CHSH polytope V representation

August 21, 2024

We use j as a label for the vertices—extremal points—of the convex polytope of the Bell scenario with two parties, each having two dichotomic measurement settings. There are 16 deterministic vertices that are also extremal points of the local polytope and 8 non-deterministic PR-boxes that are outside the local polytope.

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|-------|-------|-------|--------|
| $\{A_1, B_1\}$ | 1 | 0 | 0 | 0 |
| $\{A_1, B_2\}$ | 1 | 0 | 0 | 0 |
| $\{A_2, B_1\}$ | 1 | 0 | 0 | 0 |
| A_2, B_2 | 1 | 0 | 0 | 0 |

Table 1: j=1

| | (0,0) | (0, 1) | (1,0) | (1,1) |
|----------------|-------|--------|-------|-------|
| $\{A_1, B_1\}$ | 0 | 1 | 0 | 0 |
| $\{A_1, B_2\}$ | 0 | 1 | 0 | 0 |
| $\{A_2, B_1\}$ | 0 | 1 | 0 | 0 |
| A_2, B_2 | 0 | 1 | 0 | 0 |

Table 2: j=2

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|-------|-------|-------|--------|
| A_1, B_1 | 0 | 0 | 1 | 0 |
| $\{A_1, B_2\}$ | 0 | 0 | 1 | 0 |
| $\{A_2,B_1\}$ | 0 | 0 | 1 | 0 |
| A_2, B_2 | 0 | 0 | 1 | 0 |

Table 3: j = 3

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|-------|-------|-------|--------|
| A_1, B_1 | 0 | 0 | 0 | 1 |
| $\{A_1, B_2\}$ | 0 | 0 | 0 | 1 |
| $\{A_2, B_1\}$ | 0 | 0 | 0 | 1 |
| $\{A_2, B_2\}$ | 0 | 0 | 0 | 1 |

Table 4: j=4

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|-------|-------|-------|--------|
| A_1, B_1 | 0 | 1 | 0 | 0 |
| $\{A_1, B_2\}$ | 1 | 0 | 0 | 0 |
| $\{A_2, B_1\}$ | 0 | 1 | 0 | 0 |
| $\{A_2, B_2\}$ | 1 | 0 | 0 | 0 |

Table 5: j=5

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|-------|-------|-------|--------|
| A_1, B_1 | 1 | 0 | 0 | 0 |
| $\{A_1, B_2\}$ | 1 | 0 | 0 | 0 |
| $\{A_2, B_1\}$ | 0 | 0 | 1 | 0 |
| $\{A_2, B_2\}$ | 0 | 0 | 1 | 0 |

Table 6: j=6

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|-------|-------|-------|--------|
| A_1, B_1 | 0 | 1 | 0 | 0 |
| $\{A_1, B_2\}$ | 0 | 1 | 0 | 0 |
| $\{A_2, B_1\}$ | 0 | 0 | 0 | 1 |
| $\{A_2, B_2\}$ | 0 | 0 | 0 | 1 |

Table 7: j = 7

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|-------|-------|-------|--------|
| A_1, B_1 | 0 | 0 | 1 | 0 |
| $\{A_1, B_2\}$ | 0 | 0 | 1 | 0 |
| $\{A_2,B_1\}$ | 1 | 0 | 0 | 0 |
| $\{A_2, B_2\}$ | 1 | 0 | 0 | 0 |

Table 8: j = 8

| | (0,0) | (0,1) | (1,0) | (1,1) |
|----------------|-------|-------|-------|-------|
| $\{A_1, B_1\}$ | 0 | 0 | 0 | 1 |
| $\{A_1, B_2\}$ | 0 | 0 | 0 | 1 |
| $\{A_2, B_1\}$ | 0 | 1 | 0 | 0 |
| $\{A_2, B_2\}$ | 0 | 1 | 0 | 0 |

Table 9: j = 9

| | (0,0) | (0, 1) | (1,0) | (1,1) |
|----------------|-------|--------|-------|-------|
| A_1, B_1 | 1 | 0 | 0 | 0 |
| $\{A_1, B_2\}$ | 0 | 1 | 0 | 0 |
| $\{A_2, B_1\}$ | 1 | 0 | 0 | 0 |
| $\{A_2, B_2\}$ | 0 | 1 | 0 | 0 |

Table 10: j = 10

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|-------|-------|-------|--------|
| A_1, B_1 | 0 | 0 | 0 | 1 |
| $\{A_1, B_2\}$ | 0 | 0 | 1 | 0 |
| $\{A_2, B_1\}$ | 0 | 0 | 0 | 1 |
| $\{A_2, B_2\}$ | 0 | 0 | 1 | 0 |

Table 11: j = 11

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|-------|-------|-------|--------|
| A_1, B_1 | 0 | 1 | 0 | 0 |
| $\{A_1, B_2\}$ | 1 | 0 | 0 | 0 |
| $\{A_2, B_1\}$ | 0 | 0 | 0 | 1 |
| $\{A_2, B_2\}$ | 0 | 0 | 1 | 0 |

Table 12: j=12

| | (0,0) | (0, 1) | (1,0) | (1, 1) |
|----------------|-------|--------|-------|--------|
| $\{A_1, B_1\}$ | 0 | 0 | 1 | 0 |
| $\{A_1, B_2\}$ | 0 | 0 | 0 | 1 |
| $\{A_2, B_1\}$ | 1 | 0 | 0 | 0 |
| $\{A_2, B_2\}$ | 0 | 1 | 0 | 0 |

Table 13: j = 13

| | (0,0) | (0,1) | (1,0) | (1,1) |
|----------------|-------|-------|-------|-------|
| $\{A_1, B_1\}$ | 0 | 0 | 1 | 0 |
| $\{A_1, B_2\}$ | 0 | 0 | 0 | 1 |
| $\{A_2,B_1\}$ | 0 | 0 | 1 | 0 |
| $\{A_2, B_2\}$ | 0 | 0 | 0 | 1 |

Table 14: j = 14

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|-------|-------|-------|--------|
| $\{A_1, B_1\}$ | 1 | 0 | 0 | 0 |
| $\{A_1, B_2\}$ | 0 | 1 | 0 | 0 |
| $\{A_2,B_1\}$ | 0 | 0 | 1 | 0 |
| $\{A_2, B_2\}$ | 0 | 0 | 0 | 1 |

Table 15: j = 15

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|-------|-------|-------|--------|
| A_1, B_1 | 0 | 0 | 0 | 1 |
| $\{A_1, B_2\}$ | 0 | 0 | 1 | 0 |
| $\{A_2, B_1\}$ | 0 | 1 | 0 | 0 |
| $\{A_2,B_2\}$ | 1 | 0 | 0 | 0 |

Table 16: j=16

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|---------------|---------------|---------------|---------------|
| $\{A_1, B_1\}$ | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ |
| $\{A_1, B_2\}$ | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ |
| $\{A_2,B_1\}$ | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ |
| $\{A_2, B_2\}$ | Õ | $\frac{1}{2}$ | $\frac{1}{2}$ | Õ |

Table 17: j = 17

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|---------------|--------------------------|--------------------------|---------------|
| $\{A_1, B_1\}$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 |
| $\{A_1, B_2\}$ | 0 | $\frac{\overline{1}}{2}$ | $\frac{\overline{1}}{2}$ | 0 |
| $\{A_2, B_1\}$ | 0 | $\frac{\overline{1}}{2}$ | $\frac{\overline{1}}{2}$ | 0 |
| A_2, B_2 | $\frac{1}{2}$ | Õ | Õ | $\frac{1}{2}$ |

Table 18: j = 18

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|---------------|---------------|---------------|---------------|
| $\{A_1, B_1\}$ | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ |
| $\{A_1, B_2\}$ | $\bar{0}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\bar{0}$ |
| $\{A_2,B_1\}$ | $\frac{1}{2}$ | $\bar{0}$ | $\bar{0}$ | $\frac{1}{2}$ |
| $\{A_2, B_2\}$ | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ |

Table 19: j = 19

| | (0,0) | (0,1) | (1,0) | (1,1) |
|--------------------------|---------------|---------------|---------------|---------------|
| $\overline{\{A_1,B_1\}}$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 |
| $\{A_1, B_2\}$ | $\frac{1}{2}$ | Õ | Õ | $\frac{1}{2}$ |
| $\{A_2,B_1\}$ | Ō | $\frac{1}{2}$ | $\frac{1}{2}$ | Ō |
| $\{A_2, B_2\}$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 |

Table 20: j=20

| | (0,0) | (0,1) | (1,0) | (1,1) |
|----------------|---------------|---------------|---------------|---------------|
| A_1, B_1 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ |
| $\{A_1, B_2\}$ | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ |
| $\{A_2,B_1\}$ | $\bar{0}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\bar{0}$ |
| $\{A_2, B_2\}$ | $\frac{1}{2}$ | $\bar{0}$ | $\bar{0}$ | $\frac{1}{2}$ |

Table 21: j = 21

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|---------------|--------------------------|---------------|---------------|
| $\{A_1, B_1\}$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 |
| $\{A_1, B_2\}$ | 0 | $\frac{\overline{1}}{2}$ | $\frac{1}{2}$ | 0 |
| $\{A_2,B_1\}$ | $\frac{1}{2}$ | Ō | Ō | $\frac{1}{2}$ |
| A_2, B_2 | Õ | $\frac{1}{2}$ | $\frac{1}{2}$ | Õ |

Table 22: j=22

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|----------------|---------------|--------------------------|---------------|---------------|
| A_1, B_1 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ |
| $\{A_1, B_2\}$ | Ō | $\frac{1}{2}$ | $\frac{1}{2}$ | Ō |
| $\{A_2, B_1\}$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 |
| A_2, B_2 | 0 | $\frac{\overline{1}}{2}$ | $\frac{1}{2}$ | 0 |

Table 23: j=23

| | (0,0) | (0,1) | (1,0) | (1, 1) |
|--------------------------|---------------|---------------|---------------|---------------|
| $\overline{\{A_1,B_1\}}$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 |
| $\{A_1, B_2\}$ | $\frac{1}{2}$ | $\bar{0}$ | $\bar{0}$ | $\frac{1}{2}$ |
| $\{A_2,B_1\}$ | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ |
| A_2, B_2 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ |

Table 24: j = 24