Random Variables and Statistical Inferences

Session-4

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Agenda

Lecture -4: Random Variables and Statistical Inferences

- Use Python to apply fundamental concepts of probability theory to examine random variables and quantify probability
- Use Python to draw a random sample
- Understand Probability Density and Mass Functions (PDF and PMF)

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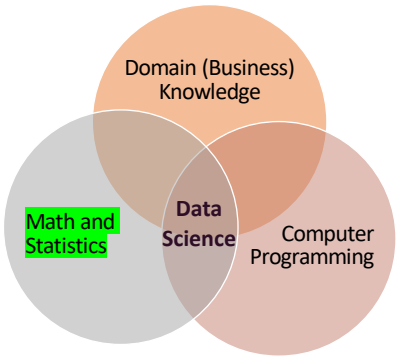
Where We Are Now

- For Data Science, you need
  - Business understanding,
  - Programming skills, and
  - Math and Statistics skills
- Today and in the next lecture we will focus on Statistics
- Python SciPy library is commonly used for statistics



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Introduction to SciPy

- SciPy stands for Scientific Python.
- It's an optimized computation library that uses Numpy underneath.
- The same NumPy's creator Travis Olliphant created SciPy.
- It has useful utility functions that are frequently used in Data Science.
- SciPy is predominantly written in Python, but a few segments are written in C.

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Statistical Significance Test With SciPy

- In Statistics, "Statistical Significance" means that the result that was produced can be explained with a reason behind it, it was not produced randomly, or by chance.
- SciPy provides us with a module called scipy.stats, which has functions for performing statistical significance tests.

Statistics and Probability

Probability Vs. Statistics

Statistics: Analyzing the frequency of **past** events

Probability: Predicting the likelihood of **future** events

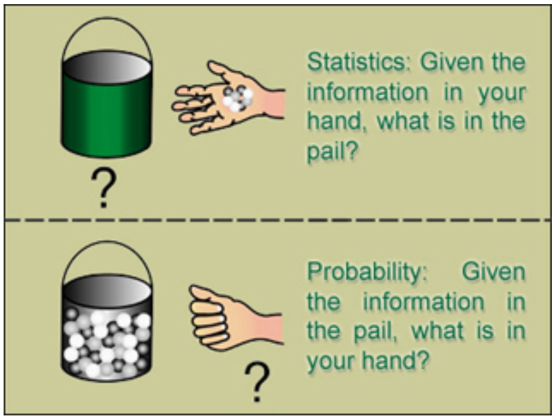

Diagram showing the difference between statistics and probability. (Image by MIT OpenCourseWare. Based on Gilbert, Norma. *Statistics.* W.B. Saunders Co., 1976.)

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Statistics in Python

Think about a coin toss:

- The **sample space** contains **two events**:
  1. Flipping a heads, and
  2. Flipping a tails
- So, the probability of getting a head is one-half (½) or 50% as tossing a **fair coin** will have a 1-in-2 chance of being heads or tails.
- However, if we perform the experiment of flipping a coin, we may get two or three heads or tails consecutively which may look different from 50% probability.
- So, we need to repeat an experiment many times to get useful **statistics** to calculate probabilities that may match with the expected or **ideal** probabilities

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Statistics in Python (Contd.)

- We can use Python **Random()** function to do this experimentation of coin flip
- The built-in function random() generates a random fraction number between 0 an 1 with equal probability of each number.
- As the 0.5 is the midpoint of 0 and 1, the numbers between 0 and 0.5 has a total of 50% probability, and, we can take this as the probability of getting a head of a coin toss.

```
import random
```

```
random.random() # Generates a number between 0 (inclusive) and 1 (exclusive)
```

Out[31]: 0.7029259239462168

Statistical Experiment in Python

[13]:
```
# A function to count the number of heads in 100 flips
def coin_toss():
    heads = 0
    for i in range(100):
        if random.random() < 0.5:
            heads +=1
    return heads
```

[14]:
```
# The expected value is 50 but a single run may not provide the value
coin_toss()
```

[14]: 51

Statistical Experiment in Python

```
def simulate(n):
    heads_in_trials = []
    for i in range(n):
        heads_in_trials.append(coin_toss())
    return(sum(heads_in_trials)/n)
```

```
# 10, 100, 1000, 10000, 100000 coin flips - what
is the average %
print("10 simulations: ", simulate(10))
print("100 simulations: ", simulate(100))
print("1000 simulations: ", simulate(1000))
print("10000 simulations: ", simulate(10000))
print("100000 simulations: ", simulate(100000))
```

As sample size increases, we get results closer to the ideal probability

Out[]:

10 simulations: 49.5

100 simulations: 49.77

1000 simulations: 49.839

10000 simulations: 50.0343

100000 simulations: 50.01029

Probability in Python

Let's use the coin toss example again:

- Tossing a coin has only two outcomes - a head or a tail
- The **binomial distribution** model deals with finding the probability of success of an event which has only two possible outcomes in a series of experiments. This model is useful to find the probability of exactly H # of heads when tossing a coin repeatedly for n times.
- In Python, we can use **binom** module from **scipy stats** package to generate the binomial distribution

```
from scipy.stats import binom
```

- **numpy random.binomial** module is also useful to draw samples from a binomial distribution

```
>>> n, p = 10, .5  # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

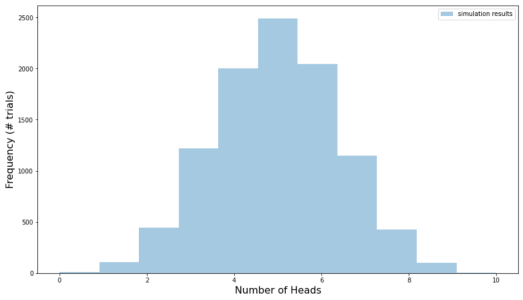
Probability in Python (Cont.)

Before using binom module, let's see how this can be done just with Python code

```
# Import libraries
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns

# Function that runs our coin toss trials
# heads is a list of the number of successes from each trial of # tosses
def run_simulated_coin_toss(trials, num_tosses):
    heads = []
    for i in range(trials):
        tosses = [np.random.random() for i in range(num_tosses)]
        heads.append(len([i for i in tosses if i>=0.50]))
    return heads
```

```
# Let's run this with high number of trials
heads = run_simulated_coin_toss(10000, 10)
# Plot the results as a histogram
```



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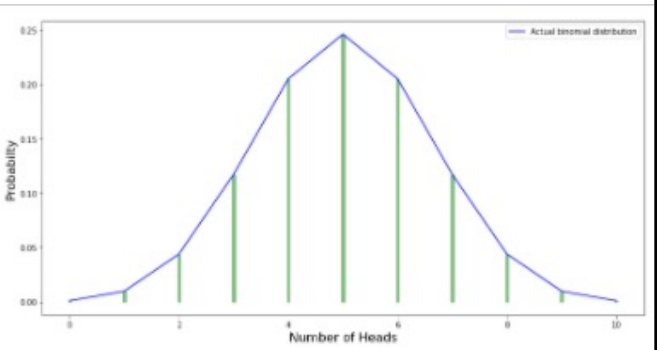
Binom module from scipy stats package

```
from scipy.stats import binom
x = range(0,11)# range is 0 to 10
n = 10  # the number of trials
p = 0.5 # the probability of a heads
```

Probability distribution of getting 0 to 10 heads (x), after 10 throws (n) with a fair coin (p=0.5)

```
# Plot the actual binomial distribution as a sanity check

# Plot the results as a histogram
fig, ax1 = plt.subplots(figsize=(14,7))
# ax1 = sns.distplot(heads, bins=11, label='simulation results')
ax1.set_xlabel("Number of Heads",fontsize=16)
ax1.set_ylabel("Probabilty",fontsize=16)
# draw the actual binomal distribution
ax1.plot(x, binom.pmf(x, n, p), 'b', label='Actual binomial distribution')
ax1.vlines(x, 0, binom.pmf(x, n, p), colors='g', lw=5, alpha=0.5)
plt.legend()
plt.show()
```



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Quiz

What will this code block will do if *np.random.binomial(n, p, size=runs)* returns a series of number between 0 and 10 (e.g. n=10, p=0.5, size=1000) ?

```
sum([1 for i in np.random.binomial(n, p, size=runs)
     if i==7])/runs
```

a. Sum of all 7s in this Binomial distribution
b. The probability of the occurrence of 7 in this Binomial distribution
c. The probability of the occurrence of 1 in this Binomial distribution
d. All of the above

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Sampling

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Sampling : What and Why

- **Sampling** is the process by which **inference** is made by examining only a part of the whole population or set.

- Purpose of Sampling:
  - In most cases, we cannot gather data from the entire population.
  - Even it is possible, an organization will not have time and money to collect data for the entire population
  - **Sampling** saves money by allowing data scientists to gather the same or similar answers from a **sample** that they **would** receive from the population.

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Sampling Process

- **Probability sampling** is based on the fact that every member of a population has a known and equal chance of being selected. For example, if you had a population of 100 people, each person would have odds of 1 out of 100 of being chosen.

- With **non-probability sampling**, those odds are not equal. For example, a person might have a better chance of being chosen if they live close to the researcher or have access to a computer.

- **Probability sampling gives you the best chance to create a sample that is truly representative of the population.**

- **Simple Random Sample:** Every member and set of members has an equal chance of being included in the sample.

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Random Samples

Random samples are usually fairly representative since they don't favor certain members.

Syntax: random.sample(population, k)

The population can be any sequence or set from which you want to select a k length numbers.

```
# Select items from a list
import random

print("random module's sample in Python ")
list = [20,40,80,100,120]
print ("choosing 3 random items from a list using sample function",
       random.sample(list,k=3))

list = [20,40,20,20,20]
print ("choosing 3 random items from a list using sample function",
       random.sample(list,k=3))
```

```
random module's sample in Python
choosing 3 random items from a list using sample function [20, 100, 80]
choosing 3 random items from a list using sample function [40, 20, 20]
```


Random samples (Contd.)

```
# select items from a set
weight_set = {40, 50, 55, 65, 75,80}
print ("choosing 4 random items from a set using sample method ",
       random.sample(weight_set, k=4))
```

choosing 4 random items from a set using sample method [40, 65, 75, 80]

```
# select items from a dictionary
marks_dict = {
  "Kelly": 55,
  "jhon": 70,
  "Donald": 60,
  "Lennin" :50
}
print ("choosing 2 random items from a dictionary using sample method ",
       random.sample(marks_dict.items(), k=2))
```

choosing 2 random items from a dictionary using sample method [('Lennin', 50), ('Kelly', 55)]

Stratified Random Sample

A stratified sample is one that ensures that subgroups (strata) of a given population are each adequately represented within the whole sample population of a research study.

Example—A student council surveys 100 students by getting random samples of 25 freshmen, 25 sophomores, 25 juniors, and 25 seniors.

Why It is Needed

A stratified sample guarantees that members from each group will be represented in the sample, so this sampling method is good when we want some members from every group.

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Example - sklearn.model_selection.StratifiedShuffleSplit

```
>>> import numpy as np
>>> from sklearn.model_selection import StratifiedShuffleSplit
>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([0, 0, 0, 1, 1, 1])
>>> sss = StratifiedShuffleSplit(n_splits=5, test_size=0.5, random_state=0)
>>> sss.get_n_splits(X, y)
5
>>> print(sss)
StratifiedShuffleSplit(n_splits=5, random_state=0, ...)
>>> for train_index, test_index in sss.split(X, y):
...     print("TRAIN:", train_index, "TEST:", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
TRAIN: [5 2 3] TEST: [4 1 0]
TRAIN: [5 1 4] TEST: [0 2 3]
TRAIN: [5 0 2] TEST: [4 3 1]
TRAIN: [4 1 0] TEST: [2 3 5]
TRAIN: [0 5 1] TEST: [3 4 2]
```


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Random Variables

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Random Variables

Random variable - a numerical description of the outcome of a statistical experiment

- **Discrete Random Variable:** A discrete random variable is one which may take on only a countable number of distinct values (or an infinite sequence of values)

  Example : The number of eggs that a hen lays in a given day

- **Continuous random variable: N**umeric **variables** that have an infinite number of values between any two positive values. For example, between 3 and 4 we can have infinite number of values like 3.1, 3.112, 3.1112 ……

  Example: We can define a continuous random variable X to be the height of students in a class.

  Since the continuous random variable is defined over an **interval** of values, it is **represented by the area under a curve (or the integral)**.

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Quiz

A random variable X to be the number which comes up when you roll a fair dice. X can take values : [1,2,3,4,5,6]

Are these discrete or continuous random variables?

a. Continuous

b. Discrete

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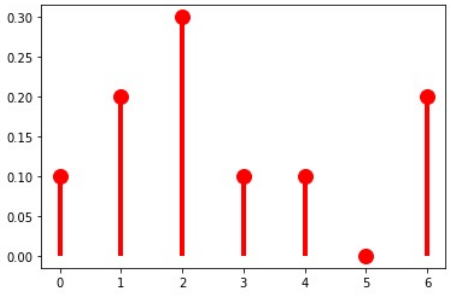
Discrete Random Variables (Optional)

rv_discrete - A base class to construct specific distribution classes and instances for discrete random variables.

Creating a discrete distribution from a list of probabilities

```
import numpy as np
from scipy import stats
xk = np.arange(7)
pk = (0.1, 0.2, 0.3, 0.1, 0.1, 0.0, 0.2)
custm = stats.rv_discrete(name='custm', values=(xk, pk))

import matplotlib.pyplot as plt
fig, ax = plt.subplots(1, 1)
ax.plot(xk, custm.pmf(xk), 'ro', ms=12, mec='r')
ax.vlines(xk, 0, custm.pmf(xk), colors='r', lw=4)
plt.show()
```



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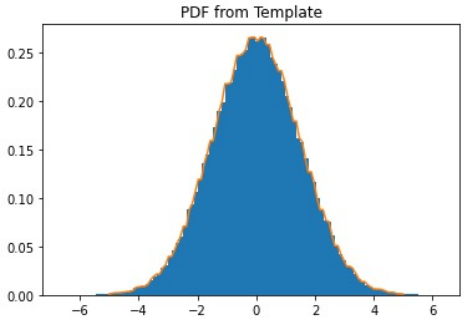
Continuous Random variables (Optional)

rv_continuous - a base class to construct specific distribution classes for continuous random variables

Python Code for Continuous Random Variable Distribution with a Histogram

```
import scipy.stats
data = scipy.stats.norm.rvs(size=100000, loc=0, scale=1.5, ran
dom_state=123)
hist = np.histogram(data, bins=100)
hist_dist = scipy.stats.rv_histogram(hist)

# this will behave according to the rv_continuous distribution
hist_dist.pdf(x)
X = np.linspace(-5.0, 5.0, 100)
plt.title("PDF from Template")
plt.hist(data, density=True, bins=100)
plt.plot(X, hist_dist.pdf(X), label='PDF')
plt.show()
```



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Probability Density and Mass Functions

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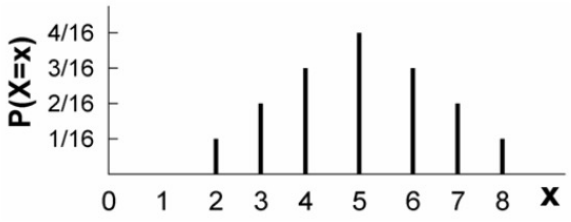
Probability Mass Function

Probability mass function (PMF) gives the probability that a discrete random variable is exactly equal to some value.

x	P(x)
2	1/16
3	2/16
4	3/16
5	4/16
6	3/16
7	2/16
8	1/16



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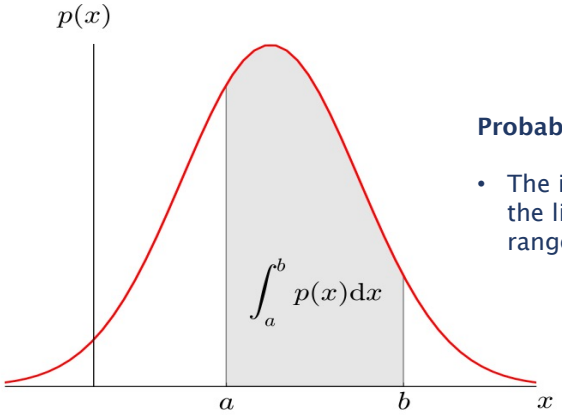
Probability Density Function

Probability density function (PDF) describes the relative likelihood for a continuous random variable (X) to take on a given value (x).



Probability Density Function (PDF) of a value x

- The integral over the PDF between a and b gives the likelihood of finding the value of x in that range.

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Quiz

The probability of a specific value of a continuous random variable will be zero or almost zero.

a. True

b. False

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Thank You

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