Design and Modeling of Fluid Power Systems

ME 597/ABE 591 Lecture 4

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Content

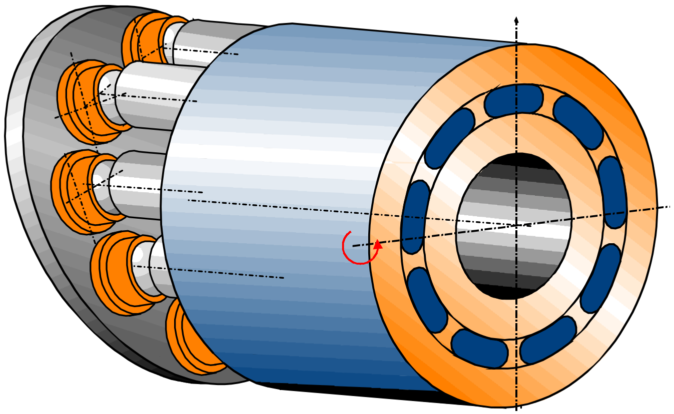
Displacement machines – design principles & scaling laws

Power density comparison between hydrostatic and electric machines

Volumetric losses, effective flow, flow ripple, flow pulsation

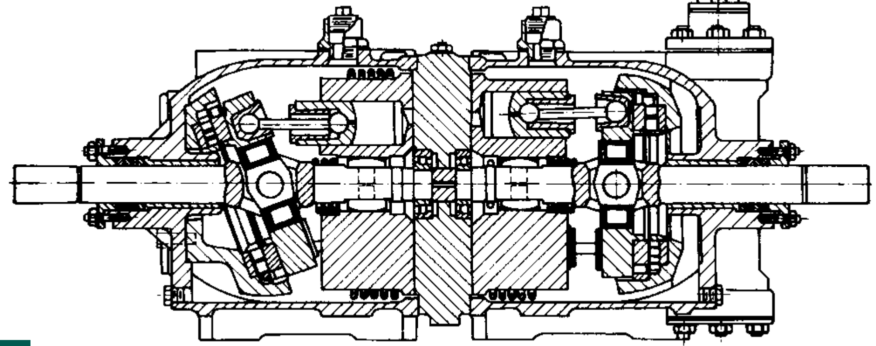
Steady state characteristics of an ideal and real displacement machine

Torque losses, torque efficiency



Historical Background

Hydrostatic transmissiom

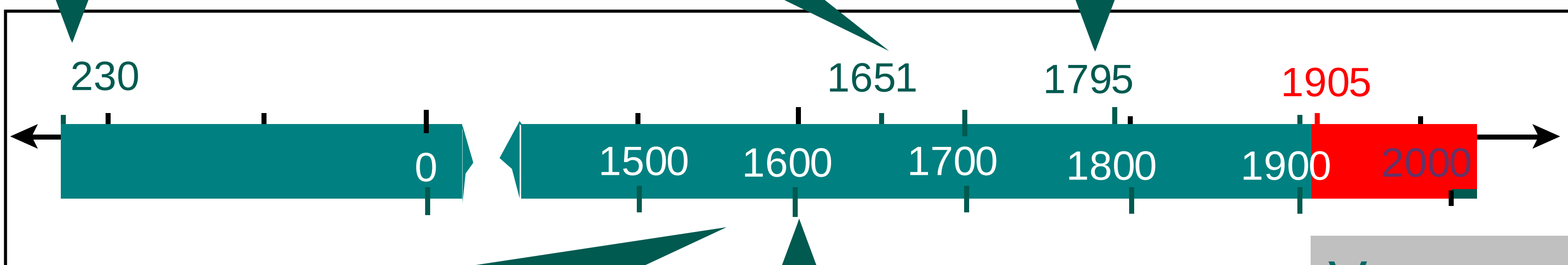


Williams und Janney

Archimedes

Pascal

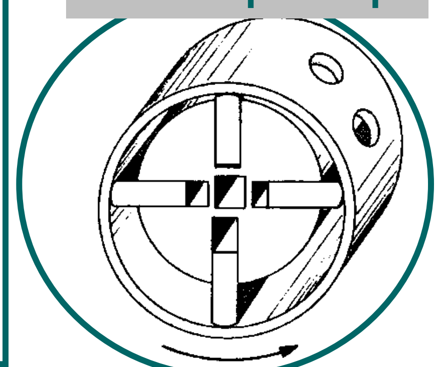
Bramah



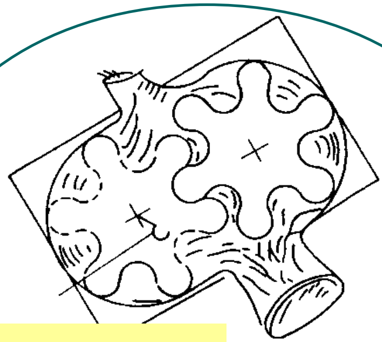
Kepler

Ramelli

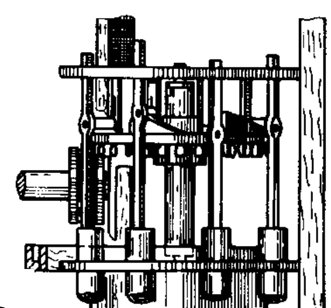
Vane pump



Gear pump

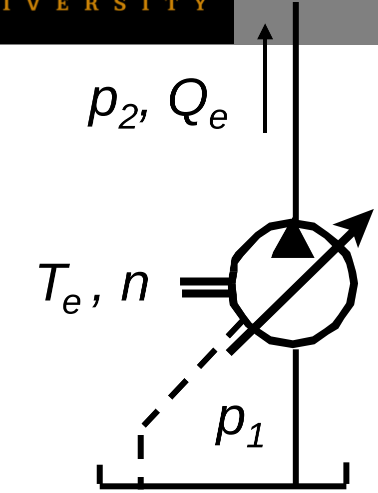


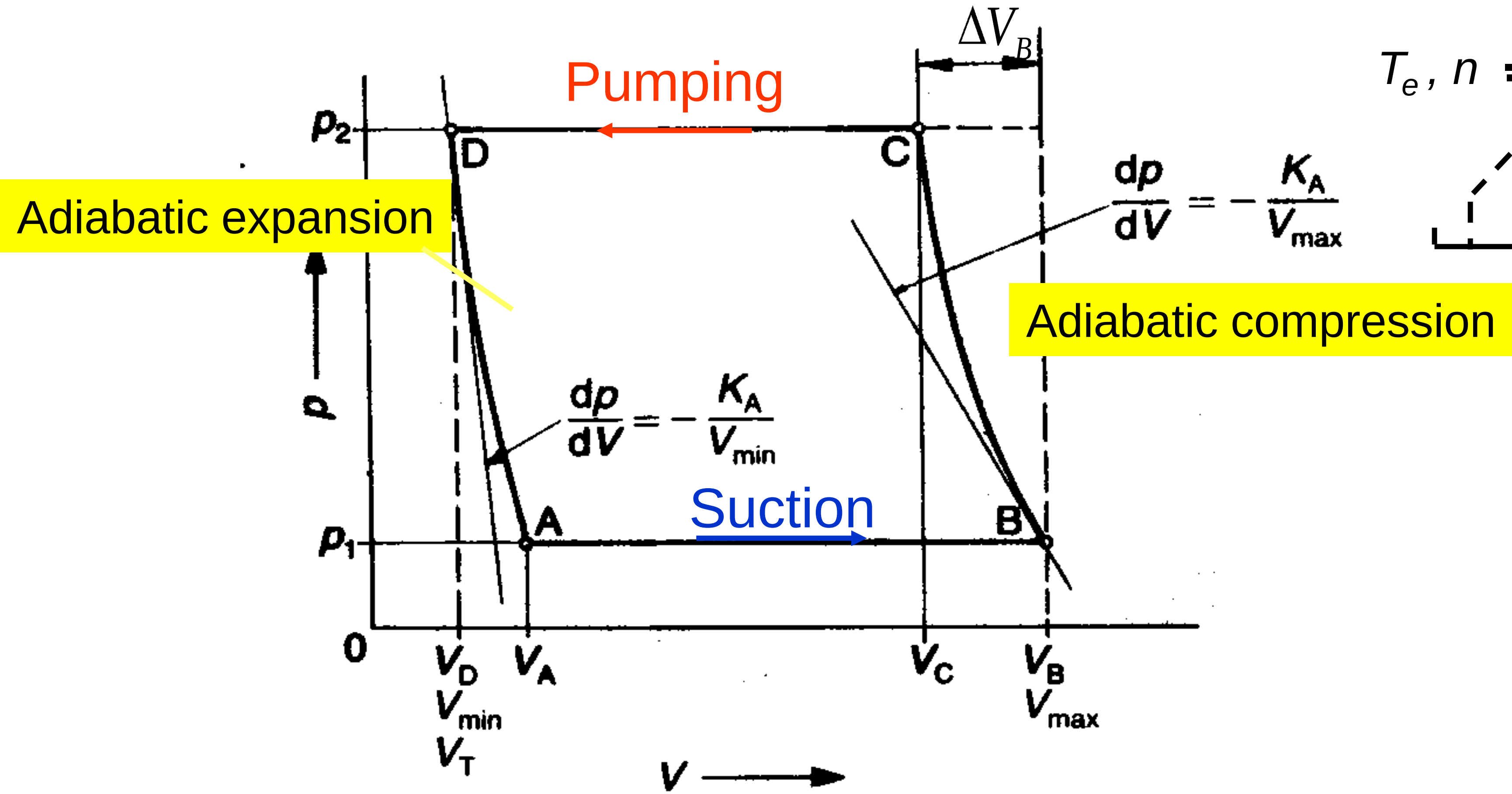
Axial Piston Pump



Displacement machine

due to compressibility of a real fluid





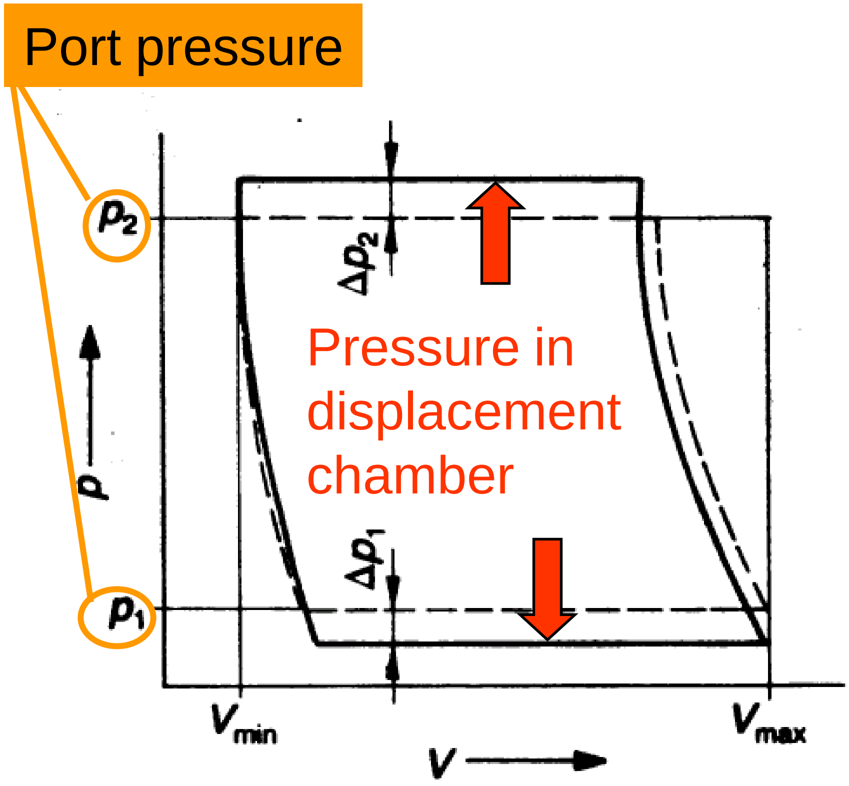
$V_{min}=V_T$ with V_T .. dead volume

K_A.. adiabatic bulk modulus

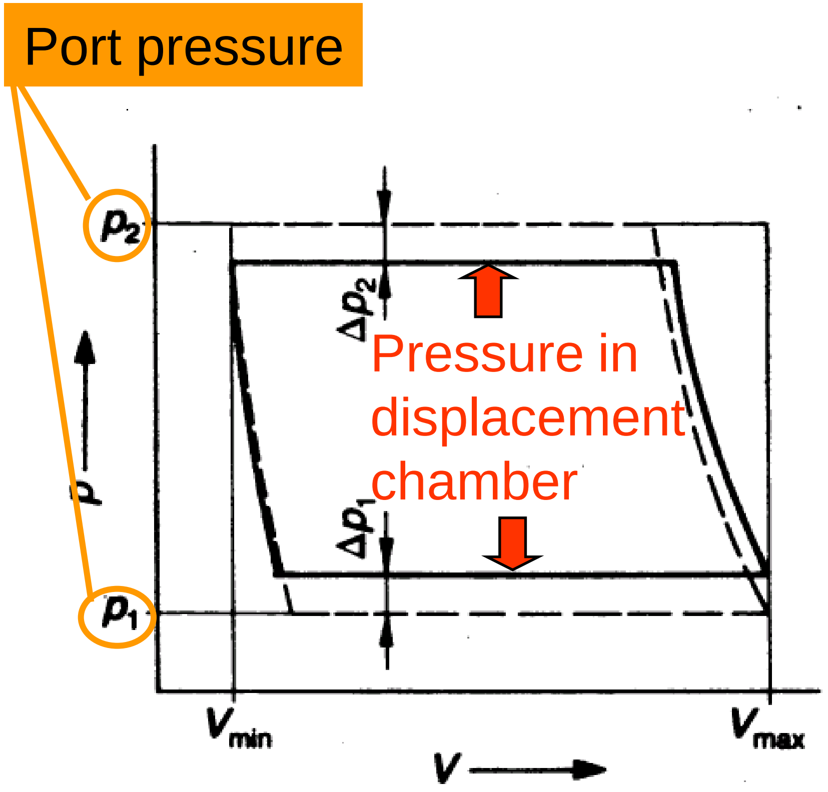
Displacement machine

due to viscosity & compressibility of a real fluid

Pressure drop between displacement chamber and port



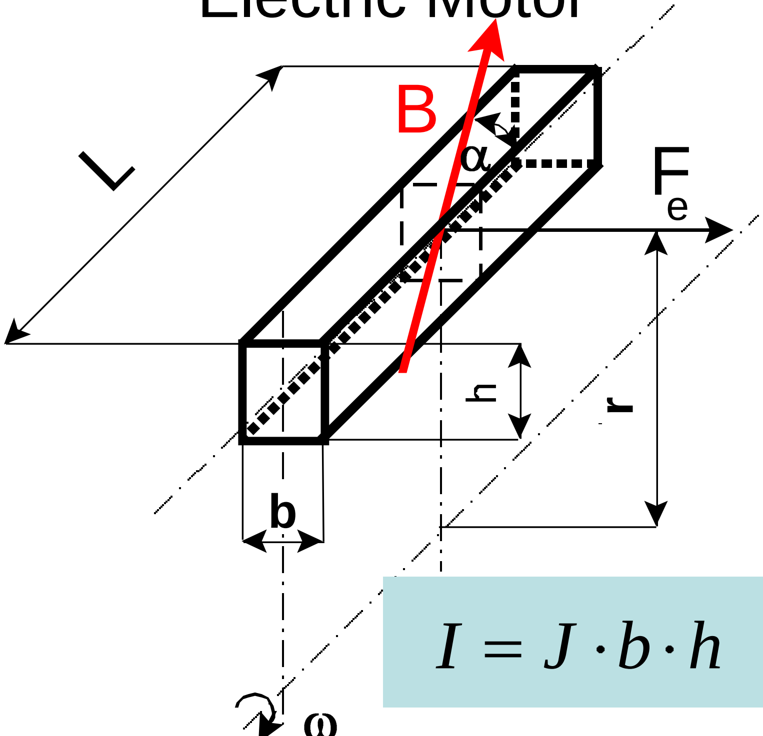
Pump



Motor

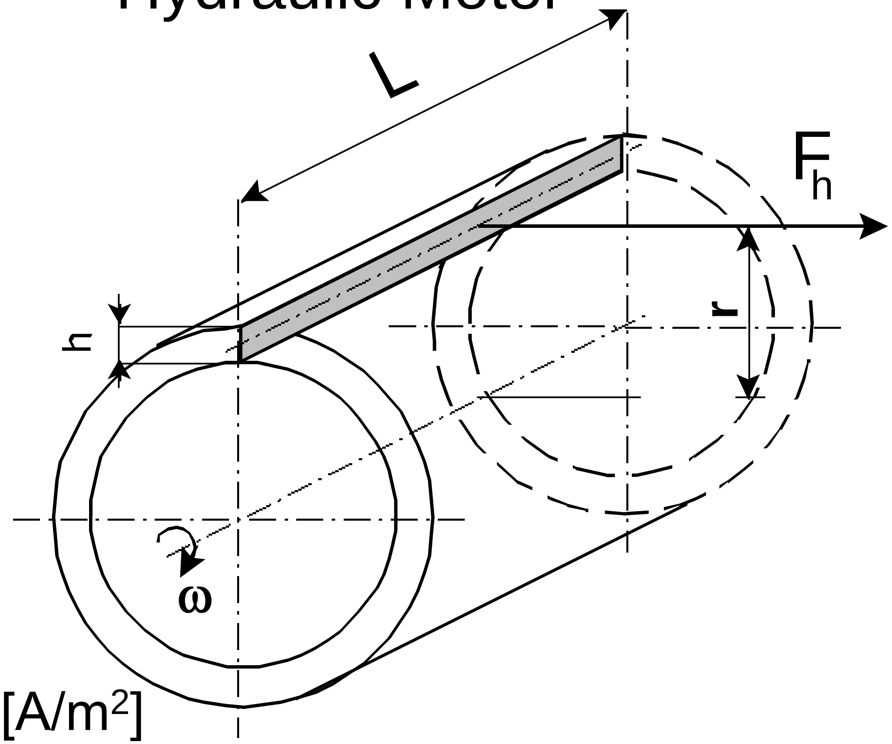
Power Density

Electric Motor



$$I = J \cdot b \cdot h$$

Hydraulic Motor



J… current density [A/m²]

$$F_e = I \cdot B \cdot L \cdot \sin\alpha$$ with *I* current [A]

$$F_h = p \cdot L \cdot h$$

B … magnetic flux density [T] or [Vs/m²]

Torque: $$T = I \cdot B \cdot L \cdot \sin\alpha \cdot r$$

$$T = p \cdot L \cdot h \cdot r$$

Example

Power: $P = T \cdot \omega = T \cdot 2 \cdot \pi \cdot n$

For electric motor follows: $P = I \cdot B \cdot L \cdot r \cdot 2 \cdot \pi \cdot n$ assuming α=90

For hydraulic motor follows: $P = p \cdot L \cdot h \cdot r \cdot 2 \cdot \pi \cdot n$

Force density:

Electric Motor

$$\frac{F_e}{L \cdot h} = \frac{J \cdot b \cdot h \cdot B \cdot L}{L \cdot h} = J \cdot b \cdot B$$

$$7.6 \cdot 10^6 \, \mathrm{A \cdot m^{-2}} \cdot 1.8 \, \mathrm{Vs \cdot m^{-2}} \cdot 3 \cdot 10^{-3} \, \mathrm{m} = 4.1 \cdot 10^4 \, \mathrm{Pa}$$

Hydraulic Motor

$$\frac{F_h}{L \cdot h} = p$$

up to $5 \cdot 10^7 \, \mathrm{Pa}$

with a cross section area of conductor: $9 \cdot 10^{-6} \, \mathrm{m^2}$

Mass / Power Ratio

Electric Machine

Positive displacement machine

$$\frac{\text{mass}}{\text{power}} = 1 \; \ldots. \; 15 \text{ kg/kW}$$

0.1 … 1 kg/kW

Positive displacement machines (pumps & motors) are:

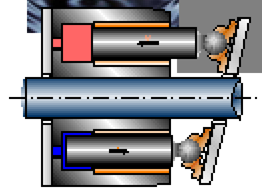
- 10 times lighter
- min. 10 times smaller
- much smaller mass moment of inertia (approx. 70 times)

much better dynamic behavior of displacement machines

Displacement Machines

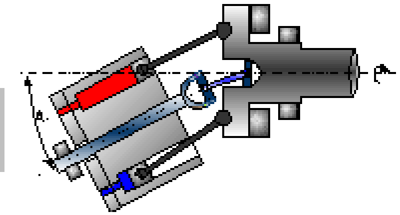
Piston Machines

Axial Piston Machines



Swash Plate Machines

Bent Axis machines



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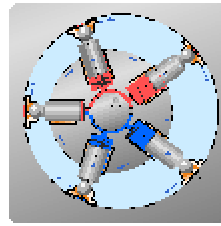
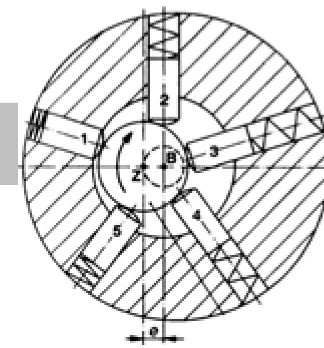
In-line Piston Machines

Radial Piston Machines



with external piston support

with internal piston support



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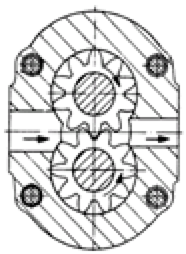
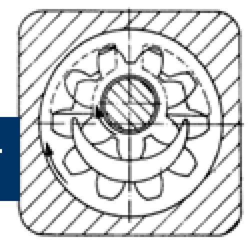
Gear Machines

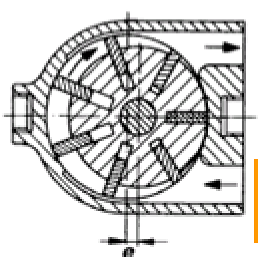


External Gear

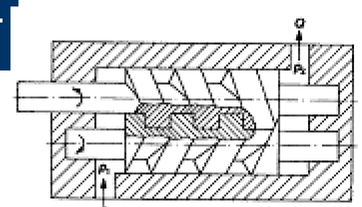
Internal Gear



Annual Gear



Vane Machines



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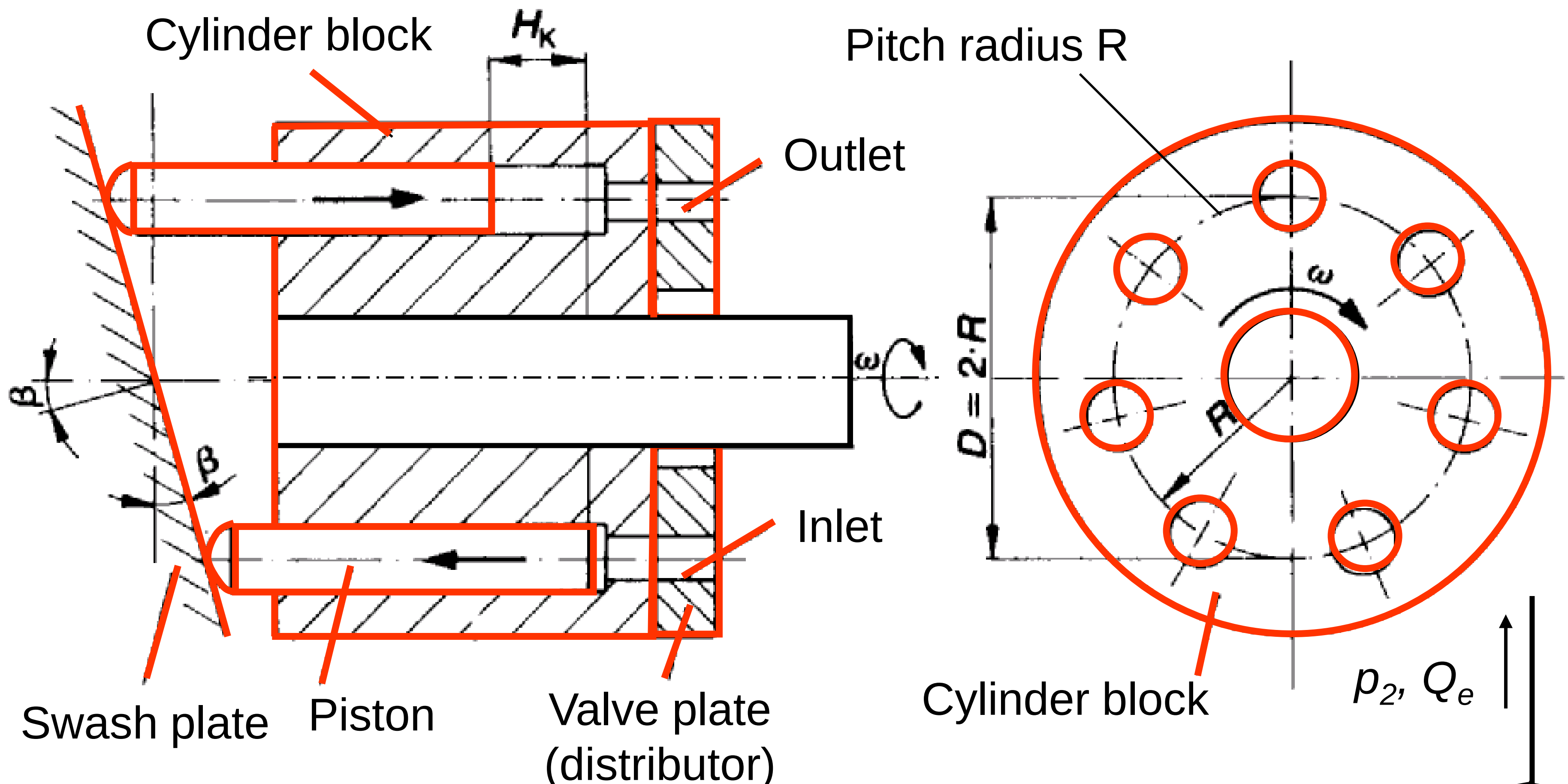
Screw Machines

others

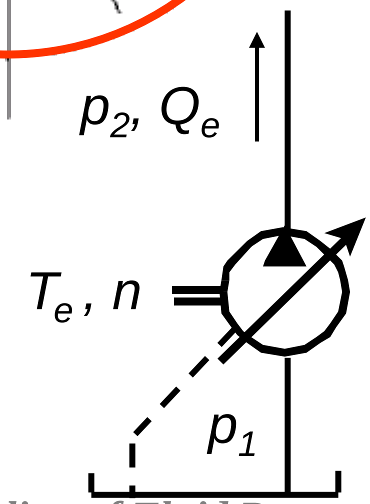
Fixed displacement machines ⟷ Variable displacement machines

Axial Piston Pumps



Piston stroke = f (ß,R)

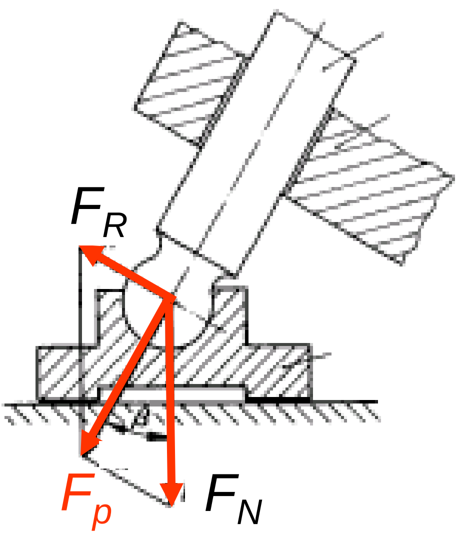
Variable displacement pump
Requires continous change of ß

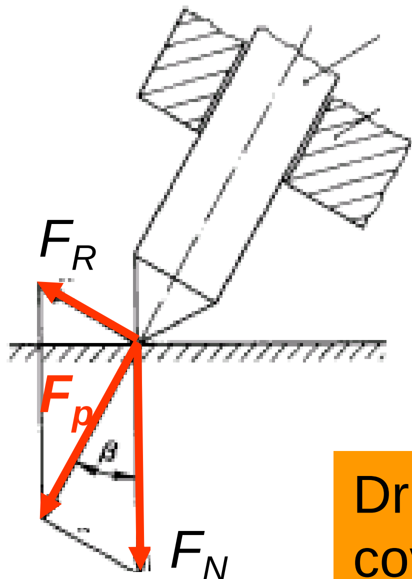


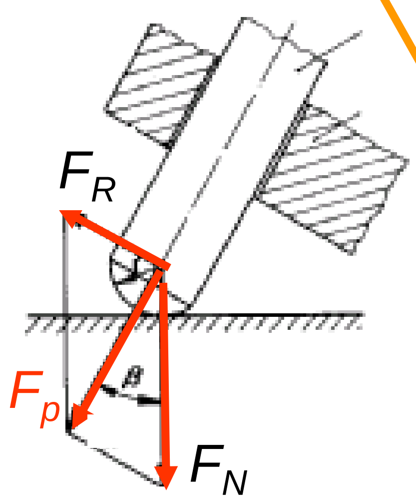
Torque generation on cylinder block

Torque generation on „swash plate“

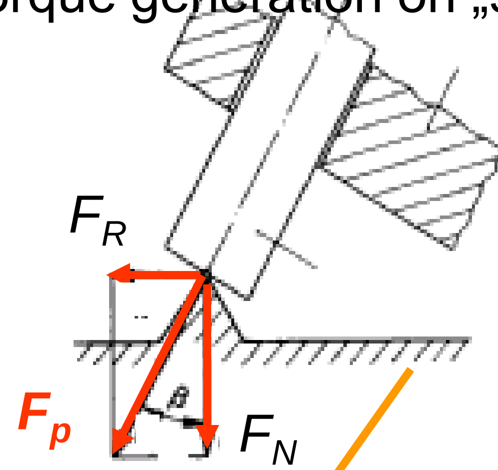
Swash plate design

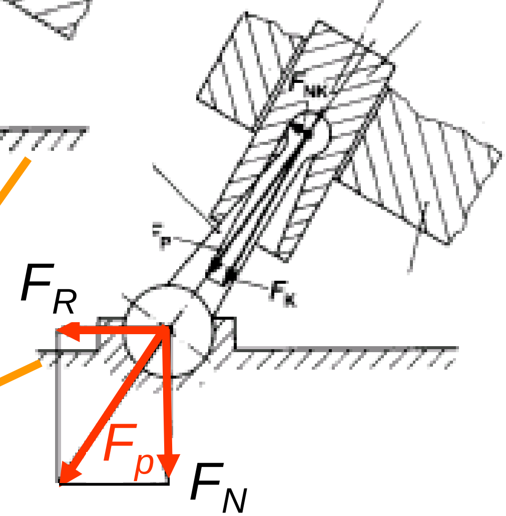




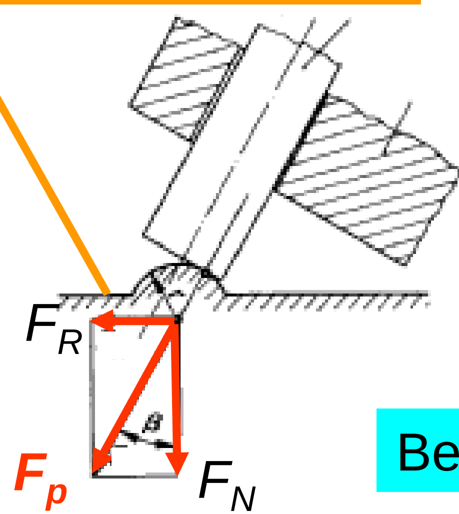


Radial force F_R exerted on piston!





Driving flange must cover radial force

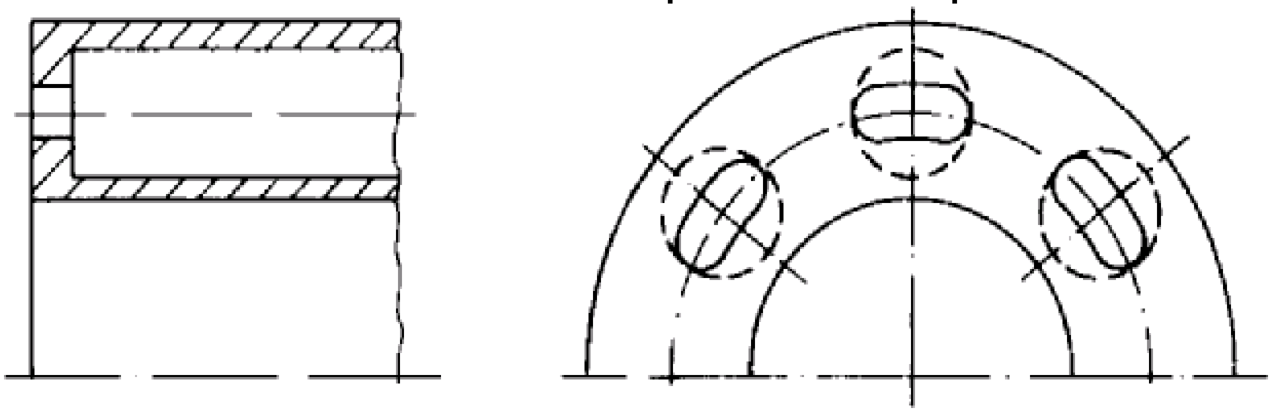


Bent axis machines

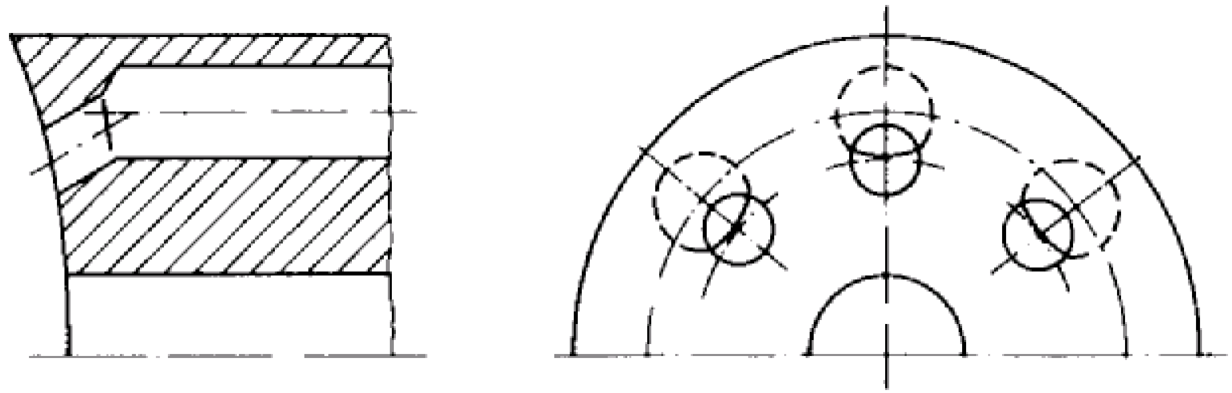
Axial Piston Pumps

Openings in cylinder bottom

In case of plane valve plate



In case of spherical valve plate

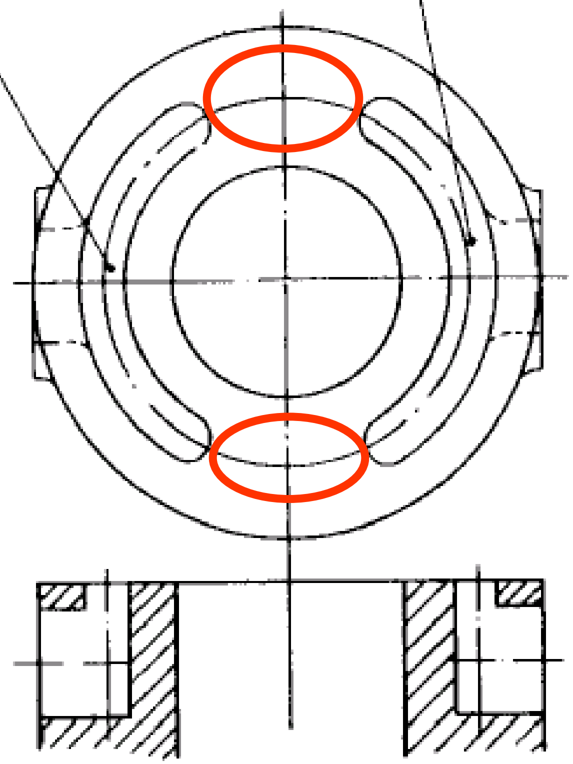


Axial Piston Pumps

Plane valve plate

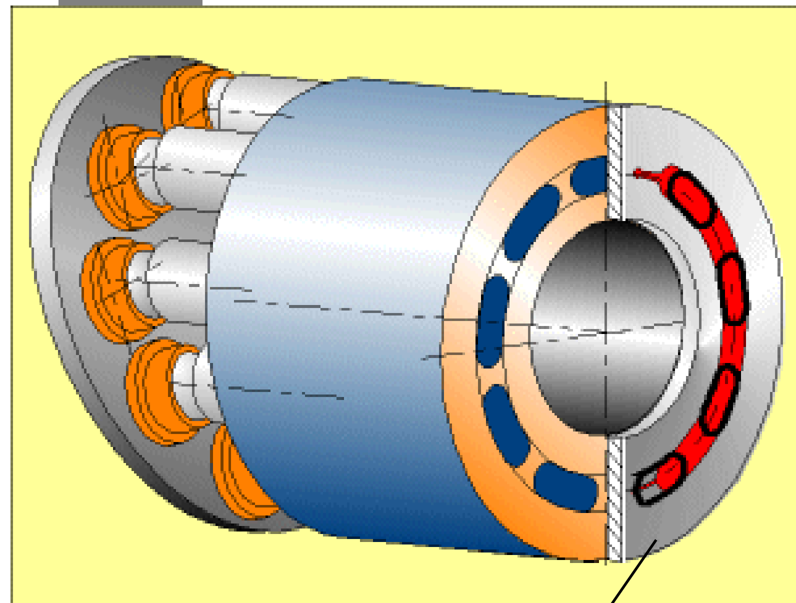
Inlet opening

Outlet opening



Inlet

Outlet



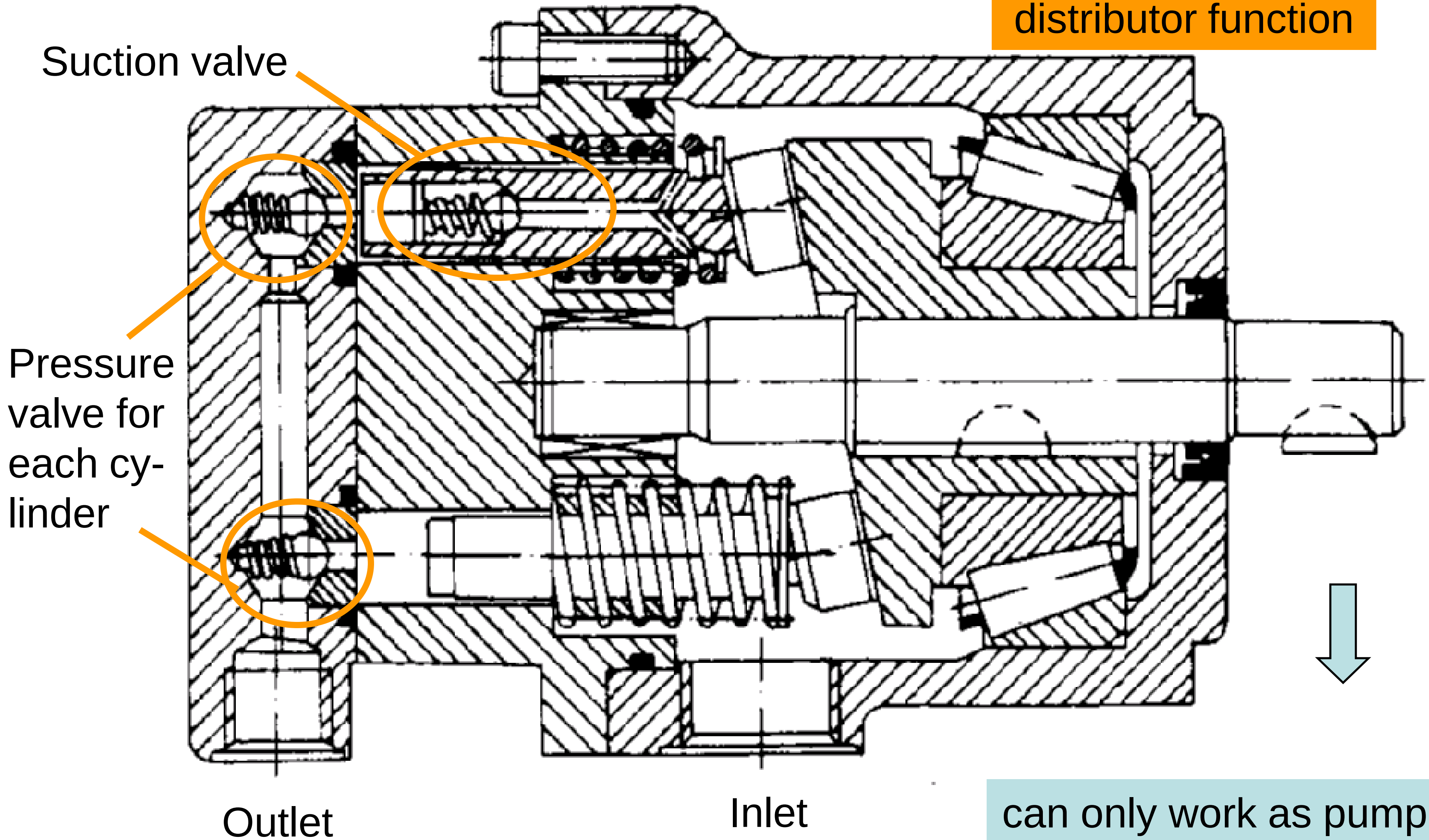
Plane valve plate

Connection of displacement chambers with suction and pressure port

Axial Piston Pumps

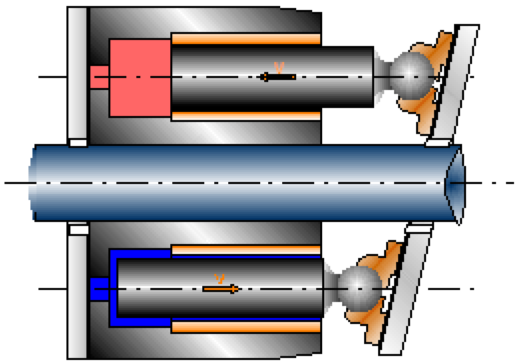
Kinematic reversal: pump with rotating swash plate

Check valves fulfill distributor function



can only work as pump

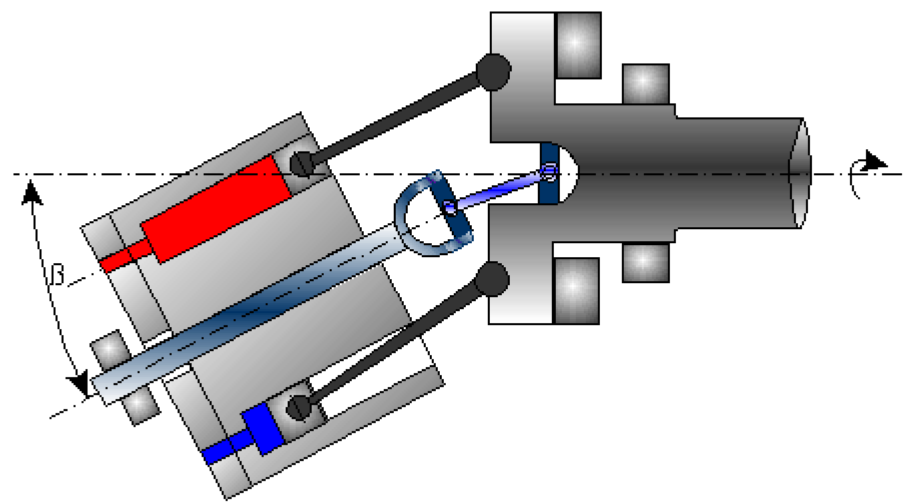
Comparison of axial piston pumps



- Simple and compact design
- Short response time, high bandwidth
- Through going shaft
- Long service life, low loaded bearings

- Limited swash plate angle $ß_{max}$ ca. 21°
- High radial piston forces

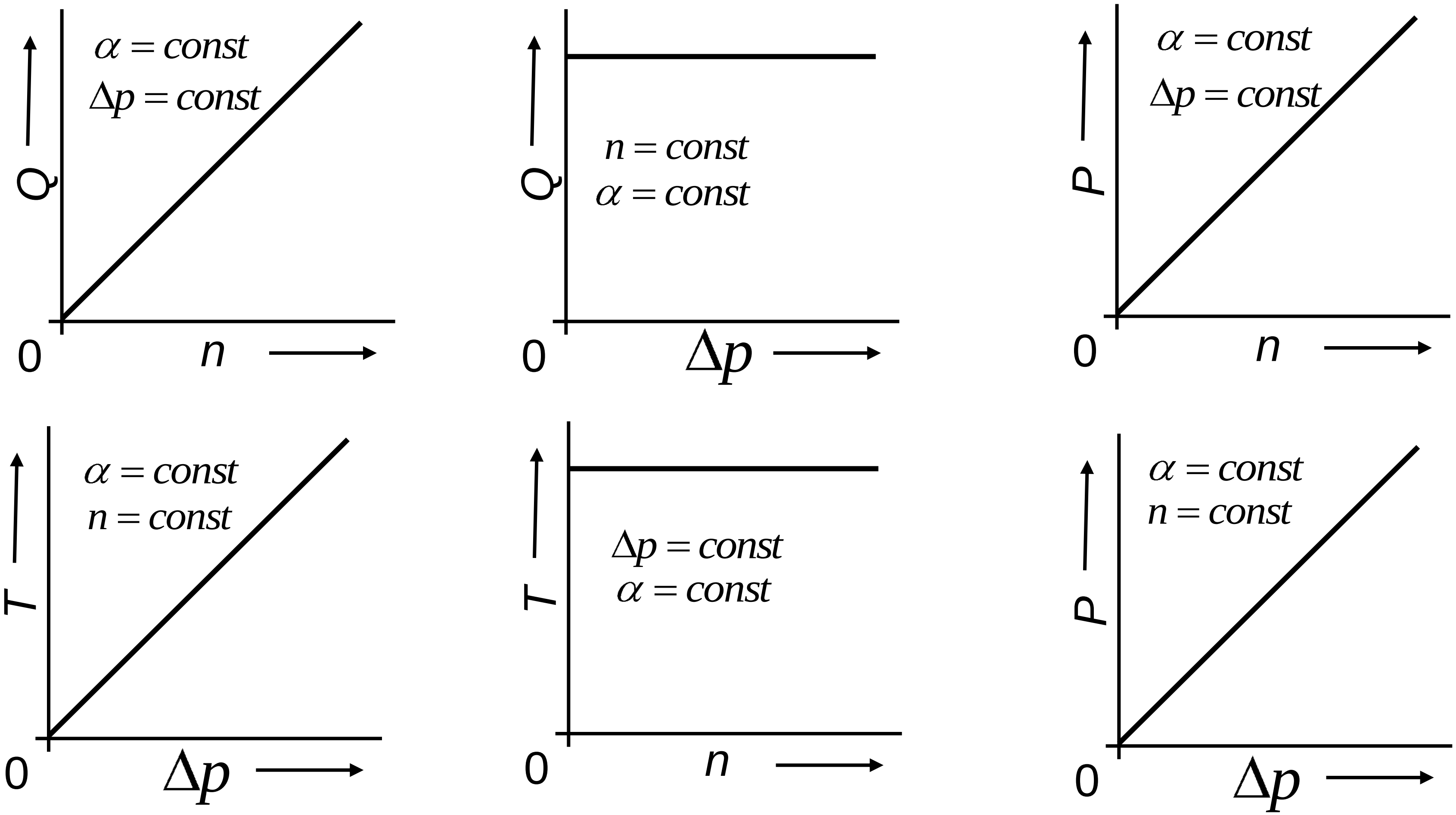
- higher max. speed
- Angle ß up to 45°
- Less losses
- High loaded bearings
- Expensive design
- Synchronisation required

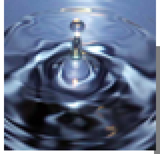


Steady state characteristics

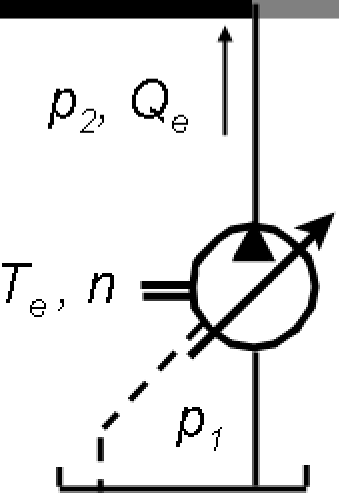
ideal displacement machine

Displacement volume of a variable displacement machine: $V = \alpha \cdot V_{\max}$





Scaling laws



The pump size is determined by the displacement volume V [cm³/rev]. Usually a proportional scaling law, conserving geometric similarity, is applied, resulting in stresses remaining constant for all sizes of units.

$$T = \frac{\Delta p \cdot V}{2 \cdot \pi} \qquad Q = V \cdot n \qquad \Delta p = p_2 - p_1$$

First Order Scaling Laws : λ … linear scaling factor

$L = \lambda \,.\, L_0$	$T = \lambda^3 .\, T_0$	$\boxed{P = \lambda^2 .\, P_0}$
$\boxed{V_i = \lambda^3 .\, V_{i0}}$	$m = \lambda^3 .\, m_0$	$\boxed{n = \lambda^{-1} .\, n_0}$

Assuming same maximal operating pressures for all unit sizes and a constant maximal sliding velocity !

Example

The maximal shaft speed of a given pump is 5000 rpm. The displacement volume of this pump is V= 40cm^3/rev. The maximal working pressure is given with 40 MPa. Using first order scaling laws, determine:

- the maximal shaft speed of a pump with 90 cm^3/rev
- the torque of this larger pump
- the maximal volume flow rate of this larger pump
- the power of this larger pump

For the linear scaling factor follows: $\lambda = \sqrt[3]{\frac{V}{V_0}} = \sqrt[3]{\frac{90}{40}} = 1.31$

Maximal shaft speed of the larger pump: $n = \lambda^{-1} \cdot n_0 = 1.31^{-1} \cdot 5000\ \text{rpm} = 3816.8\ \text{rpm}$

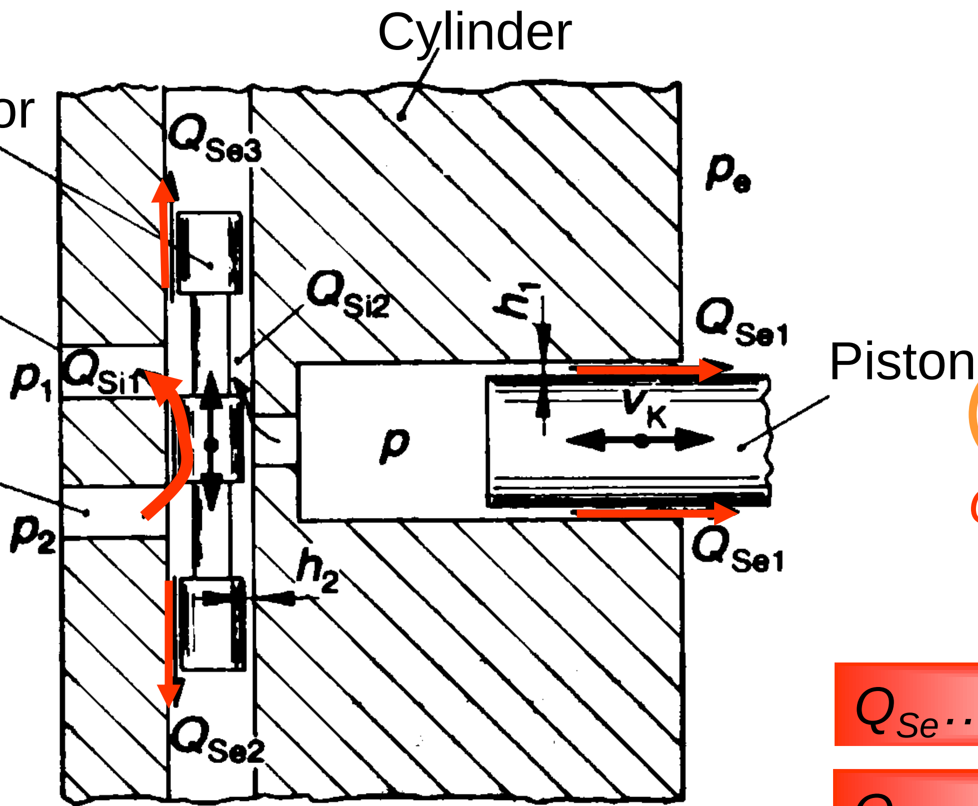
Torque of the larger pump: $T = \frac{\Delta p \cdot V}{2 \cdot \pi} = \frac{40 \cdot 10^6\ \text{Pa} \cdot 90 \cdot 10^{-6}\ \text{m}^3}{2 \cdot \pi} = 573.25\ \text{Nm}$

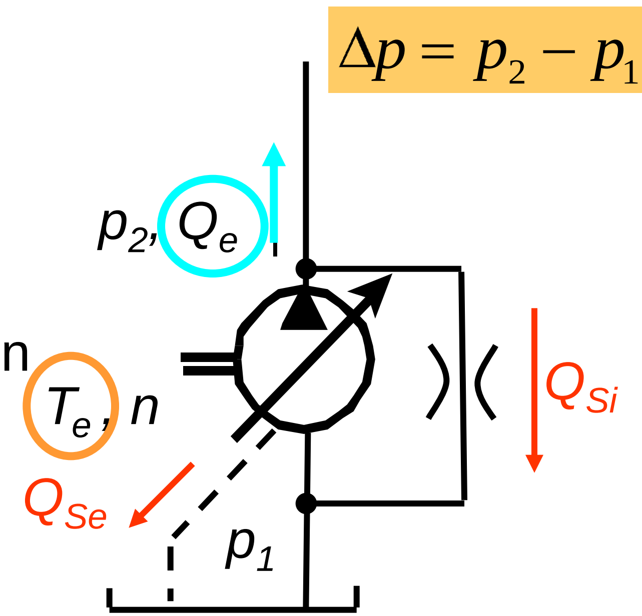
Maximal volume flow rate:

$$Q_{\max} = V \cdot n_{\max} = 90 \cdot 10^{-6}\ \text{m}^3/\text{rev} \cdot 3816.8\ \text{rpm} = 0.3435\ \text{m}^3/\text{min} = 343.5\ \text{l/min}$$

Power of the larger pump: $P = \Delta p \cdot Q = 40 \cdot 10^6\ \text{Pa} \cdot 0.3435\ \text{m}^3 \cdot \frac{1}{60}\ \text{s}^{-1} = 229\text{kW}$

Real Displacement Machine





Q_{Se}… external volumetric losses

Q_{Si}… internal volumetric losses

Effective Flow rate: $Q_e = \alpha \cdot V_{\max} \cdot n - Q_S$

Q_S … *volumetric losses*

Effective torque: $T_e = \dfrac{\Delta p \cdot \alpha \cdot V_{\max}}{2 \cdot \pi} + T_S$

T_S …*torque losses*

Volumetric Losses

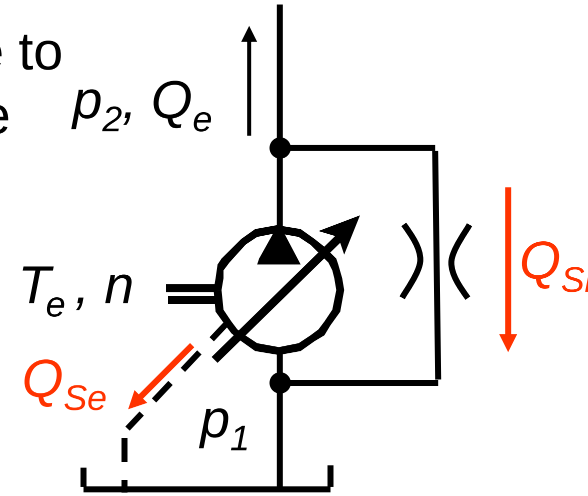
$$Q_S = \sum_{i=1}^{n} Q_{Sei} + \sum_{j=1}^{m} Q_{Sij} + Q_{SK} + Q_{Sf}$$

external internal

volumetric losses

losses due to compressibility

losses due to Incomplete filling

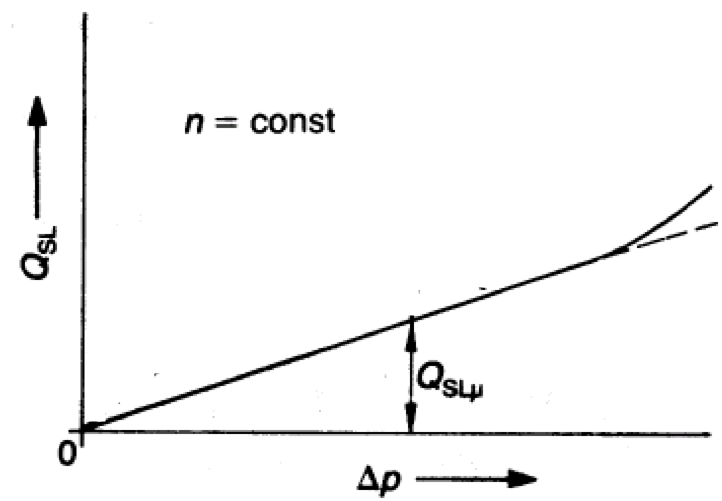


Q_{SL} external and internal volumetric losses = flow through laminar resistances:

$$Q_{SL} = C_\mu \cdot \frac{\Delta p}{\mu}$$

$$Q = \frac{b \cdot h^3 \cdot \Delta p}{12 \cdot \mu}$$

Assuming const. gap height



Dynamic viscosity $\mu = f(\theta, p)$

Volumetric Losses

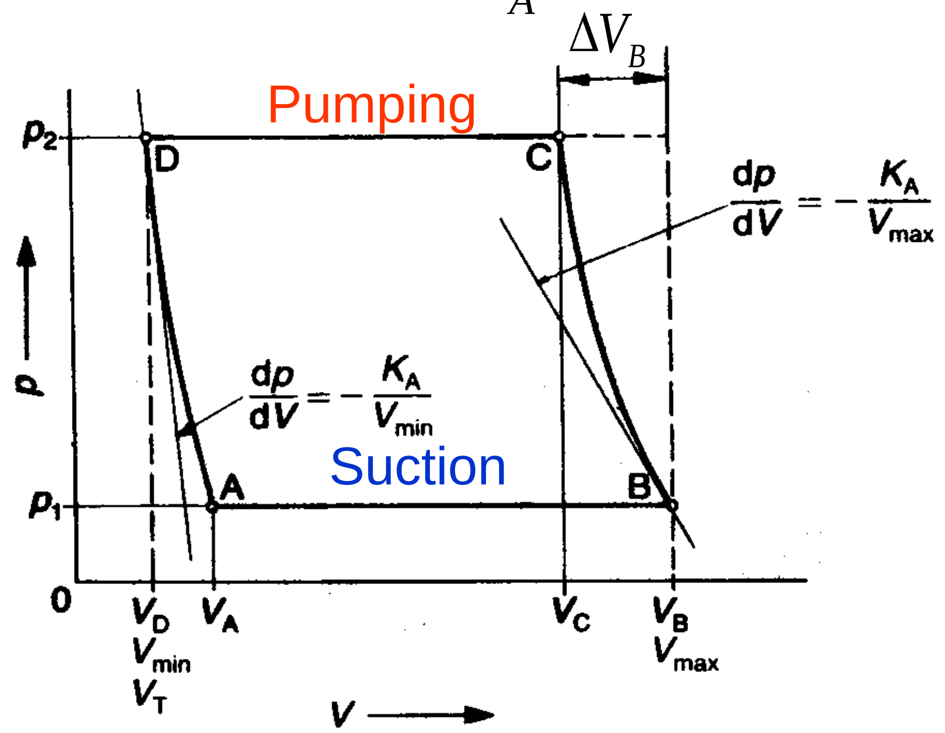
Effective volume flow rate is reduced due to compressibility of the fluid

$$\int_B^C \frac{dV}{V} = \int_B^C -\frac{1}{K_A}\,dp \quad \Rightarrow \quad \ln V_C - \ln V_B = -\frac{1}{K_A}\left(p_C - p_B\right)$$

$$\Delta V_B = V_B\left(1 - e^{-\frac{1}{K_A}\left(p_C - p_B\right)}\right)$$

simplified

$$\Delta V_B = V_B \frac{\Delta p}{K_A}$$



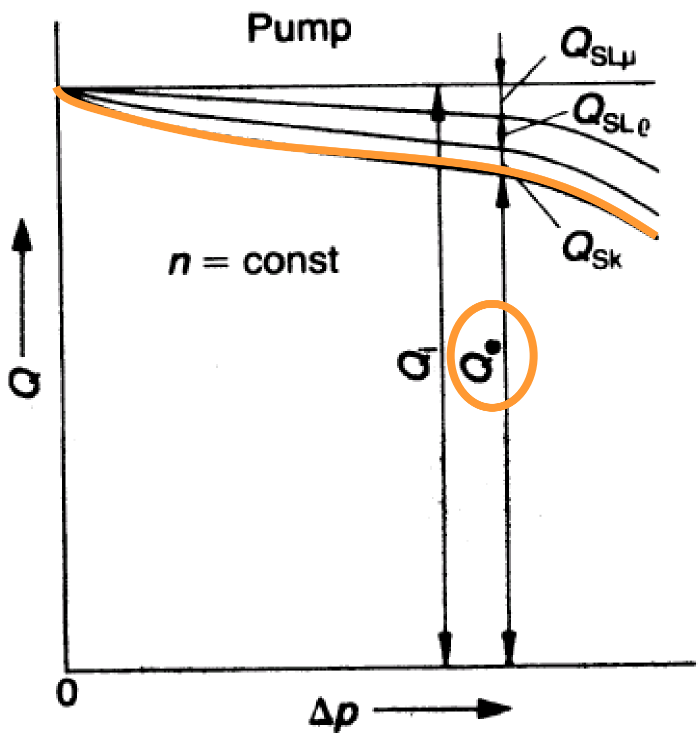
$$Q_{SK} = n \cdot \Delta V_B$$

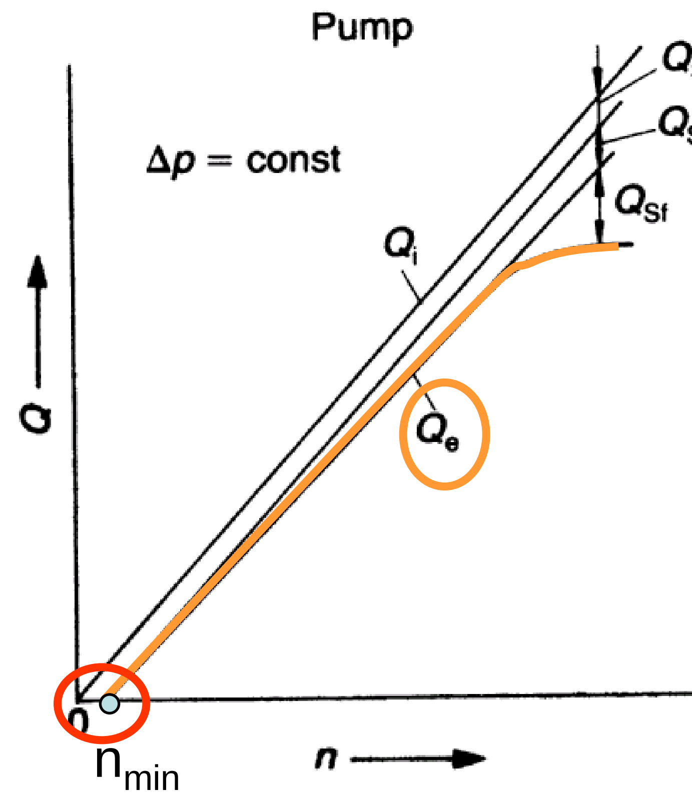
with n … pump speed

Steady state characteristics

of a real displacement machine $Q_i = V \cdot n = \alpha \cdot V_{\max} \cdot n$

Effective volumetric flow rate $Q_e = Q_i - Q_S$





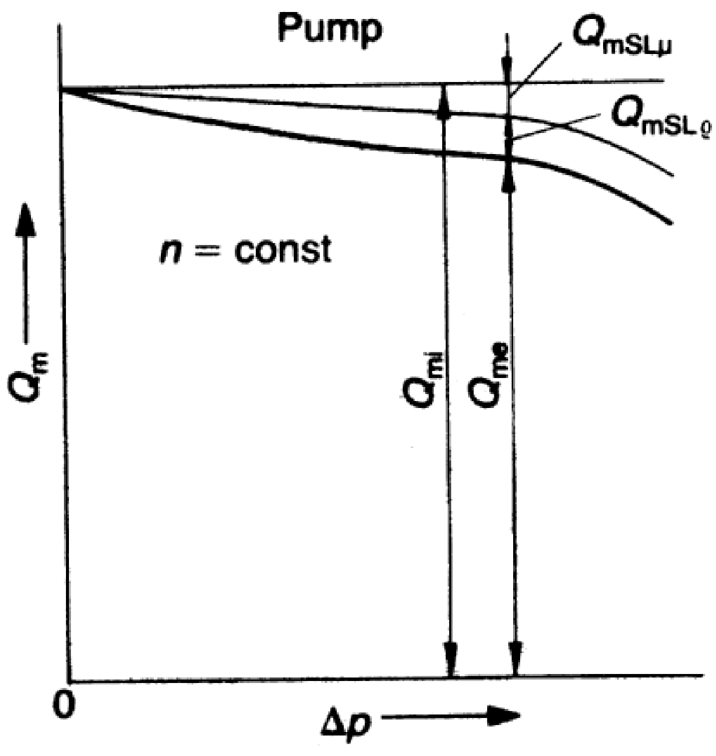
$$Q_S = f(\Delta p, n, V, \theta)$$

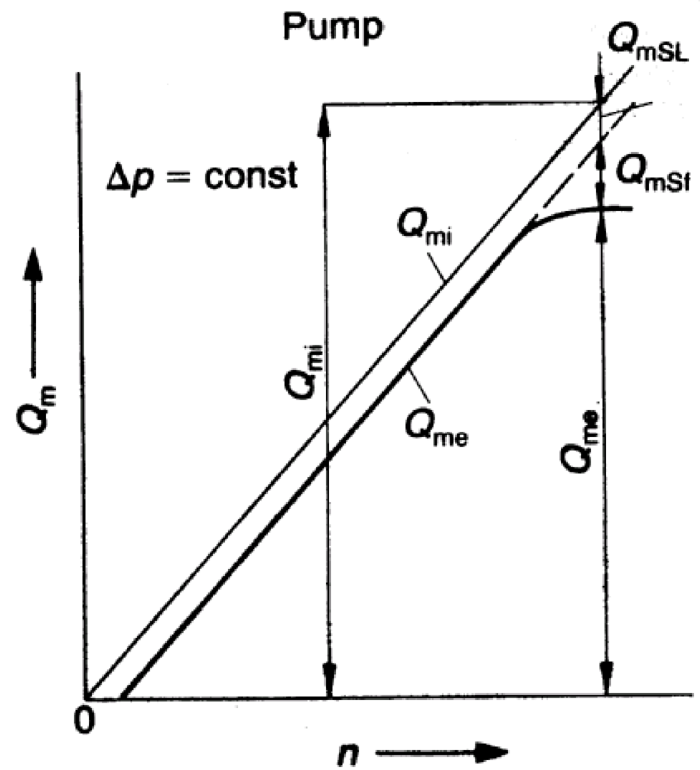
θ...temperature

Steady state characteristics

Loss component due to compressibility does not occur!

Effective mass flow at pump outlet Q_{me}





Instantaneous Pump Flow

Instantaneous volumetric flow Q_a

$$Q_a = \frac{dV}{dt} = f(\varphi)$$

Volumetric flow displaced by a displacement chamber

$$Q_{ai} = f(\varphi_i)$$

The instantaneous volumetric flow is given by the sum of instantaneous flows Q_{ai} of each displacement element:

$$Q_a = \sum_{i=1}^{k} Q_{ai}$$

k … number of displacement chambers, decreasing their volume, i.e. being in the delivery stroke

z is an even number $k = \frac{z}{2}$

z … number of displacement elements

z is an odd number $k = \frac{z}{2} + 0.5$ or $k = \frac{z}{2} - 0.5$

→ Flow pulsation of pumps → Pressure pulsation

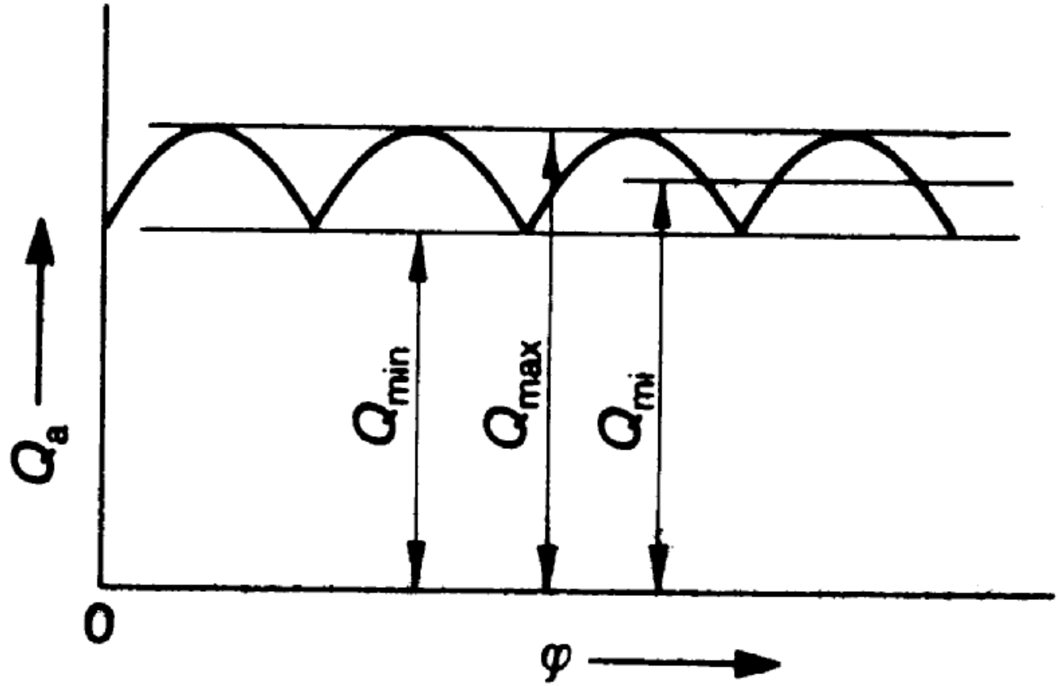
Flow pulsation

Non-uniformity grade of volumetric flow is defined:

$$\delta_Q = \frac{Q_{\max} - Q_{\min}}{Q_{mi}}$$

$$Q_{mi} = \frac{Q_{\max} + Q_{\min}}{2}$$

$$\delta_Q = 2 \cdot \frac{Q_{\max} - Q_{\min}}{Q_{\max} + Q_{\min}}$$



Torque Losses

$$T_S = T_{S\mu} + T_{S\rho} + T_{Sp} + T_{Sc} \Rightarrow$$ constant value

Torque loss due to viscous friction in gaps (laminar flow)

$$T_{S\mu} = k_{T\mu} \cdot \frac{\mu}{h} \cdot n = C_{T\mu} \cdot \mu \cdot n$$

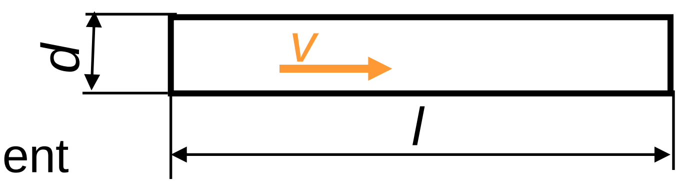
h…gap height

Torque loss to overcome pressure drop caused in turbulent resistances

$$T_{S\rho} = C_{T\rho} \cdot \rho \cdot n^2$$

$$\Delta p_s = \lambda \cdot \frac{l}{d} \cdot \rho \cdot \frac{v^2}{2} + \xi \cdot \rho \cdot \frac{v^2}{2}$$

Torque loss linear dependent on pressure



$$T_{Sp} = C_{Tp} \cdot \Delta p$$

ξ … drag coefficient

λ … flow resistance coefficient $\lambda_{turbulent} = \frac{0.3164}{\sqrt[4]{Re}}$

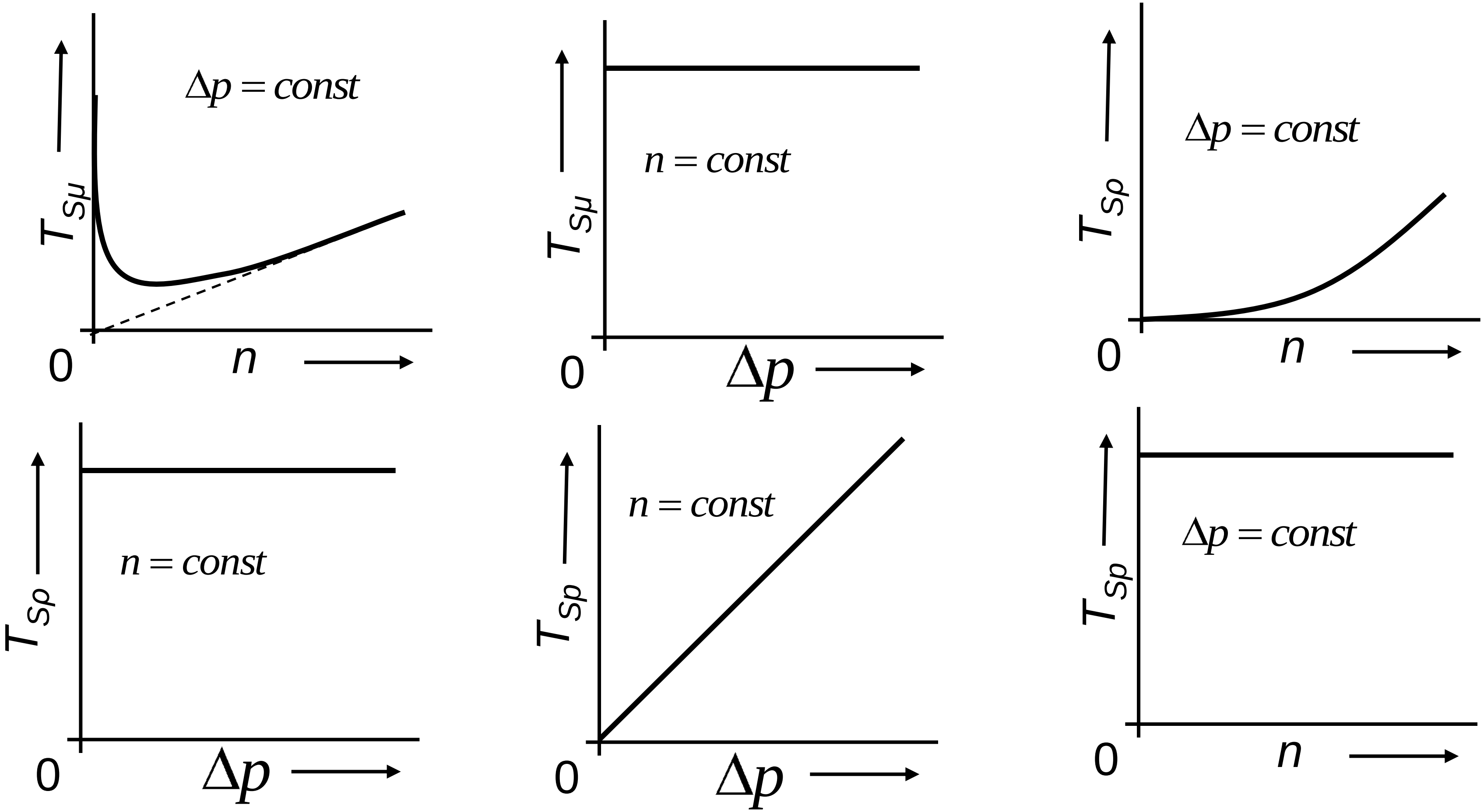
$$\Rightarrow \quad T_e = \frac{\Delta p \cdot \alpha \cdot V_{max}}{2 \cdot \pi} + T_S$$

effective torque required at pump shaft

Steady state characteristics

Torque losses

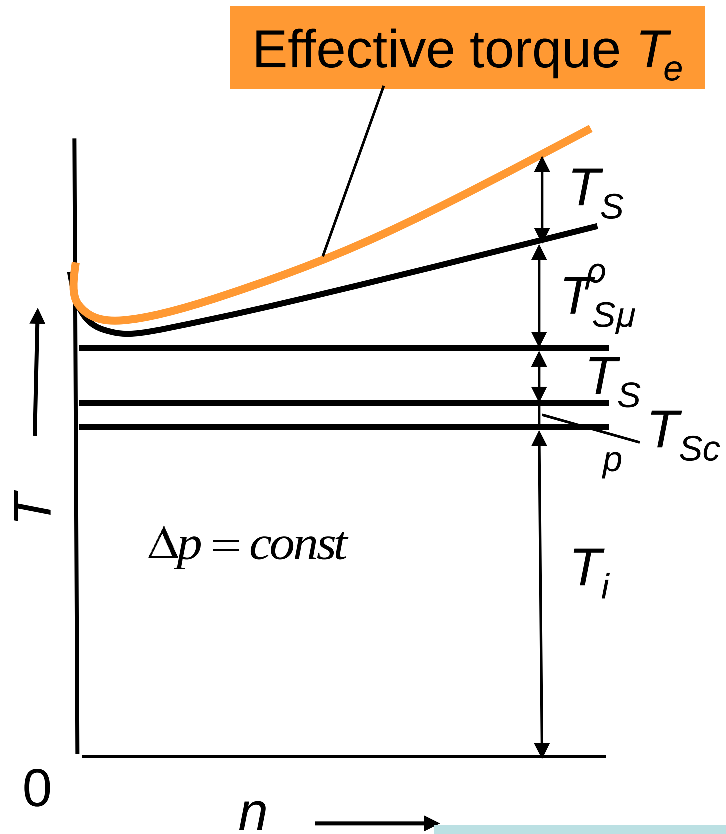
of a real displacement machine $\quad T_S = f(n, \Delta p, V, \theta)$

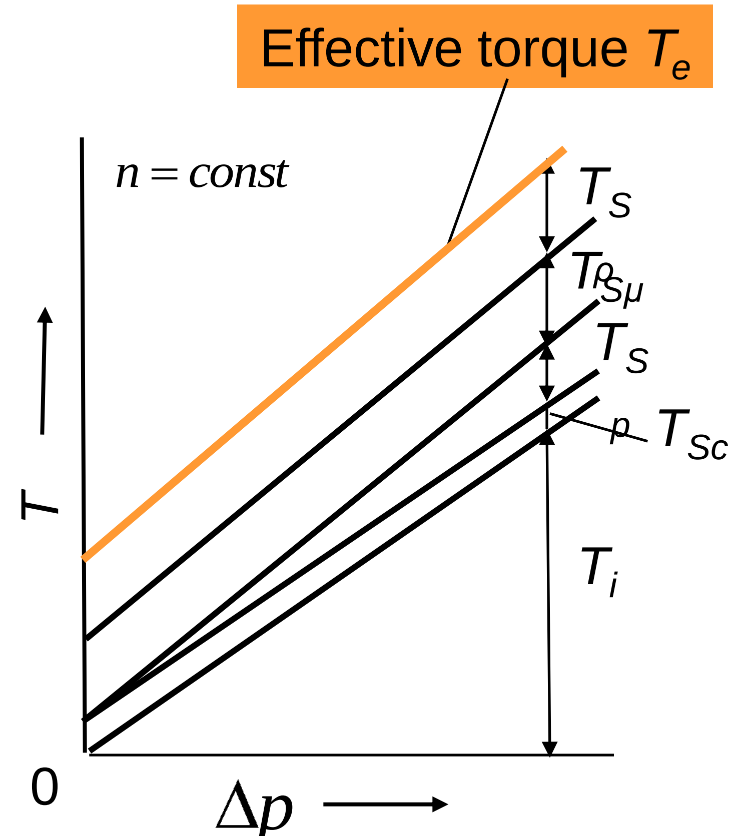


Steady state characteristics

Effective Torque

$$T_e = T_i + T_S = \frac{\Delta p \cdot \alpha \cdot V_{max}}{2 \cdot \pi} + T_S$$





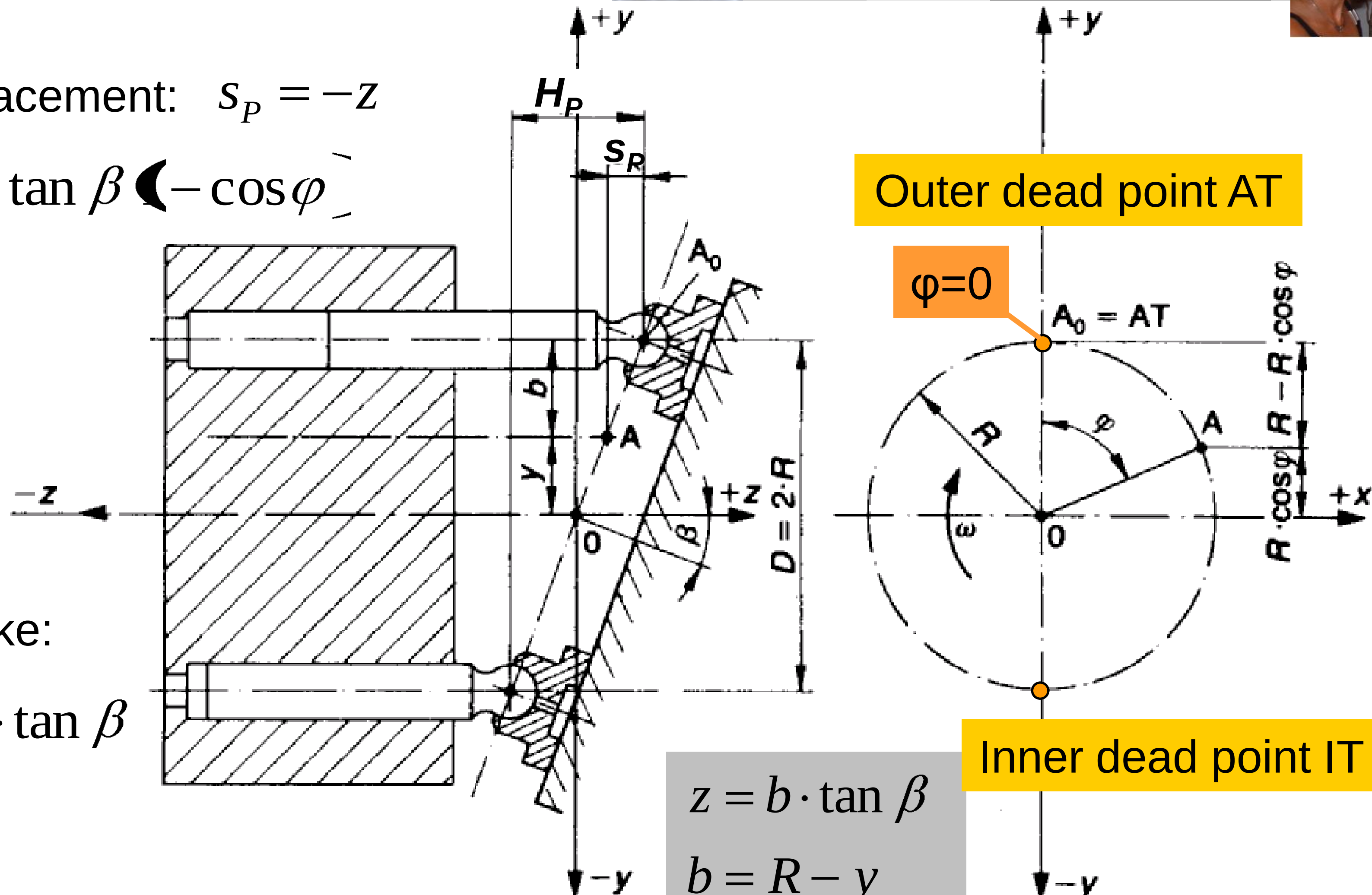
$$T_S = f(\Delta p, n, V, \theta)$$

Axial Piston Machine

Kinematics

Piston displacement: $s_P = -z$

$$s_P = -R\cdot\tan\beta\,(1-\cos\varphi)$$



Piston stroke:

$$H_P = 2\cdot R\cdot\tan\beta$$

$$z = b\cdot\tan\beta$$

$$b = R - y$$

$$y = R\cdot\cos\varphi$$

R … pitch radius

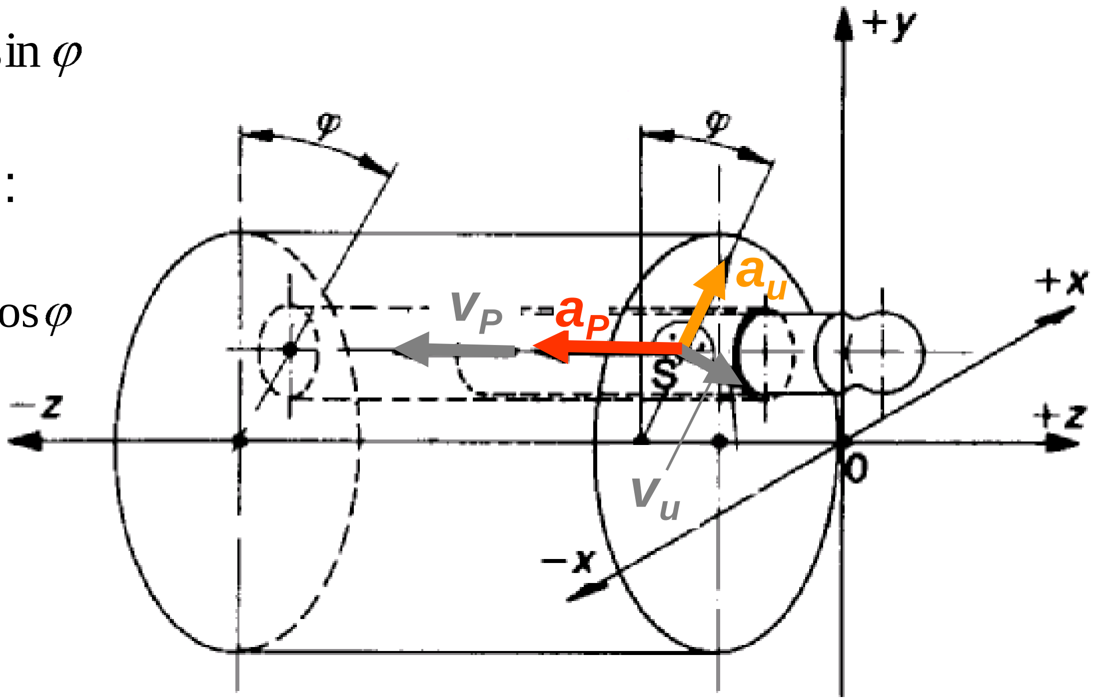
Kinematic Parameters

Piston velocity in z-direction:

$$v_P = \frac{ds_P}{dt} = \frac{ds_P}{d\varphi} \cdot \frac{d\varphi}{dt} = -\omega \cdot R \cdot \tan\beta \cdot \sin\varphi$$

Piston acceleration in z-direction:

$$a_P = \frac{dv_P}{dt} = \frac{dv_P}{d\varphi} \cdot \frac{d\varphi}{dt} = -\omega^2 \cdot R \cdot \tan\beta \cdot \cos\varphi$$



Circumferential speed

$$v_u = R \cdot \omega$$

Centrifugal acceleration:

$$a_u = R \cdot \omega^2$$

Coriolis acceleration a_c is just zero, as the vector of angular velocity ω and the piston velocity v_P run parallel

Instantaneous Volumetric Flow

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Geometric displacement volume:

$$V_g = z \cdot A_P \cdot H_P$$

z … number of pistons

For an ideal pump without losses

In case of pistons arranged parallel to shaft axis:

$$V_g = z \cdot \frac{\pi \cdot d_P^2}{2} \cdot R \cdot \tan\beta$$

Geometric flow rate: $Q_g = n \cdot z \cdot \frac{\pi \cdot d_P^2}{2} \cdot R \cdot \tan\beta$ ⇒ Mean value over time

Instantaneous volumetric flow: $\boxed{Q_a = \sum_{i=1}^{k} Q_{ai}}$

k …number of pistons, which are in the delivery stroke

with Q_{ai} instantaneous volumetric flow of individual piston

$$Q_{ai} = f(\varphi_i)$$

$$v_P = \omega \cdot R \cdot \tan\beta \cdot \sin\varphi$$

$$Q_{ai} = v_p \cdot A_P = \omega \cdot A_P \cdot R \cdot \tan\beta \cdot \sin\varphi_i$$

Instantaneous Volumetric Flow

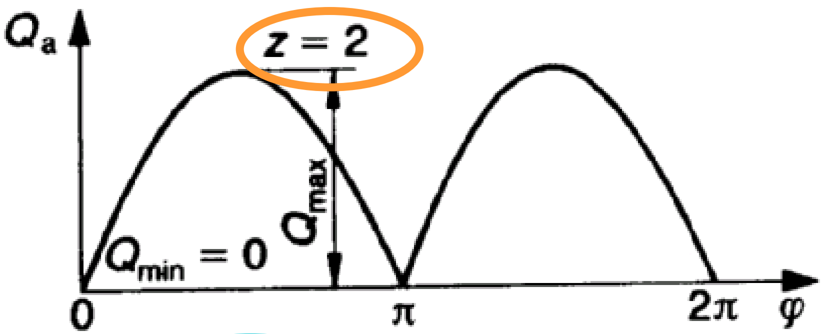
$$Q_a = \sum_{i=1}^{k} Q_{ai}$$

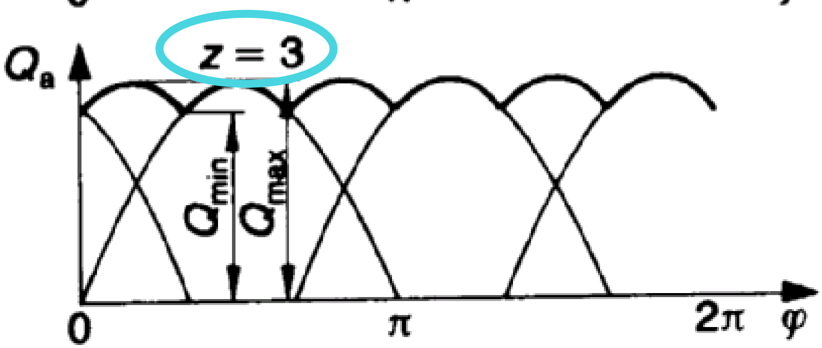
In case of even number of pistons: $k = 0.5 \cdot z$

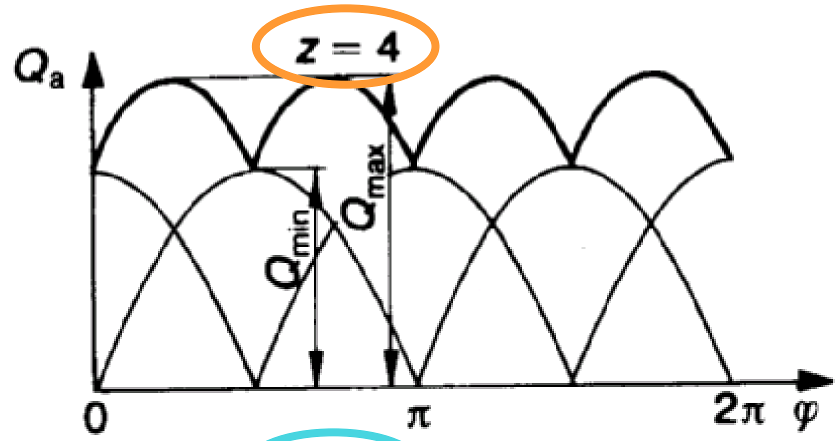
In case of odd number of pistons:

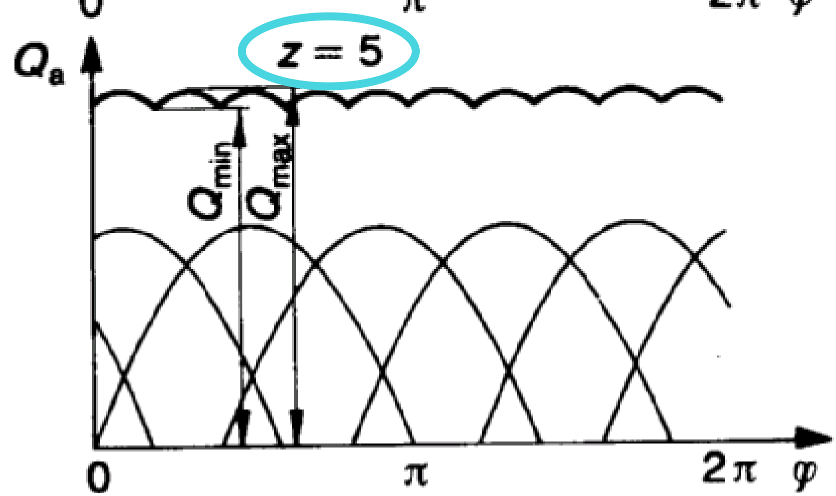
$$k_1 = \frac{z}{2} + 0.5 \quad \text{for} \quad 0 < \varphi \leq \frac{\pi}{z}$$

$$\text{and} \quad k_2 = \frac{z}{2} - 0.5 \quad \text{for} \quad \frac{\pi}{z} < \varphi \leq 2 \cdot \frac{\pi}{z}$$



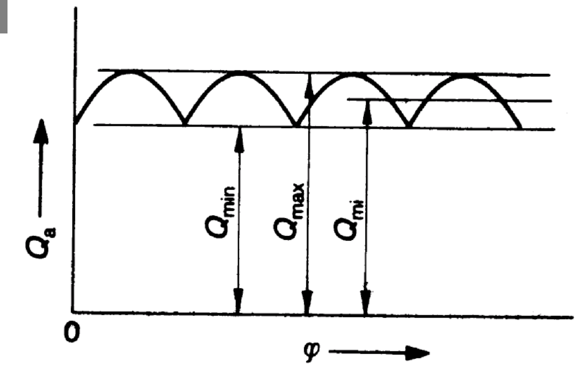






Flow & Torque Pulsation

kinematic flow and torque pulsation due to a finite number of piston



Flow Pulsation: Non-uniformity grade:

$$\delta_Q = \frac{Q_{\max} - Q_{\min}}{Q_{mi}} \quad \text{with} \quad Q_{mi} = \frac{Q_{\max} + Q_{\min}}{2}$$

Even number of pistons:

$$\boxed{\delta_Q = \frac{\pi}{2} \cdot \tan\frac{\pi}{2 \cdot z}}$$

Odd number of pistons:

$$\boxed{\delta_Q = \frac{\pi}{2 \cdot z} \cdot \tan\frac{\pi}{4 \cdot z}}$$

Torque Pulsation

$$\delta_T = \frac{T_{\max} - T_{\min}}{T_{mi}} \quad \text{with} \quad T_{mi} = \frac{T_{\max} + T_{\min}}{2}$$

kinematic flow and torque pulsation due to a finite number of piston

z… number of pistons

Non-uniformity

$$\delta_Q = \delta_T = \frac{Q_{\max} - Q_{\min}}{Q_{\text{mean}}} = \frac{T_{\max} - T_{\min}}{T_{\text{mean}}}$$

Even number of pistons:

$$\delta_Q = \frac{\pi}{z} \cdot \tan \frac{\pi}{2 \cdot z}$$

Odd number of pistons:

$$\delta_Q = \frac{\pi}{2 \cdot z} \cdot \tan \frac{\pi}{4 \cdot z}$$

NUMBER OF PISTONS	3	4	5	6	7	8	9	10	11	12	13	14
NON-UNIFORMITY of FLOW / TORQUE	0,1403	0,3253	0,0498	0,1403	0,0253	0,0781	0,0153	0,0498	0,0102	0,0345	0,0073	0,0253

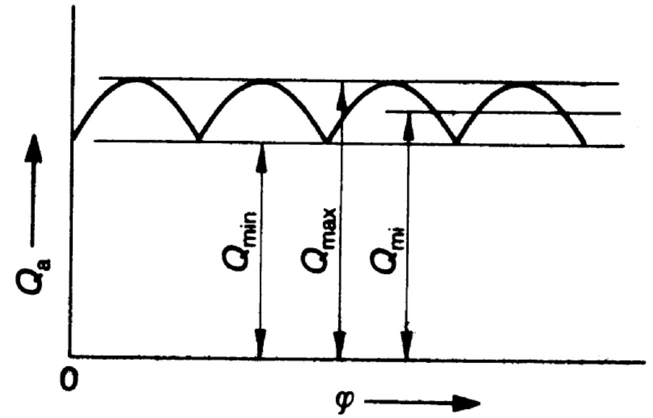
NUMBER OF PISTONS	15	16	17	18	19	20	21	22	23	24	25	26
NON-UNIFORMITY of FLOW / TORQUE	0,0055	0,0193	0,0043	0,0153	0,0034	0,0124	0,0028	0,0102	0,0023	0,0086	0,0020	0,0073

Flow and torque pulsation frequency f:

Even number of pistons: $f = z \cdot n$

Odd number of pistons: $f = 2 \cdot z \cdot n$

Flow Pulsation



Non-uniformity grade:

$$\delta_Q = \frac{Q_{\max} - Q_{\min}}{Q_{mi}} \quad \text{with} \quad Q_{mi} = \frac{Q_{\max} + Q_{\min}}{2}$$

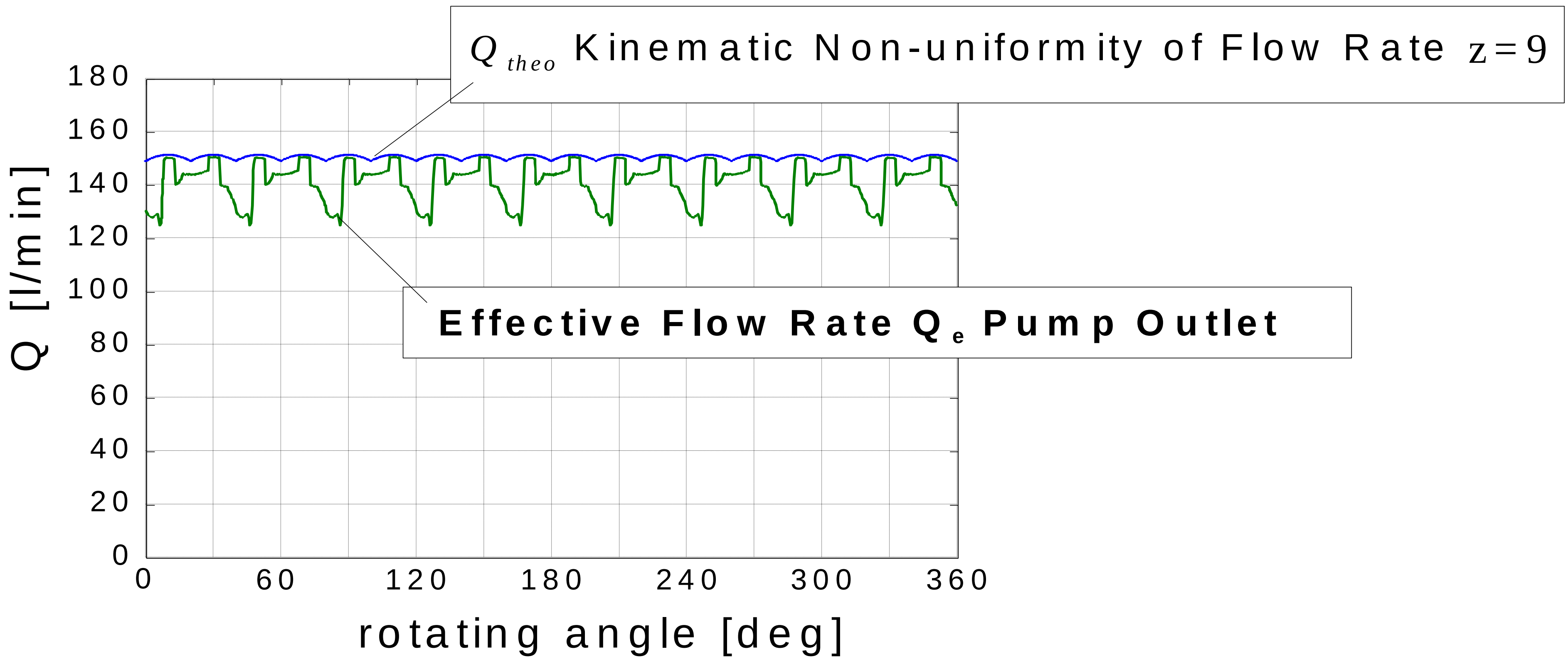
Kinematic non-uniformity grade for piston machines:

Number of pistons z	3	4	5	6	7	8	9	10	11
Non-uniformity grade δ	0.140	0.325	0.049	0.140	0.025	0.078	0.015	0.049	0.010

Volumetric losses $Q_s = f(\varphi)$ and $Q_S = f(\Delta p, n, V_i, \theta)$

Flow pulsation of a real displacement machine is much larger than the flow pulsation given by the kinematics

Flow Pulsation



Flow pulsation leads to pressure pulsation at pump outlet