

Analysis Report

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Cobb-Douglas Function of Expected GDP

In the analysis of the determinants of Gross Domestic Product (GDP), the Cobb-Douglas production function serves as a pivotal model, encapsulating the relationship between GDP and its primary inputs: capital and labor. This study employs a logarithmic transformation of the data, a standard approach in econometric analyses, to linearize the multiplicative relationships inherent in the Cobb-Douglas function. Specifically, the natural logarithms of GDP, capital stock (measured in billions of USD), and labor force (measured in thousands) are computed, yielding Log_GDP, Log_Capital, and Log_Labor, respectively.

Since Capital Stock for Australia was only available until 2019, the values for 2020, 2021 and 2022 were calculated through interpolation.

Subsequently, a linear regression model is fitted with Log_GDP as the dependent variable and Log_Capital and Log_Labor as the independent variables. The rationale behind using the logarithmic form is to align with the log-linear format of the Cobb-Douglas function, which posits that the log of output is a linear function of the logs of inputs. From this regression model, the coefficients associated with Log_Capital and Log_Labor are extracted, denoted as alpha and beta, respectively. These coefficients represent the output elasticities of capital and labor, indicating the percentage change in output resulting from a one percent change in either capital or labor.

The intercept of the regression model is exponentiated to derive the Total Factor Productivity (TFP), denoted as A. TFP is a crucial component of the Cobb-Douglas function, representing the portion of output not explained by capital and labor inputs, often interpreted as the efficiency of the combination of inputs or the impact of technology.

Each country's expected GDP is then calculated using the original Cobb-Douglas formulation: $\text{Expected_GDP} = \text{TFP} \times \text{Capital_Stock_N_Bn_USD}^{\alpha} \times \text{Labour_Force_Thousands}^{\beta}$. This formula incorporates the estimated TFP, alpha, and beta, alongside the actual values of capital stock and labor force. The Output Gap for each country is determined by subtracting the Expected_GDP from the actual GDP, providing a measure of the deviation of the actual economic output from its predicted level based on the Cobb-Douglas framework. The Output Gap thus serves as an indicator of the economy's performance relative to its potential, as envisaged by the estimated production function.

The Cobb-Douglas function is thereby expressed as:

$$\text{GDP} = A \times \text{Capital}^{\alpha} \times \text{Labor}^{\beta}$$

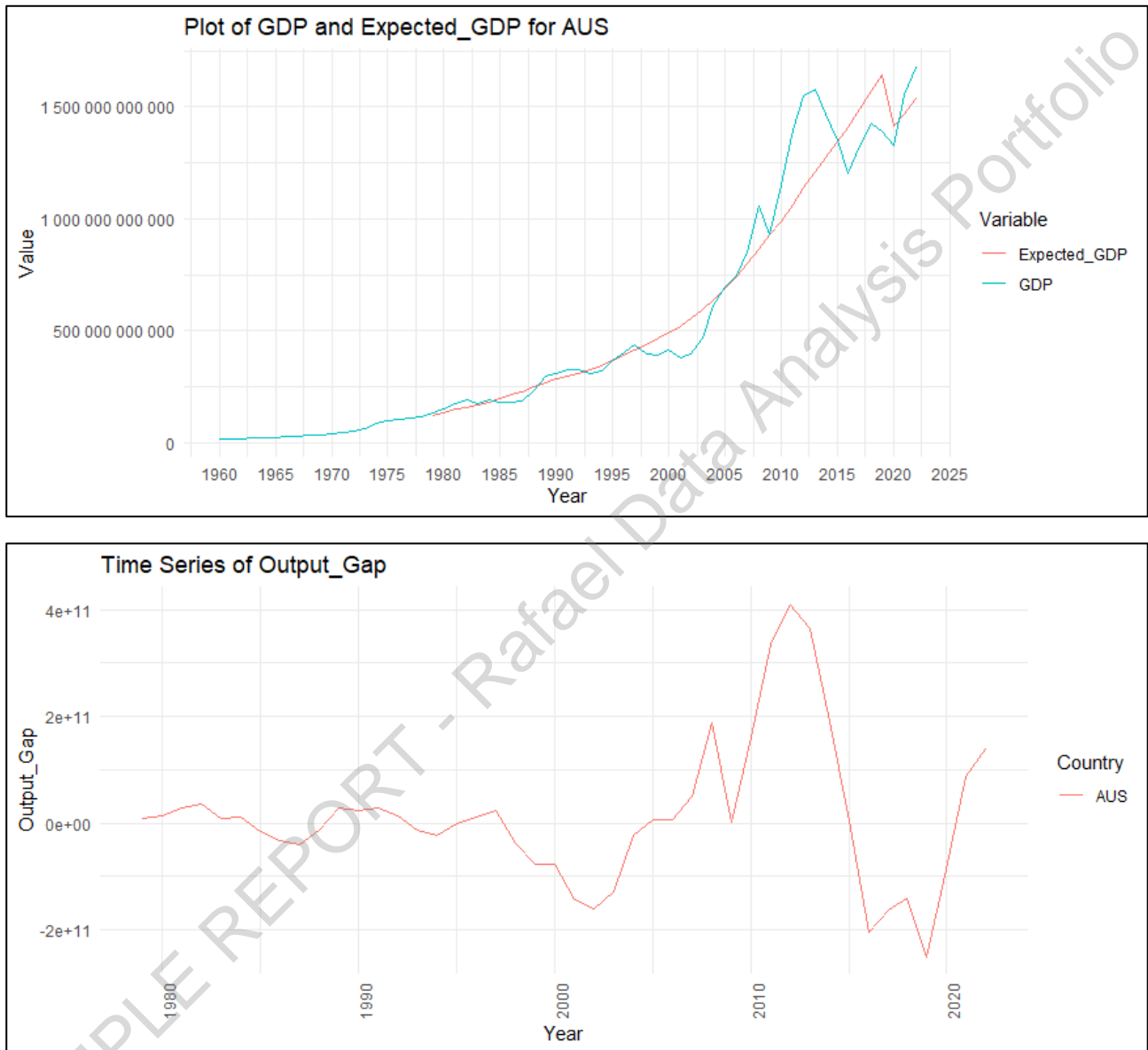
A represents the Total Factor Productivity (TFP), α is the output elasticity of capital, and β is the output elasticity of labor. Substituting the estimated values for Australia into the equation, the function becomes:

$$\text{GDP} = 5.65785 \times 10^{-6} \times \text{Capital}^{1.801608425} \times \text{Labor}^{0.740464355}$$

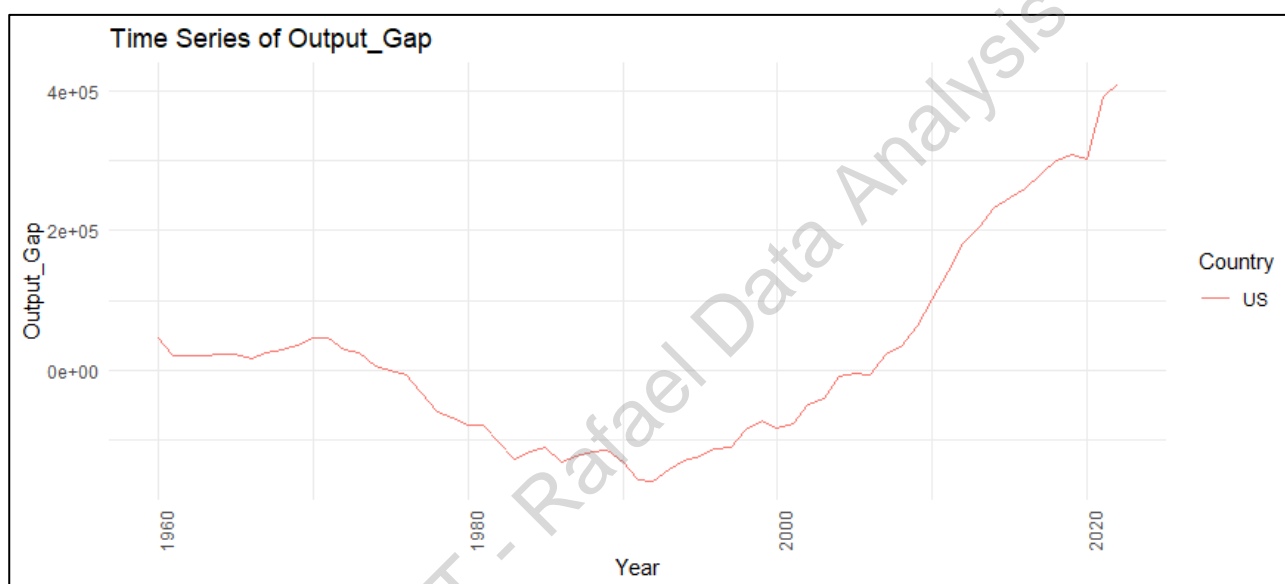
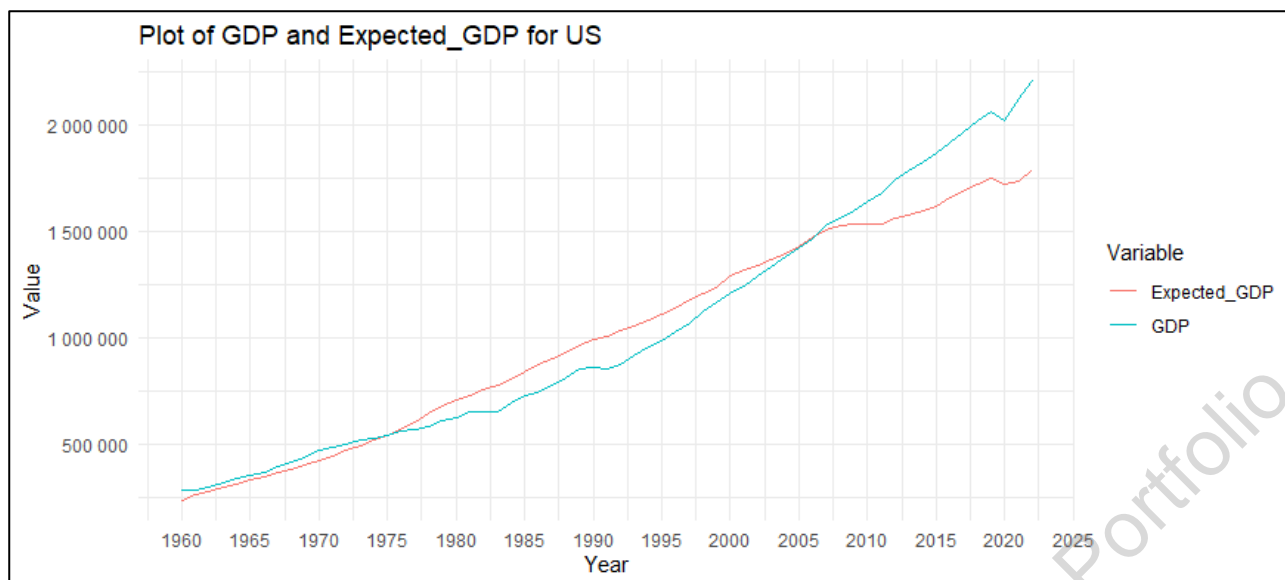
For US, the formula was:

$$\text{GDP} = 0.026315692 \times \text{Capital}^{0.131702388} \times \text{Labor}^{1.383802155}$$

The graphs below show the GDP, Expected GDP and Output Gap for Australia.



The graphs below show the GDP, Expected GDP and Output Gap for the United States.



The next step was to conduct linear models to evaluate the impact of macroeconomic factors on the output gap for US, Australia and both countries combined. These results are presented in the next section.

Linear Mixed Models for Output Gap

In the analysis of economic indicators using linear mixed models, adjustments were made for autocorrelation and non-homoscedasticity through the application of Robust Standard Errors (HC1). The assumptions pertaining to the model residuals were evaluated and confirmed via quantile-quantile plots, ensuring the reliability of the statistical inferences drawn from the models.

The results for the United States indicate significant relationships for certain variables. The log of Capital Stock (in billions of USD) showed a positive and statistically significant association with the dependent variable, with an estimated coefficient of 4.861 (p-value = 0.011). This suggests that an increase in capital stock is associated with a substantial increase in the response variable. Conversely, the log of Labour Force (in thousands) exhibited a significant negative relationship, with a coefficient of -18.468 (p-value = 0.016), indicating that increases in the labor force are associated with a decrease in the response variable. The Pandemic Period variable, introduced to control for the effects of the pandemic from 2020 onwards, was not found to be statistically significant (p-value = 0.424).

term	estimate	std.error	statistic	p.value
log_Capital_Stock_N_Bn_USD	4.861	1.246	3.900	0.011
log_Exports_N_Bn_USD	2.476	1.442	1.717	0.147
log_Imports_N_Bn_USD	-2.210	1.468	-1.506	0.192
log_Labour_Force_Thousands	-18.468	5.133	-3.598	0.016
log_Unemp_Rate_Percent	-0.110	0.340	-0.325	0.758
log_CPI_percentage	0.107	0.080	1.338	0.238
Pandemic_Period	-0.281	0.323	-0.871	0.424

The table below shows the results for Australia.

term	estimate	std.error	statistic	p.value
log_Capital_Stock_N_Bn_USD	5.081	18.062	0.281	0.782
log_Exports_N_Bn_USD	13.263	4.613	2.875	0.011
log_Imports_N_Bn_USD	-3.331	4.849	-0.687	0.501
log_Labour_Force_Thousands	-75.434	43.250	-1.744	0.099
log_Unemp_Rate_Percent	16.517	15.048	1.098	0.288
log_CPI_percentage	11.894	7.328	1.623	0.123

For Australia, the analysis revealed that the log of Exports (in billions of USD) was positively and significantly associated with the response variable, as evidenced by an estimated coefficient of 13.263 (p-value = 0.011). This finding highlights the positive impact of exports on the outcome of interest. However, other variables, including the log of Capital Stock and the log of Labour Force, did not exhibit significant associations, as indicated by their high p-values (0.782 and 0.099, respectively).

The absence of the pandemic variable from the Australian model, due to data constraints post-2018, is a notable distinction from the US model.

Lastly, the results for both countries combined are shown below.

term	estimate	std.error	statistic	p.value
log_Capital_Stock_N_Bn_USD	4.311	8.344	0.517	0.609
log_Exports_N_Bn_USD	10.643	3.624	2.937	0.007
log_Imports_N_Bn_USD	-4.890	3.921	-1.247	0.223
log_Labour_Force_Thousands	-29.522	14.766	-1.999	0.055
log_Unemp_Rate_Percent	0.704	2.930	0.240	0.812
log_CPI_percentage	-0.068	1.131	-0.060	0.953
Pandemic_Period	-0.035	2.346	-0.015	0.988

Finally, the combined model for both countries revealed a significant positive association for the log of Exports, with an estimated coefficient of 10.643 (p-value = 0.007). The log of Labour Force showed a negative relationship, although it approached the threshold of statistical significance with a p-value of 0.055. Other variables, including the log of Capital Stock and the Pandemic Period, did not demonstrate statistically significant relationships.

Difference-in-Difference Analysis

The effect of the pandemic on several dependent variables was evaluated with fixed effect models which controlled for the effect of Country and also evaluated the interaction term of Pandemic x Country, in order to check if the effect of the pandemic was different between US and Australia.

These models were specifically designed to control for country-specific effects and to investigate the interaction between the pandemic period and country indicators. This approach aims to discern if the pandemic's impact differed between the United States and Australia.

The application of Robust Standard Errors in the models is a critical methodological choice, addressing the concern of unobserved heterogeneity which could otherwise bias the estimates. The results of this analysis are both insightful and varied across different economic measures. The results are shown below.

Dependent Variable	term	estimate	std.error	statistic	p.value
log_Unemployed_Thousands	Pandemic_Period	2.147	0.440	4.880	0.000
log_Unemployed_Thousands	Pandemic_Period:Country_Indicator	-1.879	0.619	-3.037	0.003
log_Unemp_Rate_Percent	Pandemic_Period	0.048	0.105	0.461	0.646
log_Unemp_Rate_Percent	Pandemic_Period:Country_Indicator	-0.102	0.148	-0.690	0.492
log_CPI_ex_FE_percentage	Pandemic_Period	-0.204	0.261	-0.780	0.437
log_CPI_ex_FE_percentage	Pandemic_Period:Country_Indicator	0.048	0.369	0.129	0.898
log_GDP	Pandemic_Period	1.922	0.606	3.171	0.002
log_GDP	Pandemic_Period:Country_Indicator	-0.991	0.857	-1.156	0.250
log_Output_Gap	Pandemic_Period	1.665	1.158	1.438	0.156
log_Output_Gap	Pandemic_Period:Country_Indicator	0.189	1.502	0.126	0.900
GDP_Growth	Pandemic_Period	-1.347	4.495	-0.300	0.765
GDP_Growth	Pandemic_Period:Country_Indicator	0.277	6.358	0.044	0.965

For the dependent variable 'log_Unemployed_Thousands', the pandemic period showed a significant positive effect (estimate = 2.147, p-value < 0.001), indicating a substantial increase in unemployment, in thousands, during the pandemic. Interestingly, the interaction term 'Pandemic_Period:Country_Indicator' was also significant (estimate = -1.879, p-value = 0.003), suggesting a differential impact of the pandemic on unemployment between the two countries.

However, for 'log_Unemp_Rate_Percent', the impact of the pandemic, as well as its interaction with the country indicator, were not statistically significant. This suggests that the pandemic's effect on the unemployment rate percentage was not markedly different between the US and Australia, or possibly that the effect was not strong enough to be detected by this model.

In the case of 'log_CPI_ex_FE_percentage', representing the log-transformed Consumer Price Index excluding food and energy, neither the pandemic period nor its interaction with the country indicator

showed significant effects. This outcome might imply that the pandemic had a minimal or indiscernible impact on this specific measure of inflation.

Interestingly, the 'log_GDP' variable exhibited a significant increase during the pandemic period (estimate = 1.922, p-value = 0.002). However, the interaction term did not show a significant difference between the countries, suggesting a somewhat uniform impact of the pandemic on GDP across the two nations.

For 'log_Output_Gap' and 'GDP_Growth', the results did not indicate significant changes due to the pandemic or differing impacts between the US and Australia. The lack of significant findings in these variables could be attributed to various factors, including the resilience of certain economic sectors, policy responses, or insufficient data sensitivity.

Overall, this DiD analysis reveals a nuanced picture of the pandemic's economic impact. While certain variables like unemployment in thousands and GDP showed significant changes, others like the unemployment rate percentage and inflation (CPI excluding food and energy) did not exhibit substantial shifts. The mixed results highlight the complexity of the pandemic's effects, which vary not only across different economic indicators but also potentially between countries.

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