Introduction to Matlab — Problem Set II

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Exercise 1. Following the example for the computation of the Arrow-Debreu equilibrium, assume now that the utility functions are

$$u_A(x_{1,A}, x_{1,A}) = \left[\alpha x_{1,A}^{\rho} + (1-\alpha) x_{2,A}^{\rho}\right]^{\frac{1}{\rho}} \qquad u_A(x_{1,B}, x_{2,B}) = \left[\beta x_{1,B}^{\rho} + (1-\beta) x_{2,B}^{\rho}\right]^{\frac{1}{\rho}}$$

1. Compute analytically the optimal demands and the excess demand functions.
2. Create a function in Matlab that gives the excess demand for good 1.
3. Using `fzero`, find the equilibrium price for $\alpha = 0.25, \beta = 0.75, \rho = 0.25, \omega_{1,A} = 10, \omega_{2,A} = 15, \omega_{1,B} = 15$, and $\omega_{2,B} = 10$. Verify that the excess demand is, in fact, 0 or close to 0.
4. Keeping all the parameters the same, how does the solution change if $\rho = -1.5$?

Exercise 2. Suppose a consumer has \$100 to spend on two goods c_1 and c_2. The price of c_1 is \$200 per kilo while the price of c_2 is \$300 per kilo. Her utility function is given by

$$U(c_1, c_2) = \sqrt{c_1} + 2\sqrt{c_2}$$

Assume the consumer spends all her income in the two goods.

1. State this maximization problem of two variables as a **minimization problem in one variable**. Hint: Define the proportion spent on c_1 as ϕ and make a change of variables.
2. Solve this problem by hand and find the analytical solution for ϕ.
3. Solve this problem by finding the solution of the first order conditions using `fzero`.
4. Solve this problem as an optimization problem using `fminbnd`.
5. Solve this problem as an optimization problem using `fminunc`. How does the solver perform under different initial guesses?

Exercise 3. The CES production function defined as

$$F(K, L) = \left(\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

has a constant elasticity of substitution σ and nests three particular cases.

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- If $\sigma \to \infty$ the function becomes linear (perfect substitutes)

$$F(K, L) = \alpha K + (1 - \alpha) L \tag{2}$$

- If $\sigma \to 1$ the function becomes Cobb-Douglas

$$F(K, L) = K^{\alpha} L^{1-\alpha} \tag{3}$$

- If $\sigma \to 0$ the function becomes Leontief (perfect complements)

$$F(K, L) = \min\{K, L\} \tag{4}$$

1. Write a function that takes four inputs, the elasticity σ, the distribution parameter α, a vector of values for K and a vector of values for L. The function should distinguish the four different cases (1), (2), (3), (4). Show in a `subplot` the four different cases with a $3-$D figure. Hint: You might want to check `surf` command.

2. Show numerically (preferably with a plot) that as σ goes to $\{0, 1, \infty\}$ the CES approaches each case. Hint: it might be easier to rewrite the function in intensive form, i.e. divide by L and define $k = K/L$. Recall that the CES is homogeneous of degree 1.

1 Optional Exercises

The following exercises are not mandatory. However, extra credit will be granted for those who attempt to solve them. They are a good practice to go a bit deeper into the numerical analysis parts.

Exercise 4. (Optional) Given

$$f(x) = x^{1/3} e^{-x^2}$$

1. Show theoretically that if we start from an initial guess x_0 close to x^* where $f(x^*) = 0$, Newton-Raphson method diverges.

2. Show in the computer that, with each iteration, the method in fact diverges. Note that $\lim_{x \to +\infty} f(x) = 0$ but this is an asymptote!

Exercise 5. (Optional) Write a function that computes the trapezoid rule for integration. The function should take as inputs the lower and upper bounds of the interval a and b, the number of points on which to approximate the function n, and a function handle.

1. Use your function to compute

$$\int_1^5 x^2 dx \text{ and } \int_{23}^{74} \log(x) dx$$

with $n = \{1, 2, \ldots, 35\}$ and report the relative errors in $\log_{10}$ units as a function of n. If the true value of the integral is v and your estimate is $\hat{v}$, the relative error should be

$$\xi = \log_{10} \left(\frac{|v - \hat{v}|}{|\hat{v}|} \right)$$

2. Compare your results with the function `trapz` from Matlab in terms of errors. Report a plot of the errors ξ as a function of n for your function and Matlab's and compute $||\xi^M - \xi||$ where ξ^M are the errors from `trapz`.

Exercise 6. (Optional) Given the function

$$f(x) = x^2 - \log\left(\frac{x^2}{3}\right)$$

1. Find a minimum of the function using the bracketing method. State clearly which initial triplet (a, b, c) you start with and what solution you find.

2. Find a minimum of the function using `fminsearch` with initial condition $x_0 = \pm 2$.

3. Compare the two procedures.