Wood Lathe Design Project

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ENME 400 - Group 102.2
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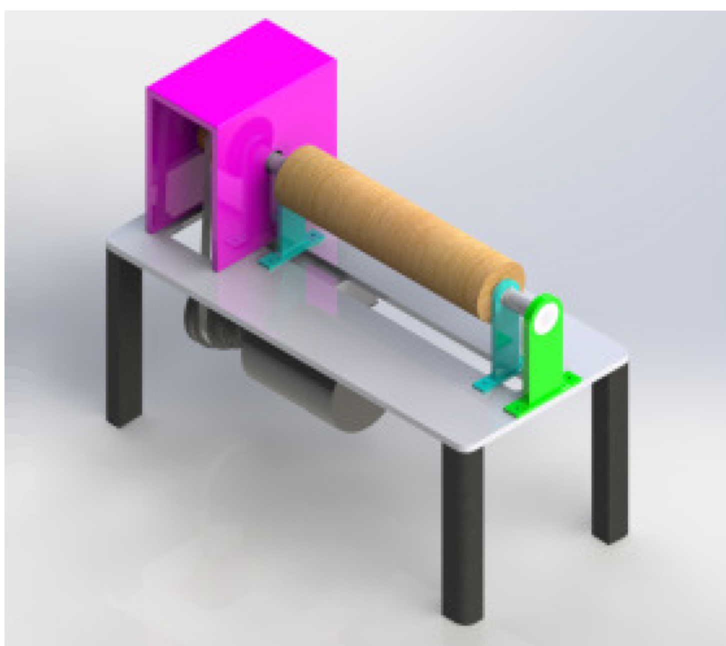
<u>Honor Code Statement:</u>
I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination.



Executive Summary

A lathe is a machine utilized to perform machining operations, in which the workpiece is subject to rotation around its longitudinal axis, and a static tool is applied against the workpiece to remove material. Lathes are used for a wide range of operations, such as sanding, wood turning, drilling and cutting. Due to its versatility, lathe machines are often utilized as an all in one machine by hobbyists and homeowners.

The purpose of this report is to demonstrate the team's ability to employ Machine Design concepts to design a real world mechanical system based on a set of user requirements and design constraints. The design will include a motor, shafts, pulleys, belts, bearings and other housing elements. The team will deliver a detailed report outlining the specifications and justifications for these design of these components.

The report consists of an introductory section, containing the machine design specifications, user's most used parameters, the layout of the overall project, and the sizing of the electrical motor required.Selecting a motor based on the design specifications and the power required was the first task performed by the team. Following the motor selection, a section dedicated to the design of critical components demonstrates the results found by the team's calculations, and the required specifications of the power transmitting belt, the main shaft, the bearings (ball and rollers), and lastly, the housing of the device. The power transmitting belt design was based on the motor specifications, designing the belt allowed the team to decide on the shaft layout. Once the team had the shaft layout they could begin the iterative process of designing the physical characteristics of the shaft and bearings simultaneously. After designing all the components the team could then focus on the actual assembly of the components based on the calculations provided below in the appendix. Finally based on the results from each section in design process and a concluding statement was provided by the team. Hand calculation, FBDs, CAD drawings and models are provided below in the appendix for reference.

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Introduction

A woodworking lathe is a machine that is used to form a piece of wood into a symmetric shape by rotating the wooden piece about its longitudinal axis. The wooden piece is cut to shape using various hand-held tools pushed into it as the wood rotates at high speeds. Smaller wood lathes are used to make items such as bowls, knobs, and rolling pins while larger wood lathes can be used for larger items such as bed frame supports.

Wood lathes have different components with each having different functions. Although modern wood lathes have extra features and accessories, all lathes share the most essential elements which are the headstock, tailstock, spindle, motor, lathe bed, and tool rest [1]. The wood piece will be arrested on the lathe by compressing it in between the spindle and the tailstock quill. Figure 1 shows the components of a basic wood lathe.

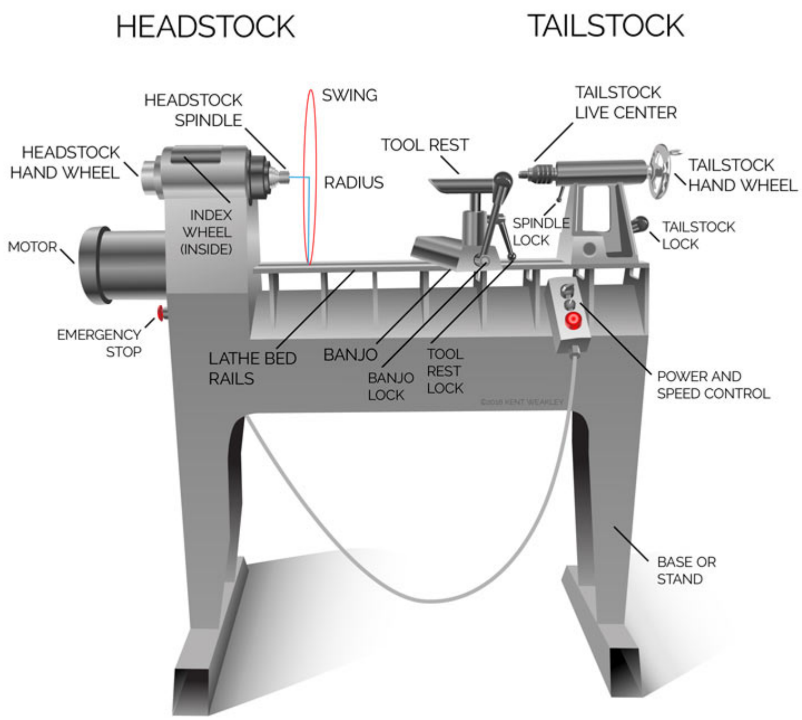


Figure 1: Basic wood lathe components [1]

Wood lathes come in different sizes. Mini and mid-size wood lathes have a 1-inch spindle which can turn wood pieces of 4 to 12 inches diameter while larger lathes can turn pieces with a diameter of 60 inches or larger [2]. Small lathes are capable of very high RPMs compared to larger lathes, however, they cannot get to lower RPM speeds that larger lathes can. The size of the lathe also affects how it will be mounted. Benchtop lathes can be clamped down on a tabletop while larger lathes already have a stand.

The objectives for the overall design are to provide an acceptable initial design that meets customer needs and design specifications, and for it to be a good base model with lots of room for improvement. For the initial design, we are focusing on making it comfortable and easy to use and give it the versatility a user may need with safety in mind. The motor size will be decided by calculating the power needed for typical operation conditions and also satisfy the design specifications provided.

Design specifications, power calculations, and motor sizing will be discussed in detail in its respective sections. The design specifications section addresses typical customer needs and the product design specification provided that the design needs to satisfy. Calculations for power required to remove wood for the most common size and material are presented in the Power Calculations section. This information is needed to select appropriate motor size for the design. Givens and assumptions are also given in detail. In the Motor Sizing section, general information and specification about the selected motor is presented.

This information serves as the foundation for the remainder of the report and the calculations. With P_{motor} calculated and also with the design limitations, we can make a preliminary design, which will then be used to find specific locations and dimensions of the lathe's subcomponents such as the belt, pulleys, bearings, and shaft. With the components' locations and sizes rigorously calculated, we can draw our lathe's components, assemble them together, and present them as one using the computer aided software, Solidworks.

Needs and Design Specification

When using a woodworking lathe, the user desires comfort, ease of use and versatility. In other words, since wood lathing can be a tedious and time consuming activity, the user wants to be able to perform the work with comfort. In a wood lathe, comfort is satisfied by the use of a tool rest. This allows the user to rest the tool down at the appropriate angle to carve the wooden piece. This also increases the precision of the carving and leads to error-free final products. With regard to ease of use, the design should be user-friendly. This means that the functionality of all of the parts are very clear to the user. Setting-up, adjusting and removing the wood from the machine should be easy to use and not lead to any confusion. In terms of versatility, the user desires a wooden lathe that can be used for all shapes and sizes. Therefore, a wooden lathe of choice would be able to accomodate wooden blocks of various dimensions without sacrificing functionality. Additionally, the wooden lathe would be able to turn at various speeds in order to accomodate for more detailed designs as well as faster cuts. The user also desires to adjust the lathe up and down as well as side to side in order to properly align the wood for the desired cut. The more adjustable the machine is, the more desirable it is to the user [3].

Other important customer needs include the foundation of the lathe, the height, and the placement of the power switch. It is important for the base of the lathe to be heavy enough in order to limit the vibrations experienced by the system. Any vibrations could affect the positioning of the tool and block. The height of the spindle should approximately be at the elbow height of the user to allow for ease of use as well as user comfort. Finally, the placement of the switch is an overlooked aspect of the design. In case of emergency, the user should be able to easily access the power switch [3].

Below is a comprehensive list of the design specifications for the wooden lathe:

- Capable of cutting maple, oak and birch
- Turning speeds: 800, 1600, 2400, and 3200 RPM
- Maximum diameter of the workpiece: 9"
- Maximum length of the workpiece without extension: 18"
- Maximum length of the workpiece with extension: 30"
- Weight ≤ 250 lbs
- Maximum size: 36" x 24" x 24"
- Manufacturing cost ≤ $200

Power Calculation

Givens and Assumptions:

Wood Piece

Minimum Diameter = 1" ~ 25.4 millimeters

Maximum Diameter = 9" ~ 228.6 millimeters

Strength ~ 10 MPa = 10×10^6 Pa

We wish to remove 1 millimeter of wood, radially. For this calculation, we will assume a wooden piece diameter of 4" ~ 101.6 millimeters (102 mm) .

Wood Piece Diameter = 102 mm

r_o = Outer Radius = 51 mm

r_i = Inner Radius = 50 mm *considering a 1 mm removal*

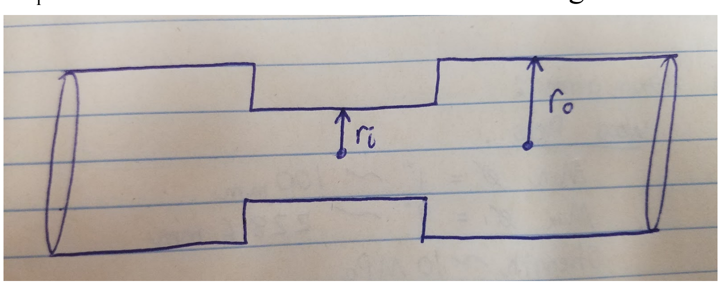


Figure 2: Schematic of the wooden piece

Assuming a cutting speed of

V = 6 m/min = 0.1 m/s

With the Power equation:

$P = T\omega$ = (Torque)(Angular Speed)

$P = FV$ = (Force)(Velocity)

F = (Strength)(Area) → [Pa][m^2]

$= (10 \times 10^6) \cdot (\Pi(0.051^2 - 0.050^2))$
= 3173 N

$P = FV$
= (3173 N)(.1 m/s)
= 317.3 W
= 0.426 HP [Horsepower]

Assuming a Safety Factor of 1.15:

$P_{Motor} = n_d P$
$P_{Motor} = (1.15)(0.426)$

$P_{Motor} = 0.490$ [HP]

Motor Sizing

To size our motor we based our calculations on the *most-used-cutting* parameters discussed above. We first began by calculating the amount of work required to reduce the original diameter of the wood by our cutting depth and then dividing the work required by the feed time to get a final motor power output. The power required for typical operation conditions was calculated to be 0.490 [HP]. Considering a factor of safety of approximately 1.15, and also to facilitate the selection process, the group decided to use a ½ HP motor.

Knowing the power translated by the motor is essential when designing for belts especially. Using the the power and torque values of the motor can be used to find the forces in the belt, which in turn, can allow us to calculate belt thickness, pulley width, and the necessary belt installation force, F_i.

The motor selected was the *Grizzly Heavy-Duty Electric Motor G2528*. The device information is provided below:

Figure 3:G2528 Grizzly Heavy-Duty Electric Motor [4]

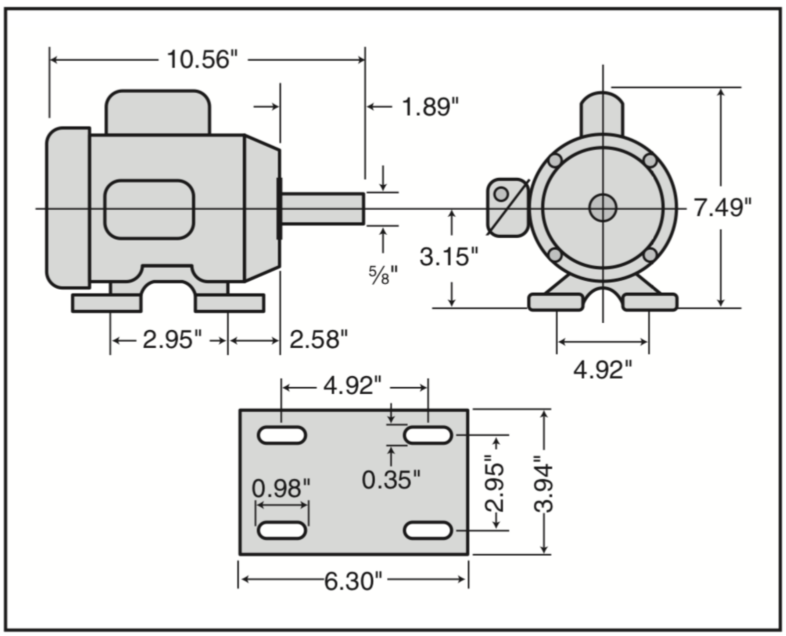


Figure 4:G2528 Grizzly Heavy-Duty Electric Motor drawings [4]

Table 1: G2528 Grizzly Heavy-Duty Electric Motor Specifications

Model	**Type**	**HP**	**Rotation**	**RPM**	**Amps at 110V/220V**
G2528	Enclosed	½	Reversible	1725	8/4

General Layout

This section of the report encompasses the team's proposed layout sketches of the wooden lathe design. The figures clearly display the dimensions of all of the necessary parts of the wooden lathe. This includes the overall dimensions of the wooden lathe, distance between the centerline of the pulleys, lengths of the shafts, thickness of the bedplate, length of the legs, location of the motor and clearance between the pulleys and bearings/housings. Additionally, all other intermediate distances are displayed on the drawings. These dimensions changed as the team progressed with the project.

Layout Sketches

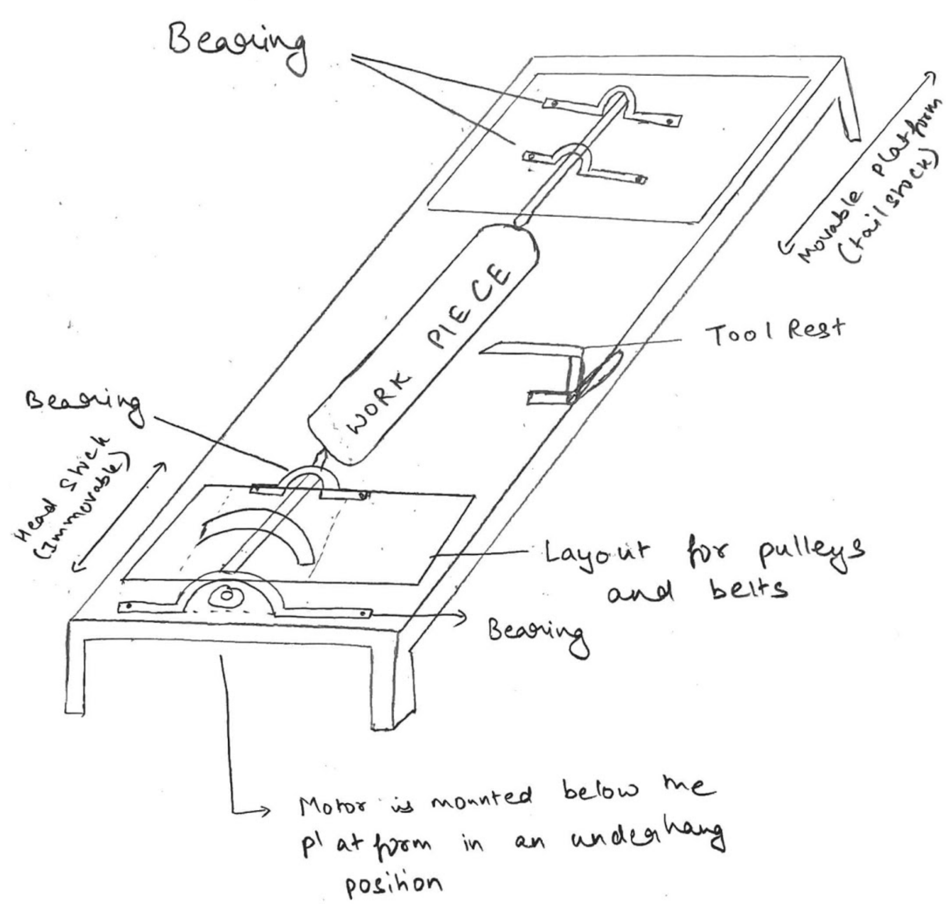


Figure 5: Layout sketch of proposed wooden lathe design

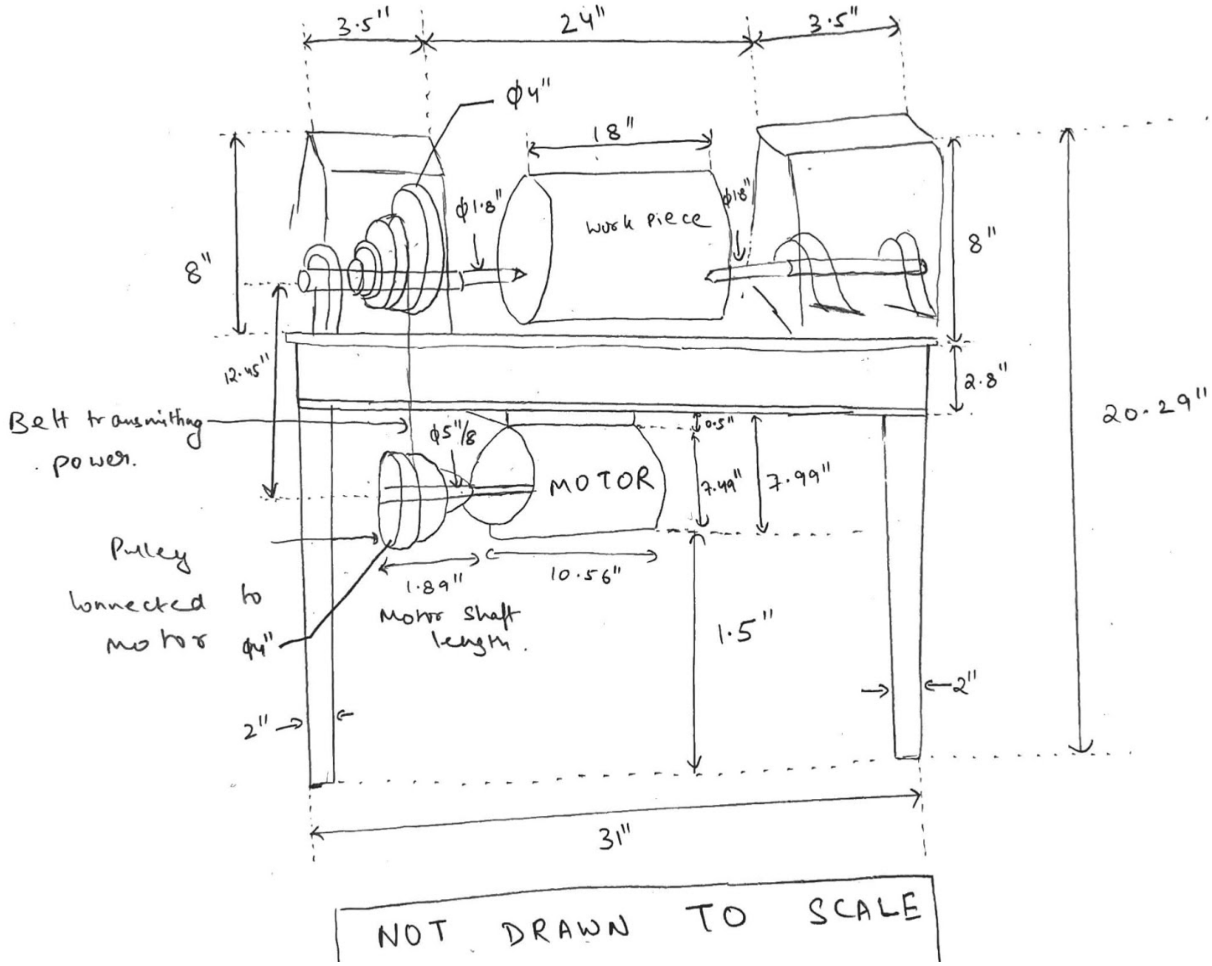


Figure 6: Side view of proposed wooden lathe with dimensions

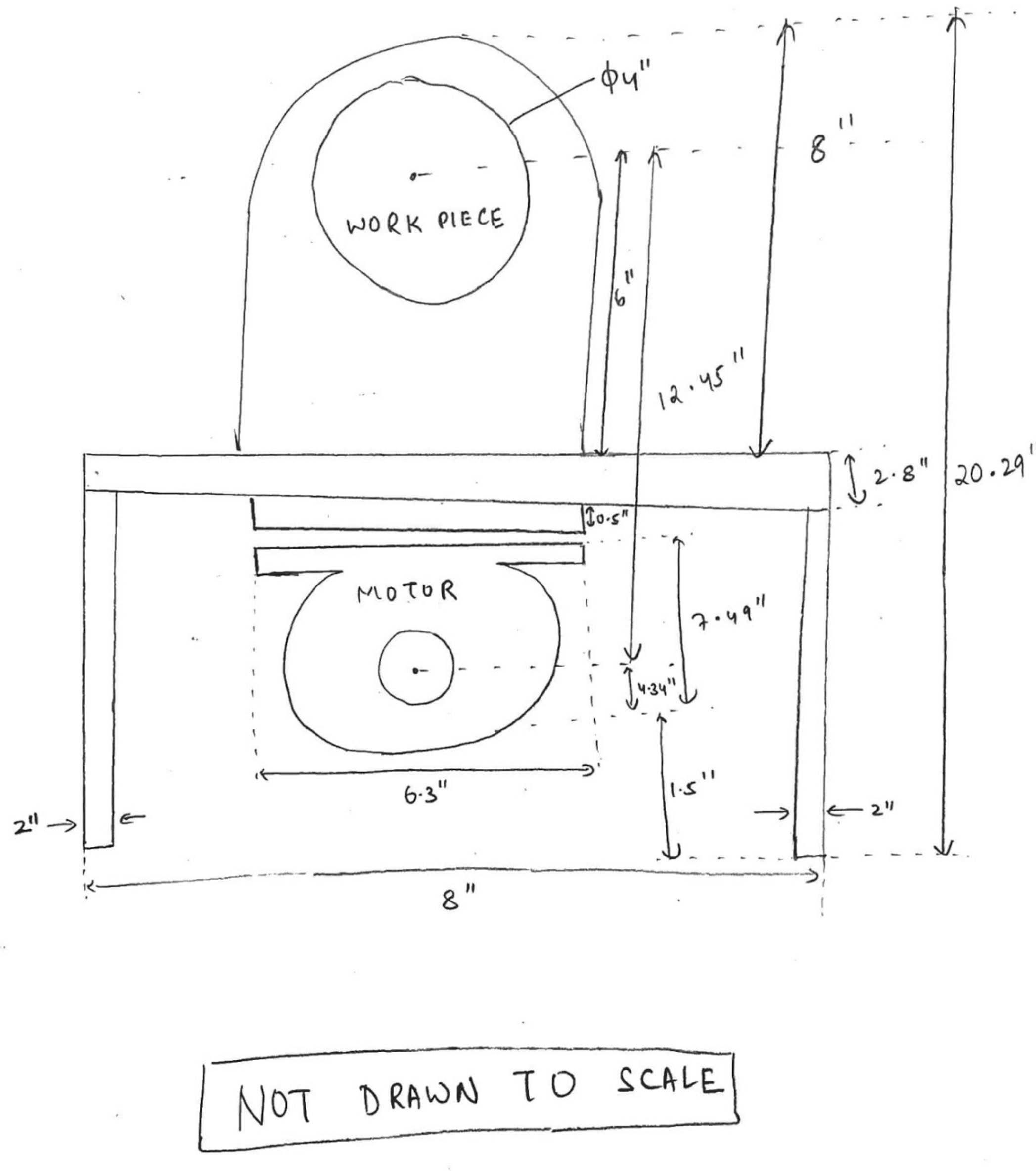


Figure 7: Front view of proposed wooden lathe design with dimensions

Rationale Behind Dimensions and Sizing

Based on the layout sketch (*Figure 1*) and 2D drawings (*Figures 2 and 3*) above, it can be seen that the proposed wooden lathe design has the pulleys inside the bearing span. The pulleys connect to the motor below through a cutout on the table. The team decided to design the lathe this way because it puts the user safety at the forefront. This exposes the pulleys the least which means that the user is less likely to come into improper contact with the pulleys when they are not manually switching the location of the belt. The team's design has a housing over the pulley and bearing on the headstock so that all the moving parts are hidden away for safety considerations. Additionally, by placing the pulleys inside of the bearing limits the deflection experienced by the workpiece.

The motor was designed to be located under the bedplate which is below the table. The motor is mounted to the bedplate which is also mounted to the table. The team chose this design because the chosen motor, the *Grizzly Heavy-Duty Electric Motor G2528 [1],* has a corresponding bedplate that easily mounts to the motor. Mounting the motor below the table also allowed the team to save space on the overall dimensions of the wooden lathe. Additionally, the motor is not as exposed to the user because when it is used for an extended period of time it can get hot which poses a safety risk to the user.

In our initial design, we expect a maximum of 4 inches diameter. However, this may change after we do more calculations which is why we made sure that the housing of the pulley can support a diameter of up to 8 inches by only adding additional height from the centerpoint of the pulley to the top of the housing. With the current pulley size, there is clearance of 4 inches from table top to the pulley and clearance of 2 inches from the top of the housing to the pulley.

There are pulley and bearing housings that help protect the respective parts of the lathe. The bearings (who help prevent the lathe shaft from moving in the vertical and outward directions) and the pulleys (who help provide a torque that is delivered to the shaft and subsequently the workpiece) are important parts of our lathe setup. Failure to protect these components can cause the lathe to unexpectedly break down and induce harm on the user and its surroundings. An 8" tall housing is used to protect the bearings and the pulleys. The advantage of using tall housings for the pulleys is that, if one wishes to use larger pulleys than the 4" inch diameter ones that we have, there will be plenty of available room within the housings to fit larger diameter pulleys.

Belt Selection

For the lathe project, we chose to select a belt/pulley system that would be responsible for rotating the workpiece. We chose to have one driver pulley (attached to the motor), one driven pulley (attached to the headstock shaft), and one belt. Having four separate pulley sizes allows our lathe to have four separate speeds, 800 RPM, 1600 RPM, 2400 RPM, and 3200 RPM. Each speed can be achieved by simply changing the location of the belt on the pulley system. This is advantageous, than say to gears, because if you wished to change rotational speeds, you would have to switch out the entire gear assembly, which could prove troublesome.

Once we calculated the required motor power output we could then begin designing the belt that would be used for all four pulley combinations to drive the workpiece shaft.We based our assumptions for belt size on the 4” work piece (2400 RPM) set up assuming that it would require the strongest belt, longest belt length due to the required size of the pulleys, and largest belt width. In order to start our calculations we had to make an initial estimate of the size of the pulleys for 4” work piece, we decided on 5 inch diameter pulleys. These sizes were chosen due to overall lathe height constraints, required pulley minimum diameters, and pulley size constraints. Knowing the size of our largest pulley combination, we could then assume a centerline distance that when summed with the radii of our pulleys would not exceed the specified maximum height of the lathe; we chose a centerline of 12.45 inches for a total height of 20.29 inches. An A-2 belt was chosen because it was the strongest belt available that fits within our pulley size limitations and it also had a 2.4 inch minimum pulley diameter. Although choosing the strongest possible belt will increase costs and belt thickness, it will reduce belt width in turn reducing pulley width and the overall length of the lathe.

After making all necessary assumptions we could start calculations regarding the actual dimensions and forces on our belt. All the calculations for this process can be found in Appendix B1-B4. The required power output of the motor could be used in conjunction with the shock factor, assumed to be 1.1 due to light shock, and the safety factor, specified at 1.1 as well, to find the power of the belt. Manufacturer motor shaft speed, belt power, and the speed reduction or amplification ratio could then be used find the torque which came out to be 22.10 lbf-in. After finding the torque we could then use equations for the tension in the left and right sides of the belt (F1 and F2), the centrifugal tension (F_c), and a third equation relating F1, F2, and Fc to an exponential function of belt friction. Each of these forces are functions of the belt thickness so they can be solved simultaneously in terms of belt width to find the belt width. The belt width could then be used to find F1 and F2, the initial belt tension, and lastly the belt dip. Using geometry, pulley size, and assumed centerline distance we could calculate the total length of the belt. The final results of our calculation were: belt width of 1.06 inches, belt length of

38.44 inches, belt thickness of 0.11 inches, initial tension of 6.188 lbf, F_1 = 10.95 lbf and F_2 = 8.29 lbf.

Since our design required a single pulley for four different shaft speeds we realized that the pulley length and centerline would remain constant for each pulley combination. In order to find the pulley size for the different work piece sizes we worked backwards from the belt length equation using the previously calculated belt length, and the assumed centerline to calculate pulley size to maintain a constant centerline and belt length. As we initially expected the required belt width for the 800 RPM was 0.8305 inches which was the largest among all the speeds and thus, shaft would be designed around the larger belt width. The entire calculation process for the 800 RPM pulley setup can be found in Appendix B.1.

Belt Length (Assembled)

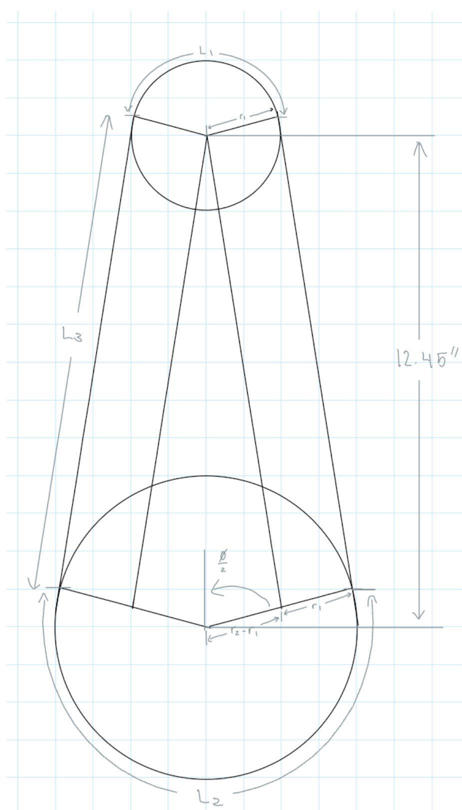


Figure 8: Belt drawing

The following procedure was used in order to find the stretched (assembled) length of the belt:

1. $cos(\varphi/2) = \frac{r_2 - r_1}{C} \Longrightarrow \varphi = 2cos^{-1}(\frac{r_2 - r_1}{C}) = 3.0286\ rad$
2. $L_1 = r_1\varphi = 5.44\ in$
3. $L_2 = r_2(2\pi - \varphi) = 8.137\ in$
4. $L_3 = \sqrt{C^2 - (r_2 - r_1)^2} = 12.43\ in$
5. $L_{assembled} = L_1 + L_2 + 2L_3 = 38.44\ in$

Pulley Selection

Using the 2400 RPM calculations as the driving factors, the group decided on choosing a Polyamide A-2 material for the belt design. With the smaller pulley diameter coming out to be 3.00 inches, Polyamide A-2 can work on pulley diameters of at least 2.4 inch. It also has the highest coefficient of friction and lowest specific weight, out of of the other usable belt materials.

The characteristics of Polyamide A-2 are as follows:

Table 2: Polyamide A-2 Characteristics

Material	Specification	Size [in] (thickness)	F' [lbf/in]	Specific weight [lbf/in^3]	Minimum Pulley Diameter [in]	Coefficient of Friction
Polyamide	A-2	t = 0.11	60	0.037	2.4	0.8

Individual Belt Calculations

See Appendix B for each team member's detailed calculation for the various workpiece speeds. *Table 3* below shows a summary of all the calculations and the resulting belt width and pulley width.

Table 3: Summary of calculations

	800 RPM**	1600 RPM	2400 RPM	3200 RPM	Summary
Belt Grade	Polyamide A-2	Polyamide A-2	Polyamide A-2	Polyamide A-2	Polyamide A-2
Stretched belt length	38.44 in	38.44 in	38.44 in	38.44 in	38.44 in
Belt Thickness	0.11 in	0.11 in	0.11 in	0.11 in	0.11 in
Belt Width	0.664 in	0.30 in	0.25 in	0.432 in	0.664 in
Pulley Width	0.8305 in	0.375 in	0.313 in	0.54 in	0.8305 in
Initial Tension	23.464 lb	7.789 lb	6.188 lb	11.57 lb	23.464 lb

**Note: The results of the 800 RPM case scenario are inaccurate due to miscalculations (found in Appendix B.1). Instead of only considering $L_{assembled}$ (from the 2400 RPM), L_1, L_2, and L_{sides} were all considered, which lead to the finding of the inaccurate pulley radii, belt width, pulley width, and tensions for the 800 RPM case. These calculations, in turn, affect the following bearing and plate deflection calculations as well. Due to lack of time and not seeing this error early on, we performed the following procedures using the numbers found in the 800 RPM column above. The CAD drawings for the pulleys reflect the correct diameters.

Belt Calculations Procedure

For identifying the appropriate belt width and initial tension required, the following procedure was used. The calculations below are based upon a motor RPM of 1725 RPM and workpiece RPM of 800 RPM for a Polyamide A-2 belt :

1. $\omega_{motor} = 2\pi \frac{N_2}{60} = 2\pi \frac{1725\ RPM}{60} = 57.5\pi\ rad/s$

2. $P_{belt} = \eta_d K_s P_{motor} = (1.1)(1.1)\ (0.5\ HP)(550 \frac{ft-lbf/s}{HP}) = 332.75\ ft - lbf/s$

3. $P_{belt} = T\omega_{motor} \Rightarrow T = \frac{P_{belt}}{\omega_{motor}} = \frac{332.75\ ft-lbf/s}{57.5\pi\ rad/s} = 1.842\ ft - lbf$

4. $T_1 = (F_1 - F_2) * r_1 \Rightarrow (F_1 - F_2) = \frac{T}{r_1} = \frac{1.842 ft-lbf}{(1.5\ in)(1 ft/12 in)} = 14.736\ lbf$

5. $F_1 = C_p C_v F'b = (0.73)(1)(60\ lbf/in)(b) = 43.8b\ lbf$

6. $F_2 = 43.8b - 14.736$

7. $F_c = m'r^2\omega^2 = (\frac{\gamma}{g})(bt)(r\omega)^2 = \frac{90.037\ lbf/in}{(32.2\ ft/s^2)(12\ in/ft)}(b)(0.11\ in)(2.77\ in * 57.5\pi\ rad/s)^2 = 2.637b\ lbf$

8. $\frac{F_1 - F_c}{F_2 - F_c} = exp(0.75\mu_s\varphi) \Rightarrow \frac{43.8b - 2.637b}{(43.8b - 14.736) - 2.637b} = exp(0.75 * 0.8 * 2.937) \Rightarrow \qquad b = 0.432\ in$

9. $Pulley\ Width = 1.25b = 1.25\ (0.432\ in) = 0.54\ in = Pulley\ Width$

10. $F_1 = 43.8b = 43.8\ lbf/in\ (0.432\ in) = 18.94\ lbf$

11. $F_2 = 43.8b - 14.736 = 43.8\ lbf/in\ (0.432\ in) - 14.736\ lbf = 4.2\ lbf$

12. $F_i = 0.5(F_1 + F_2) = 0.5(18.94\ lbf + 4.2\ lbf) = 11.57\ lbf = F_i$

Reactions at the Bearings

Before we could begin finding reactions and designing our workpiece shaft we would have to make some initial assumptions regarding the overall shaft layout. From the previous belt calculations we can find the length of the four pulley block by multiplying the width of the belt by a factor of 1.25 and then multiplying by four for the amount of pulleys on the block, this will result in a pulley block of approximately 3.322 inches long. Next, we assumed the separation between the midplanes of the two bearings on the headstock to be 3 inches with a clearance of 0.25 inches from the workpiece. On the tail stock, the pulley blocks are positioned in between the two bearings with clearances of 2 and 3 inches. Finally, the length of the workpiece was assumed to be 30 inches. A diagram of our shaft layout can be seen in *Figure 1*. Another important assumption to make to ease our calculation process is that we will be treating the shaft as a constant diameter when finding moments of inertia instead of treating it like a composite shaft. The last major assumption we made before the starting shaft calculations was shaft material, we decided on AISI 1020 Steel due to its large ultimate tensile strength and its common use in shafts.

Now that we have a basic layout of the shaft we can begin to calculate the reactions at each of the bearings. Each of the bearings will have reaction forces in the the Y and Z but only the ball bearing will have an axial force reaction, meaning we have four sets of unknown reactions.We know the forces at the pulley from the belt calculations, we know the weight of the workpiece, we can calculate the axial force to hold the workpiece in place, and we can calculate the cutting force that is applied to the workpiece.The cutting force in each direction can be found by assuming the cutting blade is angled 45° from the workpiece then multiplying the total cutting force by functions of sines and cosines depending on the the direction of force desired. After finding these three sets of forces there are still 4 sets of bearing reactions that are unknown, to solve for these reactions we will use principle of superposition since the beam is statically indeterminate. To solve for all of the superposition equations we will make use of the fact that the deflection at each of the bearings is zero. The setup to the superposition equations and resultant reactions at each of the bearings can be found in procedure below.

Bearing Reactions Procedure

The following represents a general procedure of how each of us solved for the reactions in each of the bearings. The example below is specific for calculating reactions when the motor runs at 3200 rotations per minute (RPM). Similar procedures were followed when solving reactions for the other speeds, 800, 1600, and 2400 RPM. Exact, hand written procedures of the from each of the group members were submitted separately.

Table 4: Workpiece speed and corresponding diameters

Workpiece Speed	Accompanying Workpiece Diameter	Person Assigned to
800 RPM	8”	Ed Stillwell
1600 RPM	6”	Glady Llanto
2400 RPM	4”	Raghav Agarwal
3200 RPM	2”	Bradley Berman

Initial Setup Calculations

Before we could dive into the meat of the calculations, a few preliminary calculations are needed. First, the force of the user that is applied to the workpiece, F_{user}, can be broken up into three components:

$$F_x = F_{user}\sin(45°)\cos(45°) \text{ [lbs]}$$
$$F_y = F_{user}\cos(45°) \text{ [lbs]}$$
$$F_z = F_{user}\sin(45°)\cos(45°) \text{ [lbs]}$$

We are assuming an applied user force that is 45° to each plane, x,y, and z.

Next, a Torque, T, needs to be calculated. The value will be different for each case, since the ω values vary per case. The equations is as follows:

$$P = T\omega$$

$$T = \frac{P}{\omega}$$

$$T = \frac{0.5\,[HP] \bullet 550 \frac{ft \bullet lb}{s} \bullet \frac{12\ in}{1\ ft}}{\frac{2 \bullet \pi \bullet \omega}{60}} \qquad \text{[in-lbs]}$$

With the T calculated, F_{user} can now be calculated as well. We can treat the vertical component of the F_{user}, F_y, and, multiplied by the radius, $r_{workpiece}$, can set it equal to the torque value we calculated. Again, this value will vary per case due to the different radius diameters of the workpieces.

$$F_y r_{workpiece} = T$$

$$F_{user}\cos(45°)(r_{workpiece}) = T$$

$$F_{user} = \frac{T}{cos(45°) \bullet r}$$

With F_{user} now calculated, F_x, F_y, and F_z can easily be calculated given the relationships with F_{user} defined on the previous page. The calculation for the holding force can be seen below and only applies to 3200 RPM.

$$\delta = \frac{F_x L}{EA} = \frac{(6.964\ lbf)(30\ in)}{(1.667(10^6)\ psi)(\pi(1\ in)^2)} = 4(10^{-5})\ in \approx 0.001\ mm < 2\ mm$$

Given: $\tau_s = 10\ MPa$, $d = 3\ mm$, $L = 2\ mm$

$$\tau_s = \frac{F_{def}}{A} \Rightarrow F_{def} = \tau_s(\pi d L)$$

$$F_{def} = (10(10^6)\ Pa)(\pi(3\ mm)(2\ mm))$$

$$F_{def} = 188.5\ N = 42.376\ lbf$$

$$R_x = F_{def} + F_x = 9.848\ lbf + 42.376\ lbf$$

$$R_{Bx} = 52.224\ lbf$$

Calculations for weight of workpiece, shafts and pulleys

The following table lists the various types of woods and their densities respectively. As we can see from the table oak, has the highest density hence the team decided to use a work piece made out of oak. The calculated densities below only apply to a 2 inch diameter, 3200 RPM case.

Table 5: Density of various woods

Wood	Density [lb/ft^3]
Oak	37-56
Maple	39-47
Birch	42

$Density\ of\ workpiece\ (Oak) = 56\ \frac{lb}{ft^3} = 0.03241\ lb/in^3$

$Density\ of\ pulleys\ (HDPE) = 0.95\ g/cm^3 = 0.0343\ lb/in^3$

$Density\ of\ shaft\ (AISI\ 1020\ Steel) = 0.284\ lb/in^3$

$Weight\ of\ workpiece = \varrho(\frac{\pi}{4}d^2)L = (0.03241\ lb/in^3) * (\frac{\pi}{4}2^2) * (30\ in) = 3.055\ lb$

The weight of the pulleys, shaft, headstock, and tailstock can be calculated using the equations provided below. This equations only apply for the 3200 RPM case, yet similar equations (with different diameter values) are used throughout the other cases.

$Weight\ of\ pulleys = \varrho(\frac{\pi}{4}(d_1^{\ 2} + d_2^{\ 2} + d_3^{\ 2} + d_4^{\ 2})) * thickness$

$= (0.0343\ lb/in^3) * (\frac{\pi}{4}(8.452^2 + 4.472^2 + 3.5938^2 + 3^2) * (0.8305\ in) = 2.536\ lb$

$Weight\ of\ shaft = \varrho(\frac{\pi}{4}d^2)L_{total}$

$Weight\ of\ headstock = (0.284\ lb/in^3) * (\frac{\pi}{4} * 1^2) * (2\ in + 3.322\ in + 3\ in + 0.25\ in) = 1.912\ lb$

$$Weight\ of\ tailstock = (0.284\ lb/in^3) * (\frac{\pi}{4} * 1^2)\ * (0.25\ in\ + 3\ in)\ =\ 0.725\ lb$$

The figure below shows a general layout of the lathe machine. The bearings are labelled by alphabets A,B,C and D with one roller and ball bearing on tailstock and headstock.

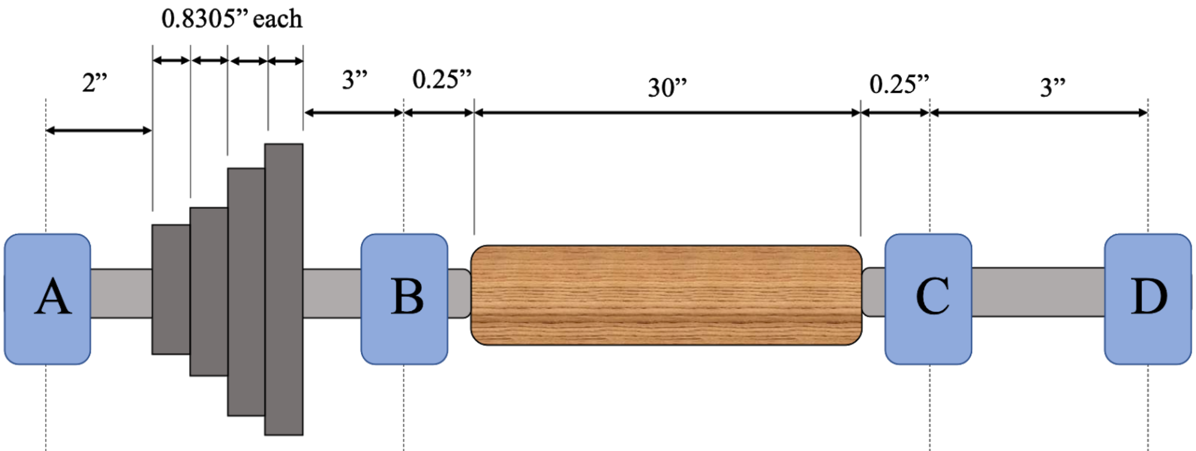


Figure 9: Lathe layout with dimensions

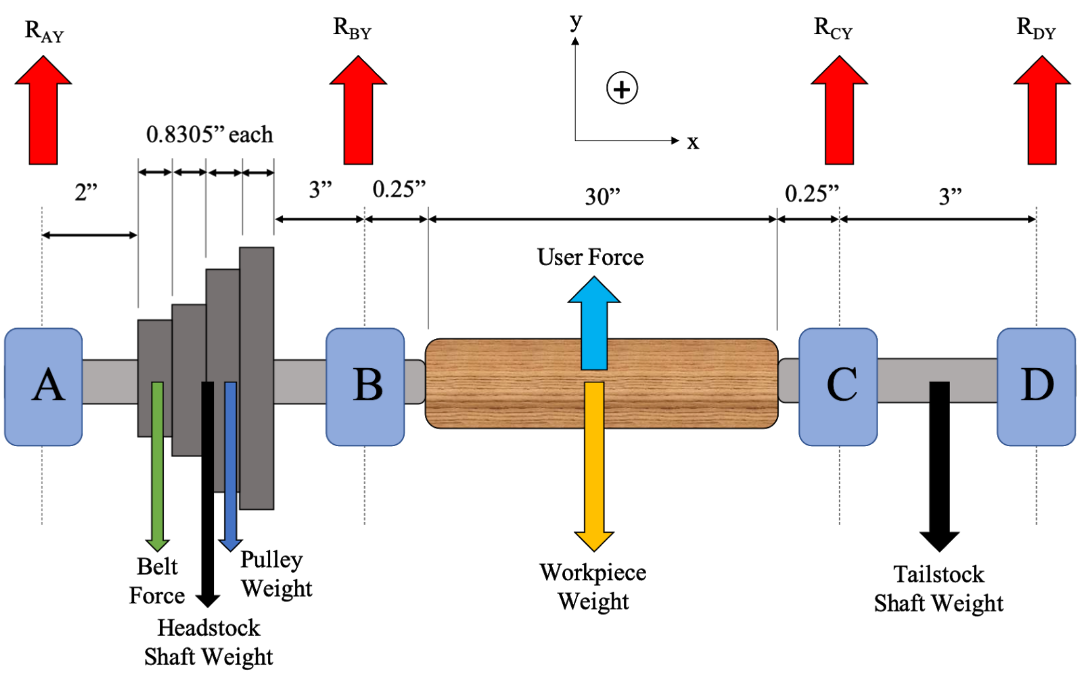


Figure 10: 2D Diagram of Lathe in XY Plane

The following values are used throughout the calculations:

$d_1 = 3$ in $\quad$ $d_2 = 3.5938$ in $\quad$ $d_3 = 4.472$ in $\quad$ $d_4 = 8.452$ in

$t = 0.8305$ in $\quad$ $L = 41.822$ in

$$\overline{x_{belt\,force}} = 2 + (t/2)$$

$$\overline{x_{belt\,force}} = 2.42\ in \quad (for\ 3200\ RPM\ only)$$

$$\overline{x_{Pulley}} = 2 + \frac{d_1^2(t/2) + d_2^2(3t/2) + d_3^2(5t/2) + d_4^2(7t/2)}{d_1^2 + d_2^2 + d_3^2 + d_4^2}$$

$$\overline{x_{Pulley}} = 4.373 \text{ in}$$

$$\overline{x_{shaft\,(headstock)}} = 0.5(2 + 4(0.8305) + 3 + 0.25)$$

$$\overline{x_{shaft\,(headstock)}} = 4.286 \text{ in}$$

$$\overline{x_{woodpiece}} = \overline{x_{user}} = 2 + 4(0.8305) + 3 + 0.25 + 0.5(30)$$

$$\overline{x_{woodpiece}} = \overline{x_{user}} = 23.572\ in$$

$$\overline{x_{shaft\,(tailstock)}} = 2 + 4(0.8305) + 3 + 0.25 + 30 + 0.5(0.25 + 3)$$

$$\overline{x_{shaft\,(tailstock)}} = 40.197\ in$$

Knowing that the deflection at points B and C is zero, the team used superposition and summed the deflections caused by each force and reaction. A sample calculation for the 3200 RPM and deflection caused by the weight of the headstock can be seen below. See Appendix C for the equations for a simply supported beam that the team used.

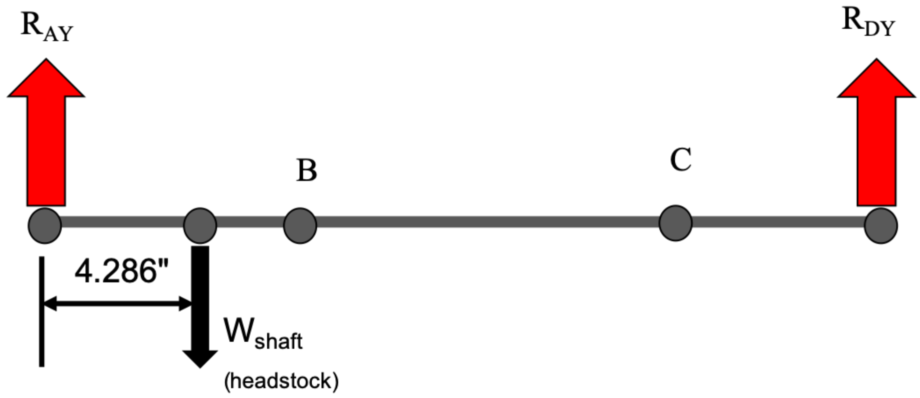


Figure 11: Superposition for weight of headstock shaft

$y_{BY1} = \frac{(F)(b)(L-x)}{6EIL}(x^2 + b^2 - 2Lx)$ where x = 8.322 in

$y_{BY1} = \frac{(W_{S,h})(4.286)(L-x)}{6EIL}(x^2 + 4.286^2 - 2Lx)$ where x = 8.322 in

$y_{BY1} = -665.7/EI$

$y_{CY1} = \frac{(W_{S,h})(4.286)(L-x)}{6EIL}(x^2 + 4.286^2 - 2Lx)$ where x = 38.822 in

$y_{CY1} = -168.7/EI$

Table 6: Superposition results in y-direction

Force due to:	**Deflection at Bearing B**	**Deflection at Bearing C**
Belt Force	$y_{BY2} = -11579.9/EI$	$y_{CY2} = -2896.1/EI$
Pulley Weight	$y_{BY3} = -899.7/EI$	$y_{CY3} = -228.2/EI$
Reaction at Bearing B	$y_{BY4} = +619.5R_{BY}/EI$	$y_{CY4} = +166.2R_{BY}/EI$
User force and Workpiece Weight	$y_{BY5} = +5537.2/EI$	$y_{CY5} = +2267.4/EI$
Reaction at Bearing C	$y_{BY6} = +166.2R_{CY}/EI$	$y_{CY6} = +108.1R_{CY}/EI$
Tailstock Shaft Weight	$y_{BY7} = -65.5/EI$	$y_{CY7} = -43.6/EI$

Summing the deflections at Bearing B yields equation 1:

$$619.5R_{BY} + 166.2R_{CY} = 7673.7$$

Summary the deflections at Bearing C yields equation 2:

$$166.2R_{BY} + 108.1R_{CY} = 1069.2$$

Summing the forces in the y-direction yields equation 3:

$$R_{AY} + R_{BY} + R_{CY} + R_{DY} - (F_1 + F_2)cos\theta - W_{s,h} - W_{pulley} + F_{y,user} - W_{wood} - W_{s,t} = 0$$

$$R_{AY} + R_{BY} + R_{CY} + R_{DY} = 56.1$$

Summing the moments in the x-direction about Bearing A yields equation 4:

$$-(F_1 + F_2)cos\theta(2.42) - W_{s,h}(4.286) - W_{pulley}(4.373) + R_{BY}(8.322) + (F_{y,user} - W_{wood})(23.572) +$$

$$R_{CY}(38.822) - W_{s,t}(40.197) + R_{DY}(41.822) = 0$$

$$8.322R_{BY} + 38.822R_{CY} + 41.822R_{DY} = 28.0$$

With 4 equations and 4 unknowns it is possible to solve for the reactions at each of the bearings using an equation solver:

$R_{AY} = 43.28\ lbs$ $\quad$ $R_{BY} = 16.57\ lbs$ $\quad$ $R_{CY} = -15.58\ lbs$ $\quad$ $R_{DY} = 11.84\ lbs$

The same procedure can then be repeated on for the z-direction. The FBD for the xz-plane can be seen below:

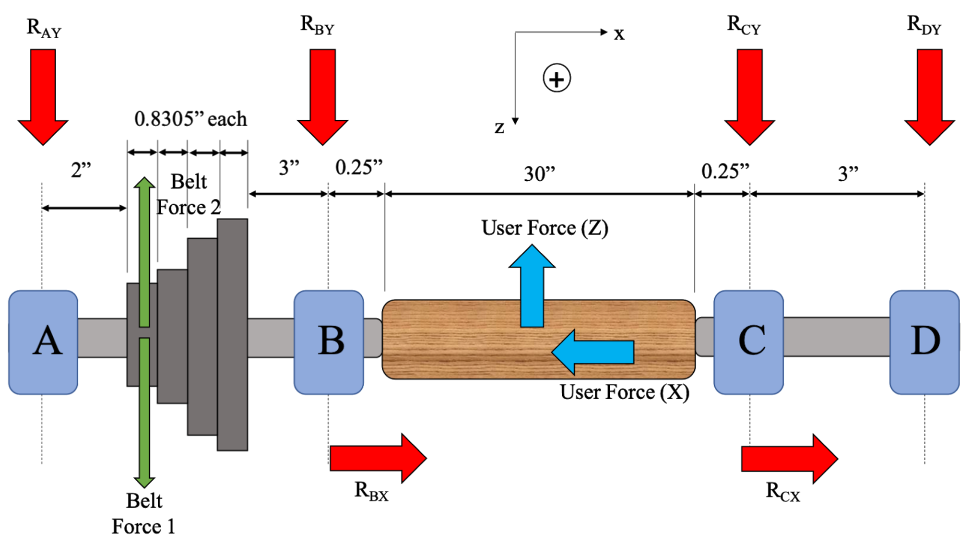


Figure 12: 2D Diagram of Lathe in XZ Plane

Table 7: Superposition results in z-direction

Force due to:	**Deflection at Bearing B**	**Deflection at Bearing C**
Belt Force	$y_{BZ1} = +300.1/EI$	$y_{CZ1} = +75.1/EI$
Reaction at Bearing B	$y_{BZ2} = +619.5R_{BZ}/EI$	$y_{CZ2} = +166.2R_{BZ}/EI$
User force	$y_{BZ3} = -5676.6/EI$	$y_{CZ3} = -2324.5/EI$
Reaction at Bearing C	$y_{BZ4} = +166.2R_{CZ}/EI$	$y_{CZ4} = +108.1R_{CZ}/EI$

Summing the deflections at Bearing B yields equation 5:

$$619.5R_{BZ} + 166.2R_{CZ} = 5376.5$$

Summary the deflections at Bearing C yields equation 6:

$$166.2R_{BZ} + 108.1R_{CZ} = 2249.4$$

Summing the forces in the z-direction yields equation 7:

$$R_{AZ} + R_{BZ} + R_{CZ} + R_{DZ} + (F_1 - F_2)sin\theta - F_{z,\,user} = 0$$

$$R_{AZ} + R_{BZ} + R_{CZ} + R_{DZ} = 5.468$$

Summing the moments in the y-direction about Bearing A yields equation 8:

$$-(F_1 + F_2)sin\theta(2.42) - R_{BZ}(8.322) + F_{z,\,user}(23.572) - R_{CZ}(38.822) - R_{DZ}(41.822) = 0$$

$$8.322R_{BZ} + 38.822R_{CZ} + 41.822R_{DZ} = 160.5$$

With 4 equations and 4 unknowns it is possible to solve for the reactions at each of the bearings using an equation solver:

R_{AZ} = - 3.50 lbs *R_{BZ} = 5.27 lbs* *R_{CZ} = 12.71 lbs* *R_{DZ} = -9.01 lbs*

Summary of Bearing Reactions

Table 8: Summary of bearing reactions

Speed	Roller Bearing A	Ball Bearing B	Ball Bearing C	Roller Bearing D
800 RPM		R_{BX} = 13.926 lbs	R_{CX} = 20.89 lbs	
	R_{AY} = -21.765 lbs	R_{BY} = 95.463 lbs	R_{CY} = 6.273 lbs	R_{DY} = 29.534 lbs
	R_{AZ} = 14.20 lbs	R_{BZ} = -15.47 lbs	R_{CZ} = 2.51 lbs	R_{DZ} = -5.38 lbs
1600 RPM		R_{BX} = 9.28 lbs	R_{CX} = 13.92 lbs	
	R_{AY} = -5.73 lbs	R_{BY} = 50.34 lbs	R_{CY} = -9.09 lbs	R_{DY} = 12.623 lbs
	R_{AZ} = 8.76 lbs	R_{BZ} = -11.57 lbs	R_{CZ} = 3.79 lbs	R_{DZ} = -5.48
2400 RPM		R_{BX} = 9.41 lbs	R_{CX} = 14.11 lbs	
	R_{AY} = 6.55 lbs	R_{BY} = 17.25 lbs	R_{CY} = -3.4 lbs	R_{DY} = 3.4 lbs
	R_{AZ} = 9.29 lb	R_{BZ} = -4.96 lbs	R_{CZ} = -1.05 lbs	R_{DZ} = -12.72 lbs
3200 RPM		R_{BX} = 52.224 lbs	R_{CX} = -42.376 lbs	
	R_{AY} = 43.28 lbs	R_{BY} = 16.57 lbs	R_{CY} = -15.58 lbs	R_{DY} = 11.84 lbs
	R_{AZ} = - 3.50 lbs	R_{BZ} = 5.27 lbs	R_{CZ} = 12.71 lbs	R_{DZ} = -9.01 lbs

**Note: Here we could have also calculated the angular deflection at each bearing, yet we decided not to. Assuming cast iron bearings along with the AISI 1020 steel shaft, we assumed deflection would be negligible. We also felt that we could ignore angular deflection because the legs of the apparatus carry a lot more weight than the bearings do, yet the legs have little to no deflection. With these assumptions in mind, and also seeing that the apparatus is over engineered, we chose to neglect the angular deflections at the bearings.

Bearing Sizing

For this studio, we were tasked with determining an appropriate size for each of the bearings, A,B,C, and D, for each of our specific cases (800 RPM, 1600 RPM, 2400 RPM, and 3200 RPM). The procedure that was used in order to figure out each bearing size is described in the next section. In short, bearing reactions that were calculated in the last section were the main determining factors that were used in order to determine the appropriate size.

A couple assumptions were made before we calculated the bearing sizes. We assumed a design life of 200 hours, which was approved by the professor. We also assumed a safety factor, n_d, of 1.1 and a reliability, R, of 99%. Finally, with the inner ring rotating, we assumed a V value of 1. With these considerations, we followed the procedure below. Our separate calculations were submitted individually.

Bearing Sizing Procedure

The general procedure used to calculate the necessary bearing size for our design is presented below. For our design, we chose a design life of 200 hours with angular speed ranging from 800 to 3200 RPM for different work piece diameters, a design factor (n_d) of 1.1, reliability of 99%, and have an inner rotating ring (V = 1). We are only using 02-Series deep-groove ball bearings and 02-series roller bearings. Reactions used for the calculations are given in Table 8 from the Bearings Reaction section. These reactions were the results of the previous studio.

Initial Calculations:

$$x_D = \frac{L_D}{L_{10}} = \frac{200hrs\,(60\frac{min}{hr})\,(speed)(\frac{rev}{min})}{10^6\ rev}$$

$$x_i = (\theta - x_0)\,(ln(\tfrac{1}{R}))^{1/b} + x_0 \quad \text{where } x_0 = 0.02,\ \theta = 4.459 \text{ and } b = 1.483$$

$$x_i = (4.459 - 0.02)\,(ln(\tfrac{1}{.99}))^{1/1.483} + 0.02 = 0.2196$$

Seen below are sample calculations for 3200 RPM:

$$L_D = 200\ hours * 60 \tfrac{min}{hour} * 3200 \tfrac{rev}{min} = 38.4(10^6)\ rev$$

$$x_D = \frac{38.4(10^6)\ rev}{10^6\ rev} = 38.4$$

<u>Roller Bearing A</u>

$R_{AX} = 0\ lbs \qquad R_{AY} = 43.28\ lbs \quad R_{AZ} = -3.50\ lbs$

$$F_a = R_x = 0\ lbs$$

$$F_r = \sqrt{(R_y)^2 + (R_z)^2} = \sqrt{43.28^2 + (-3.50)^2} = 43.42\ lbs$$

$C_{10} = F_D(\frac{x_D}{x_i})^{1/a}$ where a = 10/3 for roller bearings

$C_{10} = n_D\ (F_e)(\frac{x_D}{x_i})^{1/a}$ where $F_e = F_r$

$$C_{10} = (1.1)\ (43.42\ lbs)(\frac{38.4}{0.2196})^{3/10}$$

$$C_{10} = 224.86\ lbs \approx 1\ kN$$

Therefore, choose 02-Series 25 mm Bore Roller Bearing (Table 11-3) [2]

<u>Ball Bearing B:</u>

$R_{BX} = 52.224\ lbs \qquad R_{BY} = 16.57\ lbs \qquad R_{BZ} = 5.27\ lbs$

$$F_a = R_x = 52.224\ lbs = 0.2323\ kN$$

$$F_r = \sqrt{(R_y)^2 + (R_z)^2} = \sqrt{16.57^2 + 5.27^2} = 17.39\ lbs = 0.07735\ kN$$

$C_{10} = F_D(\frac{x_D}{x_i})^{1/a}$ where a = 3 for ball bearings

$C_{10} = n_D\, (F_e)(\frac{x_D}{x_i})^{1/a}$

$C_{10} = (1.1)\, (F_e)(\frac{38.4}{0.2196})^{1/3}$

$C_{10} = 6.151\, F_e$

Iteration 1:

$F_e = F_a + F_r = 52.224\ lbs + 17.39\ lbs = 69.6\ lbs$

$C_{10} = 6.151(69.6\ lbs) = 428.18\ lbs = 1.905\ kN$

Therefore, we guess 10 mm bore deep-groove ball bearings. From Table 11-2 [2] in Appendix A, the corresponding properties are seen below.

$$C_0 = 2.24\ kN \qquad\qquad C_{10} = 5.07\ kN$$

$\frac{F_a}{C_0} = \frac{0.2323\ kN}{2.24\ kN} = 0.1037$

Therefore, from Table 11-1 [2] in Appendix A it can be seen that the corresponding value of e is

between 0.28 and 0.30. This value is then compared to the $\frac{F_a}{VF_r}$ ratio.

$\frac{F_a}{VF_r} = \frac{0.2323\ kN}{(1)(0.07735\ kN)} = 3 > e$

Since $\frac{F_a}{VF_r}$ is greater than e, you use X_2 and Y_2 to calculate for F_e. The values for X_2 and Y_2 are taken from Table 11-1 [2] in the Appendix A.

$X_2 = 0.56$

Interpolating for Y_2:

$\frac{0.110 - 0.084}{1.45 - 1.55} = \frac{0.110 - 0.1037}{1.45 - Y_2}$

$Y_2 = 1.474$

$$F_e = X_2 V F_r + Y_2 F_a$$

$$F_e = (0.56)(1)(0.07735\ kN) + (1.474)(0.2323\ kN)$$

$$F_e = 0.386\ kN$$

Iteration 2:

$$C_{10} = 6.151(0.386\ kN) = 2.373\ kN$$

Therefore, choose 10 mm bore deep-groove ball bearings. Since the solutions converge, this is the final choice for the ball bearing.

The same procedure is then repeated for Ball Bearing C and Roller Bearing D, as well as for the other lathe speeds.

Summary of Bearing Selections

Table 9: Summary of bearing selections

Speed	Roller Bearing A	Ball Bearing B	Ball Bearing C	Roller Bearing D
800 RPM	*02-Series 25 mm Bore*	*02-Series 10 mm Bore*	*02-Series 10 mm Bore*	*02-Series 25 mm Bore*
1600 RPM	*02-Series 25 mm Bore*	*02-Series 10 mm Bore*	*02-Series 10 mm Bore*	*02-Series 25 mm Bore*
2400 RPM	*02-Series 25 mm Bore*	*02-Series 10 mm Bore*	*02-Series 10 mm Bore*	*02-Series 25 mm Bore*
3200 RPM	*02-Series 25 mm Bore*	*02-Series 10 mm Bore*	*02-Series 10 mm Bore*	*02-Series 25 mm Bore*
Final Bearing Selection	***02-Series 25 mm Bore***	***02-Series 12 mm Bore*** **	***02-Series 12 mm Bore*** **	***02-Series 25 mm Bore***

** It should be noted that the final bearing selection for both of the ball bearings was an 02-Series 12 mm bore even though everyone had selected 02-Series 10 mm bore ball bearings. The reason for this is because in the shaft design (in the following section) the shaft at Ball Bearing B was larger than 10 mm. Therefore, the team had to choose the next smallest ball bearing. The shaft design for the tailstock, at Ball Bearing C, was smaller than 10 mm so this would have been acceptable. However, in consideration of Design for Assembly (DFA), the team elected to choose the same ball bearing as Ball Bearing B (12 mm Bore).

Shaft Design

The next step of the project was to design the shaft that runs through the entirety of our lathe. The objective of this studio was to determine the necessary size(s) of the shaft such that the lathe would be able to operate safely. The shaft will have to vary in diameter in order to account for the many parts of the lathe. Practically everything found and calculated in the previous studios are needed in order properly dimension the shaft. Bore sizes, bearing reactions, pulley dimensions and pulley forces from the 2400 RPM calculations are all needed in order to size of the shaft.

The headstock and the tailstock were analyzed separately. For the headstock, three separate analyses were needed in order to accomodate for the roller bearing, retaining ring, and ball bearing. For the tailstock, only the roller and ball bearings needed to be considered. Iterations for each section were performed until the diameter value did not vary a lot.

Shaft Design Procedure

Below is the analysis used to examine the shaft diameter required when evaluating the headstock shaft after ball bearing, B. The other analyses will not be typed out due to those procedures being very similar. Those calculations can be found in Appendix E. The following procedure is also applicable to the tailstock design.

Internal moments and forces were found at a section cut taken after ball bearing B. The results of the cut are as follows:

$$N = -9.41 \text{ [lbs]}$$

$$T = -15.83 \text{ [lbs]}$$

$$M_y = 85.65 \text{ [lbs]}$$

$$M_z = 93.21 \text{ [lbs]}$$

$$M = \sqrt{M_x^{\,2} + M_y^{\,2}} = 126.59 \text{ [lbs]}$$

Next, stresses were calculated using the internal forces and moments found above. We had to consider whether each force produced an axial or bending stress along with whether or not that stress is alternating or a mean (average). The stresses are calculated below:

For reference:

A = Axial
BM = Bending Moment
a = Alternating
m = mean

$$\sigma_{xx\ A,m} = \frac{N}{A} = \frac{(-9.41\times10^{-3})}{\frac{\pi}{4}d^2} = \frac{-11.98\times10^{-3}}{d^2} \text{ [ksi]}$$

$$\sigma_{xx\ BM,m} = 0 \text{ [ksi]} \rightarrow \text{(Average is 0)}$$

$$\tau_{xy\ m} = \frac{Tr}{J} = \frac{(-15.83\times10^{-3})(d/2)}{\frac{\pi}{32}d^4} = \frac{-0.0806}{d^3} \text{ [ksi]}$$

$$\sigma_{xx\ A,a} = 0 \text{ [ksi]} \rightarrow \text{(Constant N value)}$$

$$\sigma_{xx\ BM,a} = \frac{Mr}{I} = \frac{(126.59\times10^{-3})(d/2)}{\frac{\pi}{64}d^4} = \frac{1.29}{d^3} \text{ [ksi]}$$

$$\tau_{xy\ a} = 0 \text{ [ksi]} \rightarrow \text{(Constant T value)}$$

Now, an Se value can be calculated. Se is defined as:

$$Se = k_a k_b k_c k_d k_e k_f Se'$$

Where:

$k_a = aS_{ut}^{\ b} = 27(57.25^{-.265}) = .92 \rightarrow$ (Assuming the shaft is machined)

$k_b = .879d^{-0.107}$

Where we assume d is the bore size for the roller bearing, d = .394” (10 mm)

$= .879(.394^{-0.107})$

$= .97$

$k_c = 1$ (Considering a Von Mises Analysis)

$k_d = k_e = k_f = 1$

$Se' = .5(S_{ut}) = (.5)(57.249) = 28.62$ [ksi]

Thus,

$$Se = (.92)(.97)(1)(1)(1)(1)(28.62) = 25.57 \text{ [ksi]}$$

For the first iteration, we will assume a q value of 1. (This will change with later iterations). With q = 1,

$$K_{fa} = K_{ta} = 1.7$$
$$K_{fb} = K_{tb} = 1.6$$
$$K_{fs} = K_{ts} = 1.35$$

We get the K_t constant values from given tables (found in Appendix A). We assumed a standard $\frac{D}{d}$ value of 1.2 and a $\frac{r}{d} = .1$

We use Goodman’s equation to finally evaluate a shaft diameter, d, value.

Goodman’s equation is as follows:

$$\frac{\sqrt{[(\frac{K_{fa}\sigma_{xxA,a}}{.85})+(\frac{K_{fb}\sigma_{xxBM,a}}{1})]^2+3(K_{fs}\tau_{xy,a})^2}}{Se} + \frac{\sqrt{[(\frac{K_{fa}\sigma_{xxA,m}}{1})+(\frac{K_{fb}\sigma_{xxBM,m}}{1})]^2+3(\tau_{xy,m})^2}}{K_d S_{ut}} = \frac{1}{n}$$

The equation simplifies down a lot due to $\sigma_{xx\ BM,m}$, $\sigma_{xx\ A,a}$, and $\tau_{xy\ a}$ equaling zero. Plugging in the stress equations calculated earlier into the Goodman equation yields after simplification:

$$\frac{0.00807}{d^3} + \frac{\sqrt{\frac{(4.15\times10^{-4})}{d^4} + \frac{0.0355}{d^6}}}{57.25} = .8$$

(with the n value equalling 1.2 here)

Solving the equation above above yields a d value of:

d = 0.472 [inches]

Now, with a calculated value, a second iteration can now be done. K_b can be recalculated as

$$K_b = .879(0.472^{-0.107})$$
$$K_b = 0.953$$

The Se value now has to be updated due to the new K_b value,

$$Se = \frac{25.57}{0.97} \times .953 = 25.11 \text{ [ksi]}$$

Remembering that $\frac{r}{d} = .1$, and with d = .472

$$r = 0.0472$$

With S_{ut} = 57.25 ksi and $\frac{r}{d} = .1$, and with the given graphs found in Appendix A, q values can be found by using the graphs:

$$q = .67 \text{ and } q_s = .67$$

New K_{fa}, K_{fb}, and K_{fs} values can be calculated now with actual q values:

$$K_{fa} = 1 + q(K_{ta} - 1) \rightarrow 1 + .67(1.7 - 1) = 1.469$$
$$K_{fb} = 1 + q(K_{tb} - 1) \rightarrow 1 + .67(1.6 - 1) = 1.402$$
$$K_{fs} = 1 + q(K_{ts} - 1) \rightarrow 1 + .67(1.35 - 1) = 1.23$$

Now taking the updated K values and Se, the same stress values can be replugged into the same Goodman equation stated earlier. Doing so and recalculating for the diameter value yields:

d = .454 [inches]

Since the diameter of the shaft cannot be smaller than the diameter of the ball bearing bore size (10 mm), we must increase the diameter value of the shaft to match that of the next highest bore size (12 mm).

d = 0.472 [inches]

Shaft Design Results

Calculations similar to that shown above are done for the rest of the tailstock and the head stock as well. The results are represented by the tables below:

Headstock

Table 10: Headstock shaft sizes

Shaft D due to Roller Bearing	Shaft D due to Retaining Ring	Shaft D due to Ball Bearing
.9843 [inches]	.44 [inches]	.472 [inches]

Tailstock

Table 11: Tailstock shaft sizes

Shaft D due to Roller Bearing	Shaft D due to Ball Bearing
.9843 [inches]	.472 [inches]

Bed Frame Calculations

For this section, calculations were made in order to determine the size of the bed plate which the lathe assembly will sit on to of. A number of forces were considered including the vertical bearing reaction forces, the weights of the head and tail stocks, the weight of the motor and workpiece, and finally, the weight of the bed itself. Assuming a cast iron material, a deflection calculation was used in order to see if there was a concerning amount of deflection caused by all of the weights on the bed plate. With the small amount of deflection found, we concluded that our bedplate design was was appropriate.

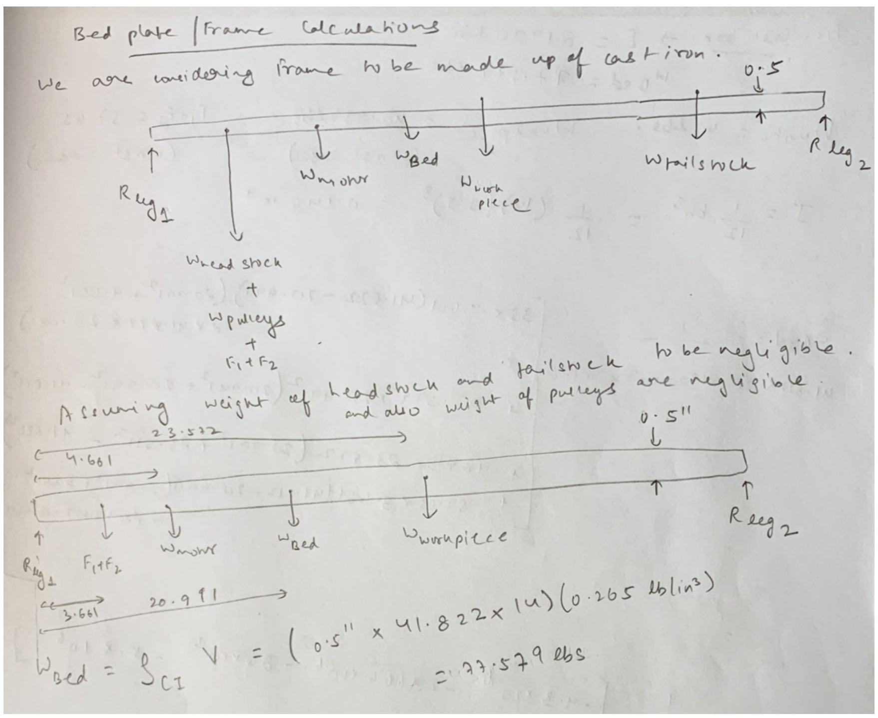


Figure 13: Free Body Diagram of the Bed Plate w/ weights

W_bed = $\varrho * V = (0.265\ lb/in^3)(0.5 * 41.822 * 14) = 77.579\ lbs$

$$\delta_{Bed} = \frac{1}{6EIL}[W_{motor}\ a_1(L-x)(x^2 + {a_1}^2 - 2Lx) + W_{bed} b_2 x(x^2 + {b_2}^2 - L^2) + W_{piece} b_3 x(x^2 + {b_3}^2 - L^2) +$$

$$(F_1 + F_2)a_4(L - x)(x^2 + a_4{}^2 - 2Lx)]$$

Here ,

$a_1 = 4.661\ in,\ b_2 = 20.991\ in,\ b_3 = 23.572\ in,\ a_4 = 3.661 in,\ L = 41.822\ in,\ x = 20.991\ in$

I = $(1/12)(bh^3)\ =\ (1/12)(14 * 0.5^3)\ = 0.146\ in^4$

For Cast iron

E = 21900 ksi

From previous calculations we know,

$W_{motor} = 40 lbs,\ W_{workpiece} = 48.873\ lbs,\ F_1 + F_2 = 57.65 lbs$

Plugging all these numbers in the equation for deflection we get :

δ_{Bed} = -0.0478 in

Deflection in legs

Similarly for the bed frame deflection calculations, deflection in the bed frame leg's should be considered as well. Again assuming a cast iron material, and a fixed pinned connection, we can conclude that the force needed to buckle one leg far exceeds any expected applied load.

About x-axis

k = $\sqrt{\frac{I}{A}} = \sqrt{\frac{(1/12)*2*1.5^3}{1.5*2}}$ = 0.433

$\frac{L}{K} = \frac{12.49}{0.433} = 28.84$

About y-axis

k = $\sqrt{\frac{I}{A}} = \sqrt{\frac{(1/12)*2^3*1.5}{1.5*2}}$ = 0.577

$\frac{L}{K} = \frac{12.49}{0.577} = 21.64$

We know

C = 2 (fixed-pinned condition)

E = 21900 ksi (Material properties for cast iron)

S_{YT} = 15250 psi (Material properties for cast iron)

$(\frac{L}{k})_1 = (\frac{2C\pi^2 E}{S_{YT}})^{1/2} = (\frac{2*2*\pi^2*21900*10^3}{15250})^{1/2} = 238.1$

So for both the cases $(\frac{L}{k})_1 > \frac{L}{K} \Rightarrow Hence\ we\ will\ use\ Johnson's\ Parabola\ method.$
The structure will fail first for lower radius of gyration i.e k value, so the team decided to use the lower k value(k=0.433) to calculate the critical load that the legs can withstand without failing.

$$P_{Cr} = (S_{YT} - (\frac{S_{YT}*L}{2\pi*k})^2 * \frac{1}{CE})A = (15250 - (\frac{15250*28.84}{2\pi})^2 * \frac{1}{2*21900*10^3}) * 2 * 1.5) = 45.41\ kips$$

Design of Housings

The pulley housing was designed in such a way that the user could easily change the belts in order to change the speed of the lathe. The pulley housing is closed on the sides, back and top. This is done to accommodate for the user safety. The front side (the side that the user is working on) is open to allow for user access. The bottom of the belt, attached to the driver pulley and motor, is exposed without any housing. The reason for this is because it is out of the way of the user and therefore does not need to be covered. The housing has three small extrusions on the left, right, and top sides as well. Here, bolts would be inserted into the housing and table so that the housing stays in place.

Conclusion

The initial design of this wood-working lathe changed after calculating for appropriate sizes of the components which was expected. Each component of the lathe must operate at the required specifications which required several calculations to be verified. The belt-driven design was chosen for its ease of use and simpler design. Initial calculations involved finding the required power of 0.5 HP for the motor that will be used for the lathe. The thickness and diameters of the driver and driven pulley for the different speeds the lathe will be running helped in finding the belt for the design. The belt needed to fit on all the different steps of the pulleys which the user will be shifting manually through the cut of the pulley housing. Finding the bearing sizes required calculations to find the reaction forces which resulted in the lowest load rating allowing for us to use a bigger bore bearing if needed. We found that since the shaft size at the ball bearing B and C of the design was bigger than the initial bearing bore size of 10mm, we had to change the bearing bore size to 12mm. Calculations were also made for the bed frame and legs, mostly to ensure it will be safe during operation and under expected loading. We simplified our design by having the tailstock fastened instead of incorporating a sliding mechanism which would allow for a wider range of woodpiece length. Having an initial thickness of 2.8" for the table required us to create a "motor extrusion" in order to mount the motor to the table for the final design.

There are some parts of the design process that we believe we could improve onto create a better design. This involves doing proper keyway calculations, considering the cut in the bed plate for deformation calculations, and adding angular deflection and fastener calculations to help with the design. The retaining ring also seemed inaccurate at first due to a very small diameter of .44 inches so further calculations may improve the result. Going back to the CAD models and improving some features will also improve its aesthetics.

Overall, the project was a success. The initial design meets the specified requirements and can be easily improved. By doing this project, we learned the design process for a working mechanical system that will be useful in the future.

References and Standards Used

[1] Thomas, et al. “Identify Wood Lathe Parts Illustrated.” *Turn A Wood Bowl*, 6 Feb. 2019, turnawoodbowl.com/identify-and-understand-parts-of-a-wood-lathe-and-accessories/.

[2] “Choosing the Best Woodworking Lathe for Your Shop.” *Woodworking | Blog | Videos | Plans | How To*, 7 Jan. 2019, www.woodworkersjournal.com/choosing-the-best-woodworking-lathe-for-your-shop/.

[3] Baylor, Chris. “The Wood Lathe Is the One Machine Every Woodturner Needs.” *The Spruce Crafts*, TheSpruceCrafts, 30 Nov. 2017, www.thesprucecrafts.com/buy-and-use-a-wood-lathe-3536927.

[4] “Grizzly ½ HP Electric Motor Manual”, from http://cdn0.grizzly.com/manuals/g2528_m.pdf

Standards Used

- AISI shaft material
- ABMA Single-Row 02-Series Deep-Groove Ball Bearings
- ABMA 02-Series Cylindrical Roller Bearings
- English Units (except for bearing selection)
- Goodman’s Criterion
- Von Mises

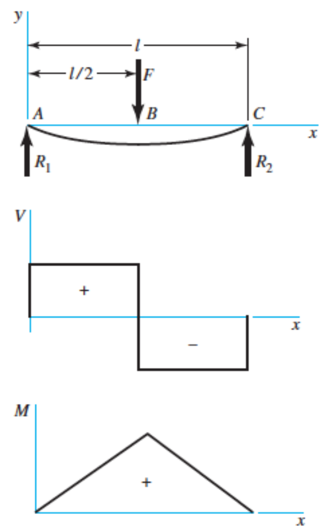
Appendices

Appendix A: References

Table A–9

Shear, Moment, and Deflection of Beams (*Note:* Force and moment reactions are positive in the directions shown; equations for shear force V and bending moment M follow the sign conventions given in Sec. 3–2.) (*Continued*)

5 Simple supports—center load



$$R_1 = R_2 = \frac{F}{2}$$

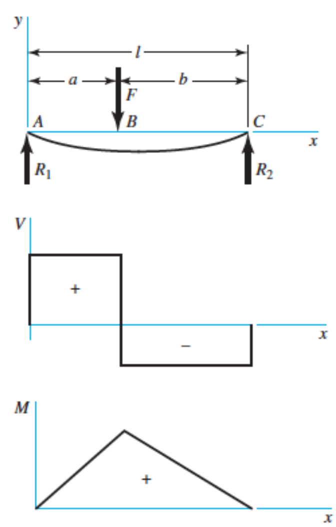
$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{Fx}{2} \qquad M_{BC} = \frac{F}{2}(l - x)$$

$$y_{AB} = \frac{Fx}{48EI}(4x^2 - 3l^2)$$

$$y_{max} = -\frac{Fl^3}{48EI}$$

6 Simple supports—intermediate load



$$R_1 = \frac{Fb}{l} \qquad R_2 = \frac{Fa}{l}$$

$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{Fbx}{l} \qquad M_{BC} = \frac{Fa}{l}(l - x)$$

$$y_{AB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2)$$

$$y_{BC} = \frac{Fa(l - x)}{6EIl}(x^2 + a^2 - 2lx)$$

(Continued)

Table 11–1

Equivalent Radial Load Factors for Ball Bearings

F_a/C_0	e	$F_a/(VF_r) \le e$ X_1	Y_1	$F_a/(VF_r) > e$ X_2	Y_2
0.014*	0.19	1.00	0	0.56	2.30
0.021	0.21	1.00	0	0.56	2.15
0.028	0.22	1.00	0	0.56	1.99
0.042	0.24	1.00	0	0.56	1.85
0.056	0.26	1.00	0	0.56	1.71
0.070	0.27	1.00	0	0.56	1.63
0.084	0.28	1.00	0	0.56	1.55
0.110	0.30	1.00	0	0.56	1.45
0.17	0.34	1.00	0	0.56	1.31
0.28	0.38	1.00	0	0.56	1.15
0.42	0.42	1.00	0	0.56	1.04
0.56	0.44	1.00	0	0.56	1.00

*Use 0.014 if $F_a/C_0 < 0.014$.

Table 11–2

Dimensions and Load Ratings for Single-Row 02-Series Deep-Groove and Angular-Contact Ball Bearings

Bore, mm	OD, mm	Width, mm	Fillet Radius, mm	Shoulder Diameter, mm d_S	d_H	Load Ratings, kN Deep Groove C_{10}	C_0	Angular Contact C_{10}	C_0
10	30	9	0.6	12.5	27	5.07	2.24	4.94	2.12
12	32	10	0.6	14.5	28	6.89	3.10	7.02	3.05
15	35	11	0.6	17.5	31	7.80	3.55	8.06	3.65
17	40	12	0.6	19.5	34	9.56	4.50	9.95	4.75
20	47	14	1.0	25	41	12.7	6.20	13.3	6.55
25	52	15	1.0	30	47	14.0	6.95	14.8	7.65
30	62	16	1.0	35	55	19.5	10.0	20.3	11.0
35	72	17	1.0	41	65	25.5	13.7	27.0	15.0
40	80	18	1.0	46	72	30.7	16.6	31.9	18.6
45	85	19	1.0	52	77	33.2	18.6	35.8	21.2
50	90	20	1.0	56	82	35.1	19.6	37.7	22.8
55	100	21	1.5	63	90	43.6	25.0	46.2	28.5
60	110	22	1.5	70	99	47.5	28.0	55.9	35.5
65	120	23	1.5	74	109	55.9	34.0	63.7	41.5
70	125	24	1.5	79	114	61.8	37.5	68.9	45.5
75	130	25	1.5	86	119	66.3	40.5	71.5	49.0
80	140	26	2.0	93	127	70.2	45.0	80.6	55.0
85	150	28	2.0	99	136	83.2	53.0	90.4	63.0
90	160	30	2.0	104	146	95.6	62.0	106	73.5
95	170	32	2.0	110	156	108	69.5	121	85.0

Table 11–3

Dimensions and Basic Load Ratings for Cylindrical Roller Bearings

	02-Series				03-Series			
Bore, mm	OD, mm	Width, mm	Load Rating, kN C_{10}	C_0	OD, mm	Width, mm	Load Rating, kN C_{10}	C_0
25	52	15	16.8	8.8	62	17	28.6	15.0
30	62	16	22.4	12.0	72	19	36.9	20.0
35	72	17	31.9	17.6	80	21	44.6	27.1
40	80	18	41.8	24.0	90	23	56.1	32.5
45	85	19	44.0	25.5	100	25	72.1	45.4
50	90	20	45.7	27.5	110	27	88.0	52.0
55	100	21	56.1	34.0	120	29	102	67.2
60	110	22	64.4	43.1	130	31	123	76.5
65	120	23	76.5	51.2	140	33	138	85.0
70	125	24	79.2	51.2	150	35	151	102
75	130	25	93.1	63.2	160	37	183	125
80	140	26	106	69.4	170	39	190	125
85	150	28	119	78.3	180	41	212	149
90	160	30	142	100	190	43	242	160
95	170	32	165	112	200	45	264	189
100	180	34	183	125	215	47	303	220
110	200	38	229	167	240	50	391	304
120	215	40	260	183	260	55	457	340
130	230	40	270	193	280	58	539	408
140	250	42	319	240	300	62	682	454
150	270	45	446	260	320	65	781	502

Fatigue Loading Formulae

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \qquad (6\text{–}8)$$

$$k_a = aS_{ut}^b \qquad (6\text{–}19)$$

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

From C.J. Noll and C. Lipson, "Allowable Working Stresses," *Society for Experimental Stress Analysis,* vol. 3, no. 2, 1946 p. 29. Reproduced by O.J. Horger (ed.) *Metals Engineering Design ASME Handbook,* McGraw-Hill, New York. Copyright © 1953 by The McGraw-Hill Companies, Inc. Reprinted by permission.

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \qquad (6\text{–}20)$$

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases} \qquad (6\text{–}26)$$

$K_f = 1 + q(K_t - 1)$

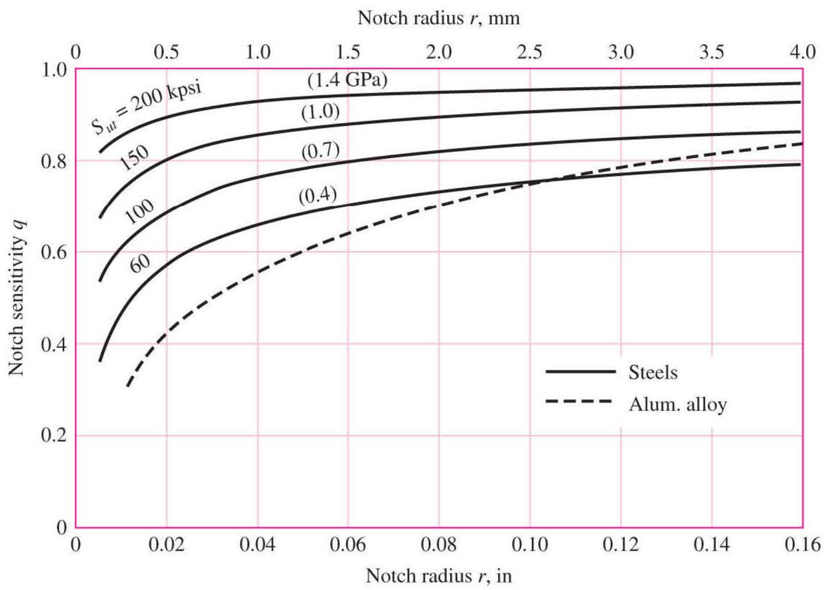
$K_{fs} = 1 + q_s(K_{ts} - 1)$

Soderberg $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n} \qquad (6\text{–}45)$$

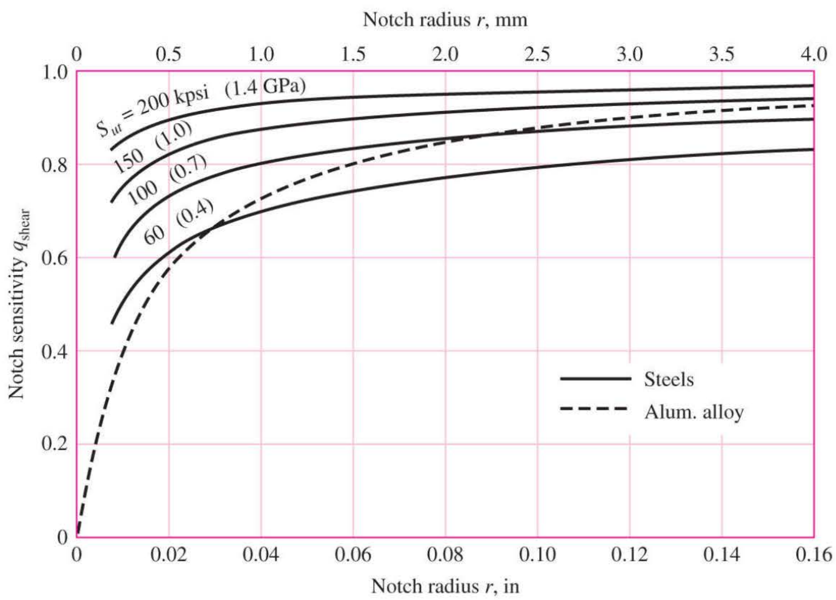
mod-Goodman $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \qquad (6\text{–}46)$$

Gerber $$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1 \qquad (6\text{–}47)$$

ASME-elliptic $$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1 \qquad (6\text{–}48)$$



Notch radius r, mm
0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
Notch sensitivity q
1.0 0.8 0.6 0.4 0.2 0
S_ut = 200 kpsi
(1.4 GPa)
150
(1.0)
100
(0.7)
60
(0.4)
Steels
Alum. alloy
0 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16
Notch radius r, in



Notch radius r, mm
0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
Notch sensitivity q_shear
1.0 0.8 0.6 0.4 0.2 0
S_ut = 200 kpsi (1.4 GPa)
150 (1.0)
100 (0.7)
60 (0.4)
Steels
Alum. alloy
0 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16
Notch radius r, in

Appendix B: Belt Calculations

Appendix B.1: 800 RPM Belt Calculations

The following pages show the procedure for calculating the belt length necessary for a 800 RPM (8” diameter workpiece) scenario.

Belt Design for 800 rpm (8" work piece) Take II

Specs

Motor:

$N_1 = 1725$ rpm

$P_{motor} = .5$ HP

Belt:

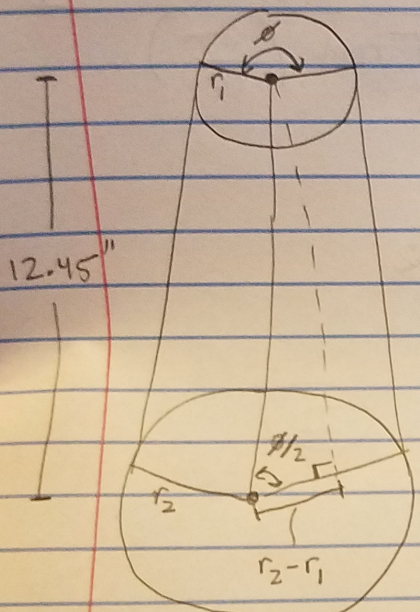
$n_d = 1.1$

$K_s = 1.1$

A-2

Pulley:

$C = 12.45$



From Raghav's calculations:

$L_1 = 5.4420"$

$L_2 = 8.1365"$

$L_{assembled} = 38.4388"$

$r_1\omega_1 = r_2\omega_2$

$r_1(1725) = r_2(800)$

$\frac{r_1}{r_2} = .464$

$r_1 = .464 r_2 \Rightarrow r_2 = 2.156 r_1$

$\cos(\phi/2) = \frac{r_2 - r_1}{12.45} \Rightarrow \phi = 2\cos^{-1}\left(\frac{r_2 - r_1}{12.45}\right)$

$= 2\cos^{-1}\left(\frac{r_2 - .464 r_2}{12.45}\right)$

$= 2\cos^{-1}(.0431 r_2)$

$L_1 = r_1 \phi$

$5.4420 = .464 r_2 (2\cos^{-1}(.0431 r_2))$

Wolfram $\Rightarrow$ $r_2 = 4.22607"$

$r_1 = 1.9609"$

$\phi = 2.775$ rad

$d_2 = 8.452"$

$d_1 = 3.922"$

$d_1 = 3.922''$
$d_2 = 8.452''$

$$P_{Belt} = P_{motor} \eta_d K_s$$
$$= (.5\ HP)(1.1)(1.1)\left(\frac{550\ ft\cdot lb/s}{1\ HP}\right)$$
$$= 332.75\ ft-lb$$

$$P = T_1 \omega_1 \Rightarrow T_1 = \frac{P}{\omega_1} = \frac{(332.75\ ft-lb)(12''/1\ ft)}{1725\ rpm\left(\frac{2\pi}{60}\right)}$$

$$T_1 = 22.105\ in-lb$$
$$T_1 = (F_1 - F_2) r_1$$
$$F_1 - F_2 = \frac{T_1}{r_1} = 11.272$$

$$F_1 - F_2 = 11.272 \qquad \text{I}$$

A2 Belt
$\mu = .8$
$C_p = .73$
$\gamma = .037\ lbf/in^3$
$C_v = 1$
$t = .11''$
$F' = 60$

$$F_1 = C_p C_v F' b$$
$$= (.73)(1)(60) b$$
$$F_1 = 43.8 b \qquad \text{II}$$

$$F_2 = 43.8b - 11.272$$

1725 rpm in rad

$$F_c = \frac{\gamma}{g} b t r_1^2 \omega_1^2 = \frac{.037}{(32.2)(12)} (b)(.11)(1.9609^2)(180.64^2)$$

$$F_c = 1.322 b$$

1.665

$$\frac{F_1 - F_c}{F_2 - F_c} = \exp\left(\cancel{.75\, \mu_s \phi}\right)$$

$$\frac{43.8b - 1.322b}{(43.8b - 11.272) - 1.322b} = 1.665$$

$$\frac{42.478\,b}{42.478b - 11.272} = 1.665$$

$$42.478b = 1.665\,(42.478b - 11.272)$$
$$42.478b = 70.726b - 18.768$$

$$\boxed{b = .6644''}$$

If we want a pulley width of 1.25 b:

$$\boxed{\text{Pulley width} = .8305''}$$

$$F_1 = 29.1007 \text{ lb}$$
$$F_2 = 17.8287 \text{ lb}$$
$$F_1 = \frac{F_1 + F_2}{2} = 23.464 \text{ lb}$$

Appendix B.2: 1600 RPM Belt Calculations

The following pages show the procedure for calculating the belt length necessary for a 1600 RPM (6” diameter workpiece) scenario.

Studio #8

$k_s = 1.1$, $n_d = 1.1$, $P_{motor} = 0.5HP$, $N_{motor} = 1725RPM$

$P_{belt} = n_d k_s P_{motor}$

$= (1.1)(1.1)(0.5HP) = 0.605HP$

Selected Belt Material: Polyamide A-2

$t = 0.11in$, $F' = 60lbf/in$, $\gamma = 0.037lbf/in$, $\mu_s = 0.8$, $C_p = 0.73$, $C_v = 1$

$\omega_1 r_1 = \omega_2 r_2 \rightarrow r_1 = \frac{\omega_2 \cdot r_2}{\omega_1} = 0.928 r_2$

$\phi = 2\cos^{-1}\left(\frac{r_2 - r_1}{12.45}\right) = 2\cos^{-1}(0.00578 r_2)$

$l_{assembled} = 38.44'' = r_1\phi + r_2(2\pi - \phi) + 2\sqrt{C^2 - (r_2 - r_1)^2}$

$r_2 = 2.24''$

$r_1 = 2.08'' \rightarrow d_1 = 4.16''$

$\phi = 3.12$ rads

$T = \frac{0.605\cancel{HP}\left(\frac{550\ ft \cdot lb}{\cancel{HP}}\right)}{\frac{2\pi(1725)}{60}} = 1.842 ft \cdot lb$

$F_1 - F_2 = \frac{1.842\ \cancel{ft} \cdot lb\left(\frac{12\ in}{\cancel{ft}}\right)}{2.08\ \cancel{in}} = 10.63 lb$

$F_1 = C_p C_v F' b = (0.73)(1)(60)b = 43.8b$

$43.8b - F_2 = 10.63 \longrightarrow F_2 = 43.8b - 10.63$

$F_C = \frac{\gamma}{g} t r^2 \omega^2 b = \frac{0.037}{32.2(12)}(0.11)(2.24)^2(180.64)^2 b = 1.725b$

$\frac{F_1 - F_C}{F_2 - F_C} = e^{(0.75(0.8)(3.12))} = \frac{43.8b - 1.725b}{43.8b - 10.63 - 1.725b} = e^{(1.872)}$

$\frac{42.075b}{42.075b - 10.63} = 6.501 \rightarrow b = 0.299''$

Pulley width $= 1.25(0.299) = 0.373''$

$F_i = \frac{F_1 + F_2}{2} = \frac{43.8(0.299) + 43.8(0.299) - 10.63}{2} = 7.78 lb$

Belt Material: Polyamide

Belt Grade: A-2

Belt Thickness: 0.11 in

Belt Width: 0.299 in

Pulley width: 0.373 in

Initial tension: 7.78 lb

Appendix B.3: 2400 RPM Belt Calculations

The following pages show the procedure for calculating the belt length necessary for a 2400 RPM (4” diameter workpiece) scenario.

Contents

Specification

```
N_1 = 1725; %RPM
N_2 = 2400; %RPM
k_s = 1.1;
n_d =1.1;
Motor_power = 0.5; %HP
c = 12.45; %inches
d_1 = 5; %inches
```

Power of Belt

```
P = n_d * k_s * Motor_power * 550 %(lb*ft)/s

w_1 = (2*pi*N_1)/60

w_2 = (2*pi*N_2)/60

d_2 = (w_1*d_1)/w_2

T_1 = (P*12)/w_1
```

```
P =

  332.7500


w_1 =

  180.6416


w_2 =

  251.3274


d_2 =

    3.5938


T_1 =

   22.1045
```

Forces

```
syms b F_1 F_2
%F_1 - F_2 = 8.84

sp_wt = 0.037
t = 0.11

F_c1 = b* (sp_wt * t * ( 2 * w_1)^2)/(32.2*12)

F_c = vpa(F_c1)

F_1 = 0.73*60*b
```

```
sp_wt =

    0.0370


t =

    0.1100


F_c1 =

(2920516513490195*b)/2124256464863232


F_c =

1.3748417678362717652849093326377*b


F_1 =

(219*b)/5
```

Assembled length

```
x = (d_1-d_2)/2
theta = 2*acos(x/c)
L_1 = (d_2/2)  * theta
L_2 = (2.5)*(2*pi-theta)
L_assembled = L_1 + L_2 + 2*(sqrt(12.45^2 - x^2))

b = 0.25 %belt width calculating by hand from belting equation

Pulley_width = b * 1.25 %pulley width

F_i = 6.1875 %lbf
```

```
x =

    0.7031
```

```
theta =

    3.0286


L_1 =

    5.4420


L_2 =

    8.1365


L_assembled =

   38.4388


b =

    0.2500


Pulley_width =

    0.3125


F_i =

    6.1875
```

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Appendix B.4: 3200 RPM Belt Calculations

The following pages show the procedure for calculating the belt length necessary for a 3200 RPM (2” diameter workpiece) scenario.

Studio #8

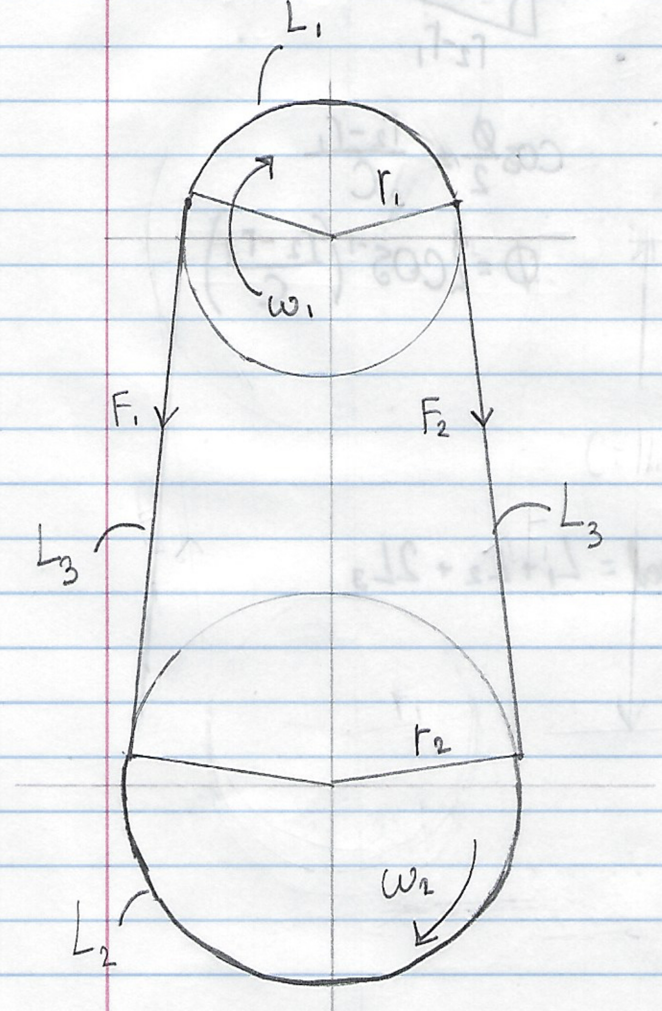
Bradley Berman
ENME400 - 0102

$K_s = 1.1$, $\eta_d = 1.1$

$P_{motor} = 0.5$ HP , $N_{motor} = N_2 = 1725$ RPM

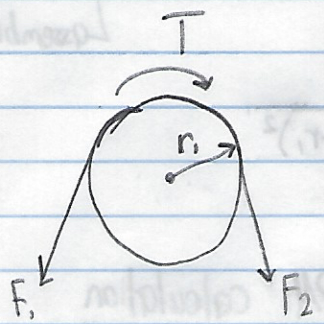
$P_{to\ be\ transmitted} = \eta_d K_s P_{motor}$

$P_{to\ be\ transmitted} = (1.1)(1.1)(0.5 \text{ HP}) = 0.605$ HP



Belt Material: Polyamide
Belt Grade: A-2

$t = 0.11$ in
$F' = 60$ lbf/in
$\gamma = 0.037$ lbf/in^3
$\mu_s = 0.8$



$\circlearrowleft + \sum M_r = 0$

$-T + F_1 r_1 - F_2 r_1 = 0$

$T = (F_1 - F_2) r_1$ — (1)

From kinematics : $v = d_1\omega_1 = d_2\omega_2$

Studio #8

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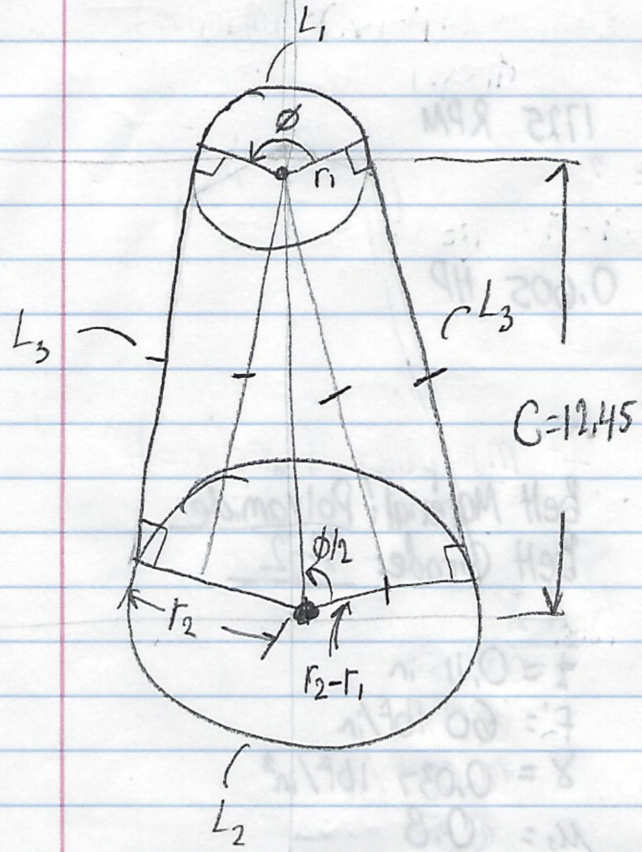
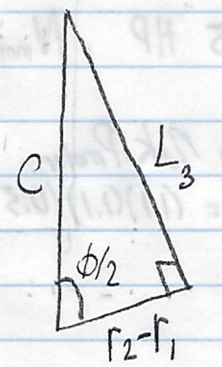
ENME400-0102





$$\cos\frac{\emptyset}{2} = \frac{r_2-r_1}{C}$$

$$\emptyset = 2\cos^{-1}\left(\frac{r_2-r_1}{C}\right)$$

$L_1 = r_1\emptyset$
$L_2 = r_2(2\pi-\emptyset)$
$L_3 = \sqrt{C^2-(r_2-r_1)^2}$

$L_{assembled} = L_1 + L_2 + 2L_3$

For 3200 RPM calculation
$\omega_1 = 3200$ RPM
$\omega_2 = 1725$ RPM

$$r_1\omega_1 = r_2\omega_2$$
$$r_1(3200) = r_2(1725)$$
$$\frac{r_1}{r_2} = 0.539$$
$$r_1 = 0.539r_2 \quad (2)$$

Studio #8

3/5

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$$L_{assembled} = r_1\phi + r_2(2\pi-\phi) + 2\sqrt{C^2-(r_2-r_1)^2}$$

$$L_{assembled} = (0.539r_2)\left(2\cos^{-1}\left(\frac{r_2-0.539r_2}{C}\right)\right) + r_2\left(2\pi - 2\cos^{-1}\left(\frac{r_2-0.539r_2}{C}\right)\right) + 2\sqrt{C^2-(r_2-0.539r_2)^2}$$

$$L_{assembled} = (1.078r_2)\cos^{-1}\left(\frac{0.461r_2}{12.45}\right) + r_2\left(2\pi-2\cos^{-1}\left(\frac{0.461r_2}{12.45}\right)\right) + 2\sqrt{(12.45)^2-(0.461r_2)^2} = 38.44 \text{ in}$$

from 2400 RPM calculation

From equation solver $\longrightarrow$ $r_2 = 2.77$ in

$d_2 = 2r_2 = 2(2.77 \text{ in}) = 5.54$ in

$\phi = 2.937$ rad

$r_1 = 0.539r_2$

$r_1 = 0.539(2.77 \text{ in}) = 1.5$ in

$d_1 = 2r_1 = 2(1.5 \text{ in}) = 3$ in

Studio #8

4/5

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A-2

$F_1 = C_P C_V F' b$

$F_1 = (0.73)(1)(60) b$

(3) — $F_1 = 43.8 b$

$\omega_2 = 2\pi \frac{N_2}{60} = 2\pi \frac{1725}{60}$

$\omega_2 = 57.5\pi \text{ rad/s}$

$P = T\omega_{motor}$ $\quad P = K_s n_d P_{motor}$

$$T = \frac{K_s n_d P_{motor}}{\omega_{motor}} = \frac{(1.1)(1.1)(0.5 \text{ HP})(550 \frac{ft \cdot lb}{s})/HP}{(57.5\pi \text{ rad/s})}$$

$T = 1.842 \text{ ft-lb}$

Combining $T = 1.842$ with equations (1) and (3):

$1.842 = (F_1 - F_2) r_1$

$1.842 = (43.8b - F_2)(1.5 \text{ in})(\frac{1 \text{ ft}}{12 \text{ in}})$

$14.736 = 43.8b - F_2$

(4) — $F_2 = 43.8b - 14.736$

Belting Equation

$F_c = m' r^2 \omega^2$

$F_c = \frac{\gamma}{g} bt (r_2 \omega_2)^2$

$$F_c = \frac{(0.037 \frac{lbf}{in})}{(32.2 \text{ ft/s}^2)(12 \text{ in/ft})} b (0.11 \text{ in}) \left(2.77 \text{ in} \cdot 57.5\pi \frac{rad}{s}\right)^2$$

$F_c = 2.637b$

$$\frac{F_1 - F_c}{F_2 - F_c} = \exp(0.75 \mu_s \phi)$$

$$\frac{43.8b - 2.637b}{(43.8b - 14.736) - 2.637b} = \exp(0.75(0.8)(2.937))$$

$$\boxed{b = 0.432 \text{ in}}$$

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5/5

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$$\text{Pulley width} = 1.25 \cdot \text{belt width}$$
$$= 1.25 \cdot 0.432 \text{ in}$$

$$\boxed{\text{Pulley width} = 0.54 \text{ in}}$$

$$F_1 = 43.8(0.432) =$$
$$F_1 = 18.94 \text{ lb}$$

$$F_2 = 43.8b - 14.736$$
$$F_2 = 43.8(0.432) - 14.736$$
$$F_2 = 4.2 \text{ lb}$$

$$F_i = \frac{F_1 + F_2}{2} = \frac{18.94 + 4.2}{2}$$

$$\boxed{F_i = 11.57 \text{ lb}}$$

Summary

Belt material = Polyamide
Belt grade = A-2
Belt thickness = 0.11 in
Belt width = 0.432 in
Pulley width = 0.54 in
Initial Tension = 11.57 lb

Appendix C: Bearing Reaction Calculations

Appendix C.1: 800 RPM Bearing Reaction Calculations

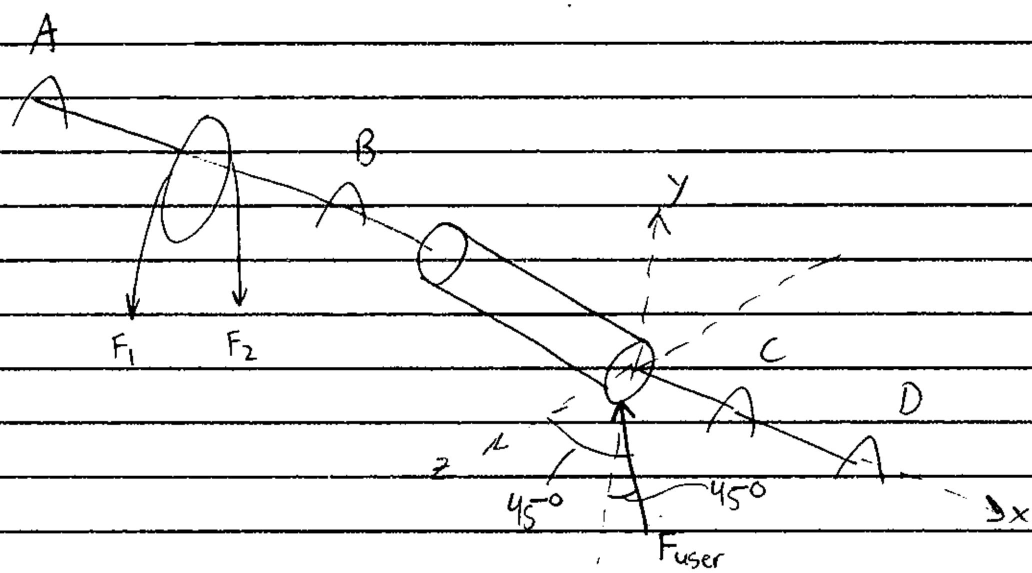
The following pages show the procedure for calculating the necessary bearing reactions for a 800 RPM (8” workpiece diameter) scenario.

Ed Stillwell ENME 400 : Spring 2019

Studio 9

ⓐ 800 RPM → Accompanying workpiece $\varnothing = 8''$

ⓑ/ⓒ



$$F_{user}: \quad F_x = F_{user} \sin 45^\circ \cos 45^\circ$$
$$F_y = F_{user} \cos 45^\circ$$
$$F_z = F_{user} \sin 45^\circ \sin 45^\circ$$

$$P = T\omega$$

$$(.5\ \text{HP})\left(\frac{550\ \frac{ft \cdot lb}{s}}{1\ \text{Hp}}\right)\left(\frac{12\ \text{in}}{1\ \text{ft}}\right) = T\left(\frac{2\pi(800)}{60}\right)$$

$$T = 39.391\ \text{in-lbs}$$

$$F_y\, r_{workpiece} = T$$
$$F_{user} \cos 45^\circ (4'') = 39.391$$

$$\boxed{F_{user} = 13.927\ \text{lbs}}$$

$F_x = 6.964$ lbs
$F_y = 9.848$ lbs → Will come in handy soon.
$F_z = 6.964$ lbs

$$\delta = \frac{F_x L}{AE} \quad \text{where} \quad F_x = .5\,F_{user}$$

$F_x = 6.963$ lbs
$L = 30''$ (length of work piece)
$A = \pi(4^2) = 50.265\ in^2$
$E = 1.667 \times 10^6$ psi

$$\delta = \frac{(6.963)(30)}{(50.265)(1.667\times10^6)} = 2.493\times10^{-6} \text{ inches}$$
$$\approx 6.332\times10^{-5}\ mm$$
$$\ll 2\,mm \checkmark$$

$F_x = .5\,F_{user} = 6.963$ lbs
$A = \pi(4^2) = 50.265\ in^2$

$$\sigma = \frac{F_x}{A} = \frac{6.963}{50.265} = .1385 \text{ psi}$$

for wood

$$\sigma = E\epsilon$$
$$.1385 = (1.667\times10^6)\epsilon$$
$$\epsilon = 8.31\times10^{-8}$$

Holding Force $= 2\epsilon = 1.66\times10^{-7}$

$$F_{Hold} = 2F_x = \boxed{13.926 \text{ lbs} = F_{Hold}}$$

on both ends

(d)

weight density

$W_{piece} = mg = \rho V$

$\rho = .03241 \ lb/in^3$

$V = (\pi)(4^2)(30") = 1507.96 \ in^3$

$W_{piece} = (.03241)(1507.96")$

$g = 32.2 \frac{ft}{s^2} \cdot \frac{12"}{1 ft} = 386.4 \frac{in}{s^2}$

$W_{piece} = 48.873 \ lbs$

Shafts

$W_{shafts} = \rho_{shaft} V_{shaft}$

$\rho_{shaft} = .284 \ lb/in^3$

$W_{Headstock} = \rho_{shaft} V_{head\ stock}$

$V_{Headstock} = (\pi)(.5^2)(8.572") = 6.732 \ in^3$

$\boxed{W_{Headstock} = 1.912 \ lbs}$

$V_{Tailstock} = (\pi)(.5^2)(3.25") = 2.553 \ in^3$

$\boxed{W_{Tailstock} = .725 \ lbs}$

Pulleys

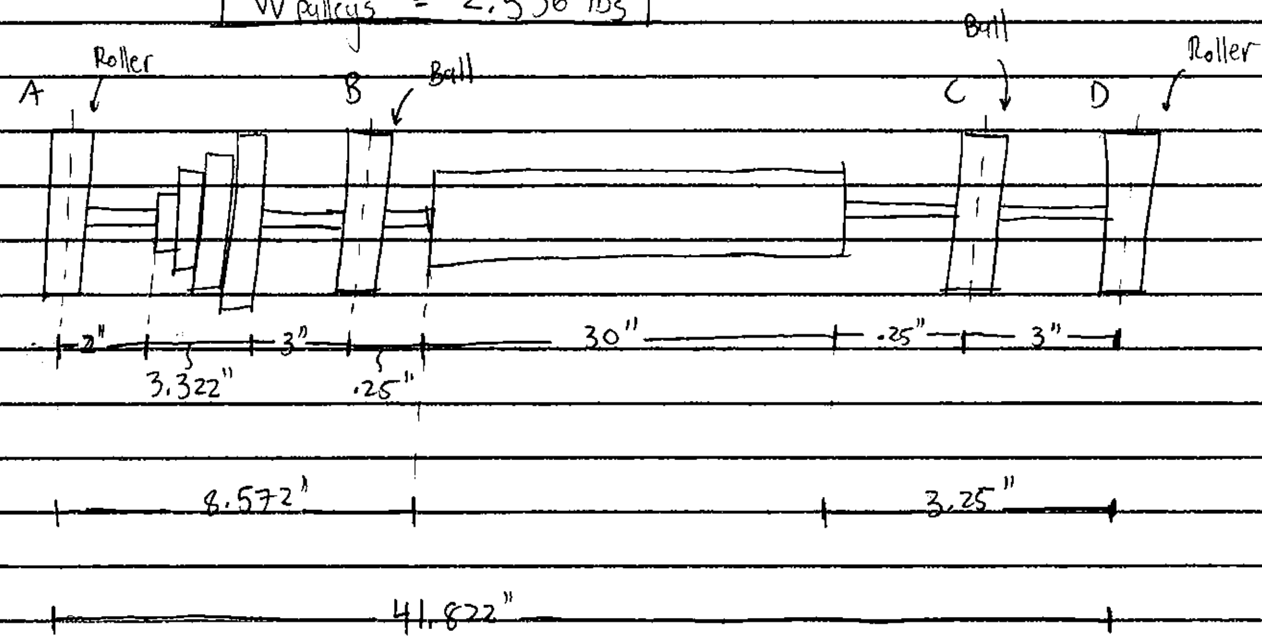
HPDE

$W_{pulleys} = \rho_{pulley} \, Area_{each\ pulley} \, t_{thickness}$

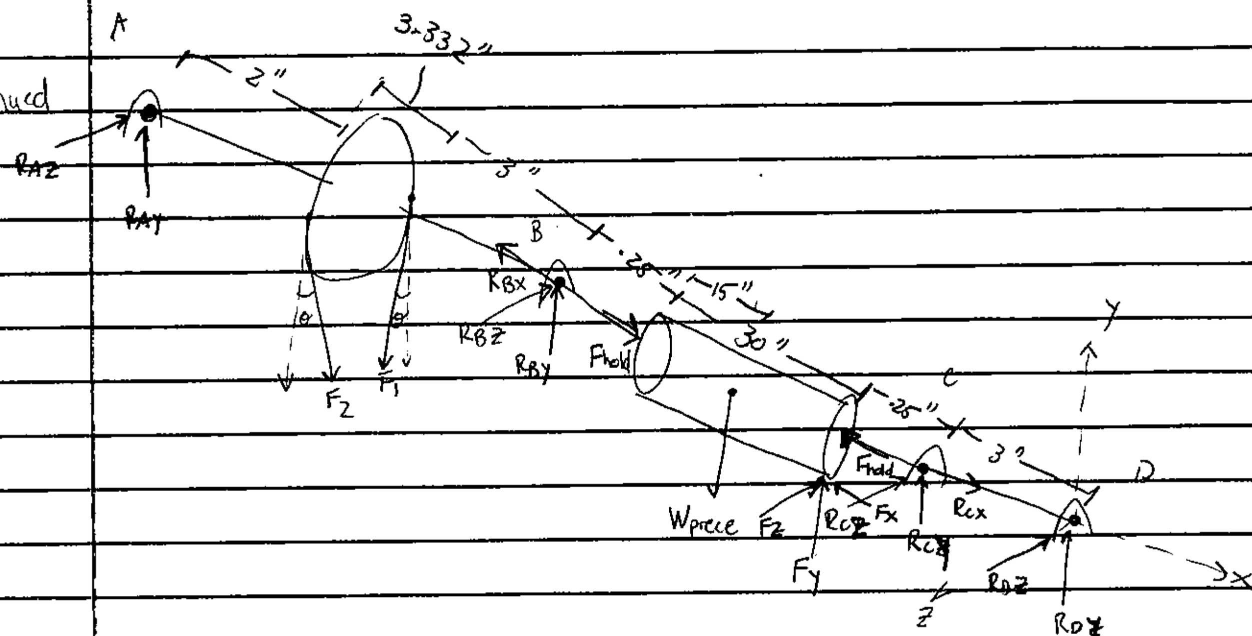
$W_{pulleys} = (.0343)\left(\frac{\pi}{4}\right)\left[8.452^2 + 4.472^2 + 3.5938^2 + 3^2\right](.8305)$

$\boxed{W_{pulleys} = 2.536 \ lbs}$

(e)



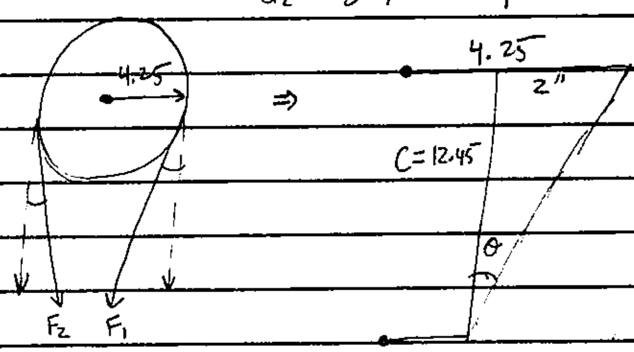
(e) continued



$F_1 = 43.8\,b$
$F_2 = 43.8\,b - 11.272$ } From pulley calc
$b = .6644''$
$F_1 = 29.10$ lbs
$F_2 = 17.83$ lbs

Pulley

$d_1 = 8.452 \approx 8.5$ $r_1 = 4.25''$
$d_2 = 3.922 \approx 4$ $r_2 = 2''$



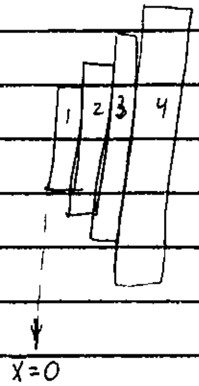
$\tan\theta = \dfrac{2}{12.45}$

$\theta = 9.126°$

$F_1 \cos\theta = 28.732$ lbs
$F_2 \cos\theta = 17.604$ lbs

$F_{Belt} = 46.336$ lbs

Pulley Weight Force Location:



each pulley width = .8305"

$\bar{x}_1 = .41525"$, $\bar{x}_2 = 1.246$, $\bar{x}_3 = 2.076"$, $\bar{x}_4 = 2.907"$

$W_1 = (.0343)(\pi/4)(3^2)(.8305) = .2014$

$W_2 = \text{''} \quad \text{''} (3.5936^2) \text{''} = .2889$

$W_3 = \text{''} \quad \text{''} (4.472^2) \text{''} = .4474$

$W_4 = \text{''} \quad \text{''} (8.452^2) \text{''} = 1.598$

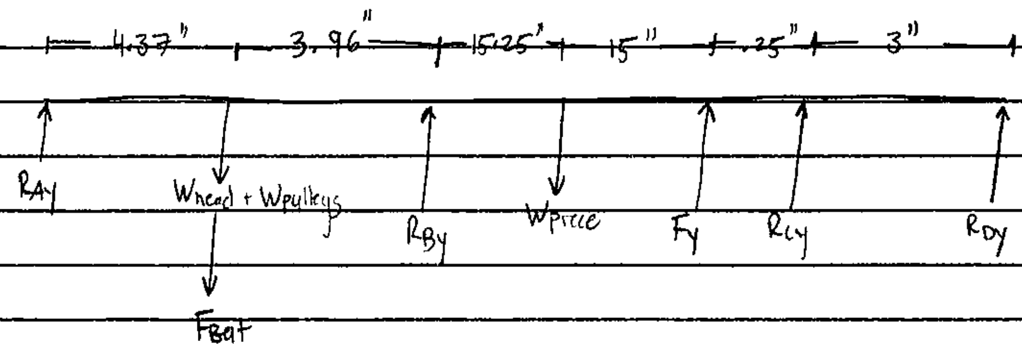
$$\bar{X} = \frac{(.2014)(.41525) + (.2889)(1.246) + (.4474)(2.076) + (2.907)(1.598)}{.2014 + .2889 + .4474 + 1.598}$$

$\bar{X} = 2.373"$

** Note → W_{pulley} force occurs 2.373" away from Left side of pulley.
W_{pulley} force distance away from A = 2 + 2.373 = 4.37" **

For calculations sake, I will assume that W_{pulley} and F_{Belt} act at the same location. In reality, this is not true.

X-Y Plane (Not drawn to scale)



$(W_{head} + W_{pulleys} + F_{Belt}) = 60.632$ lbs

$W_{piece} = 48.873$ lbs

ΣW_{piece}, pulleys, belts, F_{y}

$\Sigma F_y = 0$

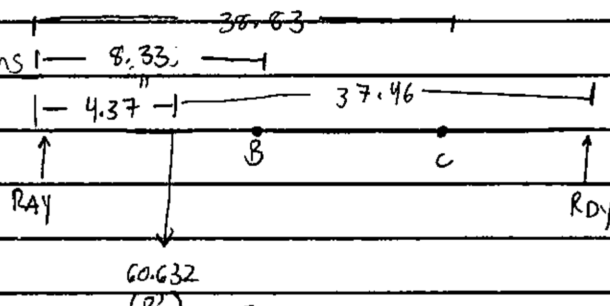
$$R_{Ay} + R_{By} + R_{cy} + R_{Dy} = 109.505 \qquad (1)$$

$\Sigma M_D = 0$

$$-(R_{Ay})(41.83) + (60.632)(37.46) - (R_{By})(33.5) + (48.87)(18.25) - (9.85)(3.25) - (R_{cy})(3) = 0$$

$$41.83 R_{Ay} + 33.5 R_{By} + 3 R_{cy} = 2306.38 \qquad (2)$$

Deflections



$l = 41.83$

$X =$ Varies

$a = 4.37$

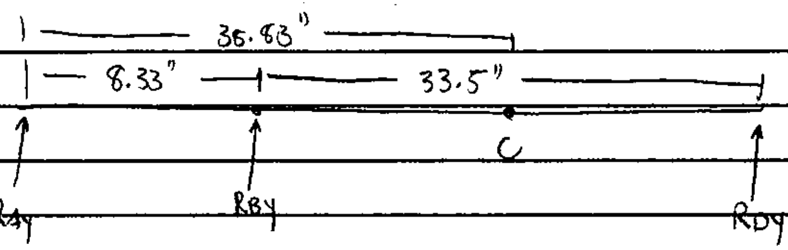
δ_{B_1} and δ_{C_1} in BC section

$$\delta_{B_1} = \frac{(60.632)(4.37)(41.83-8.33)}{6EI(41.83)}\left(8.33^2 + 4.37^2 - (2)(41.83)(8.33)\right)$$

$$\delta_{B_1} = \frac{-21516.898}{EI}$$

$$\delta_{C_1} = \frac{(60.632)(4.37)(41.83-38.83)}{6EI(41.83)}\left(38.83^2 + 4.37^2 - (2)(41.83)(38.83)\right)$$

$$\delta_{C_1} = \frac{-5452.69}{EI}$$



$l = 41.83$

$X =$ Varies

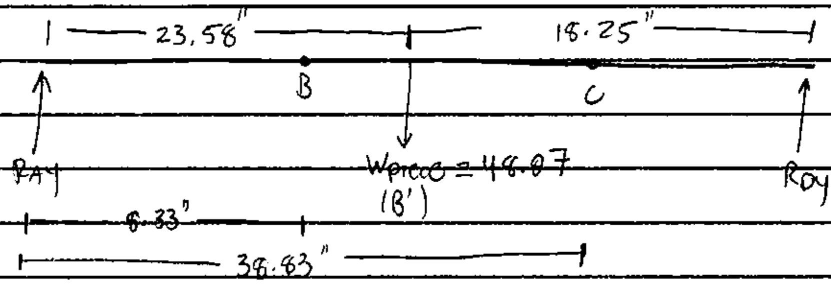
$a = 8.33$

$$\delta_{B2} = \frac{(-R_{By})(8.33)(41.83-8.33)}{6EI(41.83)} \cancel{\left(8.33^2 + 8.33^2 - 2(41.83)(8.33)\right)}$$

$$\delta_{B2} = \frac{+620.54 R_{By}}{EI}$$

$$\delta_{C2} = \frac{(-R_{By})(8.33)(41.83-38.83)}{6EI\ (41.83)} \left(38.83^2 + 8.33^2 - (2)(41.83)(38.83)\right)$$

$$\delta_{C2} = \frac{166.417 R_{By}}{EI}$$

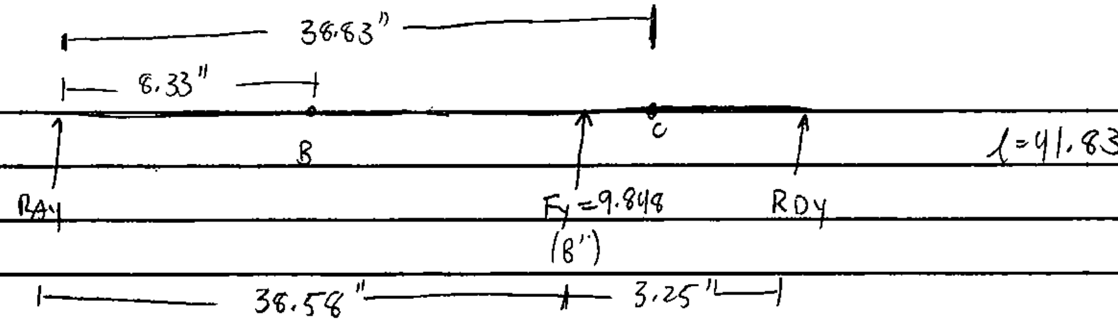


$$\delta_{B3} = \frac{(48.87)(18.25)(8.33)}{6EI\ (41.83)} \left(8.33^2 + 18.25^2 - 41.83^2\right)$$

$$\delta_{B3} = \frac{-39881.786}{EI}$$

$$\delta_{C3} = \frac{(48.87)(23.58)(41.83-38.83)}{6EI\ (41.83)} \left(38.83^2 + 23.58^2 - 2(41.83)(38.83)\right)$$

$$\delta_{C3} = \frac{-16318.81}{EI}$$

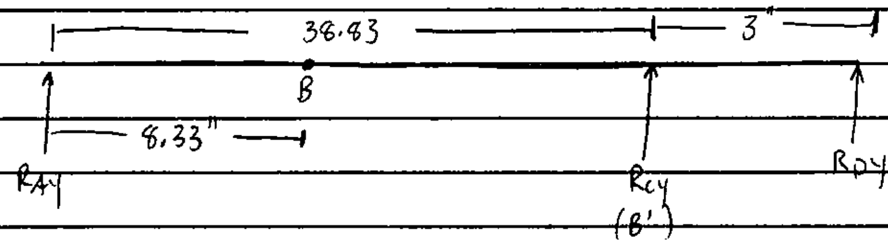


$$\delta_{By} = \frac{(-9.848)(3.25)(8.33)}{6EI(41.83)}(8.33^2 + 3.25^2 - 41.83^2)$$

$$\delta_{By} = \frac{1773.79}{EI}$$

$$\delta_{cy} = \frac{(-9.848)(38.58)(41.83 - 38.83)}{6EI(41.83)}(38.83^2 + 38.58^2 - 2(41.83)(38.83))$$

$$\delta_{cy} = \frac{1145.95}{EI}$$

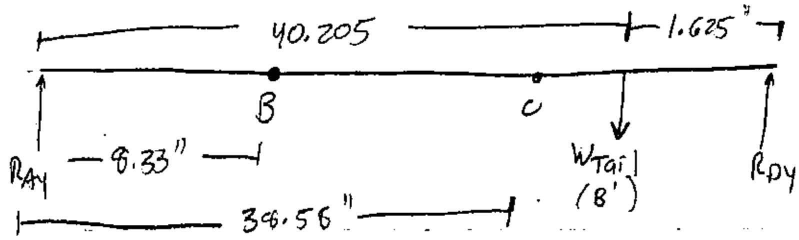


$$\delta_{B5} = \frac{(-R_{cy})(3)(8.33)}{6EI(41.83)}(8.33^2 + 3^2 - (2)(41.83)(8.33))$$

$$\delta_{B5} = \frac{+61.584\,R_{cy}}{EI}$$

$$\delta_{c5} = \frac{(-R_{cy})(3")(38.83)}{6EI(41.83)}(38.83^2 + 3^2 - (2)(41.83)(38.83))$$

$$\delta_{c5} = \frac{803.775\,R_{cy}}{EI}$$



$W_{Tail} = .725$

Both in AB.

$$\delta_{B6} = \frac{(.725)(1.625)(8.33)}{6EI\,(41.83)}\left(8.33^2 + 1.625^2 - 2(41.83)(8.33)\right)$$

$$\delta_{B6} = \frac{-24.43}{EI}$$

$$\delta_{C6} = \frac{(.725)(1.625)(38.58)}{6EI\,(41.83)}\left(38.58^2 + 1.625^2 - 2(41.83)(38.58)\right)$$

$$\delta_{C6} = \frac{-314.486}{EI}$$

$\Sigma\delta_B = 0$

(3) $$-\frac{21516.898}{\cancel{EI}} + \frac{620.54\,R_{By}}{\cancel{EI}} - \frac{39881.786}{\cancel{EI}} + \frac{1773.79}{\cancel{EI}} + \frac{61.584\,R_{cy}}{\cancel{EI}} - \frac{24.43}{\cancel{EI}} = 0$$

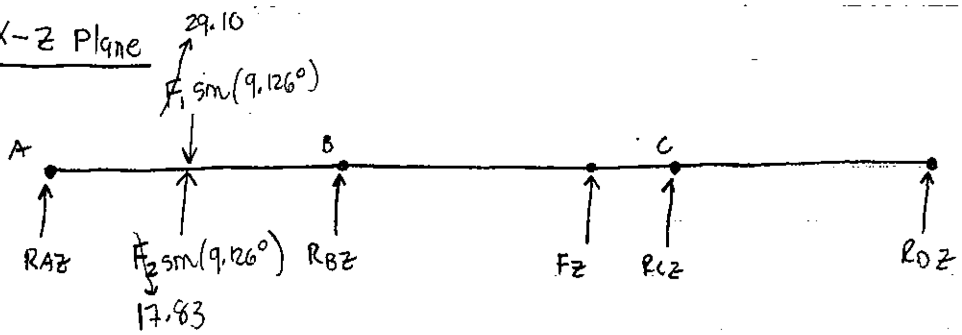
(4) $\Sigma\delta_c = 0$

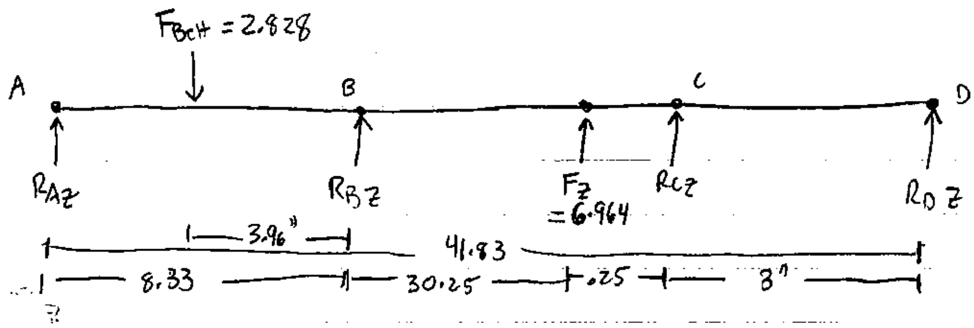
$$-\frac{5452.69}{EI} + \frac{166.417\,R_{By}}{EI} - \frac{16318.81}{EI} + \frac{1145.95}{EI} + \frac{803.775\,R_{cy}}{EI} - \frac{314.486}{EI} = 0$$

4 Eqns, 4 unknowns.

$$\boxed{\begin{aligned} R_{Ay} &= -21.765 \text{ lbs} \\ R_{By} &= 95.463 \text{ lbs} \\ R_{cy} &= 6.273 \text{ lbs} \\ R_{Dy} &= 29.534 \text{ lbs} \end{aligned}}$$

X–Z Plane





$\Sigma F_z = 0$

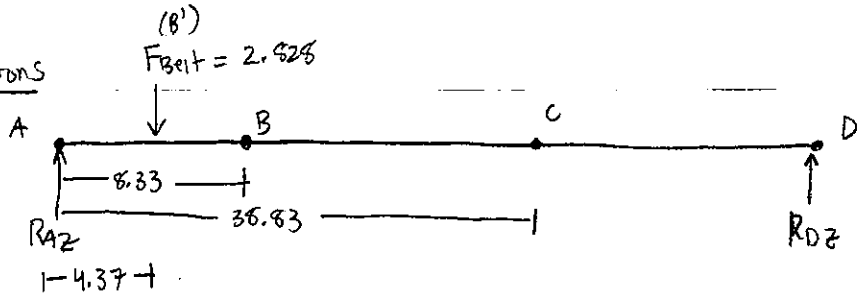
$$R_{Az} + R_{Bz} + R_{Cz} + R_{Dz} = -4.136 \qquad (1)$$

$\Sigma M_D = 0$

$$-(R_{Cz})(3) - (\overset{6.964}{F_z})(3.25) - (R_{Bz})(33.5) + (2.828)(37.46) - (R_{Az})(41.83) = 0$$

$$41.83\, R_{Az} + 33.5\, R_{Bz} + 3 R_{Cz} = 83.304 \qquad (2)$$

Deflections

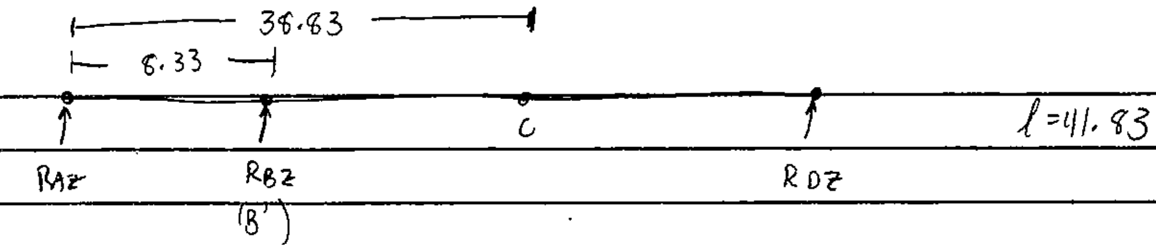


$$\delta_{B1} = \frac{(2.828)(4.37)(41.83-8.33)}{6EI(41.83)}\left(8.33^2 + 4.37^2 - 2(41.83)(8.33)\right)$$

$$\boxed{\delta_{B1} = \frac{-1003.59}{EI}}$$

$$\delta_{C1} = \frac{(2.828)(4.37)(41.83-38.83)}{6EI(41.83)}\left(38.83^2 + 4.37^2 - 2(41.83)(38.83)\right)$$

$$\boxed{\delta_{C1} = \frac{-254.32}{EI}}$$

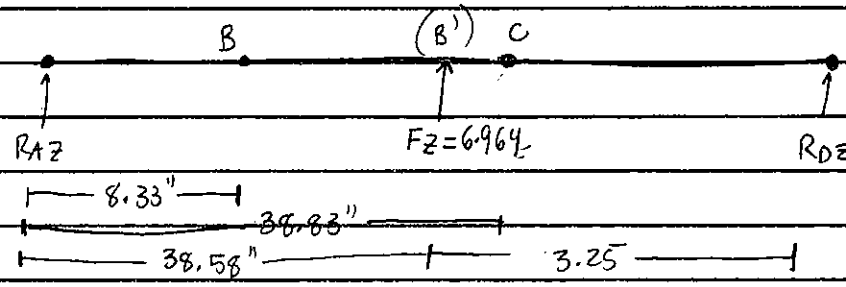


$$\delta_{Bz} = \frac{(R_{Bz})(8.33)(41.83-8.33)}{6EI}\left(8.33^2 + 8.33^2 - 2(41.83)(8.33)\right)$$

$$\delta_{Bz} = \frac{620.54\, R_{Bz}}{EI}$$

Similarly to δ_{cz} done on page 7

$$\delta_{cz} = \frac{166.417\, R_{Bz}}{EI}$$



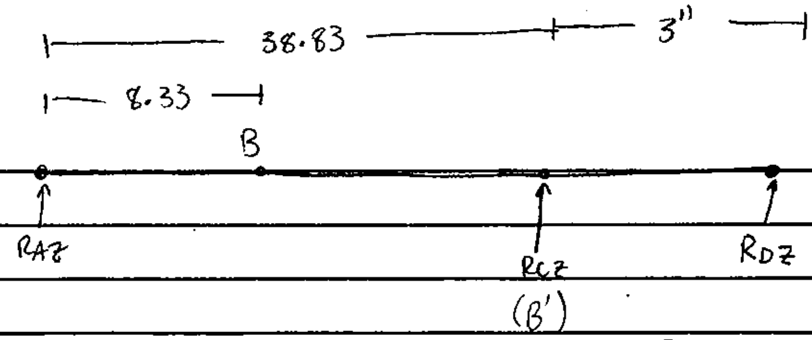
$$\delta_{B3} = \frac{(-6.964)(3.25)(8.33^2)}{6EI(41.83)}\left(8.33^2 + 3.25^2 - 41.83^2\right)$$

$$\delta_{B3} = \frac{10448.57}{EI}$$

$$\delta_{c3} = \frac{(-6.964)(38.58)(41.83-38.83)}{6EI(41.83)}\left(38.83^2 + 38.58^2 - 2(41.83)(38.83)\right)$$

$$\delta_{c3} = \frac{810.36}{EI}$$

12



Similar to δ_{B5} on page 8,

$$\boxed{\delta_{B4} = \frac{61.584\, R_{Cz}}{EI}}$$

Similar to δ_{C5} on 8,

$$\boxed{\delta_{C4} = \frac{803.775\, R_{Cz}}{EI}}$$

$\Sigma \delta_B = 0$

$$-\frac{1003.59}{\cancel{EI}} + \frac{620.54\, R_{Bz}}{\cancel{EI}} + \frac{10448.57}{\cancel{EI}} + \frac{61.584\, R_{Cz}}{\cancel{EI}} = 0 \quad (3)$$

$\Sigma \delta_C = 0$

$$-\frac{254.32}{EI} + \frac{166.417\, R_{Bz}}{EI} + \frac{810.36}{EI} + \frac{803.775\, R_{Cz}}{EI} = 0 \quad (4)$$

4 eqns, 4 unknowns

$\rightarrow$

R_{Az}	=	14.20 lbs
R_{Bz}	=	−15.47 lbs
R_{Cz}	=	2.51 lbs
R_{Dz}	=	−5.38 lbs

13

$$\boxed{R_{Bx} = F_{hold} = 13.926 \text{ lbs}}$$

$$R_{Cx} = F_{hold} + F_x$$

$$= 13.926 + 6.964$$

$$\boxed{R_{Cx} = 20.89 \text{ lbs}}$$

Appendix C.2: 1600 RPM Bearing Reaction Calculations

The following pages show the procedure for calculating the necessary bearing reactions for a 1600 RPM (6" workpiece diameter) scenario.

Studio 9

Thursday, April 25, 2019 2:13 PM

a) $d_{wp} = 6''$ (For 1600 RPM)

b) $P = T\omega = (0.5\,\cancel{HP})\left(\dfrac{550\,\frac{\cancel{ft}\cdot lb}{s}}{1\,\cancel{HP}}\right)\left(\dfrac{12\,in}{\cancel{ft}}\right) = T\left(\dfrac{2\pi(1600)}{60}\right)$

$T = 19.7$

$F_x = F_{user}\sin 45^\circ \cos 45^\circ$

$F_y = F_{user}\cos 45^\circ$

$F_z = F_{user}\sin 45^\circ \sin 45^\circ$

$F_y\, r_{workpiece} = T$

$F_{user}\cos 45^\circ\,(3'') = 19.7$

$F_{user} = 9.28$

c) $F_x = 4.64$

$F_y = 6.57$

$F_z = 4.64$

$F_{Hold} = 2F_x = 9.28$ lbs

d) ρ_{wp} (Oak) $= 56\,lb/ft^3 = 0.03241\,lb/in^3$

ρ_{pulley} (HDPE) $= 0.95\,g/cm^3 = 0.0343\,lb/in^3$

ρ_{shaft} (AISI 1020 Steel) $= 0.284\,lb/in^3$

$Weight_{wp} = \rho_{wp}\left(\frac{\pi}{4}\right)(d_{wp}^2)(L_{wp}) = (0.03241)\left(\frac{\pi}{4}\right)(6^2)(30)$

$= 27.49$ lbs

$Weight_{shaft\,(H)} = \rho_{shaft}\left(\frac{\pi}{4}\right)(d_s^2)(L_{s,H})$

$= (0.284)\left(\frac{\pi}{4}\right)(1)^2(2+4(0.8305)+3+0.25)$

$= 1.912$ lbs

$Weight_{shaft\,(T)} = \rho_{shaft}\left(\frac{\pi}{4}\right)(L_{s,T}) = (0.284)\left(\frac{\pi}{4}\right)(1)^2(0.2513)$

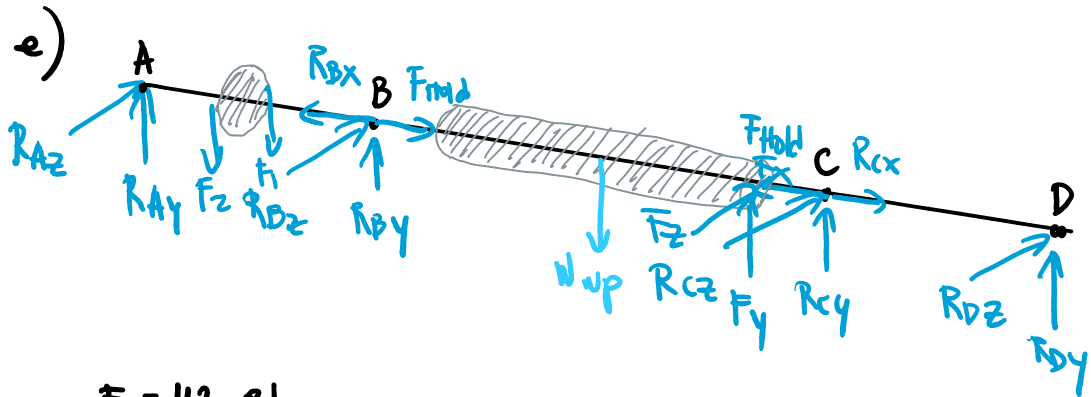
$= 0.725$ lbs

$weight_{\dots} = \rho_{pulley}\left(\frac{\pi}{\dots}\right)(d^2 + d^2 + d^2 + d_i^2)(t)$

$= 0.725 \text{ lbs}$

$$weight_{pulley} = \rho_{pulley}\left(\frac{\pi}{4}\right)\left(d_1^2 + d_2^2 + d_3^2 + d_4^2\right)(t)$$
$$= (0.0343)\left(\frac{\pi}{4}\right)\left(8.452^2 + 4.472^2 + 3.5938^2 + 3^2\right)(0.8305)$$
$$= 2.536 \text{ lbs}$$

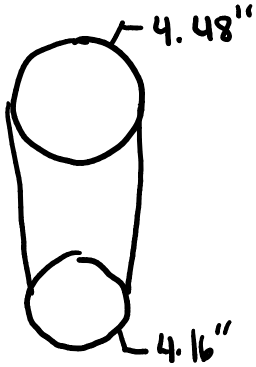
e)



$F_1 = 43.8b$

$F_2 = 43.8b - 10.63$

$b = 0.373 \longrightarrow F_1 = 16.34 \text{ lbs},\ F_2 = 5.71 \text{ lbs}$



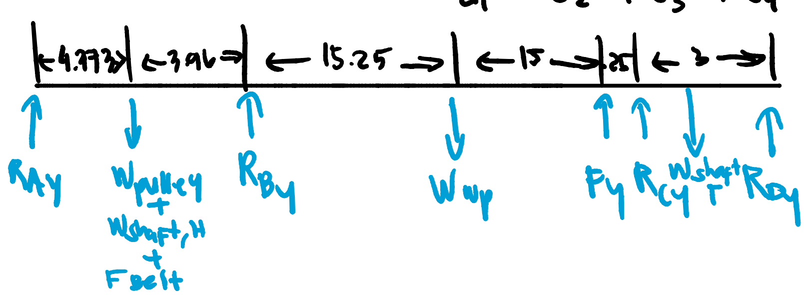
$\theta = \tan^{-1}\left(\frac{0.16}{12.45}\right) = 0.736$

$F_1 \cos\theta = 16.34 \cos(0.736) = 16.34$

$F_2 \cos\theta = 5.71 \cos(0.736) = 5.71$

$F_{belt} = 22.05 \text{ lbs}$

$$x_{pulley} = 2 + \frac{d_1^2\left(\frac{t}{2}\right) + d_2^2\left(\frac{3t}{2}\right) + d_3^2\left(\frac{5t}{2}\right) + d_4^2\left(\frac{7t}{2}\right)}{d_1^2 + d_2^2 + d_3^2 + d_4^2} = 4.373"$$



$\Sigma F_y = 0$

$R_{Ay} + R_{By} + F_y + R_{Cy} + R_{Dy} = W_{pulley} + W_{shaft,H} + F_{belt} + W_{wp} + W_{shaft,T}$

$R_{Ay} + R_{By} + R_{Cy} + R_{Dy} = 2.536 + 1.912 + 2.536 + 27.49 + 0.725 + 22.05 - F_y$

$R_{Ay} + R_{By} + R_{Cy} + R_{Dy} = 48.143$

$R_{Ay} + R_{By} + R_{Cy} + R_{Dy} = 2.536 + 1.912 + 2.536 + 27.49 + 0.725 + 22.05 - F_y$

$R_{Ay} + R_{By} + R_{Cy} + R_{Dy} = 48.143$

$\Sigma M_D = 0$ $+\circlearrowleft$

$-R_{Ay}(41.83) + (W_{S,H} + W_p + F_B)(37.46) - R_{By}(33.5) + (W_{wp})(18.25) - F_y(3.25)$
$- R_{Cy}(3) + W_{S,T}(1.625) = 0$

$-41.83 R_{Ay} + 992.6 - 33.5 R_{By} + 501.69 - 21.35 - 3R_{Cy} + 1.178 = 0$

$1474.118 = 41.83 R_{Ay} + 33.5 R_{By} - 3R_{Cy}$

Deflections:

caused by $W_{S,H}, W_P, F_B$

$$\delta_B = \frac{26.498(4.37)(41.83 - 8.33)}{6EI(41.83)}(8.33^2 + 4.37^2 - 2(41.83)(8.33))$$

$$\delta_B = \frac{-9403.53}{EI}$$

$$\delta_C = \frac{26.498(4.37)(41.83 - 38.83)}{6EI(41.83)}(38.33^2 + 4.37^2 - 2(41.83)(38.83))$$

$$= \frac{-2382.99}{EI}$$

Caused by R_{By}

$$\delta_B = \frac{-R_{By}(8.33)(41.83 - 8.33)}{6EI(41.83)}(8.33^2 + 8.33^2 - 2(41.83)(8.33))$$

$$= \frac{620.54 R_{By}}{EI}$$

$$\delta_C = \frac{-R_{By}(8.33)(41.83 - 38.83)}{6EI(41.83)}(8.33^2 + 8.33^2 - 2(41.83)(38.83))$$

$$= \frac{309.64 R_{By}}{EI}$$

Caused by W_{wp}

$$\delta_B = \frac{27.49(18.25)(8.33)}{6EI(41.83)}(8.33^2 + 18.25^2 - 41.83^2)$$

$$= \frac{-22434}{}$$

$$= \frac{-22434}{EI}$$

$$\delta_C = \frac{27.49(23.56)(41.83-38.83)}{6EI(41.83)}(38.83^2 + 23.58^2 - 2(41.83)(38.83))$$

$$= \frac{-9171.76}{EI}$$

Caused by F_y

$$\delta_B = \frac{-6.57(3.25)(8.33)}{6EI(41.83)}(8.33^2 + 3.25^2 - 41.83^2)$$

$$= \frac{1183.4}{EI}$$

$$\delta_C = \frac{-6.57(38.58)(41.83-38.83)}{6EI(41.83)}(38.83^2 + 23.58^2 - 2(41.83)(38.83))$$

$$= \frac{3589.5}{EI}$$

Caused by R_{cy}

$$\delta_B = \frac{-R_{cy}(3)(8.33)}{6EI(41.83)}(8.33^2 + 3^2 - 2(41.83)(8.33))$$

$$= \frac{61.58R_{cy}}{EI}$$

$$\delta_C = \frac{-R_{cy}(3)(38.83)}{6EI(41.83)}(38.83^2 + 3^2 - 2(41.83)(38.83))$$

$$= \frac{803.77R_{cy}}{EI}$$

Caused by $W_{s,T}$

$$\delta_B = \frac{0.725(1.625)(8.33)}{6EI(41.83)}(8.33^2 + 1.625^2 - 2(41.83)(8.33))$$

$$= \frac{-24.43}{EI}$$

$$\delta_C = \frac{0.725(1.625)(38.58)}{6EI(41.83)}(38.58^2 + 1.625^2 - 2(41.83)(38.58))$$

$$= -314.5$$

$$6EI(41.83)$$

$$= \frac{-314.5}{EI}$$

$\Sigma \delta_B = 0$

$$\frac{-9403.53}{\cancel{EI}} + \frac{620.5 R_{By}}{\cancel{EI}} - \frac{22434}{\cancel{EI}} + \frac{1183.4}{\cancel{EI}} + \frac{61.58 R_{Cy}}{\cancel{EI}} \frac{-24.43}{\cancel{EI}} = 0$$

$$30678.56 = 620.5 R_{By} + 61.56 R_{Cy}$$

$\Sigma \delta_C = 0$

$$\frac{-2382.99}{\cancel{EI}} + \frac{309.64 R_{By}}{\cancel{EI}} - \frac{9141.76}{\cancel{EI}} + \frac{3589.5}{\cancel{EI}} + \frac{803.77 R_{Cy}}{\cancel{EI}} - \frac{314.5}{\cancel{EI}} = 0$$

$$8279.75 = 309.64 R_{By} + 803.77 R_{Cy}$$

$R_{By} = 50.34$ lbs
$R_{Cy} = -9.09$ lbs
$R_{Ay} = -5.73$ lbs
$R_{Dy} = 12.623$ lbs

x-z Plane:

$\Sigma F_z = 0$

$$R_{Az} + F_2 \sin\theta + R_{Bz} + F_Z + R_{Cz} + R_{Dz} = F_1 \sin\theta$$

$$R_{Az} + R_{Bz} + R_{Cz} + R_{Dz} = F_1 \sin\theta - F_2 \sin\theta - F_Z = -4.50$$

$\Sigma M_D = 0$ +↺

$$-R_{Az}(41.83) - F_2 \sin\theta (37.46) + F_1 \sin\theta (37.46) - R_{Bz}(33.5)$$

$$- F_Z(3.25) - R_{Cz}(3) = 0$$

$$-9.97 = 41.83 R_{Az} + 33.5 R_{Bz} + 3 R_{Cz}$$

Deflections:

from belt

$$\delta_B = \frac{(0.137)\quad(4.37)(41.83-8.33)}{6EI(41.83)}(8.33^2 + 4.37^2 - 2(41.83)(8.33))$$

$$= \frac{-26.03}{EI}$$

$$\delta_C = \frac{(0.137)(4.37)(41.83-38.83)}{6EI(41.83)}(8.33^2 + 4.37^2 - 2(41.83)(38.83))$$

$$\delta_c = \frac{(0.137)(4.77)(41.83-38.83)}{6EI(41.83)}\left(8.33^2 + 4.77^2 - 2(41.83)(38.83)\right)$$

$$= \frac{-2.26}{EI}$$

From R_{Bz}

$$\delta_B = -\frac{R_{Bz}(8.33)(41.83-8.33)}{6EI}\left(8.33^2 + 8.33^2 - 2(41.83)(8.33)\right)$$

$$= \frac{620 R_{Bz}}{EI}$$

$$\delta_C = \frac{309.64 R_{Bz}}{EI}$$

From F_z

$$\delta_B = -\frac{4.64(3.25)(8.33^2)}{6EI(41.83)}\left(8.33^2 + 3.25^2 - 41.83^2\right)$$

$$= \frac{6961.71}{EI}$$

$$\delta_C = -\frac{4.64(38.58)(41.83-38.83)}{6EI(41.83)}\left(38.83^2 + 38.58^2 - 2(41.83)(38.83)\right)$$

$$= \frac{539.93}{EI}$$

From R_{Cz}

$$\delta_B = \frac{61.58 R_{Cz}}{EI}$$

$$\delta_C = \frac{803.77 R_{Cz}}{EI}$$

$$\Sigma \delta_B = 0$$

$$-26.03 + 620 R_{Bz} + 6961.71 + 61.58 R_{Cz} = 0$$

$$620 R_{Bz} + 61.58 R_{Cz} = -6935.68$$

$$\Sigma \delta_C = 0$$

$$-2.26 + 309.64 R_{Bz} + 539.93 + 803.77 R_{Cz} = 0$$

$$309.64 R_{Bz} + 803.77 R_{Cz} = -537.67$$

$R_{Cz} = 3.79$ lbs

$R_{Bz} = -11.57$ lbs

$R_{Az} = 8.76$ lbs

$R_{Dz} = -5.48$ lbs

$R_{Dz} = -5.48$ lbs

$R_{Bx} = F_{hold} = 9.28$

$R_{Cx} = F_{hold} + F_x = 13.92$

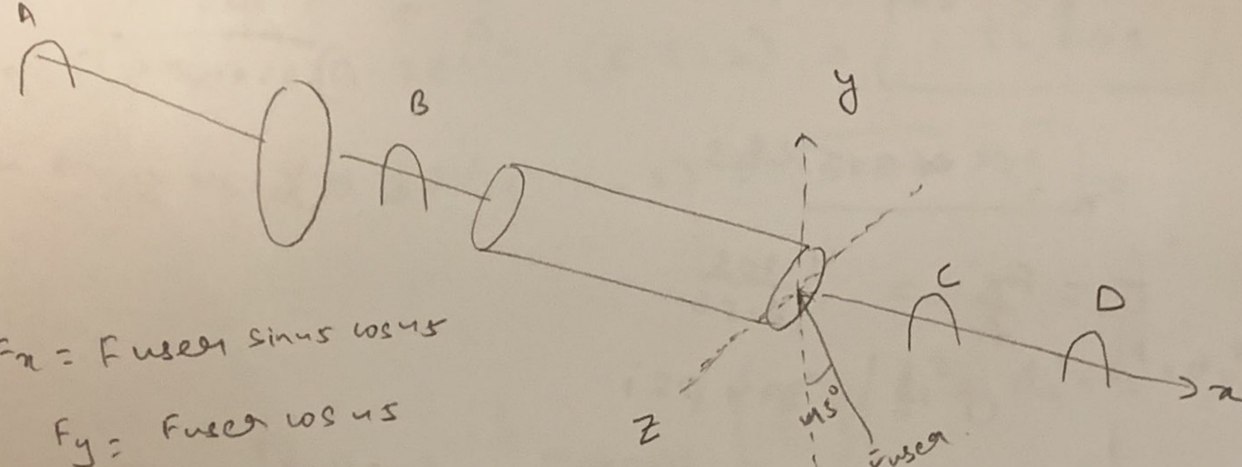
Appendix C.3: 2400 RPM Bearing Reaction Calculations

The following pages show the procedure for calculating the necessary bearing reactions for a 2400 RPM (4” workpiece diameter) scenario.

Studio 9

1) a) 2400 RPM → Accompanying workpiece $\phi = 4''$

b), c)



Fuser: $F_x = F_{user} \sin 45 \cos 45$

$F_y = F_{user} \cos 45$

$F_z = F_{user} \sin 45 \sin 45$

$P = T\omega$

$\Rightarrow (0.5)(550)(12) = T\left(\frac{2\pi}{60}(2400)\right)$

$\boxed{T = 13.13 \text{ lbs-ins.}}$

F_y at workpiece $= T$

$\Rightarrow F_{user} \cos 45 \times (2)'' = 13.13$

$\Rightarrow \boxed{F_{user} = 9.41 \text{ lbs}}$

$$\boxed{\begin{aligned} F_x &= 4.705 \text{ lbs} \\ F_y &= 6.65 \text{ lbs} \\ F_z &= 4.705 \text{ lbs} \end{aligned}}$$

$F_x = 4.705$ lbs

$L = 30''$

$A = \pi(2^2) = 12.56 in^2$

$E = 1.667 \times 10^6$ psi

$$\delta = \frac{FL}{EA}$$

$$\delta = \frac{4.705 \times 30}{1.667 \times 10^6 \times 12.56}$$

$\delta = 6.74 \times 10^{-6}$ mm $\ll 2$ mm

$F_x = 4.705$ lbs

$$\sigma = \frac{F_x}{A} = \frac{4.705}{12.56}$$

$\sigma = 0.374$ psi

$\sigma = E\epsilon$

$\Rightarrow 0.374 = (1.667 \times 10^6)\,\epsilon$

$\epsilon = 2.25 \times 10^{-7}$

Holding force $= 2\epsilon = 4.49 \times 10^{-7}$

$$\boxed{F_{hold} = 2F_x = 9.41 \text{ lbs}}$$

d) $W_{piece} = mg = \rho \forall$

$W_{piece} = (0.03241)(377)$

$$\boxed{W_{piece} = 12.21 \text{ lbs}}$$

$\rho = 0.03241$ lb/in^3

$\forall = \pi \times 2^2 \times 30$
$= 377$ in^3

$g = 32.2 \times 12$
$= 386.4 \frac{in}{s^2}$

Shafts

$W_{shafts} = \rho_{shaft} \forall_{shaft}$

$\rho_{shaft} = 0.284$ lb/in^3

$$\forall_{\text{Head stock}} = \pi(0.8)^2 \times (8.572)'' = 6.732 \text{ in}^3$$

$$\forall_{\text{Tail stock}} = \pi(0.5)^2 \times (3.25'') = 2.553 \text{ in}^3$$

$$W_{\text{Head stock}} = (0.284)(6.732) = \boxed{1.92 \text{ lbs}}$$

$$W_{\text{Tail stock}} = (0.284)(2.553) = \boxed{0.725 \text{ lbs}}$$

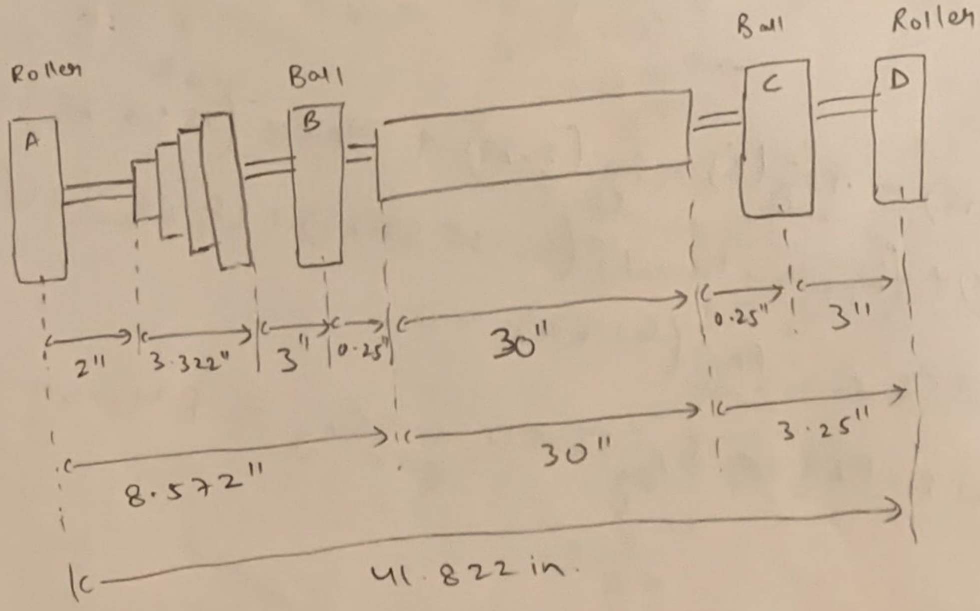
Pulleys

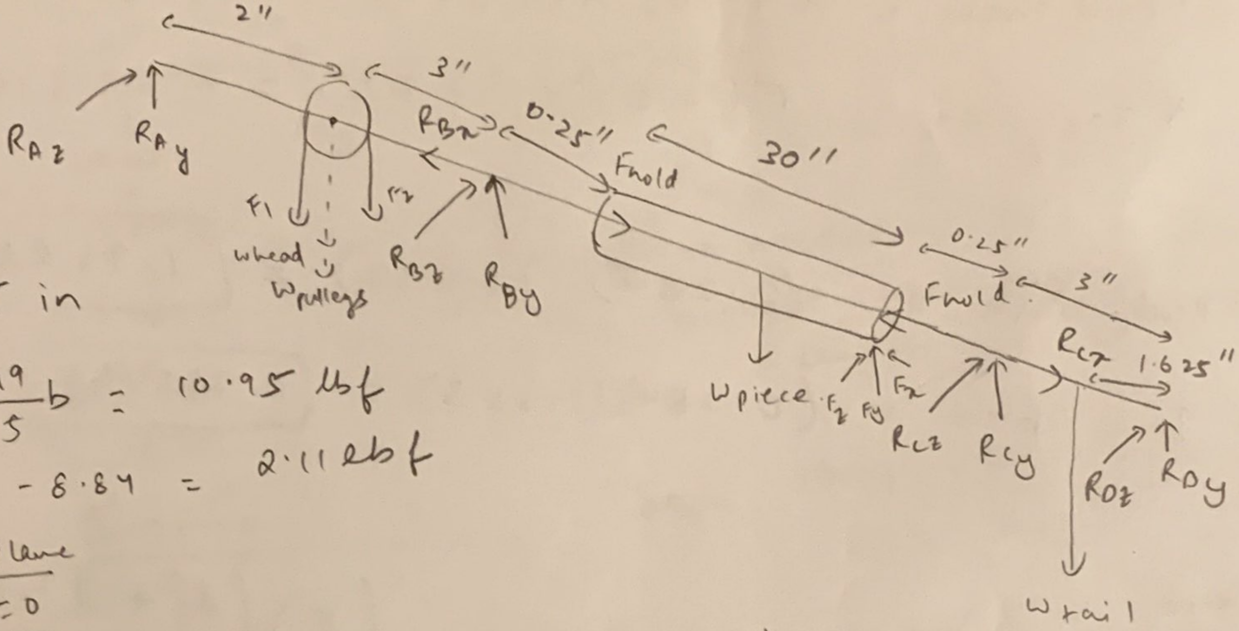
$$W_{\text{pulleys}} = \rho_{\text{pulleys}}\, t_{\text{thickness}} \left(\frac{\pi}{4}\right)\left(d_1^2 + d_2^2 + d_3^2 + d_4^2\right)$$

$$= (0.0343)(0.8305)\left(\frac{\pi}{4}\right)\left[8.452^2 + 4.477^2 + 3^2 + 3.59^2\right]$$

$$\boxed{W_{\text{pulleys}} = 2.53 \text{ lb}}$$

e)





$b = 0.25$ in

$F_1 = \frac{219}{5} b = 10.95$ lbf

$F_2 = F_1 - 8.84 = 2.11$ lbf

xy plane

$\Sigma F_y = 0$

$$R_{Ay} + R_{By} + R_{Cy} + R_{Dy} + F_y - W_{piece} - W_{head} - W_{tail} - W_{pulleys} - F_1 - F_2 = 0$$

$$\Rightarrow R_{Ay} + R_{By} + R_{Cy} + R_{Dy} = 0.725 + 1.22 + 2.53 + 12.21 + 10.95 + 2.11 - 6.65$$

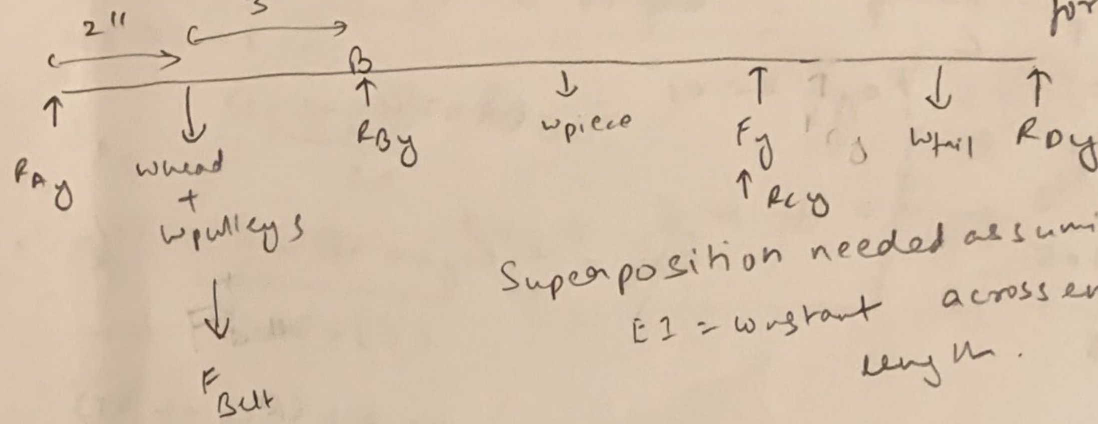
$$\Rightarrow R_{Ay} + R_{By} + R_{Cy} + R_{Dy} = 23.795 \quad - (1)$$

$\overset{+}{\curvearrowleft} \Sigma M_{Dz} = 0$

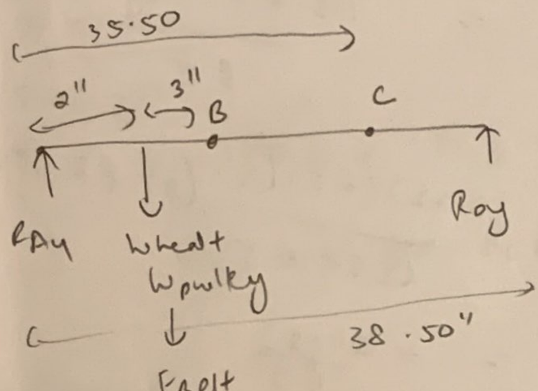
$$\Rightarrow W_{tail}(1.625) - R_{Cy}(3) - F_y(3.25) + W_{piece}(15 + 3.25) + (F_1 + F_2)(36.50) - R_{By}(30.25 + 3.25) + (W_{pulley} + W_{head})(33.25 + 3.25) - R_{Ay}(41.82) = 0.$$

$$\Rightarrow 41.82 R_{Ay} + R_{By} 33.50 + 3R_{Cy} = 841.64 \quad - (2)$$

Assuming R_{cy} & F_y act at same pt for simplicity.



Superposition needed assuming EI = constant across entire length.

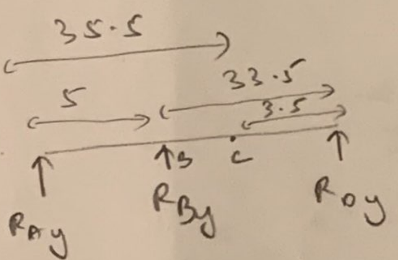


$$\delta_{B_1} = \frac{17.51(5)\times 33.50}{6EI(38.50)}\left(5^2+33.50^2-38.50^2\right)$$

$$\delta_{B_1} = \frac{-4253.37}{EI}$$

$$\delta_{C_1} = \frac{17.51(3.50)(35.50)}{6EI(38.50)}\left(35.5^2+3.5^2-38.5^2\right)$$

$$= -\frac{1975.47}{EI}$$

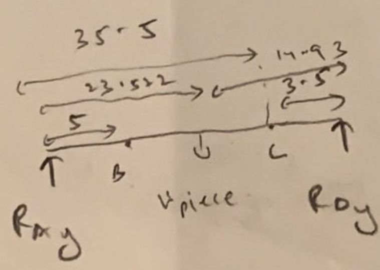


$$\delta_{B_2} = -\frac{R_{By}(33.5)(5)}{6EI(38.5)}\left(5^2+33.5^2-38.5^2\right)$$

$$= \frac{242.9\,R_{By}}{EI}$$

$$\delta_{C_2} = -\frac{R_{By}(3.5)(35.5)}{6EI(38.5)}\left(35.5^2+3.5^2-38.5^2\right)$$

$$= \frac{112.82}{EI}R_{By}$$

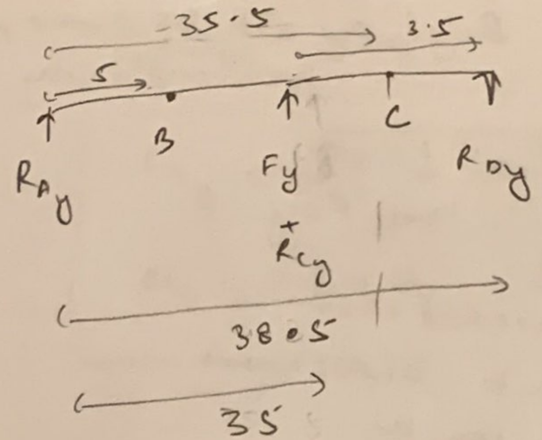


$$\delta_{B_3} = \frac{12.21(23.6)(38.5-5)}{6EI(38.5)}\left(35.5^2+23.6^2-2(38.5)(5)\right)$$

$$= -\frac{242.91}{EI}$$

$$\delta_{C_3} = \frac{12.21(14.9)(35.5)}{6EI(38.5)}\left(35.5^2+14.9^2-38.5^2\right)$$

$$= -\frac{229.6}{EI}$$

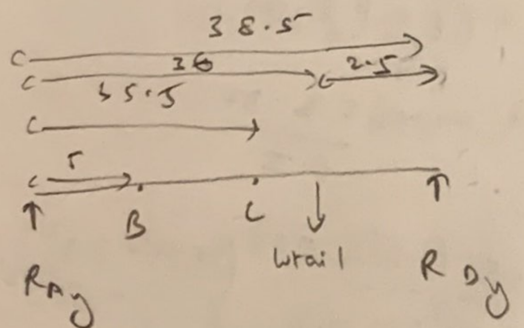


$$\delta_{By} = \frac{-(R_{cy}+6.65)(5^2+3.5^2-38.5^2)(3.5)(5)}{6EI \times 38.5}$$

$$= \frac{109.47(R_{cy}+6.65)}{EI}$$

$$\delta_{cy} = \frac{-(R_{cy}+6.65)(3.5)(35.5)(35.5^2+3.5^2-38.5^2)}{6EI\times 38.5}$$

$$= \frac{112.82(R_{cy}+6.65)}{EI}$$



$$\delta_{B5} = \frac{(0.72)(2.5)(5)(5^2+2.5^2-38.5^2)}{6EI\times 38.5}$$

$$= -\frac{56.53}{EI}$$

$$\delta_{c5} = \frac{(0.72)(2.5)(35.5)(35.5^2+2.5^2-38.5^2)}{6EI\times 38.5}$$

$$= -\frac{59.68}{EI}$$

$\delta_B = 0$

$$-\frac{4253}{EI} + \frac{243}{\cancel{EI}}R_{By} - \frac{243}{\cancel{EI}} + \frac{109R_{cy}}{\cancel{EI}} + \frac{730}{\cancel{EI}} - \frac{56.5}{\cancel{EI}} = 0 \quad \text{— (3)}$$

$$\Rightarrow 243R_{By} + 109R_{cy} = 3822.5$$

$\delta_c = 0$

$$-\frac{1975}{\cancel{EI}} + \frac{113R_{By}}{EI} - \frac{280}{\cancel{EI}} + \frac{113R_{cy}}{\cancel{EI}} + \frac{749.4}{\cancel{EI}} - \frac{59.68}{\cancel{EI}} = 0$$

$$113R_{By} + 113R_{cy} = 1565.68 \quad \text{— (4)}$$

Solving 4 eqs & 4 unknowns

$$\boxed{\begin{array}{ll} R_{Ay} = 6.55\,lb & R_{By} = 17.25\,lb \\ R_{Cy} = -3.4\,lb & R_{Dy} = 3.4\,lb \end{array}}$$

XZ plane

$\Sigma F_z = 0$

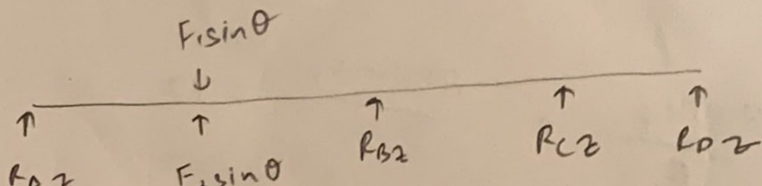
$$\Rightarrow R_{Az} + R_{Bz} + R_{Cz} + R_{Dz} + F_z + F_2 \sin(\theta) - F_1 \sin(\theta) = 0$$

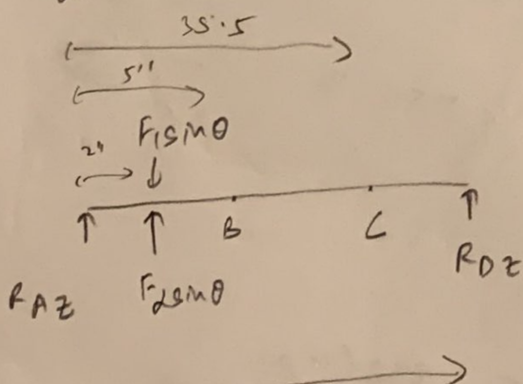
$$R_{Az} + R_{Bz} + R_{Cz} + R_{Dz} = -9.44. \quad -(1)$$

$\Sigma M_{Dy} = 0$

$$-R_{Cz}(3) - F_z(3.25) - R_{Bz}(33.5) - F_2 \sin\theta\,(36.5) + F_1 \sin\theta\,(36.5) - R_{Az}(38.5) = 0$$

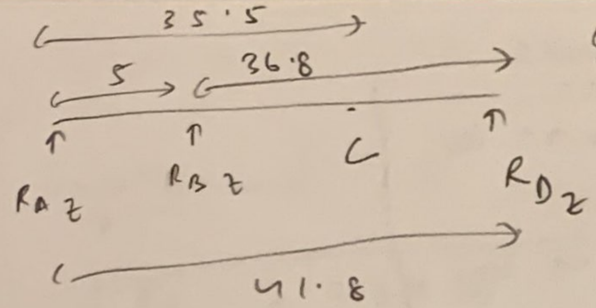
$$\Rightarrow 38.5\,R_{Az} + 33.5\,R_{Bz} + 3R_{Cz} = 188.4 \quad -(2)$$





$$\delta_{B_1} = \frac{4.74(2)(41.8-5)(5^2+2^2-2(41.8)(5))}{6EI \times 41.8} = -\frac{541}{EI}$$

$$\delta_{C_1} = \frac{4.74(2)(41.8-35.5)(35.5^2+2^2-2\times35.5\times41.8)}{6EI \times 41.8} = -\frac{405.6}{EI}$$

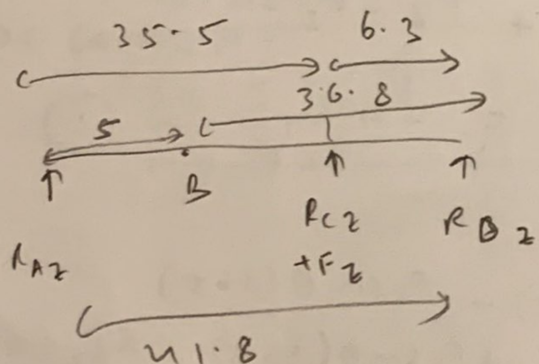


$$\delta_{B_2} = \frac{-R_{Bz}(5)(41.8-35.5)}{6EI\times 41.8}\left(5^2+5^2-2\times 5\times 41.8\right)$$

$$= \frac{46.2\,R_{Bz}}{EI}$$

$$\delta_{C_2} = \frac{-R_{Bz}(5)(41.8-35.5)}{6EI\times 41.8}\left(35.5^2+5^2-2\times 35.5\times 41.8\right)$$

$$= \frac{211.32\,R_{Bz}}{EI}$$



$$\delta_{B_3} = \frac{-(R_{Cz}+4.7)(6.3)(5)}{6EI\times 41.8}\left(5^2+6.3^2-41.8^2\right)$$

$$= \frac{211\,(R_{Cz}+4.7)}{EI}$$

$$\delta_{C_3} = \frac{-(R_{Cz}+4.7)(35.5)(41.8-35.5)}{6EI\times 41.8}\left(35.5^2+35.5^2-2(41.8)(35.5)\right)$$

$$= \frac{398\,(R_{Cz}+4.7)}{EI}$$

$\delta_B = 0$

$$\Rightarrow -\frac{541}{\cancel{EI}} + \frac{46.2\,R_{Bz}}{\cancel{EI}} + \frac{211\,R_{Cz}}{\cancel{EI}} + \frac{211(4.7)}{\cancel{EI}} = 0$$

$$\Rightarrow 46.2R_{Bz} + 211R_{Cz} = -450.7 \quad - (3)$$

$\delta_C = 0$

$$\Rightarrow -\frac{405.6}{\cancel{EI}} + \frac{211.32\,R_{Bz}}{\cancel{EI}} + \frac{398\,R_{Cz}}{\cancel{EI}} + \frac{398(4.7)}{\cancel{EI}} = 0$$

$$\Rightarrow 211R_{Bz} + 398R_{Cz} = -1465 \quad - (4)$$

We have 4 equations & 4 unknowns :-

Solving

$$\boxed{R_{Az} = 9.29\ lb \qquad R_{Cz} = -1.05\ lb \qquad R_{Bz} = -4.96\ lb \qquad R_{Dz} = -12.72\ lb}$$

$$\boxed{R_{Bx} = F_{hold} = 9.41\ lb}$$

$$\boxed{R_{Cx} = F_{hold} + F_x = 14.11\ lb}$$

Appendix C.4: 3200 RPM Bearing Reaction Calculations

The following pages show the procedure for calculating the necessary bearing reactions for a 3200 RPM (2” workpiece diameter) scenario.

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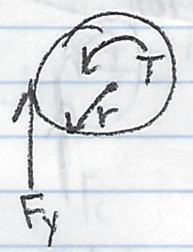
a.) 3200 RPM $\longrightarrow$ 2" Diameter or r = 1"
L = 30"

b and c) $\omega = 3200 \text{ RPM} = 2\pi \frac{3200 \text{ RPM}}{60} = 106.67\pi$ rad/s

$P = 0.5 \text{ HP} \cdot \frac{(550 \text{ ft-lbf/s})}{\text{HP}} = 275 \frac{\text{ft-lbf}}{s}$

$T = \frac{P}{\omega} = \frac{275 \text{ ft-lbf/s}}{106.67\pi \text{ rad/s}}$

$T = 0.8206$ ft-lbf

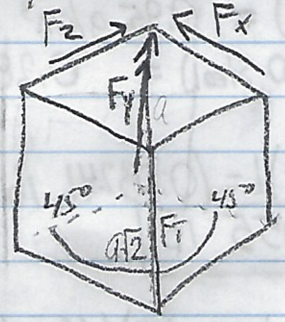


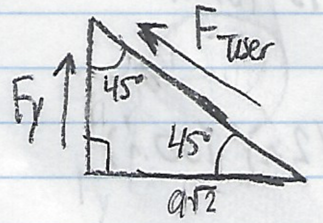
$\circlearrowleft + \Sigma M_x = 0$

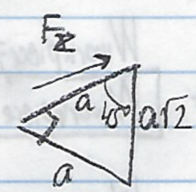
$T - F_y r = 0$

$F_y = \frac{T}{r} = \frac{0.8206 \text{ ft-lbf}}{1 \text{ in} \cdot \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)}$

$\boxed{F_y = 9.848 \text{ lbf}}$







$\cos 45° = \frac{F_y}{F_{user}}$

$F_{user} = F_y / \cos 45$

$F_{user} = 9.848 / \cos 45$

$F_{user} = 13.93$ lbf

$F_x = F_{user} \sin 45 \cos 45°$

$F_x = (13.93 \text{ lbf}) \sin 45 \cos 45$

$\boxed{F_x = 6.964 \text{ lbf}}$

$\boxed{F_y = 9.848 \text{ lbf}}$

$F_z = F_{user} \sin 45 \sin 45$

$F_z = (13.93)(\sin 45)^2$

$\boxed{F_z = 6.964 \text{ lbf}}$

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$$\delta = \frac{F_x L}{E_{wood} A} = \frac{(6.964\ \text{lbf})(30\ \text{in})}{(1.667(10^6)\ \text{lb}/\text{in}^2)(\pi(1\ \text{in})^2)}$$

Using oak b/c it is the densest $E = 1.667(10^6)$ psi

$$\delta = 3.99(10^{-5})\ \text{in} = 0.001\ \text{mm} < 2\ \text{mm}$$

$$\tau_s = \frac{F_{def}}{A} \Rightarrow F_{def} = \tau_s(2\pi r l)$$

$$F_{def} = (10(10^6)\ \tfrac{N}{m^2})(2\pi(\tfrac{3}{2000}\ m)(\tfrac{2}{1000}\ m))$$

$$F_{def} = 188.5\ N = 42.376\ \text{lbf}$$

$$R_x = F_{def} + F_x = 9.848\ \text{lbf} + 42.376\ \text{lbf}$$

$$\boxed{R_{Bx} = 52.224\ \text{lbf}}$$

d) $\rho_{workpiece}\ (\text{oak}) = 56\ \text{lb}/\text{ft}^3 = 0.03241\ \text{lb}/\text{in}^3$

$\rho_{pulleys}\ (\text{HDPE}) = 0.95\ \text{g}/\text{cm}^3 = 0.0343\ \text{lb}/\text{in}^3$

$\rho_{shafts}\ (\text{AISI 1020 Steel}) = 0.284\ \text{lb}/\text{in}^3$

$$W_{workpiece} = \rho(\tfrac{\pi}{4}d^2)L = (0.03241\ \text{lb}/\text{in}^3)(\tfrac{\pi}{4})(2\ \text{in})^2(30\ \text{in})$$

$$W_{wood} = \boxed{W_{workpiece} = 3.055\ \text{lb}}$$

$$W_{pulleys} = \rho_{pulleys}\, t_{thickness}(\tfrac{\pi}{4})(d_1^2 + d_2^2 + d_3^2 + d_4^2)$$

$$= (0.0343\ \text{lb}/\text{in}^3)(0.8305\ \text{in})(\pi/4)((8.452\ \text{in})^2 + (4.472\ \text{in})^2 + (3.5938\ \text{in})^2 + (3\ \text{in})^2)$$

$$\boxed{W_{pulleys} = 2.536\ \text{lb}}$$

$$W_{shafts} = \rho_{shaft}(\tfrac{\pi}{4})d^2(L_{tot})$$

$$W_{shaft\ (headstock)} = (0.284\ \text{lb}/\text{in}^3)(\tfrac{\pi}{4})(1\ \text{in})^2(2\ \text{in} + 3.322\ \text{in} + 3\ \text{in} + 0.25\ \text{in})$$

$$W_{s,h} = \boxed{W_{shaft\ (headstock)} = 1.912\ \text{lb}}$$

$$W_{shaft\ (tailstock)} = (0.284\ \text{lb}/\text{in}^3)(\pi/4)(1\ \text{in})^2(0.25\ \text{in} + 3\ \text{in})$$

$$W_{s,t} = \boxed{W_{shaft\ (tailstock)} = 0.725\ \text{lb}}$$

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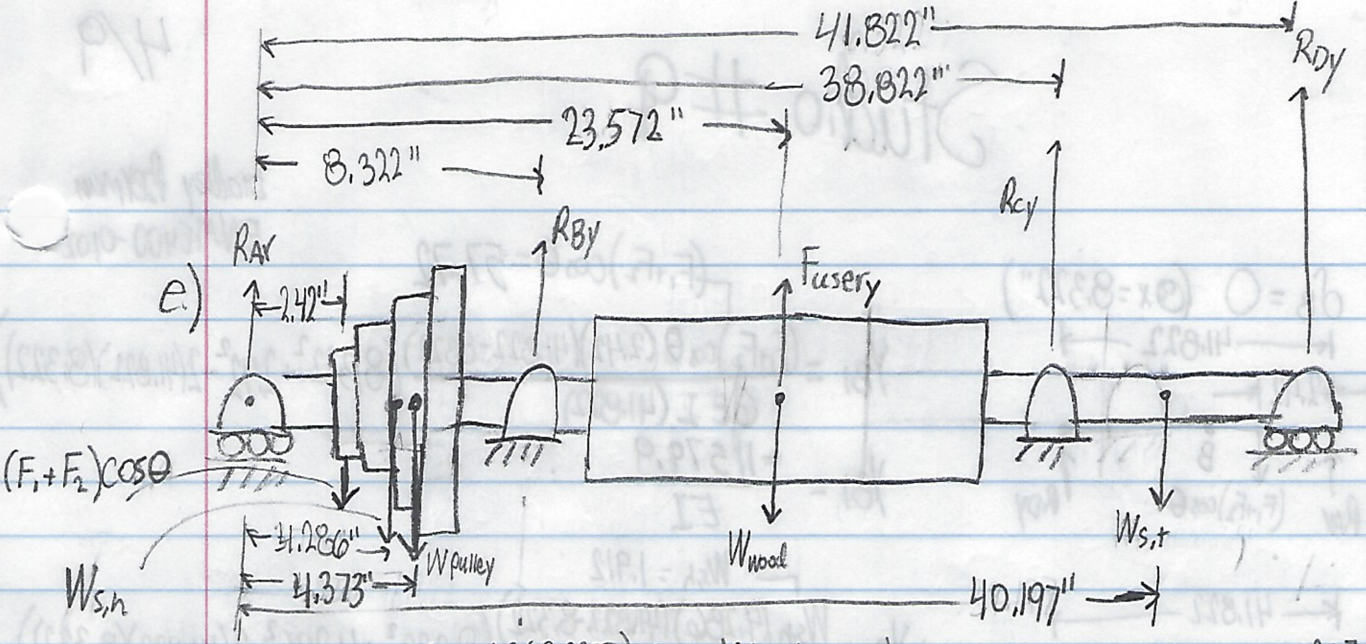
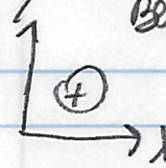




$$\bar{X}_{pulley} = 2 + \frac{(3)^2\left(\frac{0.8305}{2}\right)+(3.5938)^2\left(\frac{3}{2}(0.8305)\right)+(4.472)^2\left(\frac{5}{2}(0.8305)\right)+(8.452)^2\left(\frac{7}{2}(0.8305)\right)}{(3)^2+(3.5938)^2+(4.472)^2+(8.452)^2}$$

$\bar{X}_{pulley} = 4.373''$ from left end (bearing A)

$\bar{X}_{shaft}$ (headstock) $= 0.5(2+3.322+3+0.25 \text{ in})$

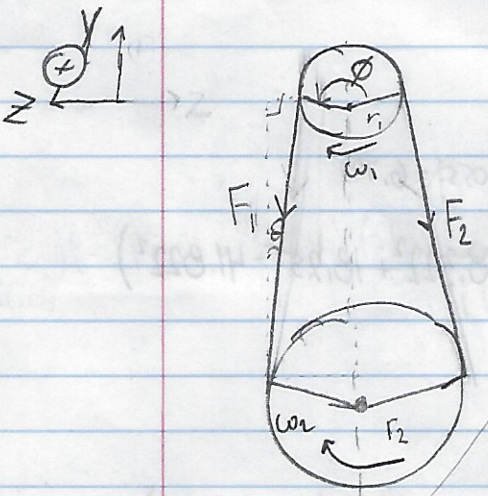
$\bar{X}_{shaft}$ (headstock) $= 4.286''$ from left end (bearing A)

$\bar{X}_{wood} = 2+3.322+3+0.25+(30/2)$

$\bar{X}_{wood} = 23.572''$ from left end $= \bar{X}_{user}$

$\bar{X}_{shaft}$ (tailstock) $= 2+3.322+3+0.25+30+0.5(0.25+3)$

$\bar{X}_{shaft}$ (tailstock) $= 40.197''$ from left end



$F_1 = 43.8D = 43.8(0.8305)$

$F_1 = 36.38$ lb

$F_2 = 43.8D - 14.736 = 43.8(0.8305) - 14.736$

$F_2 = 21.64$ lb

$\theta = \tan^{-1}\left(\frac{r_2-r_1}{c}\right) = \tan^{-1}\left(\frac{2.77-1.5}{12.45}\right) = 0.1017$ rad

$F_y = (F_1+F_2)\cos\theta = (36.38+21.64)\cos(0.1017)$

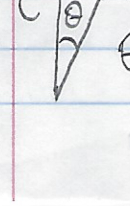
$F_y = 57.72$ lb

$F_{1z} = +F_1\sin\theta = +36.38\sin(0.1017)$

$F_{1z} = +3.692$ lb

$F_{2z} = -F_2\sin\theta = -21.64\sin(0.1017)$

$F_{2z} = -2.196$ lb



$\theta = \tan^{-1}\left(\frac{r_2-r_1}{c}\right)$

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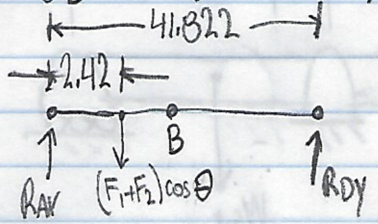
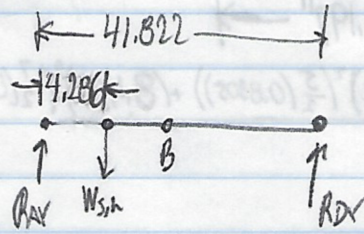
$\delta_B = 0 \quad (@x = 8.322")$



$(F_1+F_2)\cos\theta = 57.72$

$$Y_{B1} = \frac{(F_1+F_2)\cos\theta\,(2.42)(41.822-8.322)}{6EI(41.822)}\left(8.322^2+2.42^2-2(41.822)(8.322)\right)$$

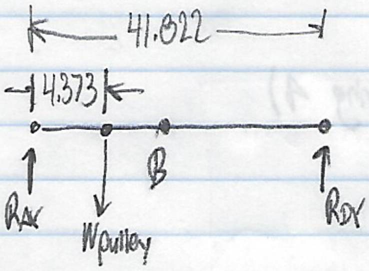
$$Y_{B1} = \frac{-11579.9}{EI}$$



$W_{s,h} = 1.912$

$$Y_{B2} = \frac{W_{s,h}(4.286)(41.822-8.322)}{6EI(41.822)}\left(8.322^2+4.286^2-2(41.822)(8.322)\right)$$

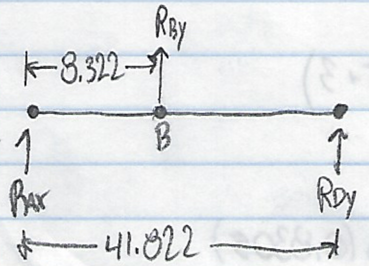
$$Y_{B2} = -\frac{665.7}{EI}$$



$W_{pulley} = 2.536$

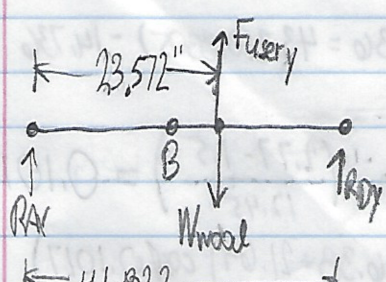
$$Y_{B3} = \frac{W_{pulley}(4.373)(41.822-8.322)}{6EI(41.822)}\left(8.322^2+4.373^2-2(41.822)(8.322)\right)$$

$$Y_{B3} = -\frac{899.7}{EI}$$



$$Y_{B4} = \frac{-R_{By}(33.5)(8.322)}{6EI(41.822)}\left(8.322^2+33.5^2-41.822^2\right)$$

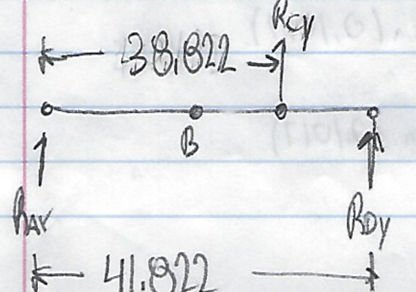
$$Y_{B4} = \frac{+619.5\,R_{By}}{EI}$$



$F_y - W_{wood} = 9.848 - 3.055 = 6.793$

$$Y_{B5} = \frac{-(F_y - W_{wood})(18.25)(8.322)}{6EI(41.822)}\left(8.322^2+18.25^2-41.822^2\right)$$

$$Y_{B5} = \frac{+5537.2}{EI}$$



$$Y_{B6} = \frac{-R_{cy}(3)(8.322)}{6EI(41.822)}\left(8.322^2+3^2-41.822^2\right)$$

$$Y_{B6} = \frac{+166.2\,R_{cy}}{EI}$$

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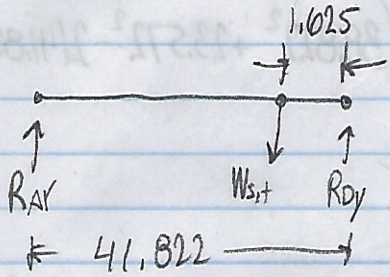


$W_{s,t} = 0.725$

$$y_{B7} = \frac{W_{s,t}(1.625)(8.322)}{6EI(41.822)}\left(8.322^2 + 1.625^2 - 41.822^2\right)$$

$$y_{B7} = \frac{-65.5}{EI}$$

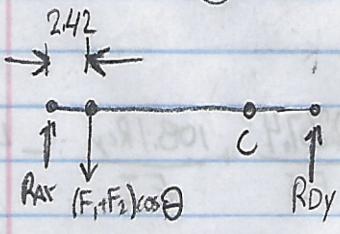
$$\delta_B = 0 = \delta_{B1} + \delta_{B2} + \delta_{B3} + \delta_{B4} + \delta_{B5} + \delta_{B6} + \delta_{B7}$$

$$0 = \frac{-11579.9}{EI} + \frac{-665.7}{EI} + \frac{-899.7}{EI} + \frac{619.5R_{By}}{EI} + \frac{+5537.2}{EI} + \frac{166.2R_{cy}}{EI} - \frac{65.5}{EI}$$

Assumption: EI is constant throughout

$$619.5R_{By} + 166.2R_{cy} = 7673.7 \quad \text{— (1)}$$

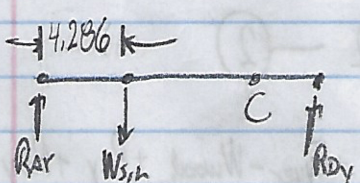
$\delta_c = 0$ (@ $x = 38.822$)



$(F_1+F_2)\cos\theta = 57.72$

$$y_{c_1} = \frac{(F_1+F_2)\cos\theta(2.42)(41.822-38.822)}{6EI(41.822)}\left(38.822^2 + 2.42^2 - 2(41.822)(38.822)\right)$$

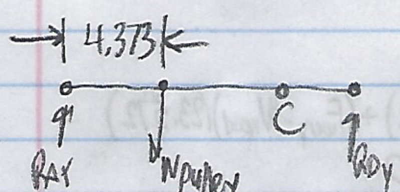
$$y_{c_1} = \frac{-2896.1}{EI}$$



$W_{s,h} = 1.912$

$$y_{c2} = \frac{W_{s,h}(4.286)(3)}{6EI(41.822)}\left(38.822^2 + 4.286^2 - 2(41.822)(38.822)\right)$$

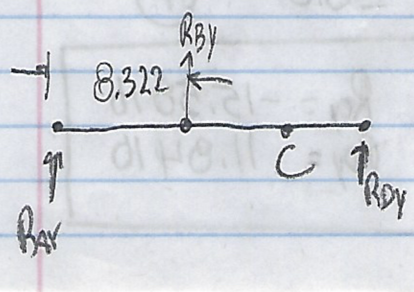
$$y_{c2} = \frac{-168.7}{EI}$$



$W_{pulley} = 2.536$

$$y_{c3} = \frac{W_{pulley}(4.373)(3)}{6EI(41.822)}\left(38.822^2 + 4.373^2 - 2(41.822)(38.822)\right)$$

$$y_{c3} = \frac{-228.2}{EI}$$



$$y_{c4} = \frac{-R_{By}(8.322)(3)}{6EI(41.822)}\left(38.822^2 + 8.322^2 - 2(41.822)(38.822)\right)$$

$$y_{c4} = \frac{+166.2R_{By}}{EI}$$

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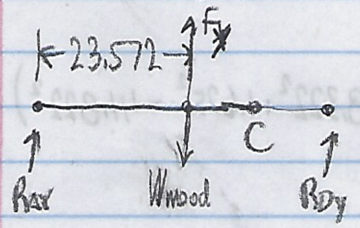
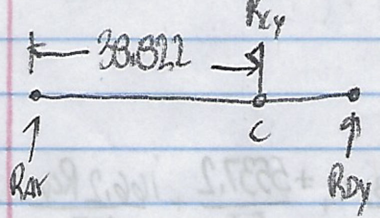


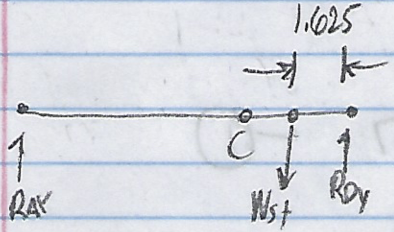
$$Y_{C5} = \frac{-\overbrace{(F_y - W_{wood})}^{6.793}(23.572)(3)}{6EI(41.822)}\left(38.822^2 + 23.572^2 - 2(41.822)(38.822)\right)$$

$$Y_{C5} = \frac{+2267.4}{EI}$$



$$Y_{C6} = \frac{-R_{Cy}(3)(38.822)}{6EI(41.822)}\left(38.822^2 + 3^2 - 41.822^2\right)$$

$$Y_{C6} = \frac{108.1R_{Cy}}{EI}$$



$$Y_{C7} = \frac{\overbrace{W_{st}}^{W_{s,t} = 0.725}(1.625)(38.822)}{6EI(41.822)}\left(38.822^2 + 1.625^2 - 41.822^2\right)$$

$$Y_{C7} = \frac{-43.6}{EI}$$

$$\delta_C = 0 = \delta_{C1} + \delta_{C2} + \delta_{C3} + \delta_{C4} + \delta_{C5} + \delta_{C6} + \delta_{C7}$$

$$0 = \frac{-2896.1}{EI} + \frac{-168.7}{EI} + \frac{-228.2}{EI} + \frac{166.2R_{By}}{EI} + \frac{+2267.4}{EI} + \frac{108.1R_{Cy}}{EI} - \frac{43.6}{EI}$$

$$166.2R_{By} + 108.1R_{Cy} = 1069.2 \quad \text{— (2)}$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow 0 = R_{Ay} - (F_1 + F_2)\cos\theta - W_{s,h} - W_{pulley} + R_{By} + F_{ywer} - W_{wood} + R_{Cy} + R_{Dy} - W_{s,t}$$

$$R_{Ay} + R_{By} + R_{Cy} + R_{Dy} = 57.72 + 1.912 + 2.536 - 9.848 + 3.055 + 0.725$$

$$R_{Ay} + R_{By} + R_{Cy} + R_{Dy} = 56.1 \quad \text{— (3)}$$

$$+\circlearrowleft \Sigma M_{x_A} = 0$$

$$-(F_1+F_2)\cos\theta(2.42) - W_{s,h}(4.286) - W_{pulley}(4.373) + R_{By}(8.322) + (F_{ywery} - W_{wood})(23.572)$$
$$+ R_{Cy}(38.822) - W_{s,t}(40.197) + R_{Dy}(41.822) = 0$$

$$8.322R_{By} + 38.822R_{Cy} + 41.822R_{Dy} = 28.0 \quad \text{— (4)}$$

4 eqs + 4 unknowns
Eq. Solver $\Rightarrow$

$$\boxed{\begin{array}{ll} R_{Ay} = 43.28 \text{ lb} & R_{Cy} = -15.58 \text{ lb} \\ R_{By} = 16.57 \text{ lb} & R_{Dy} = 11.84 \text{ lb} \end{array}}$$

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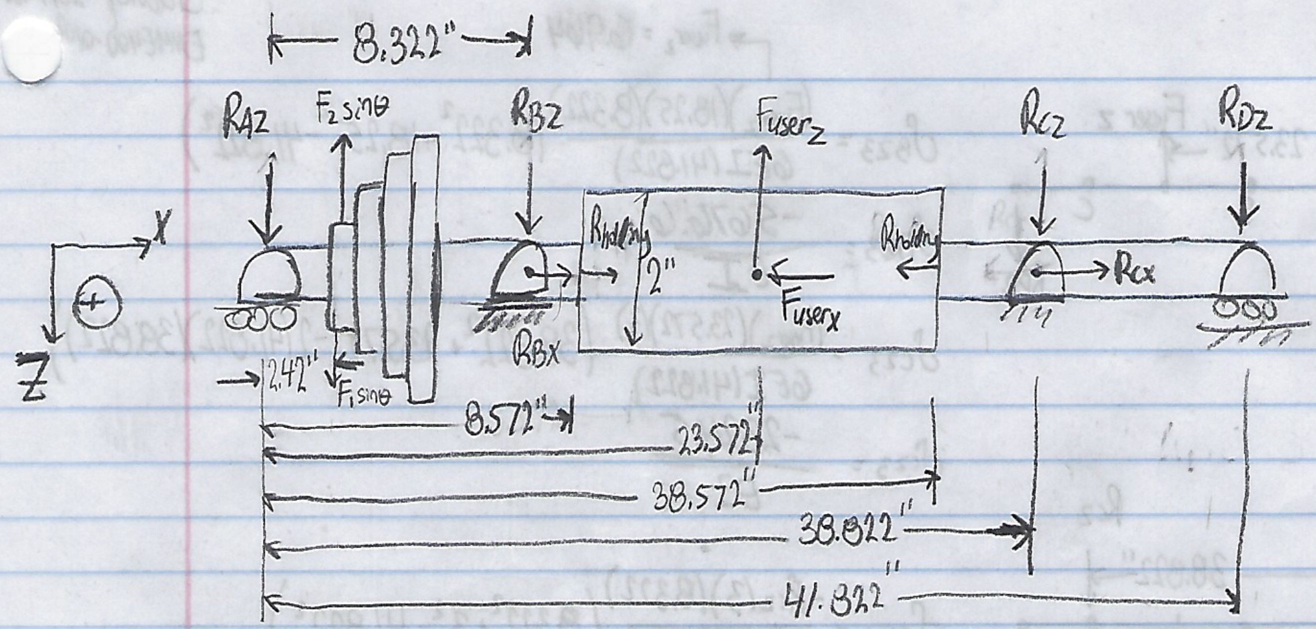
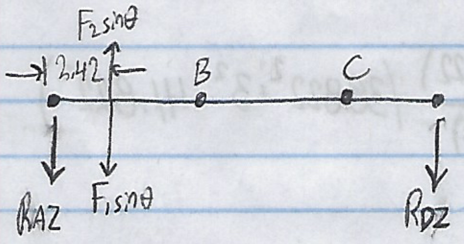


$\delta_B = 0\,(@x = 8.322)$ and $\delta_C = 0\,(@x = 38.822)$

$|F_1| > |F_2| \rightarrow (F_1+F_2)\sin\theta = 1.496$

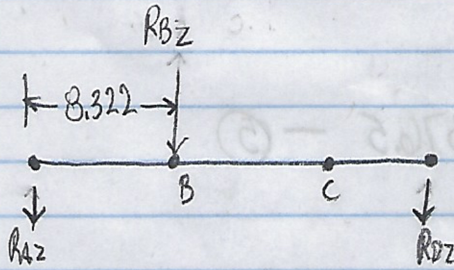


$$\delta_{BZ1} = \frac{-(F_1\sin\theta + F_2\sin\theta)(2.42)(33.5)}{6EI(41.822)}\left(8.322^2 + 2.42^2 - 2(41.822)(8.322)\right)$$

$$\delta_{BZ1} = \frac{300.1}{EI}$$

$$\delta_{CZ1} = \frac{-(F_1+F_2)\sin\theta(2.42)(3)}{6EI(41.822)}\left(38.822^2 + 2.42^2 - 2(41.822)(38.822)\right)$$

$$\delta_{CZ1} = \frac{75.1}{EI}$$



$$\delta_{BZ2} = \frac{-R_{BZ}(8.322)(33.5)}{6EI(41.822)}\left(8.322^2 + 8.322^2 - 2(41.822)(8.322)\right)$$

$$\delta_{BZ2} = \frac{+619.5 R_{BZ}}{EI}$$

$$\delta_{CZ2} = \frac{-R_{BZ}(8.322)(3)}{6EI(41.822)}\left(38.822^2 + 8.322^2 - 2(41.822)(38.822)\right)$$

$$\delta_{CZ2} = \frac{+166.2 R_{BZ}}{EI}$$

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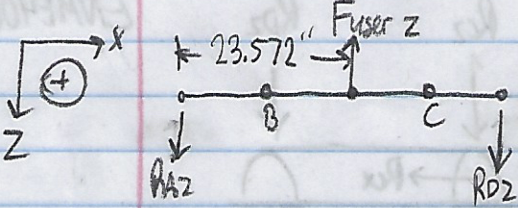


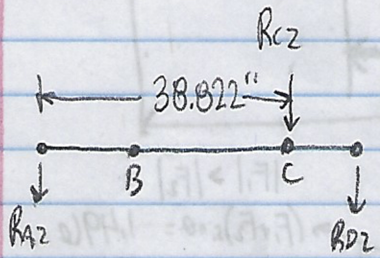
$$F_{user_z} = 6.964$$

$$\delta_{BZ3} = \frac{(F_{user_z})(18.25)(8.322)}{6EI(41.822)}(8.322^2 + 18.25^2 - 41.822^2)$$

$$\delta_{BZ3} = \frac{-5676.6}{EI}$$

$$\delta_{CZ3} = \frac{(F_{user_z})(23.572)(3)}{6EI(41.822)}\left(38.822^2 + 23.572^2 - 2(41.822)(38.822)\right)$$

$$\delta_{CZ3} = \frac{-2324.5}{EI}$$



$$\delta_{BZ4} = \frac{-R_{cz}(3)(8.322)}{6EI(41.822)}(8.322^2 + 3^2 - 41.822^2)$$

$$\delta_{BZ4} = \frac{+166.2R_{cz}}{EI}$$

$$\delta_{CZ4} \quad \frac{-R_{cz}(3)(38.822)}{6EI(41.822)}(38.822^2 + 3^2 - 41.822^2)$$

$$\delta_{CZ4} = \frac{+108.1R_{cz}}{EI}$$

$\delta_{BZ} = 0$

$$0 = \delta_{BZ1} + \delta_{BZ2} + \delta_{BZ3} + \delta_{BZ4}$$

$$0 = \frac{300.1}{EI} + \frac{619.5R_{BZ}}{EI} - \frac{5676.6}{EI} + \frac{166.2R_{cz}}{EI}$$

$$619.5R_{BZ} + 166.2R_{cz} = 5376.5 \quad - (5)$$

$\delta_{CZ} = 0$

$$0 = \delta_{CZ1} + \delta_{CZ2} + \delta_{CZ3} + \delta_{CZ4}$$

$$0 = \frac{75.1}{EI} + \frac{166.2R_{BZ}}{EI} - \frac{2324.5}{EI} + \frac{108.1R_{cz}}{EI}$$

$$166.2R_{BZ} + 108.1R_{cz} = 2249.4 \quad - (6)$$

$\circlearrowleft + \Sigma M_{Az} = 0$

$$0 = -F_1\sin\theta(2.42) + F_2\sin\theta(2.42) - R_{BZ}(8.322) + F_{user_z}(23.572) - R_{cz}(38.822) - R_{DZ}(41.822)$$

$$8.322R_{BZ} + 38.822R_{cz} + 41.822R_{DZ} = 160.5 \quad - (7)$$

$\downarrow + \Sigma F_Z = 0$

$$0 = R_{AZ} + R_{BZ} + R_{cz} + R_{DZ} + F_1\sin\theta - F_2\sin\theta - F_{user_z}$$

$$R_{AZ} + R_{BZ} + R_{cz} + R_{DZ} = 5.468 \quad - (8)$$

Studio #9

9/9

Bradley Berman
ENME400-0102

Solving eqns. 5, 6, 7 + 8 using Eq. solver $\Rightarrow$

$$\boxed{\begin{array}{ll} R_{AZ} = -3.50 \text{ lb} & R_{CZ} = +12.71 \text{ lb} \\ R_{BZ} = +5.27 \text{ lb} & R_{DZ} = -9.01 \text{ lb} \end{array}}$$

$R_{BX} = F_{def} + F_x$ ← from parts (b)(c)

$$\boxed{R_{BX} = 52.224 \text{ lbf}}$$

$R_{CX} = F_{def}$

$$\boxed{R_{CX} = 42.376 \text{ lbf}}$$

Appendix D: Bearing Sizing Calculations

Appendix D.1: 800 RPM Bearing Sizing Calculations

The following pages show the procedure for calculating the necessary bearings sizes for a 800 RPM (8" workpiece diameter) scenario. Calculations from Appendix C.1 were considered during this procedure.

Ed Stillwell
ENME 400
Studio 10

8" Ø, 800 RPM Bearing Selection

Roller Bearing A: $R_{AY} = -21.765$ lbs $R_{AZ} = 14.20$ lbs
Ball Bearing B: $R_{BX} = 13.926$ lbs $R_{By} = 95.463$ lbs $R_{BZ} = -15.47$ lbs.
Ball Bearing C: $R_{CX} = 20.89$ lbs $R_{Cy} = 6.273$ lbs $R_{CZ} = 2.51$ lbs.
Roller Bearing D: $R_{Dy} = 29.534$ lbs $R_{DZ} = -5.38$ lbs

Roller Bearing A

$R_{Ay} = -21.765 \text{ lbs} = -96.816 \text{ N} = -.096816 \text{ kN}$
$R_{AZ} = 14.20 \text{ lbs} = 63.165 \text{ N} = .063165 \text{ kN}$

$$F_R = \sqrt{.096816^2 + .063165^2}$$
$$= .1156 \text{ kN}$$

$$L_D = (200 \text{ hrs})\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)\left(\frac{800 \text{ rot}}{1 \text{ min}}\right) = 9.6 \times 10^6$$
$$L_{10} = 10^6 \text{ (always)}$$
$$X_D = \frac{L_D}{L_{10}} = 9.6$$

$$X_i = X_0 + (\theta + X_0)\left[Ln\left(\frac{1}{R}\right)\right]^{1/b}$$

where $X_0 = .02$, $\theta = 4.459$, $R = .99$, $b = 1.483$

$$X_i = .02 + (4.459 + .02)\left[Ln\left(\frac{1}{.99}\right)\right]^{1/1.483}$$

$$X_i = .2214$$

$$C_{10} = F_D\left(\frac{X_D}{X_i}\right)^{1/a} \quad \text{where } a = 10/3 \text{ (roller)}$$

$$C_{10} = F_D\left(\frac{9.6}{.2214}\right)^{3/10} = 3.098 F_D = C_{10}$$

$$F_D = n_d \cancel{F_e} \xrightarrow{} F_r$$
$$= (1.1)(.1156)$$
$$= .12716 \text{ kN}$$

Thus, $C_{10} = .394$ kN

$\boxed{\text{Roller Bearing A Bore size} \Rightarrow 25 \text{ mm for Roller Bearing A.}}$

Ball Bearing B

$R_{Bx} = 13.926 \text{ lbs} = .0619 \text{ kN}$

$R_{By} = 95.463 \text{ lbs} = .425 \text{ kN}$

$R_{Bz} = 14.20 \text{ lbs} = .0632 \text{ kN}$

$$F_R = \sqrt{.425^2 + .0632^2} = .4297 \text{ kN}$$
$$F_a = .0619 \text{ kN}$$

$$L_D = 9.6 \times 10^6 \text{ rot}$$
$$X_D = \frac{L_D}{L_{10}} = \frac{9.6 \times 10^6}{10^6} = 9.6$$
$$X_i = X_0 + (\theta + X_0) \ln\left[\frac{1}{R}\right]^{1/b} = .2214$$
$$F_D X_D^{1/a} = \overset{C_{10}}{\cancel{F_i}} X_i^{1/a}$$
$$C_{10} = F_D \left(\frac{X_D}{X_i}\right)^{1/a} \quad \text{where } a = 3$$
$$= \underset{F_D = n_d F_e}{F_D} \left(\frac{9.6}{.2214}\right)^{\frac{1}{3}} = 3.513\ F_D = C_{10}$$

Iteration 1

Say $F_e = F_a + F_r = .4916$ kN

$$C_{10} = (3.513)(1.1)(.4916) = 1.899 \text{ kN}$$

$C_{10} = 1.899$ kN

Looking @ Formula sheet: (02 Deep Groove)

↳ ($C_{10} = 1.727$ kN)

$C_{10} = 5.07$ kN

$C_0 = 2.24$ kN

10 mm bore

$\frac{F_a}{C_0} = \frac{.0619}{2.24} = .0276$ ($e \in .21$ and $.22$) → $i = 1$

$\frac{F_a}{VF_r} = \frac{.0619}{(1)(.4297)} = .1441$

$F_e = X_1 V F_r + Y_1 F_a$

$F_e = (1)(1)(.4297) + (0)(.0619)$

$F_e = .4297$

$C_{10} = (3.513)(1.1)(.4297)$

$C_{10} = 1.66$ kN

Looking @ formulae sheet:

($C_{10} = 1.66$ kN)

$C_{10} = 5.07$ kN

$C_0 = 2.24$ kN

10 mm bore

$\boxed{\text{Ball Bearing Bore size} \Rightarrow 10 \text{ mm}}$

Ball Bearing C: $R_{cx} = 20.89$ lbs $= .093$ KN
$R_{cy} = 6.273$ lbs $= .028$ KN
$R_{cz} = 2.51$ lbs $= .011$ KN

$$F_R = \sqrt{.028^2 + .011^2} = .030 \text{ KN}$$
$$F_a = .093 \text{ KN}$$
$$L_D = 9.6 \times 10^6 \text{ rot}$$
$$x_D = \frac{L_D}{L_{10}} = \frac{9.6 \times 10^6}{10^6} = 9.6$$
$$x_i = x_o + (\theta + x_o)\left[\ln\left(\frac{1}{R}\right)\right]^{1/b} = .2214$$
$$C_{10} = F_D\left(\frac{x_D}{x_i}\right)^{1/a} = F_D\left(\frac{9.6}{.2214}\right)^{1/3} = 3.513\, F_D = C_{10}$$
$$F_D = n_d F_e$$

Iteration 1

$$F_e = F_a + F_r = .123 \text{ KN}$$
$$F_D = (1.1)(.123) = .135 \text{ KN}$$
$$C_{10} = 3.513(.135) = .476 \text{ KN}$$

Looking @ formula sheet for 02 series:
$\hookrightarrow (C_{10} = .476 \text{ KN})$
$C_{10} = 5.07$ KN
$C_o = 2.24$ KN

$$\frac{F_a}{C_o} = \frac{.093}{2.24} = .0415 \quad (e \in (.22, .24))$$

$e = 2$

$$\frac{F_a}{VF_r} = \frac{.093}{(1)(.030)} = 3.1$$

Y_2

$$\frac{1.85 - Y_2}{.042 - .0415} = \frac{Y_2 - 1.99}{.0415 - .028} \rightarrow .0135(1.85 - Y_2) = 5E\text{-}4(Y_2 - 1.99)$$
$$.025 - .0135 Y_2 = 5E\text{-}4\, Y_2 - 9.95E\text{-}4$$
$$Y_2 = 1.857$$

$$F_e = X_2 V F_r + Y_2 F_a$$
$$= (.56)(1)(.030) + (1.857)(.093)$$
$$\cdot\ F_e = .1895$$

$$C_{10} = (1.1)(.1895)(3.513) = .732 \text{ kN}$$
(still under C_{10} for 10 mm bore)

$$\boxed{\text{Ball Bearing C size} \Rightarrow 10 \text{ mm}}$$

Roller Bearing D

$$R_{Dy} = 29.534 \text{ lbs} = .131 \text{ kN}$$
$$R_{Dz} = -5.38 \text{ lbs} = -.024 \text{ kN}$$

$$F_R = \sqrt{.131^2 + .024^2} = .133 \text{ kN}$$
$$L_D = 9.6 \times 10^6 \text{ rev}$$
$$L_{10} = 10^6 \text{ rev}$$
$$x_D = \frac{L_D}{L_{10}} = 9.6$$
$$x_i = x_0 + (\theta + x_0)\left[\ln\left(\frac{1}{R}\right)\right]^{1/b} = .2214$$
$$C_{10} = F_D\left(\frac{x_D}{x_i}\right)^{1/a} \qquad \text{where } a = 10/3 \text{ (roller)}$$

$$C_{10} = 3.098 F_D$$
$$F_D = n_d \cancel{F_e}\ F_r$$
$$F_D = (1.1)(.133) = .1463 \text{ kN}$$
$$C_{10} = .453 \text{ kN}$$

Looking @ Formulae sheet:

↳ $(C_{10} = .453 \text{ kN})$

$C_{10} = 16.8$ kN

$C_0 = 8.8$ kN

$$\boxed{\text{Roller Bearing D Bore size} \Rightarrow 25 \text{ mm}}$$

Appendix D.2: 1600 RPM Bearing Sizing Calculations

The following pages show the procedure for calculating the necessary bearings sizes for a 1600 RPM (6" workpiece diameter) scenario. Calculations from Appendix C.2 were considered during this procedure.

Studio 10

Saturday, April 27, 2019 5:00 PM

Design life = 200 hrs, with 1600 rev/min

Reliability = 99%

Design Factor = 1.1

Rotating ring = inner

$$x_D = \frac{L_D}{L_{10}} = \frac{200\,\cancel{hrs}\left(60\frac{\cancel{min}}{\cancel{hr}}\right)\left(1600\frac{\cancel{rev}}{\cancel{min}}\right)}{10^6\,\cancel{rev}} = 19.2$$

$$x_i = (\theta - x_0)(\ln(1/R))^{1/b} + x_0$$

$$x_i = (4.459 - 0.02)(\ln(1/0.99))^{1/1.483} + 0.02 = 0.2196$$

For bearing A (roller) $\rightarrow a = 10/3$

$R_{Az} = 8.76\text{ lbs} = 0.03897\text{ kN}$

$R_{Ay} = 5.73\text{ lbs} = 0.0255\text{ kN}$

$$F_r = \sqrt{(R_{Ay})^2 + (R_{Az})^2} = 0.0466\text{ kN}$$

$$F_a = 0\text{ kN}$$

$$F_D = n_d(F_r + F_a) = (1.1)(0.0466) = 0.05126$$

$$C_{10} = F_D\left(\frac{x_D}{x_i}\right)^{1/a} = 0.05126\left(\frac{19.2}{0.2196}\right)^{3/10} = 0.196\text{ kN}$$

$\rightarrow$ bore size = 25 mm (02-series deep-groove)

For bearing B (ball) $\rightarrow a = 3$

$R_{Bx} = 9.28\text{ lbs} = 0.04128\text{ kN}$

$R_{Bx} = 9.28 \text{ lbs} = 0.04128 \text{ kN}$

$R_{By} = 50.3 \text{ lbs} = 0.2239 \text{ kN}$

$R_{Bz} = -11.57 \text{ lbs} = -0.051466 \text{ kN}$

$F_r = \sqrt{(R_{By})^2 + (R_{Bz})^2} = 0.2297 \text{ kN}$

$F_a = R_{Bx} = 0.04128 \text{ kN}$

$F_D = (1.1)(0.2297 + 0.04128) = 0.2981 \text{ kN}$

$C_{10} = (0.2981)\left(\frac{19.2}{0.2196}\right)^{1/3} = 1.323 \text{ kN}$

$\rightarrow$ bore size = 10 mm , $C_0 = 2.24$

$\frac{F_a}{C_0} = \frac{0.04128}{2.24} = 0.0184$

$e = 0.19 + \frac{(0.0184 - 0.014)(0.21 - 0.19)}{(0.021 - 0.014)} = 0.2026$

$\frac{F_a}{VF_r} = \frac{0.04128}{(1)(0.2297)} = 0.1821$ $\leftarrow$ use X_1 & Y_1

$F_e = X_1 V F_r + \cancel{Y_1}^{0} F_a$

$F_e = 0.2297 \text{ kN}$ $\longrightarrow$ bore size = 10mm

For bearing C (ball)

$R_{Cx} = 13.92 \text{ lbs} = 0.061919 \text{ kN}$

$R_{Cy} = 9.09 \text{ lbs} = 0.040434 \text{ kN}$

$R_{Cz} = 3.79 \text{ lbs} = 0.01686 \text{ kN}$

$F_r = \sqrt{(R_{Cy})^2 + (R_{Cz})^2} = 0.04378 \text{ kN}$

$$F_r = \sqrt{(R_{Cy})^2 + (R_{Cz})^2} = 0.04378 \text{ kN}$$

$$F_a = R_{Cx} = 0.061919 \text{ kN}$$

$$F_D = (1.1)(0.04378 + 0.061919) = 0.11627 \text{ kN}$$

$$C_{10} = (0.11627)\left(\frac{19.2}{0.2196}\right)^{1/3} = 0.5160 \text{ kN}$$

$\rightarrow$ bore size = 10 mm , $C_0 = 2.24$

$$\frac{F_a}{C_0} = \frac{0.06192}{2.24} = 0.0276 \approx 0.028$$

$$e = 0.22$$

$$\frac{F_a}{VF_r} = \frac{0.06192}{(1)(0.04378)} = 1.41 > e \leftarrow \text{use } X_2 \ \& \ Y_2$$

$$F_e = (0.56)(1)(0.04378) + (1.99)(0.061919) = 0.1478$$

$\rightarrow$ bore size = 10 mm

For bearing D (roller)

$$R_{Dy} = 5.48 \text{ lbs} = 0.024376 \text{ kN}$$

$$R_{Dz} = 12.623 \text{ lbs} = 0.05615 \text{ kN}$$

$$F_r = \sqrt{(R_{Dy})^2 + (R_{Dz})^2} = 0.0612 \text{ kN}$$

$$C_{10} = (1.1)(0.0612)\left(\frac{19.2}{0.2196}\right)^{3/10} = 0.2574 \text{ kN}$$

$\rightarrow$ bore size = 25 mm

Appendix D.3: 2400 RPM Bearing Sizing Calculations

The following pages show the procedure for calculating the necessary bearings sizes for a 2400 RPM (4" workpiece diameter) scenario. Calculations from Appendix C.3 were considered during this procedure.

Studio 10

Assumptions

Design life = 200 hours , 2400 rev/min

Design reliability = 99%.

Design factor = 1.1

Rotating ring = inner.

Ball Bearing (B)

$R_y = 17.25 \text{ lbs} = 0.077 \text{ KN}$ $\quad N = 2400$ rev/min

$R_z = -4.96 \text{ lbs} = 0.022 \text{ KN}$ $\quad L = 200$ hours.

$R_x = 9.41 \text{ lbs.} = 0.042 \text{ KN}.$

$$F_r = \sqrt{0.077^2 + 0.022^2} = 0.08 \text{ KN}.$$

$F_a = 0.042 \text{KN}.$

$$L_D = (200 \times 60)(2400) = 28.8 \times 10^6 \text{ rev}.$$

$$x_D = \frac{L_D}{L_{10}} = \frac{28.8 \times 10^6}{1 \times 10^6} = 28.8$$

$$x_i = (\theta - x_0)\left[\ln\left(1/R\right)\right]^{1/b} + x_0$$

$$= (4.459 - 0.02)\left[\ln\left(1/0.99\right)\right]^{1/1.483} + 0.02$$

$$= 0.22$$

$$C_{10} = F_D\left(\frac{x_D}{x_i}\right)^{1/a} = F_D\left(\frac{28.8}{0.22}\right)^{1/3} = 5.1\, F_D$$

$C_{10} = 5.1 F_d$

$F_D = \cancel{n_d} F_e$ ← 1.1

$C_{10} = (5.1)(1.1) F_e$

$$\boxed{C_{10} = 5.61\ F_e}$$

Iteration 1

Assume $F_e = F_a + F_r = 0.042 + 0.08 = 0.122$ kN.

$C_{10} = (5.61)(0.122) = 0.68$ kN.

From Table
02 Series Deep Groove
Bore size = 10mm
$C_{10} = 5.07$
$C_o = 2.24$.

$$\frac{F_a}{C_o} = \frac{0.042}{2.24} = 0.01875$$

$\Rightarrow \boxed{e = 0.203}$
from interpolation.

$$\frac{F_a}{VF_r} = \frac{0.042}{(1)(0.08)} = 0.525$$

↳ lies to the right of e.

$F_e = X_2 V F_r + Y_2 F_a$

$Y_2 = 2.202$
from interpolation

$F_e = (0.56)(0.08) + (2.202)(0.042)$

$$\boxed{F_e = 0.137\text{ kN}}$$

Iteration 2

$F_e = 0.137$ kN

$C_{10} = (5.61)(0.137) = 0.77$ kN.

Smallest bore size for ball bearing B →

From Table
Bore size = 10mm
$C_{10} = 5.07$
$C_o = 2.24$

→ The bore size has converged hence no more iterations required.

Roller Bearing A

$R_{Ay} = 6.55$ lbs $= 0.029$ KN

$R_{Az} = 9.29$ lbs $= 0.041$ KN

$F_r = 0.05$ KN

$L_D = 28.8 \times 10^6$ rev.

$x_D = 28.8$.

$x_i = 0.22$

$$C_{10} = \underbrace{(n_d F_e)}_{F_r} \left(\frac{x_D}{x_i}\right)^{1/a} = (1.1)(0.05)\left(\frac{28.8}{0.22}\right)^{0.3}$$

$$\boxed{C_{10} = 0.24 \text{ KN}}$$

From Table

02 series

Bore size $= 25$ mm

$C_{10} = 16.8$

$C_0 = 8.8$

$\rightarrow$ We select 02 series Roller bearing of bore size 25mm

Ball Bearing C

$R_y = -3.4\ lbs = 0.015\ kN$

$R_z = -1.05\ lbs = -0.047\ kN$

$R_x = 14.11\ lbs = 0.063\ kN.$

$F_r = \sqrt{0.015^2 + 0.047^2} = 0.05\ kN$

$F_a = 0.063\ kN.$

$L_D = 28.8 \times 10^6$ rev $\quad x_D = 28.8$

$x_i = 0.22$

$C_{10} = 5.61\ Fe$

Iteration 1

Assume $F_e = F_a + F_r = 0.063 + 0.05 = 0.113\ kN$

$C_{10} = (5.61)(0.113) = 0.633$

From Table

02 series Deep Groove

Bore size = 10mm

$C_{10} = 5.07$

$C_0 = 2.24$

$\frac{F_a}{C_0} = \frac{0.063}{2.24} = 0.028$

$\Rightarrow \boxed{e = 0.22}$

$\frac{F_a}{VF_r} = \frac{0.063}{0.05} = 1.26$

↳ lies to the right of e.

$F_e = X_2 VF_r + Y_2 F_a$

$F_e = (0.56)(0.05) + (1.99)(0.063)$

$\boxed{F_e = 0.153\ kN}$

Iteration 2

$F_e = 0.153\,kN$

$C_{10} = (0.153)(5.61) = 0.86\,kN.$

Smallest bore size for ball bearing C. →

From Table
Bore size = 10 mm
$C_{10} = 0.86\,kN$
$C_0 = 2.24$

→ The bore size has converged hence no more iterations required.

Roller Bearing D

$R_{Dy} = 3.4\,lbs = 0.015\,kN$ $\quad L_D = 28.8 \times 10^6\,rev$

$R_{Dz} = -12.72\,lbs = 0.057\,kN$ $\quad X_D = 28.8$

$x_i = 0.22$

$F_r = F_e = 0.059\,kN.$

$$C_{10} = (n_d F_e)\left(\frac{X_D}{X_i}\right)^{1/a} = (1.1)(0.059)\left(\frac{28.8}{0.22}\right)^{0.3}$$

$$\boxed{C_{10} = 0.28\,kN}$$

From Table
02 series
Bore size = 25 mm
$C_{10} = 16.8$
$C_0 = 8.8$

→ We select 02 series Roller bearing of bore size 25 mm.

Appendix D.4: 3200 RPM Bearing Sizing Calculations

The following pages show the procedure for calculating the necessary bearings sizes for a 3200 RPM (2” workpiece diameter) scenario. Calculations from Appendix C.4 were considered during this procedure.

Group 102.2
HW ID: #9

Bradley Berman
ENME400-0102

Studio #10

Design life = 200 hrs , 3200 RPM
Desired Reliability = 99%
Design Factor = 1.1
Rotating Ring = Inner

$$L_{rev} = 200 \text{ hrs} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{3200 \text{ rev}}{\text{min}}$$

$$L_{rev} = 38.4(10^6) \text{ rev}$$

$$L_{10} = 10^6 \text{ rev}$$

$$x_D = \frac{L_{rev}}{L_{10}} = \frac{38.4(10^6) \text{ rev}}{10^6 \text{ rev}} = 38.4$$

for $R = 0.99$

$$x = \left[\ln\left(\frac{1}{R}\right)^{1/b}\right](\theta - x_0) + x_0 \qquad \theta = 4.459 \quad b = 1.483 \qquad x_0 = 0.02$$

$$x = \left[\ln\left(\frac{1}{0.99}\right)^{1/1.483}\right](4.459 - 0.02) + 0.02$$

$$x = 0.2196$$

Roller Bearing A

$R_{Ax} = 0$ lb $\quad F_a = 0$

$R_{Ay} = 43.28$ lb $\quad F_r = \sqrt{F_y^2 + F_z^2} = \sqrt{(43.28 \text{ lb})^2 + (-3.50 \text{ lb})^2}$

$R_{Az} = -3.50$ lb $\quad F_r = 43.42$ lb

$$C_{10} = F_D\left(\frac{x_D}{x_i}\right)^{1/a} \qquad a = 10/3 \text{ (roller brg)}$$

$$C_{10} = n_d F_r \left(\frac{x_D}{x_i}\right)^{1/a}$$

$$C_{10} = (1.1)(43.42 \text{ lb})\left(\frac{38.4}{0.2196}\right)^{3/10}$$

$$C_{10} = 224.86 \text{ lb} \approx 1 \text{ kN}$$

$\rightarrow$ $\boxed{\text{Choose 02-Series 25 mm Bore Roller Bearing}}$

Studio #10

Bradley Berman
ENME400-0102

Ball Bearing B

$R_{Bx} = 52.224$ lb $F_a = F_x = 52.224$ lb $= 0.2323$ kN
$R_{By} = 16.57$ lb $F_r = \sqrt{F_y^2 + F_z^2} = \sqrt{(16.57)^2 + (5.27)^2}$
$R_{Bz} = 5.27$ lb $F_r = 17.39$ lb $= 0.07735$ kN

$$F_i = C_{10} = F_D\left(\frac{x_D}{x_i}\right)^{1/a} \quad a = 3 \text{ (ball bearing)}$$

$$C_{10} = F_D\left(\frac{38.4}{0.2196}\right)^{1/3}$$

$$C_{10} = 5.592 F_D \qquad F_D = a_f F_e$$
$$F_D = (1.1) F_e$$

$$C_{10} = 5.592(1.1) F_e$$
$$C_{10} = 6.151 F_e$$

Iteration 1: $F_e = F_a + F_r = 52.224 + 17.39$
$F_e = 69.6$ lb
$C_{10} = 6.151 F_e = 6.151(69.6 \text{ lb})$
$C_{10} = 428.18 \text{ lb} \approx 1.905$ kN → Guess 10 mm bore (deep-groove)

$C_0 = 2.24$ kN
$C_{10} = 5.07$ kN

$$\frac{F_a}{C_0} = \frac{0.2323 \text{ kN}}{2.24 \text{ kN}} = 0.1037$$

$$e \in \{0.28, 0.30\}$$

V = 1 (inner)

$$\frac{F_a}{VF_r} = \frac{0.2323 \text{ kN}}{(1)(0.07735 \text{ kN})} = 3.00 > e \Rightarrow i = 2$$

$$X_2 = 0.56$$

Interpolating for Y_2:

$$\frac{0.110 - 0.084}{1.45 - 1.55} = \frac{0.110 - 0.1037}{1.45 - Y_2}$$

$$Y_2 = 1.474$$

Studio #10

Bradley Berman
ENME400-0102

$F_e = X_2 V F_r + Y_2 F_a$

$F_e = (0.56)(1)(0.07735 \text{ kN}) + (1.474)(0.2323 \text{ kN})$

$F_e = 0.386 \text{ kN}$

Iteration 2: $F_e = 0.386 \text{ kN}$

$C_{10} = 6.151 F_e = 6.151(0.386)$

$C_{10} = 2.373 \text{ kN}$

↳ Choose 10 mm bore

Converges ∴ choose

$\boxed{\text{02-Series, 10 mm bore deep-groove ball bearing for bearing B}}$

<u>Ball Bearing C:</u>

$R_{cx} = -42.376 \text{ lb}$ $\quad F_a = F_x = 42.376 \text{ lb} = 0.1885 \text{ kN}$

$R_{cy} = -15.58 \text{ lb}$ $\quad F_r = \sqrt{F_y^2 + F_z^2} = \sqrt{(15.58)^2 + (12.71)^2}$

$R_{cz} = 12.71 \text{ lb}$ $\quad F_r = 20.11 \text{ lb} = 0.0895 \text{ kN}$

From calc. for Ball Bearing B $\rightarrow C_{10} = 6.151 F_e$

Iteration 1: $F_e = F_a + F_r = 42.376 \text{ lb} + 20.11 \text{ lb}$

$F_e = 62.486 \text{ lb}$

$C_{10} = 6.151 F_e = 6.151(62.486)$

$C_{10} = 384.35 \text{ lb} \approx 1.710 \text{ kN}$

↳ Guess 10 mm bore (deep-groove)

↳ $C_0 = 2.24 \text{ kN}$

↳ $C_{10} = 5.07 \text{ kN}$

$\frac{F_a}{C_0} = \frac{0.1885 \text{ kN}}{2.24 \text{ kN}} = 0.0842$

$e = 0.28$

Studio #10

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ENME400-0102

$V = 1$ (inner)

$$\frac{F_a}{VF_r} = \frac{0.1885 \text{ kN}}{(1)(0.0895 \text{ kN})} = 2.107 \rightarrow e \Rightarrow i = 2$$

$$X_2 = 0.56$$
$$Y_2 = 1.55$$

$$F_e = X_2 V F_r + Y_2 F_a$$
$$F_e = 0.56(1)(0.0895 \text{ kN}) + 1.55(0.1885 \text{ kN})$$
$$F_e = 0.342 \text{ kN}$$

Iteration 2: $F_e = 0.342$ kN

$$C_{10} = 6.151 F_e = 6.151(0.342 \text{ kN})$$
$$C_{10} = 2.105 \text{ kN}$$

$\hookrightarrow$ Choose 10 mm bore

Converges ∴ Choose

$\boxed{\text{02-Series, 10 mm bore deep-groove ball bearing for bearing C}}$

Roller Bearing D

$R_{Dx} = 0$ lb
$R_{Dy} = 11.84$ lb
$R_{Dz} = -9.01$ lb

$F_a = 0$ lb
$F_r = \sqrt{F_y^2 + F_z^2} = \sqrt{(11.84)^2 + (-9.01)^2}$
$F_r = 14.878$ lb

$$C_{10} = F_D \left(\frac{X_D}{X_i}\right)^{1/a} \qquad a = 10/3 \text{ (roller brg)}$$

$$C_{10} = n_d F_r \left(\frac{X_D}{X_i}\right)^{1/a}$$

$$C_{10} = (1.1)(14.878 \text{ lb}) \left(\frac{38.4}{0.2196}\right)^{3/10}$$

$$C_{10} = 77.05 \text{ lb} = 0.34272 \text{ kN}$$

$\hookrightarrow$ $\boxed{\text{Choose 02-Series 25 mm bore Roller Brg.}}$

Appendix E: Shaft Design Calculations

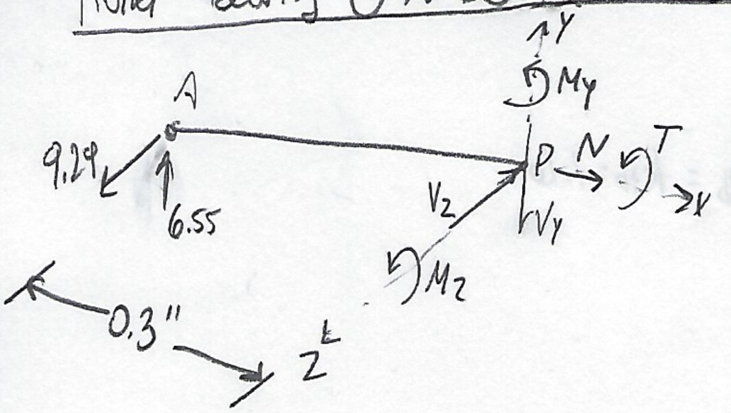
Appendix E.1: Headstock Calculations

The following pages show the procedure for determining the shaft sizes along the headstock as well as the retaining ring. Calculations from Appendix D.3 were used to size the headstock because there were the greatest belt forces during the 2400 RPM application.

1

HEADSTOCK

Roller Bearing @ A [@ 2400 RPM]



$\xrightarrow{+} \sum F_x = 0$

$N = 0$

$\circlearrowleft + \sum M_{px} = 0$

$T = 0$

$\circlearrowleft + \sum M_{P_y} = 0$

$+M_y + 9.29(0.3) = 0$

$M_y = -2.787$ lb-in

*ignoring effects of shear

$\circlearrowleft + \sum M_{P_z} = 0$

$+M_z - 6.55(0.3) = 0$

$M_z = 1.965$ lb-in

$M = \sqrt{M_y^2 + M_z^2}$

$M = \sqrt{(-2.787)^2 + (1.965)^2}$

$M = 3.41$ lb-in $= 3.41(10^{-3})$ kip-in

$\sigma_{xx_A} = 0$ (alt + mean b/c $N=0$)

$\sigma_{xx_{Bm}} = 0$

$\sigma_{xx_{Ba}} = \dfrac{Mr}{I} = \dfrac{3.41(10^{-3})(d/2)}{\frac{\pi}{64} d^4}$

$\sigma_{xx_{Ba}} = \dfrac{0.03473}{d^3}$ ksi

Material Properties

AISI 1020 Steel

$S_{ut} = 57.25$ ksi

$S_{yt} = 42.75$ ksi

Iteration 1:

$k_a = a S_{ut}^b \longrightarrow \left.\begin{matrix} a = 2.70 \\ b = -0.265 \end{matrix}\right\}$ machined

$k_a = 2.7(57.25)^{-0.265}$

$k_a = 0.9237$

Initial guess $d = 0.9843$ in

$k_b = 0.879(0.9843)^{-0.107}$

$k_b = 0.8805$

$k_c = 1$ for VM

$k_d = k_e = k_f = 1$

$S_e' = 0.5 S_{ut}$

$S_e = k_a k_b k_c k_d k_e k_f S_e'$

$S_e = (0.9237)(0.8805)(1)(1)(1)(1)(0.5 \cdot 57.25)$

$S_e = 23.28$ ksi

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2

Assume $q = 1$, $q_s = 1$

when $r/d = 0.1$, $D/d = 1.2$

not needed because only bending exists
$K_{ta} = 1.7$
$K_{tb} = 1.6$
$K_{ts} = 1.35$

$K_{fb} = K_{tb} = 1.6$

Using Goodman's equation:

$$\frac{\sigma_{VMa}}{S_e} + \frac{\sigma_{VMm}}{S_{ut}} = \frac{1}{n}$$

$$\frac{\sqrt{\left(\frac{\cancel{K_{fa}\sigma_{xxAa}}}{\cancel{0.85}} + \frac{K_{fb}\sigma_{xxBa}}{1}\right)^2 + 3\left(\cancel{K_{fs}\tau_{xya}}\right)^2}}{S_e} + \frac{\sqrt{\left(\cancel{K_{fa}\sigma_{xxAm}} + \cancel{K_{fB}\sigma_{xxBm}}\right)^2 + 3\left(\cancel{K_{fs}\tau_{xym}}\right)^2}}{k_d S_{ut}} = \frac{1}{n}$$

$$\frac{K_{fb}\sigma_{xxBa}}{S_e} + 0 = \frac{1}{n}$$

$$\frac{1.6\left(\frac{0.03473}{d^3}\right)}{23.28} = \frac{1}{1.25}$$

$$d = 0.144 \text{ in}$$

Second Iteration:

$k_b = 0.879(0.144)^{-0.107}$

$k_b = 1.082$

$S_e = \frac{23.28}{0.8805} \cdot 1.082$

$S_e = 28.60$ ksi

$\frac{D}{d} = 1.2$

$\frac{r}{d} = 0.1 \rightarrow r = 0.0144$ in

$q = 0.47$ $q_s = 0.5$

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③

$k_{fa} = 1 + q(k_{ta} - 1) = 1 + 0.47(1.7-1) = 1.329$

$k_{fb} = 1 + q(k_{tb} - 1) = 1 + 0.47(1.6-1) = 1.282$

$k_{fs} = 1 + q_s(k_{ts} - 1) = 1 + 0.5(1.35-1) = 1.175$

From Goodman's (simplified)

$$\frac{k_{fb}\,\sigma_{xxBa}}{S_e} = \frac{1}{n}$$

$$\frac{1.282\left(\frac{0.03473}{d^3}\right)}{28.6} = \frac{1}{1.25}$$

$$d = 0.124 \text{ in}$$

Third Iteration:

$k_b = 0.879(0.124)^{-0.107}$

$k_b = 1.098$

$S_e = \frac{28.60}{1.082} \cdot 1.098$

$S_e = 29.03$ ksi

$\frac{D}{d} = 1.2$

$\frac{r}{d} = 0.1 \rightarrow r = 0.0124$ in

$q = 0.46 \qquad q_s = 0.49$

k_{fa} + k_{fs} are not needed

$k_{fb} = 1 + q(k_{tb} - 1) = 1 + 0.46(1.6-1)$

$k_{fb} = 1.276$

From Goodman's

$$\frac{k_{fb}\,\sigma_{xxBa}}{S_e} = \frac{1}{n}$$

$$\frac{1.276\left(\frac{0.03473}{d^3}\right)}{29.03} = \frac{1}{1.25}$$

$$\boxed{d = 0.124 \text{ in}}$$

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(4)

Since $d <$ roller bearing bore size $(0.124" < 0.9843")$, must step up the shaft size to the bore size

$$\boxed{d \text{ @ roller bearing} = 0.9843 \text{ in}}$$

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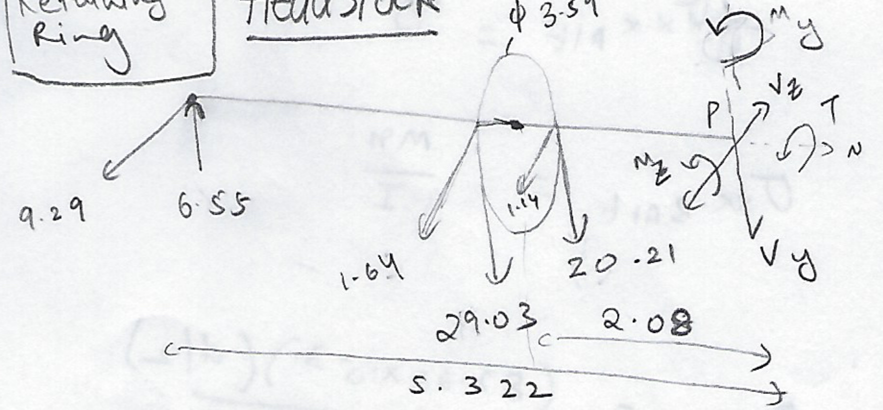
(5) Retaining Ring

Headstock



$N = 0$

$(+\uparrow)\ \Sigma F_y = 0$

$6.55 - 29.03 - 20.21 - V_y = 0$

$\Rightarrow V_y = -42.69$ lb

$(+\nearrow)\ \Sigma F_z = 0$

$-9.29 - 1.64 - 1.14 + V_z = 0$

$\Rightarrow V_z = 12.07$ lb.

$+\circlearrowleft\ \Sigma M_x = 0$

$\Rightarrow 29.03\left(\frac{3.59}{2}\right) - 20.21\left(\frac{3.59}{2}\right) + T = 0$

$\Rightarrow T = -15.83$ lb-in.

Calcs for 2400 RPM

Neglecting shear for our calculation

$+\circlearrowleft\ \Sigma M_y = 0$

$\Rightarrow +9.29 \times 5.322 + (1.64 + 1.14)(2.08) + M_y = 0$

$\Rightarrow M_y = -55.22$ lb-in.

$+\circlearrowleft\ \Sigma M_z = 0$

$-6.55 \times 5.322 + (20.21 + 29.03)(2.08) + M_z = 0$

$\Rightarrow M_z = -67.56$ lb-in.

$M = \sqrt{M_y^2 + M_z^2} = 87.25$ lb-in.

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⑥ $\sigma_{xx_{Am}} = 0$ $\sigma_{xx_{Alt}} = 0$

$\sigma_{xx_{Bm}} = 0$ $\sigma_{xx_{BAlt}} = \frac{Mr}{I}$

$$\sigma_{xx_{Balt}} = \frac{(87.25 \times 10^{-3})(d/2)}{\frac{\pi}{64}(d)^4}$$

$\tau_{xy_m} = \frac{Tr}{J}$ $\sigma_{xx_{Balt}} = \frac{0.889}{d^3}$

$\tau_{xy_m} = \frac{-15.83 \times 10^{-3} \times d/2}{\frac{\pi}{32} d^4}$ $\tau_{xy_{alt}} = 0$

$\tau_{xy_m} = -\frac{0.0806}{d^3}$

From Goodman's

$$\frac{\sigma_{vma}}{S_e} + \frac{\sigma_{vm_m}}{S_{ut}} = \frac{1}{n}$$

Using

AISI 1020 steel
$S_{ut} = 57249$ psi
$S_{yt} = 42748$ psi

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(7)

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S_e'$$

$$S_e' = 0.5\ S_{ut}$$

$$S_e' = 0.5\ (57249) = 28625\ psi$$
$$= 28.62\ ksi$$

$$k_a = a\ S_{ut}^{b}$$ [Assuming Machined]

$a = 2.7$ ksi

$b = -0.265$

$$k_a = (2.7)(57.25)^{-0.265} = 0.92$$

$$k_b = 0.879 d^{-0.107} \longrightarrow$$ [Assume d as bore size
d = 0.98 in]

$$k_b = 0.88$$

$$k_c = 1$$ [Von Mises]

$$k_d = k_e = k_f = 1$$

$$S_e = (0.92)(0.88)\ (1)\ (1)\ (1)\ (1)\ (28.62)$$

$$S_e = 23.28\ KSI$$

Assume

$\frac{a}{t} = 1$, $\frac{r}{t} = 0.1$

$K_{ta} = K_{tb} = 5.5$

$K_{ts} = 3.2$

Set $q = 1$

$K_{fa} = 1.7$

$K_{fb} = 1.6$

$K_{fs} = 1.35$

$$\frac{\sqrt{\left(\frac{\cancel{K_{fa}\ \sigma_{xxAa}}}{0.85} + \frac{K_{tb}\ \sigma_{xxBa}}{1}\right)^2 + 3(\cancel{K_{ts}\ \tau_{xya}})^2}}{S_e} + \frac{\sqrt{(\cancel{K_{fa}\sigma_{xxam}} + \cancel{K_{fb}\sigma_{xxBm}})^2 + 3(\cancel{K_{fs}\tau_{xym}})^2}}{K_d\ S_{ut}} = \frac{1}{n}$$

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⑧

$$\frac{K_{t_b}\,\sigma_{xxBa}}{S_e} + \frac{\sqrt{3}\,K_{ts}\,\tau_{aym}}{K_d\,S_{ut}} = \frac{1}{n}.$$

$$\Rightarrow \frac{(5.5)(0.889)}{d^3(23.28)} + \frac{\sqrt{3}\,(3.2)(-0.0806)}{d^3(57.25)(1)} = \frac{1}{1.25}$$

$$\Rightarrow \frac{0.21}{d^3} - \frac{7.8\times10^{-3}}{d^3} = 0.8$$

$$\Rightarrow \frac{0.2022}{d^3} = 0.8$$

$$d = 0.63''$$

Second iteration

$$K_b = 0.879\,(0.63)^{-0.107} = 0.923$$

$$S_e = \frac{23.28}{0.88} \times 0.923 = 24.41\ \text{ksi}$$

Assume

$S_{y\,string} = 40$ ksi and $n_{string} = 1.5$

$F_{a\,max} = f_{usen\,x}$
$= 4.705$ lbs

Using distortion energy theorem

$$a = \frac{\left(\frac{F_{A\,maximum}}{\pi D}\right)}{\left(\frac{\frac{S_{y\,string}}{\sqrt{3}}}{n_{string}}\right)} = \frac{\left(\frac{4.705\times10^{-3}}{\pi(0.63)}\right)}{\frac{\left(\frac{40}{\sqrt{3}}\right)}{1.5}} = \frac{2.37\times10^{-3}}{15.396}$$

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(9)

$a = 1.53 \times 10^{-4}$ m

$\frac{a}{t} = 1 \Rightarrow a = t$

$\frac{r}{t} = 0.1$

$r = 1.53 \times 10^{-5}$

$q = 0.2$ $\quad q_s = 0.2$

$k_{fa} = 1 + q(k_{ta} - 1) = 1 + 0.2(5.5 - 1) = 1.9$"

$k_{fb} = 1 + q(k_{tb} - 1) = 1 + 0.2(5.5 - 1) = 1.9$

$k_{fs} = 1 + q(k_{ts} - 1) = 1 + 0.2(3.2 - 1) = 1.44$

$$\frac{(1.9)(0.889)}{d^3(23.28)} + \frac{\sqrt{3}(1.44)(-0.08)}{d^3(57.25)} = \frac{1}{1.25}$$

$$\frac{0.069}{d^3} = \frac{1}{1.25}$$

$d = 0.44$"

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(10)

Third <u>Iteration</u>

$$k_b = 0.879(0.44)^{-0.107} = 0.959$$

$$S_e = \frac{23.28}{0.88} \times 0.959 = 25.38$$

$$a = \frac{\frac{4.705 \times 10^{-3}}{\pi(0.44)}}{\frac{\frac{40}{\sqrt{3}}}{1.5}} = \frac{3.40 \times 10^{-3}}{15.396} = 2.20 \times 10^{-4}$$

$$\frac{a}{t} = 1 \qquad \frac{r}{t} = 0.1 \Rightarrow 2.20 \times 10^{-5} = r.$$

$$q = q_s = 0.2$$

$$K_{fa} = 1.9 \quad K_{fb} = 1.9 \quad K_{fs} = 1.44$$

$$\frac{(1.9)(0.889)}{d^3(23.28)} + \frac{\sqrt{3}(3.2)(-0.0806)}{d^3(57.25)(1)} = \frac{1}{1.25}$$

$$d = 0.44'' \rightarrow \text{converges.}$$

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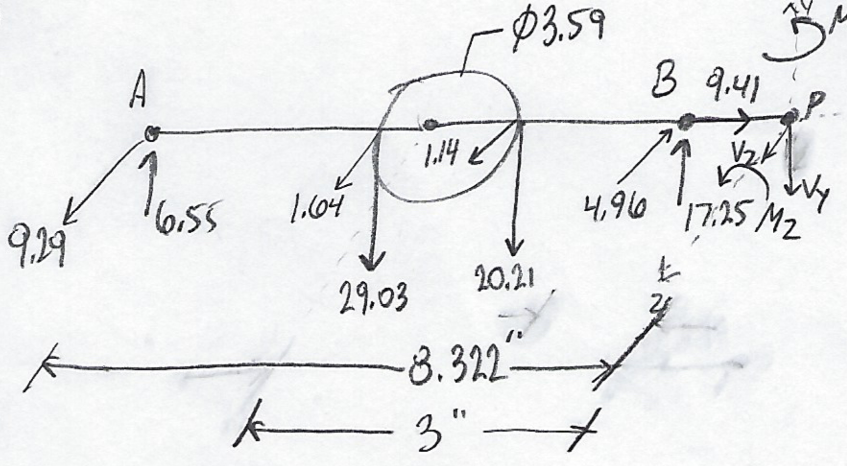

* Neglecting effects due to shear

$$\xrightarrow{+} \sum F_x = 0$$

$$N + 9.41 = 0$$

$$N = -9.41 \text{ lbs}$$

$$\circlearrowleft + \sum M_x = 0$$

$$29.03\left(\frac{3.59}{2}\right) - 20.21\left(\frac{3.59}{2}\right) + T = 0$$

$$T = -15.83 \text{ lb-in}$$

$$\circlearrowleft + \sum M_y = 0$$

$$9.29(8.322) + (1.64 + 1.14)(3) + M_y = 0$$

$$M_y = -85.65 \text{ lb-in}$$

$$\circlearrowleft + \sum M_z = 0$$

$$-6.55(8.322) + (29.03 + 20.21)(3) + M_z = 0$$

$$M_z = -93.21 \text{ lb-in}$$

$$M = \sqrt{M_y^2 + M_z^2} = 126.59 \text{ lb-in}$$

$$\sigma_{xx_{Am}} = \frac{N}{A} = \frac{-9.41(10^{-3})}{\frac{\pi}{4}d^2} = \frac{-11.98(10^{-3})}{d^2} \text{ ksi}$$

$$\sigma_{xx_{Aalt}} = 0 \text{ ksi}$$

$$\sigma_{xx_{Bm}} = 0 \text{ ksi}$$

$$\sigma_{xx_{Balt}} = \frac{Mr}{I} = \frac{126.59(10^{-3})(d/2)}{\frac{\pi}{64}d^4}$$

$$\sigma_{xx_{Balt}} = \frac{1.29}{d^3}$$

$$\tau_{xy_m} = \frac{Tr}{J} = \frac{-15.83(10^{-3})(d/2)}{\frac{\pi}{32}d^4}$$

$$\tau_{xy_m} = \frac{-0.0806}{d^3}$$

$$\tau_{xy_{alt}} = 0 \text{ ksi}$$

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(12)

$$S_e = k_a k_b k_c k_d k_e k_f \cdot S_e'$$

$$S_e' = 0.5 S_{ut} = 0.5(57249)$$

$$S_e' = 28625 \text{ psi} = 28.62 \text{ ksi}$$

$$k_a = a\, S_{ut}^{\,b}$$

$a = 27$ ksi [Assuming machined]

$b = -0.265$

$$k_a = 2.7(57.25)^{-0.265} = 0.92$$

$$k_b = 0.879 d^{-0.107} \longrightarrow \text{[Assume } d \text{ as bore size]}$$

$d = 0.394$ in

$$k_b = 0.879(0.394)^{-0.107}$$

$$k_b = 0.97$$

$k_c = 1$ [Von mises)

$k_d = k_e = k_f = 1$

$$S_e = (0.92)(0.97)(1)(1)(1)(1)(28.62)$$

$$S_e = 25.57 \text{ ksi}$$

Set $q = 1$

$k_{fa} = k_{ta} = 1.7$

$k_{fb} = k_{tb} = 1.6$

$k_{fs} = k_{ts} = 1.35$

for $\frac{D}{d} = 1.2$, $\frac{r}{d} = 0.1$

$k_{fa} = 1.7$

$k_{ts} = 1.35$

$k_{tb} = 1.6$

$$\frac{\sqrt{\left(\frac{k_{fa}\cancel{\sigma_{xxA_a}}}{\cancel{0.85}} + \frac{k_{fb}\sigma_{xxB_a}}{1}\right)^2 + 3\left(\cancel{k_{fs}\tau_{xy_a}}\right)^2}}{S_e} + \frac{\sqrt{\left(k_{fa}\sigma_{xxa_m} + \cancel{k_{fb}\sigma_{xxB_m}}\right)^2 + 3\left(k_{fs}\tau_{xy_m}\right)^2}}{k_d S_{ut}} = \frac{1}{n}$$

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(13)

$$\frac{k_{Fb}\sigma_{xxBa}}{S_e} + \frac{\sqrt{(k_{Fa}\sigma_{xxam})^2 + 3(k_{fs}\tau_{xym})^2}}{k_d S_{ut}} = \frac{1}{n}$$

$$\frac{1.6(1.29)}{(d^3)(25.57)} + \frac{\sqrt{((1.7\cdot(-11.98)(10^{-3}))/d^2)^2 + 3\left(1.35\cdot\frac{-0.0806}{d^3}\right)^2}}{(1)(57.25)} = \frac{1}{1.25}$$

$$\frac{0.0807}{d^3} + \frac{\sqrt{\left(\frac{4.15(10^{-4})}{d^4}\right) + \left(\frac{0.0355}{d^6}\right)}}{57.25} = 0.8$$

$d = 0.472$ in

Second Iteration

$k_b = 0.879(0.472)^{-0.107}$

$k_b = 0.953$

$S_e = \frac{25.57}{0.97} \cdot 0.953$

$S_e = 25.11$ ksi

$\frac{r}{d} = 0.1 \Rightarrow r = 0.0472$

From graph

$q = 0.67$ $q_s = 0.67$

$k_{Fa} = 1 + q(k_{ta} - 1) = 1 + 0.67(1.7-1)$

$k_{Fa} = 1.469$

$k_{Fb} = 1 + q(k_{tb} - 1) = 1 + 0.67(1.6-1)$

$k_{Fb} = 1.402$

$k_{Fs} = 1 + q_s(k_{ts} - 1) = 1 + 0.67(1.35-1)$

$k_{Fs} = 1.23$

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From Goodman's:

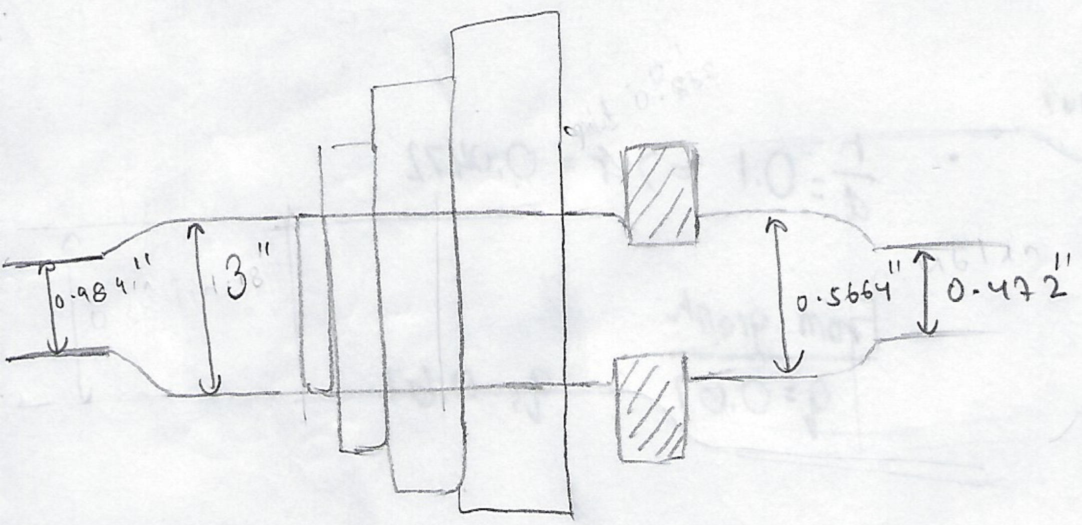
$$\frac{1.402(1.29)}{d^3(25.11)} + \frac{\sqrt{\left(\frac{(1.402)(-11.98(10^{-3}))}{d^2}\right)^2 + 3\left(1.23 \cdot \frac{(-0.0806)}{d^3}\right)^2}}{(1)(57.25)} = 0.8$$

$$\frac{0.072}{d^3} + \frac{\sqrt{\frac{2.821(10^{-4})}{d^4} + \frac{0.0295}{d^6}}}{57.25} = 0.8$$

$d = 0.454$ in

Since $d >$ ball bearing bore size, must increase to next bore size ($12\ mm = 0.472\ in$)

$$\boxed{\text{Therefore, } d \text{ @ ball bearing} = 0.472 \text{ in}}$$



The figure is not drawn to scale.

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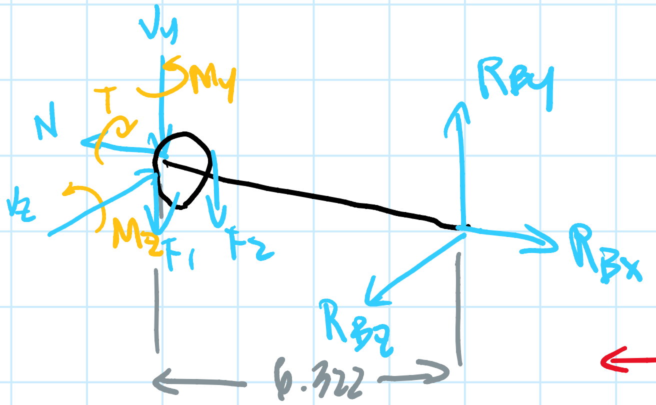
Studio



$R_{Bx} = 9.41$ lbs

$R_{By} = 17.25$ lbs

$R_{Bz} = 4.96$ lbs

$F_{belt} = 49.24$ lb

← From left of smallest pulley to bearing B

$\Sigma F_x = 0$

$N = R_{Bx} = 9.41$ lbs

$\Sigma F_y = 0$

$V_y = R_{By} + F_{belt} = 66.49$ lb

$\Sigma F_z = 0$

$V_z = R_{Bz} = 4.96$ lbs

$\Sigma M_x = 0$

$T + F_2(3.59) - F_1(3.59)$

$T = -31.66$ lb-in

$\Sigma M_y = 0$

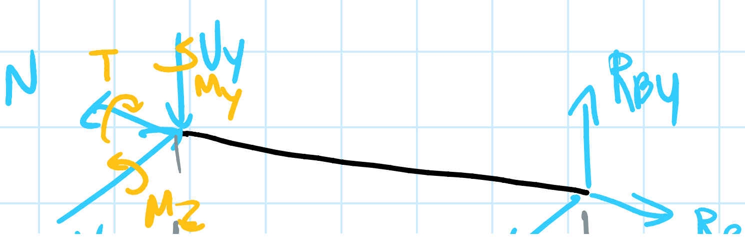
$-R_{Bz}(6\text{-}322) + M_y = 0$

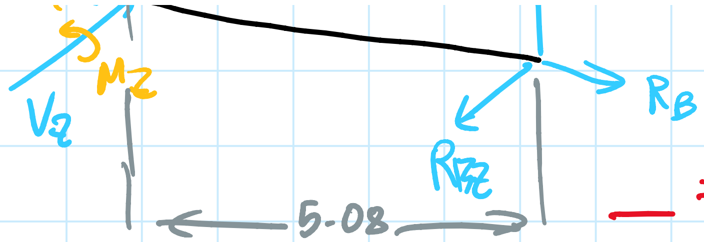
$M_y = (6\text{-}222)(4.96) = 31.2$ lb·in

$\Sigma M_z = 0$

$M_z + R_{By}(6\text{-}322) = 0$

$M_z = -109.05$ lb·in





— from center of 2nd pulley to bearing B

$\Sigma F_x = 0$

$N = R_{Bx}$

$\Sigma F_y = 0$

$V_y = R_{By} + F_{belt}$

$\Sigma F_z = 0$

$V_z = R_{Bz}$

$\Sigma M_x = 0$

$T = -31.66 \text{ lb} \cdot \text{in}$

$\Sigma M_y = 0$

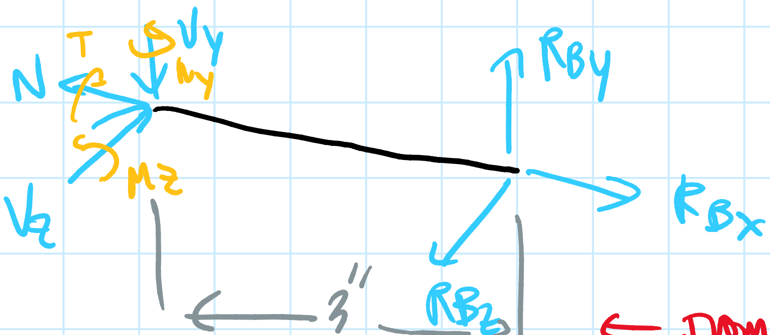
$M_y - R_{Bz}(5.08) = 0$

$M_y = 25.2 \text{ lb} \cdot \text{in}$

$\Sigma M_z = 0$

$M_z + R_{By}(5.08) = 0$

$M_z = 87.63 \text{ lb} \cdot \text{in}$



← from right of last pulley to bearing B

$\Sigma F_x = 0$

$N = R_{Bx}$

$\Sigma F_x = 0$

$N = R_{Bx}$

$\Sigma F_y = 0$

$V_y = R_{By} + F_{belt}$

$\Sigma F_z = 0$

$V_z = R_{Bz}$

$\Sigma M_x = 0$

$T = -31.66 \text{ lb in}$

$\Sigma M_y = 0$

$M_y - R_{Bz}(3) = 0$

$M_y = 14.88 \text{ lb} \cdot \text{in}$

$\Sigma M_z = 0$

$M_z + R_{By}(3) = 0$

$M_z = -51.75 \text{ lb} \cdot \text{in}$

Appendix E.2: Tailstock Calculations

The following pages show the procedure for the shaft sizes along the tailstock. Calculations from Appendix D.3 were used to size the tailstock because there were the greatest belt forces during the 2400 RPM application.

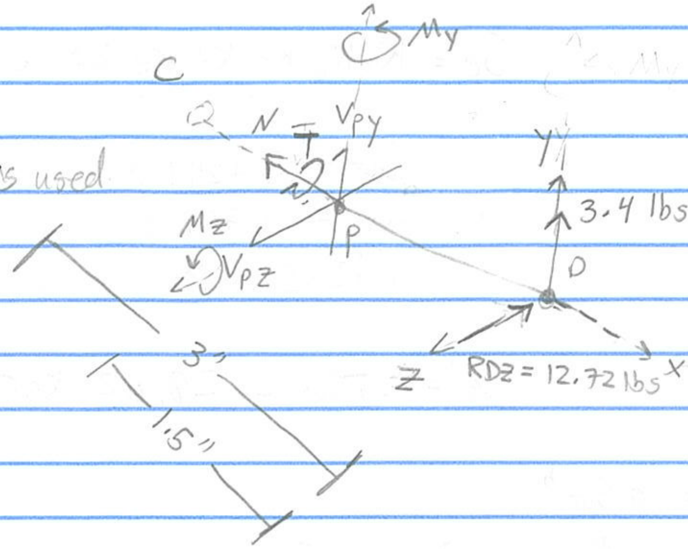
Stillwell

Tail stack (Before Bearing C)

Studio 11

Roller bearing D
Ball bearing C
2400 RPM reactions used

• Looking @ midpoint of C and D.



$\overset{+}{\rightarrow} \Sigma F_x = 0$ $\boxed{N = 0}$

$\overset{+}{\uparrow} \Sigma F_y = 0$ $\boxed{V_{py} = -3.4 \text{ lbs}}$

$\overset{+}{\swarrow} \Sigma F_z = 0$ $\boxed{V_{pz} = 12.72 \text{ lbs}}$

$\overset{+}{\circlearrowleft} \Sigma M_x = 0$ $\boxed{T = 0}$

$\overset{+}{\circlearrowleft} \Sigma M_{y_p} = 0$

$M_y = -(1.5)(12.72)$

$\boxed{M_y = -19.08 \text{ in-lbs}}$

$\overset{+}{\circlearrowleft} \Sigma M_{pz} = 0$

$M_z = -(1.5)(3.4)$

$\boxed{M_z = -5.1 \text{ in-lbs}}$

$M = \sqrt{M_y^2 + M_z^2}$

$= \sqrt{19.08^2 + 5.1^2}$

$\boxed{M = 19.75 \text{ in-lb}}$

Since no Torque $\Rightarrow \tau_{Alt} = \tau_{mean} = 0$

Since no $N \Rightarrow \sigma_{xx\,A\,alternating} = \sigma_{xx\,A\,mean} = 0$

$\sigma_{xx\,B\,mean} = 0$

$\sigma_{xx\,B\,alternation} = \dfrac{Mr}{I} = \dfrac{(19.75 \times 10^{-3})(d/2)}{\frac{\pi}{64} d^4} = \dfrac{.201}{d^3}$

$$S_e = K_a K_b K_c K_d K_e K_f S_e'$$

$S_{UT} = 57249$ psi (57.25 Kpsi)
$S_{YT} = 42748$ psi
(AISI 1020 steel)

$S_e' = .5 S_{UT} = 28.62$ ksi (28624 psi)

$K_a = a S_{UT}^{\,b}$
Machined → $a = 2.70$
$b = -.265$
$K_a = 2.70(57.249)^{-.265} = \boxed{.92 = K_a}$

$K_b = .879\, d^{-.107}$
assume $d = .394''$ (bore size) for now
$K_b = .879(.394)^{-.107}$
$\boxed{K_b = .97}$ (will change w/ iterations)

$K_c = 1$ (VM stress)
$K_D = K_e = K_f = 1$

$$S_e = (.92)(.97)(1)(1)(1)(1) = 25.57 \text{ ksi}$$

If we wish $D/d = 1.25$ and $r/d = .1$, then:
$K_{Ta} \approx 1.7$
$K_{Ts} \approx 1.35$
$K_{TB} \approx 1.6$

When $q = 1$ (which will change w/ iterations)
$K_{fa} = K_{Ta} = 1.7$
$K_{fs} = K_{Ts} = 1.35$
$K_{fB} = K_{TB} = 1.6$

Brg Eqn

$$\frac{\sqrt{\left(\frac{K_{fa}\cancel{\sigma_{xx_{Aa}}}^{0}}{.85}+\frac{K_{fb}\sigma_{xx_{Ba}}}{1}\right)^2+3\left(K_{fs}\cancel{\tau_{xy_a}}^{0}\right)^2}}{S_e}$$

$$+$$

$$\frac{\sqrt{\left(K_{fa}\cancel{\sigma_{xx_{Am}}}^{0}+K_{fb}\cancel{\sigma_{xx_{Bm}}}^{0}\right)^2+3\left(K_{fs}\cancel{\tau_{xy_m}}^{0}\right)^2}}{K_d S_{ut}}=\frac{1}{n}$$

$$\frac{\sqrt{\frac{(1.6)\left(.201/d^3\right)^2}{1}}}{25.57}=\frac{1}{1.25}$$

$$\boxed{d = .398 \text{ inches}}$$

Second iteration

$$K_b = .879\,(.398)^{-.107} = .97006623$$

$$\frac{r}{d} = .1 \Rightarrow r = .0398''$$

d ← $.398''$

$$S_e = \frac{25.57}{.97} \times .97006623 = 25.572$$

From graph ($S_{ut} = 57.25$ Kpsi and $r/d = .0398$)

↳ $q \approx .63$ (for axial / bending)

↓

No q needed for Torsion

K_{fB} is only needed:

$$K_{fB} = 1 + q(K_{TB} - 1)$$
$$= 1 + .63(1.6 - 1)$$
$$= 1.378$$

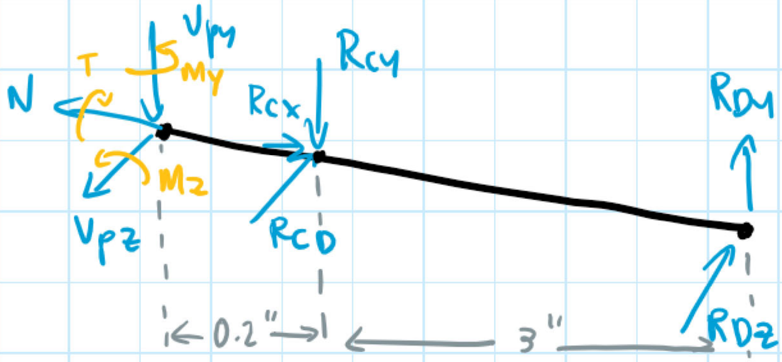
$$\frac{\sqrt{\frac{(1.378)(.201/d^3)^2}{1}}}{25.572} = \frac{1}{1.25}$$

$$\boxed{d = .388''}$$

E. J.

Studio 11

Tuesday, May 7, 2019 10:47 AM



$\Sigma F_x = 0$

$N = R_{cx} = 14.11 \text{ lbs}$

$\Sigma F_y = 0$

$R_{Dy} - R_{cy} - V_{py} = 0$

$V_{py} = 0 \text{ lbs}$

$\Sigma F_z = 0$

$R_{Dy} - R_{cz} - V_{pz} = 0$

$V_{pz} = 11.7 \text{ lbs}$

$\Sigma M_x = 0$

$T = 0$

$\Sigma M_y = 0$

$M_y + R_{Dz}(3.2) - R_{cz}(0.2) = 0$

$M_y = -40.59 \text{ in·lbs}$

$\Sigma M_z = 0$

$M_z + R_{Dy}(3.2) - R_{cy}(.2) = 0$

$M_z = -10.2 \text{ in·lbs}$

$M_a = \sqrt{M_y^2 + M_z^2} = 41.9 \text{ in·lbs}$

$\tau_a = \tau_m = 0$

$$\sigma_{xxAm} = \frac{N}{A} = \frac{14.11 \times 10^{-3}}{\pi/4 \cdot d^2} = \frac{0.018}{d^2} \text{ ksi}$$

$\sigma_{xxAa} = 0 \text{ ksi}$, $\sigma_{xxBm} = 0 \text{ ksi}$

$$\sigma_{xxBa} = \frac{M_a r}{I} = \frac{41.9 \times 10^{-3} \left(\frac{d}{2}\right)}{\pi/64 \cdot d^4} = \frac{0.427}{d^3} \text{ ksi}$$

$S_e = k_a k_b k_c k_d k_e k_f \cdot S_e'$

$S_e' = 0.5 S_{uT} = 0.5(57249) = 28624.5 \text{ psi} = 28.63 \text{ ksi}$

$S_e = k_a k_b k_c k_d k_e k_f \, S_e'$

$S_e' = 0.5 S_{UT} = 0.5(57249) = 28624.5 \text{ psi} = 28.63 \text{ ksi}$

$k_a = a S_{UT}^{\,b}$, $a = 2.7$ (machined), $b = -0.265$

$k_a = (2.7)(57.249)^{-0.265} = 0.924$

$k_b = 0.879 d^{-0.107}$, assuming $d = 0.394''$

$k_b = 0.97$

$k_c = 1$, $k_D = k_e = k_f = 1$

$S_e = (0.924)(0.97)(1)(1)(1)(1)(28.63) = 25.66 \text{ ksi}$

$D/d = 1.25$ & $r/d = 0.1 \rightarrow k_a = 1.7, k_{fb} = 1.6, k_{fs} = 1.35$

$q = 1 \rightarrow k_{fa} = 1.7, k_{fb} = 1.6, k_{fs} = 1.35$

$$\frac{\sqrt{\left(\cancel{\frac{k_{fa}\sigma_{xxAa}}{0.85}}^{\,0} + \frac{k_{fb}\sigma_{xxBa}}{1}\right)^2 + 3\left(\cancel{k_{fs}\tau_{xya}}^{\,0}\right)^2}}{S_e} + \frac{\sqrt{\left(k_{fa}\sigma_{xxam} + \cancel{k_{fb}\sigma_{xxBm}}^{\,0}\right)^2 + 3\left(\cancel{k_{fs}\tau_{xym}}^{\,0}\right)^2}}{k_d S_{UT}} = \frac{1}{n}$$

$$\rightarrow \frac{k_{fb}\sigma_{xxBa}}{S_e} + \frac{k_{fa}\sigma_{xxam}}{k_d S_{UT}} = \frac{1}{n}$$

$$\frac{(1.6)(0.427)}{d^3(25.66)} + \frac{(1.7)(0.018)}{d^2(1)(57.25)} = \frac{1}{1.25} \Rightarrow d = 0.324''$$

Second Iteration:

$r/d = 0.1 \rightarrow r = 0.0324'' \rightarrow q \approx 0.6$

$k_b = 0.879(0.324)^{-0.107}$

$k_b = 0.992$

$S_e = \frac{25.57}{0.97}(0.992) = 26.15 \text{ ksi}$

$k_{fa} = 1 + (0.6)(1.7 - 1) = 1.42$

$k_{fb} = 1 + (0.6)(1.6 - 1) = 1.36$

$K_{fa} = 1 + (0.6)(1.7-1) = 1.42$

$K_{fb} = 1 + (0.6)(1.6-1) = 1.36$

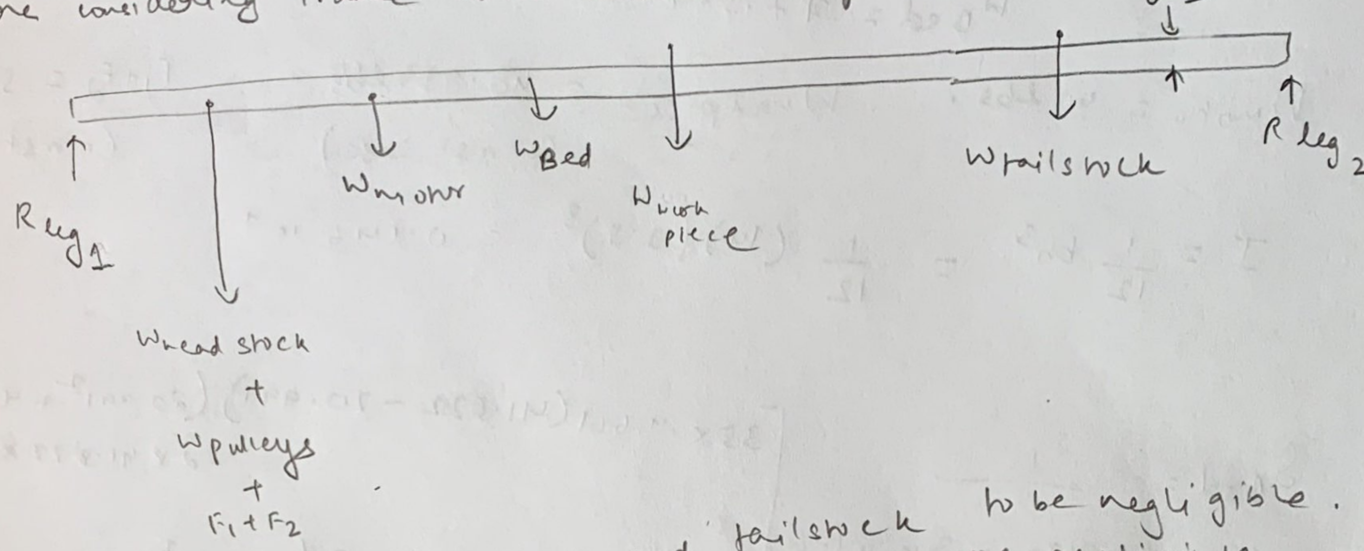
$$\frac{(1.36)(0.427)}{d^3(25.46)} + \frac{(1.42)(0.018)}{1(57.25)} = \frac{1}{1.25} \qquad d = 0.305''$$

Appendix F: Bed Plate Deformation Calculation

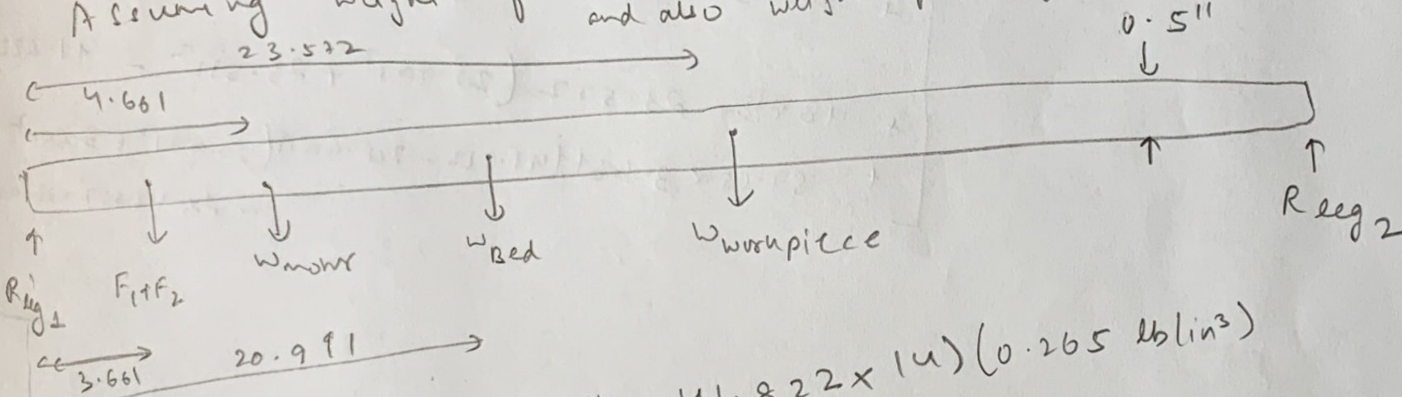
The following pages show the calculations for the bed plate deformation. It was assumed that the bed plate was a solid cast iron material with no cuts.

Bed plate / Frame Calculations

We are considering frame to be made up of cast iron.

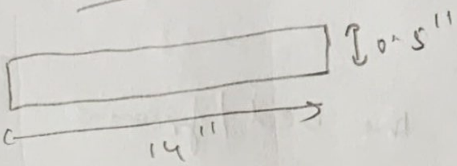


Assuming weight of headstock and tailstock to be negligible. and also weight of pulleys are negligible.



$$W_{Bed} = \rho_{CI} V = (0.5'' \times 41.822 \times 14)(0.265\ lb/in^3)$$
$$= 77.579\ lbs$$

Cross section



$$\delta_{Bed} = \frac{W_{motor}\, a_1(l-x)}{6EIl}(x^2+a_1^2-2lx) + \frac{W_{Bed}\, b_2 x}{6EIl}(x^2+b_2^2-l^2)$$
$$+ \frac{W_{piece}\, b_3 x}{6EIl}(x^2+b_3^2-l^2) + \frac{(F_1+F_2)(a_4)(l-x)}{6EIl}(x^2+a_4^2-2lx)$$

Here $a_1 = 4.661''$, $b_2 = 20.991''$, $b_3 = 23.572''$, $a_4 = 3.661''$

$l = 41.822''$, $x = 20.991''$

For Cast Iron $\rightarrow E = 21900$ ksi

$W_{Bed} = 77.579$ lbs

$W_{motor} = 40$ lbs. $\quad W_{workpiece} = 48.873$ lbs (worst case) $\quad F_1 + F_2 = 57.65$ (worst case)

$$I = \frac{1}{12} bh^3 = \frac{1}{12}(14)(0.5)^3 = 0.146 \text{ in}^4$$

$$\delta_{Bed} = \frac{1}{41.822 \times 6 \times 21900 \times 10^3 \times 0.146}\Big[35 \times 4.661(41.822 - 20.991)(20.991^2 + 4.661^2 - 2 \times 41.822 \times 20.991)$$
$$+ 77.579 \times 20.991^2 (20.991^2 + 20.991^2 - 41.822^2)$$
$$+ 48.873 \times 23.572 (20.991^2 + 23.572^2 - 41.822^2)$$
$$+ 57.65 \times 3.661 (41.822 - 20.991)(20.991^2 + 3.661^2 - 2 \times 41.822 \times 20.991)\Big]$$

$$\delta_{Bed} = \frac{1}{41.822 \times 19.18 \times 10^6}\left[-4.3 \times 10^6 - 29.66 \times 10^6 - 86 \times 10^3 - 5 \times 10^6\right]$$

$$\boxed{\delta_{Bed} = -0.0478''}$$

This deflection won't cause failure of the bed/frame made up of cast iron.

Appendix G: Buckling in Legs Calculation

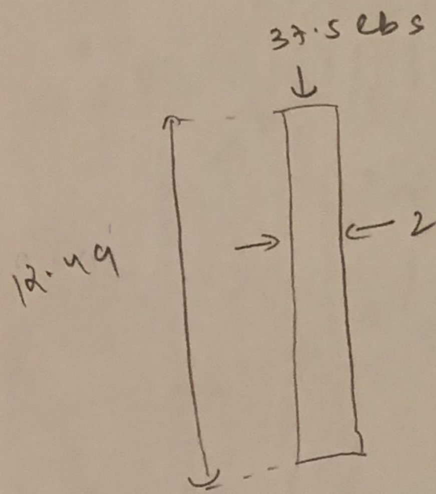
The following pages show the procedure taken when calculating the maximum force needed to buckle the bed plate legs.

Buckling in legs

We assume that the maximum force imposed on 4 legs to be 150 lbs which is an extremely conservative estimate. We are selecting <u>cast iron</u> as material for legs

150 → 4 legs

$$1 \text{ leg} \rightarrow \frac{150}{4} = 37.5 \text{ lbs}.$$



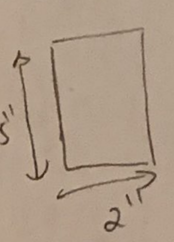
The connection between bed plate and the leg is considered to be pinned/grounded and fixed on ground.

So we have fixed-pinned condition.

Rotation about x

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{1}{12} \times 2 \times 1.5^3}{1.5 \times 2}} = 0.433$$

Cross section.



$$\frac{L}{K} = \frac{12.49}{0.433} = 28.84$$

Rotation about y

$$K = \sqrt{\frac{\frac{1}{12} \times 2^3 \times 1.5}{1.5 \times 2}} = 0.577$$

$$\frac{L}{K} = 21.64$$

$C = 2$ [Fixed Round]

$E = 21900 \text{ksi}$

$S_{yT} = 15250 \text{ psi}$

$$\left(\frac{L}{K}\right)_1 = \left(\frac{2 C \pi^2 E}{S_{yT}}\right)^{1/2}$$

$$\left(\frac{L}{K}\right)_1 = \left(\frac{2 \times 2 \times \pi^2 \times 21900 \times 10^3}{15250}\right)^{1/2}$$

$$\left(\frac{L}{K}\right)_1 = 238.1$$

So for both the cases. $\left(\frac{L}{K}\right)_1 > \left(\frac{L}{K}\right)$

we will use Johnson's Parabola method

The structure will fail first for lower K value, so we will do calculation for $K = 0.433$

$$P_{cr} = \left(S_{yT} - \left(\frac{S_{yT}}{2\pi}\frac{l}{K}\right)^2 \frac{1}{CE}\right)A$$

$$P_{cr} = \left(15250 - \left(\frac{15250}{2\pi} \times 28.84\right)^2 \times \frac{1}{2 \times 21900 \times 10^3}\right) \times 2 \times 1.5$$

$$\boxed{P_{cr} = 45.41 \text{ kips}}$$

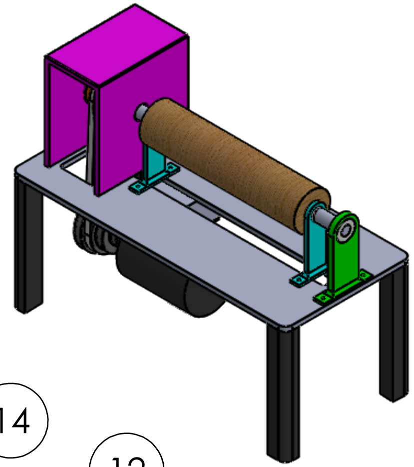
This makes sense as the legs will not buckle for a force of 150 lbs.

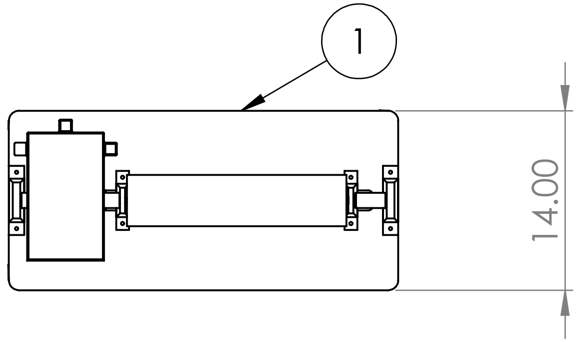
Appendix H: CAD Drawings

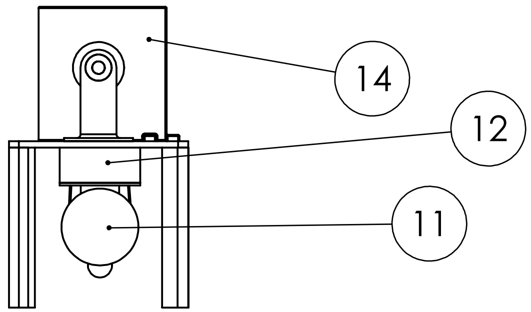
The following pages show the CAD drawings that were created using Solidworks. The drawings are ordered in the following manner:

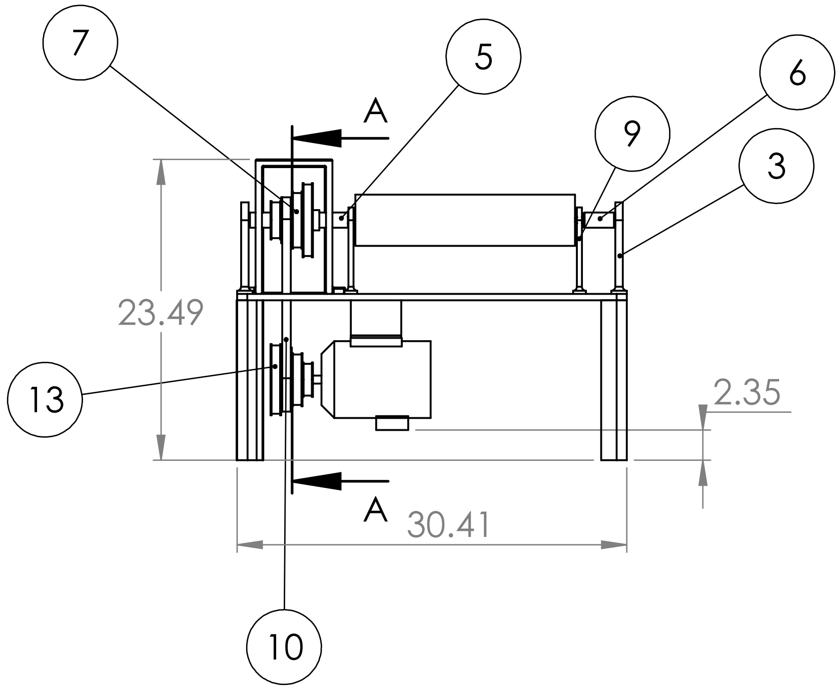
1. Assembly Drawing
2. Parts List
3. Individual Part Drawings

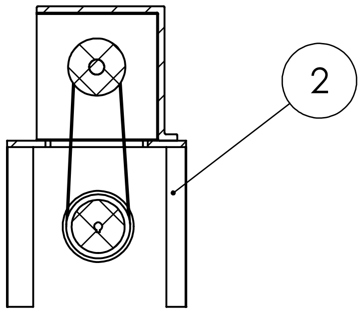
ITEM NO.	PART NUMBER	QTY.
1	Bed Plate	1
2	Leg	4
3	Roller Bearing Housing	2
4	Roller Bearing	2
5	Head Stock	1
6	TailStock Shaft	1
7	Driven_Pulley	1
8	Ball Bearing	2
9	Ball Bearing Housing	2
10	Belt	1
11	Motor	1
12	Motor Extrusion	1
13	Driver_Pulley	1
14	Pulley Housing	1











SECTION A-A
SCALE 1 : 15

Note: All Dimensions in Inches
Bearings (4/8) are hidden within housings

NAME
Ed Stillwell

DWG NO.
Assembly

A4

SCALE:1:15

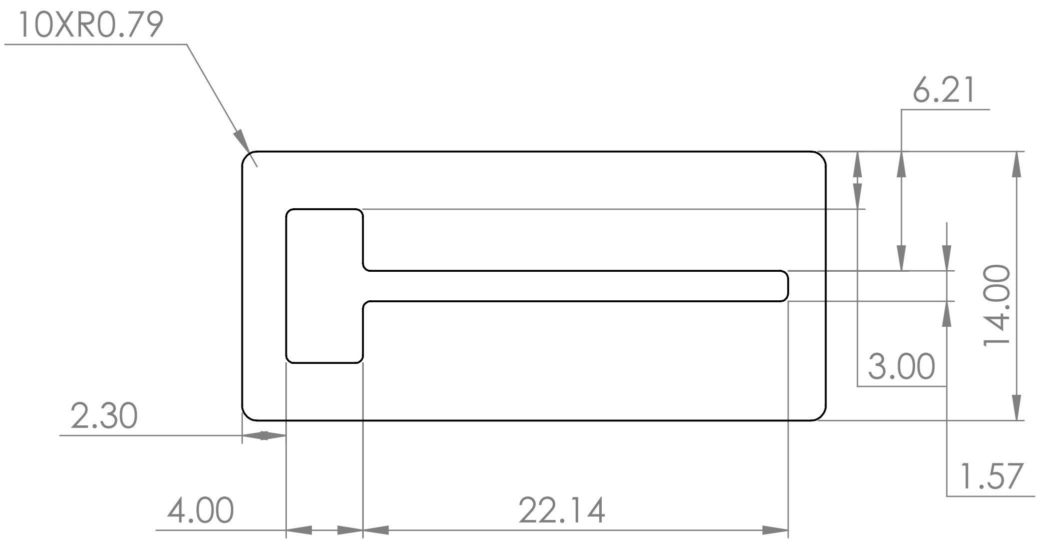
SHEET 1 OF 1

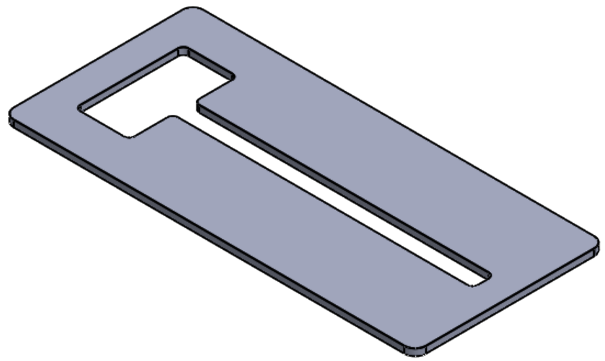
Parts List

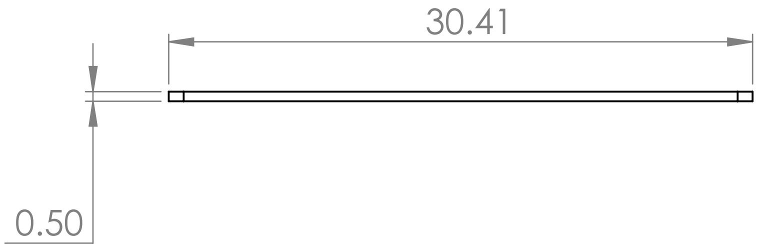
Table 12: Lathe parts list

Item Number	Part Name	Quantity	Material
1	Bed Plate	1	Cast Iron
2	Table Leg	4	Cast Iron
3	Roller Bearing Housing	2	Cast Iron
4	Roller Bearing[1]	2	Fastenal
5	Headstock Shaft	1	AISI 1020 Steel
6	Tailstock Shaft	1	AISI 1020 Steel
7	Driven Pulley	1	HDPE
8	Ball Bearing[2]	2	Fastenal
9	Ball Bearing Housing	2	Cast Iron
10	Belt	1	A-2 Polyamide
11	Motor[3]	1	Grizzly
12	Motor Extrusion	1	Cast Iron
13	Driver Pulley	1	HDPE
14	Pulley Housing	1	HDPE

[1] Roller Bearing
[2] Ball Bearing
[3] Grizzly Motor

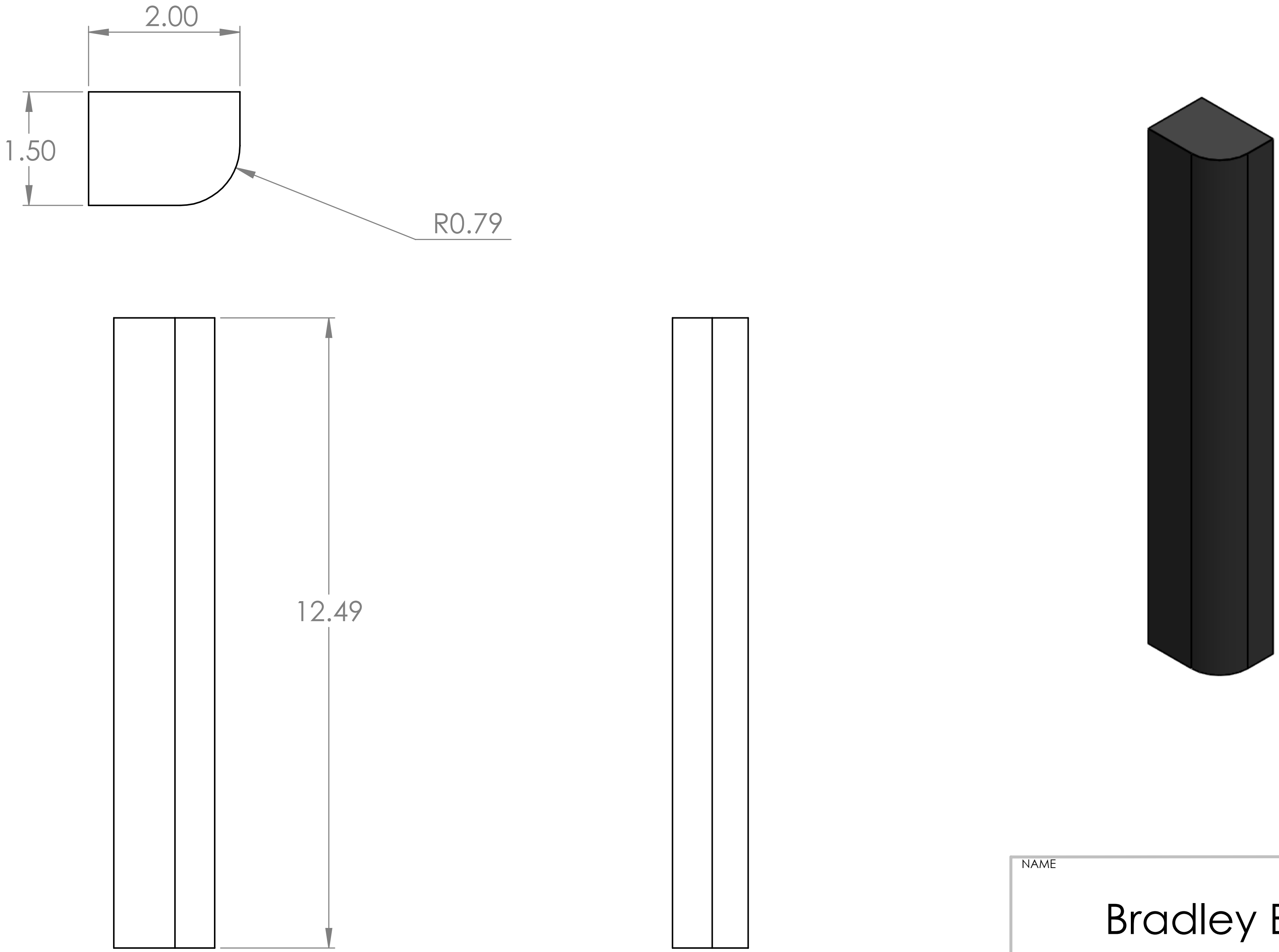






Note: All Dimensions in Inches

NAME	Ed Stillwell
DWG NO.	Bed Plate
	A4
SCALE:1:10	SHEET 1 OF 1



NAME

Bradley Berman

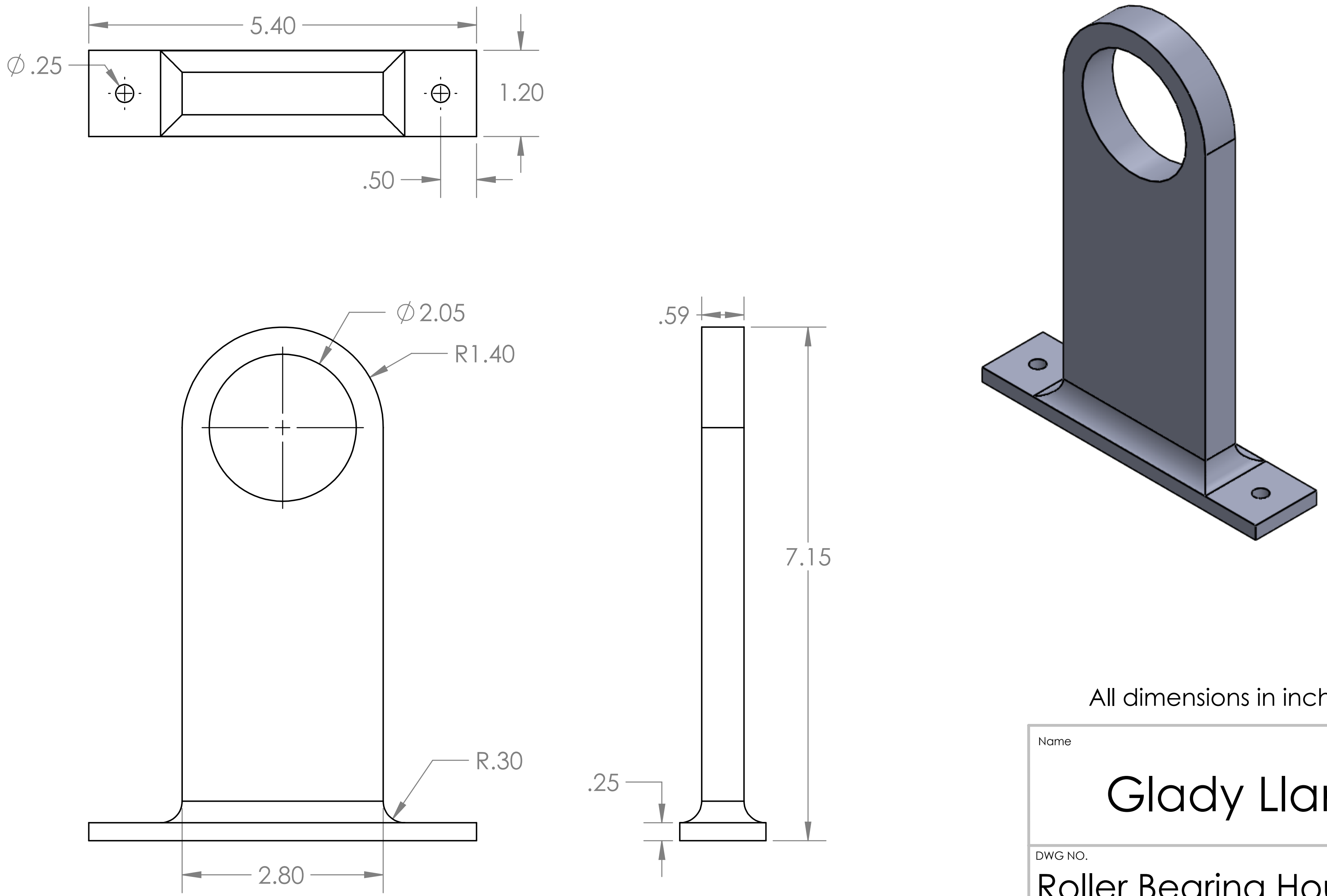
DWG NO.

Leg

A4

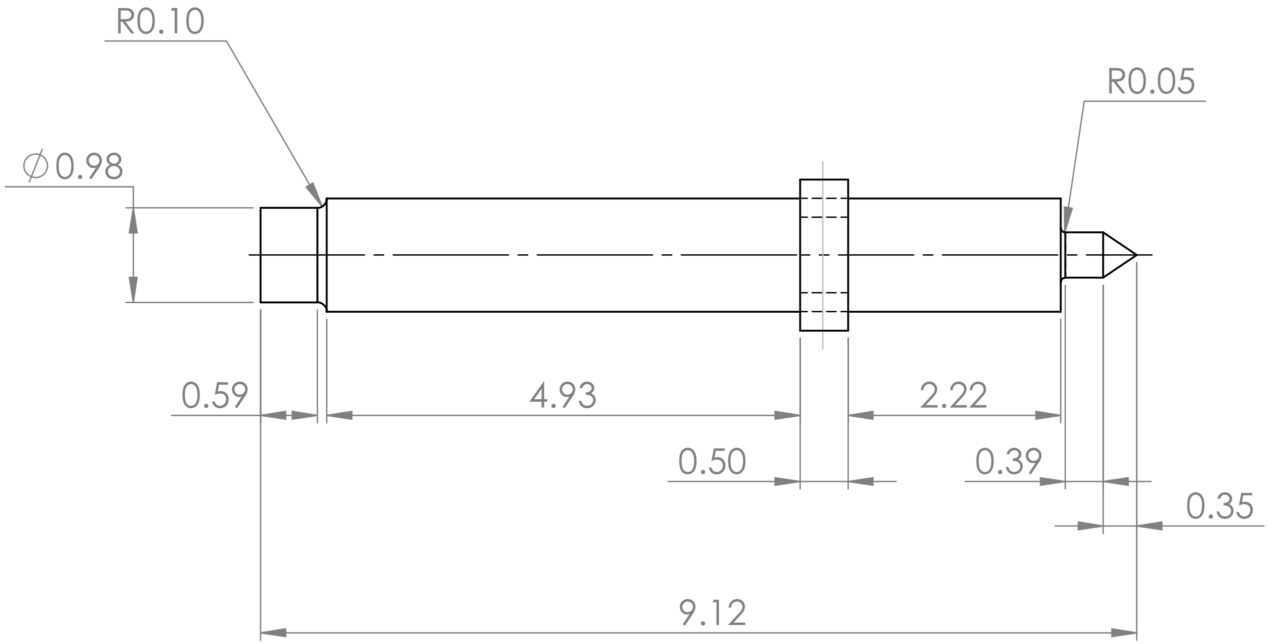
SCALE:1:3

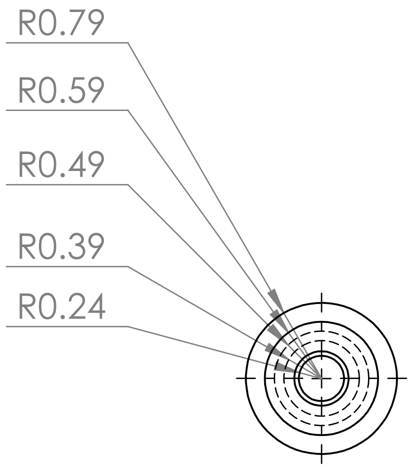
SHEET 1 OF 1

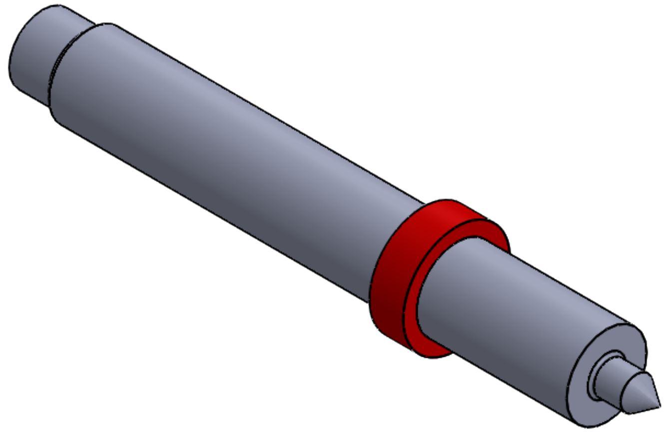


All dimensions in inches

Name Glady Llanto	
DWG NO. Roller Bearing Housing	A4
SCALE:1:2	SHEET 1 OF 1

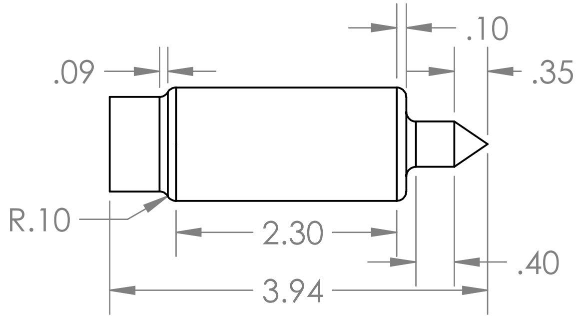


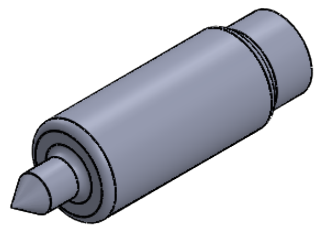


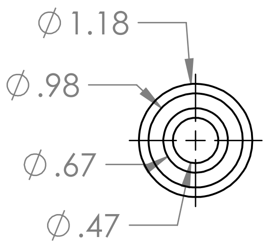


Note: Retaining Ring Included on Drawing
All Dimensions in Inches

NAME	Ed Stillwell
DWG NO.	Headstock
	A4
SCALE:1:2	SHEET 1 OF 1

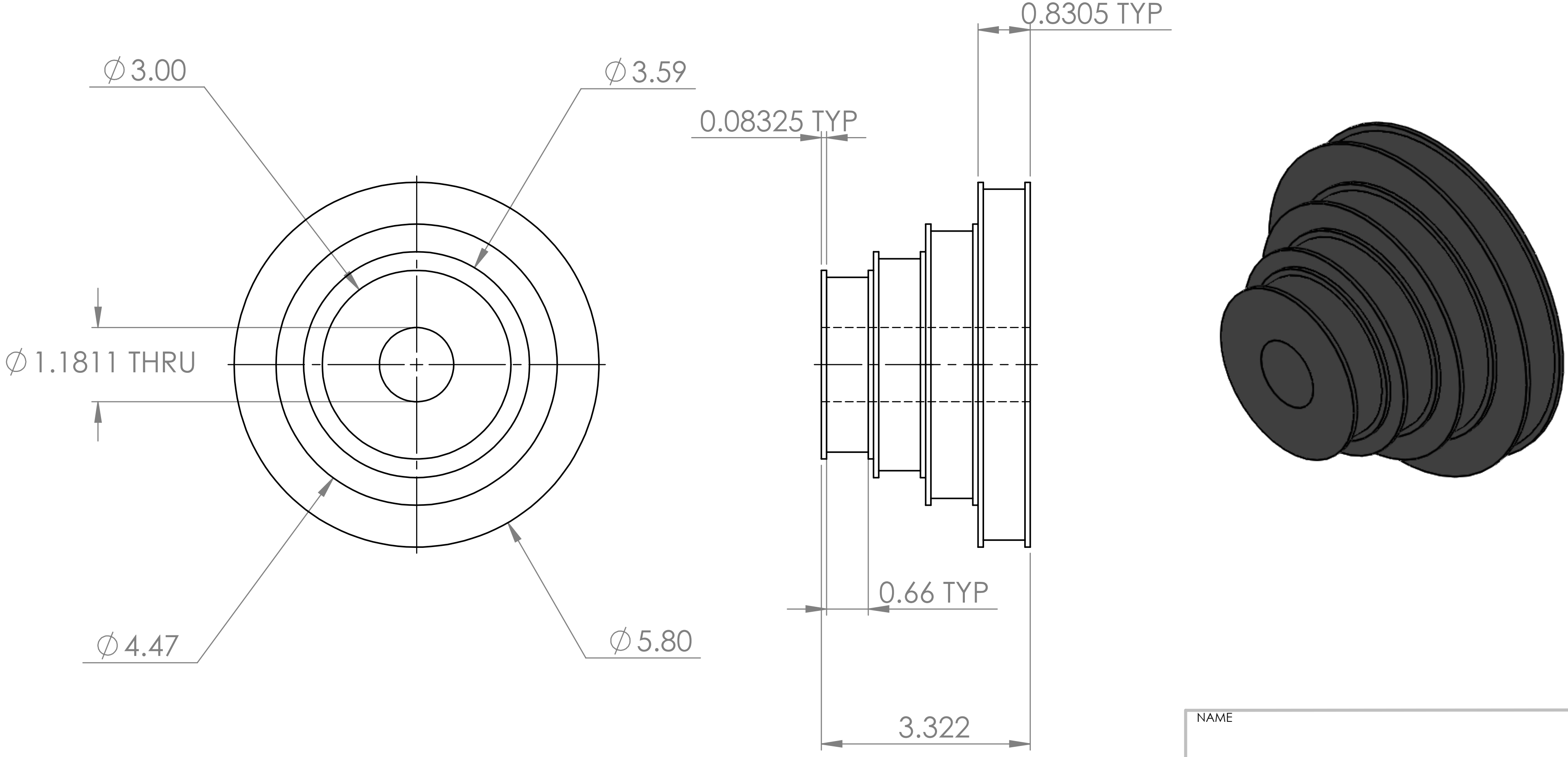






All dimensions in inches

Name Glady Llanto	
DWG NO. TailStock Shaft	A4
SCALE:1:2	SHEET 1 OF 1



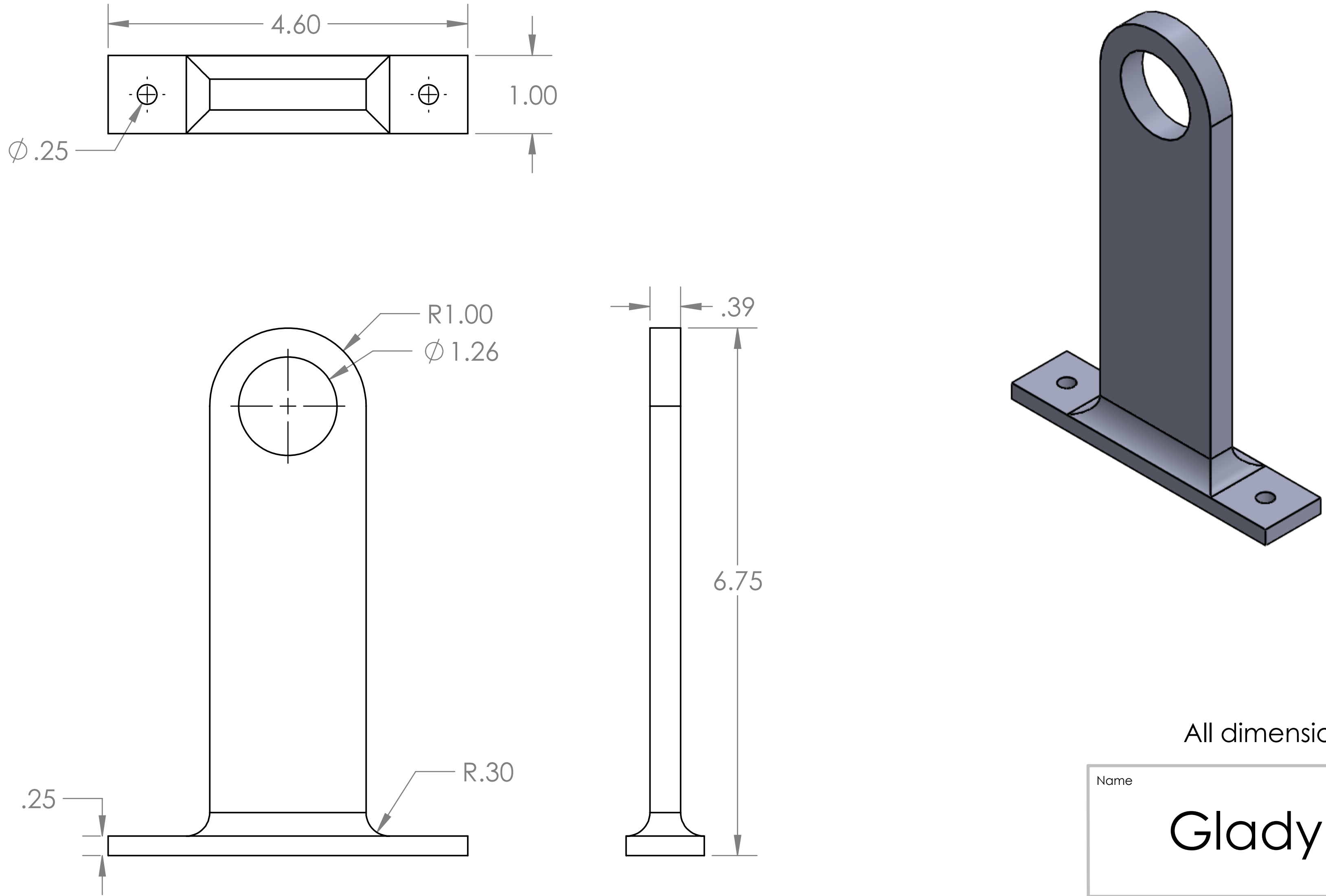
NAME
Bradley Berman

DWG NO.
Driven Pulley

A4

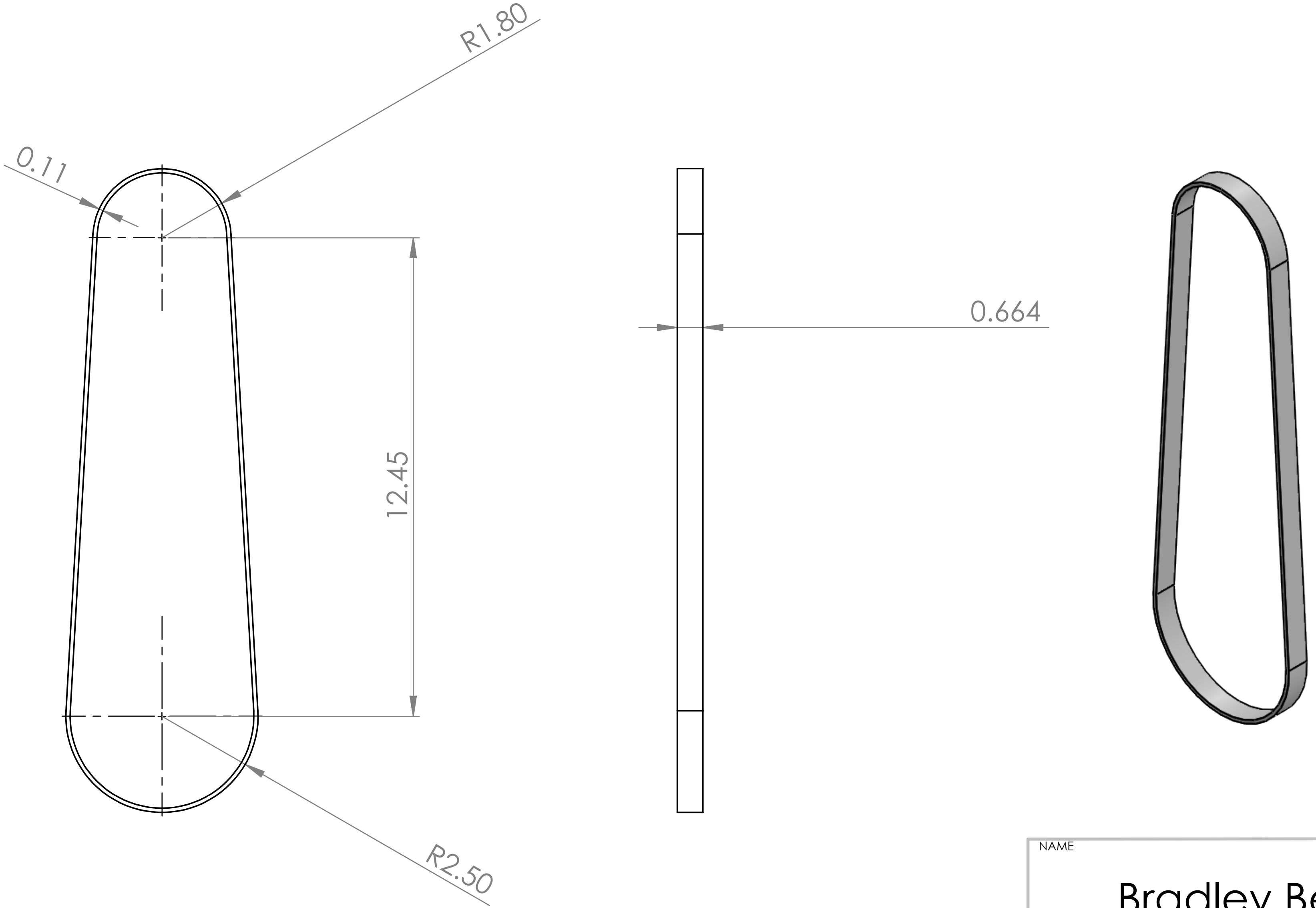
SCALE:1:3

SHEET 1 OF 1

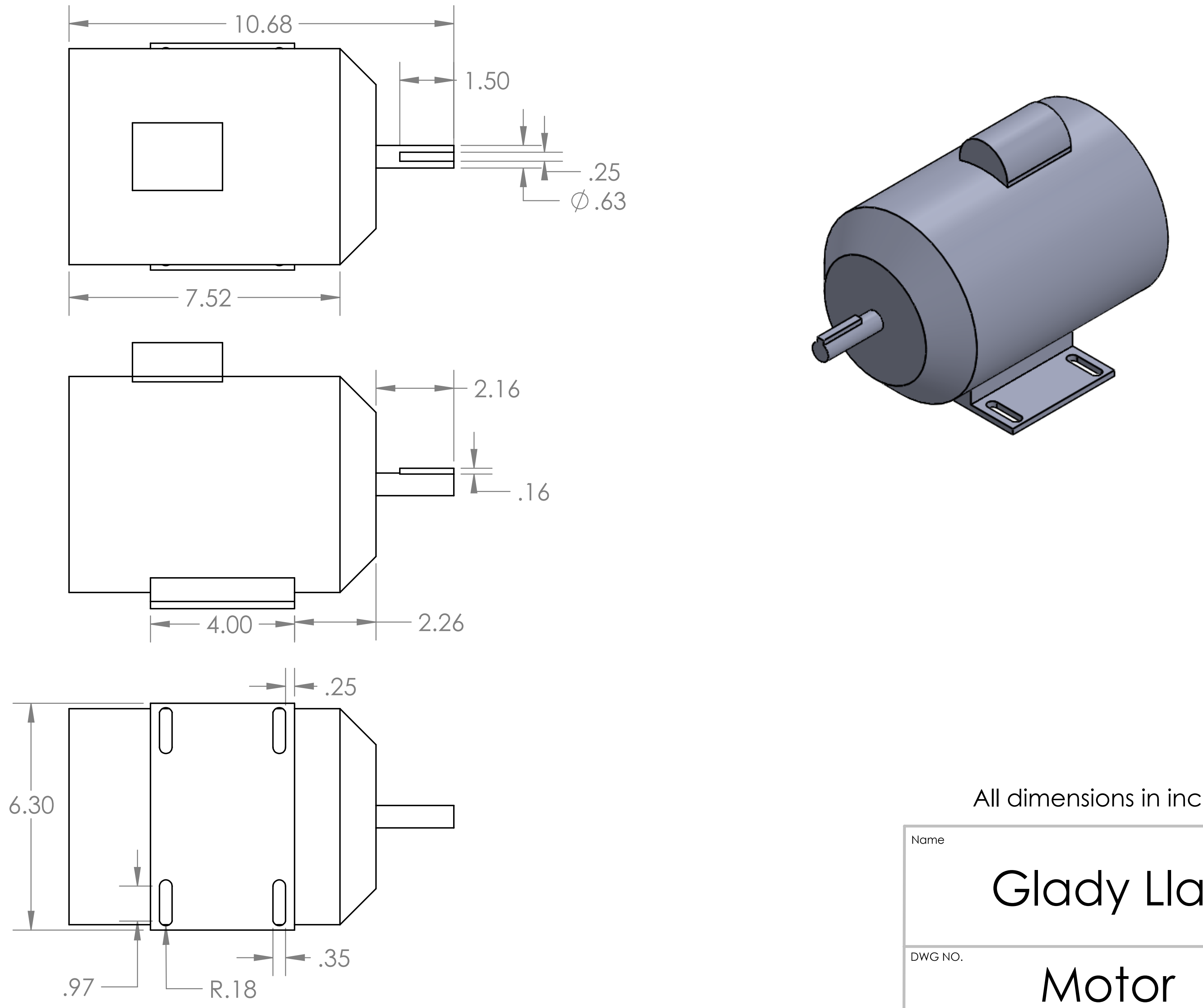


All dimensions in inches

Name	Glady Llanto
DWG NO. Ball Bearing Housing	A4
SCALE:1:2	SHEET 1 OF 1

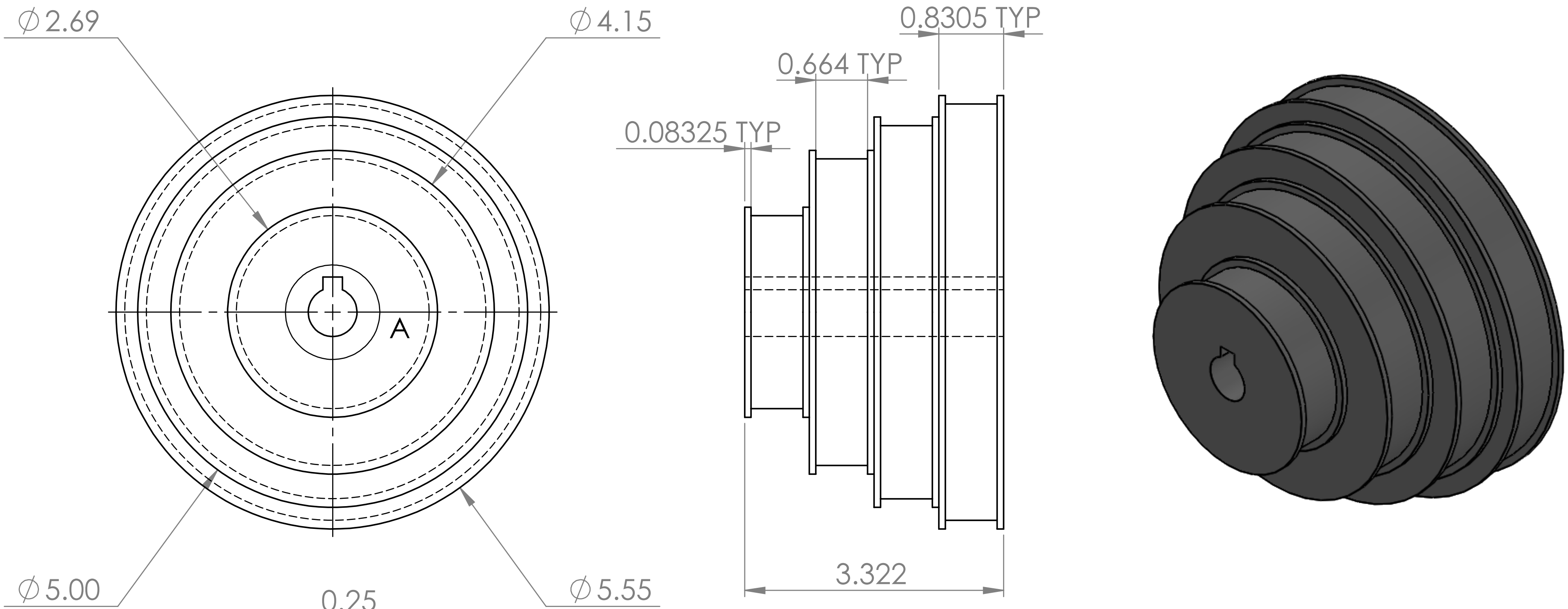


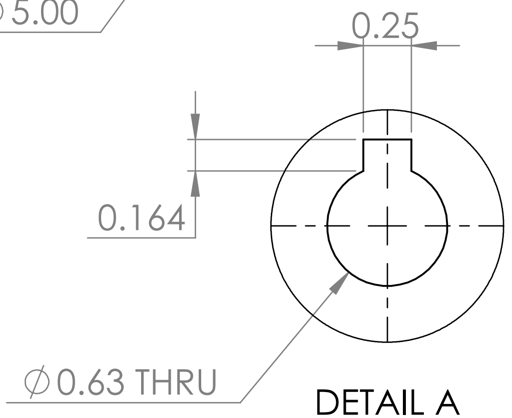
NAME		
Bradley Berman		
DWG NO. Belt		A4
SCALE:1:4	SHEET 1 OF 1	



All dimensions in inches

Name Glady Llanto	
DWG NO. Motor	A4
SCALE:1:4	SHEET 1 OF 1





DETAIL A
SCALE 1 : 1

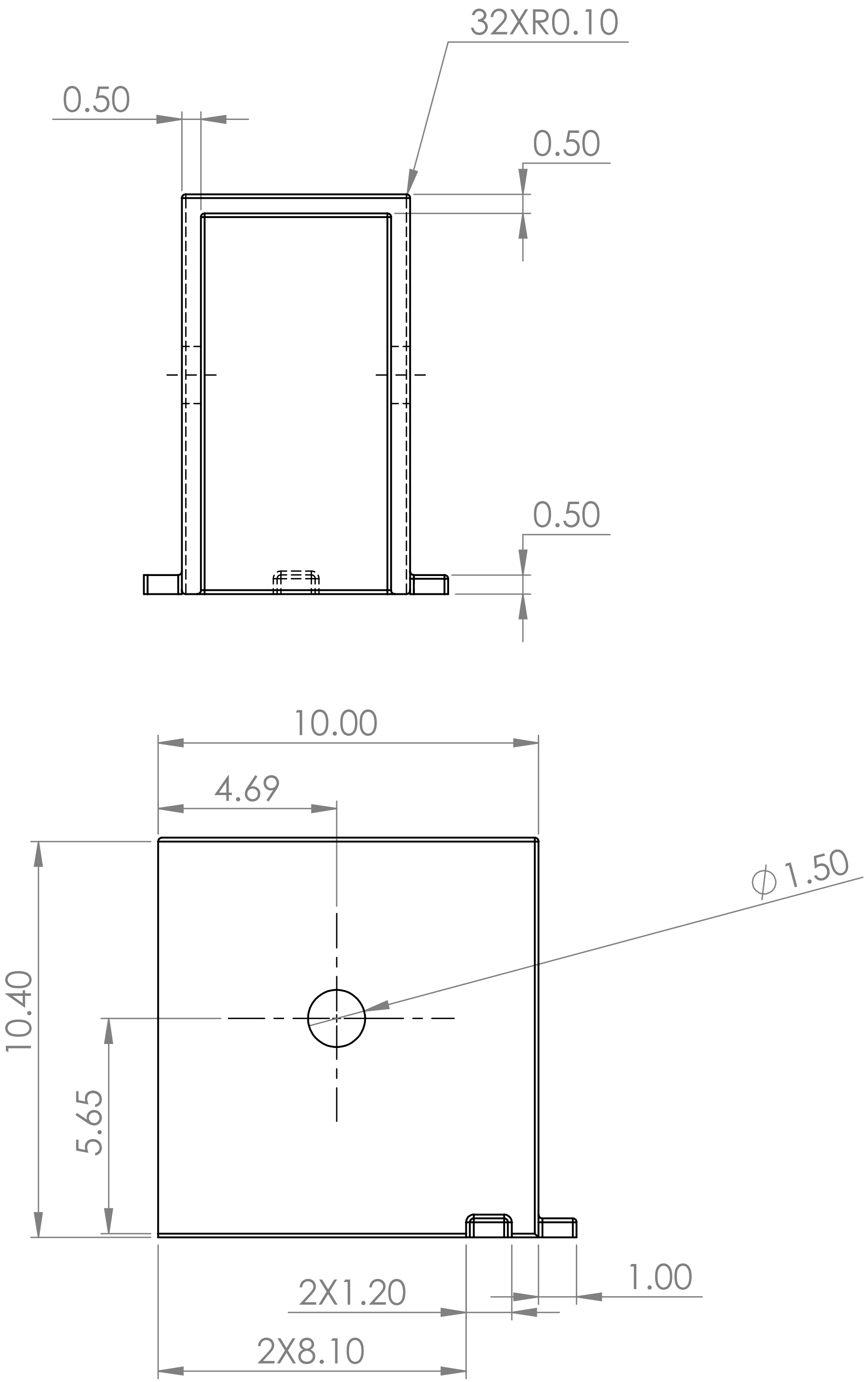
NAME
Bradley Berman

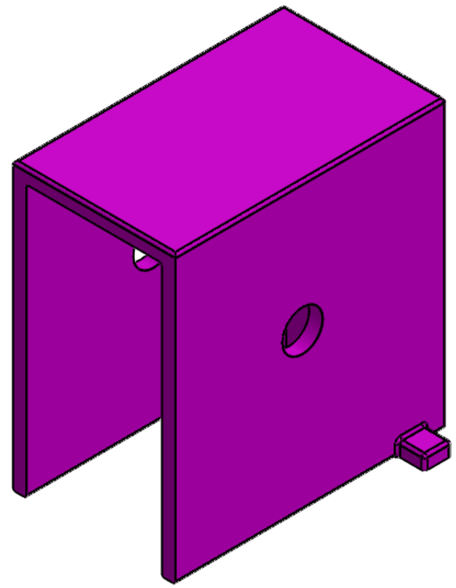
DWG NO.
Driver_Pulley

A4

SCALE:1:2

SHEET 1 OF 1





Note: All dimensions in Inches

NAME

Ed Stillwell

DWG NO.

Pulley Housing

A4

SCALE:1:5

SHEET 1 OF 1

Appendix I: Task Assignment Sheet

Name	Tasks	Signature
Raghav Agarwal	● Executive Summary ● General Layout ● Bed Frame Deflection Procedure and Calculations ● Buckling in Legs Calculation and Procedure ● All 2400 RPM Calculations	Raghav Agarwal
Bradley Berman	● Product Specification ● Belt Selection Procedure ● Bearing Reaction Procedure ● Bearing Sizing Procedure ● Design of Housings ● All 3200 RPM Calculations ● Appendix H: CAD Drawings ○ Driver Pulley ○ Driven Pulley ○ Belt ○ Legs	Bradley Berman
Glady Llanto	● Section Introductions ● Conclusion/Summary ● All 1600 RPM Calculations ● Appendix H: CAD Drawings ○ Tailstock ○ Roller Bearing Housing ○ Ball Bearing Housing	Glady Llanto
Ed Stillwell	● Problem Definition ● Shaft Design Procedure ● All 800 RPM Calculations ● Assembly Drawing ● Parts List ● Appendix H: CAD Drawings ○ Headstock ○ Pulley Housing ○ Bed Plate ○ Assembly	Ed Stillwell