Design and Modeling of Fluid Power Systems

ME 597/ABE 591 - Lecture 7

Dr. Monika Ivantysynova
MAHA Professor Fluid Power Systems

MAHA Fluid Power Research Center
Purdue University

Content

The lubricating gap as a basic design element of displacement machines

Gap flow calculation. Gap with a constant gap height

Gap with variable gap height (slipper)

Gap between piston and cylinder, numerical solution of gap flow equations

Non-isothermal gap flow

Aim: Derivation of basic equations for gap flow and calculation of Gap flow parameters (pressure and velocity distribution, load ability, gap flow and viscous friction)

Basic knowledge about the gap design and simulation models

Lubricating Gap

Basic Design Element of Displacement Machines

Design & operating parameter

Gap height

Gap flow

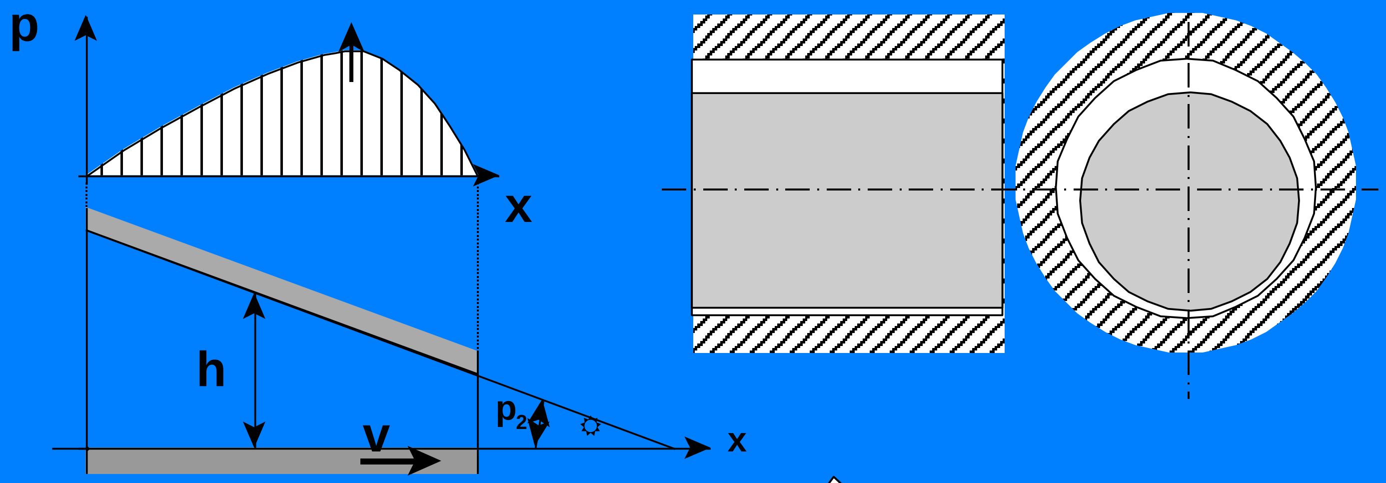
Load ability

Leakage

Viscous friction

Performance & Losses

Gaps in Displacement Machines



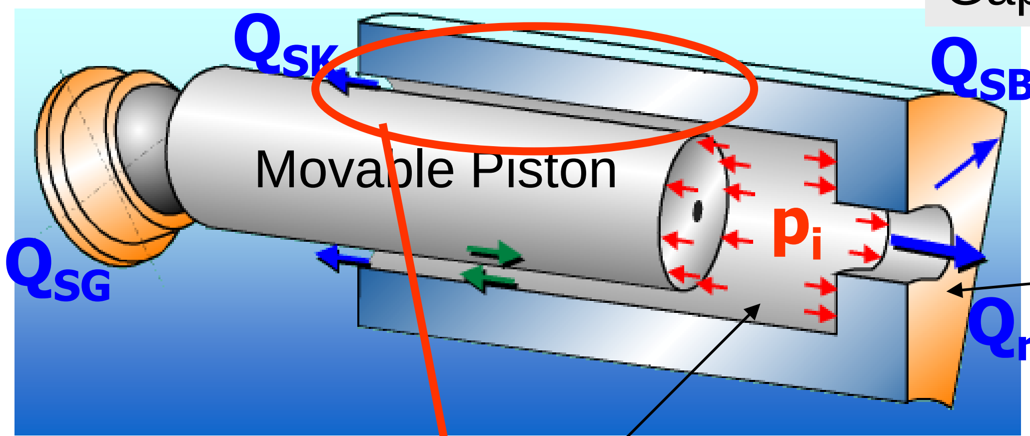
With Variable Gap Height

- Sealing Function
- Bearing Function

Lubricating Gaps -examples

Swash plate axial piston machine

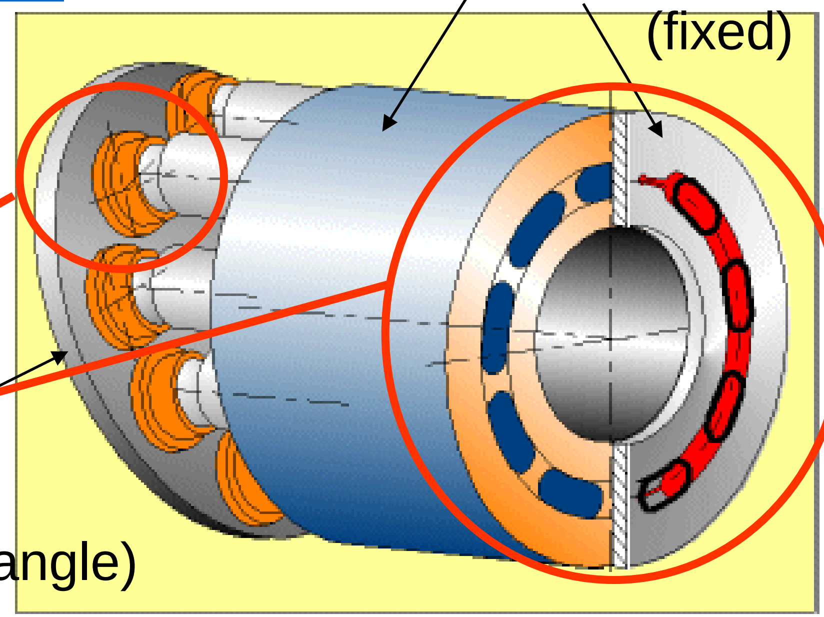
Gaps seal the displacement chamber



Rotating cylinder block

Valve plate (fixed)

Displacement chamber



Gaps between:

- piston and cylinder
- slipper and valve plate
- cylinder block and valve plate

Swash plate (fixed or with adjustable angle)

Gap Flow Calculation

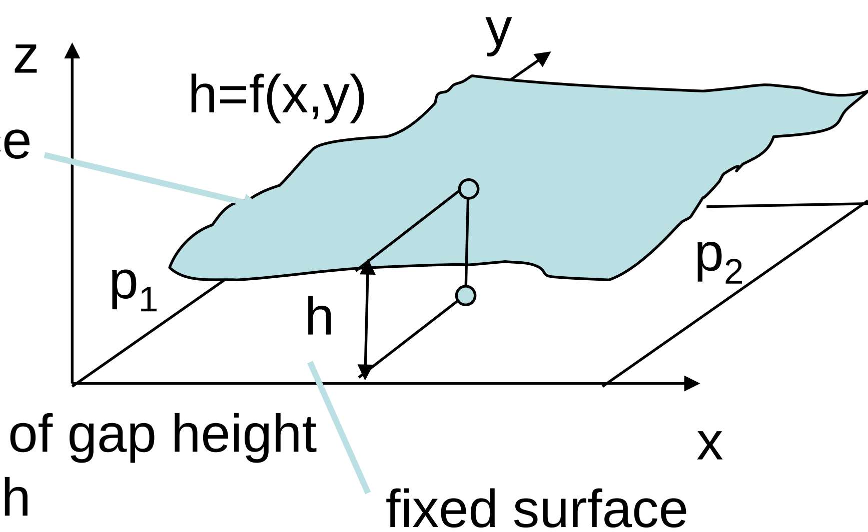
Aim: High load carrying ability

Low friction

Low leakage flow

→ Gap height lies in a range of some micrometers, whereas all other dimensions are in a range of millimeters

Laminar flow of an incompressible fluid in the gap can be described using Navier-Stokes-Equation:

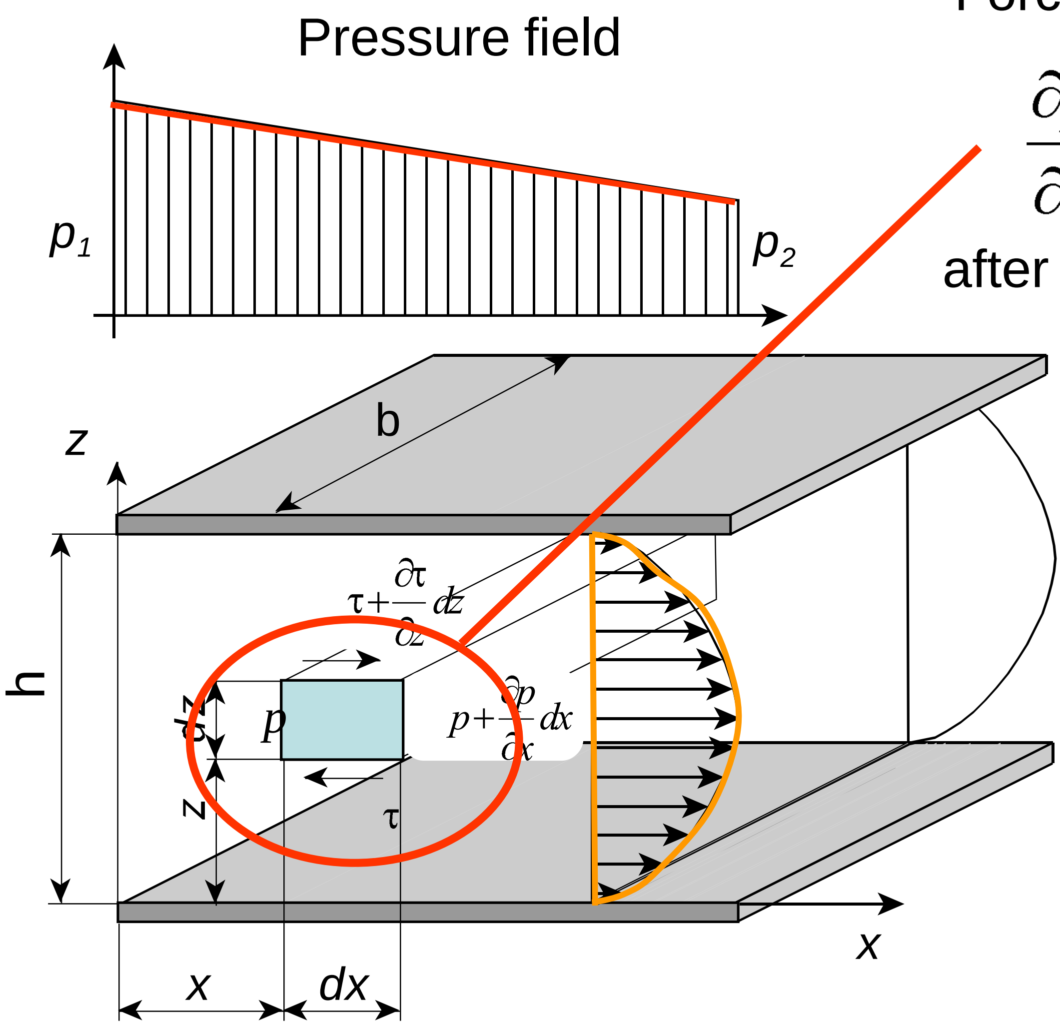


Assumptions:

- neglecting mass forces
- assuming steady state flow,
- change of fluid velocity only in direction of gap height
- pressure is not a function of gap height h

→ Pressure Force + Viscosity Force = 0

Gap with constant gap height



Forces applied on a fluid element:

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial z} \quad \text{with} \quad \tau = \mu \cdot \frac{\partial v}{\partial z}$$

after double integration velocity yields:

$$v = \frac{1}{2\cdot\mu}\frac{\partial p}{\partial x}z^2 + c_1\cdot z + c_2$$

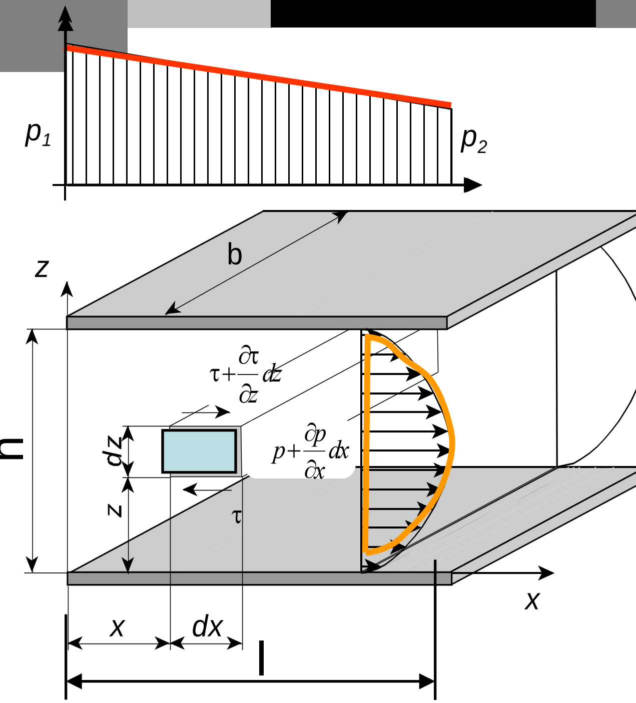
boundary conditions:

z=0 … v=0

z=h … v=0

$$v = \frac{1}{2\cdot\mu}\frac{\partial p}{\partial x}\left(z^2 - h\cdot z\right)$$

Gap with constant gap height



Gap flow: $Q = b\int_0^h v \cdot dz = -\frac{1}{12\mu}\cdot\frac{\partial p}{\partial x}\cdot b \cdot h^3$

for pressure: $p = \frac{p_2 - p_1}{l}\cdot x + p_1$

Shear stress on surfaces: $\tau = \mu \cdot \frac{\partial v}{\partial z}\Big|_{\substack{z=0\\ z=h}}$

$$\tau_{z=0} = \frac{\Delta p}{l}\cdot\frac{h}{2} \qquad \tau_{z=h} = -\frac{\Delta p}{l}\cdot\frac{h}{2}$$

$$v = \frac{1}{2\cdot\mu}\frac{\partial p}{\partial x}\left(z^2 - h\cdot z\right)$$

with: $\frac{\partial p}{\partial x} = \frac{p_2 - p_1}{l} = -\frac{\Delta p}{l}$

Viscous friction: $F_v = \tau \cdot b \cdot l$

Power loss due to gap flow: $P_{SQ} = Q\cdot\Delta p = \frac{1}{12\cdot\mu}\cdot b \cdot h^3\,\frac{\Delta p^2}{l}$

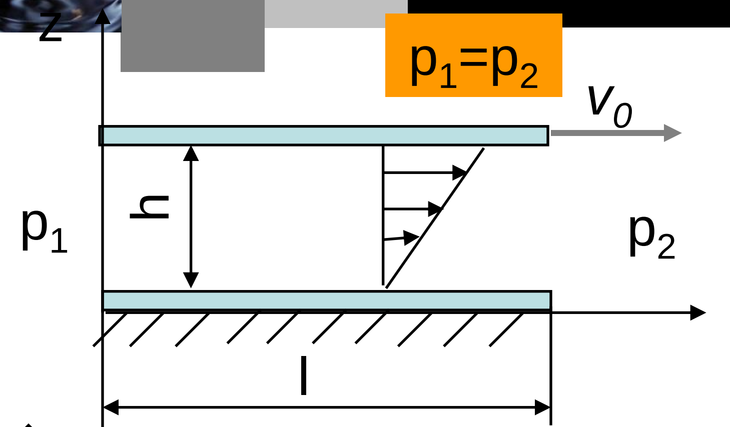
Gap with constant gap height

boundary conditions:

$z=0 \; \ldots \; v=0 \qquad x=0 \; \ldots \; p=p_1$

$z=h \; \ldots \; v=v_0 \qquad x=l \; \ldots \; p=p_2$

$p_1=p_2$



b >> h

$$\boxed{\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial z}} \Rightarrow \boxed{\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial \tau}{\partial z} = 0}$$

$$\boxed{\tau = \text{const}} \Leftarrow \boxed{\tau = \mu \cdot \frac{\partial v}{\partial z}}$$

Flow velocity: $v = \frac{v_0}{h} \cdot z$

Shear stress on surfaces: $\tau = \mu \cdot \frac{v_0}{h}$

Gap flow: $Q = b \int_0^h v \cdot dz = b \cdot \frac{v_0}{h} \int_0^h z \cdot dz = b \cdot \frac{v_0}{h} \cdot \frac{h^2}{2} = b \cdot \frac{v_0}{2} \cdot h$

Viscous friction: $F_v = \tau \cdot l \cdot b = \mu \cdot \frac{v_0}{h} \cdot l \cdot b$

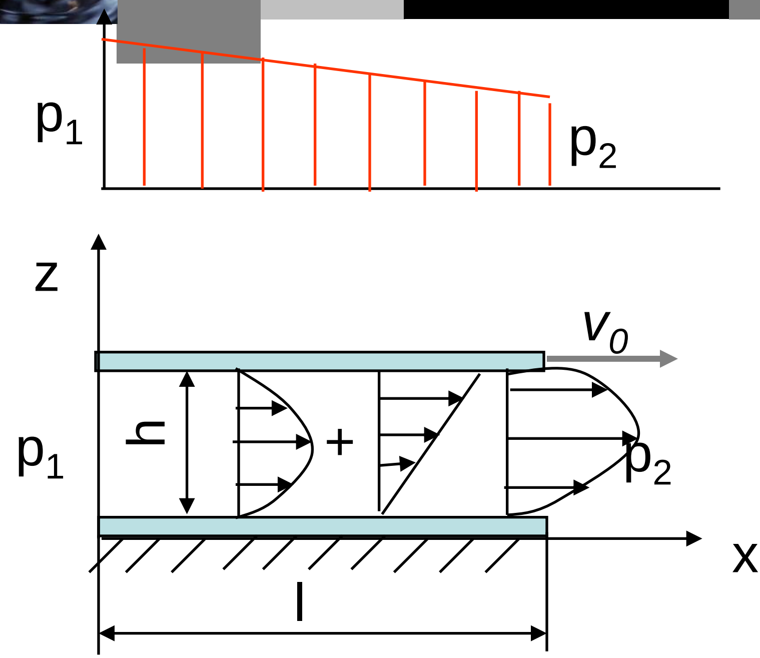
Power loss due to viscous friction: $P_{Sv} = F_v \cdot v_0 = \mu \cdot \frac{l \cdot b}{h} \cdot v_0^2$

Gap with constant gap height

Boundary conditions:

$z=0 \quad \ldots \quad v=0 \qquad x=0 \quad \ldots \quad p=p_1$

$z=h \quad \ldots \quad v=v_0 \qquad x=l \quad \ldots \quad p=p_2$



$$\frac{\partial p}{\partial x} = \mu \cdot \frac{\partial^2 v}{\partial z^2}$$

Flow velocity:

$$v = \frac{1}{2 \cdot \mu} \frac{\partial p}{\partial x} \left(z^2 - h \cdot z \right) + \frac{v_0}{h} \cdot z$$

Gap flow:

b >> h

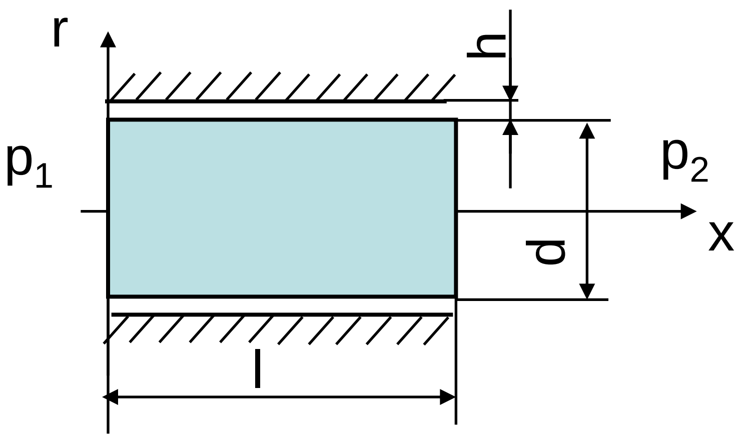
$$Q = b \int_0^h v \cdot dz = -\frac{1}{12\mu} \cdot \frac{\partial p}{\partial x} \cdot b \cdot h^3 + v_0 \cdot \frac{h}{2} \cdot b$$

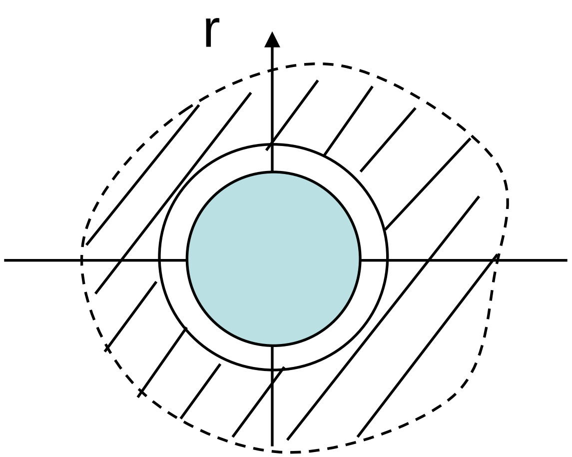
Shear stress on surfaces: $\tau = \mu \cdot \frac{\partial v}{\partial z} = \frac{p_2 - p_1}{l} \cdot \frac{1}{2} \left(2 \cdot z - h \right) + \frac{v_0}{h} \cdot \mu$

Power losses: $P_S = P_{SQ} + P_{Sv} = Q \cdot \Delta p + F_v \cdot v_0$

$$P_S = \frac{1}{12 \cdot \mu} \cdot \frac{\Delta p^2}{l} \cdot b \cdot h^3 + \mu \frac{v_0^2}{h} \cdot b \cdot l$$

Radial Gap with constant gap height



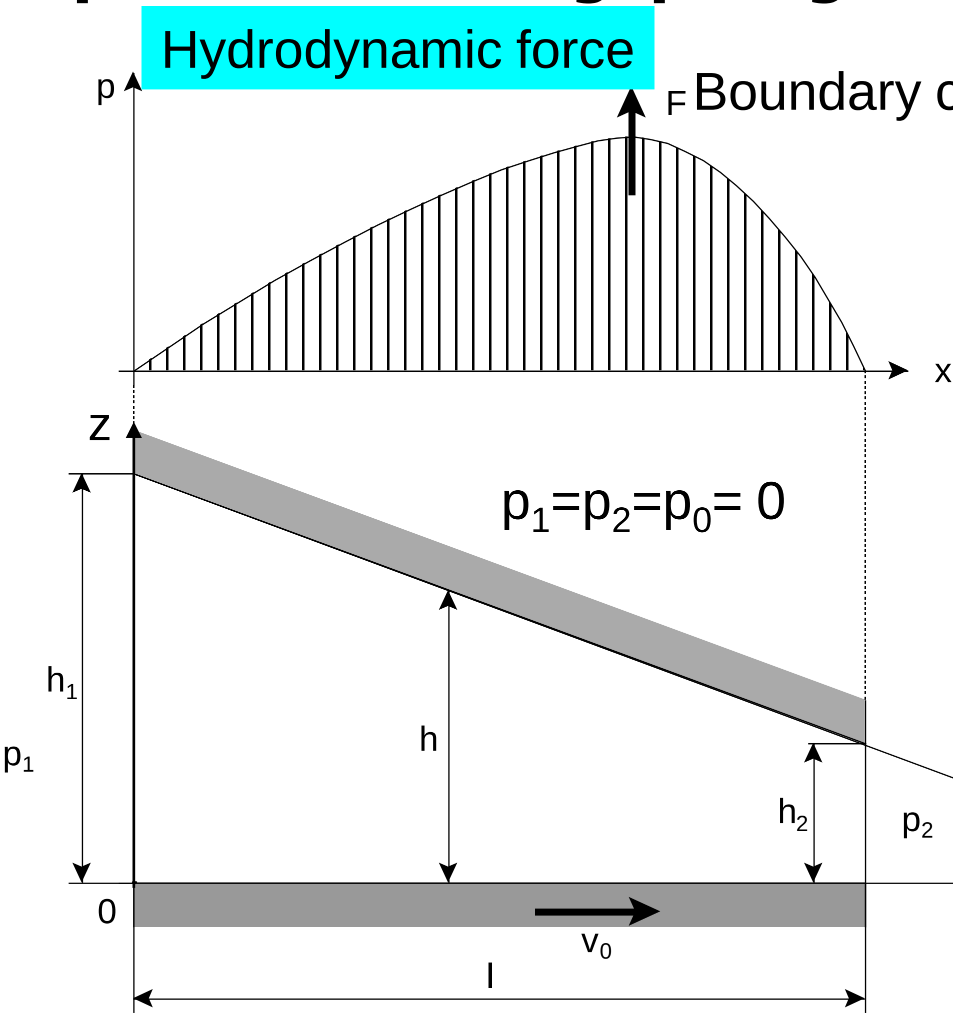


Gap width: $$b = \pi \cdot d$$

Equations from gap with constant gap height can be applied:

Gap flow: $$Q = b\int_0^h v \cdot dz = -\frac{\pi \cdot d}{12\mu} \cdot \frac{\partial p}{\partial x} \cdot h^3$$

Gap with variable gap height



Boundary conditions:

$z=0 \quad \dots \quad v=v_0 \qquad x=0 \quad \dots \quad p=p_1$

$z=h \quad \dots \quad v=0 \qquad x=l \quad \dots \quad p=p_2$

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial z} \qquad \text{with} \qquad \tau = \mu \cdot \frac{\partial v}{\partial z}$$

$$\frac{\partial p}{\partial x} = \mu \cdot \frac{\partial^2 v}{\partial z^2}$$

Flow velocity:

$$v = \frac{1}{2 \cdot \mu} \frac{\partial p}{\partial x} \left(z^2 - h \cdot z \right) - \frac{v_0}{h} \cdot z + v_0$$

Gap flow:

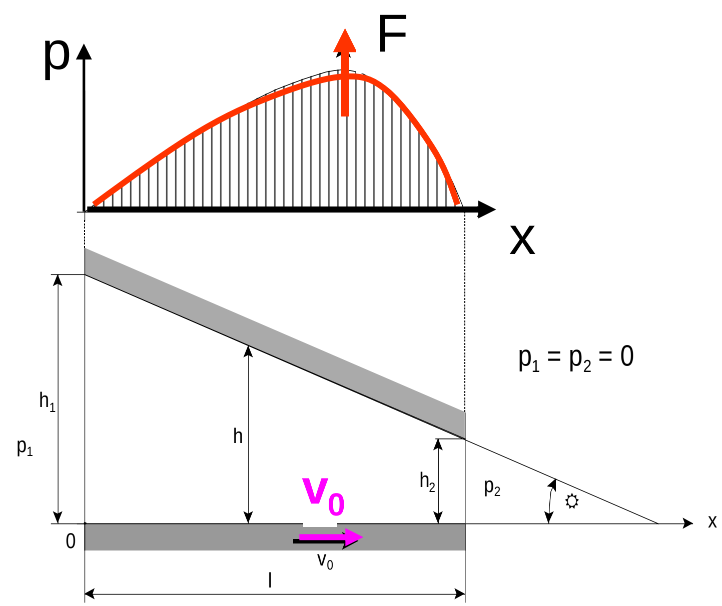
$$Q = b \int_0^h v \cdot dz = -\frac{1}{12\mu} \cdot \frac{\partial p}{\partial x} \cdot b \cdot h^3 + v_0 \cdot \frac{h}{2} \cdot b$$

$$h = \frac{h_2 - h_1}{l} x + h_1$$

Gap with variable gap height

Pressure distribution in x-direction:

$$p(x) = \int \frac{\partial p}{\partial x} dx \qquad (1)$$



from

$$Q = -\frac{1}{12 \cdot \mu} \cdot \frac{\partial p}{\partial x} \cdot b \cdot h^3 + v_0 \cdot \frac{h}{2} \cdot b \qquad (2)$$

we obtain:

$$\frac{\partial p}{\partial x} = -\frac{12 \cdot \mu \cdot Q}{b \cdot h^3} + \frac{6 \cdot \mu \cdot v_0}{h^2} \qquad (3)$$

$$p(x) = \int \left(\frac{6 \cdot \mu \cdot v_0}{h^2} - \frac{12 \cdot \mu \cdot Q}{b \cdot h^3} \right) dx \qquad (4)$$

with: $$h = \frac{h_2 - h_1}{l} x + h_1 \qquad (5)$$

Gap with variable gap height

$$p(x) = \int \left(\frac{6 \cdot \mu \cdot v_0}{h^2} - \frac{12 \cdot \mu \cdot Q}{b \cdot h^3} \right) dx \qquad (4) \qquad \text{with: } \quad h = \frac{h_2 - h_1}{l} x + h_1 \qquad (5)$$

using:

$$\int X^n dx = \frac{1}{a(n+1)} X^{n+1} \quad \text{with } X = ax + b \qquad (6)$$

in our case:

$$a = \frac{h_2 - h_1}{l} \quad \text{and} \quad b = h_1 \qquad (7)$$

where n=-2 for the first term in Eq. (4) and n= -3 for the second term in Eq. (4)

and after integration:

$$p(x) = \frac{6 \cdot \mu \cdot v_0}{\frac{h_2 - h_1}{l} \cdot (-1)} \cdot \frac{1}{\frac{h_2 - h_1}{l} \cdot x + h_1} - \frac{12 \cdot \mu \cdot Q}{b \cdot \frac{h_2 - h_1}{l} \cdot (-2)} \cdot \frac{1}{\left(\frac{h_2 - h_1}{l} \cdot x + h_1 \right)^2} + c \qquad (8)$$

h h

Boundary conditions: $x=0 \quad p=p_0$

Gap with variable gap height

$$p(x)=\frac{6\cdot\mu\cdot v_0}{\frac{h_2-h_1}{l}\cdot(-1)}\cdot\frac{1}{\underbrace{\frac{h_2-h_1}{l}\cdot x+h_1}_{h}}-\frac{12\cdot\mu\cdot Q}{b\cdot\frac{h_2-h_1}{l}\cdot(-2)}\cdot\frac{1}{\left(\frac{h_2-h_1}{l}\cdot x+h_1\right)^2}+c \qquad (8)$$

$$c=p_0+\frac{6\cdot\mu\cdot v_0\cdot l}{(h_2-h_1)\cdot h_1}-\frac{6\cdot\mu\cdot Q\cdot l}{b\cdot(h_2-h_1)\cdot h_1^2} \qquad (9)$$

$$h=\frac{h_2-h_1}{l}x+h_1 \qquad (11)$$

$$p(x)=\frac{6\cdot\mu\cdot l}{h_2-h_1}\cdot\left(-\frac{v_0}{h}+\frac{Q}{b\cdot h^2}+\frac{v_0}{h_1}-\frac{Q}{b\cdot h_1^2}\right)+p_0 \qquad (10)$$

$$h_2-h_1=(h-h_1)\cdot\frac{l}{x} \qquad (12)$$

$$p(x)=\frac{6\cdot\mu\cdot x}{h-h_1}\left[v_0\cdot\frac{h-h_1}{h_1\cdot h}+\frac{Q}{b}\cdot\frac{h_1^2-h^2}{h^2\cdot h_1^2}\right]+p_0 \qquad (13)$$

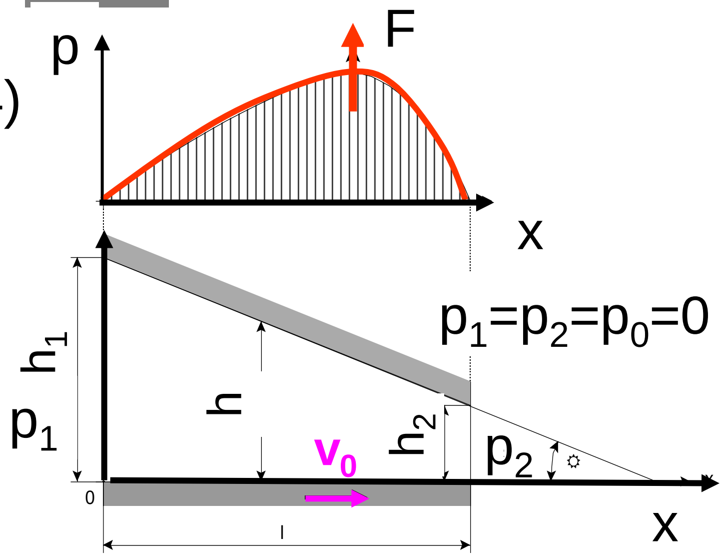
finally we get:

$$p(x)=\frac{6\cdot\mu\cdot x}{h_1\cdot h}\left[v_0-\frac{Q}{b}\cdot\frac{h+h_1}{h\cdot h_1}\right]+p_0 \qquad (14)$$

Boundary conditions: x=0 $p=p_0$

Gap with variable gap height

$$p(x) = \frac{6 \cdot \mu \cdot x}{h_1 \cdot h}\left[v_0 - \frac{Q}{b} \cdot \frac{h + h_1}{h \cdot h_1}\right] + p_0 \quad (14)$$



Using boundary conditions for x=l is $p=p_0$ and $h=h_2$ from Eq. (14) follows :

$$Q = \frac{v_0 \cdot b \cdot h_2 \cdot h_1}{h_2 + h_1} \quad (15)$$

Substituting Eq. (15) into Eq. (14) the pressure p(x) yields:

$$p(x) = \frac{6 \cdot \mu \cdot x \cdot v_0}{h^2} \cdot \frac{h - h_2}{h_2 + h_1} + p_0 \quad (16)$$

when $h=h_1=h_2$ → $p(x)=p_0$ → No load ability!

Gap with variable gap height

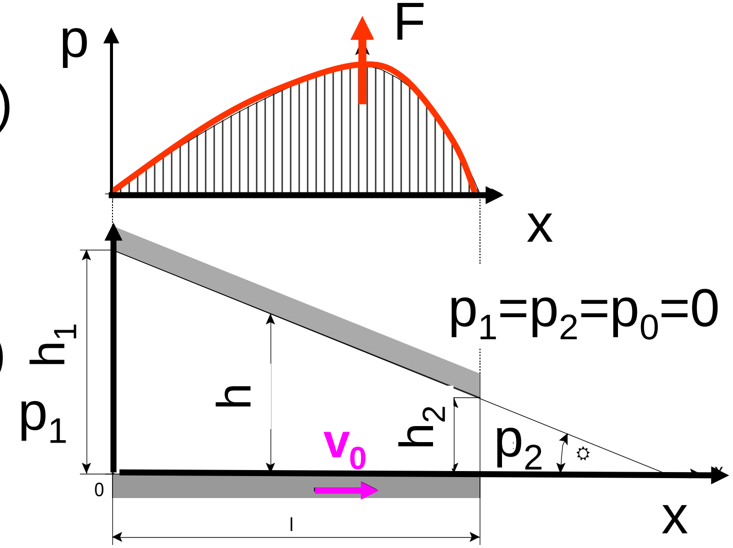
Load capacity due to hydrodynamic pressure field generated in the gap:

$$p(x) = \frac{6 \cdot \mu \cdot x \cdot v_0}{h^2} \cdot \frac{h - h_2}{h_2 + h_1} + p_0 \quad (16)$$

$$F = \int_0^l b \cdot (p - p_0)\, dx \quad (17)$$

$$F = b \int_0^l 6 \cdot \mu \cdot v_0 \cdot \frac{h - h_2}{h^2 (h_2 + h_1)} x \cdot dx \quad (18)$$

$$F = \frac{6 \cdot \mu \cdot b \cdot l^2}{(h_1 - h_2)^2} \cdot \left[\ln \frac{h_1}{h_2} - 2 \cdot \frac{h_1 - h_2}{h_1 + h_2} \right] \cdot v_0 \quad (19)$$



Maximum pressure force for $h_1/h_2 = 2.2$ yields:

$$F_{max} = \frac{6 \cdot \mu \cdot l^2}{1.2^2 \cdot h_2^2} \cdot \left[\ln 2.2 - \frac{2.4}{3.2} \right] \cdot v_0 = \frac{0.16 \cdot \mu \cdot l^2}{h_2^2} \cdot v_0 \quad (20)$$

Example

A gap between two circular plain and parallel surfaces with a central inflow as shown in Fig.1 is given. Both surfaces are fixed. The inner radius is R_1 and the outer radius is R_2.
Calculate the flow Q through the gap for a given gap height h, when the pressure at the inner radius is p_1 and the pressure at the outer radius is p_2.
And determine the pressure on the radius r= 15 mm.

The following parameters are given:

p_1=20 MPa
p_2=0.5 MPa
h = 20 µm

R_1=12 mm
R_2=25 mm

Dynamic viscosity of the fluid:
µ =0.0261 Pa· s

Boundary conditions:

z=0 … v=0   *r=R_1 … p=p_1*
z=h … v=0   *r=R_2 … p=p_2*

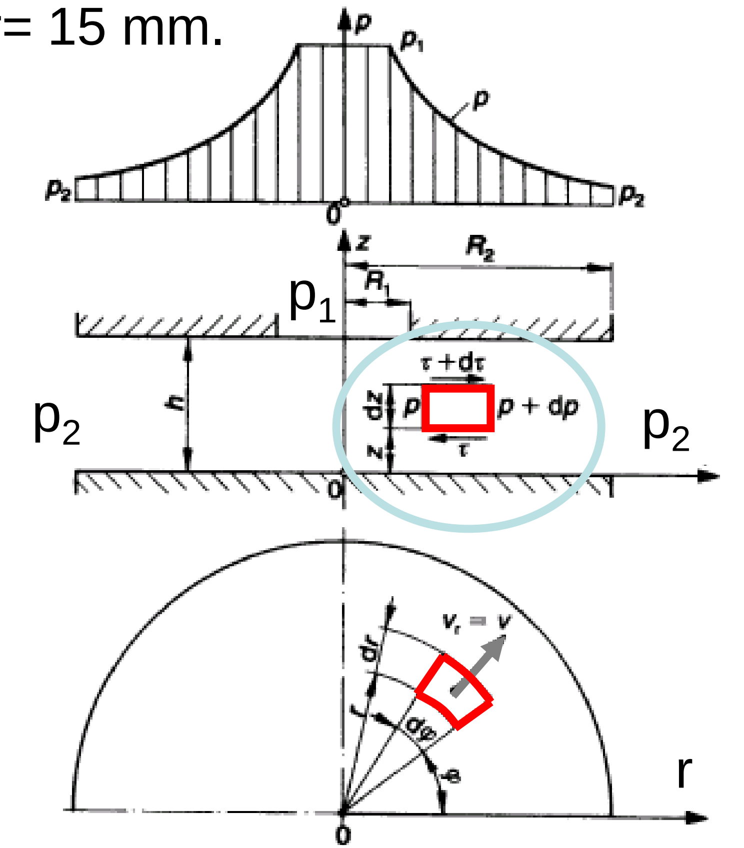
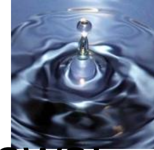


Fig.1

Example



From force balance on a fluid element follows:

$$\frac{\partial p}{\partial r} = \frac{\partial \tau}{\partial z} \quad \text{with} \quad \tau = \mu \cdot \frac{\partial v}{\partial z}$$

Boundary conditions:

z=0 … v=0 *r=R_1 … p=p_1*
z=h … v=0 *r=R_2 … p=p_2*

After double integration the velocity yields:

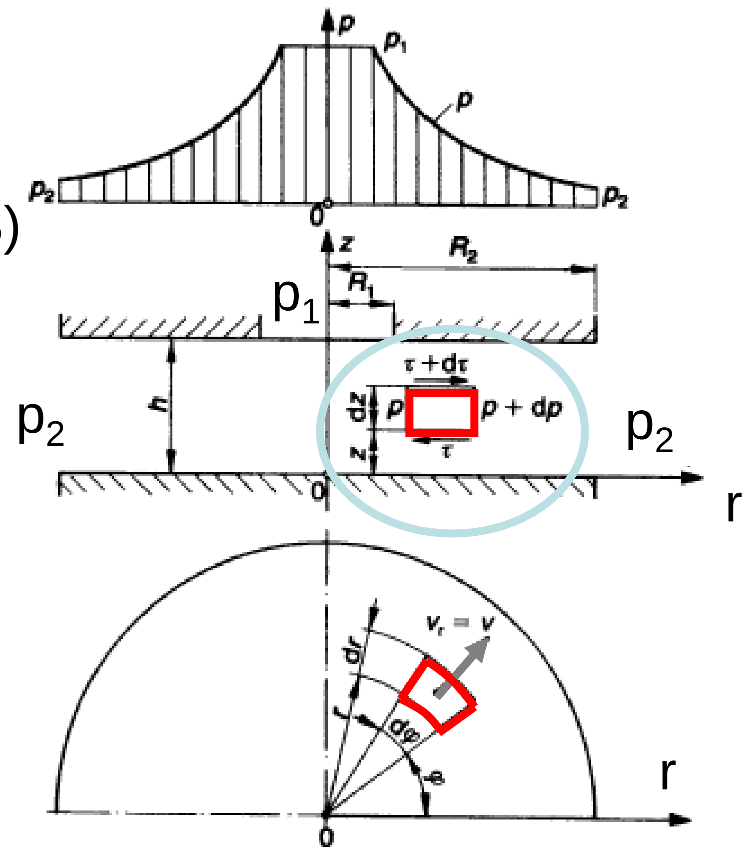
$$v = \frac{1}{2 \cdot \mu} \frac{\partial p}{\partial r} \left(z^2 - h \cdot z \right) \qquad (1)$$

Gap flow: $$Q = \int_0^h 2 \cdot \pi \cdot r \cdot v \cdot dz = -\frac{\pi \cdot r \cdot h^3}{6 \cdot \mu} \cdot \frac{\partial p}{\partial r} \qquad (3)$$

Therefore: $$\frac{\partial p}{\partial r} = -\frac{6 \cdot \mu \cdot Q}{\pi \cdot r \cdot h^3} \qquad (4)$$

and $$\int_{p_1}^{p_2} dp = \int_{R_1}^{R_2} -\frac{6 \cdot \mu \cdot Q}{\pi \cdot r \cdot h^3} \cdot dr \qquad (5)$$

$$p_2 - p_1 = -\frac{6 \cdot \mu \cdot Q}{\pi \cdot h^3} \cdot \left(\ln R_2 - \ln R_1 \right) \qquad (6)$$



Example

$$p_2 - p_1 = -\frac{6 \cdot \mu \cdot Q}{\pi \cdot h^3} \cdot \left(\ln R_2 - \ln R_1 \right) \quad (6)$$

From Eq. (6) follows for the gap flow:

$$Q = \frac{\left(p_2 - p_1 \right) \pi \cdot h^3}{6 \cdot \mu \cdot \ln \frac{R_1}{R_2}} \quad (7)$$

$$Q = \frac{\left(p_2 - p_1 \right) \pi \cdot h^3}{6 \cdot \mu \cdot \ln \frac{R_1}{R_2}} = \frac{\left(5 \cdot 10^5 \,\mathrm{Pa} - 2 \cdot 10^7 \,\mathrm{Pa} \right) \pi \cdot \left(2 \cdot 10^{-5} \right)^3 \mathrm{m}^3}{6 \cdot 0.0261 \,\mathrm{Pa} \cdot \mathrm{s} \cdot \ln \frac{12 \cdot 10^{-3}}{25 \cdot 10^{-3}}} = 4.264 \cdot 10^{-6} \,\mathrm{m}^3 \cdot \mathrm{s}^{-1} = 0.256 \,\mathrm{l/min}$$

For the pressure distribution in radial direction we obtain integrating Eq. (4):

$$p = -\frac{6 \cdot \mu \cdot Q}{\pi \cdot h^3} \cdot \ln r + c \quad (7)$$

$$\frac{\partial p}{\partial r} = -\frac{6 \cdot \mu \cdot Q}{\pi \cdot r \cdot h^3} \quad (4)$$

And using boundary conditions for c follows:

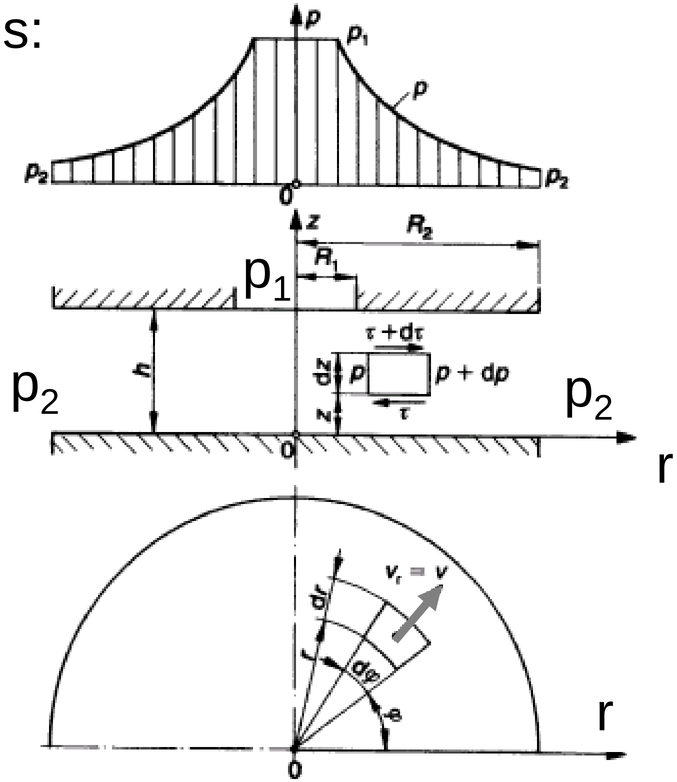
$$p_1 = -\frac{6 \cdot \mu \cdot Q}{\pi \cdot h^3} \cdot \ln R_1 + c \quad (8)$$

$$\Rightarrow \quad p = \frac{6 \cdot \mu \cdot Q}{\pi \cdot h^3} \cdot \ln \frac{R_1}{r} + p_1 \quad (9)$$

Example

The pressure distribution in radial direction yields:

$$p = \frac{6 \cdot \mu \cdot Q}{\pi \cdot h^3} \cdot \ln \frac{R_1}{r} + p_1 \qquad (9)$$



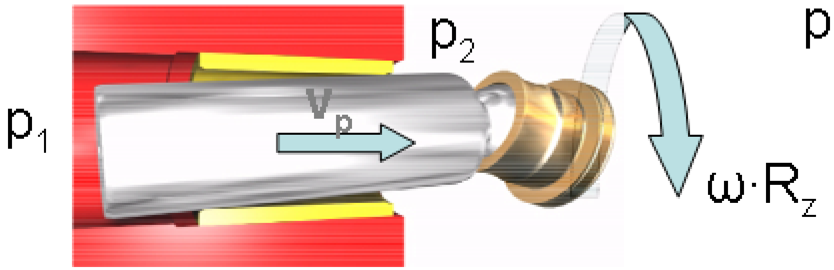
Substituting the volume flow equation Eq. (7) into Eq. (9) we obtain:

$$Q = \frac{(p_2 - p_1) \cdot \pi \cdot h^3}{6 \cdot \mu \cdot \ln \frac{R_1}{R_2}} \qquad (7)$$

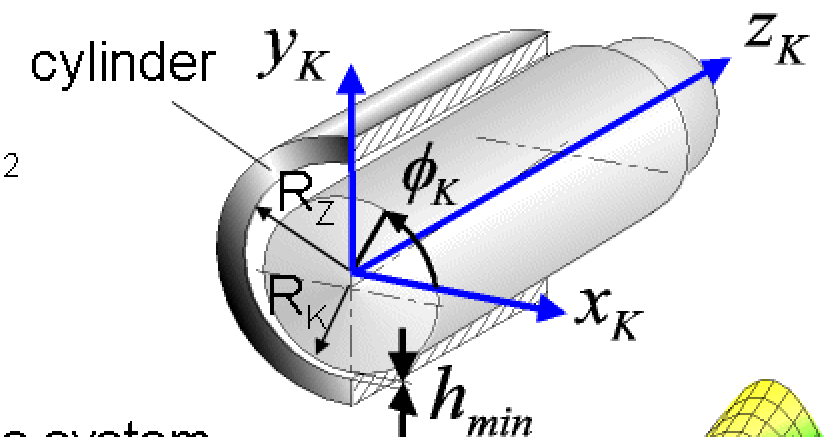
$$\boxed{p = \frac{p_2 - p_1}{\ln \frac{R_1}{R_2}} \cdot \ln \frac{R_1}{r} + p_1} = \frac{5 \cdot 10^5 \,\mathrm{Pa} - 2 \cdot 10^7 \,\mathrm{Pa}}{\ln \frac{12 \cdot 10^{-3}}{25 \cdot 10^{-3}}} \cdot \ln \frac{12 \cdot 10^{-3}}{15 \cdot 10^{-3}} + 2 \cdot 10^7 \,\mathrm{Pa} =$$

$$p = 14.071 \,\mathrm{MPa}$$

Axial and radial piston motion



$p_1 > p_2$



Unrolled gap in Cartesian co-ordinates system

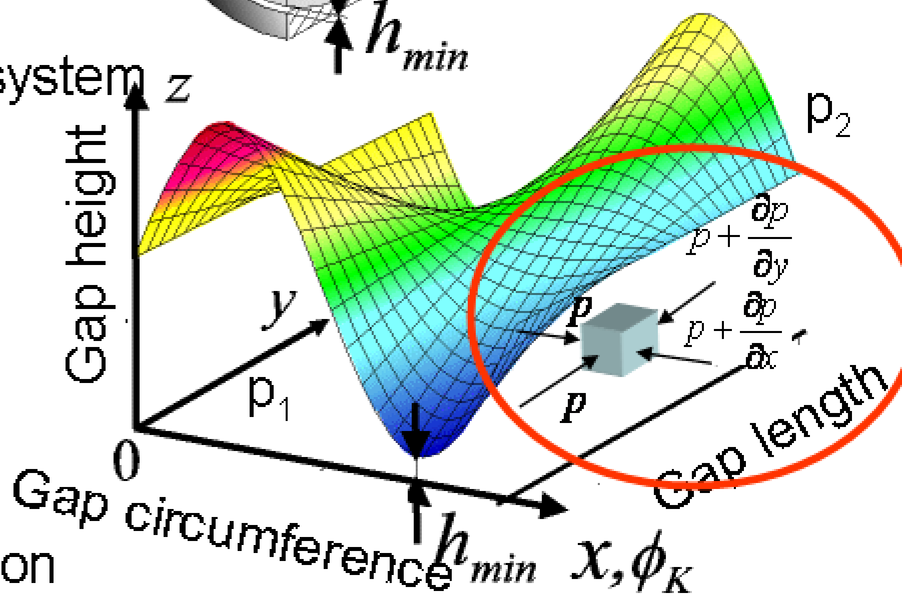
$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 v_x}{\partial z^2}$$

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v_y}{\partial z^2}$$

$x = \phi_K R_Z$

$y = z_K$

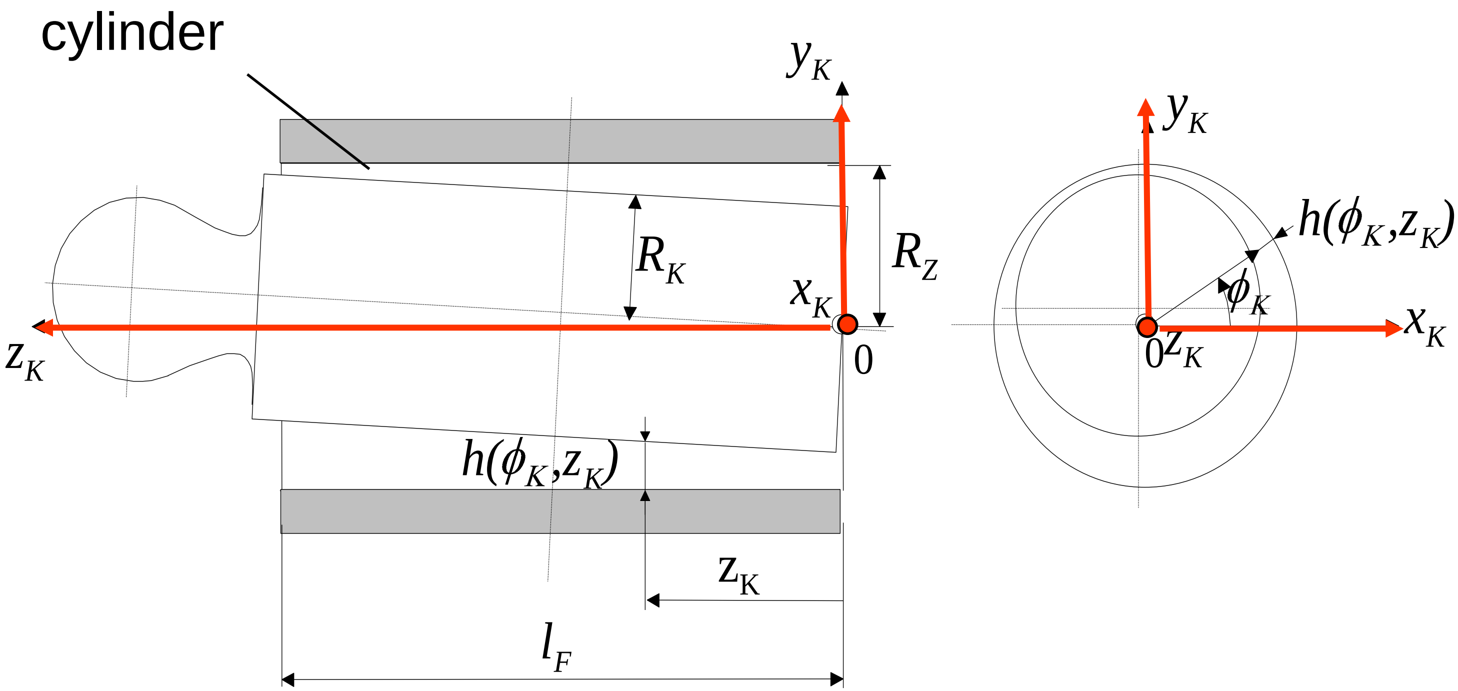
$z = h(\phi_K, z_K)$



Gap height due to inclined piston position

$$h(z_K, \phi_K) = \sqrt{(R_Z \cos\phi_K - x_m(z_K))^2 + (R_Z \sin\phi_K - y_m(z_K))^2} - R_K(z_K)$$

Gap height function



$$h(z_K,\phi_K) = \sqrt{(R_Z\cos\phi_K - x_m(z_K))^2 + (R_Z\sin\phi_K - y_m(z_K))^2} - R_K(z_K)$$

Gap between piston & cylinder

After double integration flow velocity yields:

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 v_x}{\partial z^2} \qquad \frac{\partial p}{\partial y} = \mu \frac{\partial^2 v_y}{\partial z^2}$$

$$v_x = \frac{1}{2\mu}\frac{\partial p}{\partial x} z^2 + C_1 z + C_2$$

$$v_y = \frac{1}{2\mu}\frac{\partial p}{\partial y} z^2 + C_3 z + C_4$$

Boundary conditions:

$z=0$ … $v_x=0$ $\quad v_y=0$

$z=h$ … $v_x=\omega \cdot R_z$ $\quad v_y=v_p$

$$v_x = \frac{1}{2\cdot\mu}\cdot\frac{\partial p}{\partial x}\cdot(z^2 - h\cdot z) + \omega\cdot R_z\cdot\frac{z}{h} \qquad v_y = \frac{1}{2\cdot\mu}\cdot\frac{\partial p}{\partial y}\cdot(z^2 - h\cdot z) + v_p\cdot\frac{z}{h}$$

$$v_{xmi} = \frac{1}{h}\int_0^h v_x dz = -\frac{1}{2\cdot\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{h^3}{6} + \omega\cdot R_z\cdot\frac{h}{2}$$

$$v_{ymi} = \frac{1}{h}\int_0^h v_y \cdot dz = -\frac{1}{2\cdot\mu}\cdot\frac{\partial p}{\partial y}\cdot\frac{h^3}{6} + v_p\cdot\frac{h}{2}$$

Gap between piston & cylinder

Considering the continuity equation for incompressible fluid:

$$\mathrm{div}\ \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

After substituting the mean velocities the Reynolds-equation can be derived:

$$\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x}\cdot\frac{h^3}{\mu}\right) + \frac{\partial}{\partial y}\left(\frac{\partial p}{\partial y}\cdot\frac{h^3}{y}\right) = 6\cdot\left(\omega\cdot R_z\cdot\frac{\partial h}{\partial x} + v_p\cdot\frac{\partial h}{\partial y}\right)$$

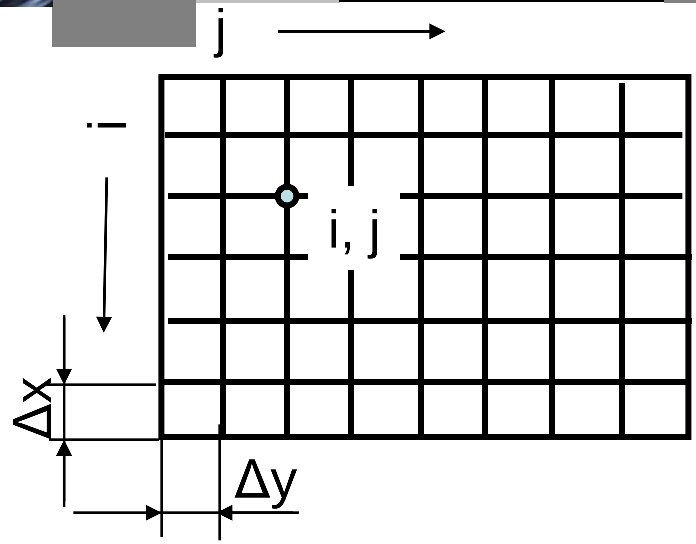
and taking into account time dependent change of gap height the Reynolds equation becomes:

$$\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x}\cdot\frac{h^3}{\mu}\right) + \frac{\partial}{\partial y}\left(\frac{\partial p}{\partial y}\cdot\frac{h^3}{y}\right) = 6\cdot\left(\omega\cdot R_z\cdot\frac{\partial h}{\partial x} + v_p\cdot\frac{\partial h}{\partial y} + 2\cdot\frac{\partial h}{\partial t}\right)$$

$\Rightarrow$ This partial differential equation has to be solved numerically

Numerical solution of gap flow

Applying the method of finite differences the Reynolds equation can be written:



$$\frac{\partial^2 p}{\partial x^2} = \frac{p_{i+1,j} - 2 \cdot p_{i,j} + p_{i-1,j}}{\Delta x^2} \quad (1)$$

$$\frac{\partial^2 p}{\partial y^2} = \frac{p_{i,j+1} - 2 \cdot p_{i,j} + p_{i,j-1}}{\Delta y^2} \quad (2)$$

$$\frac{\partial p}{\partial x} = \frac{p_{i+1,j} - p_{i-1,j}}{2 \cdot \Delta x} \quad (3) \qquad \frac{\partial p}{\partial y} = \frac{p_{i,j+1} - p_{i,j-1}}{2 \cdot \Delta y} \quad (4)$$

Different methods available:
- -finite differences
- -finite volumes
- -finite elements

Substituting Eq. (1), (2), (3) and (4) into the Reynolds- equation we obtain:

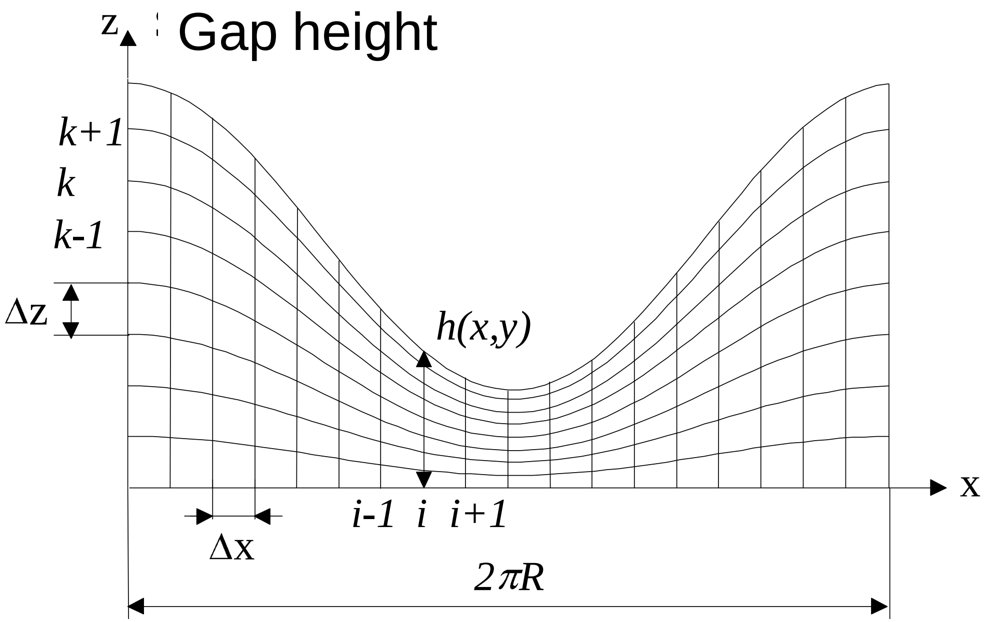
$$p_{i,j} = A \cdot p_{i+1,j} + B \cdot p_{i-1,j} + C \cdot p_{i,j+1} + D \cdot p_{i,j-1} + E \quad (5)$$

Iterative solution of Eq. (5) to calculate the pressure $p_{i,j}$

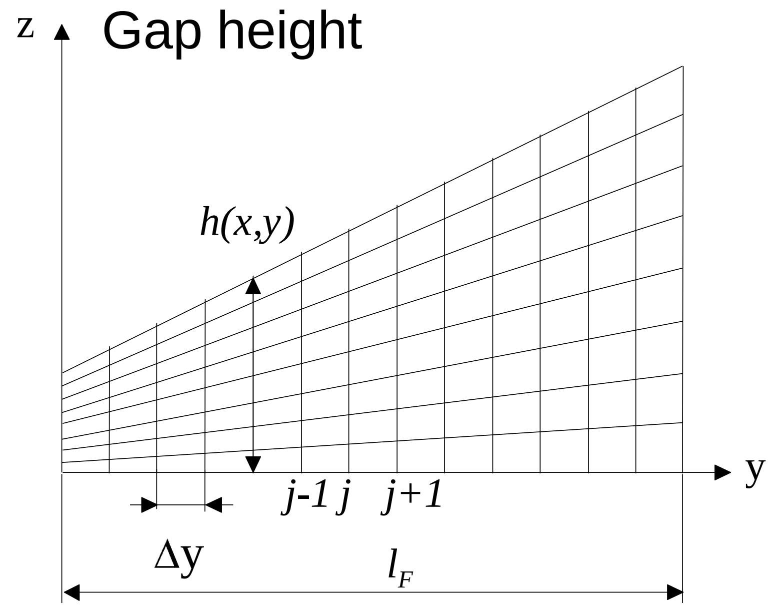
Numerical solution of gap flow

The gap grid

i=0,1,...L j=0,1,...M k=0,1,...N



Gap circumference

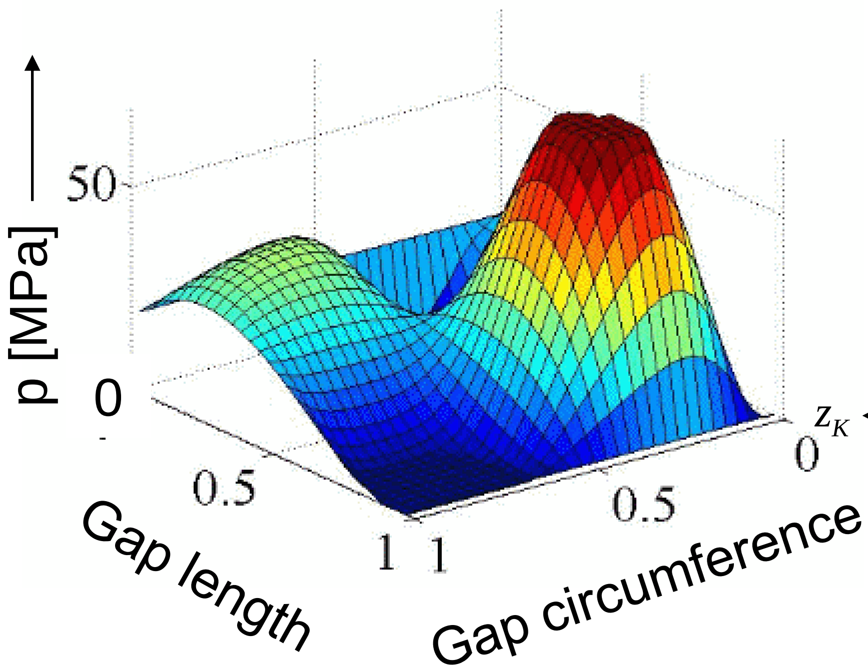


Gap length

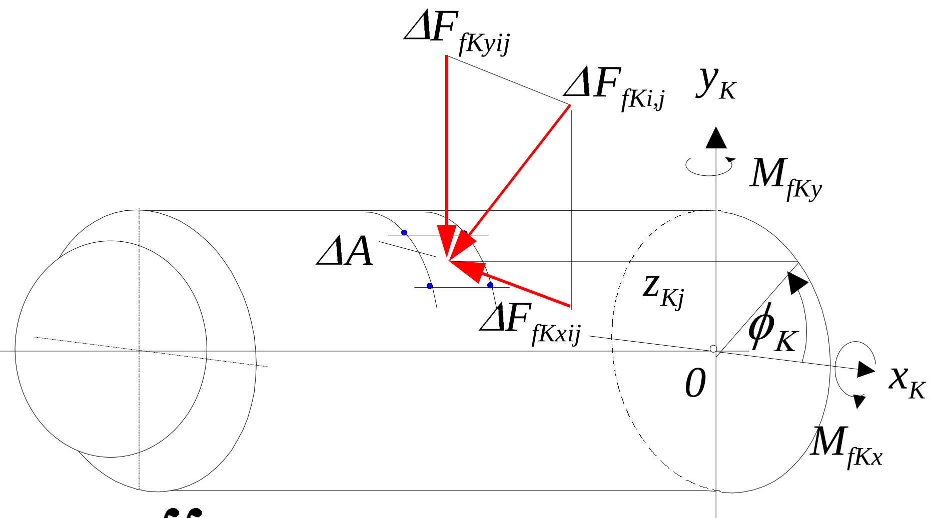
$$\Delta x = \frac{2\pi R_z}{L} \qquad \Delta y = \frac{l_F}{M} \qquad \Delta z = \frac{h(x,y)}{N}$$

Gap Parameter

Pressure field between piston and cylinder



Load capacity of the gap



$$F_{fK} = \iint\limits_A p \cdot dx \cdot dy$$

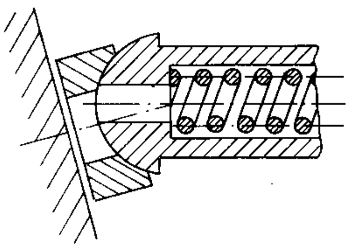
$$\Delta F_{fK_{ij}} = \bar{p}_{ij} \cdot \Delta A = \bar{p}_{ij} \cdot \Delta x \cdot \Delta y$$

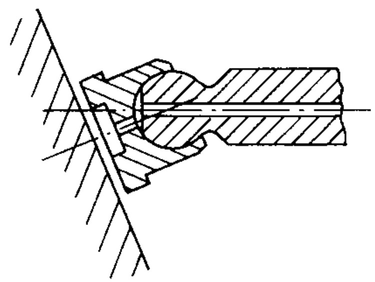
with: $$\bar{p}_{i,j} = \frac{1}{4}(p_{i,j} + p_{i+1,j} + p_{i,j+1} + p_{i+1,j+1})$$

Contact between piston and swash plate

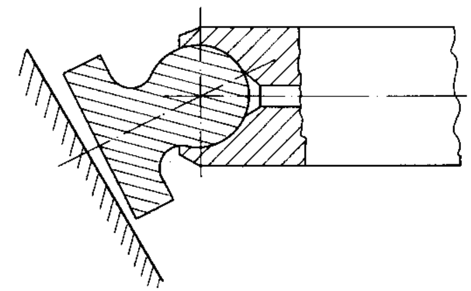
Support by slippers → Increase of volumetric losses

Hydrostatically balanced



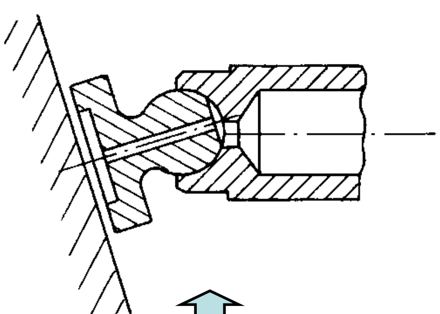


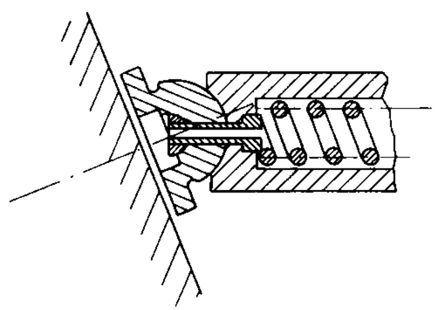
Hydrodynamically balanced



Requires oil filled case

Additional hydrodynamic effect usable





Special surface geometry can support hydrodynamic effect

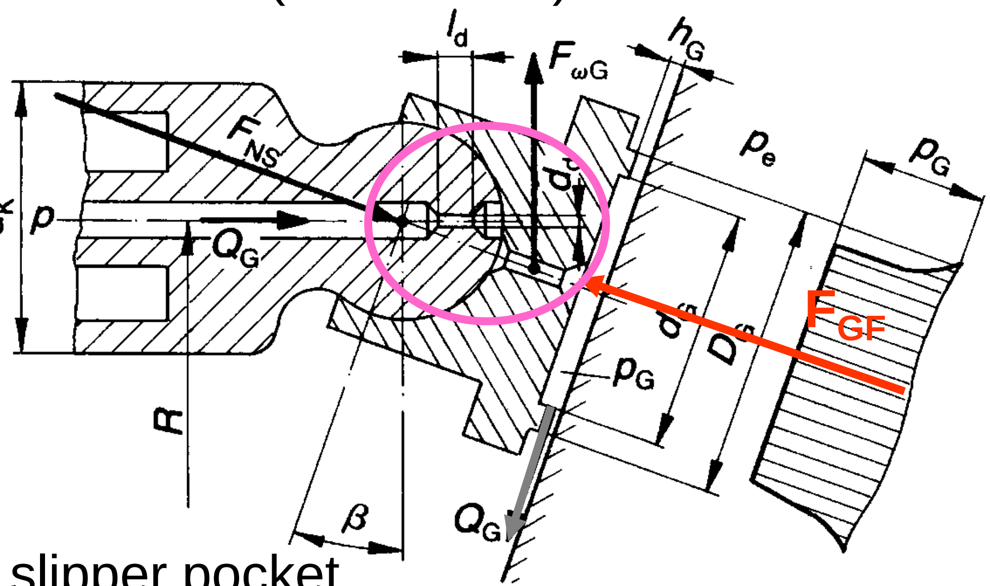
Lower loading of piston-cylinder

→ Suitable for high pressure and high speed

Slipper Design

$$F_{GF} = B_G \cdot F_{NS}$$

B_G… slipper balance (0.95 – 0.99)



Gap flow assuming constant gap height

$$Q_G = \frac{\pi \cdot h_G^3}{6 \cdot \mu \cdot \ln \dfrac{D_G}{d_G}} \cdot \left(p_G - p_e\right) \qquad (1)$$

Flow from displacement chamber to slipper pocket

Through a small orifice

$$Q_G = \frac{\pi \cdot d_d^2}{4} \cdot \alpha_D \cdot \sqrt{\frac{2}{\rho}} \cdot \sqrt{p - p_G} \qquad (2)$$

or laminar throttle

$$Q_G = \frac{\pi \cdot d_d^4}{128 \cdot \mu \cdot l_d} \cdot \left(p - p_G\right) \qquad (3)$$

$$Q_G = \frac{\pi \cdot h_G^3 \cdot d_d^4}{\mu \cdot \left(6 \cdot d_d^4 \cdot \ln \dfrac{D_G}{d_G} + 128 \cdot h_G^3 \cdot l_d\right)} \cdot \left(p - p_e\right)$$

Slipper Design

Load ability F_{GF} due to pressure field under the slipper

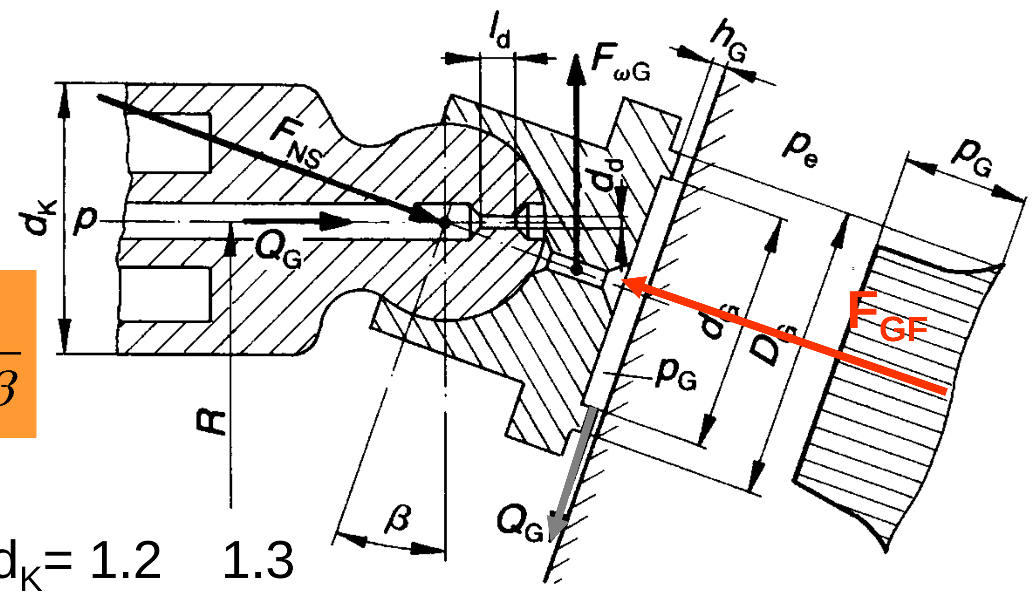
$D_G/d_G = 1.2 \quad 1.3$

$$F_{GF} = \frac{1}{8} \cdot p_G \cdot \pi \cdot \frac{D_G^2 - d_G^2}{\ln \dfrac{D_G}{d_G}} \qquad (1)$$

$$F_{GF} = B_G \cdot F_{NS} \qquad (2)$$

$$F_{NSi} = \frac{F_{AKi}}{\cos \beta}$$

$$p_G = \frac{8 \cdot F_{NS} \cdot \ln \dfrac{D_G}{d_G}}{\pi \cdot \left(D_G^2 - d_G^2\right)} \cdot B_G$$



$D_G/d_K = 1.2 \quad 1.3$

$$\tau = \mu \cdot \frac{\omega \cdot R}{h_G}$$

Viscous friction force:

$$F_{TG} = \frac{\pi}{4} \cdot \left(D_G^2 - d_G^2\right) \cdot \mu \cdot \frac{\omega \cdot R}{h_G}$$

Losses due to friction:

$$P_{ST} = F_{TG} \cdot \omega \cdot R = \frac{\pi}{4} \cdot \left(D_G^2 - d_G^2\right) \cdot \mu \cdot \frac{\omega^2 \cdot R^2}{h_G}$$

Losses due to gap flow:

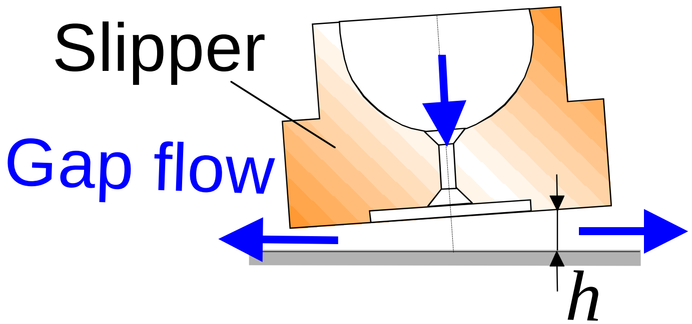
$$P_{SQ} = Q_G \cdot p$$

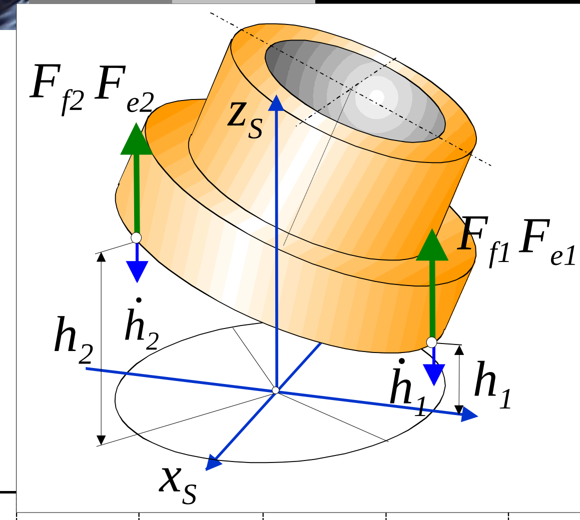
Power loss:

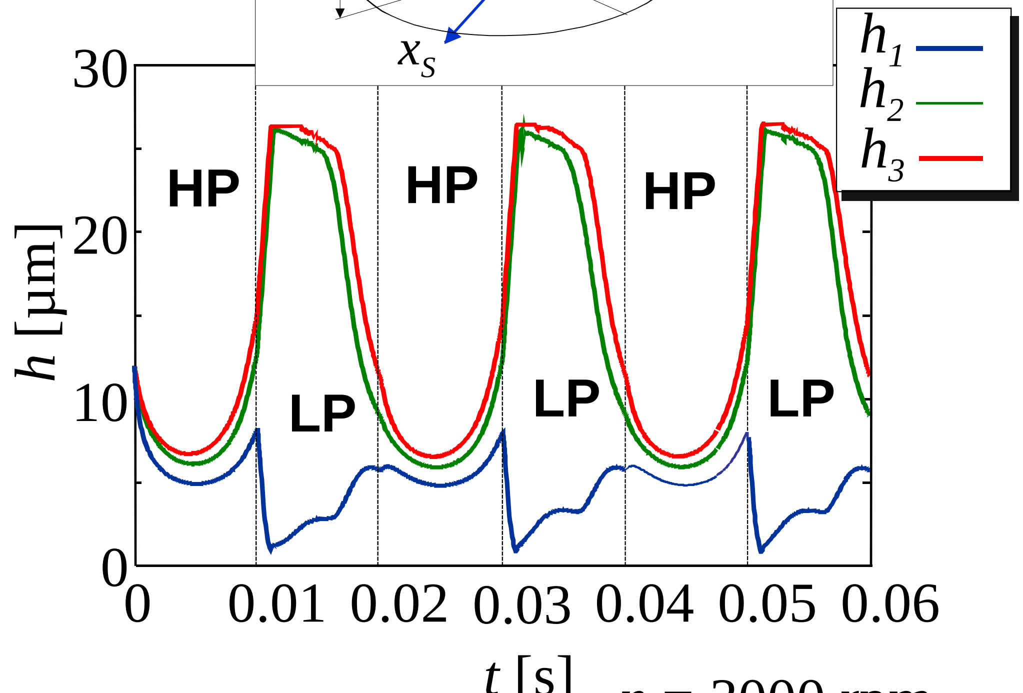
$$P_{SG} = P_{SQ} + P_{ST}$$

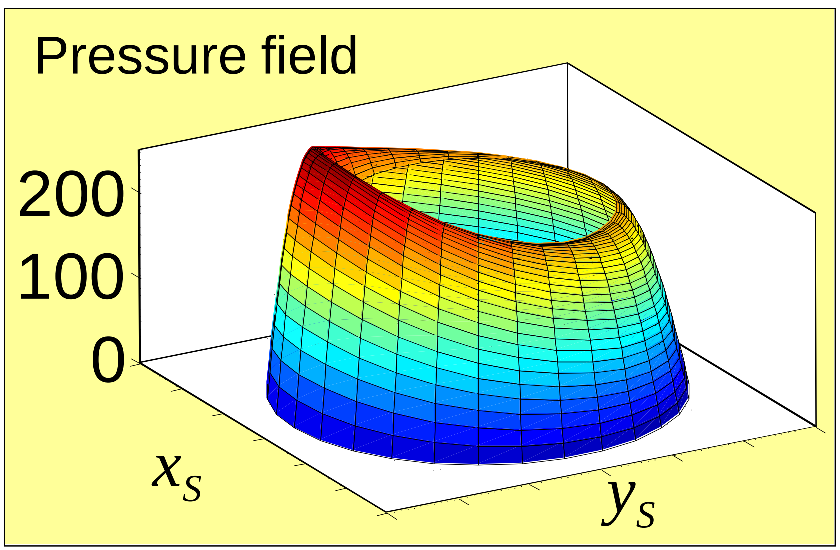
$$\left[\frac{dP_{SG}}{dh_G}\right]_{h_G = h_{Gopt}} = 0$$

Gap flow simulation requires the determination of gap height





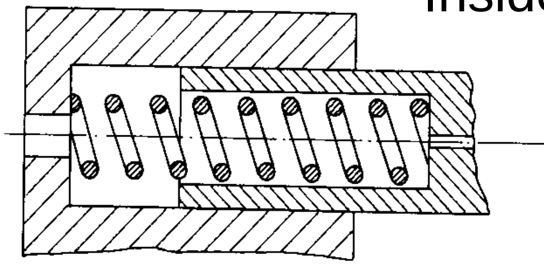




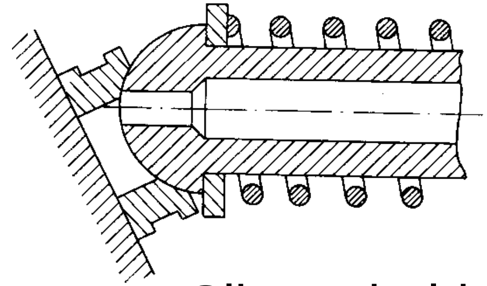
Operating parameter: $n = 3000$ rpm, $\Delta p = 200$ bar

Slipper hold down using springs

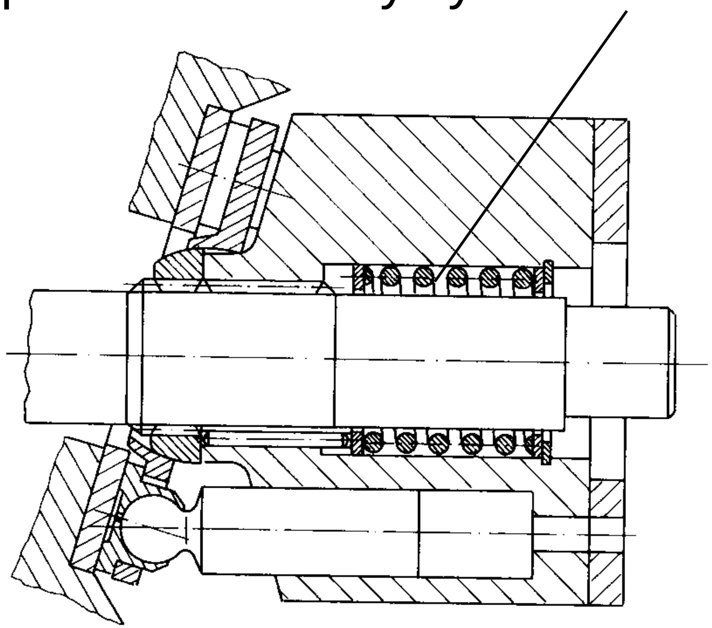
Inside the piston



Outside the piston

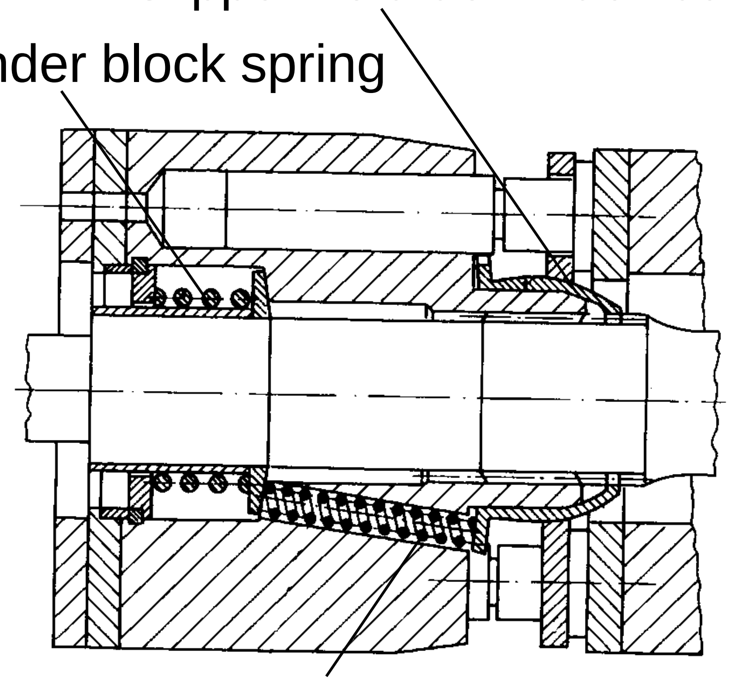


Slipper hold down by cylinder block spring



Slipper hold down device

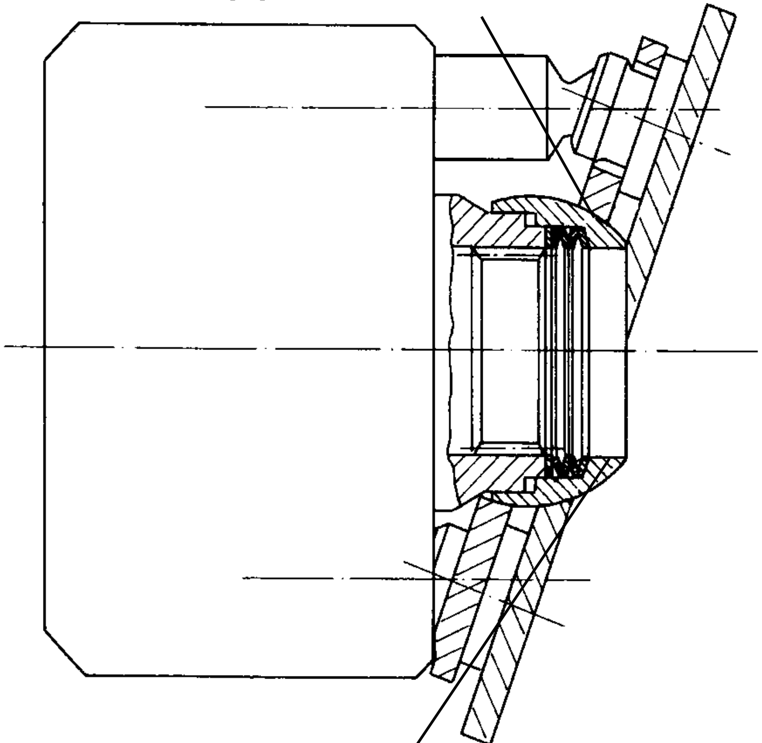
Cylinder block spring



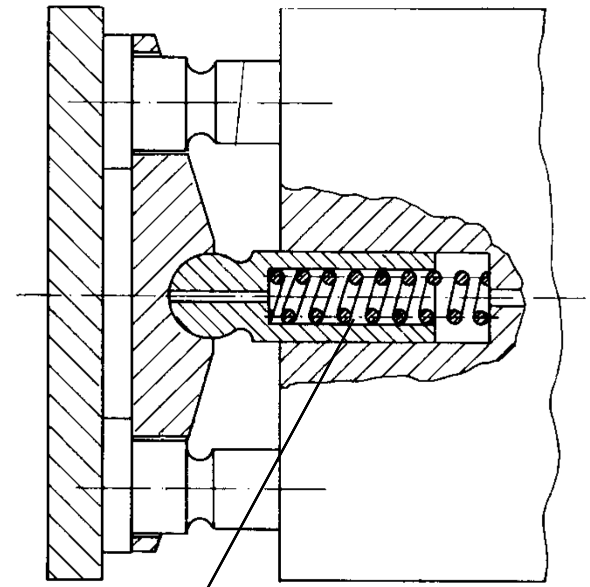
Slipper hold down spring

Slipper hold down using springs

Slipper hold down device



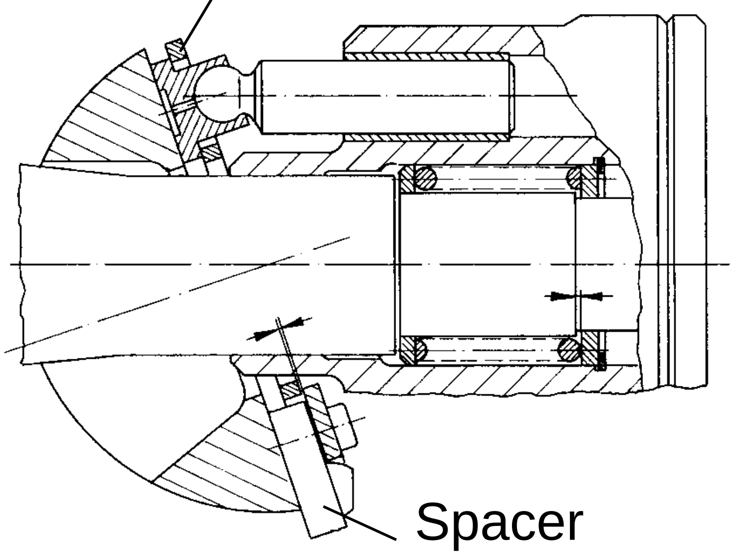
Slipper hold down spring



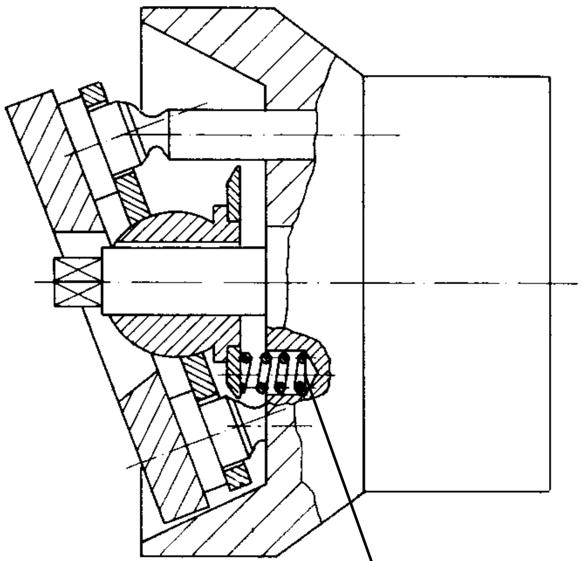
Slipper hold down spring

Slipper hold down device

Fixed slipper hold down



Spacer



Slipper hold down spring