Chapter 3

SPECIAL PROBABILITY DISTRIBUTIONS

3.1 Introduction

In chapters 1 and 2 we studied probability distributions in general. In this chapter we study some commonly occurring probability distributions and investigate their basic properties. The results of this chapter will be of considerable use in theoretical as well as practical applications. We begin with some discrete distributions and follow with some continuous distributions.

3.2 Discrete Distributions

In this section we study some well-known discrete distributions and describe their important properties.

I- The Binomial Distribution

Consider a sequence of n **independent** Bernoulli trials, each resulting in one of the two possible events, "**success**" or "**failure**". Let the probability **p** = P(success occurs at any given trial) remains constant from trial to trial. Let the r.v. **X** denote the total number of successes in the **n** trials, then **X** is said to have the binomial distribution

Definition 3.1

The r.v. X is said to have the ***binomial distribution*** and it is referred to as a binomial r.v. with parameters n and p iff its p.m.f. is given by

$$b(x;n,p) = f(x) = \binom{n}{x} p^x q^{n-x} \quad \text{for} \quad x = 0, 1, 2, \dots, n. \tag{3.1}$$

where $0 \le p \le 1$ and $q = 1 - p$.

The term "parameters" is used quite generally to refer to a set of constants in a distribution whose values may vary from one application to another. The notation **b(x;n,p)**, which have used instead of **f(x)**, reflects the dependence on the parameters **n** and **p**.

The general properties of the p.m.f. satisfied by equation (3.1), since $0 \le p \le 1$ and

$$\sum_{x=0}^{n} b(x;n,p) = \sum_{x=0}^{n} \binom{n}{x} p^x q^{n-x} = (p+q)^n = 1$$

The name "binomial distribution" derives from the preceding summation, that is the values of **b(x; n, p)** for x = 0, 1, 2,..., n are the successive terms of the binomial expansion of $\mathbf{(q + p)^n}$.

A short notation to designate that X has the **binomial distribution** with parameters n and p is

$$\mathbf{X \sim b(n,p)} \quad \text{or an alternative notation} \quad \mathbf{X \sim BIN(n,p)}$$

Theorem 3.1

The M.G.F. of the binomial distribution is given by

$$M_X(t) = \left(p\,e^t + q\right)^n \tag{3.2}$$

and the mean and the variance are given by

$\mu = np$ **and** $\sigma^2 = npq$

Proof

By definitions, we have

$$M_X(t) = E[\,e^{tX}\,] = \sum_{x=0}^{n} e^{tx} \cdot \binom{n}{x} p^x\; q^{n-x} = \sum_{x=0}^{n} \binom{n}{x} (p\,e^t)^x\; q^{n-x} = (p\,e^t + q)^n$$

Now, differentiating $M_X(t)$ w.r.t. t, then putting t=0, we get

$$\mu = E[\,X\,] = M_{X'}(0) = \left[n\,p\,e^t (p\,e^t + q)^{n-1}\right]_{t=0} = np$$

If we differentiate $M_X(t)$ twice w.r.t. t , then putting t=0, we get

$$\mu_2 = M_X'(0) = \left[n\,p\,e^t (p\,e^t + q)^{n-1} + n(n-1)\,p^2\,e^{2t}(p\,e^t + q)^{n-2}\right]_{t=0} = np + n(n-1)p^2$$

Hence, $\sigma^2 = var(X) = E(X^2) - \mu^2 = n\,p\,q$

Example 3.1

A company produces computer chips of which 40% are defective, 7 chips are chosen at random, what is the probability that:

(a) exactly 3, **(b)** at least 5 , **(c)** at most 5 chips will be non-defective.

Solution

Let X be the number of the non-defective chips, then

P(one chip will be defective) = p = 0.6 $\Rightarrow$ q = 1 - p = 0.4

number of all chips = n = 7

(a) $\mathbf{P(\text{exactly 3 chips will be non-defective}) = P(X = 3) = \binom{7}{3}(0.6)^3(0.4)^4 = 0.194}$

(b) P(at least 5 chips will be defective**) = P(X = 5) + P(X = 6) + P(X = 7)**

$$= \binom{7}{5}(0.6)^5(0.4)^2 + \binom{7}{6}(0.6)^6(0.4) + \binom{7}{7}(0.6)^7(.4)^0$$
$$= 0.261 + 0.131 + 0.028 = 0.42.$$

(c) P(at most 5 chips will be defective**) = P(X = 0) + P(X = 1) + ... + P(X = 5)**

= 1 - P(X = 6) - P(X = 7) = F(5)

$$= 1 - 0.131 - 0.028 = 0.841.$$

II- The Hypergeometric Distribution

In chapter 2 we used sampling with and without replacement to illustrate the multiplication rules for independent and dependent events. To obtain a formula analogous to that of the binomial distribution which applies to sampling without replacement , in which case the trials are not independent, let us consider a set or collection consists of a finite number of items, say N, of which M are of type 1 and the remaining N-M items are of type 2. Suppose n items are drawn at random without replacement, and denote by X the number of items of type 1 that are drawn. Then the distribution of X is called the hypergeometric distribution and its p.m.f. is given by

Definition 3.2

The r.v. X is said to have the ***hypergeometric distribution*** and it is referred to as a hypergeometric r.v. iff its p.m.f. is given by

$$h(x; n, N, M) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} \quad \text{for } x = 0, 1, 2, \ldots, n, \; x \le M \text{ and } n-x \le N-M \qquad \mathbf{(3.3)}$$

Note that the possible values of X in (3.3), in general, are

$$\max(0, n-N+M) \le x \le \min(n, M)$$

A special notation, which designates that X has the hypergeometric distribution (3.3) is

$$\mathbf{X \sim HYP(n, N, M)}$$

The hypergeometric distribution is important in applications such as deciding whether to accept a lot of manufactured items.

Example 3.2

A box contains 100 microchips, 80 good and 20 defective. The number of defectives in the box is unknown to a purchaser, who decides to select 10 microchips at random without replacement and to consider the microchips in the box acceptable if the 10 items selected include no more than 3 defectives. Find the probability of accepting this lot.

Solution

Let X be the number of defectives selected, then X has the hypergeometric distribution with n=10 , N= 100 , and M=20, and the probability of the lot being acceptable is

$$P[X \le 3] = \sum_{x=0}^{3} \frac{\binom{20}{x}\binom{80}{10-x}}{\binom{100}{10}} = 0.159$$

III- The Poisson Distribution

Definition 3.3

The r.v. X is said to have the ***Poisson distribution*** with parameter **λ** > 0 iff its p.m.f. is given by

$$f(x;\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad \text{for} \quad x = 0, 1, 2, \ldots \qquad (3.4)$$

This distribution is named due to the French mathematician Simeon Poisson (1781-1840). A special notation, which designates that X has the Poisson distribution (3.5) is

$$X \sim POIS(\lambda)$$

The general properties of the p.m.f. f(x;λ) are satisfied by (3.5) since λ > 0 implies f(x;λ) ≥ 0 and

$$\sum_{x=0}^{\infty} f(x;\lambda) = e^{-\lambda} \cdot \sum_{x=0}^{\infty} \frac{\lambda^{x}}{x!} = e^{-\lambda} \cdot e^{\lambda} = 1$$

Now we can show that the Poisson p.m.f. (3.5) is the limiting form of the binomial p.m.f., when $n \to \infty$, $p \to 0$, while np remains constant.

Thus, in the limit when n → ∞, p → 0, and np = λ remains constant, the number of successes is a r.v. having a Poisson distribution with the parameter **λ**. In general, the Poisson distribution will provide a good approximation to binomial probabilities when **n≥ 20** and **p≤ 0.05**. When **n ≥ 100** and **np < 10**, the approximation will generally be excellent.

The mean and the variance of the Poisson distribution are given by

$$\mu = \lambda \quad \text{and} \quad \sigma^2 = \lambda$$

Example 3.3

Suppose that 1% of all transistors produced by a certain company is defective. A new model of computer requires 100 of these transistors, and 100 are selected at random from the company's assembly line. Find the probability of obtaining 3 defectives.

Solution

The exact probability of obtaining 3 defectives using the binomial distribution with p=.01 and n=100 is

$$\mathbf{f(3) = \binom{100}{3} (0.01)^3 (0.99)^{97} = 0.061}$$

while the Poisson approximation with $\boldsymbol{\lambda = np = 1.0}$, is

$$\mathbf{f(3) = \frac{e^{-1}\ 1^3}{3\,!} = 0.0613}$$

Example 3.4

If 2% of the books bound at a certain bindery have defective bindings, use the Poisson approximation to the binomial distribution to determine the probability that five of 400 books bound by this bindery will have defective bindings.

Solution

Substituting $\mathbf{x = 5}$, $\boldsymbol{\lambda} = 400(0.02) = 8$, and $e^{-8} = 0.00034$ into the Poisson distribution, we get

$$\mathbf{f(5) = \frac{8^5 \cdot e^{-8}}{5!} = \frac{(32{,}768)\ (0.00034)}{120} = 0.093}$$

3.3 Continuous Distributions

In this section we study some well-known continuous distributions and describe their important properties.

I-The Uniform (Rectangular) Distribution

Suppose that a continuous r.v. X can assume values only in a bounded interval, say the open interval (α, β), and suppose that the p.d.f. is constant, say

$$\mathbf{f(x) = c} \qquad \text{for} \qquad \boldsymbol{\alpha < x < \beta}$$

property (2.3) implies $\mathbf{c = 1/(\beta - \alpha)}$, since

$$1 = \int_{-\infty}^{\infty} f(x)\, dx = \int_{\alpha}^{\beta} c\, dx = c(\beta - \alpha)$$

If we define f(x) = 0 outside the interval, then we obtain f(x) ≥ 0 for all x.

Definition 3.4

The continuous r.v. X is said to have the Uniform distribution with parameters **α** and **β** (α > β) iff its p.d.f. is given by

$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & \text{for} \quad \alpha < x < \beta \\ 0 & \text{otherwise} \end{cases} \tag{3.5}$$

A notation that designates that X has p.d.f. of the form (3.5) is

$$\mathbf{X \sim U(\alpha, \beta)}$$

This distribution provides a probability model for selecting a point "at random" from an interval (α, β). The CDF of **X ~ U(α, β)** has the form

$$F(x) = \begin{cases} 0 & x \le \alpha \\ \dfrac{x - \alpha}{\beta - \alpha} & \alpha < x < \beta \\ 1 & \beta \le x \end{cases} \tag{3.6}$$

The graphs of f(x) and F(x) are shown in Figures 3.1 and 3.2 respectively.

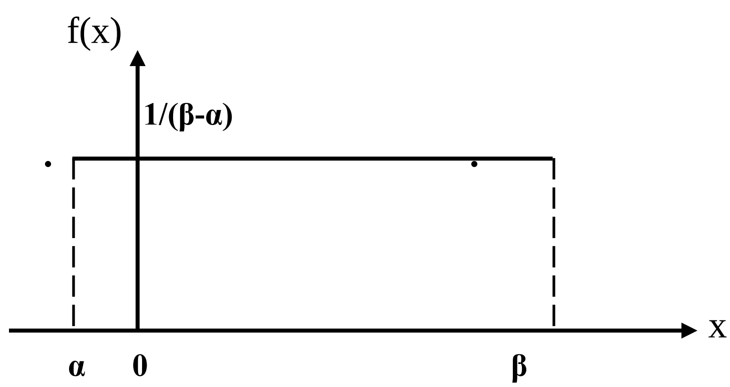
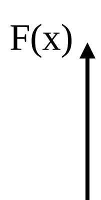


Fig. 3.1 The p.d.f. of the uniform distribution U(α,β)



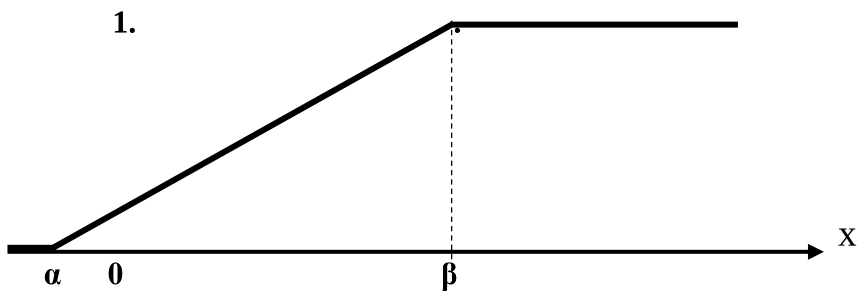


Fig. 3.2 The CDF of the uniform distribution U(α,β)

Theorem 3.1

The mean and the variance of the uniform distribution U(α, β) are given by

$$\mu = \frac{\alpha + \beta}{2} \quad \text{and} \quad \sigma^2 = \frac{(\beta - \alpha)^2}{12}$$

II-The Exponential Distribution

Definition 3.5

The continuous r.v. X is said to have the exponential distribution with parameter θ > 0 iff its p.d.f. is given by

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & \text{for} \quad x > 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.7}$$

The CDF of X is

$$F(x;\theta) = \begin{cases} 0 & x < 0 \\ 1 - e^{-x/\theta} & x \geq 0 \end{cases}$$

so that θ is a scale parameter.

A notation that designates that X has p.d.f. of the form (3.7) is

$$X \sim EXP(\theta)$$

The exponential distribution, which is an important probability model for lifetimes, is also characterized by a property that given in the following theorem.

Theorem 3.2

If the continuous r.v. $X \sim EXP(\theta)$, then

$$P[X > T + t \mid X > T] = P[X > t] \tag{3.8}$$

for all $T > 0$ and $t > 0$.

Proof

$$P[X > T+t \mid X > T] = \frac{P[X > T+t \text{ and } X > T]}{P[X > T]} = \frac{P[X > T+t]}{P[X > T]} = \frac{1-F(T+t;\theta)}{1-F(t;\theta)}$$

$$= \frac{e^{-(T+t)/\theta}}{e^{-T/\theta}} = e^{-t/\theta} = P(X > t)$$

This shows that the exponential distribution satisfies property (3.8), which is known as the **no-memory** property.

If X is the lifetime of a component, then (3.8) asserts that the probability that the component will last more than T + t time's units given that it has already lasted more than T units is the same as that of a new component lasting more than t units. In other words, an old component which is still working is just as reliable as a new component.

Theorem 3.4

The mean and the variance of the exponential distribution EXP(θ) are given by $\mu = \theta$ and $\sigma^2 = \theta^2$.

Example 3.5

Suppose a certain solid-state component has a lifetime or failure time (in hours) X ~ EXP(100). Find the probability that the component will last at least 80 hours given that it is already worked more than 30 hours.

Solution

Using formula (3.8) with $\theta = 100$, we get,

$$P[X > 80 \mid X > 30] = P[X > 50] = e^{-0.5} = 0.6065.$$

III- The Normal Distribution

The most important probability distribution in the entire field of statistic is the normal distribution. It is in many ways the cornerstone of modern statistical theory.

Definition 3.6

The continuous r.v. X is said to have the ***normal distribution*** with parameter μ and σ^2 iff its p.d.f. is given by

$$f(x,\mu,\sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for} \quad -\infty < x < \infty \qquad \textbf{(3.9)}$$

where $-\infty < \mu < \infty$ and $\sigma > 0$.

A notation that designates that X has p.d.f. of the form (3.9) is

$$X \sim N(\mu, \sigma^2)$$

The normal distribution is also frequently referred to as the **Gaussian distribution**. The graph of a normal distribution, shaped like the cross section of a **bell**, is shown in Fig 3.3.

First, let us show that the formula of definition 3.6 can serve as a p.d.f. Since the values of f(x; μ, σ) are evidently positive so long as **σ > 0**, we we can show that the total area under the curve is equal to 1.

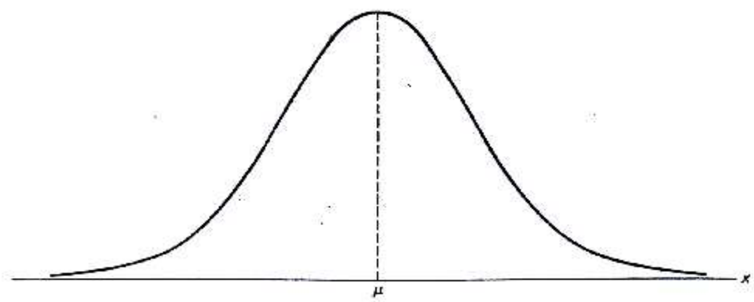


Fig. 3.3 The p.d.f. curve of the normal distribution $N(\mu , \sigma^2)$

The parameter **μ** is, in fact, **E(X)** (the mean) and that the parameter **σ** is, in fact, the square root of **var(X)** (the standard deviation), where $\mathbf{X \sim N(\mu, \sigma^2)}$.

From an inspection of Figure 3.3, we list the following properties of the normal curve.

(1) The mode, which is the point on the horizontal axis where the curve is a maximum, occurs at x = μ (i.e. mode = μ).

(2) The curve is symmetric about a vertical axis through the mean μ.

(3) The normal curve approaches the horizontal axis asymptotically as we proceed in either direction away from the mean.

(4) The total area under the curve and above the horizontal axis equals to 1.

A normal distribution is completely specified by the two parameters; the mean **μ**, and the variance $\boldsymbol{\sigma^2}$. Thus, for any given standard deviation **σ**, there are an infinite number of normal curves possible, depending on **μ**. Fig. 3.4 shows normal curves for σ = 1 and μ = 0, 1, 2.

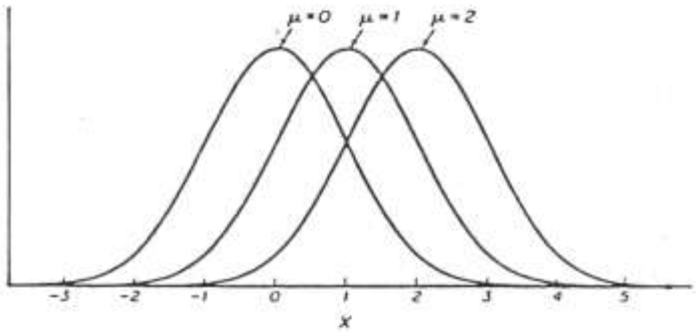


Figure 3.4 Normal distributions with σ = 1, varying in location with different means.

Likewise, for any given mean, **μ**, an infinity of normal curves are possible, each with a different value of **σ**. Fig. 3.5 shows normal curves for μ = 0 and σ = 1, 1.5, and 2.

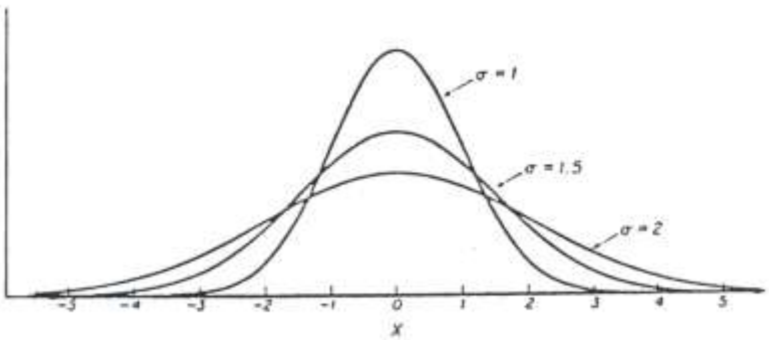


Figure 3.5 Normal distributions with μ = 0, varying in scale with different standard deviations.

Standard Normal distribution.

Since the normal distribution plays a basic role in statistics and its density cannot be integrated directly, its areas have been tabulated for the special case where μ = 0 and σ = 1. The standard normal distribution is a special case of the normal distribution. The normal distribution with μ=0 and σ=1 is called the standard normal distribution. Fig. 3.4 displays the standard normal distribution curve.

Definition 3.7

The normal distribution with **μ = 0** and **σ =1** is referred to as the ***standard normal distribution***, and its p.d.f., adopted with a special notation ϕ, is given by

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \qquad -\infty < z < \infty \tag{3.10}$$

If the r.v. **Z** has p.d.f. (3.10), then **Z ~ N(0 , 1)** , and the standard normal CDF is given by

$$\Phi(z) = \int_{-\infty}^{z} \phi(t)\,dt \qquad (3.11)$$

Z Values or Z Scores

A random variable X is said to have been standardized when it has been adjusted so that its mean is 0 and its standard deviation is 1 [i.e. N(0,1)]. Standardization can be effected by subtracting μ from X, and dividing the resulting difference by $\boldsymbol{\sigma}$; the standard

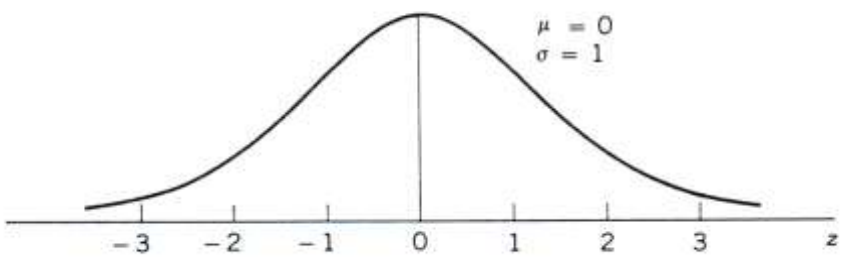


Figure 3.6 The standard normal distribution curve.

deviation of **X**, thus the variable

$$\frac{X - \mu}{\sigma}$$

is a standardized variable and is known as the standard normal variable, and is usually denoted by **Z**. One of the most important theorems of statistics states that:

If the r.v. **X** has the normal distribution with mean μ and variance $\boldsymbol{\sigma^2}$, the standardized variable

$$Z = \frac{X - \mu}{\sigma}$$

has the normal distribution with mean 0 and variance 1. In symbols;

$$\textbf{If } X \sim N(\mu, \sigma^2) \quad \textbf{then} \quad Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

Areas under the standard normal curve are frequently needed and are therefore widely tabulated. Let us denote by **Φ (z)** the area underneath the standard normal curve from -∞ to z, where z is any number positive, negative, or zero, i.e. we have for **Z ~ N(0,1)**;

$$\Phi(z) = P(Z \le z)$$

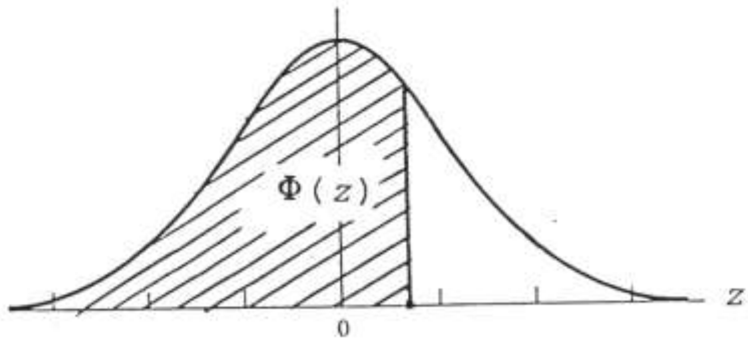
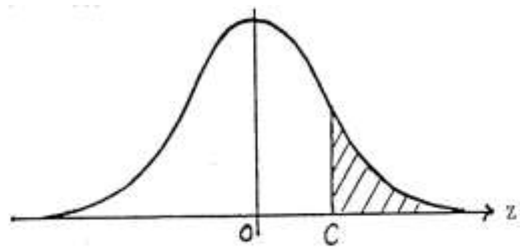


Figure 3.7

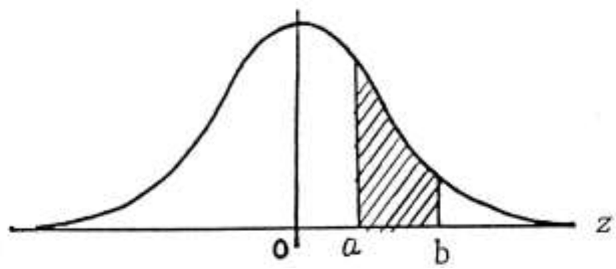
Clearly, $\Phi(z)$ has the following properties:

i) $\Phi(-\infty) = 0$, $\Phi(\infty) = 1$ **and** $\Phi(0) = \frac{1}{2}$.

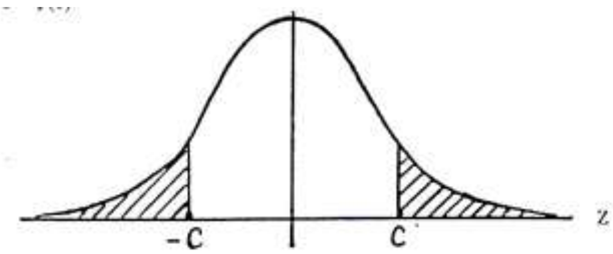
ii) $P(Z > c) = 1 - \Phi(c)$



iii) $P(a \le Z \le b) = \Phi(b) - \Phi(a)$



iv) $\Phi(-c) = 1 - \Phi(c)$



Most standard normal tables give the values of $\Phi(z)$, shown in Fig. 3.7, for positive values of z.. If z is negative we can use rule (iv) to determine $\Phi(z)$.

Example 3.6

i- $P(Z \le 1.2) = \Phi(1.2) = 0.8849$

ii-
$$\begin{aligned} P(-0.5 \le Z \le 0.9) &= \Phi(0.9) - \Phi(-0.5) \\ &= \Phi(0.9) - (1 - \Phi(0.5)) \\ &= \Phi(0.9) - 1 + \Phi(0.5) \\ &= 0.8159 - 1 + 0.6915 \\ &= 0.5074 \end{aligned}$$

$$\text{iii-}\quad P(Z \geq -1.2) = 1 - \Phi(-1.2) = 1 - (1 - \Phi(1.2)) = \Phi(1.2) = 0.8849$$

Example 3.7

If X has a normal distribution with mean 400 and standard deviation 50, find P(360 $\leq X \leq$ 469).

Solution

Here, we have $\mu = 400$, $\sigma = 50$, hence

$$P(360 \leq X \leq 469) = P\left(\frac{360-400}{50} \leq \frac{X-\mu}{\sigma} \leq \frac{469-400}{50}\right)$$
$$= P(-0.8 \leq Z \leq 1.38)$$
$$= \Phi(1.38) - \Phi(-0.8)$$
$$= \Phi(1.38) - 1 + \Phi(0.8)$$
$$= 0.9162 - 1 + 0.7881$$
$$= 0.7043.$$

Example 3.9

The heights of 1000 students in a certain college are normally distributed with a mean 68 inches and standard deviation of 3 inches. How many of these students would you expect to have heights:

a- Less than 64 inches, **b-** Between 67 and 71 inches.

Solution

Let X denotes the height of the students, then $X \sim N(68, 9)$, thus

a- $P(X < 64) = P(Z < \frac{64-68}{3}) = P(Z < -1.33) = 1 - \Phi(1.33) = 1 - 0.908 = 0.092$

Hence, the number of students having heights less than 64 inches is

$$1000(0.092) = 92 \text{ students.}$$

b- $P(67 < X < 71) = P(\frac{67-68}{3} \leq Z \leq \frac{71-68}{3}) = P(-0.33 < Z < 1)$

$$= \Phi(1) - \Phi(-0.33) = \Phi(1) - 1 + \Phi(0.33) = 0.841 - 1 + 0.629 = 0.470$$

Hence the number of students having weight between 67 and 71 inches is

$$1000(0.470) = 470 \quad \text{students.}$$

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EXERCISES

[1] Let X be a r.v. having the binomial distribution with parameters n, p such that E[X]=10 and var(X) = 6. Find n and p.

[2] If 40% of the fuses produced by a company are defective, what is the probability that at least 2 out of 5 fuses chosen at random will be defective?

[3] If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random,

a- one will be defective, **b-** at most 3 will be defective.

[4] The probability that a patient recovers from a delicate heart operation is 0.9. What is the probability that exactly 5 of the next 7 patients who have this operation survive?

[5] The probability that a certain kind of component will survive a given shock test is 0.7. Find the probability that exactly 2 of the next 4 components tested survive.

[6] If is known that 60% of mice inoculated with a serum are protected from a certain disease. If 5 mice are inoculated, what is the probability that

(a) exactly 3, **(b)** at least 2 , **(c)** at most 4

of the mice contract the disease?

[7] Suppose the mortality rate for a certain disease is 0.10, and suppose 60 people in a community contract the disease. What is the probability that:

a- None will survive? **b-** Fifty percent will die?

c- At least ten will die? **d-** Exactly three will die?

[8] If the probability is 0.75 that a person will believe a rumor about the transgressions of a certain politician, find the probabilities that five from eight persons hear the rumor will believe it.

[9] If the r.v. X has the Poisson distribution with P(X=1) = 2P(X=0), find P(X > 1).

[10] Suppose 2% of the people on the average are left handed. Find the probability that at least four are left handed among 200 people.

[11] A box of fuses contains 40 fuses, of which 8 are defective. If 5 of the fuses are selected at random and removed from the box in succession without replacement, what is the probability that at most two fuses will be defective?

[12] The number of monthly breakdown of a computer is a r.v. having a Poisson distribution with $\lambda = 1.8$. Find the probabilities that this computer will function for a month:

a- without a breakdown; **b-** with only one breakdown.

[13] In certain experiments, the error made in determining the density of a substance is a r.v. having a uniform density with $\alpha = -0.015$ and $\beta = 0.015$. Find the probabilities that an error will be

a- between -0.002 and 0.003; **b-** exceed 0.005 in absolute value.

[14] Assume the length X in minutes of a particular type of telephone conversation is a r.v. with p.d.f

$$f(x) = \begin{cases} \frac{1}{5} e^{-x/5} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Determine:

a- The mean length E(X) of this type of telephone conversation.

b- Find the variance and S.D. of X.

c- Find $E(X+5)^2$.

[15] Suppose it is known that the life X of a particular compressor in hours has the p.d.f.

$$f(x) = \begin{cases} \frac{1}{900} e^{-x/900} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

a- Find the mean life of the compressor. **b-** Find the S.D. of the r.v. X.

[16] Given that X has the normal distribution with mean 50 and variance 100, find

i- $P(X \geq 60)$ **ii-** $P(45 \leq X \leq 65.3)$

[17] Given that X is normally distributed with mean 18 and standard deviation 2.5, find

a- The value of k such that $P(X \geq k) = 0.2236$ **b-** $P(X < 15)$

c- $P(17 < X < 21)$.

[18] Given that $X \sim N(\mu, \sigma^2)$ with $\sigma^2 = 25$ and $P(X > 14) = 0.8849$, find the value of μ.

[19] The speeds of motorists passing a particular point on a motorway are found to be normally distributed with mean 115 km/h and standard deviation 8 km/h.

i. Find the percentage of motorists whose speeds exceed 120 km/h.

ii. Find v such that the speeds of 20% of the motorists do not exceed v km/h.

[20] The lengths of components produced by a machine are normally distributed with standard deviation 1.5 mm. At a certain setting 25% of the components are longer than 100 mm, calculate the mean value of the lengths at this setting?

[21] A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a normal distribution and can be measured to any degree of accuracy, what percentage of resistors will have a resistance that exceeds 43 ohms?. [Ans. 6.68%]

[22] An electrical firm manufactures light bulbs that have a length of life that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

[Ans. 0.511]

[23] The volume (in liters) of liquid in bottles filled by a machine is normally distributed with mean 1.02 and standard deviation 0.01 liter.

a- What is the probability that a bottle, selected at random, contains less than 1 liter.

b- To what value must the mean be altered to reduce the probability in (a) to 1% (assuming the standard deviation is unaltered)?

[24] The length of time between breakdowns of an essential piece of equipment is important in the decision of the use of auxiliary equipment. An engineer thinks that the best model for time between breakdowns of a generator is the exponential distribution with a mean of 15 days.

(a) If the generator has just broken down, what is the probability that it will break down in the next 21 days?

(b) What is the probability that the generator will operate for 30 days without a breakdown?

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