Sheet 1

Sequence and series (part one)

(1) Which of the following sequences $\{a_n\}$ is converge and which is diverge:

1) $a_n = n$

2) $a_n = \dfrac{n}{n-1}$

3) $a_n = \left(\dfrac{1}{3}\right)^{2n}$

4) $a_n = \left(\dfrac{5}{4}\right)^{n-1}$

5) $a_n = \dfrac{n+1}{n^2+3n}$

6) $a_n = \dfrac{n^2+1}{\sqrt{n^2+3n}}$

7) $a_n = \dfrac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}$

8) $a_n = \left(1+\dfrac{1}{n}\right)^n$

9) $a_n = \left(\dfrac{n-3}{n}\right)^{2n}$

10) $a_n = \dfrac{\ln(n)}{\ln(2n)}$

11) $a_n = (-1)^n$

12) $a_n = \sqrt[n]{n}$

(2) find the sum of the following series if it exists

1) $\sum_{n=0}^{\infty} \left(\dfrac{1}{2}\right)^n$

2) $\sum_{n=0}^{\infty} \dfrac{2^{n+3}}{3^n}$

3) $\sum_{n=0}^{\infty} (-1)^n \dfrac{5}{4^n}$

4) $\sum_{n=2}^{\infty} \dfrac{2^{n-1}}{(-3)^n}$

5) $\sum_{n=0}^{\infty} (-2)8^n 3^{-2n}$

6) $\sum_{n=0}^{\infty} \dfrac{\cos(n\pi)}{-3^n}$

7) $\sum_{n=0}^{\infty} \dfrac{2^{2n}+3^{n+1}}{5^n}$

8) $\sum_{n=1}^{\infty} 9^{-n+2}.4^{n+1}$

9) $\sum_{n=1}^{\infty} \dfrac{3^n.2^{n+1}}{9^n}$

10) $\sum_{n=1}^{\infty} \dfrac{3^n}{2^{n+1}}$

11) $\sum_{n=0}^{\infty} \left(\dfrac{1}{n+2} - \dfrac{1}{n+3}\right)$

12) $\sum_{n=1}^{\infty} \dfrac{1}{n(n+2)}$

13) $\sum_{n=0}^{\infty} \frac{1}{n^2+3n+2}$

14) $\sum_{n=1}^{\infty} \frac{n+1}{(n+2)!}$

15) $\sum_{n=1}^{\infty} \log(\frac{n}{n+1})$

16) $\sum_{n=1}^{\infty} \ln(1 + \frac{1}{n})$

17) $\sum_{n=1}^{\infty} \left(\cos\left(\frac{1}{n^2}\right) - \cos(\frac{1}{n+1})^2\right)$

18) $\sum_{0}^{\infty} \frac{1}{n!}$