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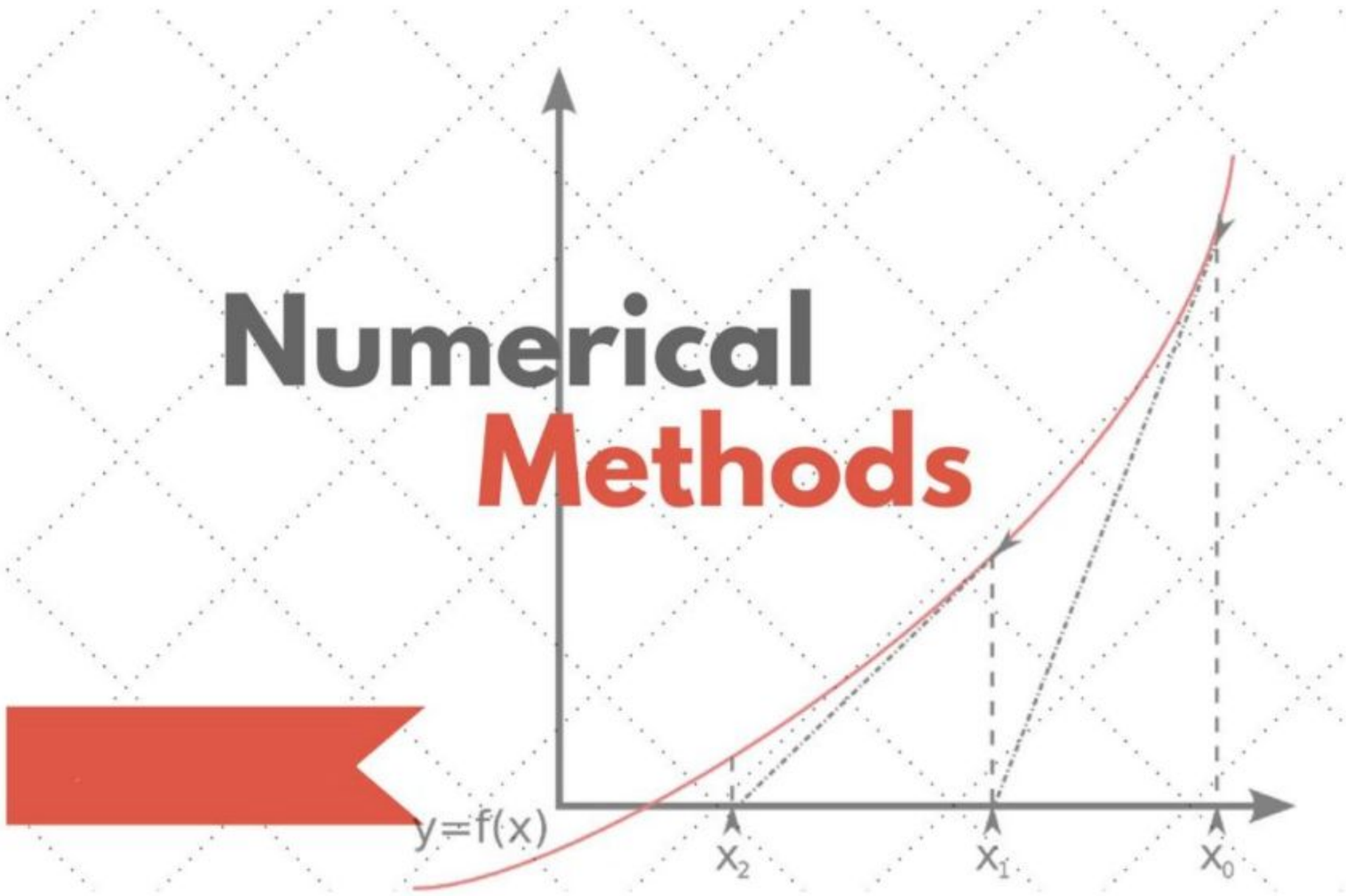


Numerical Methods

Interpolation Exercises

Interpolation

Exercises:- Using Lagrange interpolating Polynomials to approximate the following.

(1) Find $f(2.5)$

x	2	2.2	2.4	2.6	2.8	
$f(x)$	0.52	0.63	0.85	1.1	1.3	

$\rightarrow 5 \rightarrow L_4$

$\rightarrow x = 2.5$

$$L_0(x_0) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} = \frac{(x-2.2)(x-2.4)(x-2.6)(x-2.8)}{(2-2.2)(2-2.4)(2-2.6)(2-2.8)} = \frac{3}{128}$$

$$L_1(x_1) = \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} = \frac{(x-2)(x-2.4)(x-2.6)(x-2.8)}{(2.2-2)(2.2-2.4)(2.2-2.6)(2.2-2.8)} = \frac{-5}{32}$$

$$\therefore L_2(x_2) = \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} = \frac{(x-2)(x-2.2)(x-2.6)(x-2.8)}{(2.4-2)(2.4-2.2)(2.4-2.6)(2.4-2.8)} = \frac{45}{64}$$

$$L_3(x_3) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} = \frac{(x-2)(x-2.2)(x-2.4)(x-2.8)}{(2.6-2)(2.6-2.2)(2.6-2.4)(2.6-2.8)} = \frac{15}{32}$$

$$L_4(x_4) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} = \frac{(x-2)(x-2.2)(x-2.4)(x-2.6)}{(2.8-2)(2.8-2.2)(2.8-2.4)(2.8-2.6)} = \frac{-5}{128}$$

$$f(2.5) = \frac{3}{128}(0.52) + \left(\frac{-5}{32}\right)(0.63) + \left(\frac{45}{64}\right)(0.85) + \left(\frac{15}{32}\right)(1.1)$$

$$+ \left(\frac{-5}{128}\right)(1.3) = 0.9765425 \simeq 0.9765.$$

(2) Find Cosh(1.1)

x	1	1.2	1.3	1.4	1.5
Cosh(x)	1.543	1.811	1.971	2.151	2.352

$$L_0(x_0) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x-x_3)(x-x_4)} = \frac{(x-1.2)(x-1.3)(x-1.4)(x-1.5)}{(1-1.2)(1-1.3)(1-1.4)(1-1.5)} = \frac{1}{5}$$

$$L_1(x_1) = \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x-x_4)} = \frac{(x-1)(x-1.3)(x-1.4)(x-1.5)}{(1.2-1)(1.2-1.3)(1.2-1.4)(1.2-1.5)} = 2$$

$$L_2(x_2) = \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} = \frac{(x-1)(x-1.2)(x-1.4)(x-1.5)}{(1.3-1)(1.3-1.2)(1.3-1.4)(1.3-1.5)} = -2$$

$$L_3(x_3) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} = \frac{(x-1)(x-1.2)(x-1.3)(x-1.5)}{(1.4-1)(1.4-1.2)(1.4-1.3)(1.4-1.5)} = 1$$

$$L_4(x_4) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} = \frac{(x-1)(x-1.2)(x-1.3)(x-1.4)}{(1.5-1)(1.5-1.2)(1.5-1.3)(1.5-1.4)} = \frac{-1}{5}$$

$$f(1.1) = \left(\frac{1}{5}\right)(1.543) + (2)(1.811) + (-2)(1.971) + (1)(2.151) + \left(\frac{-1}{5}\right)(2.352)$$
$$= 1.6692$$

(3) Find $e^{2.2}$

x	1.7	1.9	2	2.1	2.3
e^x	5.474	6.686	7.389	8.166	9.974

$$L_0(x_0) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} = \frac{(x-1.9)(x-2)(x-2.1)(x-2.3)}{(1.7-1.9)(1.7-2)(1.7-2.1)(1.7-2.3)} = \frac{-1}{24}$$

$$L_1(x_1) = \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} = \frac{(x-1.7)(x-2)(x-2.1)(x-2.3)}{(1.9-1.7)(1.9-2)(1.9-2.1)(1.9-2.3)} = \frac{5}{8}$$

$$L_2(x_2) = \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} = \frac{(x-1.7)(x-1.9)(x-2.1)(x-2.3)}{(2-1.7)(2-1.9)(2-2.1)(2-2.3)} = \frac{-5}{3}$$

$$L_3(x_3) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} = \frac{(x-1.7)(x-1.9)(x-2)(x-2.3)}{(2.1-1.7)(2.1-1.9)(2.1-2)(2.1-2.3)} = \frac{15}{8}$$

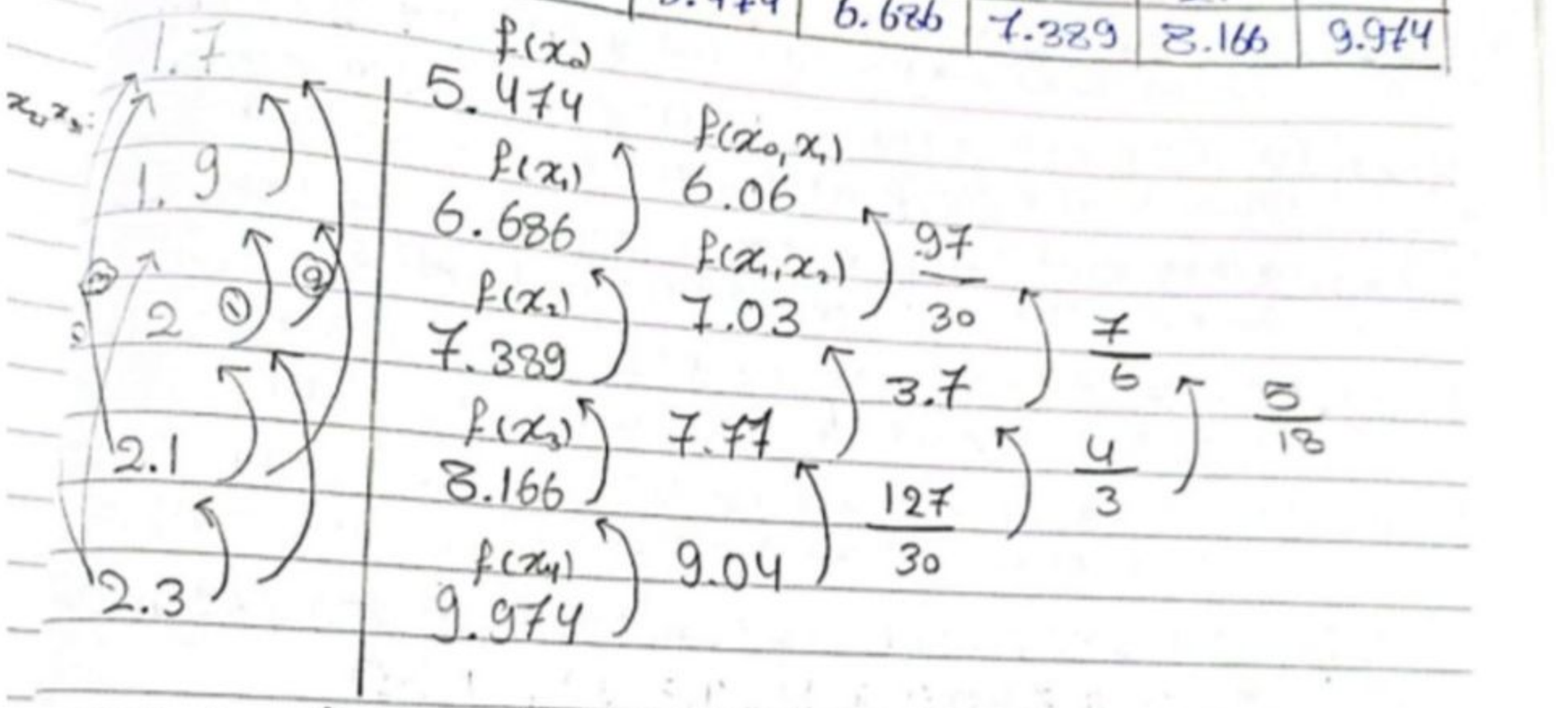
$$L_4(x_4) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} = \frac{(x-1.7)(x-1.9)(x-2)(x-2.1)}{(2.3-1.7)(2.3-1.9)(2.3-2)(2.3-2.1)} = \frac{5}{24}$$

$$f(e^{2.2}) = \left(\frac{-1}{24}\right)(5.474) + \left(\frac{5}{8}\right)(6.686) + \left(\frac{-5}{3}\right)(7.389) + \left(\frac{15}{8}\right)(8.166) + \left(\frac{5}{24}\right)(9.974) = 9.0248$$

*Exercises: Using Newton's interpolation method to evaluate the following:

(i) Find $e^{1.8}$

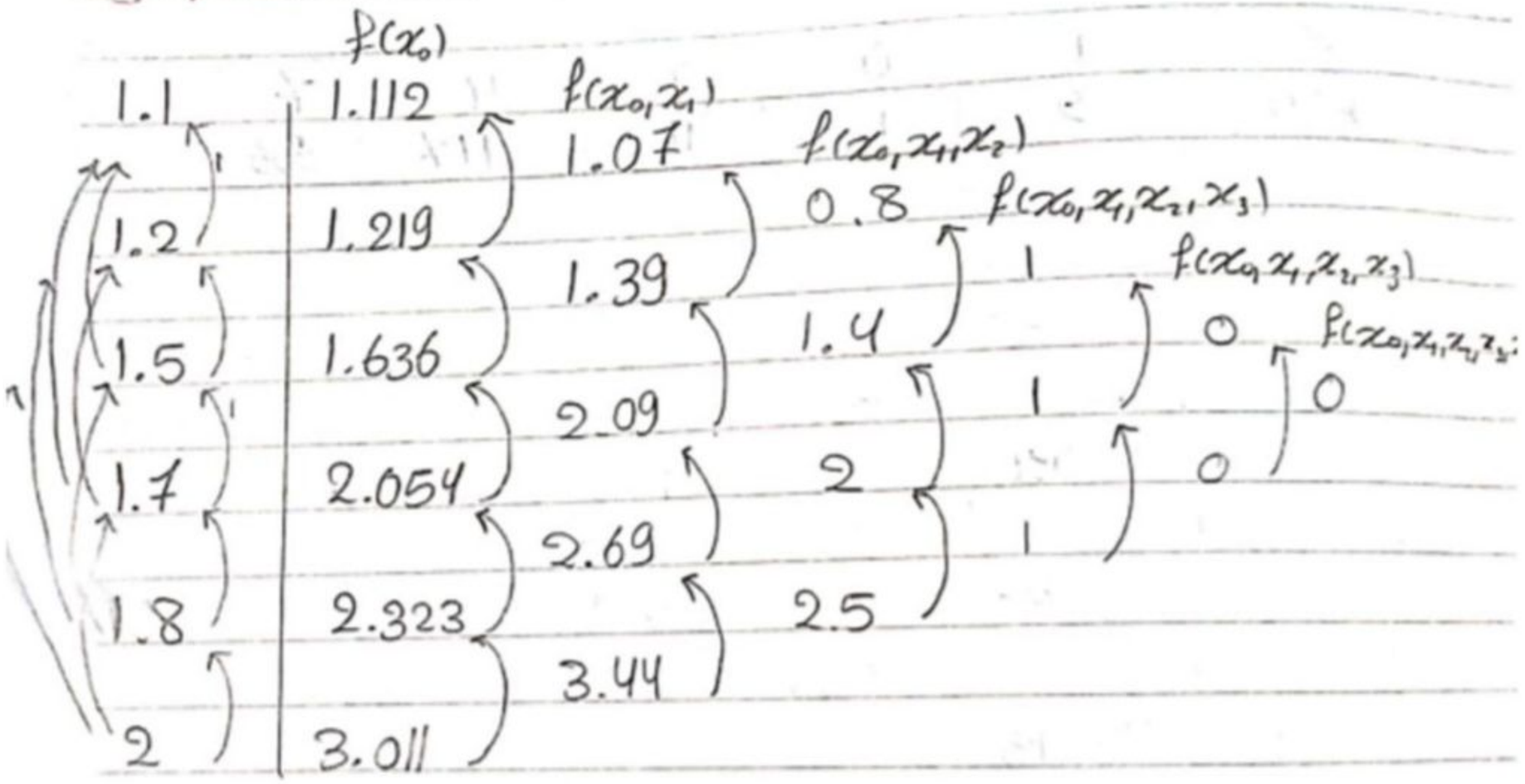
x	1.7	1.9	2	2.1	2.3
e^x	5.474	6.686	7.389	8.166	9.974



$$P(x) = f(x_0) + f(x_0,x_1)(x-x_0) + f(x_0,x_1,x_2)(x-x_0)(x-x_1) + f(x_0,x_1,x_2,x_3)(x-x_0)(x-x_1)(x-x_2) + f(x_0,x_1,x_2,x_3,x_4)(x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

$$f(1.8) = 6.04983$$

[2] Find f(1.3) and f(1.95)



$f(1.3) = 1.112 + (1.07)(1.3-1.1) + (0.8)(1.3-1.1)(1.3-1.2)$
$+ (1)(1.3-1.1)(1.3-1.2)(1.3-1.5) = 1.338$

$f(1.95) = (1.112) + (1.07)(1.95-1.1) + (0.8)(1.95-1.1)(1.95-1.2)$
$+ (1)(1.95-1.1)(1.95-1.2)(1.95-1.5) = 3.566375$

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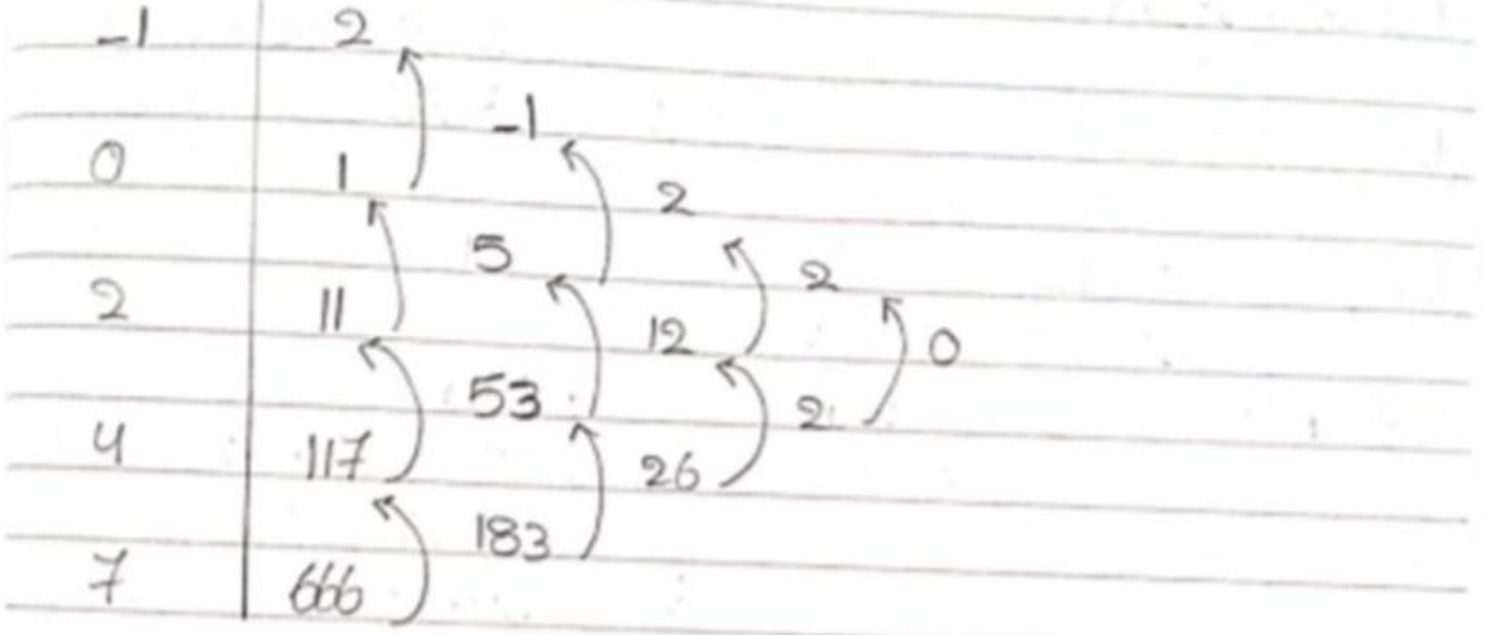
[3] Find f(2.5)

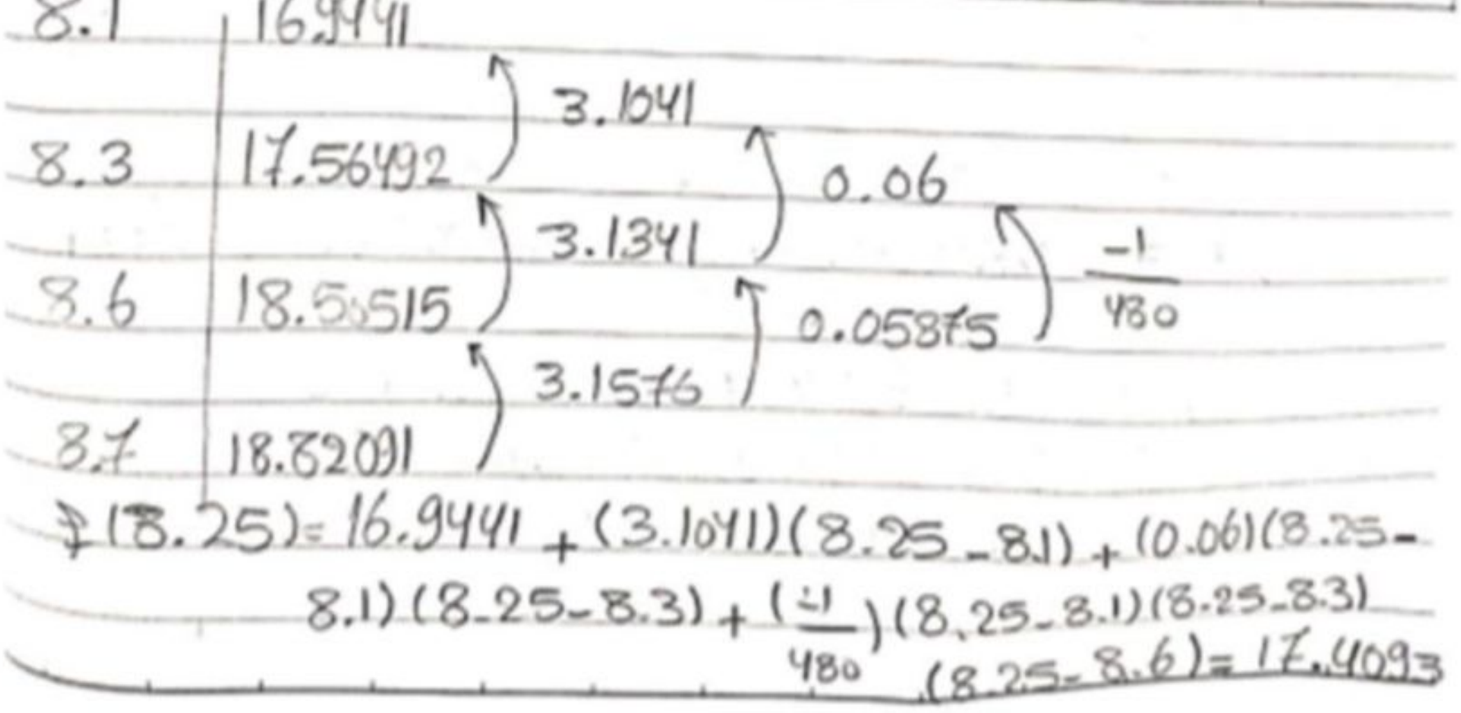
x	-1	0	2	4	7
f(x)	2	1	11	117	666



$f(2.5) = 2 + (-1)(2.5+1) + (2)(2.5+1)(2.5-0) + (2)(2.5+1)(2.5-0)(2.5-2) = 24.75$

[4] Find f(8.25)

x	8.1	8.3	8.6	8.7
f(x)	16.9441	17.56492	18.58615	18.82091



$f(8.25) = 16.9441 + (3.1041)(8.25-8.1) + (0.06)(8.25-8.1)(8.25-8.3) + (\frac{-1}{480})(8.25-8.1)(8.25-8.3)(8.25-8.6) = 17.4093$

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[5] Find f(0.05) and f(0.75)

x:	0	0.2	0.4	0.6	0.8
f(x)	1	1.2214	1.4918	1.8221	2.2255

$x = x_0 + sh$

$0.05 = 0 + s(0.2)$

$\therefore s = 0.25$

$x = x_0 + sh$

$0.75 = 0 + s(0.2)$

$\therefore s = 3.75$

$f(x_0)$

$f(x_0)$	$\Delta f(x_0)$	$\Delta^2 f(x_0)$	$\Delta^3 f(x_0)$	$\Delta^4 f(x_0)$
1	0.2214			
1.2214		0.049		
	0.2704		0.0109	
1.4918		0.0599		$\frac{23}{10000}$
	0.3303		0.0132	
1.8221		0.0731		
	0.4034			
2.2255				

$$f(0.05) = f(x_0) + \Delta f(x_0)s + \frac{\Delta^2 f(x_0)}{2!}s(s-1) + \frac{1}{3!}\Delta^3 f(x_0)s(s-1)(s-2) + \frac{1}{4!}\Delta^4 f(x_0)s(s-1)(s-2)(s-3)$$

$$= (1) + (0.2214)(0.25) + \frac{1}{2}(0.049)(0.25)(0.25-1) + \frac{1}{6}(0.0109)(0.25)(0.25-1)(0.25-2)(0.25-3)$$

$$= 1.04912$$

$$f(0.75) = f(x_n) + \nabla f(x_n)s + \frac{1}{2!}\nabla^2 f(x_n)s(s+1) + \frac{1}{3!}\nabla^3 f(x_0)s(s+1)(s+2) + \frac{1}{4!}\nabla^4 f(x_0)s(s+1)(s+2)(s+3) =$$

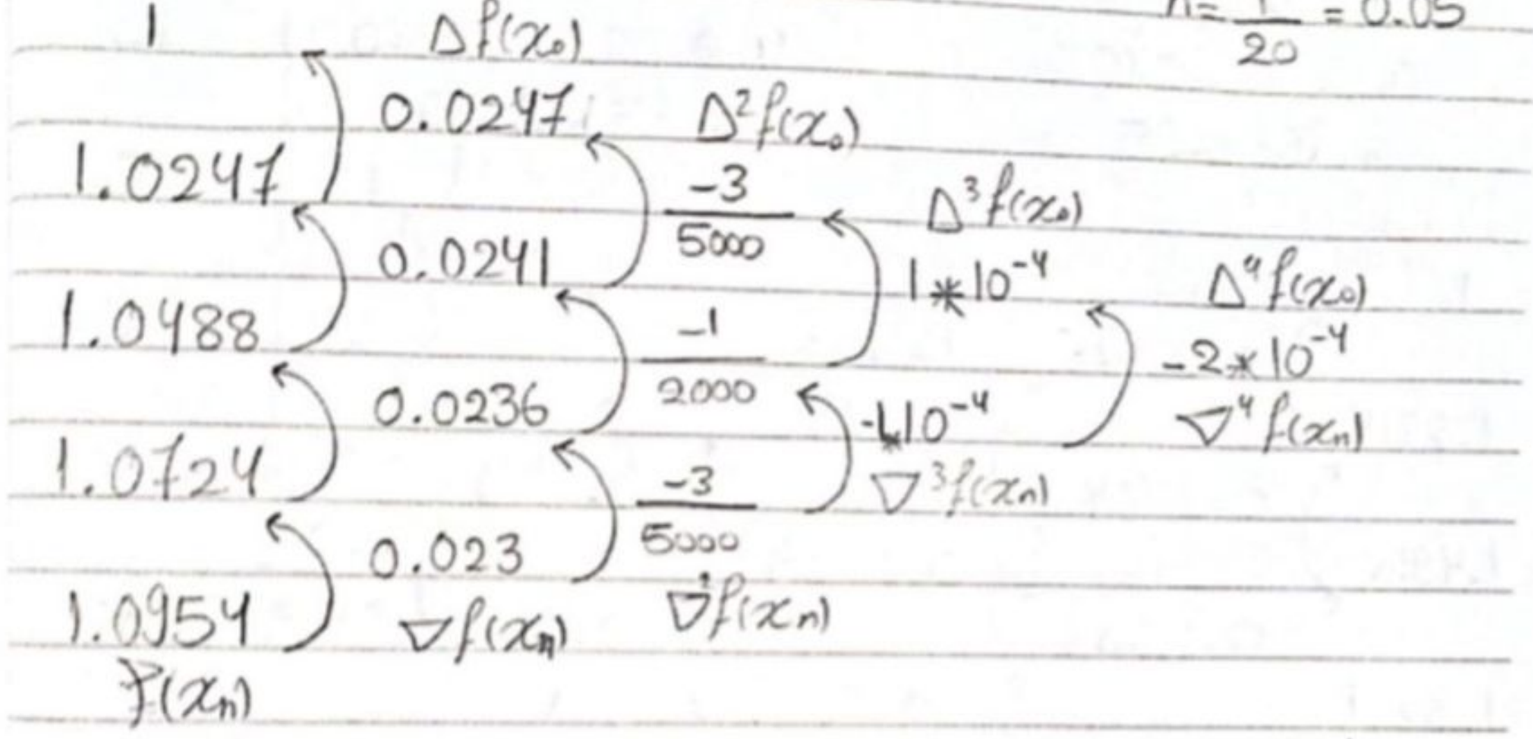
$$= 2.2255 + (0.4034)(3.75) + \frac{1}{2}(0.0731)(3.75)(3.75+1) + (0.0132)(3.75)(3.75+1)(3.75+2) + \left(\frac{23}{10000}\right)(3.75)(3.75+1)(3.75+2)(3.75+3) =$$

$$5.9487$$

6] Find $(\sqrt{1.03})$ & $(\sqrt{1.23})$

x	1	1.05	1.1	1.15	1.2
$f(\sqrt{x})$	1	1.0247	1.0488	1.0724	1.0954

$h = \frac{1}{20} = 0.05$



$x = x_0 + sh$ $\quad$ $x = x_n + sh$

$1.03 = 1 + s(0.05)$ $\quad$ $1.23 = 1.2 + s(0.05)$

$\therefore s = 0.6$ $\quad$ $\therefore s = -0.6$

$$f(\sqrt{1.03}) = f(x_0) + \Delta f(x_0)s + \frac{\Delta^2 f(x_0)}{2!}s(s-1) + \frac{\Delta^3 f(x_0)}{3!}s(s-1)(s-2) + \frac{\Delta^4 f(x_0)}{4!}s(s-1)(s-2)(s-3) =$$

$$= 1.0149$$

$$f(\sqrt{1.23}) = f(x_n) + \nabla f(x_n)s + \frac{\nabla^2 f(x_n)}{2!}s(s+1) + \frac{\nabla^3 f(x_n)}{3!}s(s+1)(s+2) + \frac{\nabla^4 f(x_n)}{4!}s(s+1)(s+2)(s+3) =$$

$$= (1.0954) + (0.023)(0.6) + \frac{1}{2}\left(\frac{-3}{5000}\right)(0.6)(0.6+1) + \frac{1}{3!}(-1*10^{-4})(0.6)(0.6+1)(0.6+2) + \frac{1}{4!}(-2*10^{-4})(0.6)(0.6+1)(0.6+2)(0.6+3) = 1.168795$$

7. Find ln(4.1) & ln(4.9)

x	2	2.2	2.4	2.6	2.8	3
ln(x+2)	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094

$x = x_n + sh$

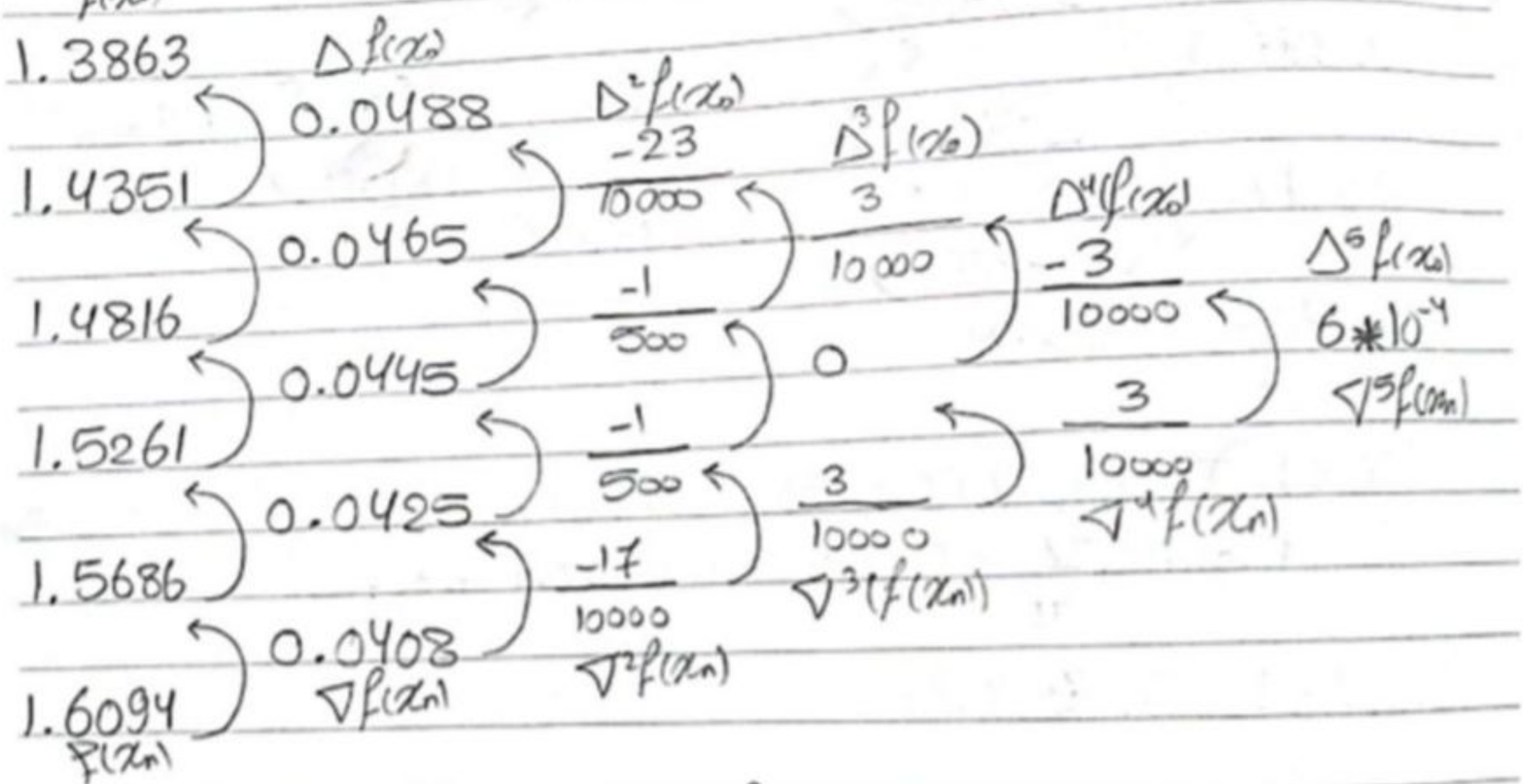
$4.1 = 3 + (0.2)s$

$\therefore s = 5.5$

$x = x_n + sh$

$4.9 = 3 + (0.2)s$

$s = 9.5$



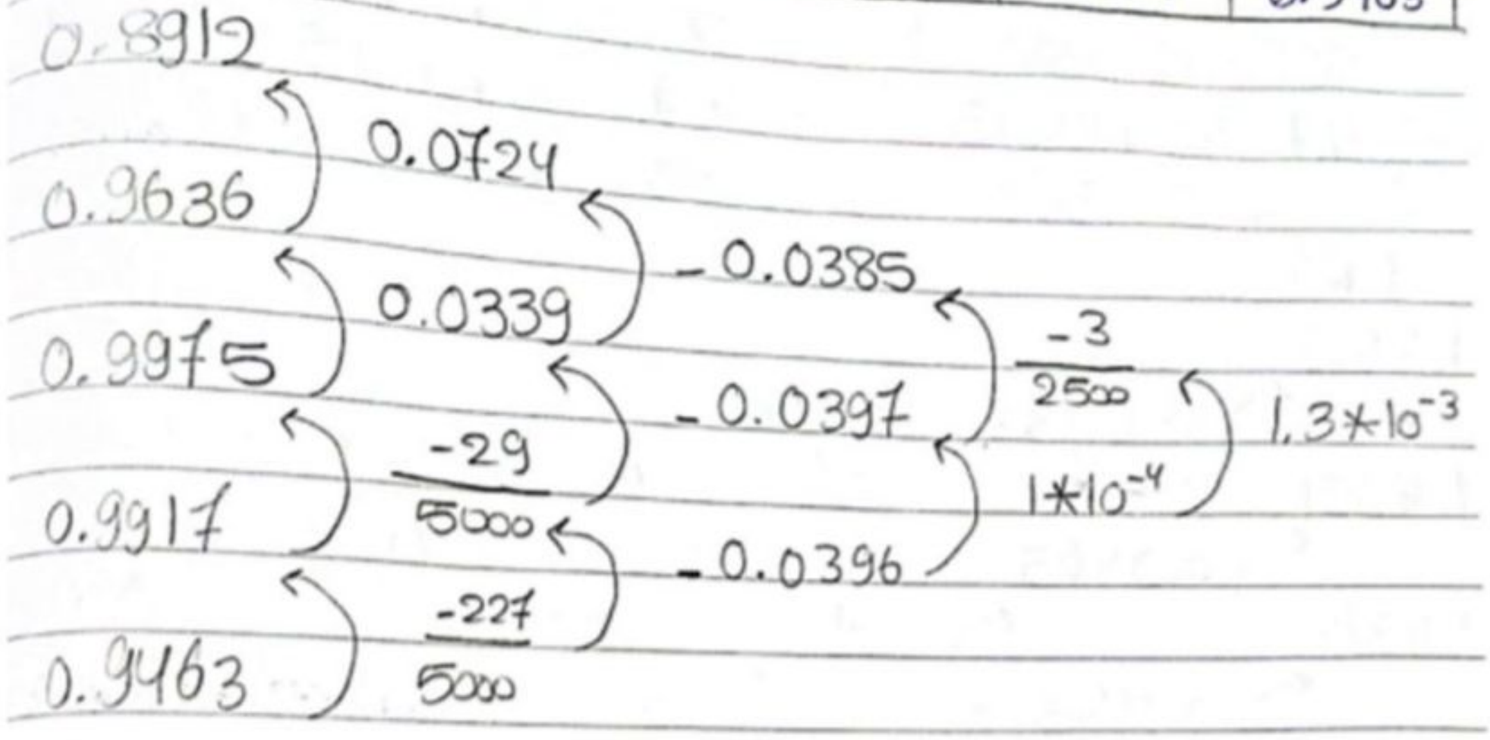
$$\ln(x+2) = f(x_n) + \nabla f(x_n)s + \frac{\nabla^2 f(x_n)s(s+1)}{2!} + \frac{\nabla^3 f(x_n)s(s+1)(s+2)}{3!} + \frac{\nabla^4 f(x_n)s(s+1)(s+2)(s+3)}{4!} + \frac{\nabla^5 f(x_n)s(s+1)(s+2)(s+3)(s+4)}{5!} =$$

$\ln(4.1) = 1.680079$

$\ln(4.9) = 3.11669$

[5] Find Sin(1.2) & Sin(1.85)

x	0.1	0.3	0.5	0.7	0.9
Sin(x)11	0.8912	0.9636	0.9975	0.9917	0.9463



$$f(x_n) = f(x_n) + \nabla f(x_n) S + \frac{\nabla^2 f(x_n) S(S+1)}{2!} + \frac{\nabla^3 f(x_n) S(S+1)(S+2)}{3!} + \frac{\nabla^4 f(x_n) S(S+1)(S+2)(S+3)}{4!} =$$

Sin(1.2) = 0.80736

Sin(1.85) = 0.270327

$x_1 = x_n + Sh$
$1.2 = 0.9 + (0.2)S$
$S = 1.5$

$x = x_n + Sh$
$1.85 = 0.9 + (0.2)S$
$S = 4.75$

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