Path towards the extraction of the scattering amplitude through angular asymmetries in TCS

Rafayel Paremuzyan

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This document describes proposed steps towards the extraction of the real part of the helicity conserving scattering amplitude $Re\tilde{M}^{--}$ (look [1] for details) for the Timelike Compton scattering (TCS) process. Discussions in this note are based on studies which are presented in a workshop at Bochum in Feb 2014 [2]. At the time of this studies the CLAS12 reconstruction software was not ready, and therefore acceptance and resolution effects were studied using the CLAS12 FASTMC package. It is also discussed here the selection of kinematic points assuming 120 days of beam time with $10^{35} cm^{-2}s^{-1}$ luminosity electron beam. Unfortunately all the codes and data files are lost, therefore almost all plots in this document will be snippets from above mentioned slides [2].

1 Accessing the scattering amplitude

Experimentally TCS is accessible through $\gamma p \to e^- e^+ p$ reaction. Main contributions to this reaction are the Bethe Heitler (BH), TCS and their interference term.

$$\sigma(\gamma p \to e^- e^+ p) = \sigma_{TCS} + \sigma_{BH} + \sigma_{Int} \tag{1}$$

As discussed in [1], the TCS cross-section is very small (Fig. 10 of [1]) wrt BH cross-section, therefore in the following discussion the TCS part will be neglected. From remaining temrs, BH is well known (calculable within 1%-2% precision). The interference term (eq.30 from [1]) can be represented in a following way.

$$\sigma_{Int} = a \cdot M^{--} \cdot \frac{L_0(\theta)}{L(\theta, \phi)} \cdot cos(\phi) \tag{2}$$

where

$$a = -\frac{\alpha_{em}^3}{4\pi s^2} \frac{1}{-t} \frac{M}{Q'} \frac{1}{\tau\sqrt{1-\tau}} \frac{1 + cos^2(\theta)}{sin(\theta)} \tag{3}$$

As in the paper [1], here too, we will use the weighted cross section (WCRS), $\frac{dS}{dQ'^2 dt d\phi}$, which is obtained from the differential cross section $\frac{d\sigma}{dQ'^2 dt d(cos(\theta)) d\phi}$ weighted by $\frac{L}{L_0}$ and integrated over θ (in [1] from $\pi/4$ to $3\pi/4$). The main reason for not integrating over the whole θ range (0 to π) is because BH dominates over the interference term at $\theta \sim 0^\circ$ and $\theta \sim 180^\circ$ (the $\frac{\sigma_{BH}}{\sigma_{int}} \sim \frac{1}{\sin(\theta)}$). The exact values of integration limits is a subject for studies, and these limits must be chosen to minimize uncertainties on the extracted scattering amplitude M^{--}.

The WCRS has the following shape

$$\frac{dS_{Tot}}{dQ'^2 dt d\phi} = \int_{\pi/4}^{3\pi/4} d\theta \frac{d\sigma}{dQ'^2 dt d(cos(\theta)) d\phi} \cdot \frac{L}{L_0} = S_{\rm BH} + S_{Int} = S_{BH} + A \cdot ReM^{--} \cdot cos(\phi) \tag{4}$$

Here $A = \int_{\pi/4}^{3\pi/4} d\theta \cdot a = -\frac{\alpha_{em}^3}{4\pi s^2}\frac{1}{-t}\frac{M}{Q'}\frac{1}{\tau\sqrt{1-\tau}}\int_{\pi/4}^{3\pi/4}(1+cos^2(\theta))d\theta$

Now, if one subtracts the weighted BH cross section from the total weighted cross section, then the result will be the weighted interference term which has a cosine dependence on the angle ϕ. With this proposed method we should divide the $\phi \in (0. - 2\pi)$ range into N_ϕ bins (N_ϕ to be determined), and for each ϕ bin subtract calculated weighted BH cross section from the measured total weighted cross section. The resulting distribution should have a cosine dependence on the angle ϕ, moreover the amplitude of the modulation is proportional to the scattering amplitude ReM^{--}. In order to extract the ReM^{--}, this distribution should be fitted with a function

$$f = P \cdot cos(\phi) \tag{5}$$

As an example the plot in Fig.1 is an illustration of the extraction of P. Points in the plot are generated using a Gaussian function with the mean equal to interference part of the weighted cross section in a given kinematic bin (t, Q'^2, ϕ), and sigma equal to the expected statistical uncertainty (estimated using CLAS12 FASTMC) of the cross section in the given bin for running 120 days @$10^{35} cm^{-2}s^{-1}$. As soon the parameter

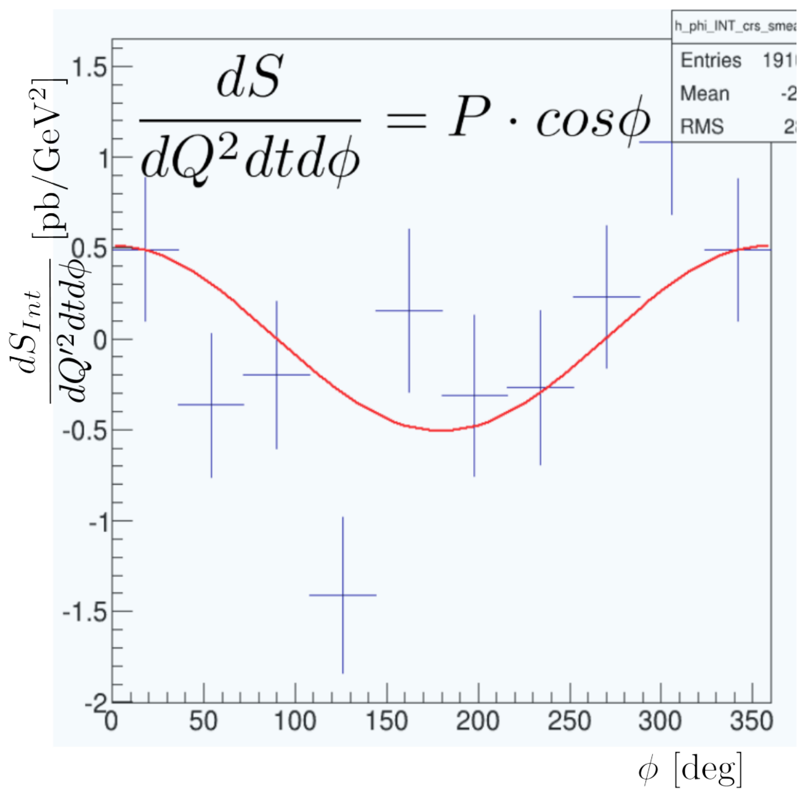

Figure 1: Illustration plot: The ϕ angular dependence of the weighted interference term cross section is fitted with cosine function.

P will be extracted from the fit, one then can calculate the real part of M^{--}

$$ReM^{--} = -\frac{P}{\frac{\alpha_{em}^3}{4\pi s^2} \cdot \frac{1}{-t}\frac{M}{Q'}\frac{1}{\tau\sqrt{1-\tau}}\int_{\pi/4}^{3\pi/4}(1+cos^2(\theta))d\theta} \tag{6}$$

2 The Effect of the CLAS(12) acceptance and the extrapolation

It is important to mention that in the above described method, integration limits over θ should not be changed for different ϕ bins. Varying integration limits, the ϕ dependence of the weighted cross section will not be directly related to M^{--}.

The CLAS and CLAS12 detector acceptances have strong θ and ϕ dependence, as it is shown in Fig.2

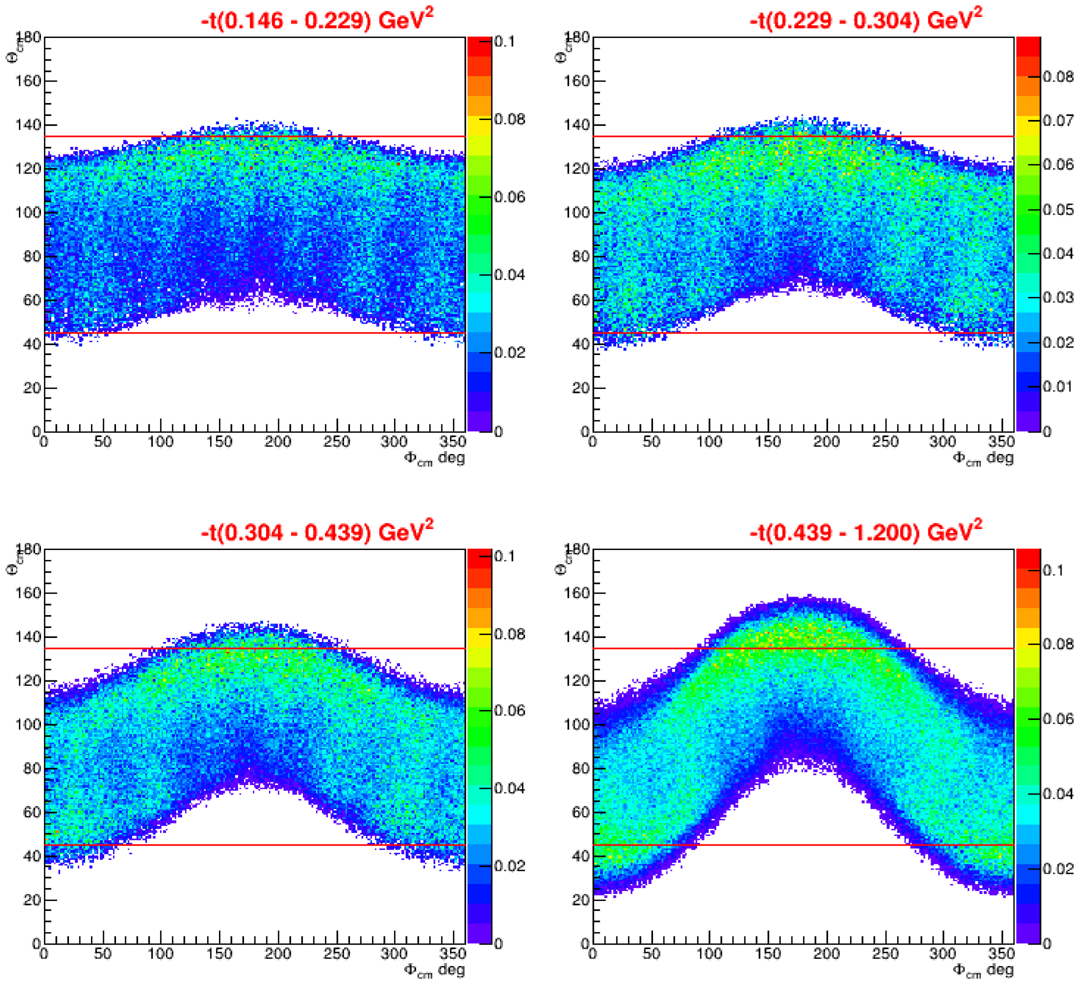

Figure 2: CLAS12 acceptance obtained in different $-t$ bins. Acceptance are calculated through CLAS12 FASTMC package.

for CLAS12 acceptance obtained in different t bins. One can see that the CLAS12 acceptance doesn't cover the full θ range, and moreover, the θ range varies a lot as a function of the angle ϕ. This means that if in the data we integrate over θ in the available θ range, then the resulting ϕ dependence will be distorted.

Now let's look into the equation (4). It shows that the WCRS of interference term has as $1 + cos^2(\theta)$ dependence on the angle θ for any angle ϕ, and if one measures the interference term WCRS in a given range $\theta \in (a, b)$, ${}_a^bS_{Int}$, then one can uniquely calculate the interference term WCRS in any $\theta \in (A, B)$ range.

$${}_A^BS_{Int} ={}_a^b S_{Int} \frac{\displaystyle\int_A^B (1 + cos^2(\theta))d\theta}{\displaystyle\int_a^b (1 + cos^2(\theta))d\theta} \qquad (7)$$

The interference term WCRS can be calculated by subtracting the BH WCRS from the total measured WCRS.

$$ {}_a^b S_{Int} = {}_a^b S_{Tot} - {}_a^b S_{BH} \tag{8} $$

In short: the ${}_a^b S_{Tot}$ will be measured from the experiment, the ${}_a^b S_{BH}$ will be calculated by integrating the analytic form of the BH cross section over the $\theta \in (a, b)$, and therefore the interference term WCRS will be measured using the eq. (8). Then using equations (8) and (7), the interference term WCRS will be

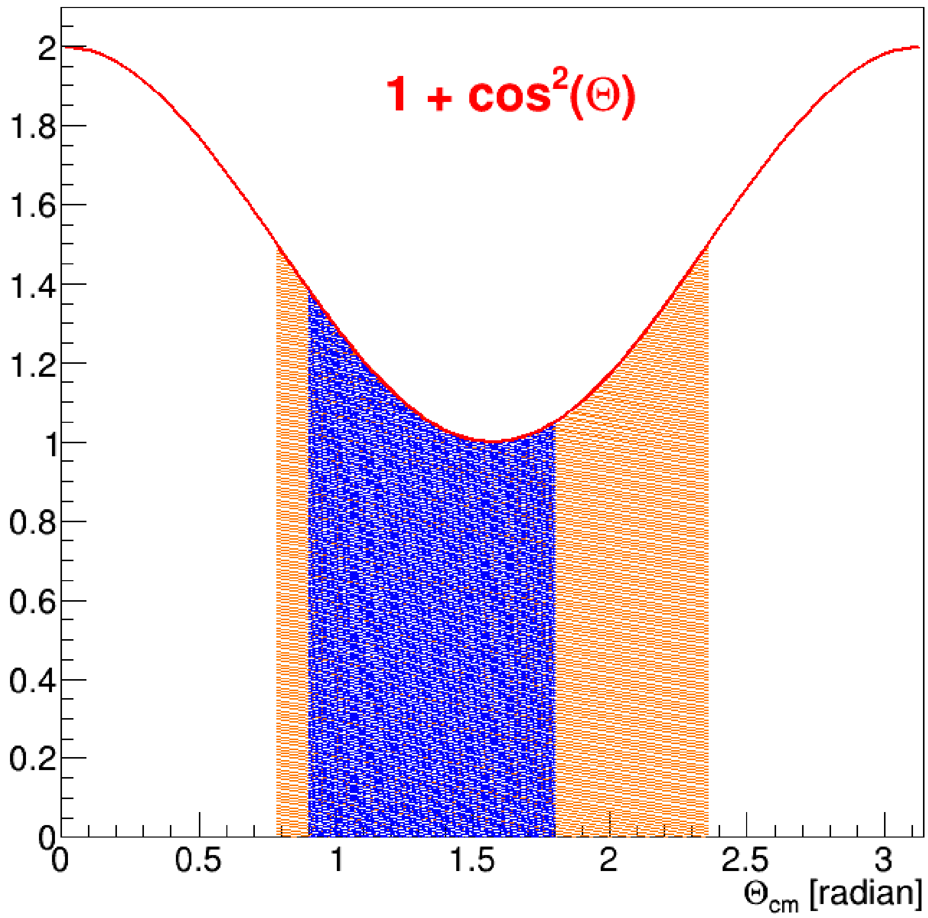

Figure 3: The interference term has a $(1 + cos^2(\theta))$ dependence on the angle θ, for any ϕ bin.

calculated for any $\theta \in (A, B)$ range for any given ϕ bin. As an illustration, in Fig.3 shown the function $(1 + cos^2(\theta))$. Knowing only the integral of 'blue' shaded area, the integral of the 'Orange' area can be determined using the equation (7). In our case the blue shaded area will correspond the CLAS(12) acceptance, and the orange shaded area will correspond to the $\pi/4$ to $3\pi/4$ range.

In order to demonstrate steps towards the extraction of the ReM^{--}, let's assume that we have divided the $(0, 2\pi)$ region into 10 bins in ϕ, and for each ϕ bin we have measured the WCRS

$$ \frac{dS}{dQ'^2 dt d\phi} = \frac{\sum_{i=1}^{N_{ev}} \frac{L_i}{L_{0\,i}}}{\mathcal{L} \cdot Acc \cdot \Delta Q'^2 \Delta t \Delta\phi} \frac{\int_A^B (1 + cos^2(\theta)) d\theta}{\int_a^b (1 + cos^2(\theta)) d\theta} \tag{9} $$

where N_{ev} is the number of events in the given bin, $\frac{L_i}{L_{0\,i}}$ is the weighting factor for a given event, $\mathcal{L}$ is the luminosity, Acc is the acceptance in the given (Q'^2, t, ϕ) bin, $\Delta Q'^2$, Δt and $\Delta\phi$ are correspondingly widths of Q'^2, t and ϕ in the given bin, a and b are CLAS(12) acceptance limits on θ for the current kinematic bin, A and B are limits of the extrapolated θ range which is the same for all ϕ bins of the given kinematic bin (Q'^2, τ, t). For simplicity, let's assume number of events in each ϕ bin is 100. This assumption will not change conclusions but will simplify codes which generates following plots, and at the end kinematic bins will be chosen such, that N_{bin} to not be much different from 100.

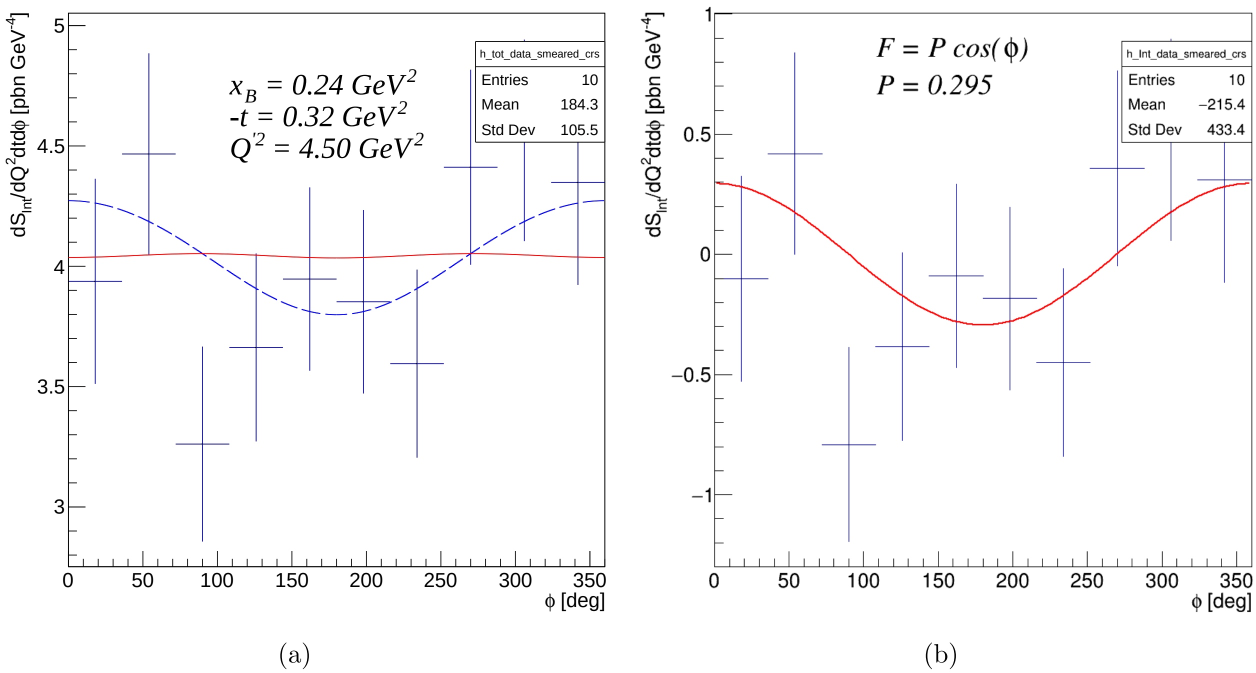

Figure 4: Left: Blue dashed line represent the sum WCRS (BH + Interference) for a kinematic point quoted in the picture, Red solid line represent the BH WCRS, and points with error bars are the generated from the sum WCRS assuming to the 10% statistical uncertainties. Right: points with error bars are obtained from points in the left plot by subtracting the BH WCRS (red cure in the left plot), the red curve is the fit of points with $P \cdot cos(\phi)$ function.

In the Fig.4a dashed blue line represent the sum WCRS (BH + Interference) for a kinematic point quoted in the picture, Red solid line represent the BH WCRS, and points with error bars represent WCRS that will be calculated with the eq. (9). In this plot these points were generated with a Gaussian mean equal to the WCRS in the middle of ϕ bin (value of blue dashed curve), and sigma equal to the statistical uncertainty (in this particular case it is 10%).

In the 2nd step we should subtract BH WCRS from the measured WCRS. The subtracted WCRS is shown in Fig.4b. This distribution is fitted with a $P \cdot cos(\phi)$ function (the red curve). As P will be obtained from the fit, the ReM^{--} can be calculated directly using the eq. (6).

3 Estimation of statistical uncertainties

The statistical uncertainty on the ReM^{--} can be obtained in a following way. Let's assume the final distribution of the interference term WCRS is the one depicted on Fig.4b. Using this distribution as a source, we will generate a big number (e.g. 10000 or more) of similar distributions, in a way that a WCRS in a given ϕ bin will be a Gaussian random number with a mean equal to the WCRS and sigma equal to the statistical error of corresponding bin of the source distribution. Then each of generated distribution will be fitted with a cosine function in a same way as the source distribution, and each fit will yield a certain P. The distribution of P is expected to be a Gaussian with a mean equal to the value of P obtained from the source distribution, and the σ of it will show the statistical uncertainties of the measured P. After 10000 such fits, the resulting distribution of the P is shown in Fig.5. The dashed blue vertical line shows

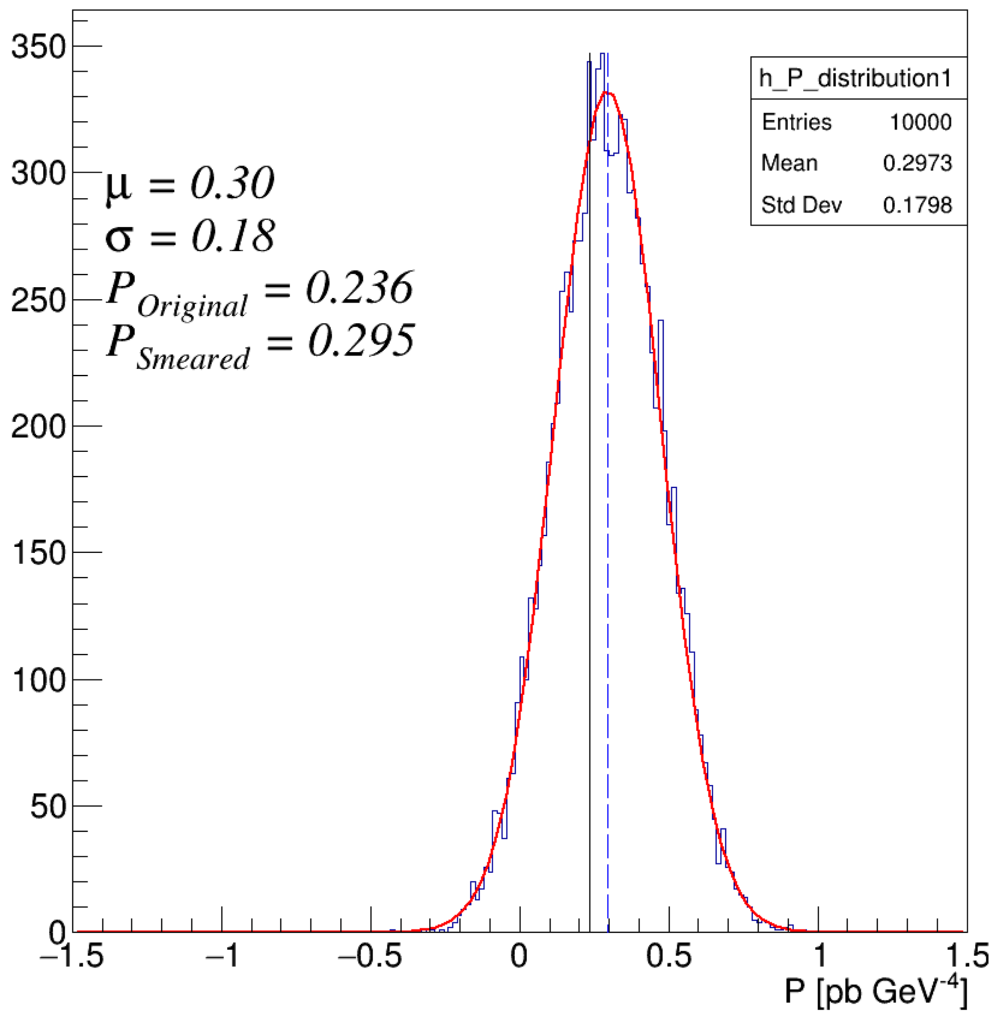

Figure 5: distribution of fit parameter P obtained from 10000 generated distributions.

the value of P obtained from the fit of the source distribution, in Fig.4b. The width of this distribution will represented the statistical uncertainty of P which is related to ReM^{--} by eq(6). As one can see the Gaussian distribution is well centered on the value of P from the source distribution fit. The black vertical line is the theoretical value of P for the given kinematic point. and as one can see the extracted value of P is consistent with the theoretical value of P within the statistical uncertainty.

4 selection of kinematic points

In this section we will discuss selection of TCS kinematic points, and compare TCS phase space to the DVCS phase space which is approved for Hall-C kinematics [3]. In TCS, the corresponding variables of

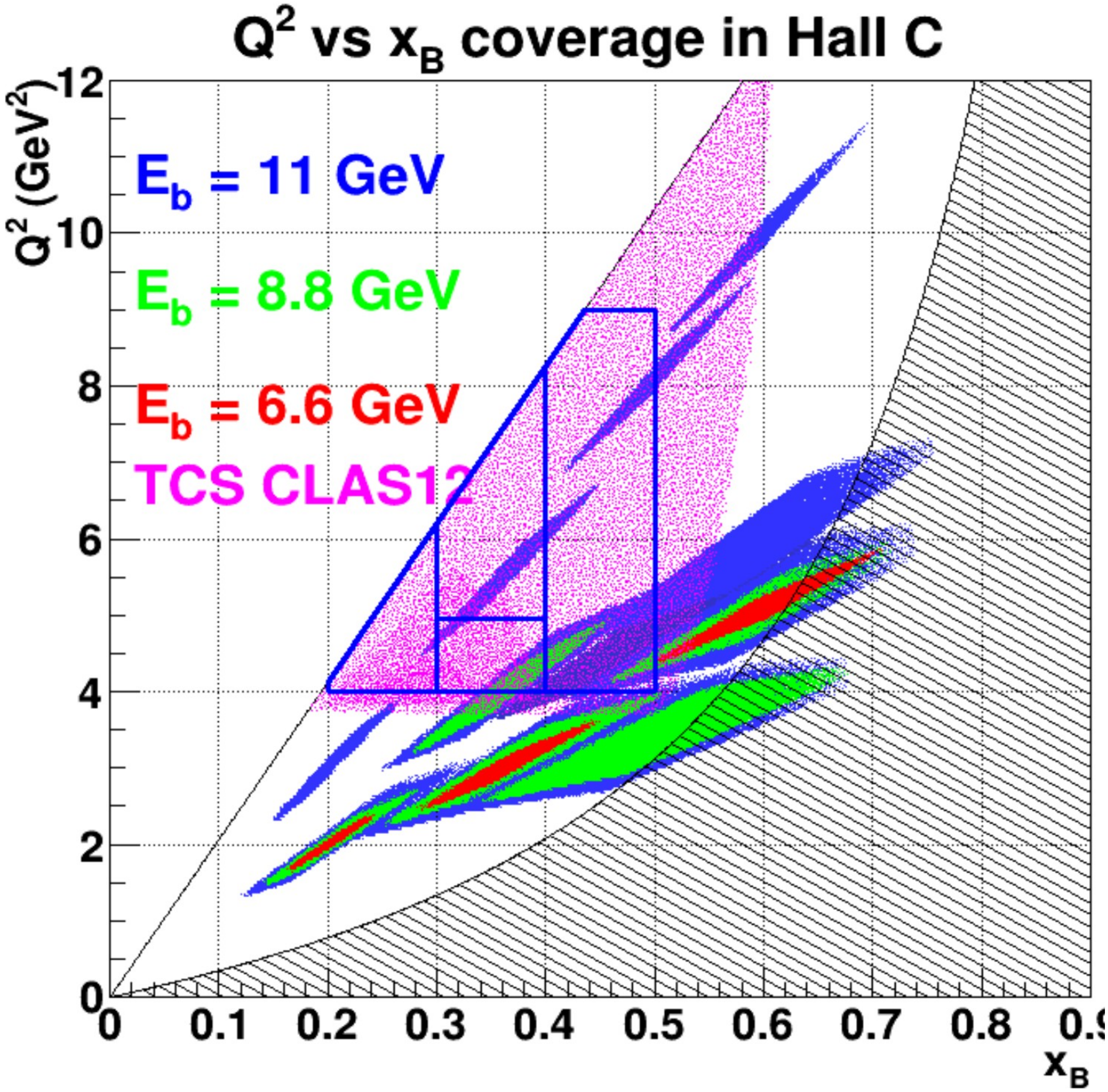

Figure 6: The Q^2 vs x_B distribution for the approved Hall-C kinematics (blue, green, and red areas), and TCS kinematics with CLAS12 acceptance (pink area).

$Q^2 \equiv (p_e - p_e'^2)$ ans $x_B \equiv Q^2/2pq$, are $Q'^2 = M^2_{e^-e^+}$ and $\tau = Q'^2/2pq = \dfrac{Q^2}{s - M^2}$, where M is the mass of the nucleon. In Fig.6 shown the Q^2 vs x_B distribution for the approved Hall-C kinematics (blue, green, and red areas), and TCS kinematics with CLAS12 acceptance (pink area).

In that plot the black solid line corresponds to the maximum achievable Q^2 for a given beam energy. Usually in DVCS experiments maximum Q^2 is not accessible experimentally, since it requires detection of a very low momentum electron, which are usually out of the acceptance, therefore as one can see DVCS kinematic coverage doesn't reach to the maximum accessible Q^2 values. In TCS however maximum Q^2 is well inside the the acceptance region for CLAS12 detector (both leptons are high energy and have large scattering angles).

In the fig.6, areas inside solid blue lines are proposed kinematic regions for TCS analysis. In particular the lowest x_B region ($x_B \in (0.2 - 0.3)$) can be complementary to DVCS, since this Q^2, x_B point is not accessible in any JLab DVCS experiments. Other points can serve as a test of universality of GPDs. I want to mention that these regions are selected in a way to have roughly equal statistics in all bins, and in total there was about 6000 expected events in each Q'^2, x_B bin.

5 Summary and outlook

It was demonstrated here that, the real part of the scattering amplitude ReM^{--} can be extracted by fitting the WCRS as a function of ϕ angle. This is somewhat different from the calculation of the cosine moment which is proposed in paper. [1]. The procedure of estimation of statistical uncertainties is also discussed.

The CLAS(12) acceptance imposes significant ϕ dependent variation of θ range, however because of the known θ depence of the cross section, we can extrapolate the cross section to a fixed range in all ϕ bins, however proper determination of θ range needs more studies.

Based on old FASTMC code and 120 days of full luminosity, this study chose 4 kinematic bins in Q'^2, x_B phase space, and the lowest x_B can serve as a complementary to DVCS program since it has potential to cover a phase space not accessible in DVCS experiments at JLab. However situation now is different, and in order to estimate rates more realistically these studies should be repeated with GEMC and be reconstructed with the current CLAS12 reconstruction code, and we need to use the currently available luminosity to estimate expected statistics.

References

[1] E. R. Berger, M. Diehl and B. Pire, Eur. Phys. J. C **23**, 675 (2002) doi:10.1007/s100520200917 [hep-ph/0110062].

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[3] DVCS Experiment proposal in Hall-C. https://www.jlab.org/exp_prog/proposals/13/PR12-13-010.pdf