Problem Set II

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Exercise 1. Given

$$f(x) = x^{1/3}e^{-x^2}$$

1. Show theoretically that if we start from an initial guess x_0 close to x^* where $f(x^*) = 0$, Newton-Raphson method diverges.

2. Show in the computer that, with each iteration, the method in fact diverges. Note that $\lim_{x\to+\infty} f(x) = 0$ but this is an asymptote!

Exercise 2. Write a function that computes the trapezoid rule for integration. The function should take as inputs the lower and upper bounds of the interval a and b, the number of points on which to approximate the function n, and a function handle.

1. Use your function to compute

$$\int_1^5 x^2 dx \text{ and } \int_{23}^{74} \log(x)dx$$

with $n = \{1, 2, \ldots, 35\}$ and report the relative errors in $\log_{10}$ units as a function of n. If the true value of the integral is v and your estimate is $\hat{v}$, the relative error should be

$$\xi = \log_{10}\left(\frac{|v - \hat{v}|}{|\hat{v}|}\right)$$

2. Compare your results with the function `trapz` from Matlab in terms of errors. Report a plot of the errors ξ as a function of n for your function and Matlab's and compute $||\xi^M - \xi||$ where ξ^M are the errors from `trapz`.

Exercise 3. Given the function

$$f(x) = x^2 - \log\left(\frac{x^2}{3}\right)$$

1. Find a minimum of the function using the bracketing method. State clearly which initial triplet (a, b, c) you start with and what solution you find.

2. Find a minimum of the function using `fminsearch` with initial condition $x_0 = \pm 1$.

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3. Compare the two procedures.

Exercise 4. Suppose a consumer has \$100 to spend on two goods c_1 and c_2. The price of c_1 is \$200 per kilo while the price of c_2 is \$300 per kilo. Her utility function is given by

$$U(c_1, c_2) = \sqrt{c_1} + 2\sqrt{c_2}$$

Assume the consumer spends all her income in the two goods.

1. State this maximization problem of two variables as a **minimization problem in one variable**. Hint: Define the proportion spent on c_1 as ϕ and make a change of variables.

2. Solve this problem by hand.

3. Solve this problem as a problem in one variable using the bracketing method. How can you find the initial triplet (a, b, c) **without** the routine explained in class?

4. Solve this problem as a problem in one variable using the Newton-Raphson method. Give a reasoning for your choice of the initial guess.

5. Compare the results of both methods with the true solution, the number of iterations needed, and the tolerance set.

Exercise 5. The CES production function defined as

$$F(K, L) = \left(\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

has a constant elasticity of substitution σ and nests three particular cases.

- If $\sigma \to \infty$ the function becomes linear (perfect substitutes)

$$F(K, L) = \alpha K + (1-\alpha)L \tag{2}$$

- If $\sigma \to 1$ the function becomes Cobb-Douglas

$$F(K, L) = K^{\alpha} L^{1-\alpha} \tag{3}$$

- If $\sigma \to 0$ the function becomes Leontief (perfect complements)

$$F(K, L) = \min\{\alpha K, (1-\alpha)L\} \tag{4}$$

1. Write a function that takes three inputs, the elasticity σ, a vector of values for K and a vector of values for L. The function should distinguish the three four different cases (1), (2), (3), (4).

2. Show numerically (preferably with a plot) that as σ goes to $\{0, 1, \infty\}$ the CES approaches each case. *Hint: it might be easier to rewrite the function in intensive form, i.e. divide by L and define $k = K/L$.*