**Problem 1**

Initial tree

Steps :

1. Compare 13 with root 26, since its less than 26 go to the left
2. Compare 13 with 10 its more so go to the right
3. Compare 13 with 15, since its less go to the left
4. Compare 13 with 12, since its more , it will become the right child

Final tree:

**Problem 2**

Initial tree

Steps

1. Remove the node 42
2. Locate the predecessor of 42 , in this case 42 has two internal nodes , so we find the rightmost node in the left subtree (36)
3. Replace the node 42 with 36
4. Bring the node 31 to the position of 36

Final tree

**Problem 3**

Initial tree

First rotation

Second rotation

**Problem 4**

Initial tree: height shown in bracket

After inserting 54

Steps

1. Traverse up the tree and check the nodes to understand which one breaks the AVL tree property
2. The immediate parent node 50 is fine since its children only differ by 1
3. The parent of 50 ,59 also is height balanced
4. The parent of 59, node 42 also is height
5. The root node however is not height balanced, the left child has a height of 2 while the right child has a height of 4

We can mark the node as :

Double rotation:

**Problem 5**

1. Encoding the string “BDEGC”

The encoding of different letters:

B – 1111

D – 011

E – 010

G – 10

C – 00

The encoding is: 11110110101000

1. Decode the bit pattern “010111010011” to the original string

010 – E

1110 – A

10 – G

011 - D

The decoding is: EAGD

**Problem 6**

The matrix looks like this

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | **P(I,j)** |  |  |  |  |  |
| 1/2 |  |  |  |  |  | 1 | 6 |  |
|  | 1/2 |  |  |  |  | 1 | 5 |  |
|  |  | 1/2 |  | **13/16** | 15/16 | 1 | 4 |  |
|  |  |  | 1/2 | 11/16 | 7/8 | 1 | 3 |  |
|  |  |  |  | 1/2 | 3/4 | 1 | 2 | j |
|  |  | **1/16** | 1/8 | 1/4 | 1/2 | 1 | 1 |  |
| 0 | 0 | 0 | 0 | 0 | 0 |  | 0 |  |
| 6 | 5 | 4 | 3 | 2 | 1 | 0 |  |  |
|  |  |  | i |  |  |  |  |  |

**Calculations for P(4,1):**

P(2,1) = (P(1,1) + P(2,0))/2 = P(1,1)/2 = (1/2)/2 = 1/4

P(3,1) = ((P(2,1) + P(3,0))/2 = 1/8

P(4,1) = (P(3,1) + P(4,0))/2 = (P(3,1) + 0)/2 = (1/8)/2 = **1/16**

**Calculations for P(2,4):**

P(2,4) = (P(1,4) + P(2,3))/2 = (15/16 + 11/16)/2 = **13/16**

P(1,4) = (P(0,4) + P(1,3)) /2 = (1 + P(1,3)) /2 = (1 + 7/8)/2 = 15/16

P(1,3) = (P(0,3) + P(1,2))/2 = (1 + P(1,2))/2 = (1+ P(1,2))/2 = (1 + 3/4) /2 = 7/8

P(1,2) = (P(0,2) + P(1,1))/2 = (1 + ½)/2 = 3/4

P(2,3) = (P(1,3) + P(2,2))/2 = (7/8 + P(2,2))/2 = (7/8 + 1/2)/2 = 11/16

P(2,2) = (P(1,2) + P(2,1))/2 =(3/4 + 1/4) /2 = 1/2

**Problem 7**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **n**  **Algorithm** | 10000 | 20000 | 30000 | 40000 | 50000 | 60000 | 70000 | 80000 | 90000 | 100000 |
| Insertion | 425 | 1545 | 1630 | 1931 | 2768 | 4314 | 6438 | 9882 | 12619 | 17308 |
| merge | 36 | 33 | 6 | 8 | 11 | 13 | 23 | 39 | 20 | 22 |
| quick | 7 | 6 | 10 | 13 | 16 | 20 | 39 | 48 | 33 | 16 |
| heapsort | 20 | 10 | 11 | 20 | 17 | 21 | 27 | 31 | 35 | 41 |