

# Macroeconomics II

Lesson 01 — Preliminaries. Stylized Facts and Measurement of Business Cycles

Rafael Serrano-Quintero

Department of Economics  
University of Barcelona

# Preliminaries

## The Course

- Macroeconomics II
- Check the [syllabus](#)
- Instructors: [Alessandro Di Nola](#) and [Rafa Serrano-Quintero](#).

## Me

- [rafael.serrano@ub.edu](mailto:rafael.serrano@ub.edu)
- Office hours: Send me an email and we can arrange a meeting.

# Syllabus Highlights

## What you have to do

1. Three problem sets
2. Midterm exam. 50% of the grade of the final exam. Other 50% will be Alessandro's part.

## Materials

- In class notes and slides. Most of the work will be in the blackboard.
- Useful references:
  - Cooley (1995)
  - King and Rebelo (1999)
  - Judd (1998)
  - Ljungqvist and Sargent (2004)

## Preliminary Concepts

# Preliminary Concepts

Modern macroeconomics is

- **dynamic:** things change and are related over time.
- **stochastic:** there are random events. Expectations matter.
- We denote  $X_t$  the realization of variable  $X$  at time  $t$ .
- Models are stochastic and people try to guess what will happen in the future. We call that expectations.

## Notation and Expectations

- If we do not know anything about the current state of the system, we call it **unconditional expectation** and express it as  $\mathbb{E}(X_{t+1})$ .
- If we condition on what we know at time  $t$ , we call the **conditional expectation** and we express it as  $\mathbb{E}(X_{t+1}|\Omega_t)$  where  $\Omega_t$  is what we know at time  $t$ . Usually, we shorten it by writing  $\mathbb{E}_t(X_{t+1})$ .
- Note that  $\mathbb{E}_t(X_t) = X_t$  and  $\mathbb{E}_t(X_{t-k}) = X_{t-k}$  for all  $k > 0$ .

### Theorem 1 (Law of Iterated Expectations)

*Let  $Y$  and  $Z$  be two arbitrary random variables, then*

$$\mathbb{E}(Y) = \mathbb{E}(\mathbb{E}(Y|Z))$$

*i.e. the expected value of the conditional expected value of  $Y$  given  $Z$  is the same as the expected value of  $Y$ .*

# Notation and Expectations

- The LIE (Theorem 1) has the following implication

$$\mathbb{E}_t(\mathbb{E}_{t+1}(X_{t+2})) = \mathbb{E}_t(X_{t+2})$$

- Rational expectations (Muth 1961) is a stronger than simple expectations. Two conditions on expectations of future realizations of variables
  1. Correct on average.
  2. Unpredictable forecasting errors given current info.
- This implies the agents *"know the model and use it to act."*
- Does this imply agents do not make mistakes? **No!!**

# Stochastic Processes

## Definition 2 (Markov Property)

A stochastic process has the Markov property if

$$\Pr [X_t = x | X_0, \dots, X_{t-1}] \equiv \Pr [X_t = x | X_{t-1}]$$

i.e. that the conditional probability of future states depends upon the present state only. If you know  $X_{t-1}$  knowing  $X_0, \dots, X_{t-2}$  does not give you extra information.

## Definition 3 (ARMA( $p, q$ ) Process)

An ARMA( $p, q$ ) process can be expressed as

$$X_t = c + \sum_{i=1}^p \rho_i X_{t-i} + \sum_{i=1}^q \theta_i \varepsilon_{t-i}$$

where  $c$  is a constant,  $\rho_i$  are the autorregressive parameters, and  $\theta_i$  are the moving average parameters.



# Stochastic Processes

- ARMA processes can sometimes be approximated with sufficiently long AR processes.
- An  $\text{AR}(p)$  process **does not** strictly have the Markov property. However

$$\begin{pmatrix} s_t \\ s_{t-1} \\ \vdots \\ s_{t-p+1} \end{pmatrix} = \begin{pmatrix} \rho_1 & \rho_2 & \cdots & \rho_p \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} s_{t-1} \\ s_{t-2} \\ \vdots \\ s_{t-p} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

which reduces to

$$\mathbf{s}_t = \Lambda \mathbf{s}_{t-1} + \varepsilon_t$$

# Impulse Response Functions

If we give a shock to some exogenous variable. How do the endogenous variables react?

$$\text{IRF}(h) = \mathbb{E}_t(X_{t+h}) - \mathbb{E}_{t-1}(X_{t+h} | \varepsilon_t = \varepsilon)$$

## Example 4

Take the AR(1) process

$$X_t = \rho X_{t-1} + \varepsilon_t$$

suppose  $\varepsilon \sim \mathcal{N}(0, 1)$  and suppose at time  $t$  there is a shock of  $\varepsilon_t = 1$ . Compute the IRF after  $h$  periods.

# Impulse Response Functions

Take a multivariate process and assume  $\mathbf{X}$  is a  $2 \times 1$  vector

$$\mathbf{X}_t = \mathbf{A}\mathbf{X}_{t-1} + \mathbf{B}\boldsymbol{\varepsilon}_t$$

Assume also that off-diagonals elements of  $\mathbf{A}$  and  $\mathbf{B}$  are not zero. The IRF is now a vector

$$\text{IRF}_1(h) = \mathbf{A}^{h-1}\mathbf{B}_1$$

$$\text{IRF}_2(h) = \mathbf{A}^{h-1}\mathbf{B}_2$$

## Measurement and Stylized Facts of Business Cycles

# Measurement

# Trend and Cycle

- In Macro I you have studied the macroeconomics of the long-run. Why countries grow.
- Now we will focus on booms and busts or expansions and recessions.
- We first need to separate trend and cycle. Then, identify what are expansion and recession periods and define them formally.

# Trend and Cycle

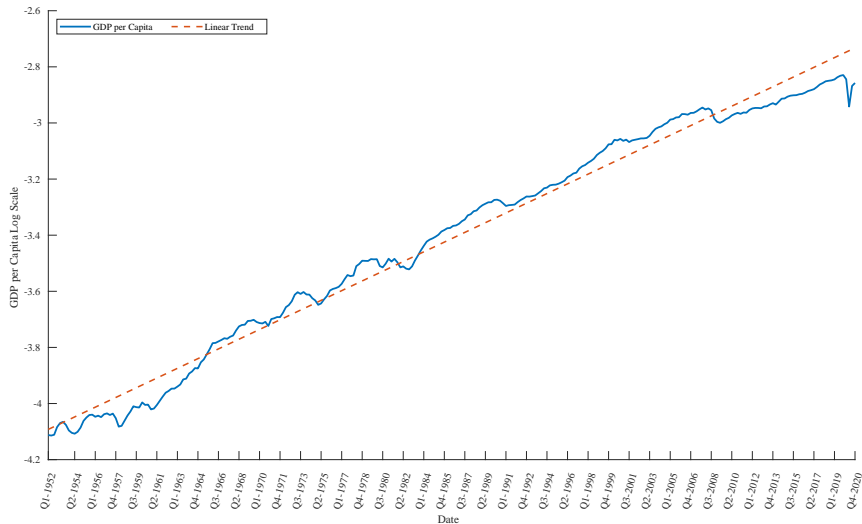


Figure 1: U.S. GDP per capita in Log Scale

# Trend and Cycle

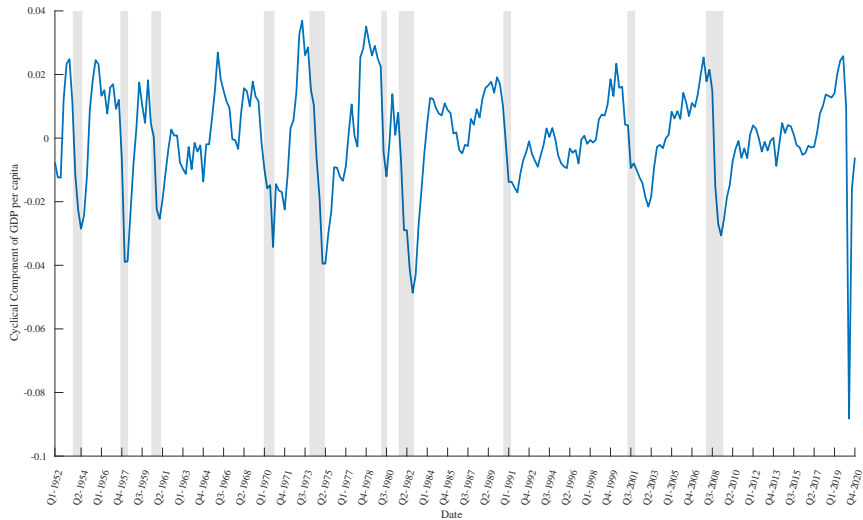


Figure 2: Cycles of U.S. GDP per capita in Log Scale



## Trend and Cycle — Identification

- How can we go from trend to cycle? How do we remove this?
- Let  $y_t$  be GDP per capita and decompose it into a trend and a cycle component.

$$y_t = y_t^c + y_t^g$$

- How can we estimate these two? Several options:
  - Spectral analysis (see [Hamilton 1994](#), Chapter 6)
    - Describe  $y_t$  as a weighted sum of periodic functions like  $\cos(\omega t)$  and  $\sin(\omega t)$ . Identify peaks and troughs and call that a cycle.
  - Detrending methods.
    - Linear detrending, first differences...
    - Filters: band-pass ([Baxter and King 1999](#)), [Hodrick and Prescott \(1997\)](#).

# Hodrick-Prescott Filter

The method separates the trend component from the solution of the problem

$$\min_{\{y_t^g\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (y_t - y_t^g)^2 + \lambda [(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)]^2$$

where  $\lambda$  is a smoothing parameter. The cycle is thus

$$y_t^c = y_t - y_t^g$$

Widely used method but with many critiques. From [Hamilton \(2018\)](#):

1. Introduces spurious dynamic relations w/o basis in the underlying DGP and filtered values at the end are very different from the middle.
2. Choosing  $\lambda$  from a statistical formalization implies  $\neq$  to the typically applied.
3. Not good for prediction, double-sided.

# Hodrick-Prescott Filter

## Alternatives

- **Hamilton (2018)** proposes instead a regression approach.
- How different is the value at date  $t + h$  from the **expected value** at  $t$ ?
- Suggests to estimate by OLS

$$y_{t+h} = \beta_0 + \beta_1 y_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_4 y_{t-3} + v_{t+h}$$

- The residuals

$$\hat{v}_{t+h} = y_{t+h} - \hat{\beta}_0 - \hat{\beta}_1 y_t - \hat{\beta}_2 y_{t-1} - \hat{\beta}_3 y_{t-2} - \hat{\beta}_4 y_{t-3}$$

# Filtering Techniques

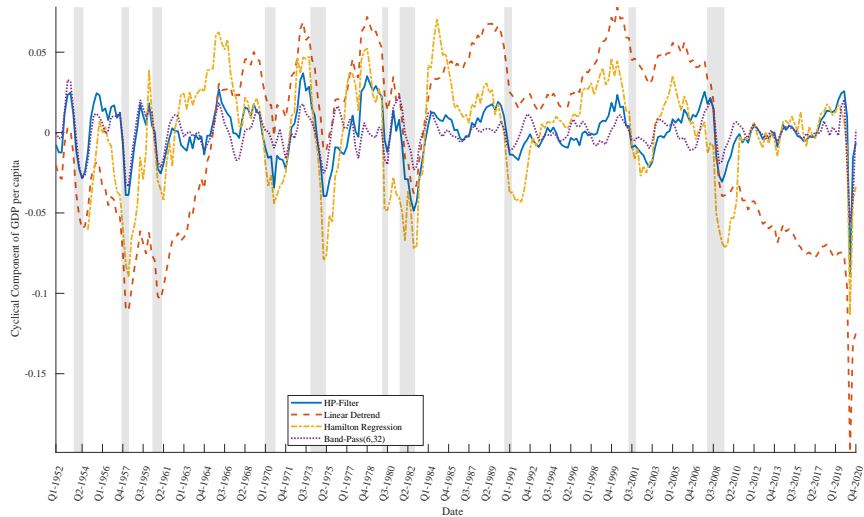


Figure 3: Cycles of U.S. GDP per capita in Log Scale

## Stylized Facts

# Business Cycle Statistics

When we study empirical properties of business cycles we typically look at three moments of the data

1. **Volatility:** Relative standard deviation  $sd(x) / sd(gdp)$ . Amplitude of fluctuations of  $x_t$  relative to GDP.
2. **Cyclicity:** the correlation between  $x_t$  and GDP  $\rho(x_t, GDP)$ 
  - If  $\rho(x_t, GDP) > 0$  then  $x_t$  is **procyclical**.
  - If  $\rho(x_t, GDP) < 0$  then  $x_t$  is **countercyclical**.
  - If  $\rho(x_t, GDP) = 0$  then  $x_t$  is **acyclical**.
3. **Persistence:** autocorrelation  $\rho(x_t, x_{t-1})$

# Business Cycle Statistics

Table 1: Business Cycle Statistics — Sims (2017)

Series	Std. Dev.	Rel. Std. Dev.	Corr $w/y_t$	Autocorr	Corr $w/Y_{t-4}$	Corr $w/Y_{t+4}$
Output	0.017	1.00	1.00	0.85	0.07	0.11
Consumption	0.009	0.53	0.76	0.79	0.07	0.22
Investment	0.047	2.76	0.79	0.87	-0.10	0.26
Hours	0.019	1.12	0.88	0.90	0.29	-0.03
Productivity	0.011	0.65	0.42	0.72	-0.50	0.35
Wage	0.009	0.53	0.10	0.73	-0.10	0.10
1+ Interest Rate	0.004	0.24	0.00	0.42	0.27	-0.25
Price Level	0.009	0.53	-0.13	0.91	0.09	-0.41
TFP	0.012	0.71	0.76	0.75	-0.34	0.34

# Business Cycle Stylized Facts

## Volatility:

- Investment  $\gg$  GDP. Consumption  $\ll$  GDP. Hours  $\approx$  GDP. Prices  $\ll$  GDP.

## Cyclicalities:

- Consumption, investment, hours are **procyclical**.
- Real wages and real interest rate **acyclical**. If used other expectations, real interest rate is slightly **countercyclical**. Modern macro models struggle to match this feature of the data.

**Persistence:** typically, all variables are strongly persistent.