

Problem Set I

Rafael Serrano Quintero *
University of Barcelona

Exercise 1. Simulate an AR(1) process. To do so, construct a function called `my_ar_process` that takes as arguments the initial condition of the AR(1) (y_0), the autoregressive parameter (ρ), the length of the simulation (T), and the variance of the error term (σ^2). Recall an AR(1) takes the form:

$$y_{t+1} = \rho y_t + \varepsilon_t ; \varepsilon \sim \mathcal{N}(0, \sigma^2)$$

The function should return the vector y_t .

Hint: loops might be useful in these cases. Check [randn](#) function to generate random numbers.

1. Test the function with $T = 100$, $\rho = 0.95$, $y_0 = 0$, and $\sigma = 0.5$ and make a plot.
2. Run 20 different simulations and plot them together in a graph. Keep all parameters the same except the initial condition y_0 which should be drawn from a uniform distribution $U(10, 15)$. Can you explain what happens with all the series?

Exercise 2. Read about [recursion](#). Recall the Fibonacci sequence is defined as

$$F_n = F_{n-1} + F_{n-2} \text{ where } F_0 = 0, F_1 = 1$$

1. Write a function `fibonacci` that takes as input an integer $n \geq 1$ and returns the value of F_n .
2. Write a function `fibonacci_recursive` in a recursive way.
3. Using [tic](#) and [toc](#) report the relative time for $n = \{10, 30, 50\}$.

Exercise 3. Create a function `my_polynomial` that evaluates a polynomial of degree n given its coefficients. That is, let a polynomial $p(x)$ be defined as:

$$p(x) = \sum_{i=1}^n a_i x^{i-1}$$

Write a function that takes as inputs a vector of coefficients a_i and a value for x , then compute the value of the polynomial at that point x given the coefficients. Do not use built-in functions such as [polyval](#).

*Department of Economics. Email: r.serrano@ua.es

Exercise 4. Download the series for real GDP per capita in quarterly basis from the Federal Reserve Bank of St. Louis (you can download them from [here](#)). The purpose of this exercise is that you familiarize yourself with extracting data from files and manipulate it in Matlab. Start this exercise in a new script, start by clearing the workspace and the command window.

1. As a zero step, input the data as a variable called Y_t , and take the length of the series as a variable T . Compute also the growth rate of GDP per capita in this step and save it as another variable, for example g_Y .
2. First, suppose we want to just fit a time trend. To do so, suppose the model we have for the evolution of output is:

$$Y_t = e^{\phi_1 t + \phi_2} + \varepsilon_t ; \varepsilon_t \underset{iid}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2) \quad (1)$$

where t is a time trend, ϕ_1 and ϕ_2 are the parameters of interest, and ε is white noise. Your task is to estimate parameters ϕ_1 and ϕ_2 using `lsqcurvefit` or `lsqnonlin`. Explain why you choose those initial values. Plot the data, and the fitted curve.

3. We will fit now the **growth rate** of GDP per capita using an $AR(2)$ specification.¹

$$g_{Y,t} = \alpha + \rho_1 g_{Y,t-1} + \rho_2 g_{Y,t-2} + u_t ; u_t \underset{iid}{\sim} \mathcal{N}(0, \sigma_u^2) \quad (2)$$

Where α is a constant, ρ_1 and ρ_2 are the autoregressive parameters of the model, and u_t is white noise. Estimate this model via OLS. The estimator should be programmed **by yourselves**, do not use built-in functions or other user-defined functions not written by yourselves. Obtain the parameters ρ_1 , ρ_2 , an estimate of $\hat{\sigma}_u^2$, and the variance-covariance matrix of the OLS estimator. Recall that:

$$\mathbb{E} [(\hat{\rho} - \rho)(\hat{\rho} - \rho)'] = \sigma^2 (X'X)^{-1} \quad (3)$$

$$\hat{\sigma}^2 = \frac{\hat{u}'\hat{u}}{n - k} \quad (4)$$

Where $(n - k)$ denotes the degrees of freedom, (3) gives the Variance-Covariance Matrix for the OLS estimator, and (4) is the estimator for the variance of the residuals with k the number of regressors.

4. Plot the predicted values for the OLS estimates and the data for comparison. Clarify which series is which in a legend.
5. Check `arima` and `estimate` (check the first example!) and compare the OLS results you obtained with the ones from Matlab.

¹Please, take into account this is an exercise, this is not a good way to forecast GDP nor almost any economic variable.