Chapter 2

Lecture 5: Sets and Functions

26/3/2022

Book: Section 2.2

2.2 Set Operations

Set Operations

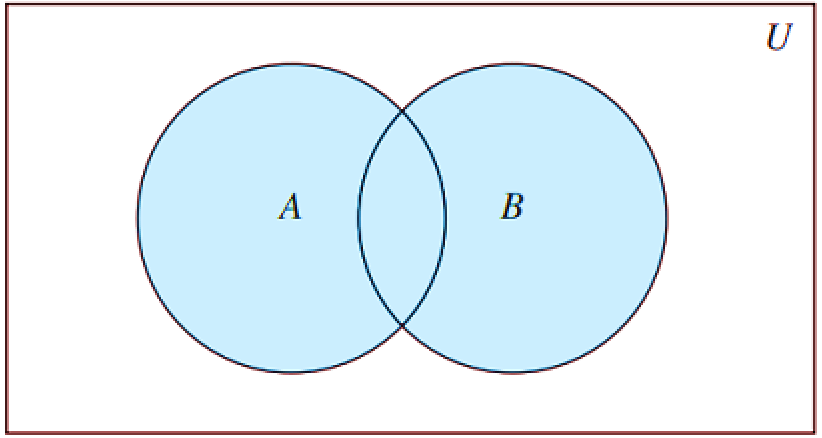
- Union
- Intersection
- Difference
- Disjoint
- Complement
- Generalized Unions
- Generalized Intersections

2.2 Set Operations

Set Operations: Union

- Let A and B be sets.
- The union of the sets A and B, denoted by $A \cup B$.
- $A \cup B$ is the set that contains those elements that are either in A or in B , or in both.

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$



$A \cup B$ is shaded.

2.2 Set Operations

Set Operations: Union

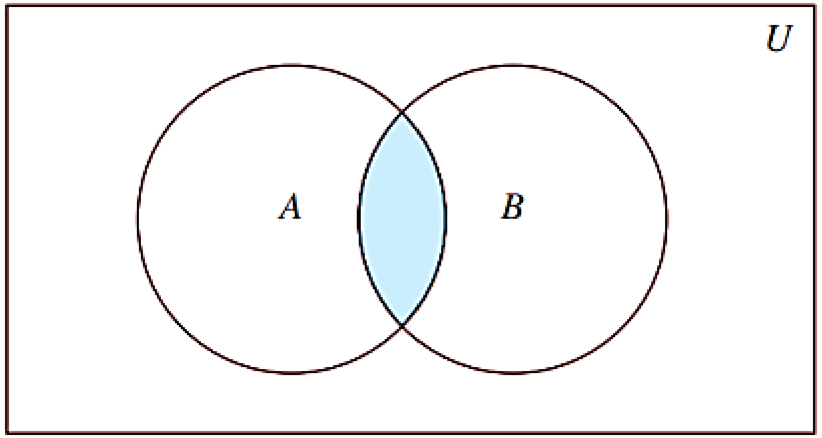
- The union of the sets {1, 3, 5} and {1, 2, 3}
- The union is the set {1, 2, 3, 5}

2.2 Set Operations

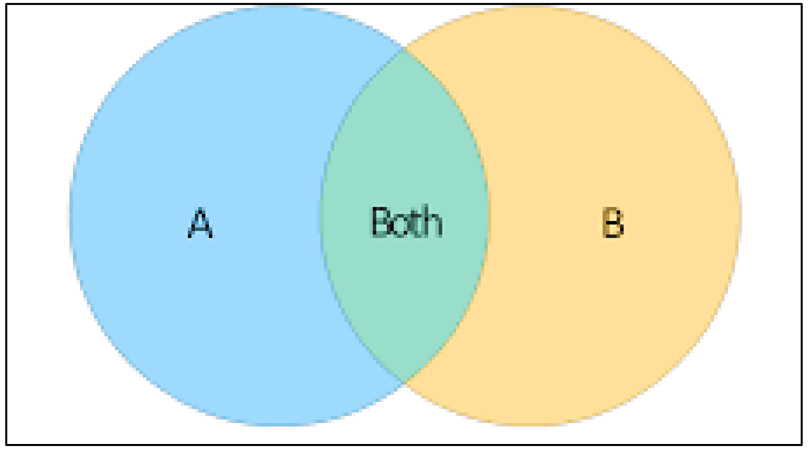
Set Operations: Intersection

- Let A and B be sets.
- The intersection of the sets A and B, denoted by $A \cap B$.
- $A \cap B$ is the set that contains those elements that are in both A and B.

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

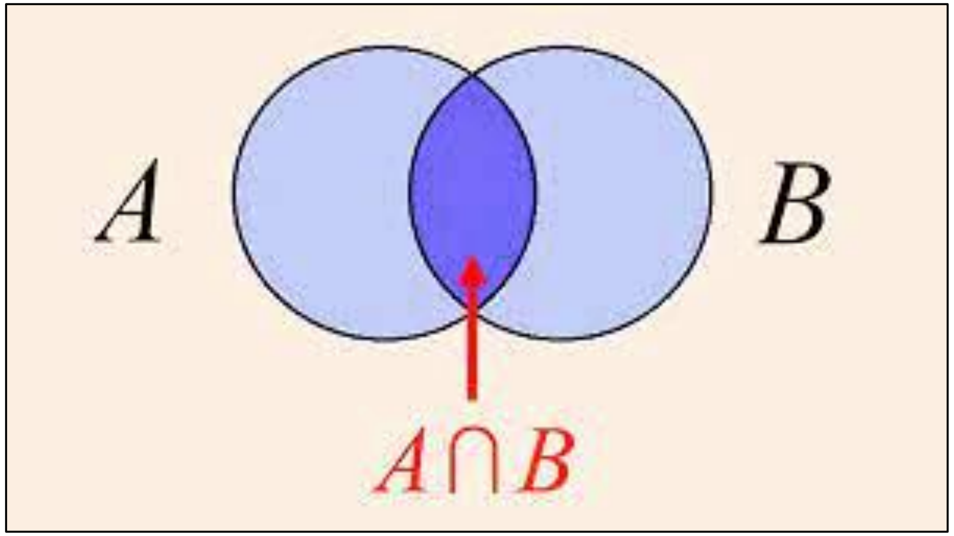


$A \cap B$ is shaded.



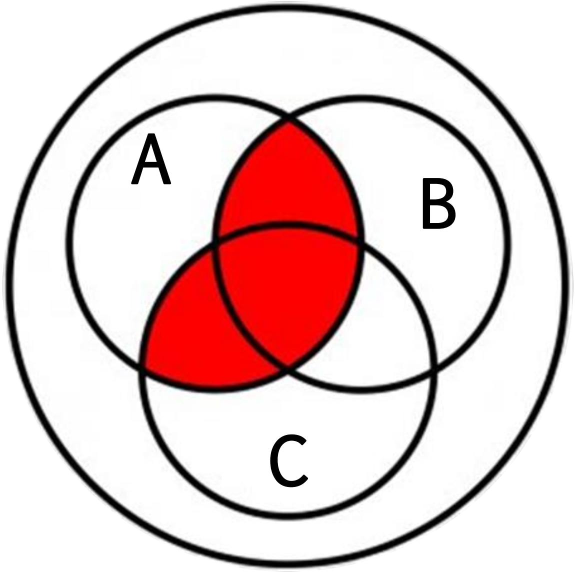
2.2 Set Operations

Set Operations: Intersection



- The Intersection of the sets {1, 3, 5} and {1, 2, 3}
- The Intersection is the set {1, 3}

- Name are the shaded regions

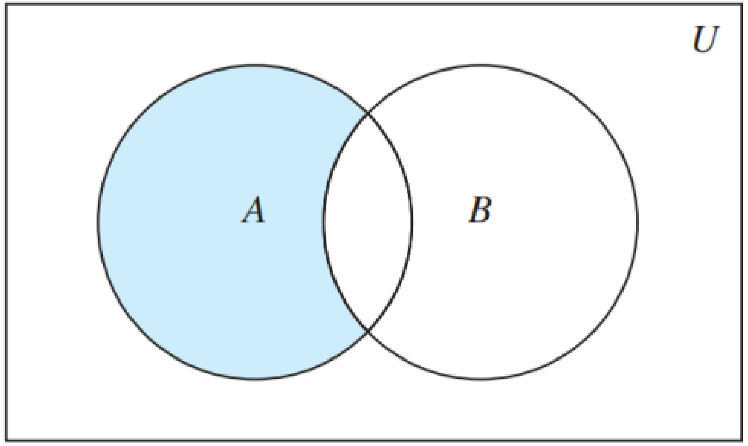


2.2 Set Operations

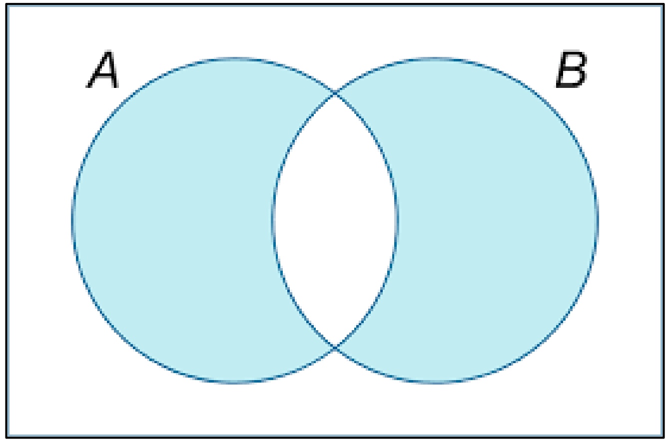
Set Operations: Difference

- Let A and B be sets.
- The difference of A and B, denoted by $A - B$
- $A - B$ is the set containing those elements that are in A but not in B.

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$



$A - B$ is shaded.



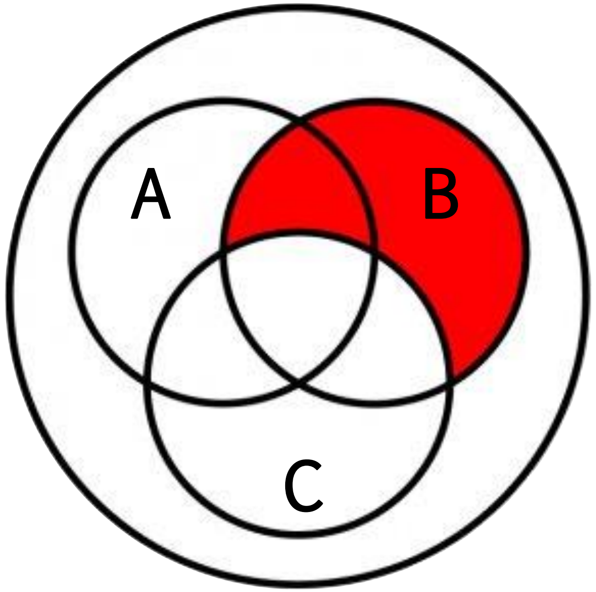
2.2 Set Operations

Set Operations: Difference

A = {1,3,5} , B = {1,2,3}

$A - B$ = {5}

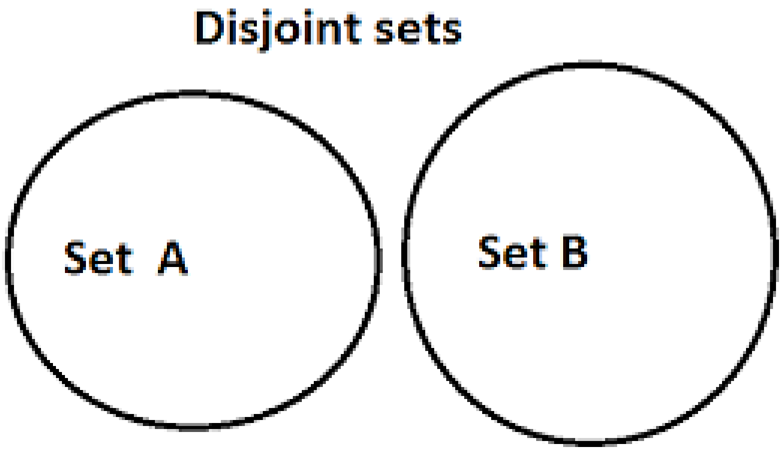
- Name are the shaded regions

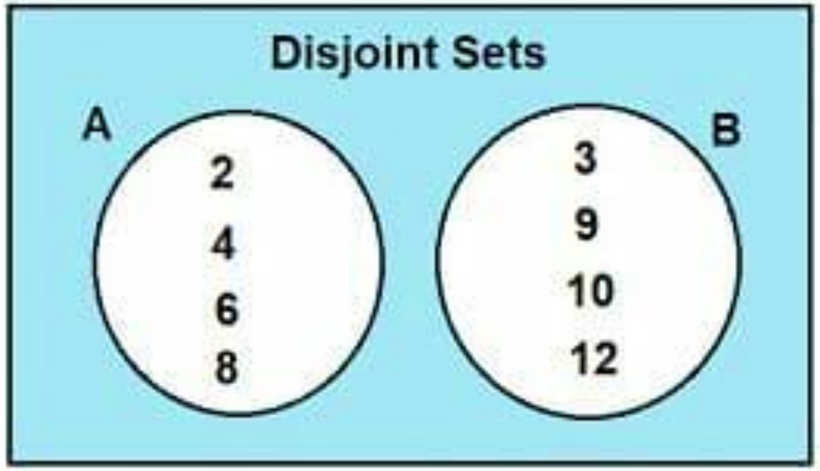


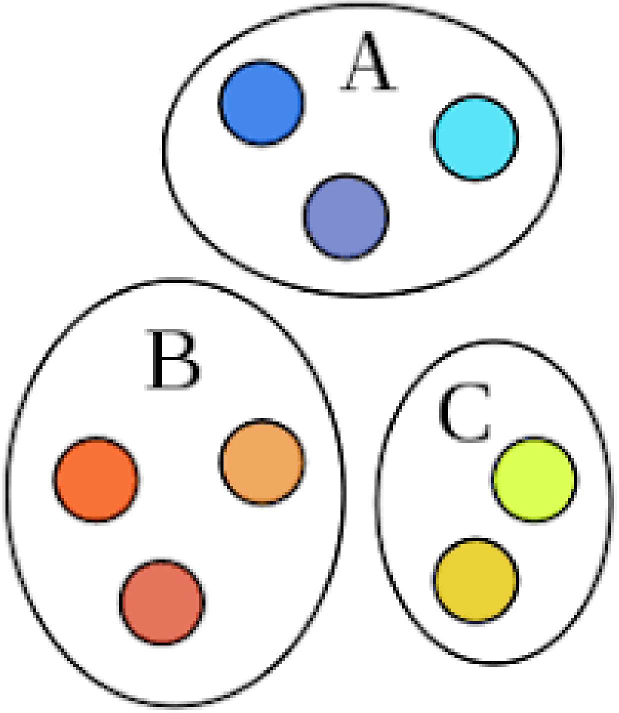
2.2 Set Operations

Set Operations: Disjoint

- Two sets are called disjoint if their intersection is the empty set.
- $A \cap B = \emptyset$





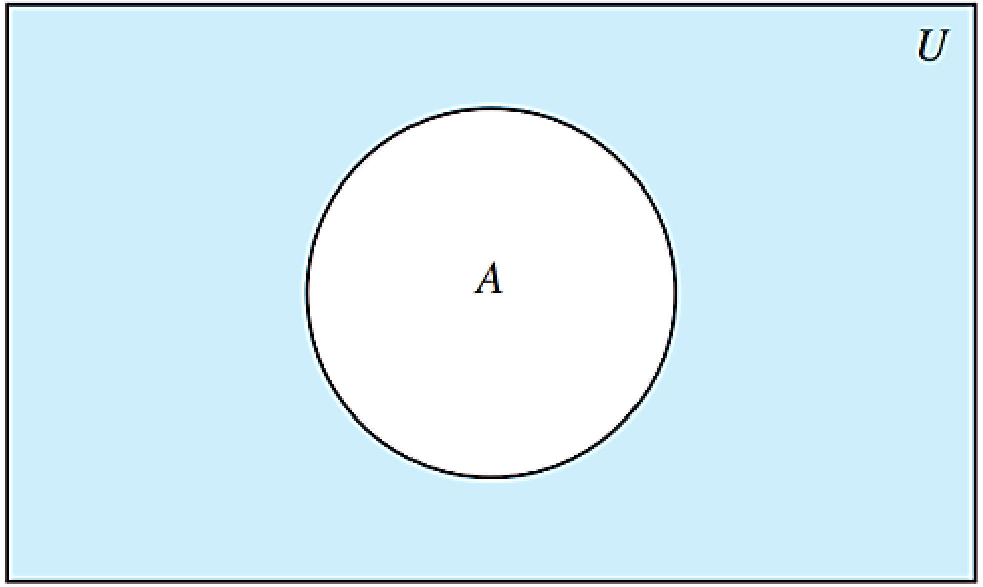


2.2 Set Operations

Set Operations: Complement

- Let U be the universal set.
- The complement of the set A, denoted by $\overline{\mathbf{A}}$
- An element x belongs to U if and only if $x \notin A$.

$$\overline{A} = \{x \in U \mid x \notin A\}$$

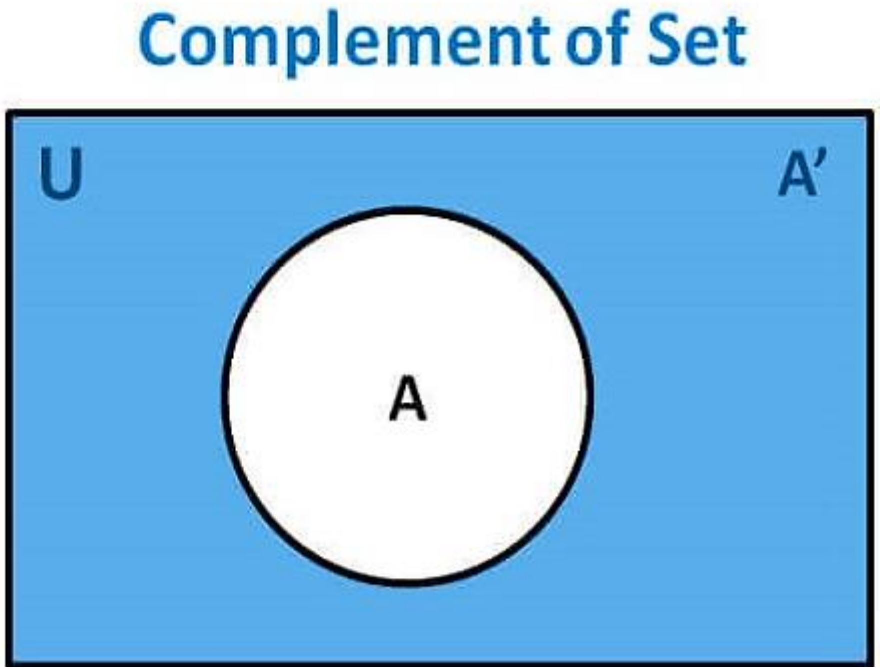


$\overline{A}$ is shaded.

2.2 Set Operations

Set Operations: Complement

- U = {1,2,3,4,5} , A = {1,3}
- $\overline{\mathbf{A}}$ = {2,4,5}



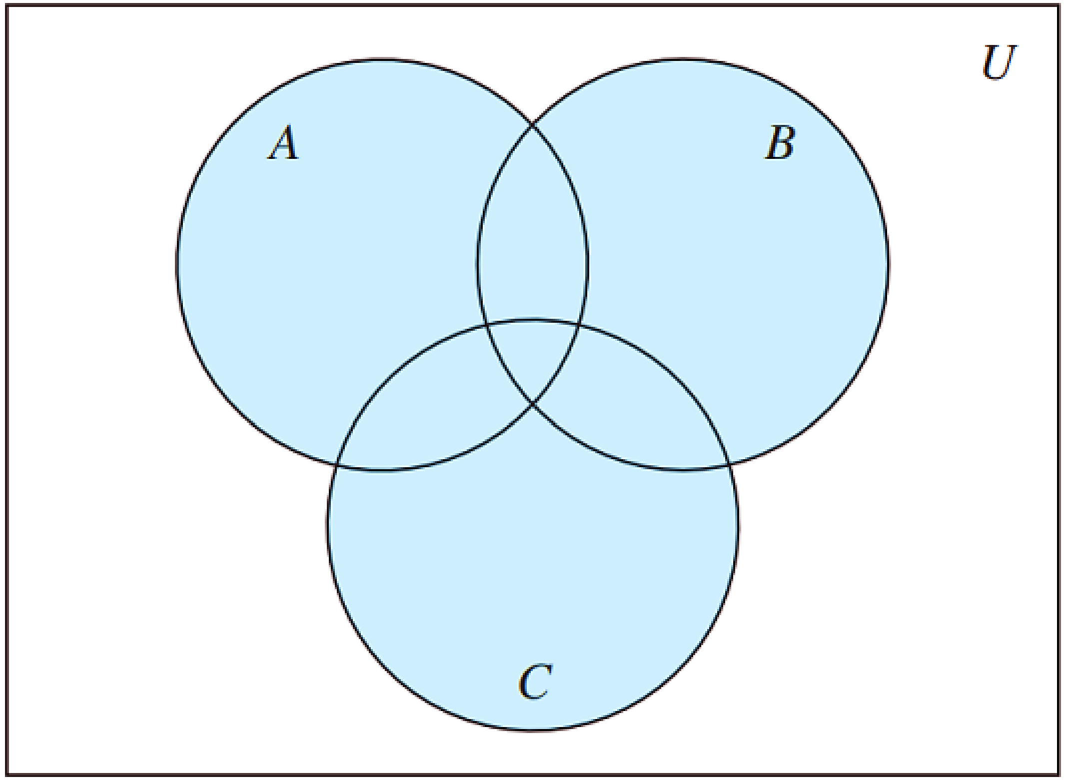
2.2 Set Operations

Set Operations: Generalized Unions

We use the notation

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$$

to denote the union of the sets $A_1, A_2, \ldots, A_n$.



$A \cup B \cup C$ is shaded.

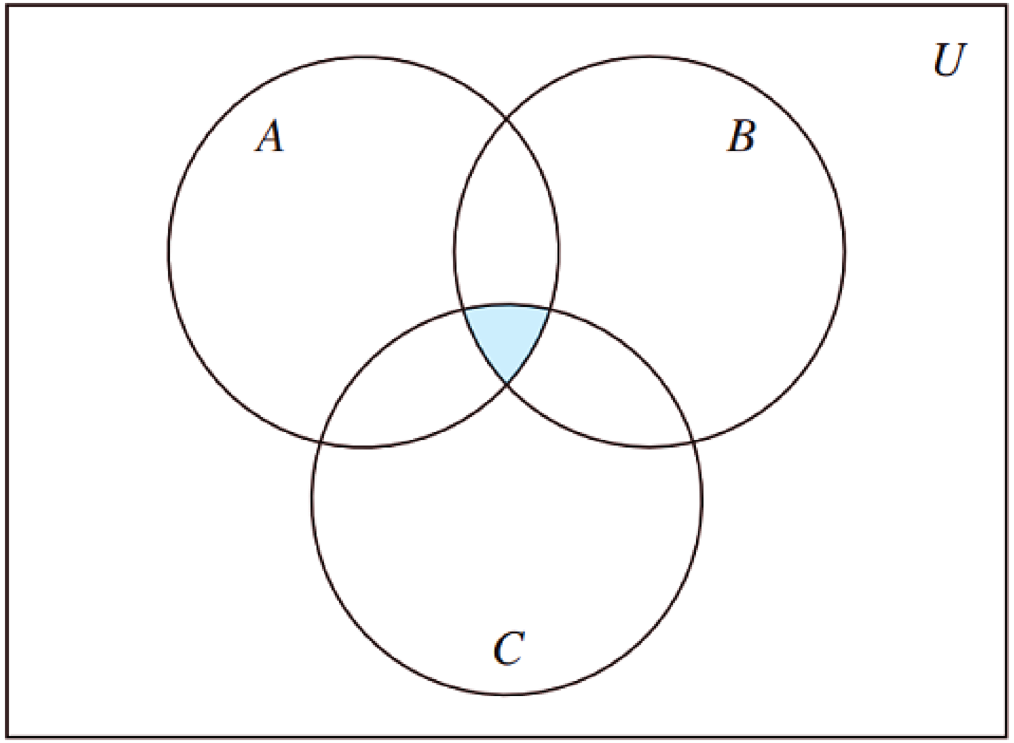
2.2 Set Operations

Set Operations: Generalized Intersections

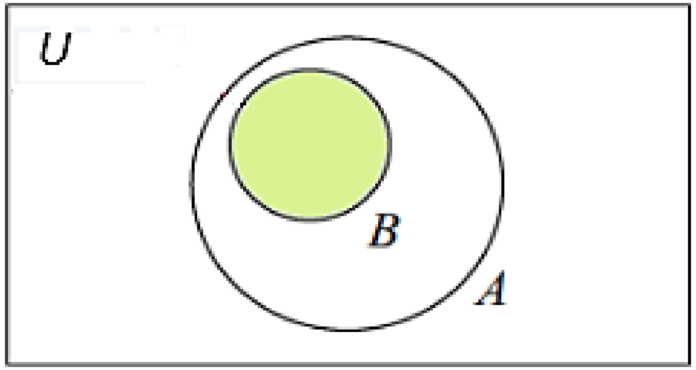
We use the notation

$$A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$$

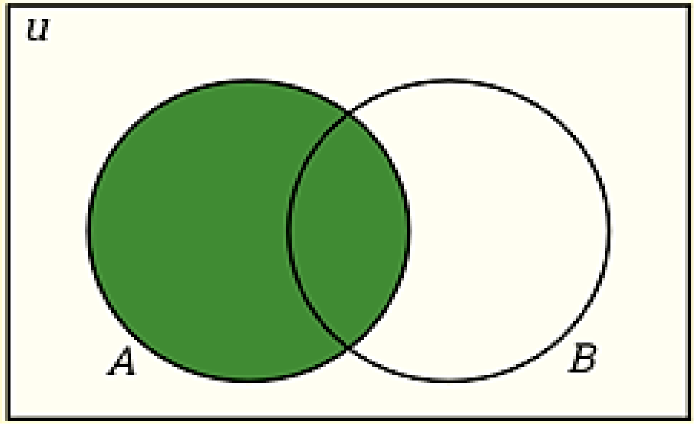
to denote the intersection of the sets $A_1, A_2, \ldots, A_n$.



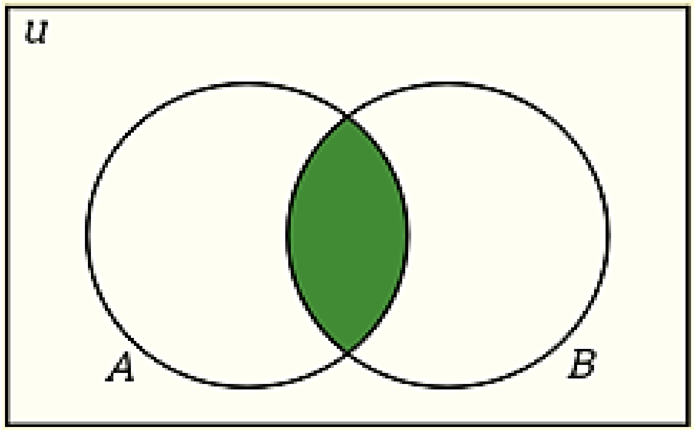
$A \cap B \cap C$ is shaded.



B is proper subset of *A* $B \subset A$

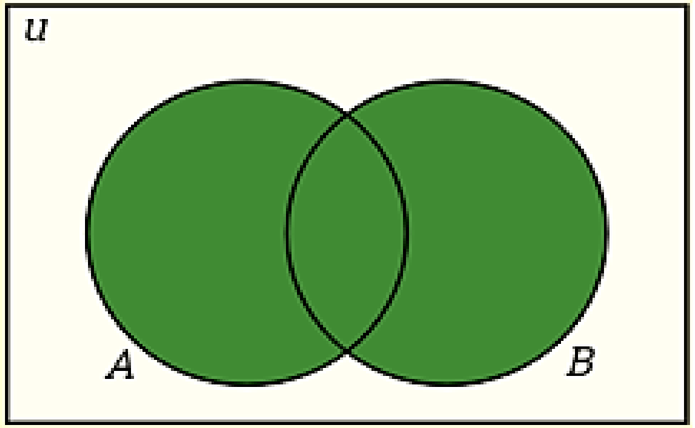


Set *A*



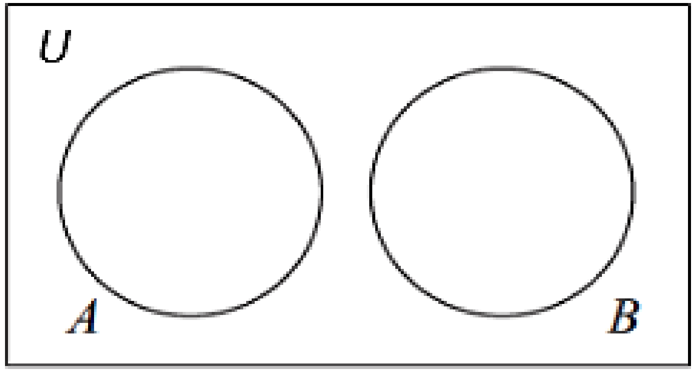
Both A and B $A \cap B$

A intersect *B*

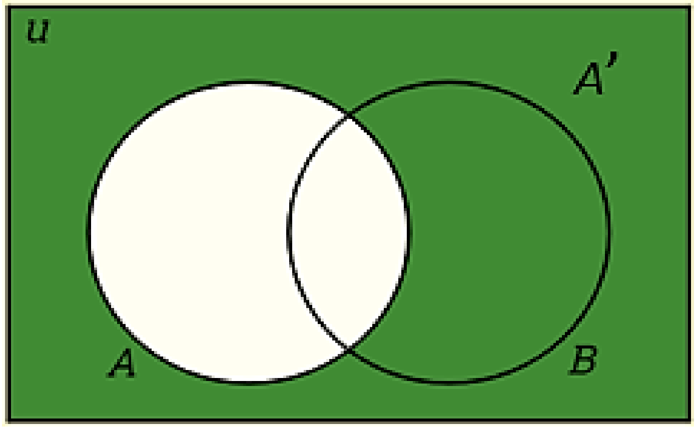


Either A or B $A \cup B$

A union *B*



A and *B* are disjoint sets



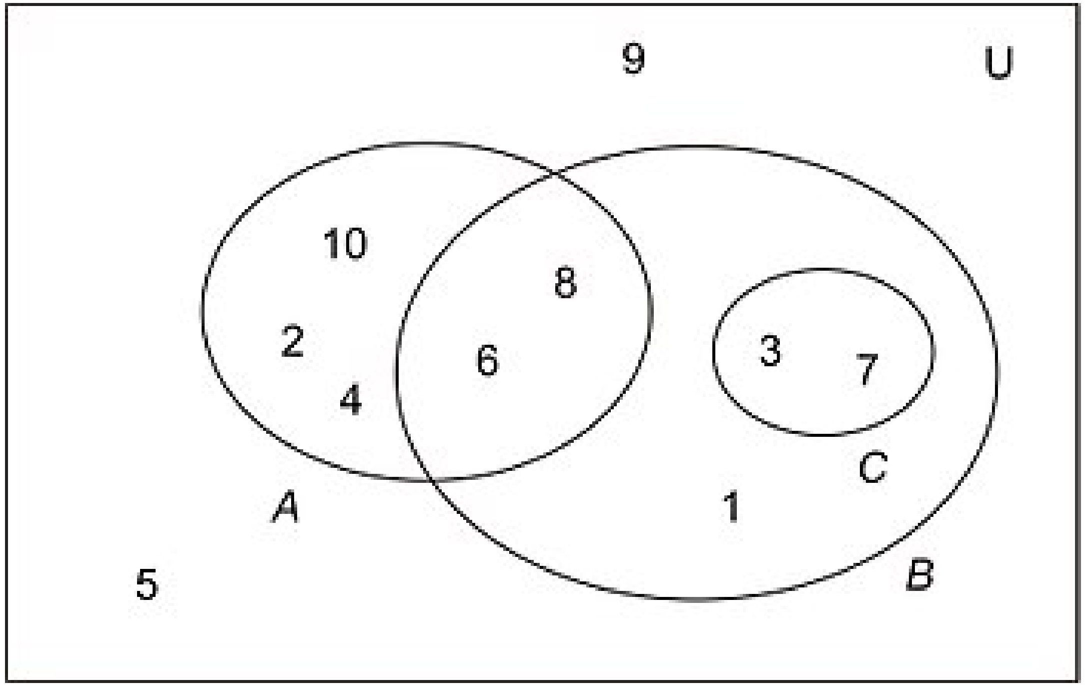
A’ the complement of *A*

Solve

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{2, 4, 6, 8, 10\}$
$B = \{1, 3, 6, 7, 8\}$
$C = \{3, 7\}$

(a) Illustrate the sets U, A, B and C in a Venn diagram and find:

1. $A \cap B$
2. $A \cup C$
3. $\bar{A}$
4. $\bar{B}$
5. $B \cap \bar{A}$
6. $B \cap \bar{C}$
7. $A - B$



1. $A \cap B = \{6, 8\}$
2. $A \cup C = \{2, 3, 4, 6, 7, 8, 10\}$
3. $\bar{A} = \{1, 3, 5, 7, 9\}$
4. $\bar{B} = \{2, 4, 5, 9, 10\}$
5. $B \cap \bar{A} = \{1, 3, 7\}$
6. $B \cap \bar{C} = \{1, 6, 8\}$
7. $A - B = \{2, 4, 10\}$

Solve

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {2, 4, 6, 8, 10}
B = {1, 3, 6, 7, 8}
C = {3, 7}

(b) Complete the statement using a single symbol: C - B = ...

C - B = Ø

Solve

Let $U = \{0,1,2,3,4,5,6,7,8,9,10\}$; $A = \{0,1,2,3,5,8\}$; $B=\{0,2,4,6\}$; $C = \{1,3,5,7\}$

1. $A \cup B =$ $\{0, 1, 2, 3, 4, 5, 6, 8\}$
2. $\overline{B} =$ $\{1, 3, 5, 7, 8, 9, 10\}$
3. $A \cap \overline{B} =$ $\{1, 3, 5, 8\}$ Hint: List the elements in $\overline{B}$ first
4. $B \cup C =$ $\{0, 1, 2, 3,4, 5, 6, 7\}$
5. $B \cup \overline{C} =$ $\{0, 2, 4, 6, 8, 9, 10\}$ Hint: list the elements of $\overline{C}$ first
6. $\overline{A} \cup C =$ $\{1, 3, 4, 5, 6, 7, 9, 10\}$ Hint: list the elements of $\overline{A}$ first
7. $(\overline{A} \cap C) \cup B =$ $\{0, 2, 4, 6, 7\}$ Hint: list the elements of $\overline{A}$, then $\overline{A} \cap C$ first
8. $\overline{(A \cup B)} =$ $\{7, 9, 10\}$
9. $(A \cup C) \cap B =$ $\{0, 2\}$

2.2 Set Operations

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cap \overline{B}$	$A \cup B$	$\overline{A \cup B}$
1	1	0	0	0	1	0
1	0	0	1	0	1	0
0	1	1	0	0	1	0
0	0	1	1	1	0	1

2.2 Set Operations

Example:

- **10 1010 1010.**
- **11 1110 0000.**

Find:

The bit string for the **union** of these sets.

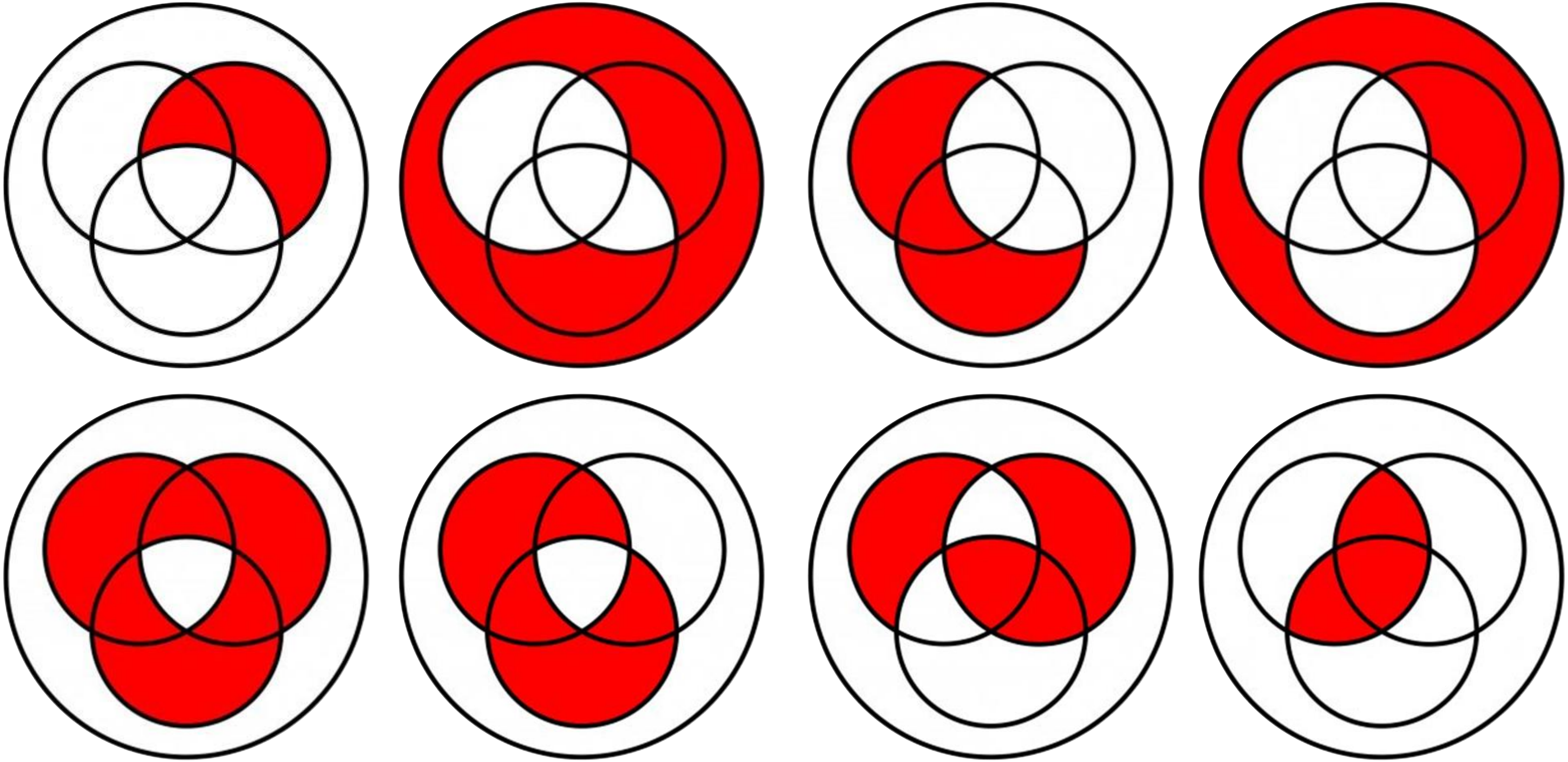
The bit string for the **intersection** of these sets.

Solution

- The bit string for the **union** of these sets is:
  - 11 1110 0000 $\vee$ 10 1010 1010 = 11 1110 1010
  - Corresponds to the set {l, 2, 3,4, 5, 7, 9}.
- The bit string for the **intersection** of these sets is:
  - 11 11100000 $\wedge$ 10 1010 1010 = 10 1010 0000
  - Corresponds to the set {l, 3, 5}.

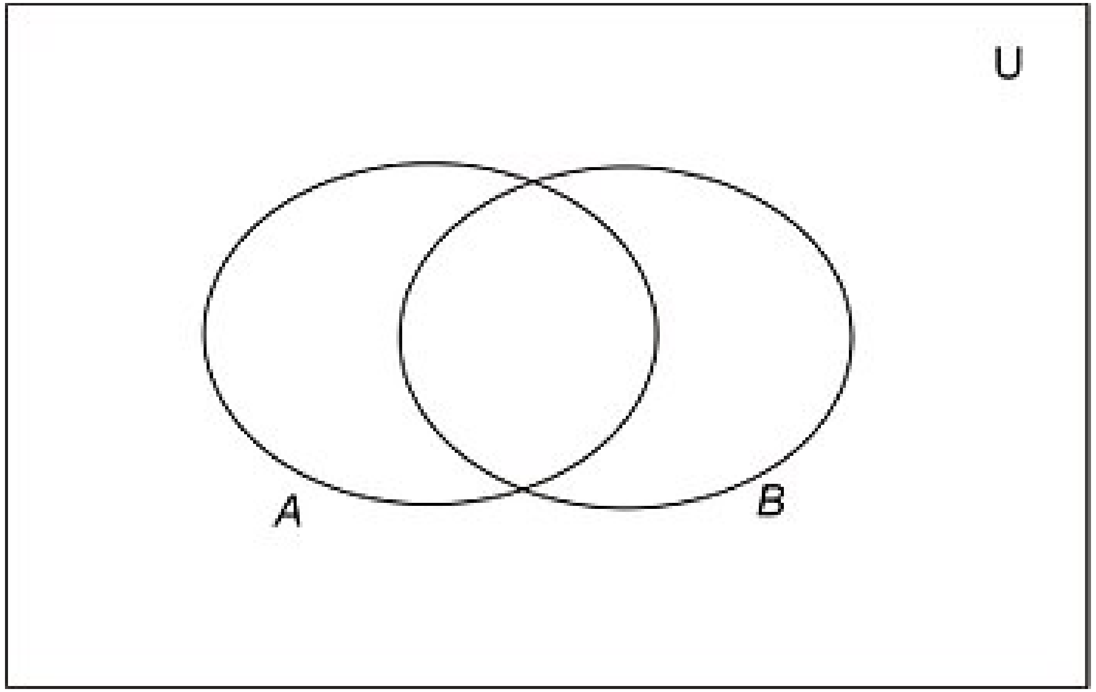
2.2 Set Operations

Homework



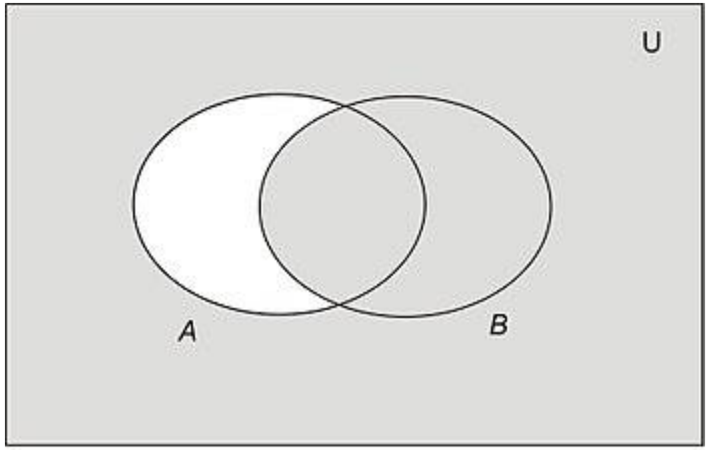
Make six copies of the Venn diagram shown alongside, and then shade the areas represented by:

(a) $A' \cup B$
(b) $A \cap B'$
(c) $(A \cap B)'$
(d) $A' \cup B'$
(e) $(A \cup B)'$
(f) $A' \cap B'$

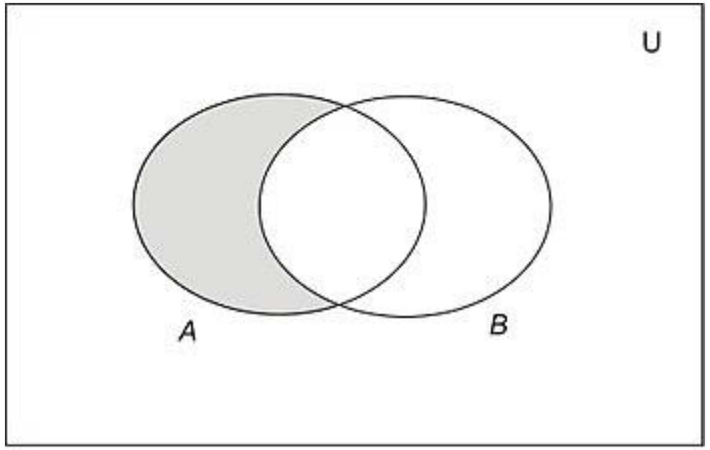


Solution

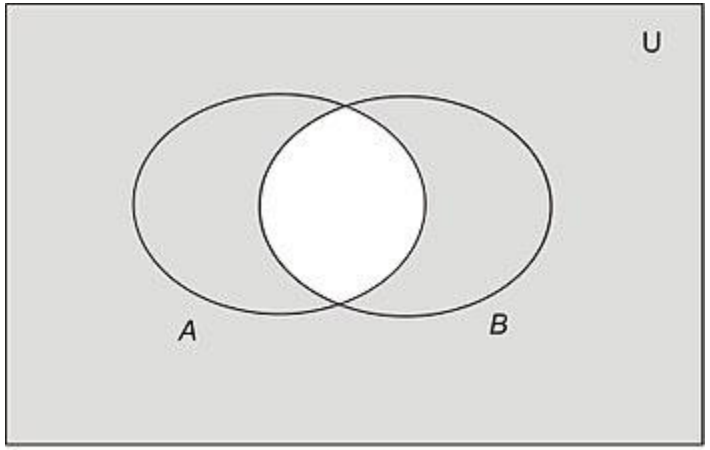
Make six copies of the Venn diagram shown alongside, and then shade the areas represented by:



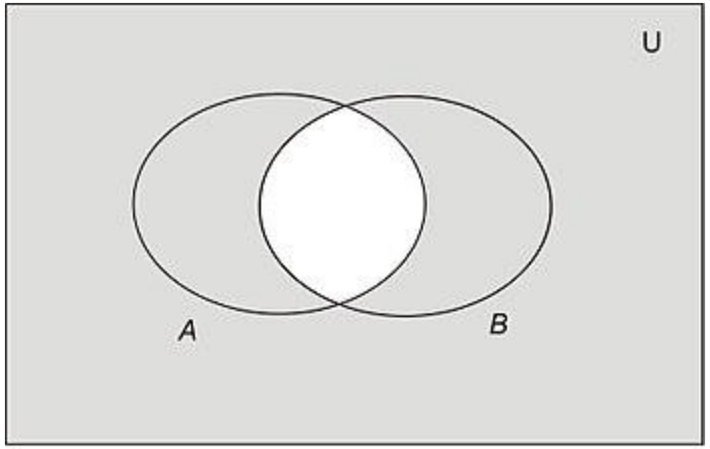
$A' \cup B$



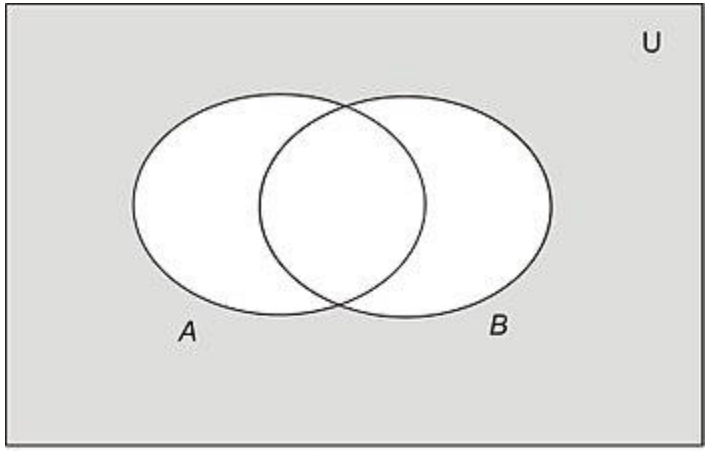
$A \cap B'$



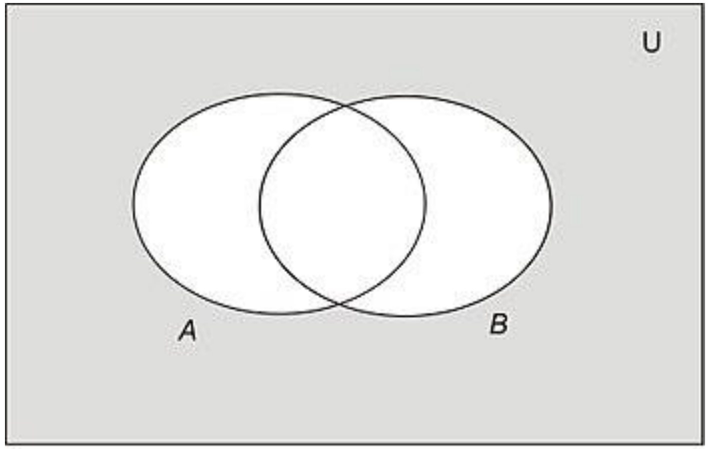
$(A \cap B)'$



$A' \cup B'$



$(A \cup B)'$



$A' \cap B'$

Thank you