Introduction to Matlab — Problem Set I

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Exercise 1. Simulate an AR(1) process. To do so, construct a function called `my_ar_process` that takes as arguments the initial condition of the AR(1) (y_0), the autoregressive parameter (ρ), the length of the simulation (T), and the variance of the error term (σ^2). Recall an AR(1) takes the form:

$$y_{t+1} = \rho y_t + \varepsilon_t \,;\, \varepsilon \sim \mathcal{N}(0, \sigma^2)$$

The function should return the vector y_t.

Hint: loops might be useful in these cases. Check `randn` *function to generate random numbers.*

1. Test the function with $T = 100$, $\rho = 0.95$, $y_0 = 0$, and $\sigma = 0.5$ and make a plot.
2. Run 20 different simulations and plot them together in a graph. Keep all parameters the same except the initial condition y_0 which should be drawn from a uniform distribution $U(10, 15)$. Can you explain what happens with all the series?

Exercise 2. Create a function `my_polynomial` that evaluates a polynomial of degree n given its coefficients. That is, let a polynomial $p(x)$ be defined as:

$$p(x) = \sum_{i=1}^{n} a_i x^{i-1}$$

Write a function that takes as inputs a vector of coefficients a_i and a value for x, then compute the value of the polynomial at that point x given the coefficients. Do not use built-in functions such as `polyval`.

Exercise 3. Download the series for real GDP per capita in quarterly basis from the Federal Reserve Bank of St. Louis (you can download them from here). The purpose of this exercise is that you familiarize yourself with extracting data from files and manipulate it in Matlab. Start this exercise in a new script, start by clearing the workspace and the command window.

1. As a zero step, input the data as a variable called Y_t (in Matlab, `Yt`), and take the length of the series as a variable T (`T`). Compute also the growth rate of GDP per capita in this step and save it as another variable, for example g_Y (`gY`). Compute the growth rate as the difference in logs.

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2. First, suppose we want to just fit a time trend. To do so, suppose the model we have for the evolution of output is:

$$Y_t = e^{\phi_1 t + \phi_2} + \varepsilon_t \ ; \ \varepsilon_t \underset{iid}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2) \tag{1}$$

where t is a time trend, ϕ_1 and ϕ_2 are the parameters of interest, and ε is white noise. Your task is to estimate parameters ϕ_1 and ϕ_2 using `lsqcurvefit` or `lsqnonlin`. Explain why you choose those initial values. Plot the data, and the fitted curve.

3. We will fit now the **growth rate** of GDP per capita using an $AR(2)$ specification.[1]

$$g_{Y,t} = \alpha + \rho_1 g_{Y,t-1} + \rho_2 g_{Y,t-2} + u_t \ ; \ u_t \underset{iid}{\sim} \mathcal{N}(0, \sigma_u^2) \tag{2}$$

Where α is a constant, ρ_1 and ρ_2 are the autoregressive parameters of the model, and u_t is white noise. Estimate this model via OLS. The estimator should be programmed **by yourselves**, do not use built-in functions or other user-defined functions not written by yoursleves. Obtain the parameters ρ_1, ρ_2, an estimate of $\hat{\sigma}_u^2$, and the variance-covariance matrix of the OLS estimator. Recall that:

$$\mathbb{E}\left[(\hat{\rho} - \rho)(\hat{\rho} - \rho)'\right] = \sigma^2 \left(X'X\right)^{-1} \tag{3}$$

$$\hat{\sigma}^2 = \frac{\hat{u}'\hat{u}}{n-k} \tag{4}$$

Where $(n-k)$ denotes the degrees of freedom, (3) gives the Variance-Covariance Matrix for the OLS estimator, and (4) is the estimator for the variance of the residuals with k the number of regressors.

4. Plot the predicted values for the OLS estimates and the data for comparison. Clarify which series is which in a legend.

5. Check `arima` and `estimate` (check the first example!) and compare the OLS results you obtained with the ones from Matlab.

[1] Please, take into account this is an exercise, this is not a good way to forecast GDP nor almost any economic variable.