Macroeconomics II

Lesson 04 — Extensions of the Basic RBC Framework

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Introduction

Introduction

- We saw in last lecture that the RBC performs well in some dimensions.

- Far from perfect in matching features of the data.

- More sophisticated DSGE models build upon the basic RBC to match particular features of the data.

- Extensions we will focus on:
  1. Indivisible labor
  2. Investment adjustment costs
  3. Variable capital utilization
  4. Intertemporal non-separability (in the Problem Sets)

Indivisible Labor

Indivisible Labor

- The basic RBC does not generate sufficient volatility in hours of work.
- One possibility is to increase the Frisch elasticity of labor supply $\Rightarrow$ at odds with the data.
- Variation in hours in the basic RBC come from the **intensive margin** only.
- In reality people face the decision to work or not (extensive margin) and how much to work (intensive margin).
- Most of the fluctuations come from the **extensive margin** (King and Rebelo 1999, Figure 4).

Indivisible Labor

- Rogerson (1988) develops a model with a continuum of ex-ante identical agents

- Let household preferences be

$$u(c_t, h_t) = \log(c_t) + \xi \frac{(1 - h_t)^{1-\eta} - 1}{1 - \eta}$$

- Each individual works $\bar{h} \in (0, 1)$ hours. Labor is indivisible, i.e.

$$h_t = \begin{cases} \bar{h} & \text{if the individual works} \\ 0 & \text{otherwise} \end{cases}$$

The decision set is non-convex (it could be optimal for an individual to work $h_t < \bar{h}$ but that is not allowed)

Indivisible Labor

- In each period, there is a probability of working p_t
- Households will choose this probability but not how much can work if it ends up working
- In this way, the **expected hours of work** is $h_t = p_t \bar{h}$
- Assume there is perfect insurance (as in Arrow-Debreu equilibrium) $\Rightarrow$ competitive equilibrium is Pareto Optimal
- The expected flow utility can be written as

$$u(c_t, h_t) = \log(c_t) + p_t \xi \frac{(1 - \bar{h})^{1-\eta} - 1}{1 - \eta} + (1 - p_t) \xi \frac{(1)^{1-\eta} - 1}{1 - \eta}$$

Indivisible Labor

- Collecting terms and noting that $p_t = h_t/\bar{h}$

$$u(c_t, h_t) = \log(c_t) + \frac{h_t}{\bar{h}}\xi\left(\frac{(1-\bar{h})^{1-\eta}-1}{1-\eta} - \frac{(1)^{1-\eta}-1}{1-\eta}\right) + \xi\frac{(1)^{1-\eta}-1}{1-\eta}$$

- For $\eta > 0$ it is satisfied that $\frac{1^{1-\eta}}{1-\eta} > \frac{(1-\bar{h})^{1-\eta}}{1-\eta}$

$$u(c_t, h_t) = \log(c_t) - h_t\underbrace{\frac{\xi}{\bar{h}}\left(\frac{(1)^{1-\eta}-1}{1-\eta} - \frac{(1-\bar{h})^{1-\eta}-1}{1-\eta}\right)}_{B} + \underbrace{\xi\frac{(1)^{1-\eta}-1}{1-\eta}}_{D}$$

- We can drop D since it is just a constant and rewrite it as

$$u(c_t, h_t) = \log(c_t) - Bh_t$$

Indivisible Labor

- In this framework, utility is linear in hours.
- This holds for **any value of η**.
- In this framework, we can think of the choice of hours as **effort** agents put into finding a job.
- Then work 8 fixed hours. If they don't find a job, they earn unemployment benefits.
- The equilibrium is characterized by the exact same equations as in the standard RBC but with $\eta = 0$

Indivisible Labor

- We need to calibrate B, how?

- From the Euler equation we can get a value for the capital-labor ratio in the steady state

$$\frac{k^*}{h^*} = \left(\frac{\alpha}{\frac{(1+g)(1+n)}{\beta} - (1-\delta)} \right)^{\frac{1}{1-\alpha}}$$

- From the intratemporal condition

$$c^* = \frac{1}{B}(1-\alpha)\left(\frac{k^*}{h^*}\right)^{\alpha}$$

- From the resource constraint

$$c^* = h^* \left(\left(\frac{k^*}{h^*}\right)^{\alpha} - \delta \frac{k^*}{h^*} \right)$$

Indivisible Labor

- Combining last two equations

$$B = \frac{(1-\alpha)\left(\frac{k^*}{h^*}\right)^{\alpha}}{h^*\left(A\left(\frac{k^*}{h^*}\right)^{\alpha} - \left(\frac{k^*}{h^*}\right)\left[(1+n)(1+g)-(1-\delta)\right]\right)}$$

- I solve now the basic RBC with log-log preferences and the indivisible labor model

- I target $h^* = 1/3$ then

$$\left(\frac{k^*}{h^*}\right) = 24.07 \qquad B = 2.71$$

Indivisible Labor

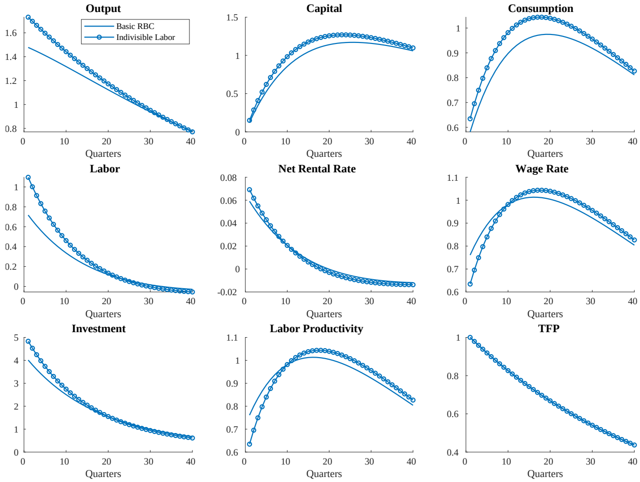

Figure 1: Basic RBC vs Indivisible Labor IRFs

- More amplification in labor than in log-log preferences.
- $\uparrow \{y_t, h_t\}$ more on impact.
- $\Rightarrow \{c_t, i_t\} \uparrow$

Investment Adjustment Costs

Investment Adjustment Costs

- The standard RBC cannot generate hump-shaped responses to most shocks
- This is a feature of the data (VAR literature)
- Convex adjustment costs of capital or investment induce sluggish adjustments in those endogenous variables $\Rightarrow$ effectively increases persistence of the shock.
- We focus on investment adjustment costs as in Christiano et al. (2005)
- Basically, these costs affect the rate of transformation of investment into capital

Investment Adjustment Costs

- The presence of investment adjustment costs affects the law of motion for capital

$$(1+g)(1+n)k_{t+1} = (1-\delta)k_t + \left[1 - \frac{\phi}{2}\left((1+g)\frac{i_t}{i_{t-1}} - (1+g)\right)^2\right] i_t$$

- Note that, in the steady state, $i_t = i_{t-1}$ and the cost vanishes.

- Out of the steady state, with $\phi > 0$ there is a cost of transforming investment into capital.

- $(1+g)$ enters the adjustment costs to guarantee that a BGP exists.

Investment Adjustment Costs

- Note that investment becomes an endogenous variable we cannot remove.
- This implies we will get an additional Euler equation for investment.
- The two Euler equations we will get are (Details in class)

$$q_t = \frac{\beta}{(1+g)(1+n)}\mathbb{E}_t\left\{\frac{c_t}{c_{t+1}}\left(r^k_{t+1} + q_{t+1}(1-\delta)\right)\right\}$$

$$1 = q_t\left[1 - \phi\left((1+g)\frac{i_t}{i_{t-1}} - (1+g)\right)(1+g)\frac{i_t}{i_{t-1}} - \frac{\phi}{2}\left((1+g)\frac{i_t}{i_{t-1}} - (1+g)\right)^2\right]$$

$$+ \beta\mathbb{E}_t\left\{\frac{c_t}{c_{t+1}}q_{t+1}\phi(1+g)\left((1+g)\frac{i_{t+1}}{i_t} - (1+g)\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right\}$$

Investment Adjustment Costs

- We have defined $q_t \equiv \frac{\mu_t}{\lambda_t}$
- Where λ_t is the multiplier associated to the budget constraint and μ_t the multiplier associated to the law of motion for capital.
- q_t denotes how much consumption the agent is willing to give up to get a unit of installed capital.
- This is the relative price of capital in terms of consumption.

Investment Adjustment Costs

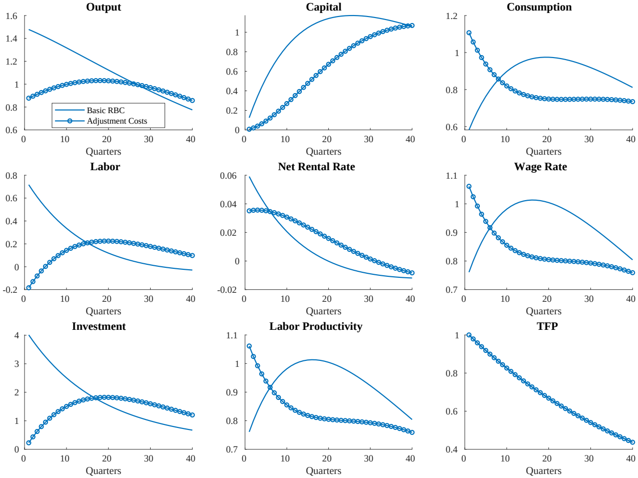

Figure 2: Basic RBC vs Adjustment Costs IRFs

- Smoother response of investment.
- Slower capital accumulation.
- Investment and output are hump-shaped $\Rightarrow$ autocorrelated growth rates.
- Adjustment costs $\Rightarrow$ consumption increases more.

Variable Capital Utilization

Variable Capital Utilization

- Capital is a predetermined variable, however, the **intensity** with which capital is used, can change from one period to the next.

- Think of electricity consumption in manufacturing $\Rightarrow$ procyclical at the business cycle frequencies.

- How does correcting for cyclical variations in capital services affect the statistical properties of estimated aggregate technology shocks? Burnside et al. (1995) say *"a lot"*

Variable Capital Utilization

- To model variable capital utilization, we assume firms need **capital services**

- Capital services are a function of the capital stock and utilization.

- We assume
  1. Households own the capital stock
  2. Households choose the level of utilization
  3. Households lease capital services $\hat{k}_t \equiv u_t k_t$
  4. Capital depreciates with utilization

$$\delta(u_t) = \delta_0 + \phi_1 (u_t - 1) + \frac{\phi_2}{2} (u_t - 1)^2$$

Variable Capital Utilization

Household's Problem

$$\max_{\{c_t, h_t, k_{t+1}, u_t\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t)^{1-\sigma} - 1}{1-\sigma} + \xi \frac{(1-h_t)^{1-\eta} - 1}{1-\eta} \right] \right\}$$

$$\text{subject to} \quad c_t + (1+n)(1+g)k_{t+1} = w_t h_t + \left(r_t^k u_t + (1 - \delta(u_t)) \right) k_t \ \forall t \geq 0$$

$$k_0 > 0 \text{ given}$$

- We have an additional control variable u_t
- A different net rental rate $r_t \equiv r_t^k u_t - \delta(u_t)$

Variable Capital Utilization

Equilibrium Conditions

$$\lambda_t = c_t^{-\sigma} \tag{VKU1}$$

$$\xi(1-h_t)^{-\eta} = w_t c_t^{-\sigma} \tag{VKU2}$$

$$\lambda_t = \frac{\beta}{(1+n)(1+g)} \mathbb{E}_t \left\{ \lambda_{t+1} \left(r_{t+1}^k u_{t+1} + 1 - \delta(u_{t+1}) \right) \right\} \tag{VKU3}$$

$$r_t^k = \phi_1 + \phi_2(u_t - 1) \tag{VKU4}$$

$$r_t^k = \alpha \frac{y_t}{u_t k_t} \tag{VKU5}$$

Variable Capital Utilization

Calibration

- Parameters in the utilization cost function are not fully free.
- Normalize $u = 1$ in the steady state $\Rightarrow r_t^k = \frac{(1+g)(1+n)}{\beta} - 1 + \delta_0$
- But FOC for utilization implies $\phi_1 = \frac{(1+g)(1+n)}{\beta} - 1 + \delta_0$
- Parameter ϕ_2 is a free parameter so we need a target. Typically, target

$$\varepsilon_{\delta',u_t} = \frac{u\delta''(1)}{\delta'(1)} = \frac{\phi_2}{\phi_1}$$

Evidence points to $\varepsilon_{\delta',u_t} = 1$.

Variable Capital Utilization

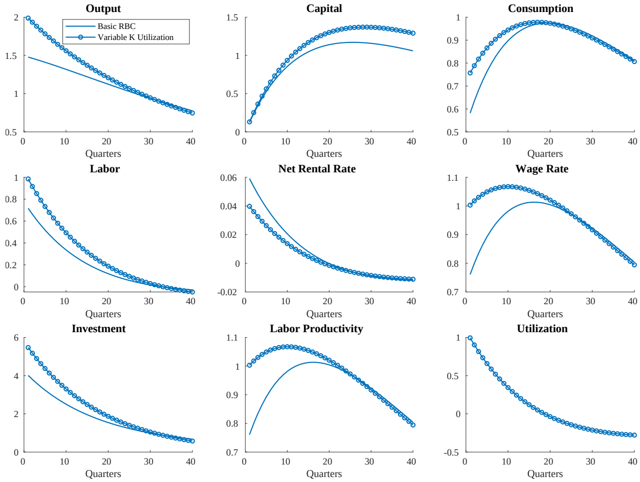

Figure 3: Basic RBC vs Variable Capital Utilization IRFs

- Utilization increases after positive TFP shock.
- Significant amplification of the shock.
- Output, employment, consumption, and investment increase more.