More on coin tosses and the beta distributions.

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From the class, we know that if prior is $Beta(\beta_H, \beta_T)$, and the likelihood is $Bernoulli(\alpha_H, \alpha_T)$, we expect our posterior to be $Beta(\alpha_H + \beta_H, \alpha_T + \beta_T)$. In the class we solved a problem with $\beta_H =, \beta_T = 2, \alpha_H = 18, \alpha_T = 28$. The solution we arrived had a mistake that the formula for variance was wrong (It should have had a '- 1' on the denominator instead of a '+'.)

The actual posterior distribution for the same prior is shown in figure 1 along with the MLE, mean and variance. The figure 1 can be replicated by the code 'problem_from_class.m'.

Online coin tosses

Let's now observe how the probability changes over time. Let's assume that we have a prior parameterized by β_H and β_T. Now lets generate a random sequences of heads and tails with some probability θ for head and lets make it binomial. After every flip of a coin, lets construct the likelihoods, priors and posteriors and observe how they change along. One can notice that after sufficient number of trials, the likelihood flattens out

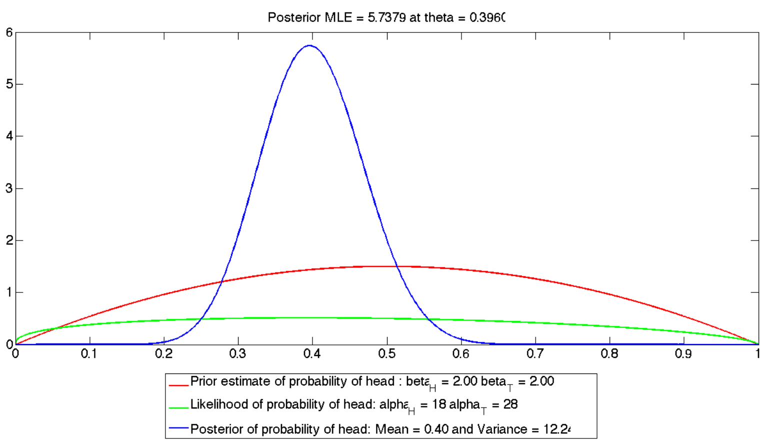

Figure 1: Solution for the problem we solved in class. MLE mean

(if unbiased coin) the posterior peaks and settles at a constant value. You will also observe that the MLE estimate of theta and the MAP estimate of theta becomes equal.

The above can be observed using the code 'beta_distribution.m'. You may try and vary the parameters and observe how the various functions get affected. You may also use this as a tool to verify the problems you practice on. Feel free to make additions and post some good plots on the discussion boards.