Introduction to Matlab — Problem Set I

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Exercise 1. Simulate an AR(1) process. To do so, construct a function called `my_ar_process` that takes as arguments the initial condition of the AR(1) (y_0), the autoregressive parameter (ρ), the length of the simulation (T), and the variance of the error term (σ^2). Recall an AR(1) takes the form:

$$y_{t+1} = \rho y_t + \varepsilon_t \; ; \; \varepsilon \sim \mathcal{N}(0, \sigma^2)$$

The function should return the vector y_t.
Hint: loops might be useful in these cases. Check `randn` function to generate random numbers.

1. Test the function with $T = 100$, $\rho = 0.95$, $y_0 = 0$, and $\sigma = 0.5$ and make a plot.

2. Run 20 different simulations and plot them together in a graph. Keep all parameters the same except the initial condition y_0 which should be drawn from a uniform distribution $U(10, 15)$. Can you explain what happens with all the series?

Exercise 2. Create a function `my_polynomial` that evaluates a polynomial of degree n given its coefficients. That is, let a polynomial $p(x)$ be defined as:

$$p(x) = \sum_{i=1}^{n} a_i x^{i-1}$$

Write a function that takes as inputs a vector of coefficients a_i and a value for x, then compute the value of the polynomial at that point x given the coefficients. Do not use built-in functions such as `polyval`.

Exercise 3. We are going to explore ways to approximate functions. Given the function

$$f(x) = e^{-sx} \tag{1}$$

where $s \in \mathbb{R}$ is a given constant. Let us denote the Taylor expansion of order n of a function around x_0 as $T_n(x_0)$. The Taylor expansion is computed as

$$T_n(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \frac{f^{iv}(x_0)}{4!}(x - x_0)^4 + \cdots$$

where f' denotes the first derivative, f'' the second...

Throughout this exercise, you are not allowed to use the functions `taylor`, `pade` or any other function that uses symbolic calculus.

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1. Write a Matlab function that approximates function (1) using a Taylor expansion of order 4 around $x = 0$. The function should take as inputs an interval (a, b) where it is approximated, and the value of the constant s. Test it with several values of the constant s. Compute the error of the approximation in the extremes of the interval for $a = 0, b = 3$. The Taylor expansion of order 4 around $x = 0$ for $f(x)$ is given by

$$T_4(x) = 1 - sx + \frac{s^2}{2}x^2 - \frac{s^3}{3!}x^3 + \frac{s^4}{4!}x^4.$$

2. We are going to approximate now the function using a Padé approximant. Padé approximations are rational approximations. We are going to simplify our lives and consider an approximation of our function (1) of order $(2, 2)$ around $x = 0$ which corresponds to

$$R_2^2(x) = \frac{a_0 + a_1 x + a_2 x^2}{1 + b_1 x + b_2 x^2}$$

where

$$a_0 = 1, a_1 = \frac{s}{2}, a_2 = -\frac{5s^2}{12}, b_1 = \frac{3s}{2}, b_2 = \frac{7s^2}{12}$$

Write a Matlab function that takes as input the value of s and the interval around we want to approximate. Remember this expression approximates around $x = 0$. Plot the original function together with the Padé approximation, and the Taylor approximation in the interval $[0, 3]$.

3. Which method approximates better $f(x)$ around $x = 0$? To answer, compute the average absolute error for both approximations over the interval $[0, 3]$. Why do you think this is?