# Chapter 1

## Lecture 2

26/2/2022

Book: Sections 1.1.6 to 1.3.2

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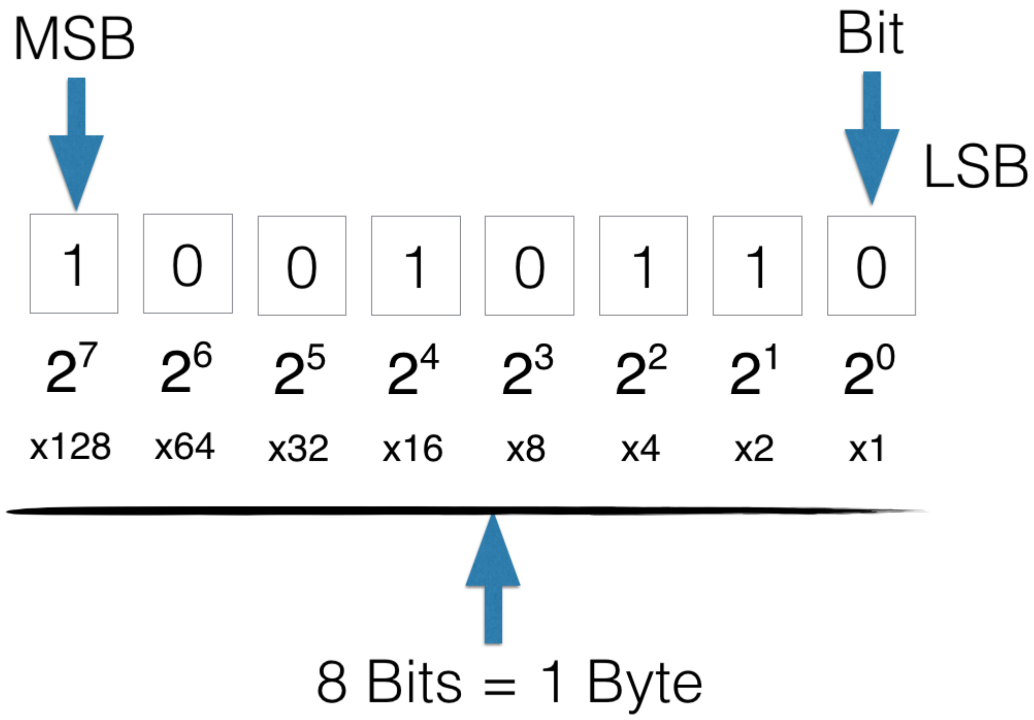
# 1.1.6. Bit Operations

- Computers represent information using bits. A bit is a symbol with two possible values: 0 (zero) and 1 (one).

  - **1** represents **T** (true)
  - **0** represents **F** (false)

- A variable is called a **Boolean variable** if its value is either **true** or **false**.

- A Boolean variable can be represented using a bit.



# 1.1.6. Bit Operations

**Bit string**

- A bit string is a sequence of zero or one bits.
- The length of bit string is the number of bits in the string.

- **Example**

**101010011** is a bit string of length nine.

# 1.1.6. Bit Operations

| *Truth Value* | *Bit* |
| --- | --- |
| T | 1 |
| F | 0 |

| $x$ | $y$ | $x \vee y$ | $x \wedge y$ | $x \oplus y$ |
| --- | --- | --- | --- | --- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

# 1.1.6. Bit Operations

## Example

- Find the bitwise OR, bitwise AND, and bitwise XOR of the bit strings:
  01 1011 0110 - 11 0001 1101

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| AND | $\wedge$ | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| OR | $\vee$ | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| XOR | $\oplus$ | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |

# 1.1.6. Bit Operations

- **Which of the following bits is the negation of the bits "010110"?**

a) 111001 b) 001001 $\boxed{\text{c) 101001}}$ d) 111111

- **If A is "001100" and B is "010101" then what is the value of A (Ex-or) B?**

a) 000000 b) 111111 c) 001101 $\boxed{\text{d) 011001}}$

- **Which of the following option is suitable, if A is "10110110", B is"11100000" and C is"10100000"?**

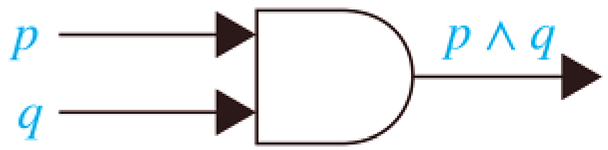
a) C=A or B b) C=¬A c) C=¬B $\boxed{\text{d) C=A and B}}$

# 1.2.6. Logic circuits (digital circuit)

- A logic circuit (or digital circuit) receives input signals $p_1, p_2, \ldots, p_n$, each a bit [either 0 (off) or 1 (on)], and produces output signals $s_1, s_2, \ldots, s_n$, each a bit.

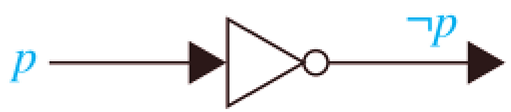
- In this course, we will restrict our attention to logic circuits with a single output signal; in general, digital circuits may have multiple outputs.

- Complicated digital circuits can be constructed from three basic circuits, called **gates**.



AND gate

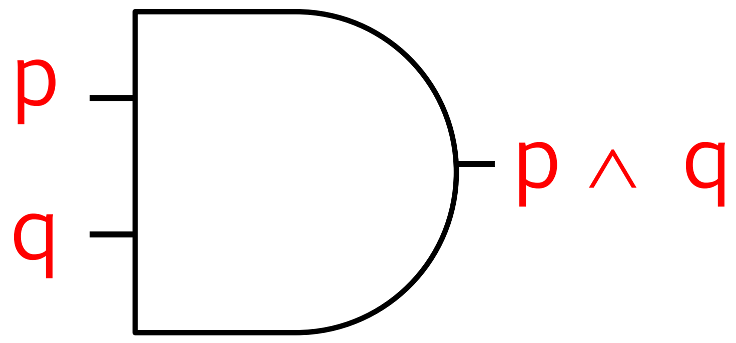
OR gate



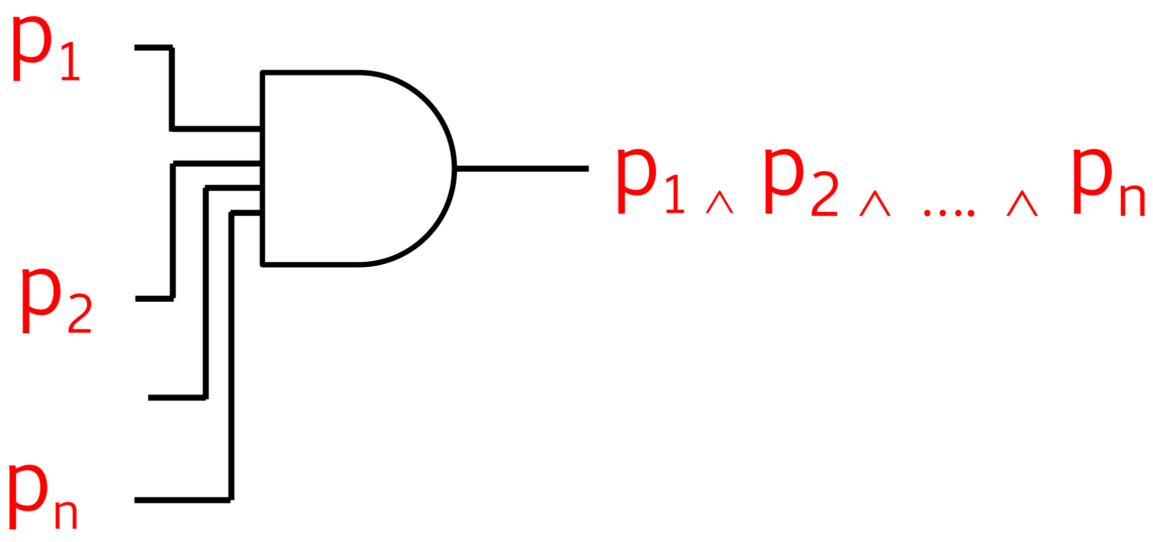
Inverter
NOT gate

# 1.2.6. Logic circuits (digital circuit)

## AND gate (Boolean Product)

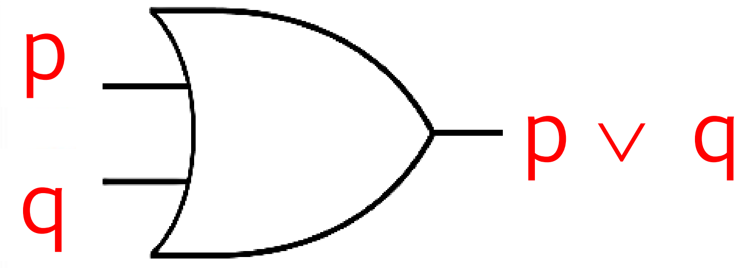



AND gates can be extended to arbitrary n inputs.

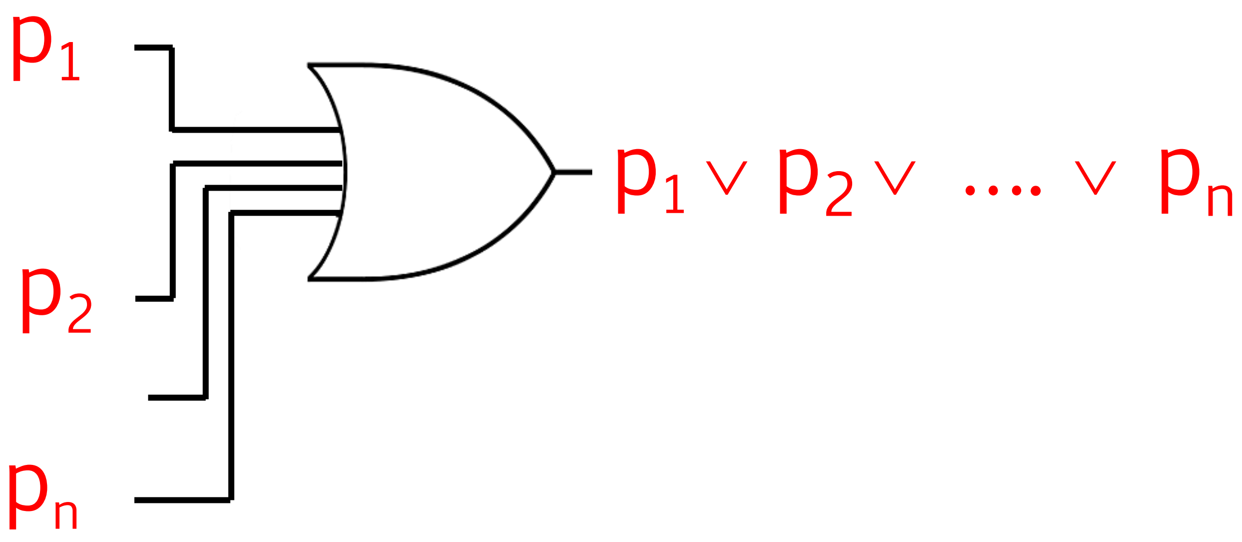


# 1.2.6. Logic circuits (digital circuit)

## OR gate (Boolean Sum)

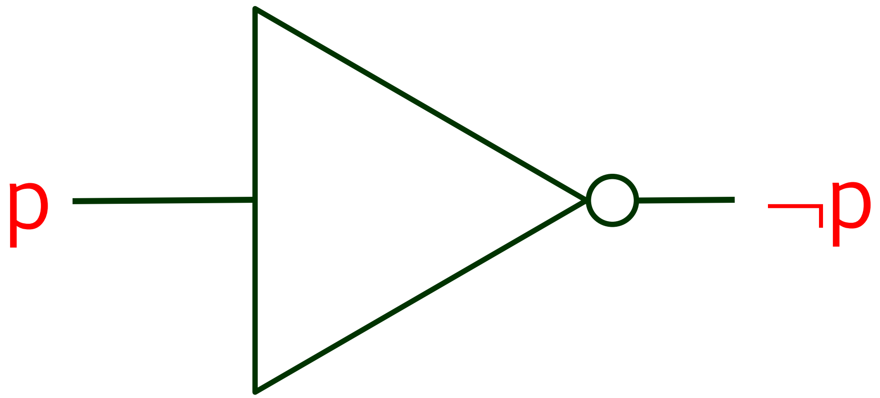



<u>OR gates can be extended to arbitrary n inputs.</u>



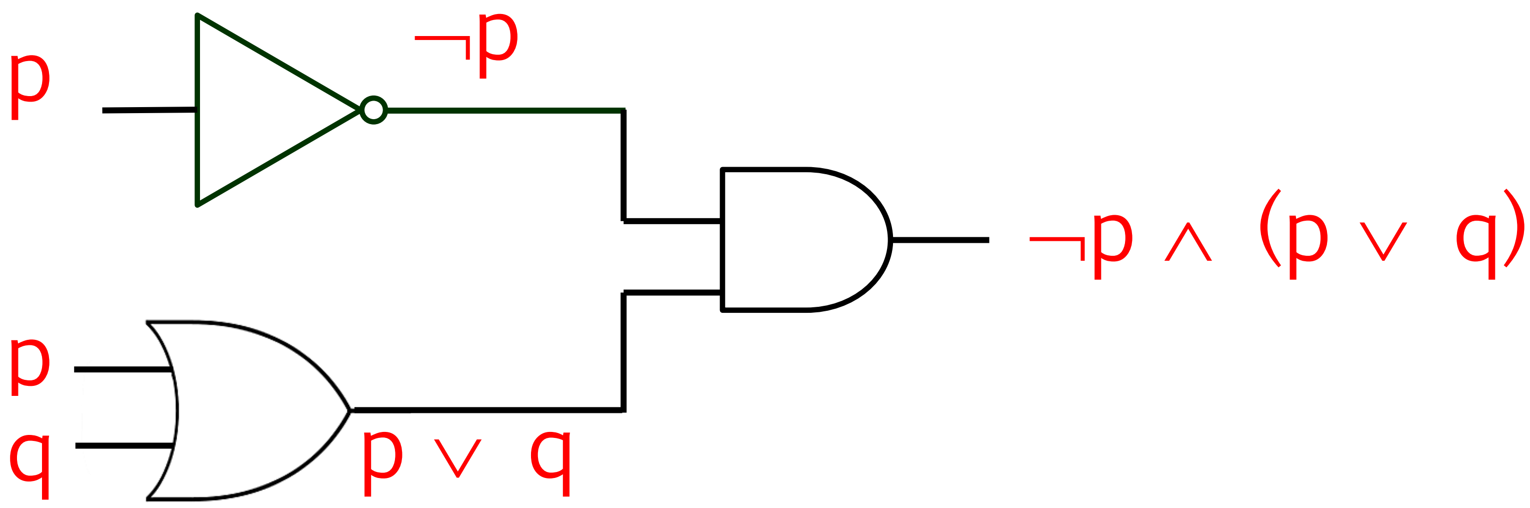
# 1.2.6. Logic circuits (digital circuit)

NOT gate (Boolean Complement)



# 1.2.6. Logic circuits (digital circuit)

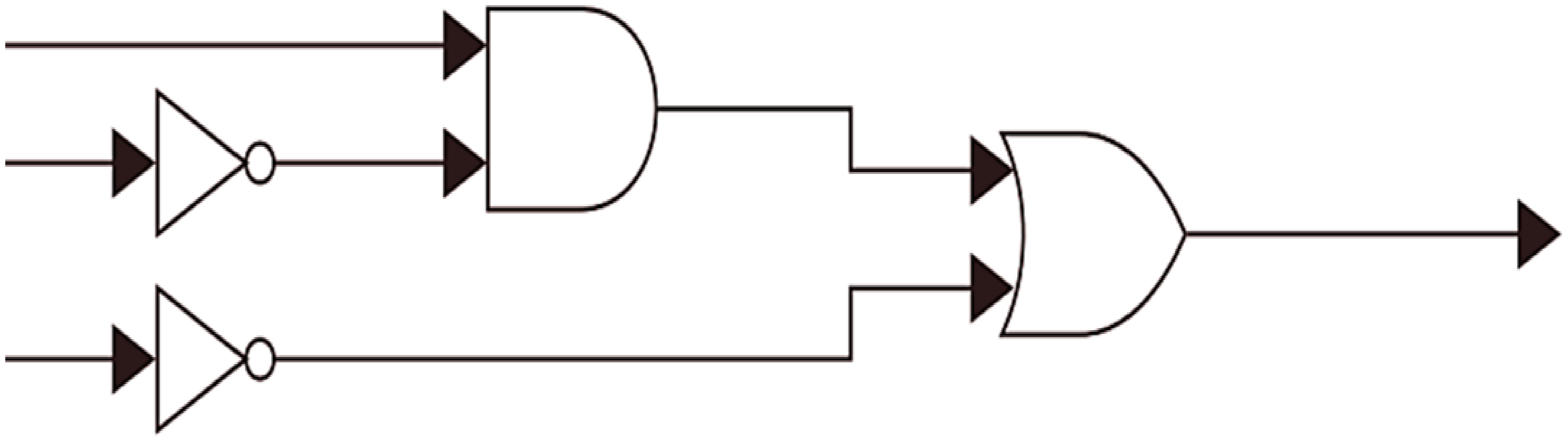
## Combination of Gates



# 1.2.6. Logic circuits (digital circuit)

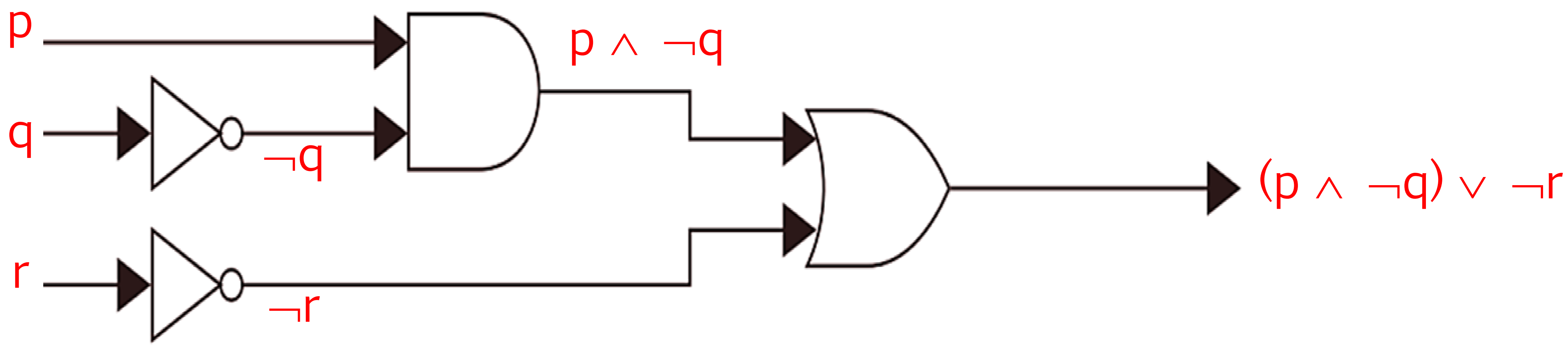
## Example (1)

- Determine the output for the combinatorial circuit in the following figure.



# 1.2.6. Logic circuits (digital circuit)

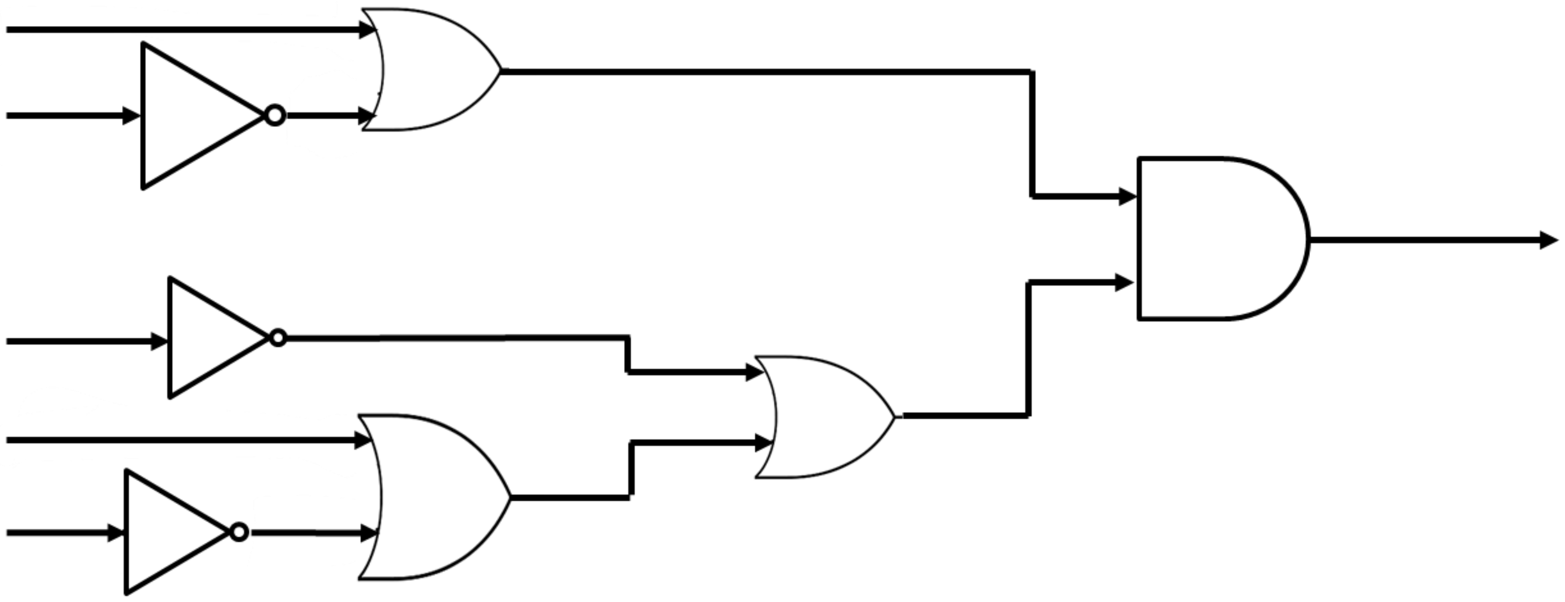
**Solution**



# 1.2.6. Logic circuits (digital circuit)

## Example (2)

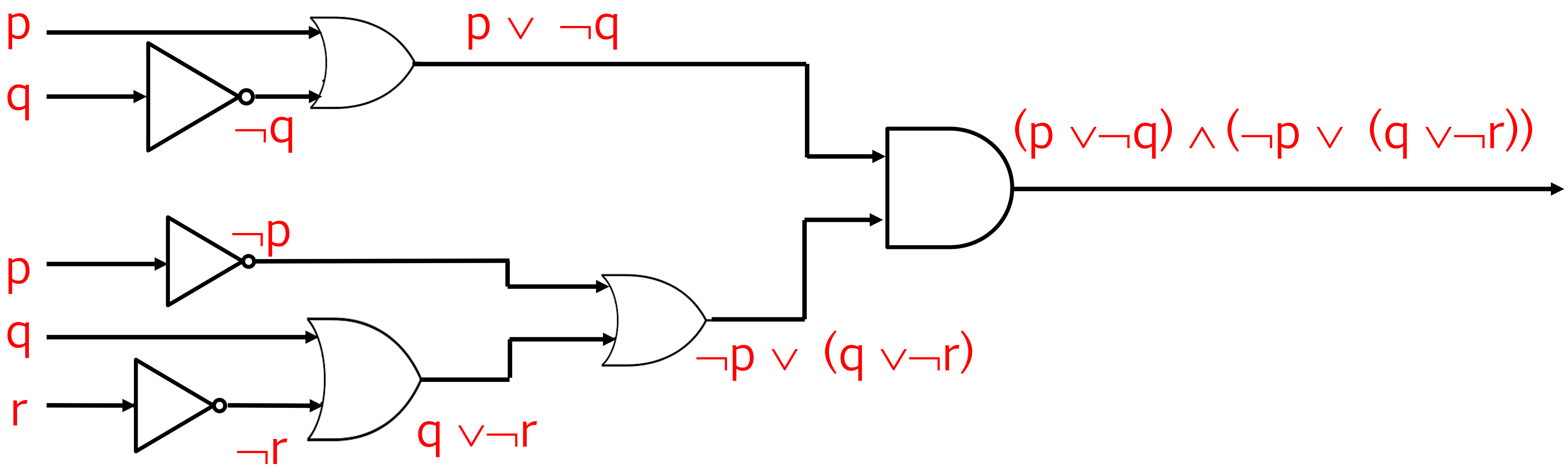
- Determine the output for the combinatorial circuit in the following figure.



# 1.2.6. Logic circuits (digital circuit)

## Example (2)

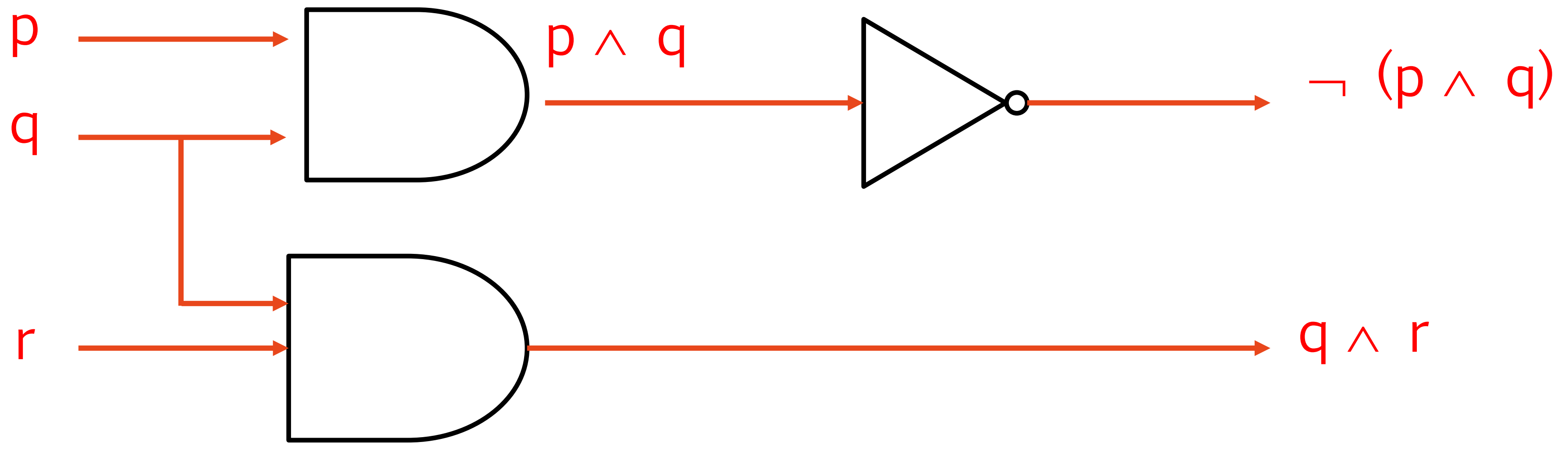
- Determine the output for the combinatorial circuit in the following figure.



# 1.2.6. Logic circuits (digital circuit)

## Example (3)

- Determine the output for the combinatorial circuit in the following figure.



# Homework (2)

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# 1.2.6. Logic circuits (digital circuit)

## Problem (1)

- Build a digital circuit that produces the output:

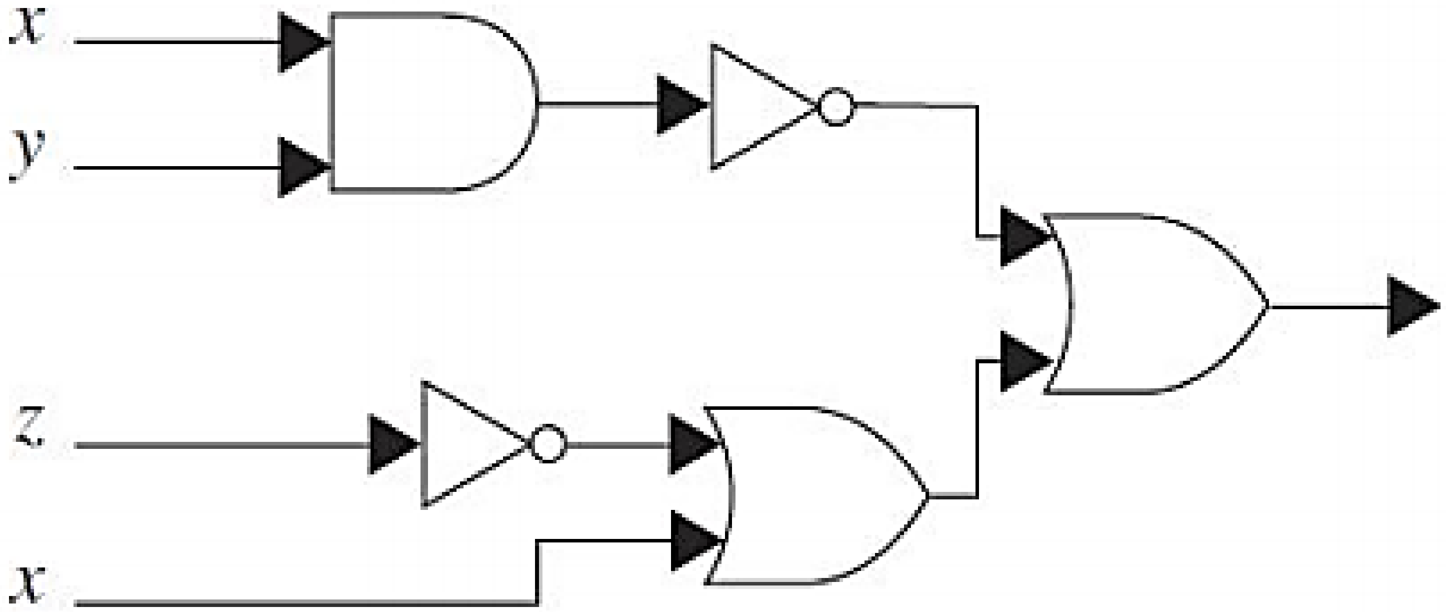
$$(p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r))$$

when given input bits $p$, $q$, and $r$.

# 1.2.6. Logic circuits (digital circuit)

## Problem (2)

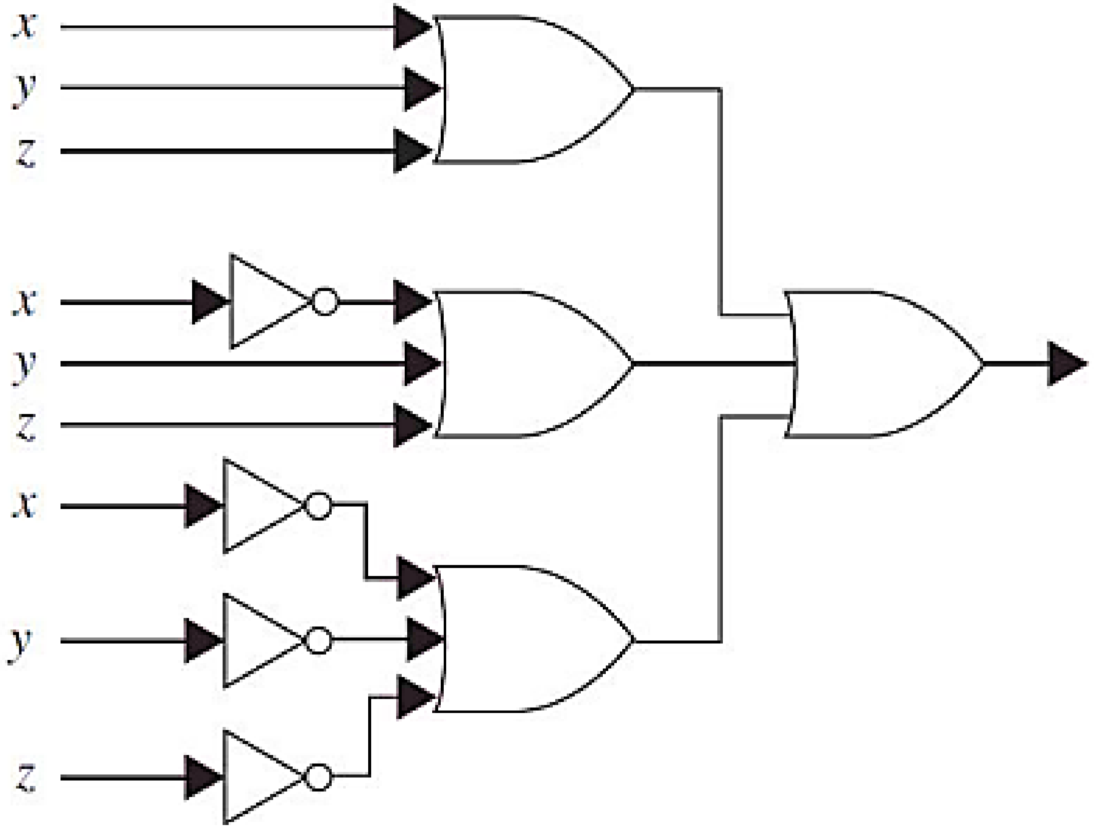
- Find the output of the given circuit.



# 1.2.6. Logic circuits (digital circuit)

## Problem (3)

- Find the output of the given circuit.



# 1.3. Propositional equivalences

The compound propositions can be classified according to their possible truth values into three types:

**Compound Propositions**

- Tautology
- Contradiction
- Contingency

# 1.3. Propositional equivalences

**Propositional Equivalences**

- **Tautology**: **A compound proposition that is always true.**
- **Contradiction**: A compound proposition that is always false.
- **Contingency**: A compound proposition that is neither a tautology nor a contradiction.

# 1.3. Propositional equivalences

## Example (1)

Show that following conditional statement is a tautology by using truth table:

- $p \vee \neg p$
- $p \wedge q \rightarrow p$

| $p$ | $\neg p$ | $p \vee \neg p$ |
|---|---|---|
| T | F | T |
| F | T | T |
| T | F | T |
| F | T | T |

| $p$ | $q$ | $p \wedge q$ | $p \wedge q \rightarrow p$ |
|---|---|---|---|
| T | T | T | T |
| F | T | F | T |
| T | F | F | T |
| F | F | F | T |

- $p \vee \neg p$ is a tautology
- $p \wedge q \rightarrow p$ is a tautology

# 1.3. Propositional equivalences

## Logical Equivalences

- Compound propositions that **<u>have the same truth values in all possible cases</u>** are called logically equivalent.
- The compound propositions p and q are called ***logically equivalent*** if $p \leftrightarrow q$ is a tautology.
- The notation $p \equiv q$ denotes that p and q are logically equivalent.

$\equiv$

$\Leftrightarrow$

# 1.3. Propositional equivalences

## Example (1)

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

| $p$ | $q$ | $p \vee q$ | $\neg(p \vee q)$ | $\neg p$ | $\neg q$ | $\neg p \wedge \neg q$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| F | T | T | F | T | F | F |
| T | F | T | F | F | T | F |
| F | F | F | T | T | T | T |

So, $\neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$ is logically equivalent

# 1.3. Propositional equivalences

## Example (2)

Show that $\neg p \vee q$ and $p \rightarrow q$ are logically equivalent.

| $p$ | $q$ | $\neg p$ | $\neg p \vee q$ | $p \rightarrow q$ |
|---|---|---|---|---|
| T | T | F | T | T |
| F | T | T | T | T |
| T | F | F | F | F |
| F | F | T | T | T |

So, $\neg p \vee q$ and $p \rightarrow q$ is logically equivalent

# 1.3. Propositional equivalences

## Example (3)

Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

| $p$ | $q$ | $r$ | $q \wedge r$ | $p \vee (q \wedge r)$ | $p \vee q$ | $p \vee r$ | $(p \vee q) \wedge (p \vee r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| F | T | T | T | T | T | T | T |
| T | F | T | F | T | T | T | T |
| F | F | T | F | F | F | T | F |
| T | T | F | F | T | T | T | T |
| F | T | F | F | F | T | F | F |
| T | F | F | F | T | T | T | T |
| F | F | F | F | F | F | F | F |

So, $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ is logically equivalent

# 1.3. Laws of Propositional Logic

| Logical Equivalences. | |
|---|---|
| $p \wedge \mathbf{T} \equiv p$<br>$p \vee \mathbf{F} \equiv p$ | Identity laws |
| $p \vee \mathbf{T} \equiv \mathbf{T}$<br>$p \wedge \mathbf{F} \equiv \mathbf{F}$ | Domination laws |
| $p \vee p \equiv p$<br>$p \wedge p \equiv p$ | Idempotent laws |
| $\neg(\neg p) \equiv p$ | Double negation law |
| $p \vee q \equiv q \vee p$<br>$p \wedge q \equiv q \wedge p$ | Commutative laws |

T compound proposition is always true
F compound proposition is always false

# 1.3. Propositional equivalences

| Logical Equivalences. | |
|---|---|
| $(p \vee q) \vee r \equiv p \vee (q \vee r)$<br>$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ | Associative laws |
| $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$<br>$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ | Distributive laws |
| $\neg(p \wedge q) \equiv \neg p \vee \neg q$<br>$\neg(p \vee q) \equiv \neg p \wedge \neg q$ | De Morgan's laws |
| $p \vee (p \wedge q) \equiv p$<br>$p \wedge (p \vee q) \equiv p$ | Absorption laws |
| $p \vee \neg p \equiv \mathbf{T}$<br>$p \wedge \neg p \equiv \mathbf{F}$ | Negation laws |

# Thank you