

# Introduction to Matlab — Problem Set II

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**Exercise 1.** Following the example for the computation of the Arrow-Debreu equilibrium, assume now that the utility functions are

$$u_A(x_{1,A}, x_{2,A}) = \left[ \alpha x_{1,A}^\rho + (1 - \alpha) x_{2,A}^\rho \right]^{\frac{1}{\rho}} \quad u_B(x_{1,B}, x_{2,B}) = \left[ \beta x_{1,B}^\rho + (1 - \beta) x_{2,B}^\rho \right]^{\frac{1}{\rho}}$$

1. Compute analytically the optimal demands and the excess demand functions.
2. Create a function in Matlab that gives the excess demand for good 1.
3. Using `fzero` find the equilibrium price for  $\alpha = 0.25, \beta = 0.75, \rho = 0.25, \omega_{1,A} = 10, \omega_{2,A} = 15, \omega_{1,B} = 15$ , and  $\omega_{2,B} = 10$ . Verify that the excess demand is, in fact, 0 or close to 0.
4. Keeping all the parameters the same, how does the solution change if  $\rho = 1.5$ ?

**Exercise 2.** Suppose a consumer has \$100 to spend on two goods  $c_1$  and  $c_2$ . The price of  $c_1$  is \$200 per kilo while the price of  $c_2$  is \$300 per kilo. Her utility function is given by

$$U(c_1, c_2) = \sqrt{c_1} + 2\sqrt{c_2}$$

Assume the consumer spends all her income in the two goods.

1. State this maximization problem of two variables as a **minimization problem in one variable**. Hint: Define the proportion spent on  $c_1$  as  $\phi$  and make a change of variables.
2. Solve this problem by hand and find the analytical solution for  $\phi$ .
3. Solve this problem by finding the solution of the first order conditions using `fzero`.
4. Solve this problem as an optimization problem using `fmincon`.
5. Compare the results of both methods with the true solution, the number of iterations needed, and the tolerance set.

**Exercise 3.** The CES production function defined as

$$F(K, L) = \left( \alpha K^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) L^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (1)$$

has a constant elasticity of substitution  $\sigma$  and nests three particular cases.

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- If  $\sigma \rightarrow \infty$  the function becomes linear (perfect substitutes)

$$F(K, L) = \alpha K + (1 - \alpha)L \quad (2)$$

- If  $\sigma \rightarrow 1$  the function becomes Cobb-Douglas

$$F(K, L) = K^\alpha L^{1-\alpha} \quad (3)$$

- If  $\sigma \rightarrow 0$  the function becomes Leontief (perfect complements)

$$F(K, L) = \min\{K, L\} \quad (4)$$

1. Write a function that takes four inputs, the elasticity  $\sigma$ , the distribution parameter  $\alpha$ , a vector of values for  $K$  and a vector of values for  $L$ . The function should distinguish the four different cases (1), (2), (3), (4). Show in a subplot the four different cases with a 3-D figure. Hint: You might want to check `surf` command.
2. Show numerically (preferably with a plot) that as  $\sigma$  goes to  $\{0, 1, \infty\}$  the CES approaches each case. Hint: it might be easier to rewrite the function in intensive form, i.e. divide by  $L$  and define  $k = K/L$ . Recall that the CES is homogeneous of degree 1.

# 1 Optional Exercises

The following exercises are not mandatory. However, extra credit will be granted for those who attempt to solve them.

**Exercise 4. (Optional)** Given

$$f(x) = x^{1/3}e^{-x^2}$$

1. Show theoretically that if we start from an initial guess  $x_0$  close to  $x^*$  where  $f(x^*) = 0$ , Newton-Raphson method diverges.
2. Show in the computer that, with each iteration, the method in fact diverges. Note that  $\lim_{x \rightarrow +\infty} f(x) = 0$  but this is an asymptote!

**Exercise 5. (Optional)** Write a function that computes the trapezoid rule for integration. The function should take as inputs the lower and upper bounds of the interval  $a$  and  $b$ , the number of points on which to approximate the function  $n$ , and a function handle.

1. Use your function to compute

$$\int_1^5 x^2 dx \text{ and } \int_{23}^{74} \log(x) dx$$

with  $n = \{1, 2, \dots, 35\}$  and report the relative errors in  $\log_{10}$  units as a function of  $n$ . If the true value of the integral is  $v$  and your estimate is  $\hat{v}$ , the relative error should be

$$\xi = \log_{10} \left( \frac{|v - \hat{v}|}{|\hat{v}|} \right)$$

2. Compare your results with the function `trapz` from Matlab in terms of errors. Report a plot of the errors  $\xi$  as a function of  $n$  for your function and Matlab's and compute  $\|\xi^M - \xi\|$  where  $\xi^M$  are the errors from `trapz`.

**Exercise 6. (Optional)** Given the function

$$f(x) = x^2 - \log \left( \frac{x^2}{3} \right)$$

1. Find a minimum of the function using the bracketing method. State clearly which initial triplet  $(a, b, c)$  you start with and what solution you find.
2. Find a minimum of the function using `fminsearch` with initial condition  $x_0 = \pm 2$ .
3. Compare the two procedures.