**Problem 1**

The up-heap bubbling is demonstrated below:

***Step 0****: Initial heap*

***Step1****: Adding 27 to heap*

***Step2****: Compare 27 and 56 , since 27<56 swap*

***Step3****: Compare 27 and 42 , since 27<42 swap, this is final state of heap*

**Problem 2**

The down-bubbling of the heap is demonstrated below:

***Step 0****: Initial heap*

***Step 1****: Remove 5 move last node(34) to top*

***Step 2****: Swap the root with the smaller of the 2 children*

***Step 3****: Swap the node again with the smaller of its children*

*The above is the final state*

**Problem 3**

Using the division method for hashing for inserting the sequence:

5, 8, 44, 23, 30, 34, 52, 32, 15, 16

First number insert

5 mod 11 = 5

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|  |  |  |  |  | \* |  |  |  |  |  |
|  |  |  |  |  | 5 |  |  |  |  |  |

Second number insert

8 mod 11 = 8

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|  |  |  |  |  | \* |  |  | \* |  |  |
|  |  |  |  |  | 5 |  |  | 8 |  |  |

Third number insert

44 mod 11 = 0

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| \* |  |  |  |  | \* |  |  | \* |  |  |
| 44 |  |  |  |  | 5 |  |  | 8 |  |  |

Fourth number insert

23 mod 11 = 1

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| \* | \* |  |  |  | \* |  |  | \* |  |  |
| 44 | 23 |  |  |  | 5 |  |  | 8 |  |  |

Fifth number insert

30 mod 11 = 8

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| \* | \* |  |  |  | \* |  |  | \* |  |  |
| 44 | 23 |  |  |  | 5 |  |  | 8 |  |  |
|  |  |  |  |  |  |  |  | 30 |  |  |

Sixth number insert

34 mod 11 = 1

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| \* | \* |  |  |  | \* |  |  | \* |  |  |
| 44 | 23 |  |  |  | 5 |  |  | 8 |  |  |
|  | 34 |  |  |  |  |  |  | 30 |  |  |

Seventh number insert

52 mod 11 = 8

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| \* | \* |  |  |  | \* |  |  | \* |  |  |
| 44 | 23 |  |  |  | 5 |  |  | 8 |  |  |
|  | 34 |  |  |  |  |  |  | 30 |  |  |
|  |  |  |  |  |  |  |  | 52 |  |  |

Eighth number insert

32 mod 11 = 10

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| \* | \* |  |  |  | \* |  |  | \* |  |  |
| 44 | 23 |  |  |  | 5 |  |  | 8 |  | 32 |
|  | 34 |  |  |  |  |  |  | 30 |  |  |
|  |  |  |  |  |  |  |  | 52 |  |  |

Nineth number insert

15 mod 11 = 4

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| \* | \* |  |  |  | \* |  |  | \* |  |  |
| 44 | 23 |  |  | 15 | 5 |  |  | 8 |  | 32 |
|  | 34 |  |  |  |  |  |  | 30 |  |  |
|  |  |  |  |  |  |  |  | 52 |  |  |

Tenth number insert

16 mod 11 = 5

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| \* | \* |  |  |  | \* |  |  | \* |  |  |
| 44 | 23 |  |  | 15 | 5 |  |  | 8 |  | 32 |
|  | 34 |  |  |  | 16 |  |  | 30 |  |  |
|  |  |  |  |  |  |  |  | 52 |  |  |

**Problem 4**

The position is determined by: A[(j+i) mod N]

Where j = k mod N where k is the key.

The step by step building of the table is illustrated below

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Step** | **Hash Table** | | | | | | | | | | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 |  |  |  |  |  | 5 |  |  |  |  |  |
| 2 |  |  |  |  |  | 5 |  |  | 8 |  |  |
| 3 | 44 |  |  |  |  | 5 |  |  | 8 |  |  |
| 4 | 44 | 23 |  |  |  | 5 |  |  | 8 |  |  |
| 5 | 44 | 23 |  |  |  | 5 |  |  | 8 | 30 |  |
| 6 | 44 | 23 | 34 |  |  | 5 |  |  | 8 | 30 |  |
| 7 | 44 | 23 | 34 |  |  | 5 |  |  | 8 | 30 | 52 |
| 8 | 44 | 23 | 34 | 32 |  | 5 |  |  | 8 | 30 | 52 |
| 9 | 44 | 23 | 34 | 32 | 15 | 5 |  |  | 8 | 30 | 52 |
| 10 | 44 | 23 | 34 | 32 | 15 | 5 | 16 |  | 8 | 30 | 52 |

**Problem 5**

The formulae is: A[(h(k) + i \* h’(k)) mod N], for i = 0, 1, 2, . . ., N – 1

**Iteration 1 (i=0):**

h(k) = *k* mod 13 = 15 mod 13 = 2

h’(k) = 1 *+* (*k mod* 11) = 1+(15 mod 11) = 5

So the position is ((2 + 0\* 5) mod 13) = 2, since the position 2 is occupied move to next iteration

**Iteration 2 (i=1):**

h(k) = 2 (doesn’t change)

h’(k) = 5(doesn’t change)

Position = ((2 + 1\* 5) mod 13) = 7 mod 13 = 7, since position 7 also is occupied move to next iteration

**Iteration 3 (i=2):**

h(k) = 2 (doesn’t change)

h’(k) = 5(doesn’t change)

Position = ((2 + 2\* 7) mod 13) = 16 mod 13 = 3, since position 3 is not occupied we can place the key at this position.

**Problem 6**