Design and Modeling of Fluid Power Systems

ME 597/ABE 591 - Lecture 6

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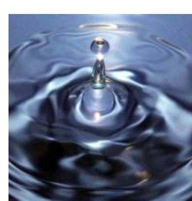


Content

Steady state charcteristics, measurement and modeling

Volumetric and torque efficiency

Measurement of steady state characteristics

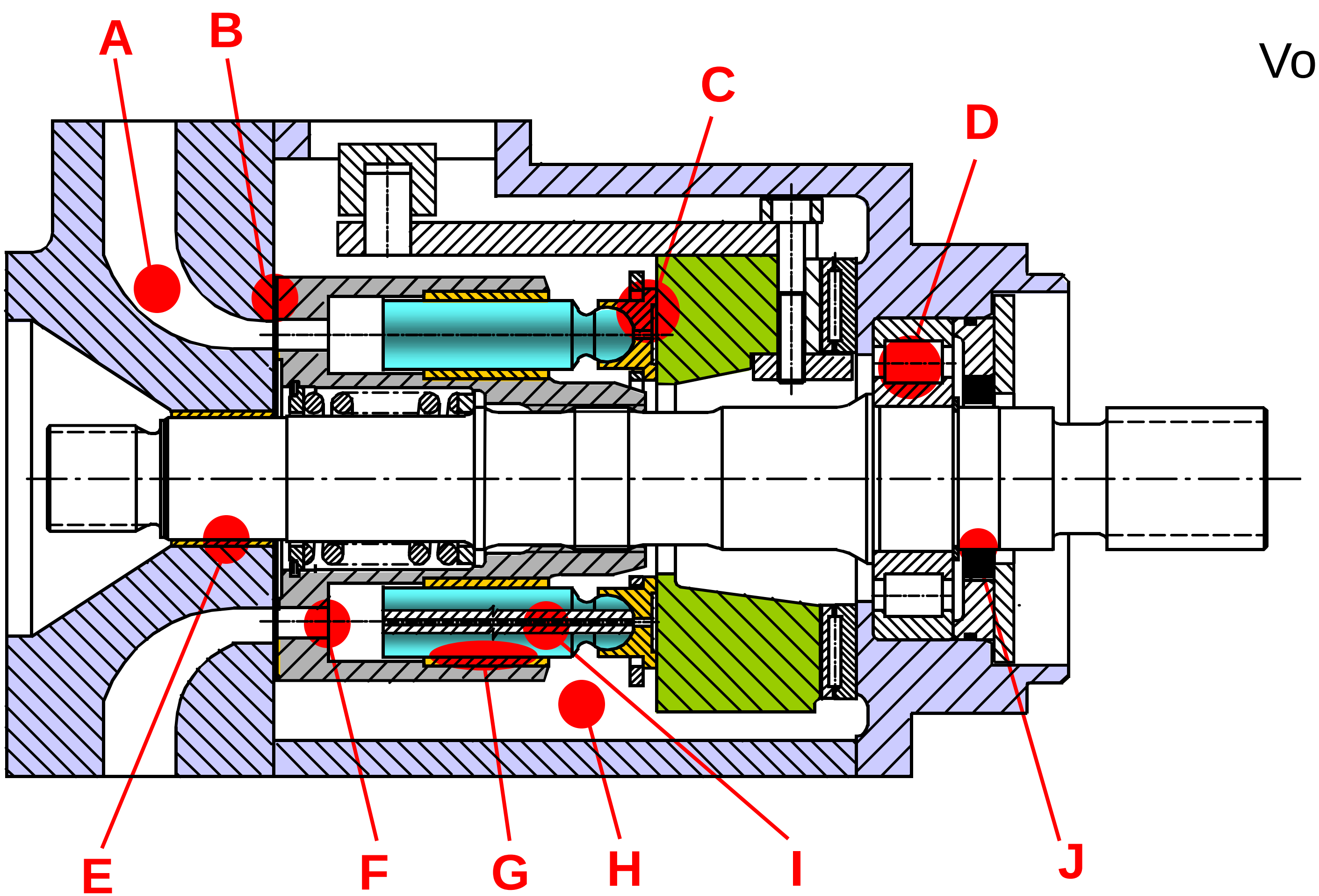
Determination of derived displacement volume

Measurement based steady state models

Aim: Application of basic equations for calculation of performance data of pumps
Performance test circuit design
Basic knowledge about the generation of steady state models

Losses - displacement machines

Axial piston machine – swash plate design



Volumetric Losses

B C F G I

Torque Losses

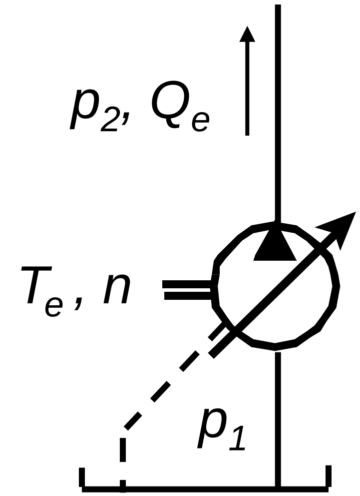
A B C D E

G H J

Pump Efficiency

Volumetric efficiency: $$\eta_v = \frac{Q_e}{n \cdot V_i}$$

where V_i …. represents the derived displacement volume



Torque efficiency
(hydraulic-mechanical efficiency): $$\eta_{hm} = \frac{T_i}{T_e} = \frac{\Delta p \cdot V_i}{2 \cdot \pi \cdot T_e}$$

$$\Delta p = p_2 - p_1$$

Total efficiency: $$\eta_t = \frac{P_{out}}{P_{in}} = \frac{Q_e \cdot \Delta p}{T_e \cdot \omega} = \eta_v \cdot \eta_{hm}$$

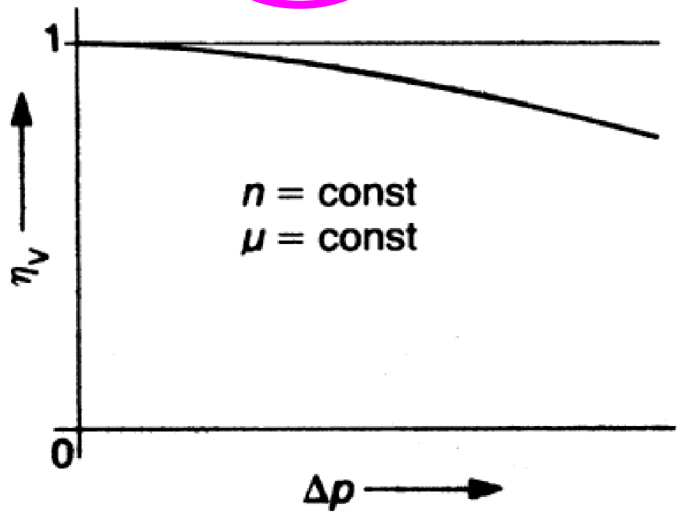
The derived displacement volume can only be determined by measurement

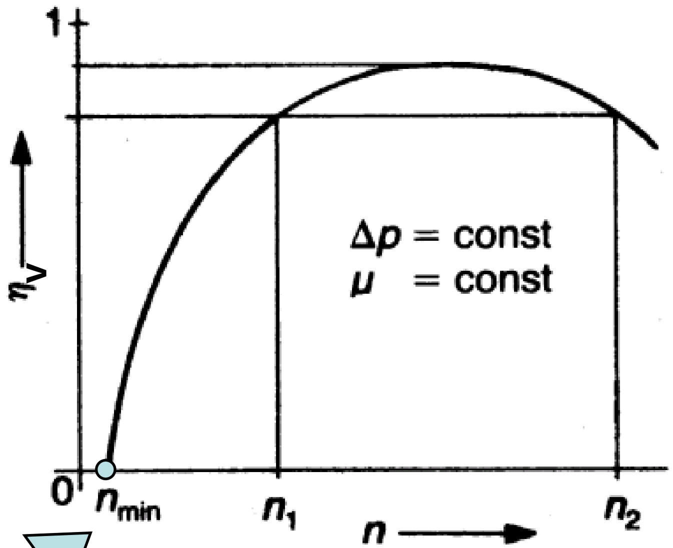
Volumetric Efficiency of a Pump

Volumetric efficiency $\eta_v = f(\Delta p, n, V_i, \mu)$

$$\eta_v = \frac{Q_e}{n \cdot V_i}$$

$$Q_e = \alpha \cdot V_{\max} \cdot n - Q_S$$





$$\alpha = \frac{V}{V_{\max}}$$

$$V_i \cdot n_{\min} = Q_S$$

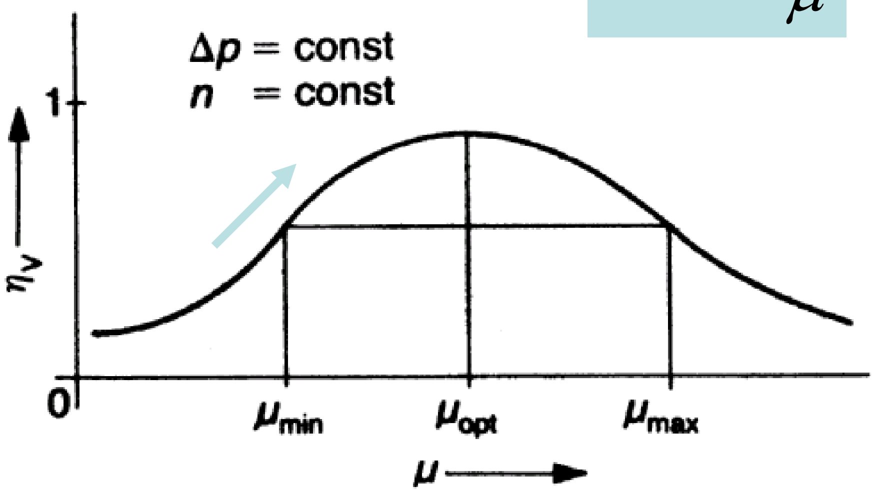
Dynamic viscosity of fluid: $\mu = f(\theta, p)$ [Pa·s]

Volumetric Efficiency of a Pump

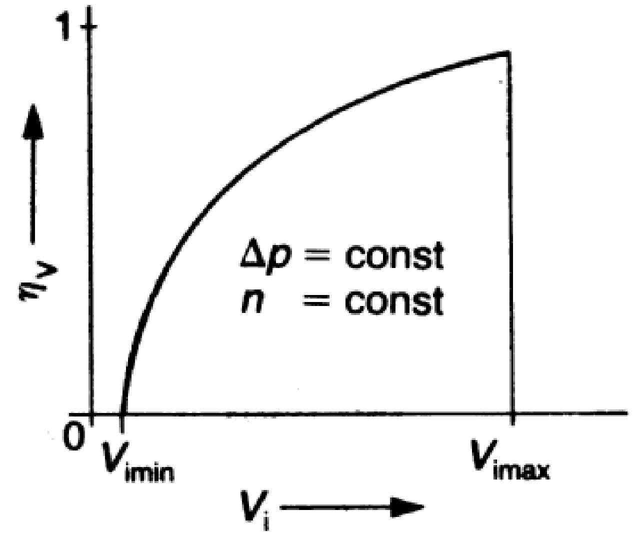
Volumetric efficiency $\eta_v = f(\Delta p, n, V_i, \mu)$ $\eta_v = \frac{Q_e}{n \cdot V_i}$

$$Q_e = \alpha \cdot V_{\max} \cdot n - Q_S$$

$$Q_{SL\mu} \approx \frac{1}{\mu}$$



Variable displacement pump



Typical values of dynamic viscosity
used in displacement machines: $0.0435\ \mathrm{Pa \cdot s} \div 0.0087\ \mathrm{Pa \cdot s}$

with: Kinematic viscosity ν [cSt, mm²/s] and $\mu = \nu \cdot \rho$

$$\nu = 10\ \mathrm{mm}^2 \cdot s^{-1} \div 50\ \mathrm{mm}^2 \cdot s^{-1}\ \text{with}\ \ \rho = 870\ \mathrm{kg} \cdot \mathrm{m}^{-3}$$

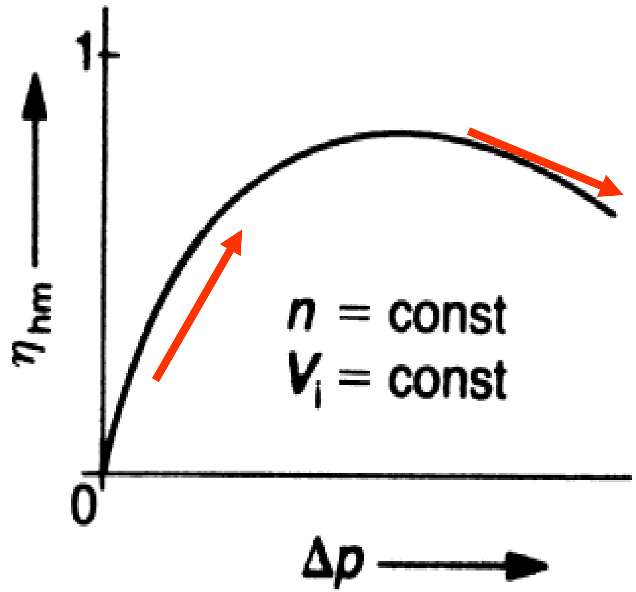
Torque Efficiency of a Pump

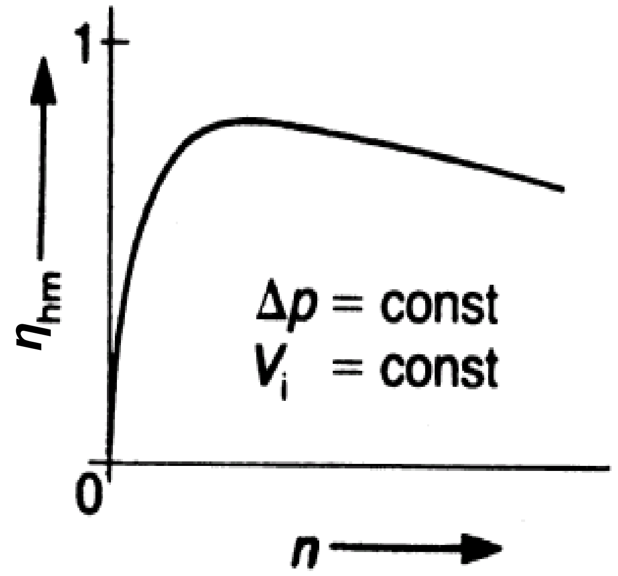
$$\eta_{hm} = f(\Delta p, n, V_i, \mu)$$

$$\eta_{hm} = \frac{T_i}{T_e} = \frac{\Delta p \cdot V_i}{2 \cdot \pi \cdot T_e} = 1 - \frac{T_S}{T_e}$$

$$T_e = \frac{\Delta p \cdot \alpha \cdot V_{\max}}{2 \cdot \pi} + T_S$$

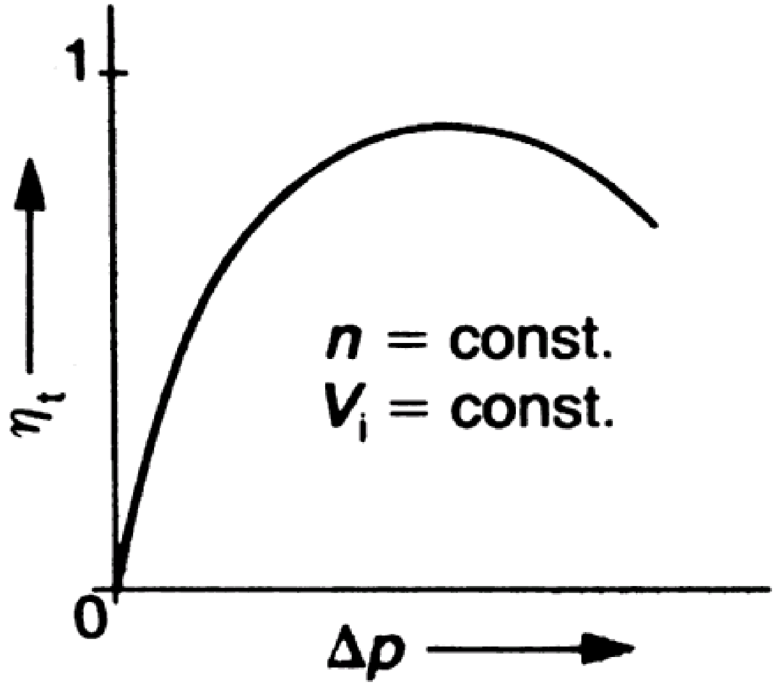
$$T_S = T_{S\mu} + T_{S\rho} + T_{Sp} + T_{Sc} = C_\mu \cdot \mu \cdot n + C_\rho \cdot \rho \cdot n^2 + C_p \cdot \Delta p + T_{Sc}$$

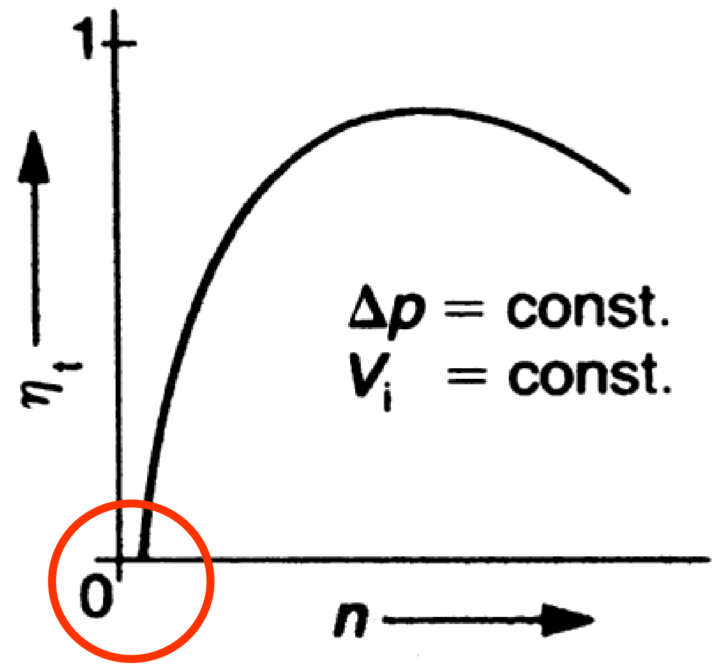




Total Efficiency of a Pump

$$\eta_t = f(\Delta p, n, V_i, \mu) \qquad \eta_t = \frac{P_{out}}{P_{in}} = \frac{Q_e \cdot \Delta p}{T_e \cdot \omega} = \eta_v \cdot \eta_{hm}$$

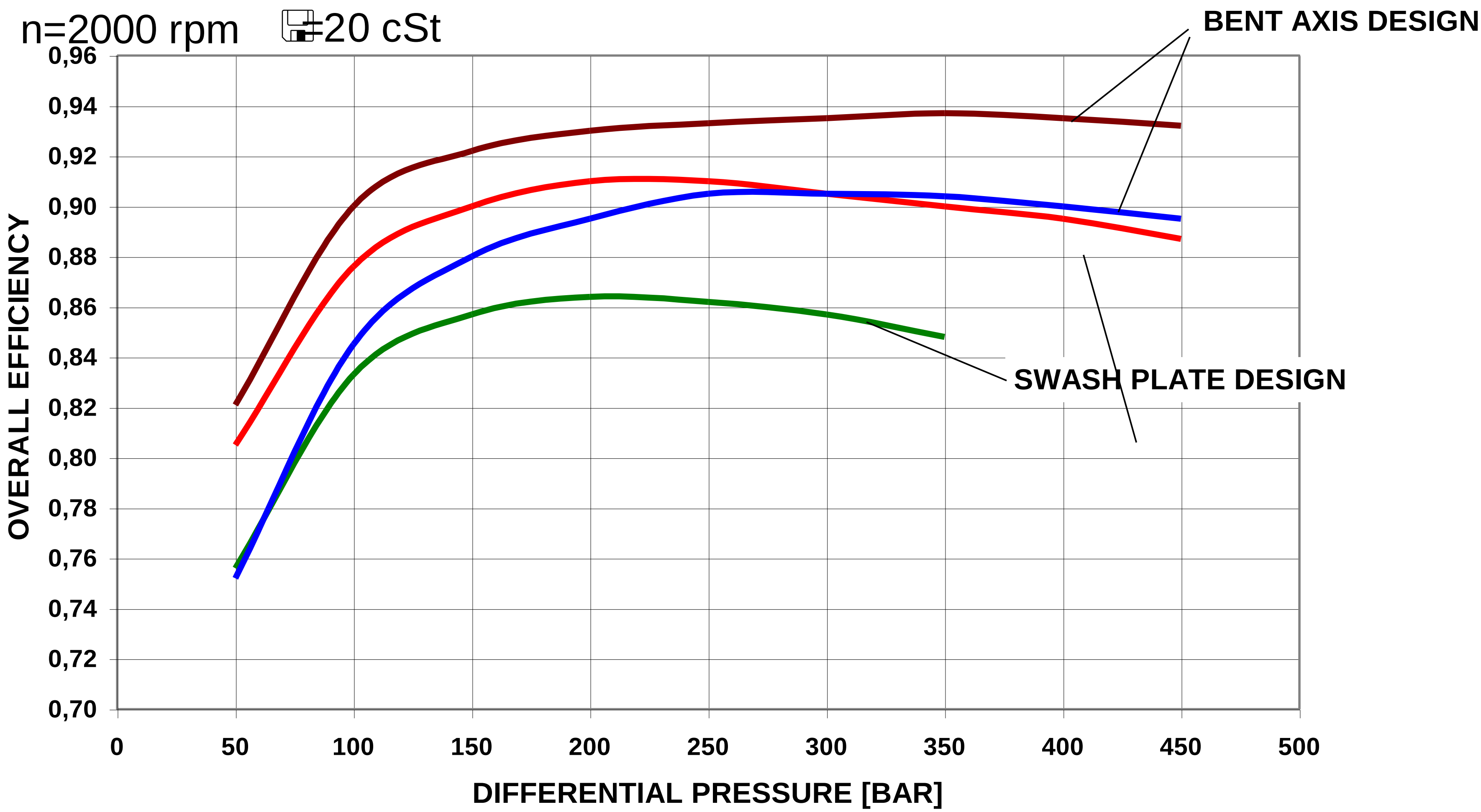




Steady State Characteristics

Comparison of Efficiencies of Axial Piston Pumps

n=2000 rpm =20 cSt

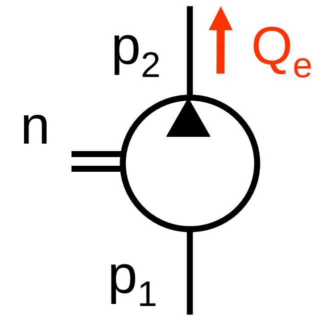


Example Lecture 2

The displacement pump with a displacement volume of V=100 cm³/rev is driven by an electric motor at 1800 rpm. At steady state conditions the pressure difference across the pump is Δp= 380 bar. Determine the effective flow rate at the pump outlet Q_e and the power required to drive this pump. Assume the following values for efficiency:

- volumetric efficiency η_v=0.87
- torque efficiency η_{hm}=0.95

$$\Delta p = p_2 - p_1$$



The effective volume flow rate:

$$Q_e = V \cdot n \cdot \eta_v = 100 \cdot 10^{-6}\,\mathrm{m}^3 \cdot 1800 \cdot \frac{1}{60} \cdot s^{-1} \cdot 0.87 = 0.00261\,\mathrm{m}^3 \cdot s^{-1} = 156.6\ \mathrm{l} \cdot \mathrm{min}^{-1}$$

The effective torque yields:

$$T_e = \frac{T_i}{\eta_{hm}} = \frac{\Delta p \cdot V_i}{2 \cdot \pi \cdot \eta_{hm}} = \frac{380 \cdot 10^5\ \mathrm{Pa} \cdot 100 \cdot 10^{-6}\ \mathrm{m}^3}{2 \cdot \pi \cdot 0.95} = 636.62\ \mathrm{Nm}$$

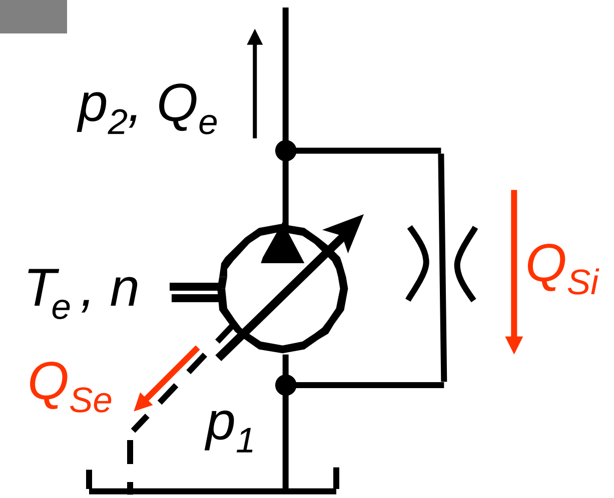
The power required to drive the pump:

$$P = T_e \cdot \omega = T_e \cdot 2 \cdot \pi \cdot n = 636.62\ \mathrm{Nm} \cdot 2 \cdot \pi \cdot \frac{1800}{60} \cdot s^{-1} = 120\ \mathrm{kW}$$

Steady State Measurement

The aim of steady state measurements is determination of steady state characteristics of pumps.

- Losses and their dependency on operating parameters
- Efficiency and its dependency on operating parameters
- Effective torque T_e and effective volumetric Flow rate Q_e in the whole parameter range



Under stable conditions, all parameters remain constant, including temperatures !

Parameters to be measured:

Inlet pressure p_1

Outlet pressure p_2

Temperatures $\theta_1, \theta_2\, \theta_{Se}$,

Torque T_e

Shaft speed n

Volume flow rate at pump outlet $Q_e = Q_2$

Steady State Characteristics

V_i derived displacement volume to be determined during measurements

$$Q_e = \alpha \cdot V_{max} \cdot n - Q_S$$

Q_S must be calculated using measurement results

Measured value

$$\Delta p = p_2 - p_1$$

$$T_e = \frac{\Delta p \cdot \alpha \cdot V_{max}}{2 \cdot \pi} + T_S$$

T_S must be calculated using measurement results

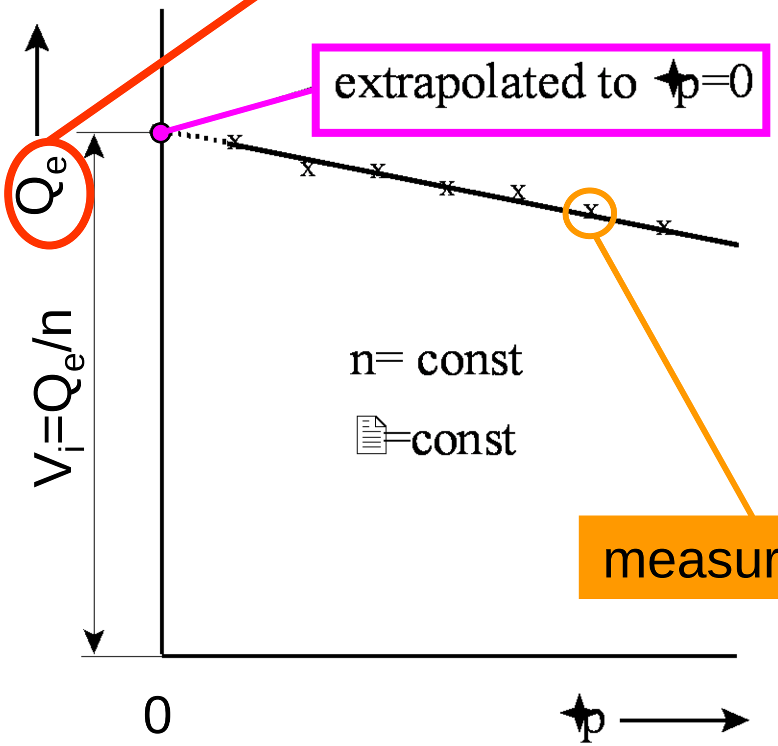
$$P_{in} = P_{out} + P_S = \Delta p \cdot Q_e + P_S = T_e \cdot 2 \cdot \pi \cdot n$$

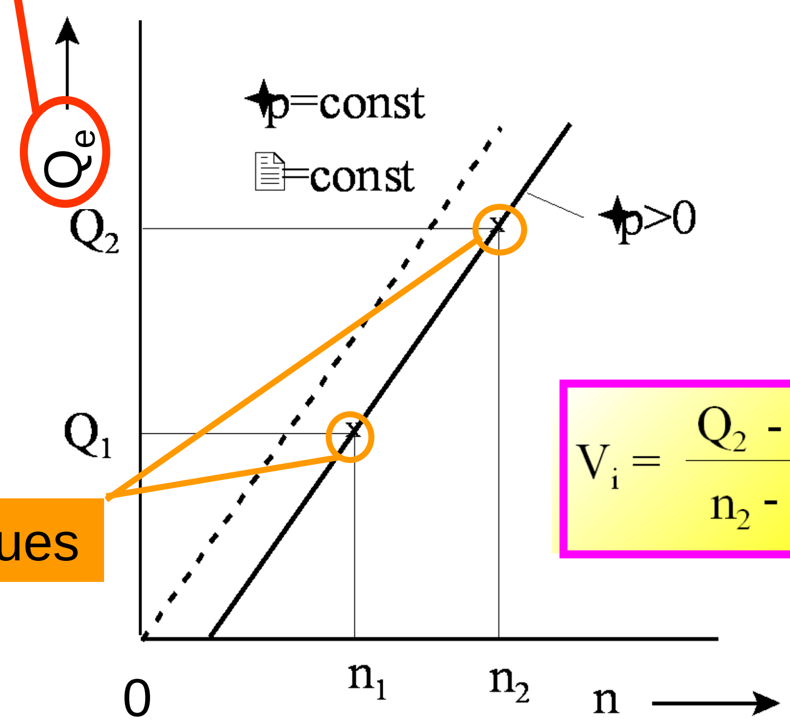
P_S must be calculated using measurement results

Derived displacement volume V_i

Measurement of effective volume flow rate at pump outlet under defined conditions

Method by Toet	Method definied in ISO 8426





$$V_i = \frac{Q_2 - Q_1}{n_2 - n_1}$$

V_i required for determination of losses and volumetric and torque efficiency

Test Circuit

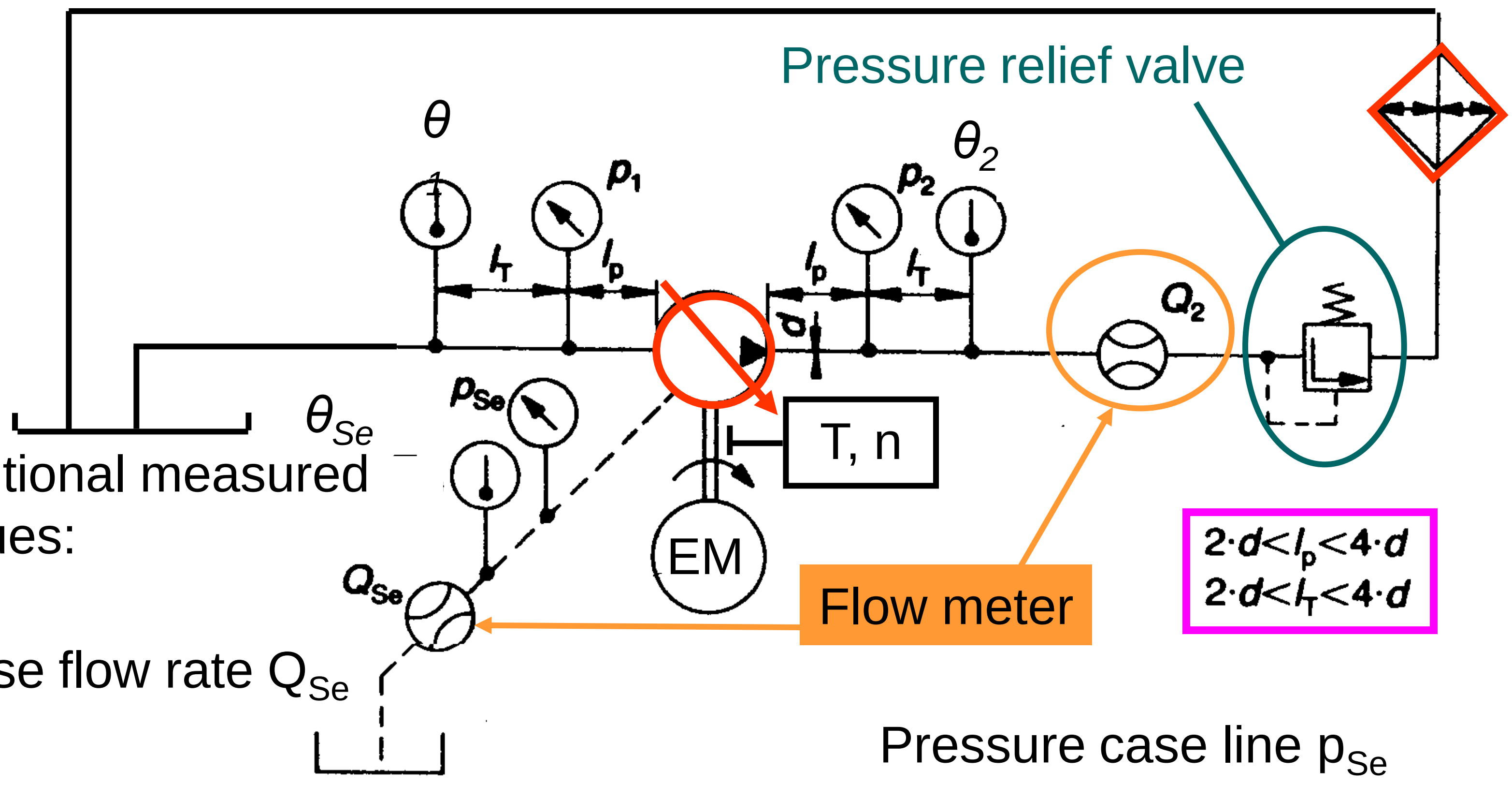
Measured Values: Inlet pressure p_1 Torque T Shaft speed n

Outlet pressure p_2 Temperature $\theta_1, \theta_2\, \theta_{Se}$

Volume flow rate at pump outlet Q_2 ,

ISO 4409

Temperature θ_1 must remain constant during measurements



Additional measured values:

Case flow rate Q_{Se}

Pressure case line p_{Se}

Steady State Measurement

In case that Q_2 is measured in low pressure line, the measured value must be corrected with respect to p_2 and θ_2

$$Q_2 = Q_3\left[1 - \frac{p_2 - p_3}{K} + \beta_\theta\left(\theta_2 - \theta_3\right)\right]$$

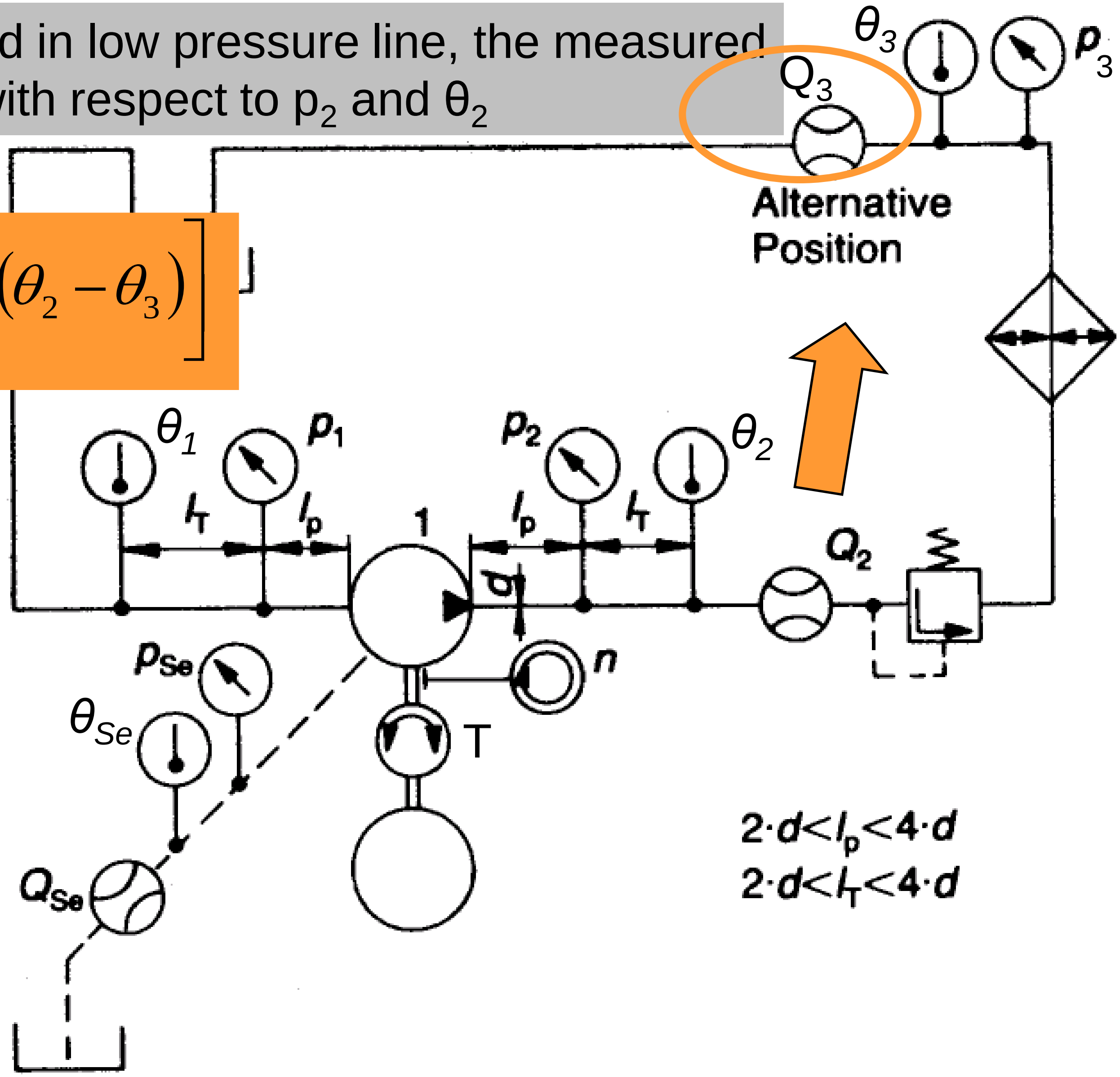
with

K … bulk modulus

$ß_\theta$ …thermal volumetric expansion coefficient

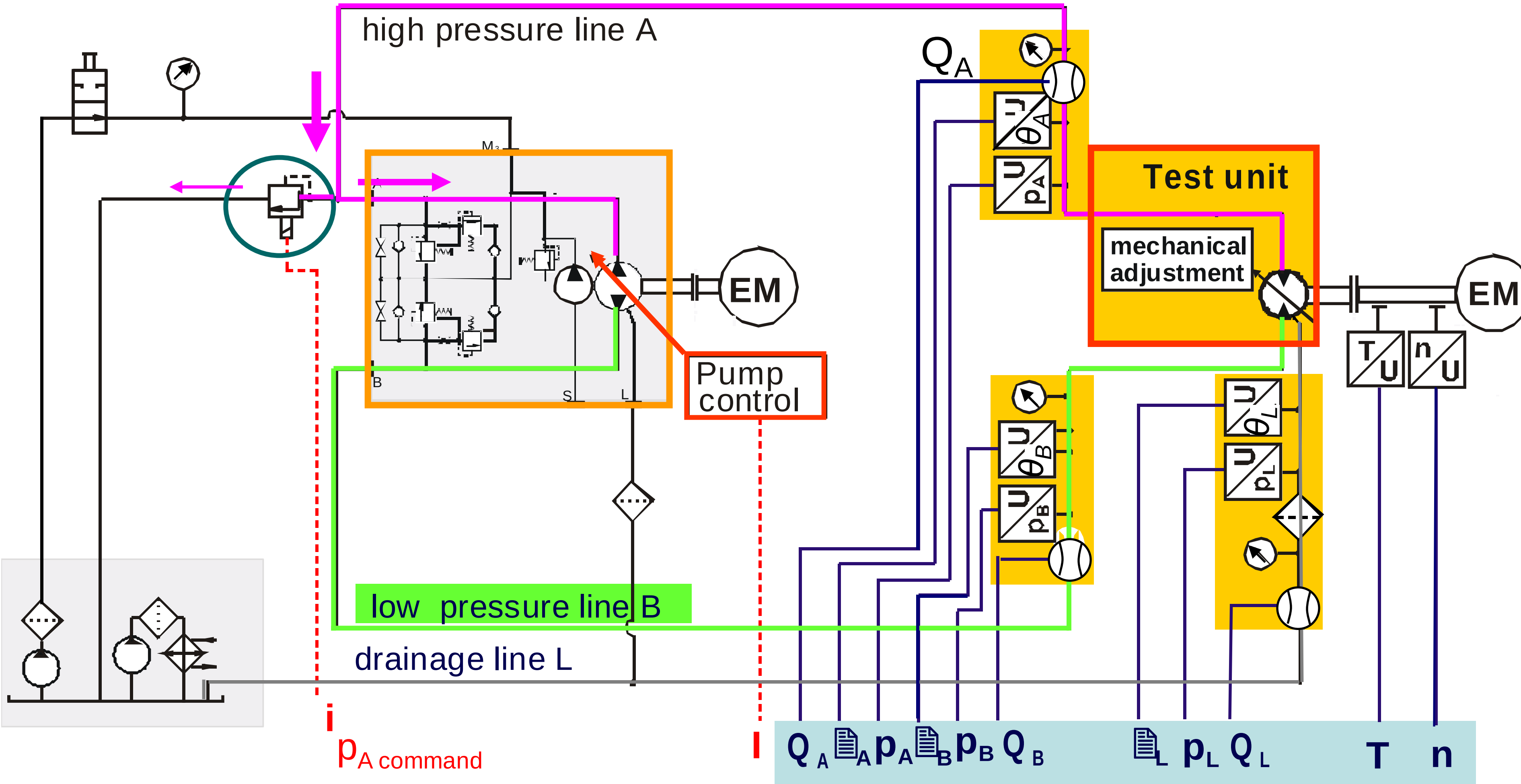
$K = 2 \cdot 10^9$ Pa

$ß_\theta = 0.65 \cdot 10^{-3}$ K^{-1}



Steady State Measurement

Measurement in pumping and motoring mode



Less temperature problems, because only a small amount of volume flow is throttled in pressure relief valve

Steady State Measurement

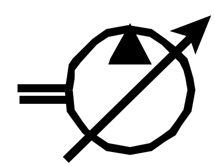
ISO measurement accuracy classes

Table 1: Permissible systematic errors of measuring instruments as determined during calibration

Parameter of measuring instrument	***Permissible systematic errors for classes of measurement accuracy***		
	A	**B**	**C**
Rotational frequency [%]	0.5	1.0	2.0
Torque [%]	0.5	1.0	2.0
Volume flow rate [%]	0.5	1.5	2.5
Pressure below 2 bar gauge [bar]	0.01	0.03	0.05
Pressure greater than or equal to 2 bar gauge [%]	0.5	1.5	2.5
Temperature [C]	0.5	1.0	2.0

Permissible temperature variation

Accuracy class	A	B	C
Temperature variation [K]	1.0	2.0	4.0



Pressure Measurement

Types of pressure

The different types of pressure differ only with respect to their reference point.

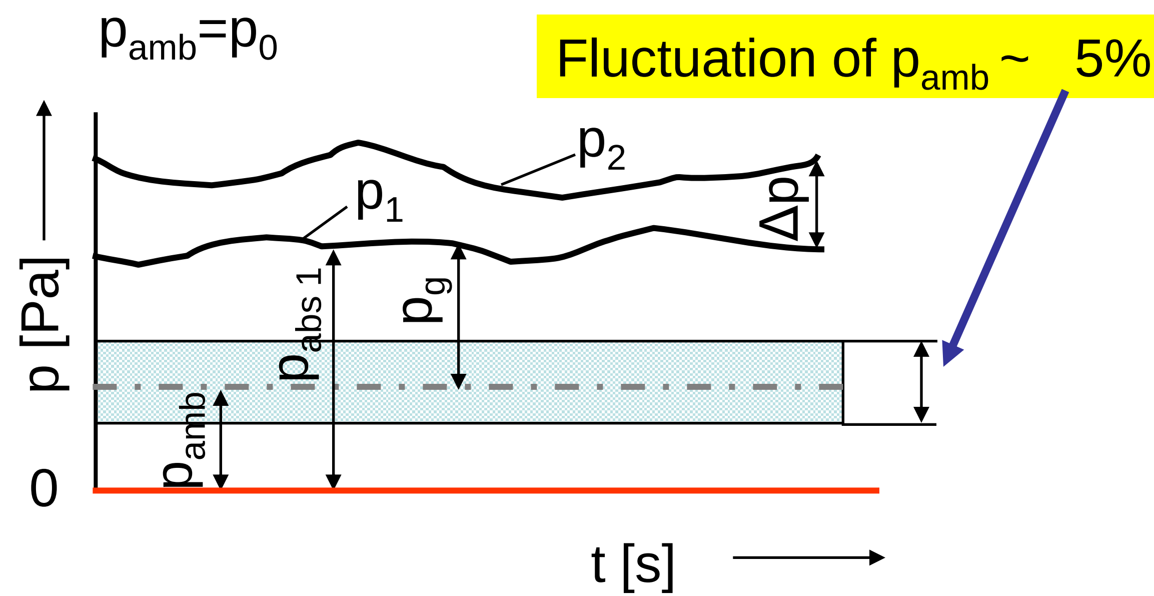
Absolute pressure p_{abs}

Atmospheric pressure p_{amb}

Differential pressure Δp

Gauge pressure p_g

$p_g = p_{measured} - p_{amb}$

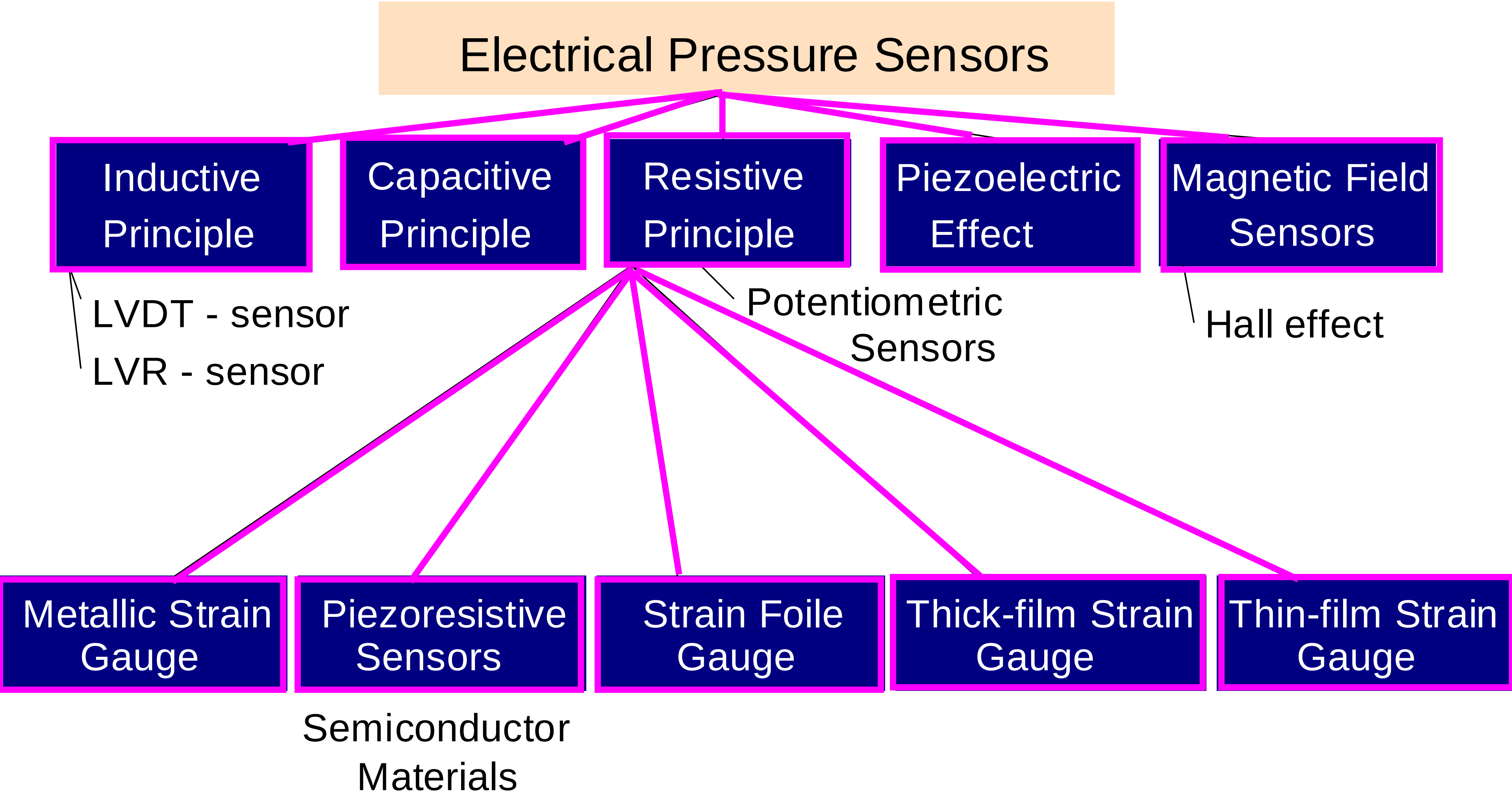


Direct measuring pressure instruments — Using a liquid column

Indirect measuring pressure instruments — Using the effect of a pressure acting on a material or on bodies of a certain shape

Electrical pressure sensors

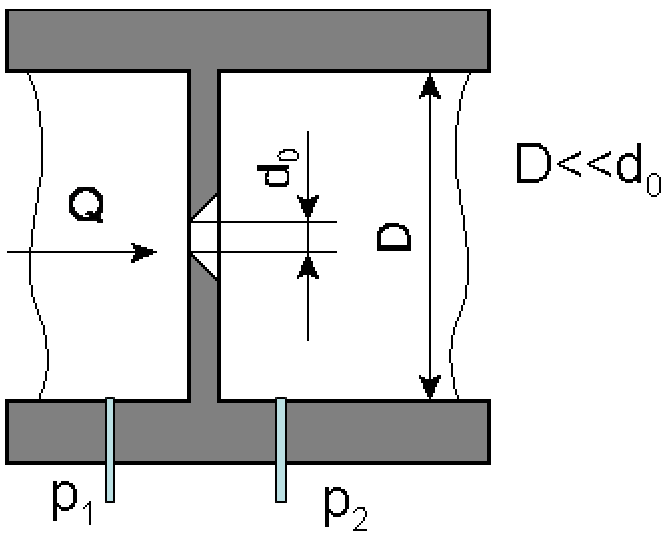
Electrical Pressure Sensors



Flow measuring instrument is defined as device which measures the flow rate of a fluid.

Pressure difference across an orifice

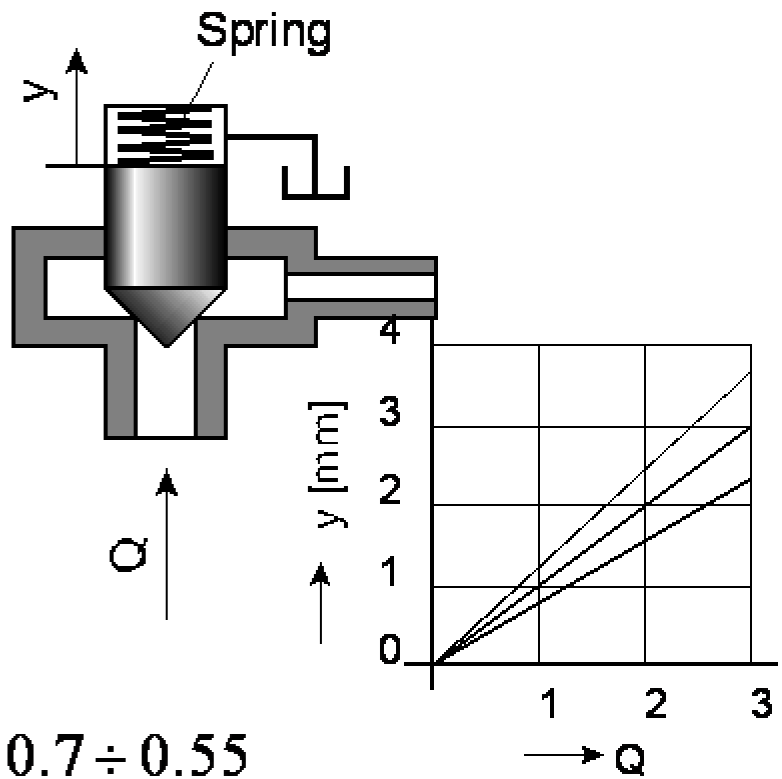
Displacement of a spring loaded floated element



$$Q = \alpha_D \cdot \frac{\pi \cdot d^2}{4} \sqrt{\frac{2 \cdot (p_1 - p_2)}{\rho}}$$

α_D…flow discharge coefficient

$$\alpha_D \approx 0.7 \div 0.55$$



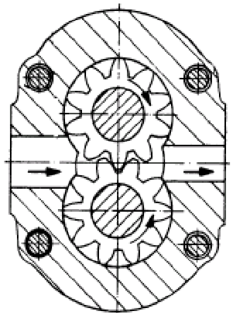
Should not be used for determination of steady state characteristics!

Flow Measurement

Flow meter – device which directly indicates the rate of flow of a fluid

Displacement principle

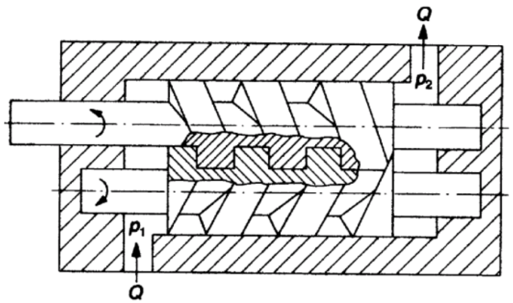
Hydrodynamic principle

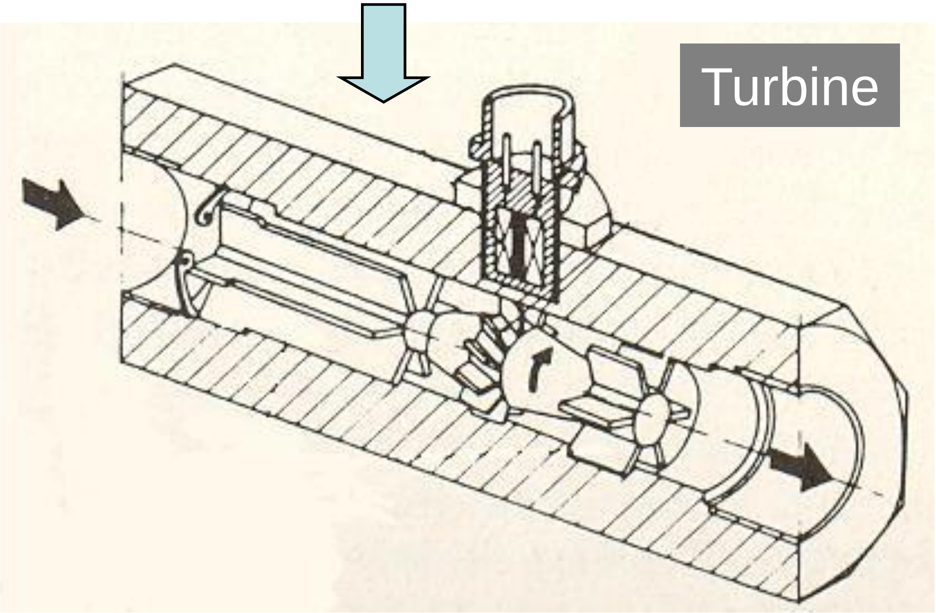


Gear flow meter

Using kinetic energy of a fluid to drive a rotating system of blades (an impeller), whereas the rotational speed of the rotor is measured with an electric speed sensor (frequency measurement device)

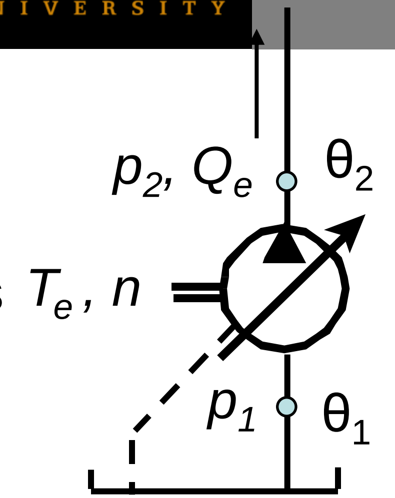
Screw flow meter





Turbine

Test Procedure



1. Keep p_1 and θ_1 constant
2. Adjust different pressure levels at constant shaft speed
3. Record all measured values under steady state conditions

⇨ Repeat measurements for different speed settings

⇨ In case of variable displacement pump repeat measurements for 75%, 50% and 25% of V_{max}

⇨ Repeat measurements for different temperatures θ_1

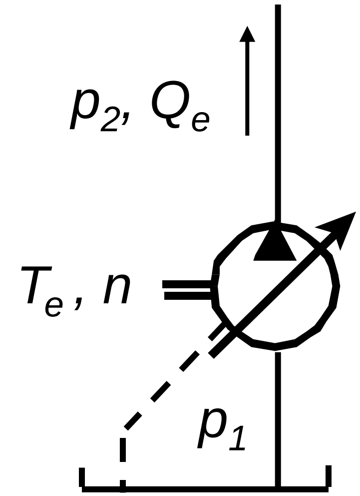
Provide table with measurement results

No.	n [RPM]	T [Nm]	p_1 [bar]	θ_1 [C]	p_2 [bar]	Q_2 [l/min]	θ_2 [C]	p_{Se} [bar]	Q_{Se} [l/min]	θ_{Se} [C]
1	2001.46	15.03	19.77	50.50	25.11	145.13	61.20	1.20	0.21	65.30
2	2001.63	68.78	19.79	50.30	69.65	144.02	60.30	1.25	0.43	66.00
3	2001.79	130.59	19.86	50.30	120.82	142.92	60.40	1.28	0.56	65.20

Measurement Results

Determination of derived displacement volume

$$V_i = \frac{1}{n} \cdot \frac{\sum_{j=1}^{k} Q_{ej} \cdot \sum_{j=1}^{k} \Delta p_j^2 - \sum_{j=1}^{k} \Delta p_j \cdot \sum_{j=1}^{k} \Delta p_j \cdot Q_{ej}}{k \cdot \sum_{j=1}^{k} \Delta p_j^2 - \left(\sum_{j=1}^{k} \Delta p_j \right)^2}$$



k ...number of measurements and $\Delta p = p_2 - p_1$

In case of variable displacement pumps, the derived displacement volume must be determined for each adjusted value!

Determination of Losses

$$Q_S = V_i \cdot n - Q_e$$

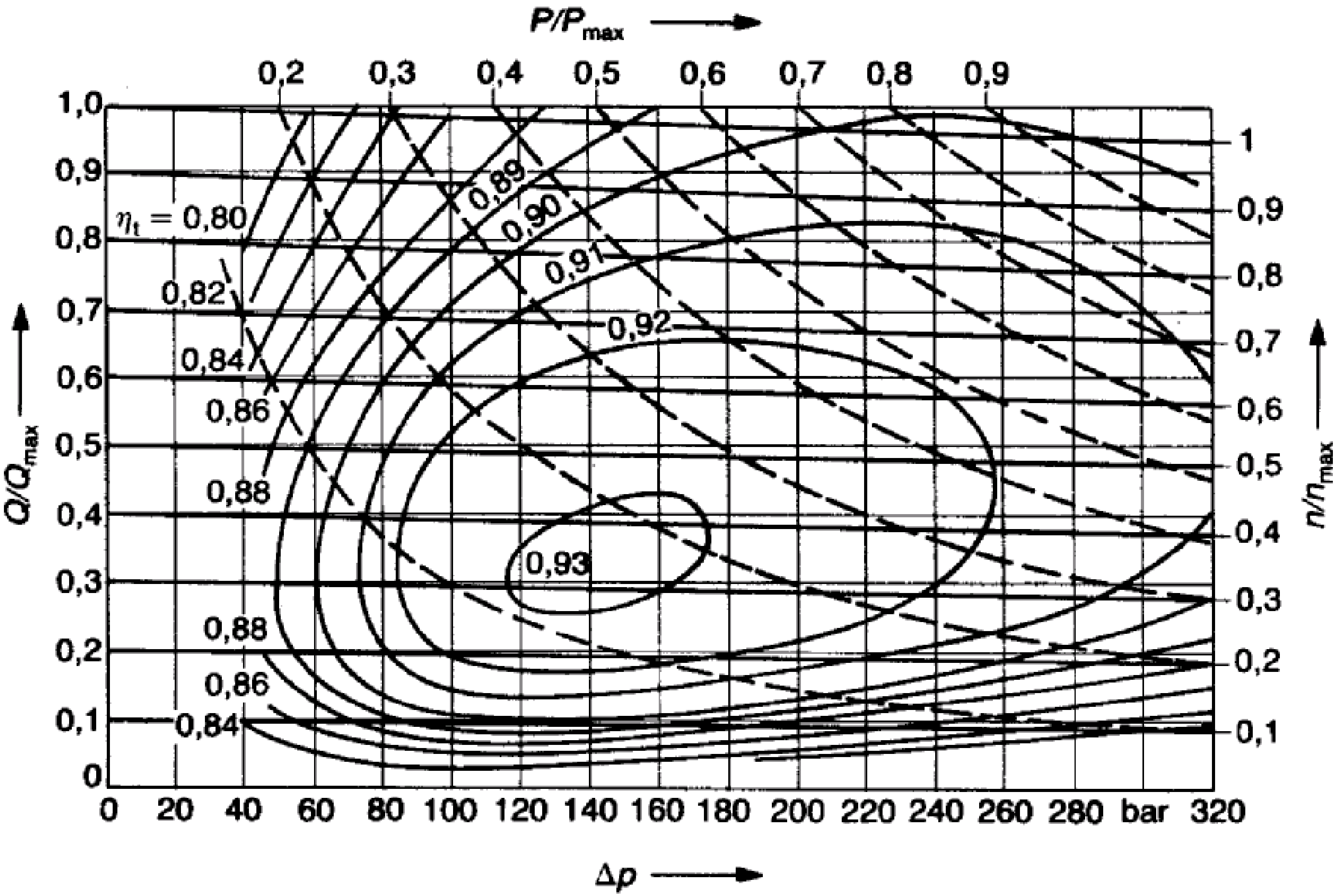
$$\eta_t = \eta_v \cdot \eta_{hm}$$

$$T_S = T_e - \frac{V_i \cdot \Delta p}{2 \cdot \pi}$$

$$\eta_v = \frac{Q_e}{n \cdot V_i}$$

$$\eta_{hm} = \frac{\Delta p \cdot V_i}{2 \cdot \pi \cdot T_e}$$

Steady State Characteristics



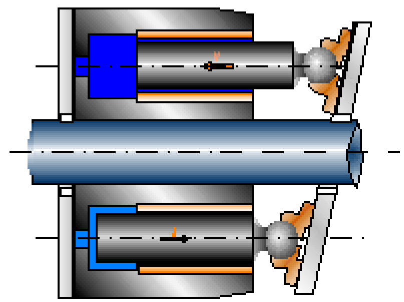
Measurement Based Steady State Model

Why do we need models?

Modelling of Losses

Design Engineer

- Where do they come from?
- How to improve ?



System Engineer

Prediction of :

- System behaviour
- Energy consumption
- Thermal behaviour

Precise Model of the Machine

not available today → Measurements required!

Steady State Models

What are the approaches?

Models are based on use of data found by steady state measurements. The main difference of existing models is given by the type of mathematical description.

Use of physical laws of known physical processes to find a basic mathematical expression

Pure mathematical approach for approximation of measured curves

Who were the authors in the past?

- Wilson, 1948 - introduction of T_S and Q_S

$$Q_S = C_\mu \cdot \Delta p + Q_{Sc}$$

$$T_S = C_f \cdot \Delta p + C_\mu \cdot n + T_{Sc}$$

- Schlösser, 1961 – extension of Wilson model
- Thoma, 1969

turbulent flow

$$Q_{S\rho} = C_{Q\rho} \cdot \sqrt{\Delta p}$$

$$T_{S\rho} = C_{T\rho} \cdot n^2$$

Steady State Models

Increase of the number of terms

Zarotti and Nervegna, 1981 — Non-linear terms for both Q_S and T_S

Rydberg, 1983 — Introduction of new terms without physical background

Dorey, 1988 — Non-linear terms for both Q_S and T_S, using also variable coefficients

Bavendiek, 1987 — Totally 22 loss terms for Q_s and T_S

Ivantysyn and Ivantysynova, 1993 — First pure mathematical approach

Huhtala, 1997 – two line model

Ivantysynova, 1999 - Polymod

POLYMOD - Steady State Model

Measured Data

Least Square Method

Coefficients & Exponents

Polynom (integer exponents)

or

Function with real exponents

$$
\begin{aligned}
F(x,y,z) = {} & k_{000} + k_{100}\cdot x + k_{200}\cdot x^2 + \ldots + k_{p00}\cdot x^p \\
& + (k_{010} + k_{110}\cdot x + k_{210}\cdot x^2 + \ldots + k_{p10}\cdot x^p)\cdot y \\
& + \ldots + (k_{0q0} + k_{1q0}\cdot x + k_{2q0}\cdot x^2 + \ldots + k_{pq0}\cdot x^p)\cdot y^q \\
& + [k_{001} + k_{101}\cdot x + k_{201}\cdot x^2 + \ldots + k_{p01}\cdot x^p \\
& + (k_{011} + k_{111}\cdot x + k_{211}\cdot x^2 + \ldots + k_{p11}\cdot x^p)\cdot y \\
& + \ldots + (k_{0q1} + k_{1q1}\cdot x + k_{2q1}\cdot x^2 + \ldots + k_{pq1}\cdot x^p)\cdot y^q]\cdot z \\
& + \ldots + [k_{00r} + k_{10r}\cdot x + k_{20r}\cdot x^2 + \ldots + k_{p0r}\cdot x^p \\
& + (k_{01r} + k_{11r}\cdot x + k_{21r}\cdot x^2 + \ldots + k_{p1r}\cdot x^p)\cdot y \\
& + \ldots + (k_{0qr} + k_{1qr}\cdot x + k_{2qr}\cdot x^2 + \ldots + k_{pqr}\cdot x^p)\cdot y^q]\cdot z^r
\end{aligned}
$$

with: $x = V_i$, $y = n$, $z = \Delta p$

Approximation

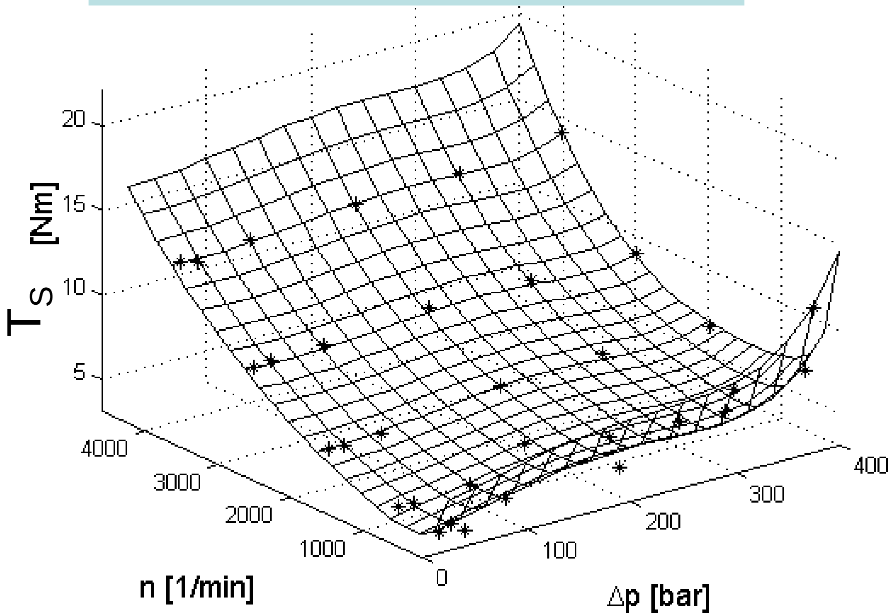
$$Q_S = f(V_i, n, \Delta p)$$

$$M_S = f(V_i, n, \Delta p)$$

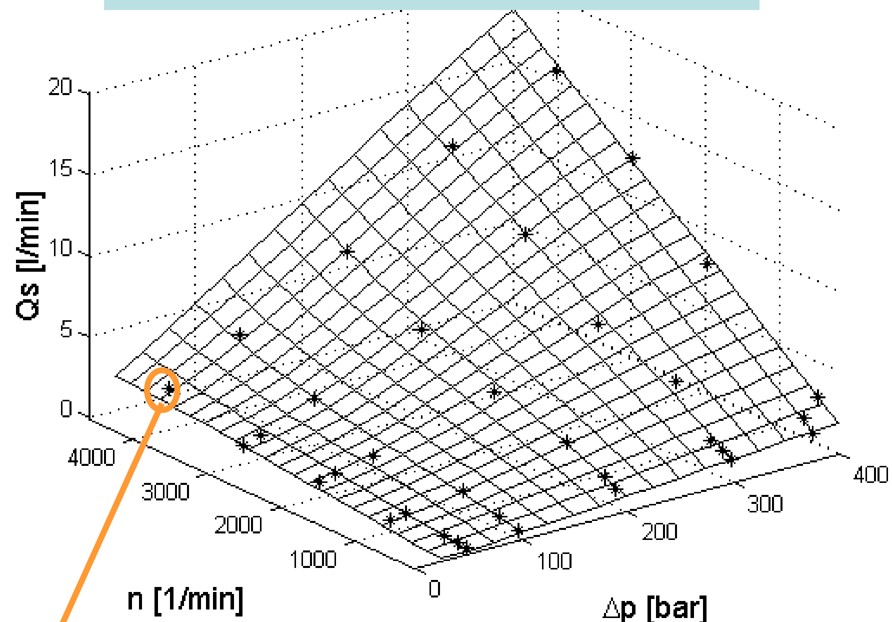
Steady State Characteristics

$$T_S = f\left(\Delta p, n, V_i, \theta\right) \qquad Q_S = f\left(\Delta p, n, V_i, \theta\right)$$

V_i=100%



V_i=100%



Measured points

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