Introduction to Matlab — Problem Set I

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Exercise 1. Simulate an AR(1) process. To do so, construct a function called `my_ar_process` that takes as arguments the initial condition of the AR(1) (y_0), the autoregressive parameter (ρ), the length of the simulation (T), and the variance of the error term (σ^2). Recall an AR(1) takes the form:

$$y_{t+1} = \rho y_t + \varepsilon_t \; ; \; \varepsilon \sim \mathcal{N}(0, \sigma^2)$$

The function should return the vector y_t.

Hint: loops might be useful in these cases. Check `randn` *function to generate random numbers.*

1. Test the function with $T = 100$, $\rho = 0.95$, $y_0 = 0$, and $\sigma = 0.5$ and make a plot.

2. Run 20 different simulations and plot them together in a graph. Keep all parameters the same except the initial condition y_0 which should be drawn from a uniform distribution $U(10, 15)$. Can you explain what happens with all the series?

Exercise 2. Create a function `my_polynomial` that evaluates a polynomial of degree n given its coefficients. That is, let a polynomial $p(x)$ be defined as:

$$p(x) = \sum_{i=1}^{n} a_i x^{i-1}$$

Write a function that takes as inputs a vector of coefficients a_i and a value for x, then compute the value of the polynomial at that point x given the coefficients. Do not use built-in functions such as `polyval`.

Exercise 3. Let $f(x) = e^{-sx}$. For this exercise **you cannot use** the functions `taylor`, `pade`, or any other function that uses symbolic calculus.

1. Write a Matlab script that produces a plot of the function $f(x)$ in the interval $x \in [0, 3]$ for $s = \{0.75, 1, 1.25\}$. Evaluate the function in a vector with 1000 points, i.e. `x = linspace(0, 3, 1000)`.

2. Compute the Taylor expansion of order 4 for function $f(x)$ around $x = 0$. For the rest of the exercise, fix the value of s to $s = 1.25$, plot the function $f(x)$ together with its Taylor expansion. Evaluate the Taylor expansion in the same points as in the previous part. *Hint: Set s as a parameter, do not carry the value of s throughout your calculations or the code.*

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3. We are going to approximate now the function using a Padé approximant. Padé approximations are rational approximations.[1] The Padé approximation of $f(x)$ around 0 of order $[2,2]$ is given by

$$R_2^2(x) = \frac{a_0 + a_1 x + a_2 x^2}{1 + b_1 x + b_2 x^2}$$

where the coefficients $\{a_0, a_1, a_2, b_1, b_2\}$ are given by

$$a_0 = 1, a_1 = \frac{s}{2}, a_2 = -\frac{5s^2}{12}, b_1 = \frac{3s}{2}, b_2 = \frac{7s^2}{12}$$

In the same figure, plot $f(x)$, its Taylor expansion around 0 of order 2, and its Padé approximation of order $[2,2]$.

4. Which method approximates better $f(x)$ around $x = 0$? To answer, compute the average absolute error for both approximations over the interval $[0,2]$. Why do you think this is? An informed guess is fine, I do not require any formal proof.

[1] I will not ask anything about computing the Padé approximation but it is a relatively interesting exercise for simple functions. Check the Wikipedia if you're interested.