660	1.222	0.466	0.650	0.962	0.925	0.809
VAR(G)	0.128	0.301	0.423	0.663	0.825	0.526	0.611	0.703	0.717	0.853
VECM	0.085	1.735	1.247	3.525	8.384	1.499	1.311	3.549	7.066	1.283
LAR	0.065	0.206	0.176	0.652	0.785	0.498	0.541	0.520	0.933	0.798
Panel B: Mean Absolute Deviation										
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	0.056	0.089	0.094	0.195	0.244	0.169	0.240	0.274	0.273	0.382
AR	0.057	0.097	0.092	0.187	0.251	0.200	0.243	0.270	0.279	0.307
ARMA	0.060	0.097	0.093	0.189	0.237	0.192	0.243	0.267	0.290	0.342
AR(G)	0.057	0.090	0.094	0.174	0.217	0.196	0.212	0.235	0.247	0.315
VAR	0.105	0.134	0.126	0.219	0.311	0.225	0.260	0.301	0.322	0.329
VAR(G)	0.063	0.105	0.117	0.189	0.257	0.206	0.235	0.268	0.270	0.352
VECM	0.061	0.197	0.153	0.496	0.995	0.344	0.400	0.616	0.760	0.483
LAR	0.045	0.078	0.079	0.173	0.226	0.193	0.227	0.232	0.265	0.313
Panel C: Mean Correct Prediction %										
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
AR	52.80	52.98	58.58	56.84	61.49	57.33	59.97	64.03	62.22	66.81
ARMA	58.87	54.49	60.00	56.65	59.16	60.02	60.55	64.33	63.38	68.89
AR(G)	58.53	54.69	60.47	61.57	62.82	59.88	62.60	66.45	66.60	69.56
VAR	54.28	53.64	57.61	57.55	59.13	59.50	61.73	65.00	61.40	62.96
VAR(G)	63.06	59.55	64.67	61.67	62.06	64.36	65.95	62.58	63.73	65.33
VECM	61.65	59.89	62.46	60.69	57.44	60.74	59.89	59.55	59.96	59.29
LAR	71.33	68.93	69.13	67.55	66.96	67.67	66.71	70.16	68.38	68.95

More interestingly, if we examine the MCPs between RNV and RNS forecasting, forecasting skewness is shown to produce better accuracy. This is shown in aggregated form in Table 3 where there is an average MCP of over 70% for AR, AR(G), and LAR models in skewness forecasting, but only 58% to 69% in the volatility forecasting. In Table 5, this phenomenon is uniform across all maturities. Thus again, the Table 2 indications of higher autocorrelation in RNV than in RNS must not be taken as an interpretation of better predictions for the RNV.

In Table 6, for kurtosis, forecasting with LAR outperforms with AR(G) and AR coming in as close seconds. This is the case for MAD and MCP error metrics, although AR and AR(G) perform better in RMSE. As with RNV and RNS, the VECM, VAR(G), and ARMA models do not perform well relative to the others. Distinctly, unlike the RNV and RNS, the forecast for RNK improves with maturity. Across all models, RMSE reduces from a magnitude of 4–5 in one-day maturity to a magnitude of 2+ in ten-days maturity. Similarly, MAD reduces from 2 to 3 in one-day maturity to 1+ in ten-day maturity. The MCPs remain steady at about 69% across the maturities, excluding the case of VECM with slightly lower MCPs. Thus, for RNK, the longer the horizon, there appears to be some mean reversion and thus better forecasting performance. This is consistent with the Table 1 finding that over longer maturities, kurtosis shrinks on average. If we examine the MCPs between RNK, RNV, and RNS forecasting, forecasting kurtosis is shown to produce accuracies in between the cases of RNV and RNS. This is shown in aggregated form in Table 3 where there is an average MCP of about 68% for LAR, AR(G), and AR models in kurtosis forecasting, but only 58% to 69% in the volatility forecasting, and over 70% in skewness forecasting. In Table 6, this phenomenon is uniform across all maturities.

5. Options trading strategies

Using the forecasts generated by the seven competing models of AR lag-one, ARMA(1,1), AR(1) with GARCH error, VAR lag-one, VAR(1) with GARCH errors, VECM, and LAR, we attempt to construct a trading strategy to benefit from the accurate forecast

Table 5

Out-of-Sample Error Metrics for Forecasting Risk-Neutral Skewness. This table reports the statistical performances of the Random Walk (RW), the Autoregressive (AR) model, the Autoregressive Moving Average (ARMA) model, the Autogressive with Garch error (AR(G)) model, the Vector Autogressive (VAR) model, the Vector Autoregressive with Garch errors (VAR(G)) model, the Vector Error Correction model (VECM), and the Local Autoregressive (LAR) model. The respective autoregressive models are lag-one models. A separate regression is performed for moments data corresponding to each time-to-maturity. Every trade day from August 24, 2009 to December 31, 2012, the risk-neutral moments computed in each 10-min intervals from 8:40 a.m. to 12:00 p.m. are used to estimate the parameters in each regression. The estimated or fitted model is then used to forecast the risk-neutral moments for each 10-min interval from 12:10 p.m. to 2:50 p.m. The error metrics or loss functions of root mean square error (RMSE), mean absolute deviation (MAD), and mean correct prediction (MCP) % are shown in Panels A, B, and C respectively. The MCP is the % of times that the forecast of directional change in the risk-neutral moment is correct.

Time-to-Maturity n	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
Panel A: Root-Mean-Square Error										
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	0.720	0.647	0.628	0.627	0.704	0.594	0.573	0.606	0.641	0.696
AR	0.566	0.508	0.500	0.508	0.551	0.473	0.461	0.479	0.496	0.530
ARMA	0.629	0.577	0.577	0.565	0.649	0.532	0.516	0.564	0.573	0.604
AR(G)	0.574	0.525	0.508	0.526	0.554	0.481	0.463	0.484	0.493	0.568
VAR	0.579	0.519	0.516	0.591	0.629	0.477	0.483	0.503	0.534	0.541
VAR(G)	0.688	0.627	0.656	0.593	0.734	0.575	0.527	0.568	0.574	0.600
VECM	0.742	1.455	1.011	1.559	1.798	0.668	1.582	1.085	2.435	0.829
LAR	0.610	0.529	0.540	0.515	0.592	0.489	0.475	0.492	0.494	0.550
Panel B: Mean Absolute Deviation										
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	0.539	0.477	0.456	0.444	0.486	0.421	0.416	0.432	0.419	0.503
AR	0.422	0.377	0.369	0.377	0.395	0.346	0.339	0.355	0.348	0.379
ARMA	0.470	0.428	0.419	0.414	0.455	0.393	0.380	0.408	0.404	0.451
AR(G)	0.428	0.388	0.371	0.385	0.395	0.351	0.340	0.350	0.342	0.394
VAR	0.430	0.384	0.379	0.402	0.420	0.348	0.353	0.364	0.369	0.388
VAR(G)	0.491	0.443	0.437	0.429	0.467	0.405	0.382	0.406	0.395	0.418
VECM	0.539	0.538	0.484	0.559	0.653	0.459	0.557	0.527	0.617	0.570
LAR	0.435	0.376	0.375	0.364	0.395	0.348	0.341	0.348	0.333	0.386
Panel C: Mean Correct Prediction %										
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
AR	71.24	70.41	70.80	69.29	72.55	69.35	71.56	71.29	69.40	72.16
ARMA	69.48	69.94	68.68	68.15	71.22	68.21	70.84	69.91	68.21	70.15
AR(G)	70.87	70.02	70.48	69.73	71.95	68.36	71.31	70.00	68.24	71.46
VAR	70.80	69.17	69.92	68.67	71.43	69.47	69.68	70.16	66.74	73.02
VAR(G)	67.76	66.92	66.28	65.60	69.92	68.00	70.38	66.29	68.03	72.52
VECM	65.13	65.20	63.95	61.90	63.22	61.09	63.46	61.46	60.36	62.84
LAR	70.98	71.16	71.06	71.22	72.05	68.15	72.24	70.32	72.07	71.31

of the various future moment changes. RW is excluded as it has served its purpose for benchmark comparison in the forecast assessments shown in Tables 3–6. Besides, our seven competing models outperform RW in terms of MCP. We now add the benchmark case of perfect knowledge forecast (PK) whereby prediction of moment increase or decrease is 100% correct.

We construct three different trading strategies corresponding to the forecasts of the three RNMs. The trading strategies are designed to capture option price changes consistent with the forecast changes in the RNMs, and to capture them in a way that beats the market. For example, the forecast could be faster than what the market realizes, and options could be bought or sold at better prices in the next few minutes before market demand and supply align those prices. And holding over the next period would turn in a profit when prices eventually move according to forecast, and the portfolio is liquidated. Although the direction and not the magnitude of the RNM prediction is key to our options portfolio strategy, we also incorporate some consideration of the magnitude of the forecast changes – this is called the signal threshold. We consider different thresholds for different RNMs as some like RNS and RNK have forecast ranges that can be much larger compared to RNV. In Table 1, standard deviation divided by the absolute value of the mean averages about 0.4 for RNV, 0.6 for RNS, and 0.65 for RNK, indicating the kind of ranges. If a threshold is $x\%$, we will execute the options portfolio strategy at a particular 10-min interval $(t, t+1]$ when the forecast of the RNM exceeds the RNM in the last interval by at least $x\%$, i.e., $E_t(RNM_{t+1}(\tau))/RNM_t \geq 1 + x\%$.

Also, unlike some studies that do not explicitly account for transaction cost for each trade, our option trading strategy needs to weigh the cost of a trade versus the ex-ante signal of gain provided by the forecast of future moment changes. If we do not set a signal threshold > 0, then any forecast signal no matter how small, would trigger a trade, and this could be very costly. On the other hand, if we set a threshold to be too high, then most forecast moments changes would not trigger a trade, and there would be too few trades and the sample of trade profits would be too small and carry large standard errors.

In practice, using thresholds based on expected forecast changes is much more parsimonious and clearer to the trader than trying to use statistical significance levels on the parameters of each regressions or using other forecast metrics as decision thresholds to trade or not. However, setting thresholds cannot be arbitrary from the point of view of ex-ante research, Based

Table 6

Out-of-Sample Error Metrics for Forecasting Risk-Neutral Kurtosis. This table reports the statistical performances of the Random Walk (RW), the Autoregressive (AR) model, the Autoregressive Moving Average (ARMA) model, the Autogressive with Garch error (AR(G)) model, the Vector Autogressive (VAR) model, the Vector Autoregressive with Garch errors (VAR(G)) model, the Vector Error Correction model (VECM), and the Local Autoregressive (LAR) model. The respective autoregressive models are lag-one models. A separate regression is performed for moments data corresponding to each time-to-maturity. Every trade day from August 24, 2009 to December 31, 2012, the risk-neutral moments computed in each 10-min intervals from 8:40 a.m. to 12:00 p.m. are used to estimate the parameters in each regression. The estimated or fitted model is then used to forecast the risk-neutral moments for each 10-min interval from 12:10 p.m. to 2:50 p.m. The error metrics or loss functions of root mean square error (RMSE), mean absolute deviation (MAD), and mean correct prediction (MCP) % are shown in Panels A, B, and C respectively. The MCP is the % of times that the forecast of directional change in the risk-neutral moment is correct.

Time-to-Maturity n	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
Panel A: Root-Mean-Square Error										
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	5.224	4.245	3.987	4.085	4.471	3.432	2.939	2.968	3.521	2.867
AR	4.261	3.444	3.293	3.535	3.594	2.667	2.345	2.400	2.627	2.194
ARMA	4.689	3.875	3.724	3.870	3.895	3.094	2.512	2.657	3.153	2.392
AR(G)	4.221	3.466	3.254	3.558	3.518	2.660	2.365	2.322	2.542	2.228
VAR	4.406	3.541	3.427	4.068	3.997	2.696	2.553	2.553	2.915	2.278
VAR(G)	4.641	3.679	3.356	3.617	3.595	2.692	2.392	2.418	2.678	2.314
VECM	5.343	6.819	6.520	10.591	13.931	3.687	8.889	5.607	9.228	2.973
LAR	4.327	3.731	3.383	3.787	3.690	2.772	2.494	2.405	2.682	2.384
Panel B: Mean Absolute Deviation										
Time-to-Maturity n	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	3.309	2.677	2.560	2.616	2.813	2.183	1.902	1.926	1.944	1.854
AR	2.817	2.343	2.271	2.431	2.365	1.891	1.672	1.699	1.701	1.511
ARMA	3.082	2.583	2.472	2.605	2.561	2.127	1.762	1.823	1.960	1.646
AR(G)	2.760	2.331	2.200	2.403	2.318	1.857	1.638	1.642	1.633	1.518
VAR	2.856	2.420	2.348	2.594	2.524	1.920	1.752	1.757	1.784	1.542
VAR(G)	2.963	2.433	2.264	2.433	2.359	1.916	1.686	1.676	1.722	1.551
VECM	3.443	3.284	2.930	3.598	4.206	2.472	2.754	2.478	2.749	1.990
LAR	2.739	2.251	2.167	2.403	2.324	1.826	1.656	1.596	1.630	1.531
Panel C: Mean Correct Prediction %										
Time-to-Maturity n	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
AR	69.76	70.33	68.60	67.65	70.43	67.79	69.00	69.52	66.32	68.95
ARMA	68.00	68.38	66.93	67.44	69.74	67.30	69.39	69.27	65.79	69.31
AR(G)	70.43	69.26	68.96	69.73	70.18	68.85	71.31	71.29	66.60	68.71
VAR	68.44	68.51	67.72	67.76	69.32	68.39	69.54	68.87	66.53	68.95
VAR(G)	68.74	66.50	67.26	68.39	69.67	67.64	69.97	70.32	65.16	70.40
VECM	63.13	60.95	62.98	61.69	61.87	60.02	63.59	64.97	57.14	62.42
LAR	72.03	70.74	69.39	69.59	70.68	68.75	70.49	71.45	67.76	70.24

on examination of the sampling data from the first couple of months, suitable thresholds yielding enough trades and producing positive profits are chosen for the rest of the sampling period. Based on a range of such suitable thresholds, profitability results are reported; however, some of these results are not necessarily significant from an ex-post point of view.

For trading, the portfolio involves typically some E-mini calls, puts, and also the E-mini futures contracts. With our dataset of actual transacted prices in calls, puts, and futures, we are able to micmic closely actual trading possibilities. We assume that in the transaction at 10-min time intervals $(t, t+1]$, based on forecast at time t, we are able to trade with market orders at transacted prices in interval $(t, t+1)$. We use only actual traded option prices and do not use synthetic prices that we created for the purpose of only developing the forecasting equations in the morning. In the variance and skewness cases, we typically have enough of options, although for the kurtosis case, as it involves four options within a 10-min interval, there were some days where there were signals but no trades due to shortages of transactions prices. We also assume there is trading in a small number of portfolio units in each time interval in order not to impact the spread. Our results show profitable per round trip trade.

For the forecast on risk-neutral volatility, the trading strategy involves creating a volatility portfolio for each 10-min interval as follows: long an OTM call and short delta amount of the underlying asset, together with long an OTM put and short a delta amount of underlying index futures. The respective deltas are based on the strike prices of the call and the put of the same maturity, and are computed using the Black-Scholes model. Since the delta of a put is negative, shorting the delta related to a put amounts to buying the underlying index futures asset. Each trading interval is 10-min intraday. At the end of the interval, the positions are liquidated at actual market prices. Prediction is done on moments with the same maturity. If the predicted next interval risk-neutral volatility is higher than the current risk-neutral volatility by at least the threshold percentage, the above portfolio of long call and long put is executed.

Table 7
Profitability of Trading Strategy using Risk-Neutral Volatility Prediction. The table reports the average $ trading profit per trade according to the different forecasting methods and threshold signals. Trading cost per option contract is $0.225, and this has been deducted to arrive at the net trading profit. The trading strategy involves creating a volatility portfolio each 10-min interval as follows: long an OTM call and short delta amount of underlying asset, together with long an OTM put and short a delta amount of underlying index futures. The respective deltas are based on the strike prices of the call and the put, and are computed using the Black-Scholes model. At the end of the interval the positions are liquidated at market prices. Prediction is done on moments with the same maturity. If the predicted next interval risk-neutral volatility is higher (lower) than the current risk-neutral volatility by at least the threshold percentage, the above portfolio of long (short) call and long (short) put is executed. The execution now and liquidation next interval constitute one trade. There can be more than one trade per interval if different maturity moment forecasts exceed the threshold, or there may be no trade in a particular interval if a non-zero threshold signal is used. The portfolio has zero cost as the net balance of the cost in the call, put, and underlying asset is financed by borrowing at risk-free rate. The overall cost of the portfolio can be expressed as:

$$\pi_t^{Vola} = C_{t,OTM} - \Delta_{C_{t,OTM}} F_t + P_{t,OTM} - \Delta_{P_{t,OTM}} F_t - B_t.$$

B_t is chosen such that $\pi_t^{Vola} = 0$. Outlay for F_t the index futures is assumed to be zero for the initial futures position. The numbers within the parentheses are the t-statistics based on bootstrapped variances calculated for the average profit. The bootstrap is carried out over 2000 iterations. **, * denote significance at the 1-tail 1%, and 2.5% significance levels respectively. #Trades refers to number of trades.

Threshold	Signal 0.0%		Signal 5.0%		Signal 7.5%		Signal 10.0%	
	Profit	#Trades	Profit	#Trades	Profit	#Trades	Profit	#Trades
PK	1.91**	7137	2.56**	2152	3.39**	1264	4.31**	773
	(7.86)		(4.51)		(3.98)		(4.50)	
AR	−1.01	7137	−0.32	1403	0.46	921	0.87	664
	(−4.14)		(−0.52)		(0.56)		(0.90)	
ARMA	−0.96	7137	−1.40	1175	−0.54	719	−0.56	457
	(−3.92)		(−2.08)		(−0.55)		(−0.46)	
AR(G)	−1.06	6455	−0.90	1243	−0.21	806	0.24	570
	(−4.14)		(−1.34)		(−0.23)		(0.24)	
VAR	−0.71	7137	−0.61	3121	−0.05	1962	0.59	1296
	(−2.88)		(−1.66)		(−0.09)		(0.91)	
VAR(G)	−0.69	6455	−0.69	2817	−0.07	1754	0.61	1144
	(−2.68)		(−1.80)		(−0.14)		(0.90)	
VECM	−0.87	7137	−0.80	2175	−0.64	1265	−1.30	954
	(−3.57)		(−1.26)		(−0.74)		(−1.31)	
LAR	−0.20	7137	0.09	2050	1.33	1235	0.84	884
	(−0.80)		(0.15)		(1.55)		(0.98)	

The execution and liquidation at end of an interval or the start of the next interval at $t + 1$ constitute one trade. There can be more than one trade per interval if different maturity moment forecasts exceed the threshold, or there may be no trade in an interval if a non-zero threshold signal is used. The portfolio has zero cost as the net balance of the cost in the call, put, and underlying asset is financed by borrowing at the risk-free rate. In the opposite case when predicted risk-neutral volatility is lower than the current risk-neutral volatility by at least the threshold percentage, the portfolio is short. In this case, a call and a put are sold together with associated positions in the underlying asset and risk-free bond. The overall cost of the portfolio can be expressed as:

$$\pi_t^{Vola} = C_{t,OTM} - \Delta_{C_{t,OTM}} F_t + P_{t,OTM} - \Delta_{P_{t,OTM}} F_t - B_t.$$

B_t is chosen such that $\pi_t^{Vola} = 0$. The outlay for F_t the index futures is assumed to be zero for the initial futures position. The profit measure can be evaluated as a change in the above cost. We impute a trading commission cost of $0.225 per option contract or $0.45 per round trip of an option contract position. This is an average figure obtained from brokers dealing with large orders or familiar clients.[2] Trading takes place from 12:10 p.m. till 3:00 p.m. each trading day based on the forecasts.

Table 7 reports the average trading profit per round-trip trade net of commission costs according to the different forecasting methods on RNV and the various threshold signals of 0.0%, 5.0%, 7.5%, and 10.0%. As the theoretical probability distribution of the average profit is not known, and the sample size is not very large, we employ the bootstrap method suggested by Efron (1979) to compute the standard errors. The bootstrap is carried out over 2000 iterations of the sample indicated by the total number of trades. The number of trades (#Trades) refers to all possible trades where there are signals given enough options during 8:40 a.m. until 12:00 p.m. for making forecasts and where there are enough calls and puts from 12:10 p.m. until 2:50 p.m. for execution of each 10-min interval. The number of trades for AR(Garch) and VAR(Garch) are less because of some cases of non-convergence in the estimation algorithm. Generally, higher thresholds produced fewer trades because there are less cases where signals exceeded the thresholds for trade execution.

Table 7 shows that for the 0% and 5% thresholds, trading the volatility induces losses except for the LAR method. If we add back the commission costs that were deducted, most of the models yield a before-cost payoff of close to zero per trade. Thus, greater than 50% MCPs are not a guarantee for a profitable options strategy on volatility. However, when the threshold

[2] For example, the Internet-based trading firm "TradeStation" offers flat-fee trading of $4.99 plus $0.20 per option trade for trading volumes of 200 or more contracts per month. This works out to about $ 0.20 + $4.99/200 = $ 0.225 per option trade. Trading costs are even lower for institutional brokers or exchange members.

Table 8
Profitability of Trading Strategy using Risk-Neutral Skewness Prediction. The table reports the average $ trading profit per trade according to the different forecasting methods and threshold signals. Trading cost per option contract is $0.225, and this has been deducted to arrive at the net trading profit. The trading strategy involves creating a skewness portfolio each 10-min interval as follows: long an OTM call and short a number of OTM puts equal to the ratio of the call vega to put vega. Also short a number of underlying assets equal to the call delta less the same vega ratio times put delta. The respective vegas and deltas are based on the strike prices and other features of the call and the put and are computed using the Black-Scholes model. At the end of the interval the positions are liquidated at actual market prices. Prediction is done on moments with the same maturity. If the predicted next interval risk-neutral skewness is higher (lower) than the current risk-neutral skewness by at least the threshold percentage, the above portfolio of long (short) call and short (long) puts is executed. The execution and liquidation next interval constitute one trade. There can be more than one trade per interval if different maturity moment forecasts exceed the threshold, or there may be no trade in a particular interval if a non-zero threshold signal is used. The portfolio has zero cost. The overall cost of the portfolio can be expressed as:

$$\pi_t^{Skew} = C_{t,OTM} - \left(\frac{\upsilon_{C_{t,OTM}}}{\upsilon_{P_{t,OTM}}}\right) P_{t,OTM} - \left(\Delta_{C_{t,OTM}} - \left(\frac{\upsilon_{C_{t,OTM}}}{\upsilon_{P_{t,OTM}}}\right)\Delta_{P_{t,OTM}}\right) F_t - B_t.$$

B_t is chosen such that $\pi_t^{Skew} = 0$. Outlay for F_t the index futures is assumed to be zero for the initial futures position. The numbers within the parentheses are the *t*-statistics based on bootstrapped variances calculated for the average profit. The bootstrap is carried out over 2000 iterations. **, * denote significance at the 1-tail 1%, and 2.5% significance levels respectively. #Trades refers number of trades.

Threshold	Signal 0.0%		Signal 10.0%		Signal 20.0%		Signal 50.0%	
	Profit	#Trades	Profit	#Trades	Profit	#Trades	Profit	#Trades
PK	2.81**	7137	2.97**	5657	3.00**	4291	2.68**	2406
	(9.33)		(8.98)		(7.96)		(5.56)	
AR	1.42**	7137	1.74**	5178	1.88**	3290	1.16	1507
	(4.76)		(5.01)		(4.30)		(1.81)	
ARMA	1.39**	7137	2.01**	4101	1.84**	2645	0.52	1384
	(4.65)		(5.04)		(3.73)		(0.76)	
AR(G)	1.48**	6455	1.80**	4661	1.97**	2928	1.03	1317
	(4.66)		(4.85)		(4.21)		(1.46)	
VAR	1.59**	7137	1.94**	5246	1.75**	3336	1.02	1562
	(5.28)		(5.58)		(4.18)		(1.67)	
VAR(G)	1.62**	6455	1.91**	4710	1.78**	2961	0.91	1372
	(5.19)		(5.23)		(3.96)		(1.39)	
VECM	0.45	7137	0.91**	5115	0.53	3439	0.02	1685
	(1.50)		(2.55)		(1.21)		(0.03)	
LAR	1.40**	7137	1.62**	5458	1.83**	3647	1.06	1622
	(4.65)		(4.67)		(4.43)		(1.67)	

is 7.5% or larger, the stronger and larger signals for change in volatility yields some profits, although there are not statistically significant for some methods such as AR and LAR. Overall, risk-neutral volatility forecasts cannot lead to a profitable options trading strategy. This is similar to the results using daily RNMs as in Neumann and Skiadopoulos (2013).

For the forecast on skewness, the trading strategy involves creating a skewness portfolio for each 10-min interval as follows: long an OTM call and short a number of OTM puts equal to the ratio of the call vega to put vega. Also short a number of underlying assets equal to the call delta less the same vega ratio times put delta. The respective vegas and deltas are based on the strike prices and other features of the call and the put. The vegas and deltas are computed based on the Black-Scholes model. Each trading interval is ten minutes intraday. At the end of the interval, the positions are liquidated at market prices. If the predicted next interval risk-neutral skewness is higher than the current risk-neutral skewness by at least the threshold percentage, the above portfolio of long call and short put of the same maturity is executed. The execution and liquidation for the next interval constitute one round-trip trade. There can be more than one trade per interval if different maturity moment forecasts exceed the threshold, or there may be no trade in a particular interval if a non-zero threshold signal is used. The portfolio has zero cost as the net balance of the cost in the call, puts, and underlying asset is financed by borrowing at a risk-free rate. In the opposite case, when predicted risk-neutral skewness is lower than the current risk-neutral skewness by at least the threshold percentage, the portfolio is short. In this case, there is a short call and a long put, together with associated positions in the underlying asset and risk-free bond. The overall cost of the portfolio can be expressed as:

$$\pi_t^{Skew} = C_{t,OTM} - \left(\frac{\upsilon_{C_{t,OTM}}}{\upsilon_{P_{t,OTM}}}\right) P_{t,OTM} - \left(\Delta_{C_{t,OTM}} - \left(\frac{\upsilon_{C_{t,OTM}}}{\upsilon_{P_{t,OTM}}}\right)\Delta_{P_{t,OTM}}\right) F_t - B_t.$$

B_t is chosen such that $\pi_t^{Skew} = 0$. Outlay for F_t the index futures is assumed to be zero for the initial futures position. The profit measure can be evaluated as a change in the above cost. Similarly, we impute a trading commission cost of $0.225 per option contract or $0.45 per round trade of an option contract position.

Table 8 reports the average trading profit in $ per round-trip trade net of commission costs according to the different forecasting methods on RNS and the various threshold signals of 0.0%, 10.0%, 20.0%, and 50.0%. As the theoretical probability distribution of the average profit is not known, and the sample size is not very large, we employ the bootstrap method suggested by Efron (1979) to compute the standard errors for the statistics. The bootstrap is carried out over 2000 iterations of the sample size indicated by the total number of trades.

In the skewness case as shown in Table 8, except for VECM forecasting, all the other forecasting models using thresholds less than 50% produce significantly positive profits per trade, significant at the 1% level. The AR and AR(G) methods consistently produce a high $ average trading profit per trade of $1.40 to $2 for thresholds from 0% up to 20%. Since the results are obtained using transactions prices and not the mid of bid-ask prices, the profits already accounted for the spread cost. Moreover, trading commissions costs are also deducted. Thus, the intraday 10-min forecasts and trades produce significant profits not seen in the daily trading results found elsewhere, such as in Neumann and Skiadopoulos (2013).

The possibility of a risk-neutral skewness forecast on index futures return to yield profitable trading strategies using options in the next 10-min interval indicates that the E-mini options market is not informationally efficient. This could be the reason why this market experiences very active high-frequency trading by speculators as well as hedgers. Informational inefficiency is construed in the sense of one being able to use available information at t to profit from trades in $(t, t+1]$. This skewness profitability anomaly may be an indication of informational market inefficiency in intraday S&P 500 futures options markets. This is not tantamount to inferring inefficiency in the index futures markets since most of the trades in a portfolio execution are positions in the futures options. If a positive skewness forecast results in a long position in OTM calls and a short position in OTM puts, being able to profit implies that the calls are underpriced relative to the puts or the puts are overpriced relative to the calls. This in itself does not however imply that there is breach to no-arbitrage conditions on the option prices. An example to illustrate could be that an OTM call on strike 1400 is valued using a no-arbitrage model at 1.935, with underlying index futures at 1300. An OTM put with strike 1,200, same maturity, is priced at 1.353. One can profit by buying the OTM call, selling the OTM put, hedging the vega and delta, if the skewness indeed increases with index futures rising to 1350. However, the calls and puts themselves are non-arbitrageable and their prices follow put-call parities.

For forecasts of kurtosis, the trading strategy involves creating a kurtosis portfolio for each 10-min interval as follows: long X number of ATM calls and X number of ATM puts and simultaneously short one OTM call and one OTM put, where $X = (C_{t,OTM} + P_{t,OTM})/(C_{t,ATM} + P_{t,ATM})$. Each trading interval is ten minutes intraday. At the end of the interval, the positions are liquidated at market prices. Prediction is done on moments with the same maturity. If the predicted next interval risk-neutral kurtosis is higher than the current risk-neutral kurtosis by at least the threshold percentage, the above portfolio of long ATM call and put, and short OTM call and put, all of the same maturity, is executed. The execution and liquidation next interval constitutes one trade. There can be more than one trade per interval if different maturity moment forecasts exceed the threshold, or there may be no trade in a particular interval. The portfolio is self-financing and has zero cost.

In the opposite case when predicted risk-neutral kurtosis is lower than the current risk-neutral kurtosis by at least the threshold percentage, the portfolio is short. In this case, there is short X numbers of ATM calls and ATM puts, and long one of OTM call and OTM put. The overall cost of the portfolio can be expressed as:

$$X(C_{t,ATM} + P_{t,ATM}) - (C_{t,OTM} + P_{t,OTM}) = 0.$$

Table 9 reports the average trading profit per round-trip trade net of commission costs according to the different forecasting methods on RNK and the various threshold signals of 0.0%, 10.0%, 20.0%, and 50.0%. For a predicted increase in kurtosis, Aït-Sahalia et al. (2001)'s study on S&P 500 options indicated that the risk-neutral probabilities would increase in prices closer to the ATM strike while they would decrease in prices at far-OTM. Therefore the trading strategy as indicated above would be to buy ATM or near ATM options and sell OTM options. Table 9 shows that there are far fewer trades possible under this kurtosis strategy since four options with the same maturity have to be traded in 10-min intervals and there are fewer of such intervals. There are only two cases of significant profits. At the 20% threshold, the AR model yields a significant profit at the 2.5% level. At the 10% threshold, the VECM yields a significant profit at the 1% level. However, all the models do not produce statistically significant profits that are consistent across all thresholds.

5.1. Daily profit comparison

We also attempt an approximate comparison of our trading strategies with the daily trading profitability reported in Neumann and Skiadopoulos (2013). We qualify the approximation as the data used in our study are different in several aspects, even as both studies analyze the RNMs of distributions related to the S&P 500 Index. Neumann and Skiadopoulos use S&P 500 Index quote and mid-point of bid-ask quoted index option prices as the underlying. We use actual transaction prices on the index futures and the index futures options (E-mini futures options). They use end of trading day option prices in OptionMetrics; we use CME intraday transaction prices in the last ten minutes of the trading hours each trading day. The OptionMetrics data spanned 30 days and so on, while our data for EW options are meant for short maturities of one to ten days. It is more difficult to find enough transacted option prices within a 10-min end-of-trading day span, so our daily data points are more limited. Neumann and Skiadopoulos remove data where implied volatility exceeds 100%; we do not, as long as all data meet no-arbitrage conditions. This may allow higher volatility and kurtosis to show up in our data.

There may be another possible difference in the results based on index options versus that based on index futures options in our study, due to an index portfolio yielding dividends whereas a futures position in the index does not. We explain if there is any difference as follows. Let F_t and S_t denote the maturity τ index futures price and spot index price at time t respectively. $F_t = S_t E_t^Q\{\exp[(r-d)\tau]\}$, where r is the continuously compounded risk-free rate, and d is the continuous aggregate dividend yield from stocks of the index. E_t^Q is the conditional expectation on information at t taken with respect to the risk-neutral

Table 9

Profitability of Trading Strategy using Risk-Neutral Kurtosis Prediction. The table reports the average $ trading profit per trade according to the different forecasting methods and threshold signals. Trading cost per option contract is $0.225, and this has been deducted to arrive at the net trading profit. The trading strategy involves creating a kurtosis portfolio each 10-min interval as follows: long X ATM calls and X ATM puts and simultaneously short one OTM call and one OTM put, where $X = (C_{t,OTM} + P_{t,OTM})/(C_{t,ATM} + P_{t,ATM})$. The ATM (OTM) options are chosen as far as possible to have similar strikes. At the end of the interval the positions are liquidated at actual market prices. Prediction is done on moments with the same maturity. If the predicted next interval risk-neutral kurtosis is higher (lower) than the current risk-neutral kurtosis by at least the threshold percentage, the above portfolio of long (short) ATM calls and puts, and short (long) OTM call and put is executed. The execution and liquidation next interval constitute one trade. There can be more than one trade per interval if different maturity moment forecasts exceed the threshold, or there may be no trade in a particular interval if a non-zero threshold signal is used. The portfolio has zero cost. The numbers within the parentheses are t-statistics based on bootstrapped variances calculated for the average profit. The bootstrap is carried out over 2000 iterations. **, * denote significance at the 1-tail 1%, and 2.5% significance levels respectively. #Trades refers to number of trades.

Threshold	Signal 0.0%		Signal 10.0%		Signal 20.0%		Signal 50.0%	
	Profit	#Trades	Profit	#Trades	Profit	#Trades	Profit	#Trades
PK	2.92**	682	2.70*	377	3.53	228	4.35	87
	(2.98)		(2.01)		(1.86)		(1.15)	
AR	0.71	682	2.82	319	5.78*	137	4.00	33
	(0.71)		(1.72)		(2.09)		(0.63)	
ARMA	0.55	682	2.85	241	1.99	128	5.33	41
	(0.55)		(1.44)		(0.68)		(0.82)	
AR(G)	0.60	623	2.57	286	5.72	121	4.64	29
	(0.56)		(1.43)		(1.85)		(0.63)	
VAR	1.37	682	2.16	328	4.42	142	4.18	38
	(1.39)		(1.33)		(1.52)		(0.74)	
VAR(G)	1.36	623	2.22	301	3.93	129	4.52	36
	(1.28)		(1.27)		(1.25)		(0.76)	
VECM	0.79	682	3.82**	374	3.75	192	1.10	60
	(0.79)		(2.69)		(1.75)		(0.32)	
LAR	−0.89	682	1.64	374	3.65	188	2.25	45
	(−0.88)		(1.17)		(1.65)		(0.38)	

measure Q. $(r - d)\tau$ is the net cost of carry. Also, since the futures price converges to the spot at maturity, $F_{t+\tau} = S_{t+\tau}$. Then, $\ln(F_{t+\tau}/F_t) = \ln(S_{t+\tau}/S_t) - \ln(E_t^Q\{\exp[(r-d)\tau]\})$.

Let $R^*(t,\tau) = \ln(S_{t+\tau}/S_t)$ be the continuously compounded rate of return on index S_t at time t over time period τ. Therefore, with our futures continuously compounded rate of return $R(t,\tau) = \ln(F_{t+\tau}/F_t)$, $E_t^Q[R(t,\tau)] = E_t^Q[R^*(t,\tau)] - (r-d)\tau$ if r, d are constants. Then the kth central moment of the returns are related as:

$$E_t^Q\{R(t,\tau) - E_t^Q[R(t,\tau)]\}^k = E_t^Q\{R^*(t,\tau) - E_t^Q[R^*(t,\tau)]\}^k .$$

Even if the dividend yield $d_{t,\tau}$ over (t,τ) is stochastic, assuming $E_t^Q\{\exp[(r-d_{t,\tau}\tau]\} > 0$ exists, which is adapted to information at t, then $E_t^Q\{\ln(F_{t+\tau}/F_t)\} = E_t^Q\{\ln(S_{t+\tau}/S_t)\} - \ln E_t^Q\{\exp[(r-d_{t,\tau})\tau]\}$, which implies again that $E_t^Q\{R(t,\tau) - E_t^Q[R(t,\tau)]\}^k = E_t^Q\{R^*(t,\tau) - E_t^Q[R^*(t,\tau)]\}^k$.

Thus the instantaneous RNMs at each time t obtained from index futures options with continuously compounded futures returns over horizon τ as the underlying are identical to RNMs obtained from index options with continuously compounded cash index returns over horizon τ as the underlying as long as the futures price is a derivative of underlying cash index.

However, according to the literature, there is a drag on the cash index in catching up with leading index futures that evidenced more price discovery, e.g., Chan (1992) and Ira et al. (1988). Index futures volatility is also higher than cash index volatility as reported in Ira et al. (1990). The above empirical facts are consistent with the faster reaction of futures returns than index returns to news including dividend news. They can also arise when we consider the RNMs to be measured across discrete time intervals and not measured instantaneously. As an illustration, suppose the RNM is measured at time t for prices that are not all transacted simultaneously and instantaneously at t but over a short time interval $(t, t + \triangle]$. Over this interval, even as index futures prices absorb and reflect new information about dividends, it is assumed that the cash index is stale and does not change and remains at S_t during this interval. Let the index portfolio aggregate dividend yield d_t at any t follow a stochastic process $d_t = \theta_t + e_t$ where both θ_t and e_t are stationary random variables and are independent of each other. e_t has mean 0, variance σ_e^2, and $E_t^Q e_t^3 > 0$ under the Q-measure. Unconditional mean $E^Q(\theta_t) = \alpha$, a constant. It is assumed θ_t has a well-defined probability distribution with finite moments up to the fourth moment. Moreover, $\theta_t + e_t > 0$.

The dividend news at any time t^* is represented by θ_{t^*}. Other factors affecting dividend yield are not observed and cause the random noise e_t. As we are isolating dividend effect for consideration, we do not model the news affecting prices $F_T = S_T$ at maturity. Daily one-month Treasury rates from 2009 to 2012 were mostly less than 0.1% p.a. and thus r is treated as constant here. We assume no-arbitrage in the cost of carry model, $F_{t^*} = S_t E_{t^*}^Q\{\exp[(r - d_{t^*})\tau]\}$ for $t < t^* \le t + \triangle$, where conditioning on t^* refers to conditioning on news θ_{t^*}. For any time during a day or $\triangle \le$a day, the arbitrageur deems time-to-maturity as the same τ. Without loss of generality, let t be the start of day d, and $t^* > t$. Then, $\ln F_{t^*} = \ln S_t + r\tau - \theta_{t^*}\tau + \ln E_{t_*}^Q[\exp(-e_{t^*}\tau)] = \ln S_t + (r - \theta_{t^*})\tau + h(\tau, \sigma_e)$, where $h(\tau,\sigma_e) \equiv \ln E_{t^*}^Q[\exp(-e_{t^*}\tau)]$ is a deterministic function in τ and σ_e.

Thus, $\ln F_T/F_{t*} = \ln S_T/S_t - (r - \theta_{t*})\tau - h(\tau, \sigma_e)$.

We consider for illustration $\triangle$ equivalent to a day. Thus $\tau = n\triangle$ or n number of days to maturity. For traded futures prices F_{t*} and stale price S_t for every t^* within day d, the computation of mean of $R(t^*, \tau)$ and $R^*(t, \tau)$ at day d (or conditional at t, which is de facto conditional on prices observed at day d) is done using statistical averaging over the discrete interval $\triangle$. This is represented by the expectation as $E^Q[\ln F_T/F_{t*}] = E^Q[\ln S_T/S_t] - (r - \alpha)\tau - h(\tau, \sigma_e)$. Then, for any t^* in day d, $R(t^*, \tau) - E^Q[R(t^*, \tau)] = R^*(t, \tau) - E^Q[R^*(t, \tau)] + (\theta_{t*} - \alpha)\tau$. Therefore at day d,

$$E^Q\{R(t^*, \tau) - E^Q[R(t^*, \tau)]\}^2 = E^Q\{R^*(t, \tau) - E^Q[R^*(t, \tau)]\}^2 + \tau^2 E^Q(\theta_{t*} - \alpha)^2 .$$

The risk-neutral third central moment on futures return is computed as:

$$E^Q\{R(t^*, \tau) - E^Q[R(t^*, \tau)]\}^3 = E^Q\{R^*(t, \tau) - E^Q[R^*(t, \tau)]\}^3 + \tau^3 E^Q(\theta_{t*} - \alpha)^3 .$$

And the risk-neutral fourth central moment on futures return is computed as:

$$E^Q\{R(t^*, \tau) - E^Q[R(t^*, \tau)]\}^4 = E^Q\{R^*(t, \tau) - E^Q[R^*(t, \tau)]\}^4 + 6\tau^2 E^Q(\theta_{t*} - \alpha)^2 E^Q\{R^*(t, \tau) - E^Q[R^*(t, \tau)]\}^2 + \tau^4 E^Q(\theta_{t*} - \alpha)^4.$$

From these central moments, the comparisons of risk-neutral skewness and kurtosis can be analysed. We approximate the risk-neutral dividend moments using the empirical measure, employing the S&P 500 daily index and the S&P 500 daily total index return data from September 2009 to December 2012 collected from the CBOE public website (https://www.cboe.com). The daily dividend yield over interval $\triangle$ of a day can be determined. We use the daily dividend yield to approximate the stationary d_t. From this time series, the mean of daily d_t is 8.383×10^{-5} and its standard deviation is 1.0641×10^{-4}.

The risk-neutral variance at day d obtained from the index futures options differs from the risk-neutral variance obtained from the index options over maturity τ by an excess of $\tau^2 E^Q(\theta_{t*} - \alpha)^2$, which is bounded above by $\tau^2 \text{var}(d_{t*})$. Using $\tau = 10$ days, $\tau^2 \text{var}(d_{t*}) = 10^2 \times 0.00010641^2 = 1.1323 \times 10^{-6}$. This implies an upper bound of 0.000003 in the difference between RNV of the futures return and RNV of the index return. The difference is thus smaller than 0.0013% of the RNV of futures return of the 0.21 shown in Table 1. The risk-neutral third central moment at day d obtained from index futures options differs from the same obtained from index options over maturity τ by an excess of $\tau^3 E^Q(\theta_{t*} - \alpha)^3$, which is bounded above by $\tau^3 E^Q(d_{t*} - \alpha)^3$. Using $\tau = 10$ days, $\tau^3 E^Q(d_{t*} - \alpha)^3 = 10^3 \times 2.5321 \times 10^{-12} = 2.5321 \times 10^{-9}$. From Table 1, employing a RNS (skewness) of −1 and RNV of 0.21 indicates a third central moment of −0.009261 for index futures returns. This is bounded above by the third central moment for index option plus 2.5321×10^{-9}. The difference in RNS between futures returns and index returns is thus bounded above by $2.5321 \times 10^{-9}/0.21^3 = 2.7342 \times 10^{-7}$. This is about 0.000027% of the RNS of futures returns. The risk-neutral fourth central moment at day d obtained from index futures options differs from the same obtained from index options over maturity τ by an excess of $6\tau^2 E^Q(\theta_{t*} - \alpha)^2 E^Q\{R^*(t, \tau) - E^Q[R^*(t, \tau)]\}^2 + \tau^4 E^Q(\theta_{t*} - \alpha)^4$, which is bounded above by $6\tau^2 \text{var}(d_{t*}) \times E^Q\{R^*(t, \tau) - E^Q[R^*(t, \tau)]\}^2 + \tau^4 E^Q(d_{t*} - \alpha)^4 = 6 \times 1.1323 \times 10^{-6} \times 0.00010641^2 + 10^4 \times 8.3682 \times 10^{-16} = 8.4451 \times 10^{-12}$. The difference in RNK between futures returns and index returns is thus bounded above by $8.4451 \times 10^{-12}/0.21^4 = 4.3424 \times 10^{-9}$. This is about 8.7×10^{-10}% of the RNK of futures returns of about 5 from Table 1. Thus in this sub-section, via the illustration, we can reasonably provide a level comparison of the daily risk-neutral moments from CME futures options and those from CBOE index options.

Unlike use of the S&P 500 Index, which tends to be traded by larger institutional funds on position taking, our E-mini futures is more a day trading vehicle, which suits the essence of our study.[3]

Our daily data of 506 points starts basically in 2011–2102 as dense option trading at the end of day is not available in the earlier period. We use for each day, RNM computed on the longest maturity contracts, and average maturity is just over five days. In our daily one-day ahead forecast, we employ a constant rolling 60-day window of daily computed RNMs. Similar skewness and kurtosis trading strategies based on RNM signals with no threshold (i.e., 0%, as in Tables 7–9) are employed over one day. The Sharpe ratios of the trading results based on daily trading profits are reported as follows. Following Neumann and Skiadopoulos (2013), we perform a similar bootstrap and report the 95% confidence in the pair of numbers in parentheses below the averaged Sharpe ratio as shown in Table 10.

In the case of skewness, as in the Neumann and Skiadopoulos (2013) study, our Sharpe ratios are positive, although not significant at the 1-tail 2.5% level. For the case of kurtosis, our forecasting methods yield positive Sharpe ratios in the use of ARMA, AR(G), and VAR(G) models, whereas Neumann and Skiadopoulos' kurtosis strategy does not appear to perform as well, making mostly losses. We do not attempt to compare daily trading outcomes in risk-neutral volatility as our study does not show intraday profitability based on RNV forecast and similarly Neumann and Skiadopoulos find that the predictability of the risk-neutral moments does not lead to profitable outcomes in volatility trading.

Our results show no profitability net of trading cost in all forecasts of RNMs, whereas Neumann and Skiadopoulos' (2013) Table 10 shows some profitability in skewness forecasts before deducting transaction costs. They are not significantly larger

[3] Ed Tilly, Barclays 2012 Global Financial Services Conference, September 11, 2012, stated, "SPX is the index option of choice for customers trading large and complex orders. Buy-side institutions typically have tremendous exposure to the S&P 500 Index, and SPX options enable them to cost efficiently manage that exposure". Internet Broker Cannon Trading, https://www.cannontrading.com/tools/trading-weekly-mini-sp500-options-contracts stated, "Weekly mini SP500 options (have) been around for the last few years and provide traders with the ability of speculating with options rather than day trade as well as hedging their day trades if they choose to do so." "In general, equity products such as the E-mini NASDAQ contracts that hedge market risks different than those of the E-mini S&P 500, our most liquid equity product, do not tend to benefit from macro-level events or increased volatility to the same extent," was expressed in CME GROUP article "The Power of Risk Management," p.48, 2012 Annual Report.

Table 10
Approximate Comparison of Daily Trading Profitability. In this table we compare our results with the daily trading profitability reported in Neumann and Skiadopoulos (2013). Our daily data of 506 points start basically in 2011–2102 as dense option trading at end of day is not available in the earlier period. We use for each day, RNM computed on the longest maturity contracts, and average maturity is just over five days. Our daily RNM is computed from CME intraday option transaction prices in the last ten minutes of the regular trading hours each day. In our daily one-day ahead forecast, we employ a constant rolling sixty days window of daily computed RNMs. Similar skewness and kurtosis trading strategies based on RNM signals with no threshold i.e. 0%, as in Tables 7–9, are employed over one day. The Sharpe ratios of the trading results based on daily trading profits are reported. We perform similar bootstrap and report the 95% confidence in the pair of numbers within the parentheses below the averaged Sharpe ratio. Our results show no profitability net of trading cost at 1% significance level whereas Neumann et al.'s Table 10 displayed for comparison shows profitability before deducting transaction costs.

Our Forecasting Models	Sharpe Ratio	N&S Forecasting Models	Sharpe Ratio
Skewness Trading Strategies			
ARMA	0.94	ARIMA(1,1,1)	2.58
	(−0.62,2.44)		(2.03,3.15)
AR(G)	0.70	ARIMAX(1,1,1)	2.81
	(−0.85,2.18)		(2.24,3.39)
VECM	0.24	VECM(1)	1.94
	(−1.36,1.72)		(1.35,2.50)
VAR(G)	0.83	VECM-X(1)	2.16
	(−0.72,2.33)		(1.59,2.77)
Kurtosis Trading Strategies			
ARMA	2.30	ARIMA(1,1,1)	−0.22
	(−2.26,7.23)		(−0.64,0.32)
AR(G)	3.61	ARIMAX(1,1,1)	−0.05
	(−0.85,8.26)		(−0.52,0.62)
VECM	−1.67	VECM(1)	0.01
	(−5.95,3.17)		(−0.52,0.69)
VAR(G)	0.60	VECM-X(1)	0.52
	(−4.17,5.10)		(−0.14,1.31)

than 0 at the 1-tail 2.5% level, but are reported in Neumann and Skiadopoulos as significantly larger than 0, before spread costs, at the 1-tail 5% level. The conclusions from both studies with respect to daily trading profitability look similar.

The sharp difference between extant studies and ours is that we test for intraday options trading, whereas almost all significant published studies concentrated on trading over at least a one-day interval if not weeks or months. The longer trading intervals are not typical of a trader with a smart strategy and higher trading frequencies in a highly liquid market such as S&P 500 derivatives instruments. Also, there is a marked difference in our options trading strategies, such as in kurtosis, compared to those for example in the Neumann and Skiadopoulos (2013) study. They used forecasts of 60-day and 90-day implied moments compared to our intraday moments; their study also employed delta and vega hedged portfolios to attempt to extract profits from kurtosis changes. For trading intervals longer than a day, such hedging ratios may contain higher sampling risks, so their use may not be as effective in capturing advantages from a change in kurtosis. On the other hand, we rely on a keen observation reported in Aït-Sahalia et al. (2001) and develop a more effective kurtosis strategy as is evident in our comparative trading results.

5.2. Persistence of moments and profitability

In this subsection we discuss the persistence of the RNMs and the relation with our forecasting and portfolio performances. As observed in Table 2, the slope coefficient estimates of base-case AR(1) regression across 10-min intervals are on average all positive for risk-neutral volatility, skewness, and kurtosis. The estimates are larger for risk-neutral volatility. For risk-neutral volatility, this persistence over the 10-min intervals is stronger for shorter maturities. The percentage of the coefficient estimates being different from zero at the 10% significance level is also higher for risk-neutral volatility compared to risk-neutral skewness and risk-neutral kurtosis. The significant positive correlation finding is similar to that in the time series investigation by Neumann and Skiadopoulos (2013) using SPX daily option prices.

The strong persistence and significant explanation would imply high R^2 in the linear AR(1) regressions. Strong in-sample persistence would also imply higher forecasting or predictability effectiveness where the coefficients do not randomly change. In our study, every trade day the RNMs from 8:40 a.m. to 12:00 p.m. are used to estimate the parameters in the AR(1) regression, which is one of the forecasting models. This fitted model is then used to forecast the RNMs for each 10-min interval from 12:10 p.m. to 2:50 p.m. The RMSE and MCP of the forecasts for each day are matched with the R^2 of the AR(1) fitted regression. The results of RMSE versus R^2 and MCP versus R^2 for the RNV, RNS, and RNK are shown in Figs. 1A, 1B, 2A, 2B, 3A, and 3B.

Let the RNM at time $t + 1$ be Y_{t+1}. In the AR(1) statistical model,

$$Y_{t+1} = b_0 + b_1 Y_t + e_{t+1} \ ,$$

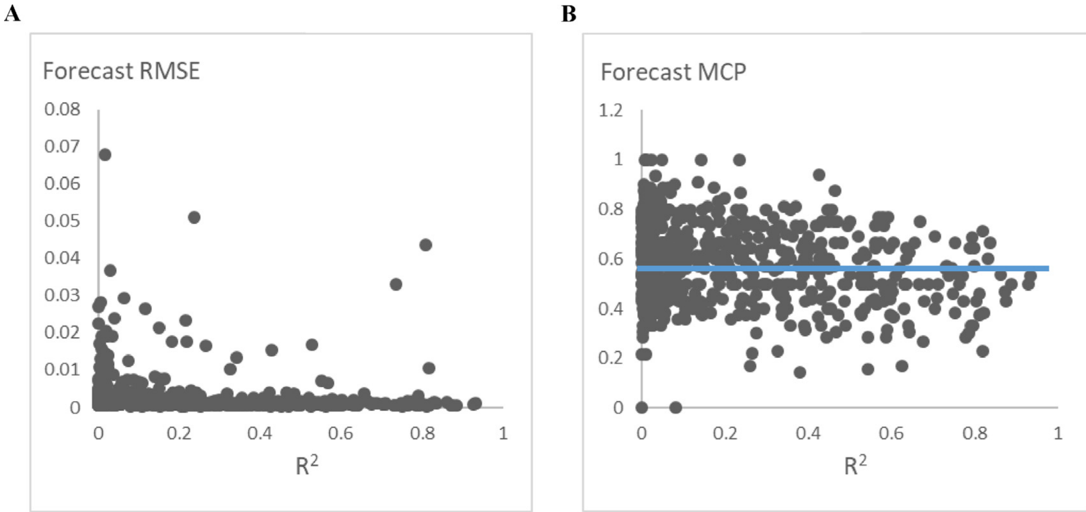

Fig. 1. **(A)** Risk-neutral volatility forecast RMSE vs. R^2. **(B)** Risk-neutral volatility forecast MCP vs. R^2.

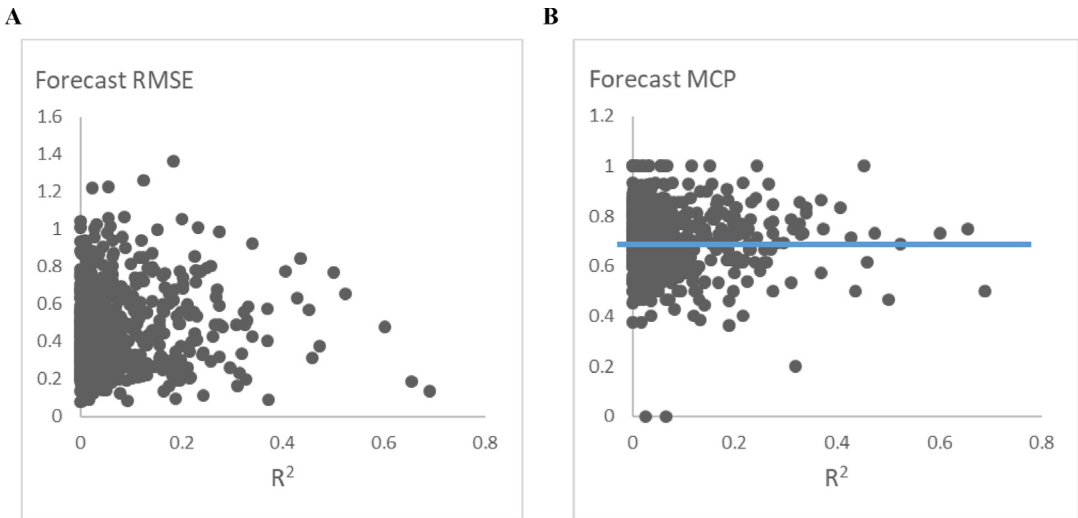

Fig. 2. **(A)** Risk-neutral skewness forecast RMSE vs. R^2. **(B)** Risk-neutral skewness forecast MCP vs. R^2.

where e_{t+1} has zero mean and is i.i.d., and b_0, b_1 are constants. Fitted $\widehat{Y}_{t+1} = \widehat{b}_0 + \widehat{b}_1 Y_t$. This fitted value is the same as the conditional expectation $E_t(Y_{t+1})$ given the AR(1) model if $\widehat{b}_0 = b_0$ and $\widehat{b}_1 = b_1$. In AR(1), the fitted R^2 is also the square of estimated slope, $\widehat{b}_1^2$. From Table 2, the average slopes of the AR(1) regressions are positively higher for RNV than for RNS and RNK, hence R^2's are generally higher for RNV. This can be seen in Figs. 1A, 2A and 3A. The slope estimates vary from day to day.

The forecast error at $t + 1$ is $Y_{t+1} - \widehat{Y}_{t+1} = (b_0 - \widehat{b}_0) + (b_1 - \widehat{b}_1)Y_t + e_{t+1}$. In large sample theory, we expect the estimates $\widehat{b}_0$ and $\widehat{b}_1$ to asymptotically approach b_0 and b_1, so the forecast error is just a random disturbance e_{t+1}. However, in the finite sample, the sampling errors of estimates $\widehat{b}_0$ and $\widehat{b}_1$ add to the forecast error of Y_{t+1}; hence RMSE can sometimes be considerably larger than the standard deviation of e_{t+1}. In particular, for the same R^2 value for two different trading days with different parameters b_0 and b_1, the forecast error can vary widely, as shown in Figs. 1A, 2A and 3A. However, the figures show that generally a higher R^2 is associated with lower forecast RMSE.

As for MCP, we count the percentage of times $Y_{t+1} - Y_t$ is of the same sign (in the same direction) as $\widehat{Y}_{t+1} - Y_t$. Now, $Y_{t+1} - Y_t = b_0 - (1 - b_1)Y_t + e_{t+1}$, whereas $\widehat{Y}_{t+1} - Y_t = \widehat{b}_0 - (1 - \widehat{b}_1)Y_t$. Let $\Pi_{t+1} = (Y_{t+1} - Y_t)(\widehat{Y}_{t+1} - Y_t)$. Then $\Pi_{t+1} = (b_0 - (1 - b_1)Y_t + e_{t+1})(\widehat{b}_0 - (1 - \widehat{b}_1)Y_t)$. Assuming the estimates are approximately the parameter values, then $\Pi_{t+1} \approx Z_t^2 +$

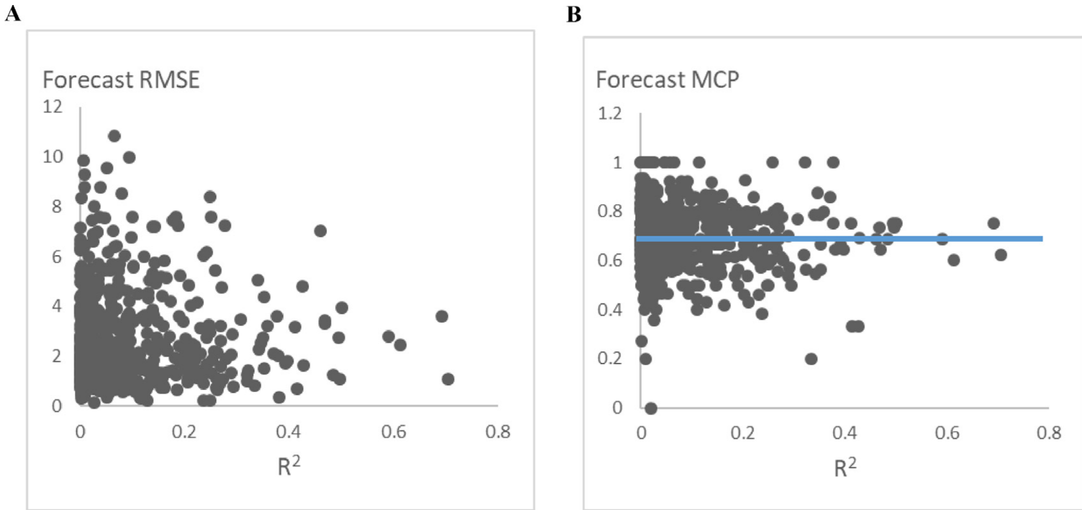

Fig. 3. **(A)** Risk-neutral kurtosis forecast RMSE vs. R^2. **(B)** Risk-neutral kurtosis forecast MCP vs. R^2.

$Z_t e_{t+1}$, where $Z_t = b_0 - (1 - b_1)Y_t$. Or,

$$\Pi_{t+1} \approx b_0^2 - 2b_0(1-b_1)Y_t + (1-b_1)^2Y_t^2 + e_{t+1}(b_0 - (1-b_1)Y_t)\ .$$

$$\begin{aligned}\Pi_{t+1}^2 \approx\ & b_0^4 + 4b_0^2(1-b_1)^2Y_t^2 + (1-b_1)^4Y_t^4 + e_{t+1}^2(b_0-(1-b_1)Y_t)^2 - 4b_0^3(1-b_1)Y_t + 2b_0^2(1-b_1)^2Y_t^2 \\ & + 2e_{t+1}b_0^2(b_0-(1-b_1)Y_t) - 4b_0(1-b_1)^3Y_t^3 - 4b_0(1-b_1)e_{t+1}Y_t(b_0-(1-b_1)Y_t) + 2(1-b_1)^2e_{t+1}Y_t^2(b_0-(1-b_1)Y_t).\end{aligned}$$

Now $E[\Pi_{t+1}] \approx b_0^2 - 2b_0(1-b_1)E[Y_t] + (1-b_1)^2E[Y_t^2]$. And

$$\begin{aligned}E[\Pi_{t+1}^2] \approx\ & b_0^4 + 4b_0^2(1-b_1)^2E[Y_t^2] + (1-b_1)^4E[Y_t^4] + E[e_{t+1}^2][b_0^2 - 2b_0(1-b_1)E[Y_t] + (1-b_1)^2E[Y_t^2]] \\ & - 4b_0^3(1-b_1)E[Y_t] + 2b_0^2(1-b_1)^2E[Y_t^2] - 4b_0(1-b_1)^3E[Y_t^3].\end{aligned}$$

If Π_{t+1} is i.i.d., we employ the sample averages $\overline{\Pi}_{t+1}$ and $\overline{\Pi_{t+1}^2}$ as estimates of $E[\Pi_{t+1}]$ and $E[\Pi_{t+1}^2]$. In the above, we replace $E[Y_{t+1}^k]$ by $\overline{Y_t^k} = N^{-1}\sum_{j=1}^N Y_j^k$ or the sampling moments, where N is sample size and $k = 1,2,3,4$. $E(e_{t+1}^2)$ is replaced by $(1-\widehat{b}_1)^2[\overline{Y_t^2} - (\overline{Y_t})^2]$. For illustration purposes, we approximate Π_{t+1} using normal distribution N (μ, σ^2), where $\mu = E[\Pi_{t+1}]$, and $\sigma^2 = E[\Pi_{t+1}^2] - \mu^2$. Then we can estimate MCP using Prob $(\Pi_{t+1} > 0) = 1 - \Phi(-\mu/\sigma)$. Using the sample moments of RNMs computed in Table 1 and average constant and slope estimates in Table 2 for computational illustration, we show the differences in MCP outcomes for RNV, RNS, and RNK. These are shown in Table 11 below.

The illustration shows that RNS obtains higher estimated MCP than RNV and RNK. In the empirical results in Fig. 2B, the average MCP of out-of-sample forecast of RNS is 70.75% (see Table 3). The average MCP of an out-of-sample forecast of RNV in Fig. 1B is 58.15% and for RNK in Fig. 3B is 68.98%. Thus high persistence, such as in high $\widehat{b}_1 = 0.6$ for RNV, is not necessarily the only condition for high predictability as in high MCP. From the illustration, higher MCP depends on higher $E[\Pi_{t+1}]$ and smaller var$[\Pi_{t+1}]$. High persistence for RNV reduces var$[\Pi_{t+1}]$ but also leads to smaller $E[\Pi_{t+1}]$. Hence although RMSE is smaller for RNV, as shown in Fig. 1A, the forecast change is not always on the same side of the actual change, although the error is small. On the other hand, for RNS, negative $\widehat{b}_0$, averagely smaller positive $\widehat{b}_1$, and a negative skew of skewness lead to a smaller var$[\Pi_{t+1}]$. The latter produces a higher MCP. For example, when skewness is near to being negative, predicting negative changes due to the negative skew of skew on average becomes more accurate, even though the fitted change may differ from actual change by a larger error. For RNS, the terms in $\overline{\Pi_{t+1}^2}$ such as $-4b_0(1-b_1)^3\overline{Y_t^3}$ and $E[e_{t+1}^2][-2b_0(1-b_1)E[Y_t]]$ can be seen to reduce var$[\Pi_{t+1}]$.

Increasing MCP is a critical condition for making a profit before transaction costs. This is because the options trading strategies we use make money if the changes in future moments are predicted correctly. Therefore, understanding how MCP is related to the parameters b_0, b_1, the various moments of the RN moments, and the persistence and fit of the regression model, is key to understanding why some models predict more profitable trades than others.

6. Ex-ante moments and subsequent returns

In single period asset pricing or intertemporal asset pricing with stationary stochastic investment opportunities, there is a majority of models where higher systematic covariance, systematic co-kurtosis, and lower co-skewness are compensated

Table 11
Illustration of Relationship between Persistence and Mean Correct Prediction. This table illustrates the mean correct prediction outcomes of forecasting intraday risk-neutral moments using an AR(1) model. Inputs to MCP computation proxied by Prob($\Pi_t > 0$) are moments of the RNM estimated in Table 1 and also constant b_0 and slope b_1 of the regression estimated in Table 2. The MCP outcomes are shown for different days to maturity n. $\overline{Y_t^k}$ denotes kth sample non-central moment of the respective RNM at time t.

Time-to-Maturity n	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
Risk-Neutral Volatility										
$\overline{Y_t}$	0.2105	0.1985	0.1897	0.1907	0.2049	0.2031	0.2035	0.2089	0.2043	0.2141
$\overline{Y_t^2}$	0.0511	0.0453	0.0407	0.0417	0.0482	0.0479	0.0468	0.0504	0.0488	0.0539
$\overline{Y_t^3}$	0.0146	0.0123	0.0101	0.0109	0.0134	0.0135	0.0123	0.0142	0.0138	0.0159
$\overline{Y_t^4}$	0.0049	0.0041	0.0029	0.0035	0.0045	0.0046	0.0037	0.0047	0.0046	0.0054
b_0	0.007	0.082	0.093	0.106	0.124	0.127	0.137	0.152	0.153	0.169
b_1	0.60	0.52	0.45	0.42	0.35	0.32	0.27	0.27	0.24	0.18
$\overline{\Pi_t}$	0.0011	0.0015	0.0016	0.0018	0.0027	0.0032	0.0030	0.0036	0.0041	0.0055
$\overline{\Pi_t^2} \times 10^4$	0.1192	0.2810	0.1940	0.4670	0.8920	1.0800	7.3000	1.0400	1.2400	1.8000
Prob($\Pi_t > 0$)	0.635	0.619	0.648	0.609	0.619	0.628	0.646	0.648	0.656	0.672
Risk-Neutral Skewness										
$\overline{Y_t}$	−1.01	−1.03	−1.07	−1.17	−1.26	−1.12	−1.08	−1.05	−1.03	−1.02
$\overline{Y_t^2}$	1.405	1.400	1.517	1.741	1.972	1.525	1.397	1.363	1.331	1.354
$\overline{Y_t^3}$	−2.293	−2.247	−2.556	−3.012	−3.527	−2.346	−2.031	−1.982	−2.014	−2.047
$\overline{Y_t^4}$	4.363	4.283	5.004	5.961	7.199	4.047	3.302	3.240	3.580	3.601
b_0	−0.922	−0.932	−0.919	−0.997	−1.115	−0.971	−0.969	−0.924	−0.890	−0.929
b_1	0.13	0.11	0.08	0.16	0.12	0.06	0.05	0.09	0.15	0.12
$\overline{\Pi_t}$	0.2928	0.2667	0.3192	0.2628	0.2977	0.2456	0.2112	0.2164	0.1956	0.2438
$\overline{\Pi_t^2}$	0.4066	0.4066	0.5331	0.3360	0.4738	0.3060	0.2348	0.2414	0.2335	0.3303
Prob($\Pi_t > 0$)	0.697	0.677	0.687	0.694	0.684	0.690	0.686	0.688	0.671	0.680
Risk-Neutral Kurtosis										
$\overline{Y_t}$	6.28	5.72	5.76	6.28	6.53	5.16	4.68	4.49	4.37	4.14
$\overline{Y_t^2}$	58.89	48.24	49.18	57.08	61.13	36.17	28.66	27.02	27.92	24.11
$\overline{Y_t^3}$	795.04	595.43	601.34	711.15	790.93	341.25	230.28	215.89	274.81	203.52
$\overline{Y_t^4}$	14005	9852	9554	11105	13007	4148	2353	2202	3960	2461
b_0	5.388	4.972	4.786	5.324	5.616	4.263	3.989	3.635	3.440	3.643
b_1	0.16	0.16	0.14	0.15	0.16	0.11	0.12	0.19	0.23	0.15
$\overline{\Pi_t}$	13.74	10.98	11.86	12.75	13.06	7.67	5.25	4.50	5.24	5.05
$\overline{\Pi_t^2}$	1785	1304	1395	1254	1386	590	261	182	461	371
Prob($\Pi_t > 0$)	0.634	0.625	0.631	0.650	0.646	0.630	0.634	0.639	0.599	0.607

by higher expected returns. Theoretical papers include Kraus and Litzenberger (1976) and Harvey and Siddique (2000) on co-skewness, and Dittmar (2002) on co-kurtosis. In a somewhat similar setup, across portfolios of stocks that are left-skewed, have high variance, and high kurtosis, their average returns will also show risk compensations. The latter could include compensations for idiosyncratic risks that may not be fully diversified away. A number of other papers have presented some evidence of ex-ante skewness being negatively related to stock returns. These relations are about equilibrium asset pricing and not directly about informational market efficiency.

As discussed earlier, Chang et al. (2013) find daily aggregate market skewness from S&P 500 Index option data to be a significant risk factor in explaining cross-sectional stock returns. Ratcliff (2013) finds daily ex-ante aggregate risk-neutral skewness levels and innovations to have a positive impact on S&P 500 Index returns the following day, although Lynch and Panigirtzoglou (2008) find no significant correlation between such daily skewness and index returns. Ratcliff (2013) suggests the relation between risk-neutral index skewness and the next-day index return is not likely due to a risk premium but may be a risk factor such as sentiment. It is interesting in our study to add a result about the intraday relation between risk-neutral index futures returns moments in a 10-min interval and the index futures return in the next 10-min interval. There is so far scant research on the forecast of intraday index return or intraday change in log futures price based on its intraday past risk-neutral moments. Such a regression would involve the explanatory RNMs as intraday risk factors. For the forecast regression, we can write:

$$\ln\left(\frac{F_{t+1}(\tau)}{F_t(\tau)}\right) = \beta_0 + \beta_1 \ln\left(\frac{F_t(\tau)}{F_{t-1}(\tau)}\right) + \beta_2\left(E_t\left[RNM_{t+1}(\tau)\right] - RNM_t(\tau)\right) + \epsilon_{t+1}(\tau),$$

where $t = 1, \ldots, T$ are the different 10-min intervals in any trading day, $\epsilon_{t+1}(\tau)$ is the i.i.d. residual error, τ indicates the correspondence of the futures price and RNM to futures and futures options with maturity τ. The second explanatory variable is the ex-ante RNM innovation. This innovation is based on an AR(1) forecast of $RNM_{t+1}(\tau)$. Note that we add the lagged futures return

Table 12
Intraday Ex-Ante Risk-Neutral Moments and Subsequent Returns. The table provides results of regressions of futures return or change in log futures price on previous 10-min interval risk-neutral moments as intraday risk factors:

$$\ln\left(\frac{F_{t+1}(\tau)}{F_t(\tau)}\right) = \beta_0 + \beta_1 \ln\left(\frac{F_t(\tau)}{F_{t-1}(\tau)}\right) + \beta_2\left(E_t\left[RNM_{t+1}(\tau)\right] - RNM_t(\tau)\right) + \epsilon_{t+1}(\tau),$$

where $t = 1, \ldots, T$ are the 10-min intervals in any trading day, $\epsilon_{t+1}(\tau)$ is the i.i.d. residual error, τ indicates the correspondence of the futures price and RNM to futures and futures options with maturity τ. The second explanatory variable is the ex-ante RNM innovation. This innovation is based on an AR(1) forecast of $RNM_{t+1}(\tau)$. The lagged futures return is added to capture any bid-ask bounce trades. RNM in Models 1,2, and 3 denotes respectively RNV, RNS, and RNK. For Model 4:

$$\ln(\frac{F_{t+1}}{F_t}) = \beta_0 + \beta_1 \ln(\frac{F_t}{F_{t-1}}) + \beta_2\left(E_t\left[RNV_{t+1}(\tau)\right] - RNV_t(\tau)\right)$$
$$+\beta_3\left(E_t\left[RNS_{t+1}(\tau)\right] - RNS_t(\tau)\right) + \beta_4\left(E_t\left[RNK_{t+1}(\tau)\right] - RNK_t(\tau)\right) + \epsilon_{t+1}(\tau).$$

Results are displayed for $\tau = 1, 5$ days. Over trading days where the regression is performed, the estimated coefficients are averaged to obtain their means. The t-statistics are reported within parentheses. Bootstrap is carried out over 2000 iterations. $^+$ denotes 2-tail 1% significance level.

τ	Coefficient Estimate (Mean)	Model 1	Model 2	Model 3	Model 4
1 day	$\widehat{\beta}_0$	0.00002	0.00002	0.00002	0.00000
		(0.32)	(0.24)	(0.28)	(0.07)
	$\widehat{\beta}_1$	-0.1110^+	-0.0944^+	-0.0922^+	-0.1183^+
		(−4.02)	(−3.11)	(−3.12)	(−3.94)
	$\widehat{\beta}_2$	−0.1020	−0.0001	0.0000	−0.2453
		(−1.12)	(−0.65)	(0.731)	(−1.67)
	$\widehat{\beta}_3$				−0.0003
					(−1.57)
	$\widehat{\beta}_4$				−0.0000
					(−0.39)
5 days	$\widehat{\beta}_0$	−0.00001	−0.00001	−0.00000	−0.00002
		(−0.22)	(−0.13)	(−0.01)	(−0.35)
	$\widehat{\beta}_1$	-0.1297^+	-0.1335^+	-0.1287^+	-0.1381^+
		(−3.08)	(−2.80)	(−3.01)	(−2.51)
	$\widehat{\beta}_2$	−0.0211	0.0001	0.0000	−0.0086
		(−0.66)	(0.39)	(0.18)	(−0.23)
	$\widehat{\beta}_3$				0.0003
					(0.65)
	$\widehat{\beta}_4$				0.0001
					(0.56)

in order to capture any bid-ask bounce trades.

A separate forecasting regression involving all three RNM's as explanatory variables is also run:

$$\ln(\frac{F_{t+1}}{F_t}) = \beta_0 + \beta_1 \ln(\frac{F_t}{F_{t-1}}) + \beta_2\left(E_t\left[RNV_{t+1}(\tau)\right] - RNV_t(\tau)\right)$$
$$+\beta_3\left(E_t\left[RNS_{t+1}(\tau)\right] - RNS_t(\tau)\right) + \beta_4\left(E_t\left[RNK_{t+1}(\tau)\right] - RNK_t(\tau)\right) + \epsilon_{t+1}(\tau).$$

In Table 12, we display the results for $\tau = 1, 5$ days. (Results for the other τs are basically similar.) For each trading day where there are many trades throughout the day, the regression is performed and the coefficient estimates are collected. Over the total number of such trading days, the estimated coefficients are averaged to obtain their means. Their t-statistics indicates if their average deviates significantly from zero. The results for the above regressions are reported in each column for models 1 to 4. In models 1, 2, and 3, we employ RNV,RNS, and RNK respectively. In model 4, all the RNMs are used as explanatory variables. A bootstrap is carried out over 2000 iterations of the sample size.

The results show that as in other studies there is a 2-tail 1% statistically significant negative bid-ask bounce in the estimated coefficients of the lagged futures returns. The risk-neutral volatility contributes negatively to the next interval futures returns, although the estimated coefficients are not significant with low t-statistics based on the bootstrapped distribution. Ex-ante risk-neutral skewness innovation contributes negatively to next interval returns for one-day maturity but positively for five-day maturity. In all cases, the estimated coefficients are not significant. Ex-ante risk-neutral kurtosis has close to zero effect and is insignificant, like all the other ex-ante RNMs. In Model 4, the lagged futures return has a significant negative coefficient estimate, but all RNMs have coefficient estimates that are not significant at the 2-tail 5% significance level. Thus RNMs have no predictive power or significance on the next 10-min futures return or change in log price. While we note that predictability may or may not indicate market informational efficiency, non-predictability would typically provide statistical evidence of futures market efficiency with respect to past intraday information from options since the RNMs were derived from option prices.

7. Conclusions

As far as we know, ours is one of the first studies on the intraday implied moments of S&P 500 Index futures returns using intraday or high-frequency futures option prices and the E-mini Index futures prices. The E-mini S&P 500 Index futures options are European-style and the data are actual transactions prices on the CME Exchange. First, we improve on existing techniques by using cubic hermite interpolation with better smoothing properties to extract the first four moments of the risk-neutral return distribution. Secondly, we perform intraday out-of-sample forecasting or prediction, and document the intraday dynamics of the index futures return risk-neutral moments. We also introduce a novel local autoregression method that allows a variable window in fitting the autoregressive parameters. This is particularly useful in situations when there may be intraday news that cause structural changes in the returns or price distributions.

Based on the forecasts of the risk-neutral moments on each 10-min interval during a trading day, option portfolios are constructed, executed for trade, and then liquidated 10-min later to take advantage of the forecast information. The trades are performed on market orders and trading commission costs are also charged. Our key results are that there is profitability after transaction costs in the trading strategies involving risk-neutral skewness, and that autoregressive models including those modeling GARCH errors on the moments innovations are the most accurate in terms of less mean absolute errors and higher directional accuracy in forecasting. An improved kurtosis strategy recognizing the empirical observation of higher risk-neutral densities around ATM relative to OTM option prices when forecast kurtosis increases leads to very marginal cases of profitable trading based on kurtosis forecasts. This is quite interesting as past studies typically find that trading on kurtosis leads to losses.

The positive profitability after transaction costs in skewness trading indicates an anomaly, which may be an indication of market inefficiency in the intraday S&P 500 futures options markets. It could be due to information inefficiency within short time spans, such as a 10-min interval. We thus provide results for intraday trading contrary to the findings of informational efficiency in daily trading by studies covering the 2009 to 2012 period. The notable exception is Noh et al.'s (1994) study. Our result also provides an explanation why traders continue to trade intraday when research has suggested that the market is informationally efficient.

Amongst practitioners, it is widely known that trading strategies relying on intraday news and changes in intraday prices. Trading would best be executed on an intraday basis. It is thus critical to complement existing studies with a focus on intraday dynamics and intraday options trading. In particular, the option strategies we invoke depend on the forecasts of future moments, and our trading profits take into account bid-ask costs and exogenous transaction costs. However, we assume small amounts of trading with a few portfolio units so as not to impact the bid-ask spread, but this would strongly limit the size of the trading profits.

Finally we also examine whether our autoregressive models of ex-ante risk-neutral moments forecasts could help predict the next 10-min underlying index futures returns. We find the expected bid-ask bounce effects, but no predictive ability by the expected moments innovations. This is different from the Ratcliff (2013) study using daily data. Thus the S&P 500 Index futures market itself, unlike the index futures options market, appears to pass the test of efficiency in intraday trading using moments information. This is not surprising given that the underlying S&P 500 Index futures has been the most actively traded index futures contract in the world.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.finmar.2018.10.001.

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