18. Quantum teleportation

What is quantum teleportation?

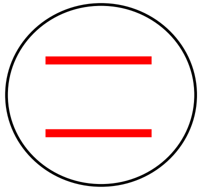
The aim of quantum teleportation is to send an unknown quantum state held by Alice to Bob.

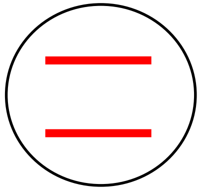
At the end of the process Bob should have the unknown quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Alice and Bob never need to know what the state is.

Initial state

$|\psi\rangle$





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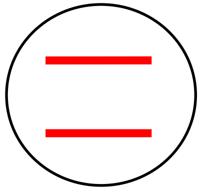
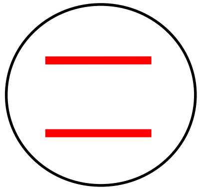
Final state



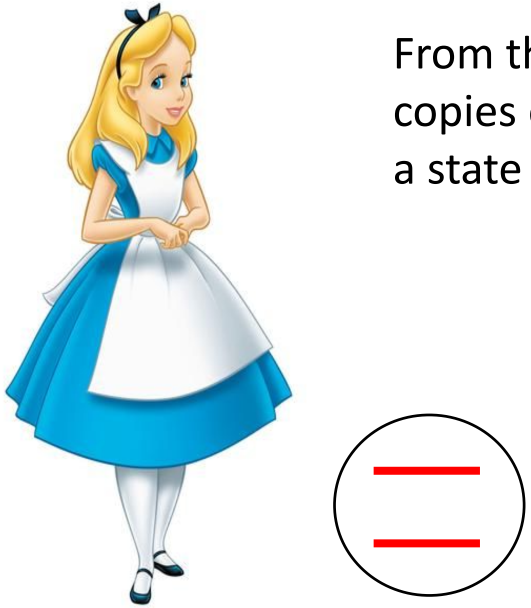
$|\psi\rangle$



What quantum teleportation is and is not

The atom itself doesn’t disintegrate and get sent somewhere. So its not quite like the Star Trek-like teleporter.

But the quantum state is the most fundamental representation of the state, and if the qubits are atoms, then they are fundamentally identical. So if you say the quantum state is the most basic reality, then it is quite close to teleportation.



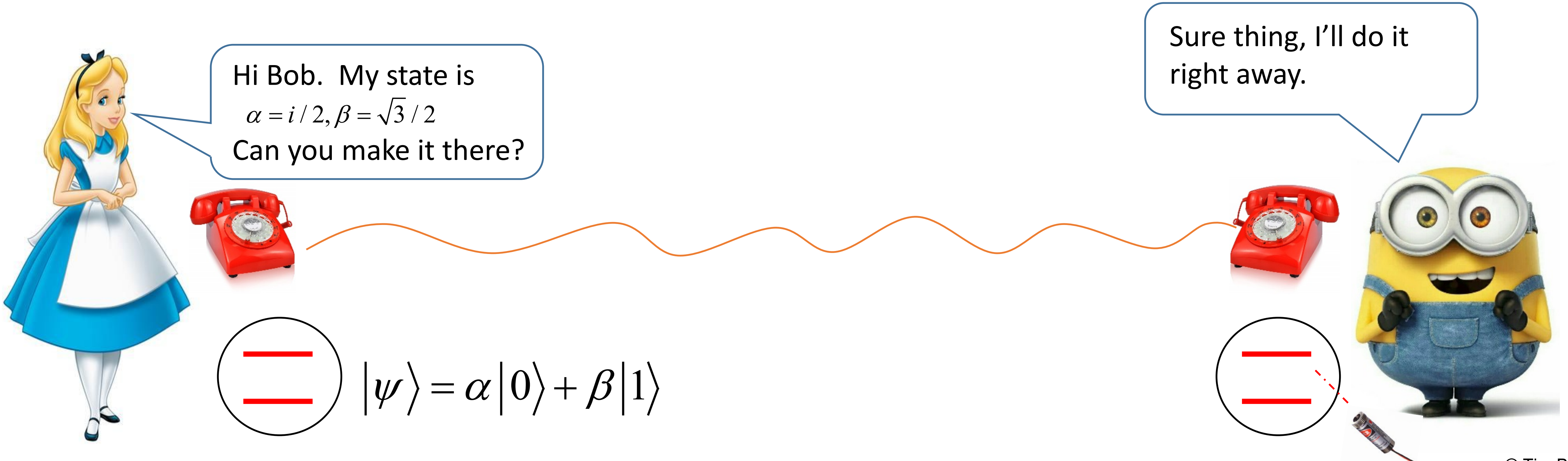
From the no cloning theorem, we can never make two copies of $|\psi\rangle$. But teleportation allows one to destroy a state on one side and create it elsewhere.

$|\psi\rangle$

Why is teleportation amazing?

There is an emphasis on the fact that Alice and Bob do not know the state. Why is this important?

If Alice knew what the state was, she could just tell Bob what the state is, and Bob could make that state. No need for teleportation.



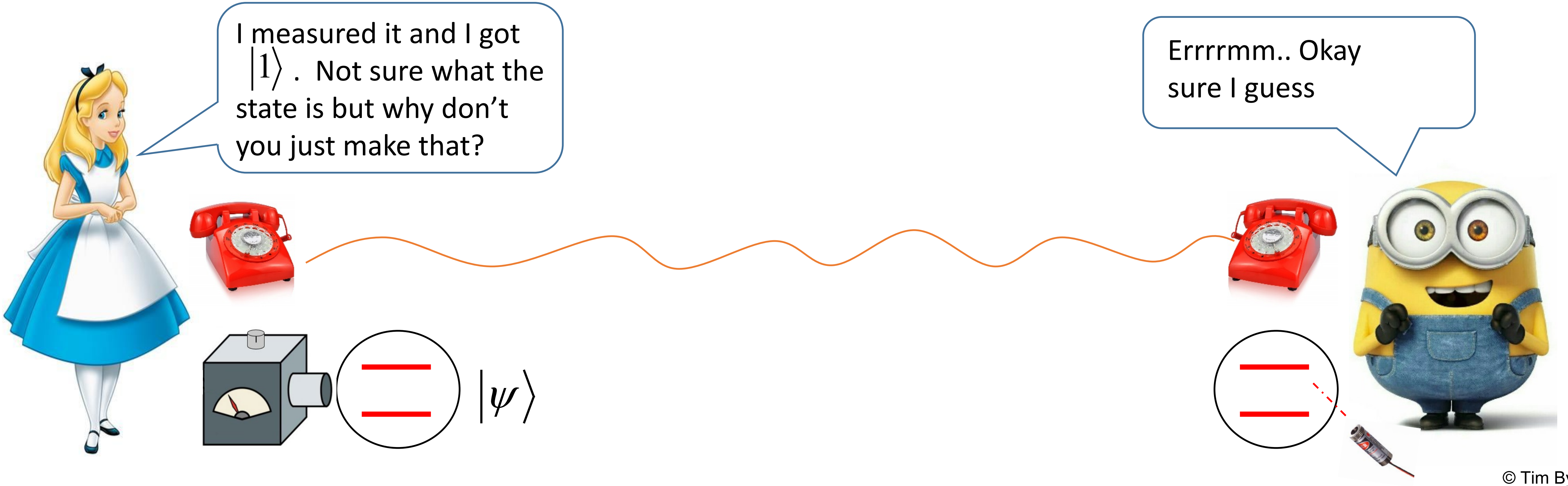
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Measuring an unknown state

Even if Alice didn't know her state initially, couldn't she just figure this out and tell Bob?

With only one copy of the state, it is impossible to figure out an unknown state. There are an infinity of basis choices that could be made, and since the measurement outcome is random, this doesn't give much information.

e.g. Measuring $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ gives the outcome $|1\rangle$. All this says for sure is that $\beta \neq 0$



Theoretical limit to estimate state

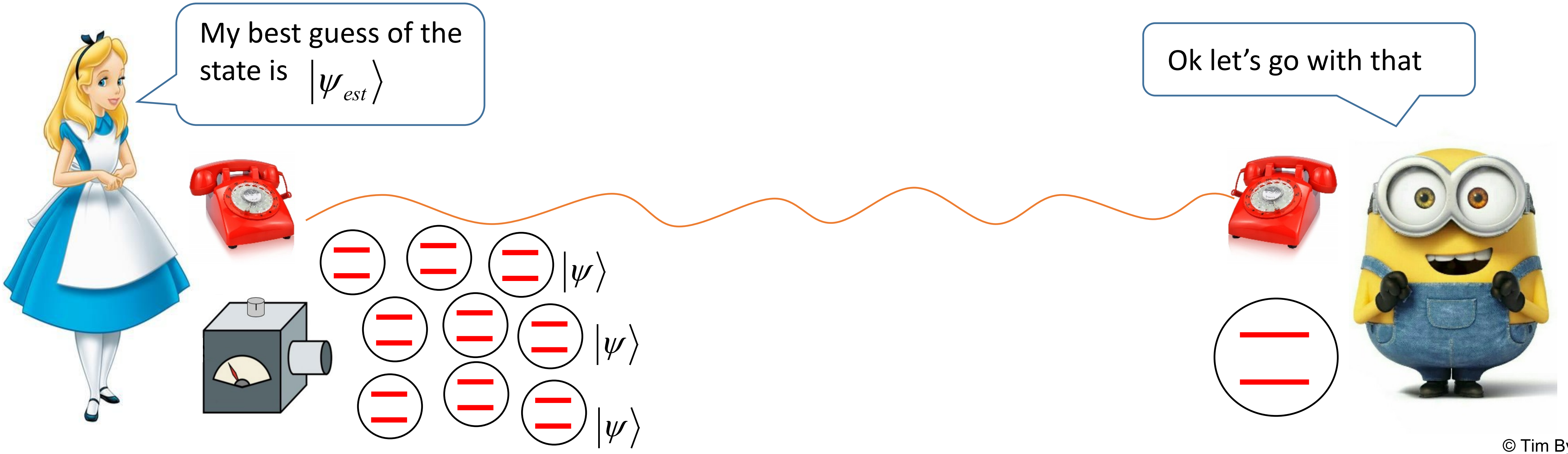
It was shown that even with the fanciest possible measurements, the best estimate you can get of N unknown copies of a state is fidelity

$$F = |\langle \psi | \psi_{est} \rangle|^2 = \frac{N+1}{N+2}$$

Massar & Popescu Phys. Rev. Lett. 74, 1259 (1995)

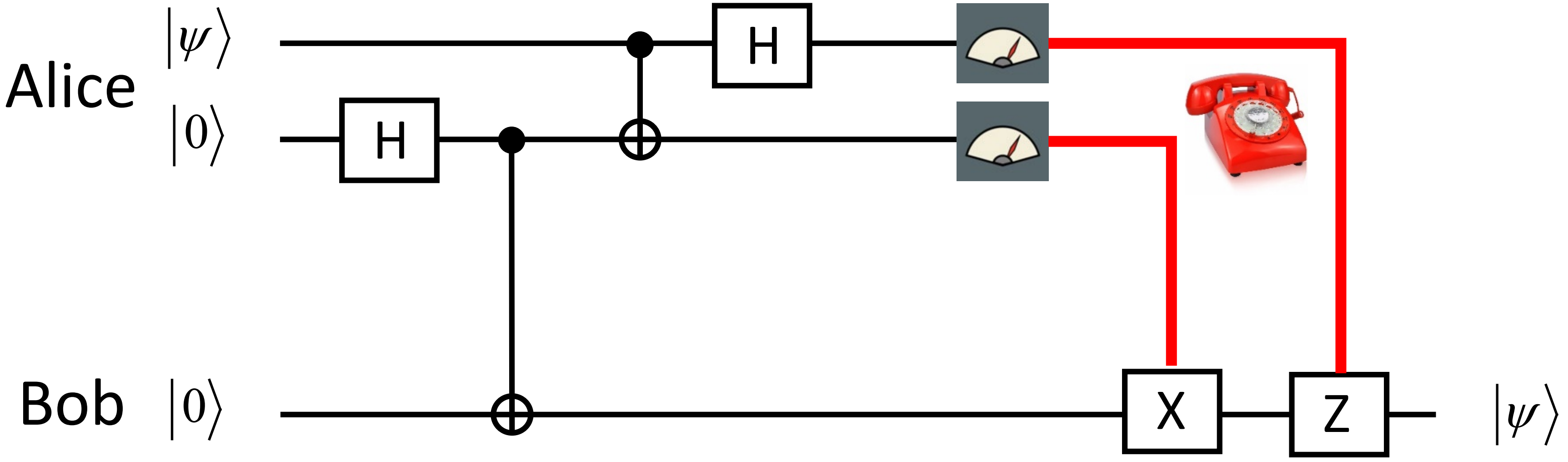
For one qubit N=1, the best fidelity is F=2/3=0.666.

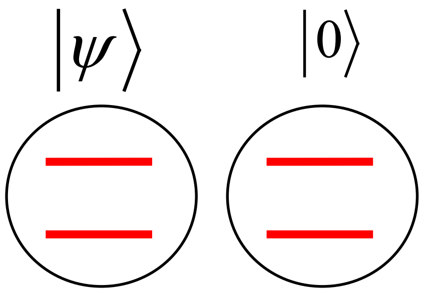
If Alice doesn't know her state its impossible to just tell Bob exactly what it is with one copy.

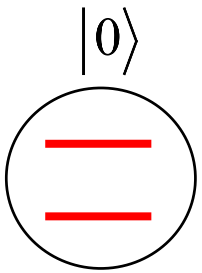


The teleportation circuit

The teleportation circuit allows you to overcome this and (under ideal conditions) get perfect transfer of the state!

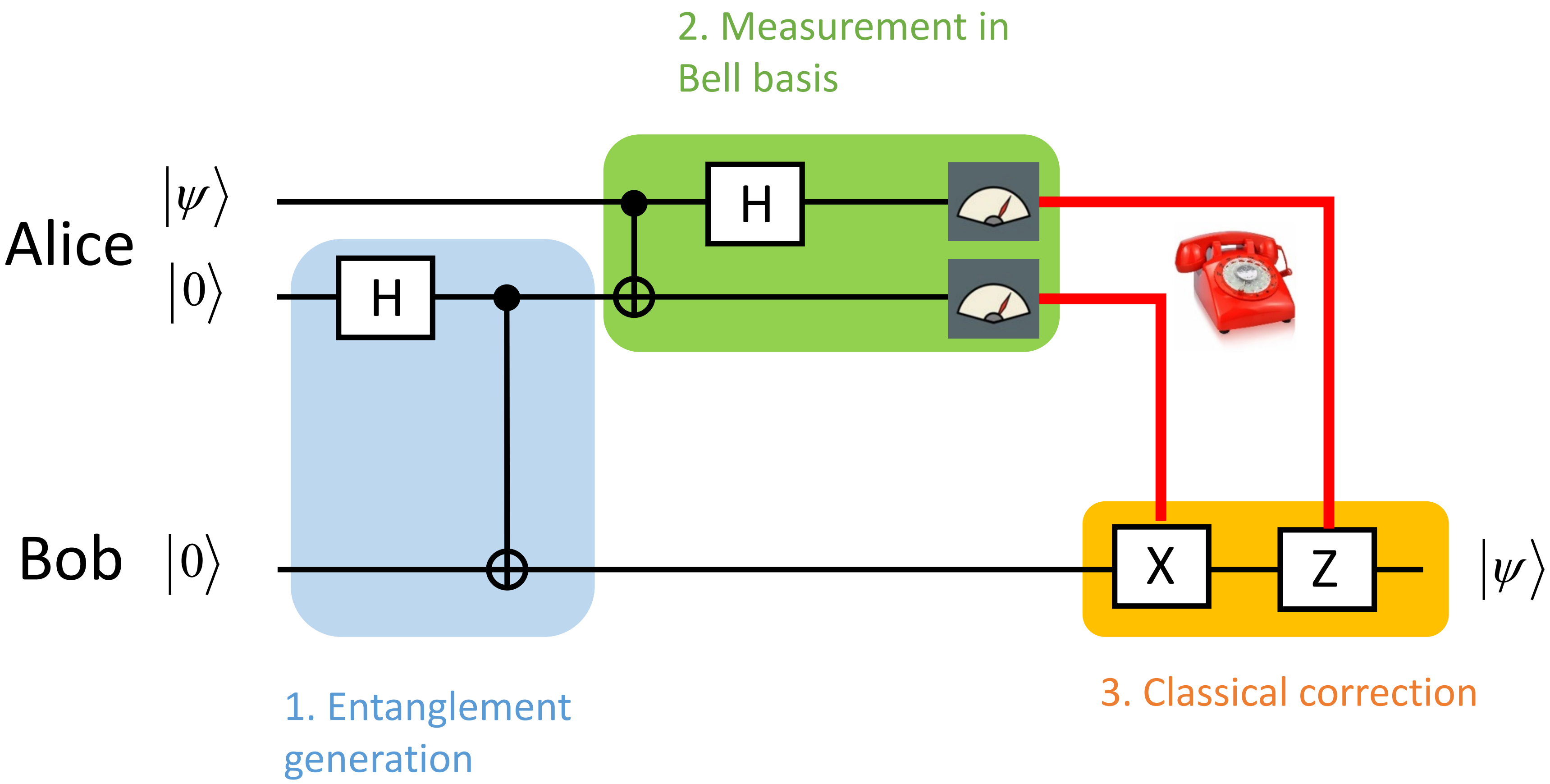






The teleportation circuit

The main steps of the teleportation circuit are



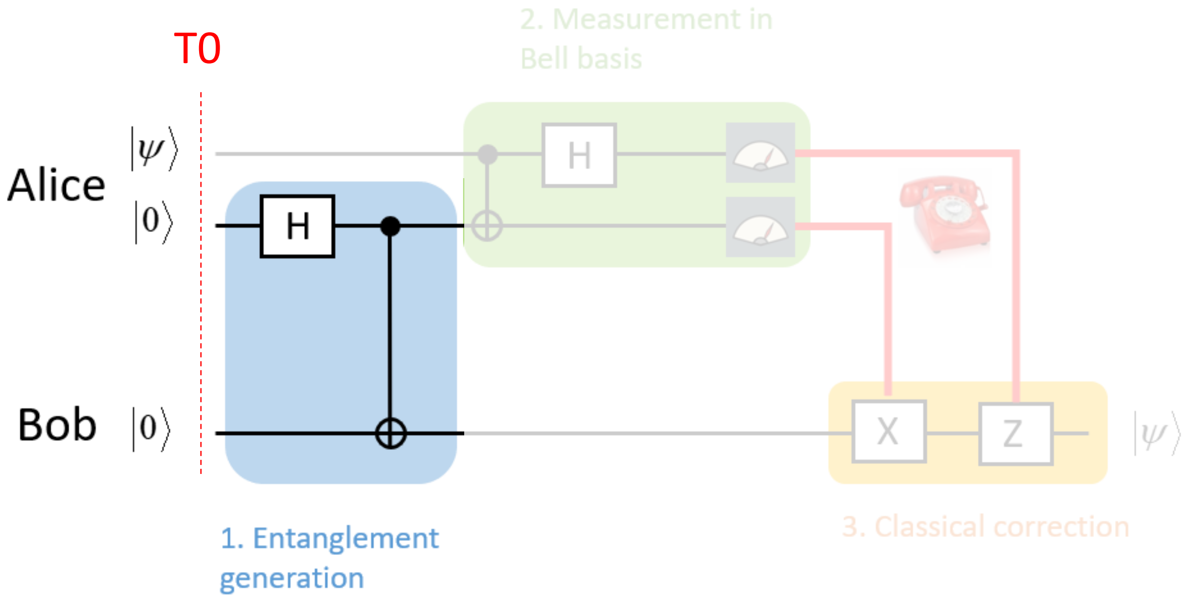
Part 1: Entanglement generation

The first part of the circuit produces an entangled state.

Working through the circuit step by step we have

Time T0:

$$|T0\rangle = |\psi\rangle|0\rangle|0\rangle$$



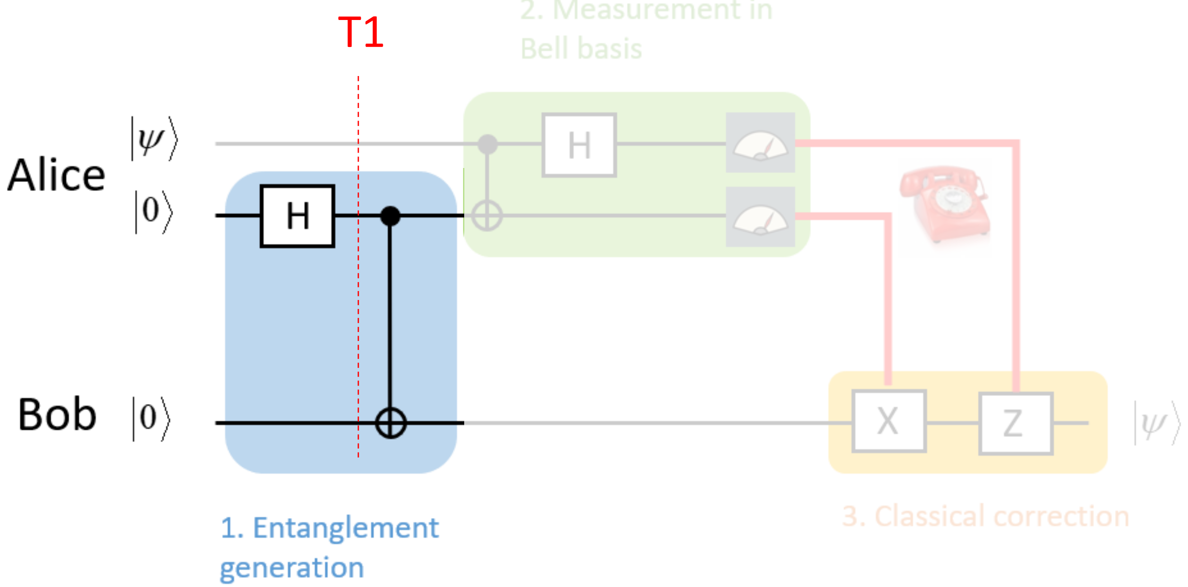
Part 1: Entanglement generation

Apply a Hadamard on qubit 2:

Time T1:

$$|T1\rangle = H_2|T0\rangle = |\psi\rangle \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)|0\rangle$$

$$= |\psi\rangle \frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)$$



$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$H|0\rangle = |+\rangle$$
$$H|1\rangle = |-\rangle$$

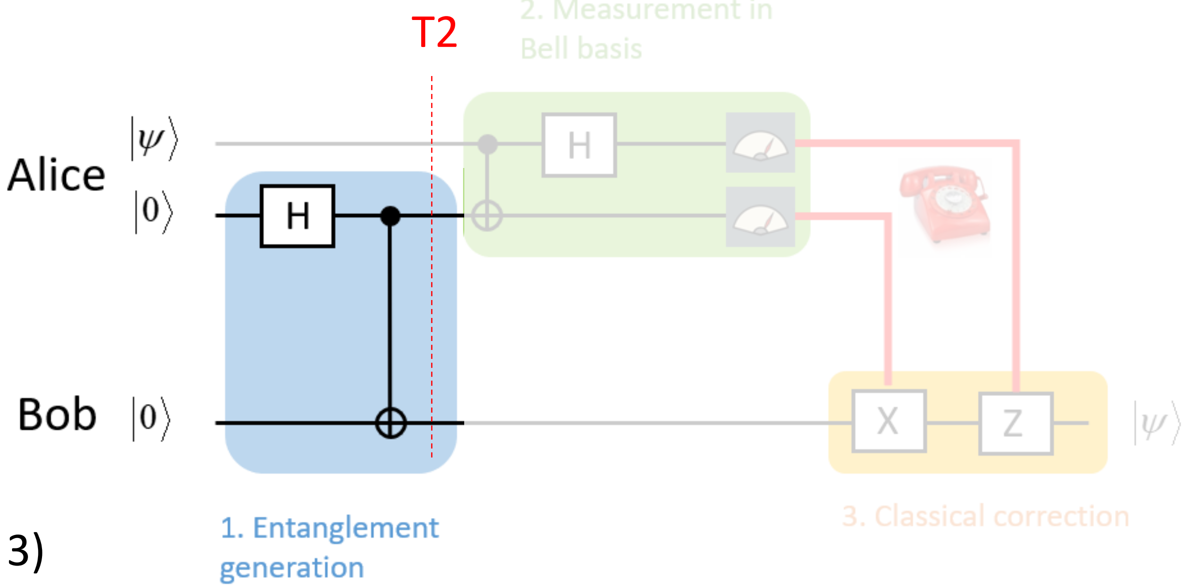
Part 1: Entanglement generation

Apply a CNOT between qubits 2 and 3

Time T2:

$$|T2\rangle = U_{CNOT}^{(ctrl=q2)} |T1\rangle = U_{CNOT}^{(ctrl=q2)} |\psi\rangle \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

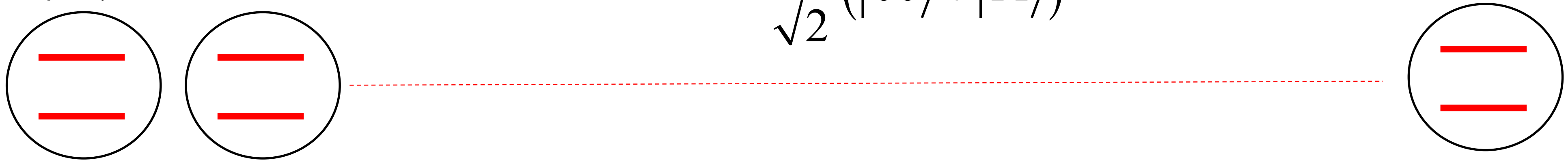
$$= |\psi\rangle \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



This is a Bell state (entangled state on qubits 2 & 3)

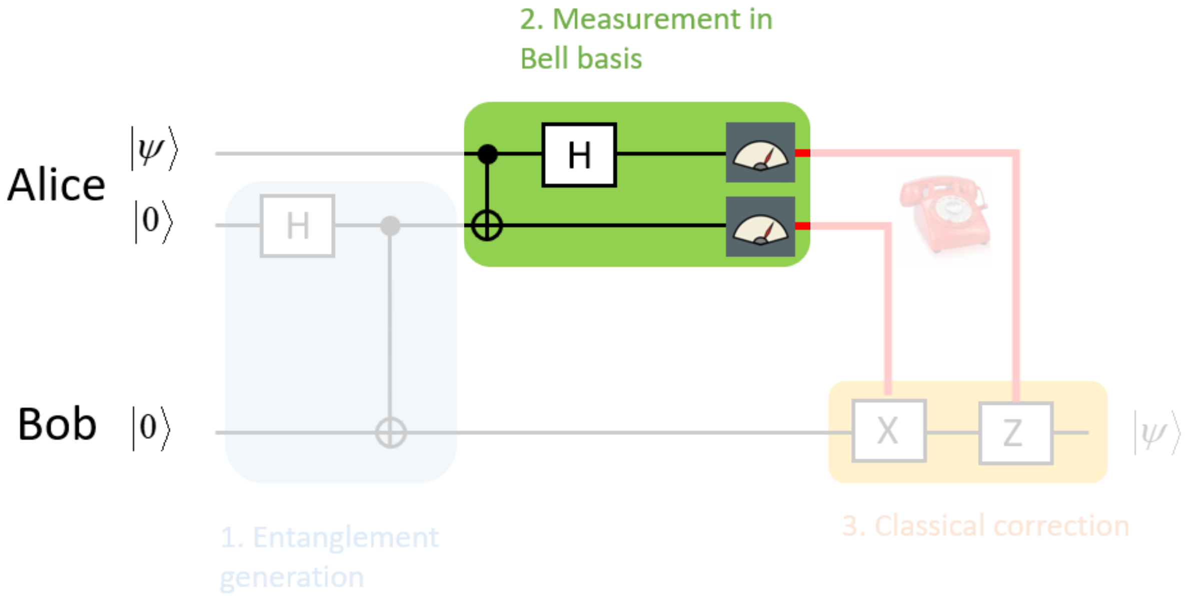
$|\psi\rangle$

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

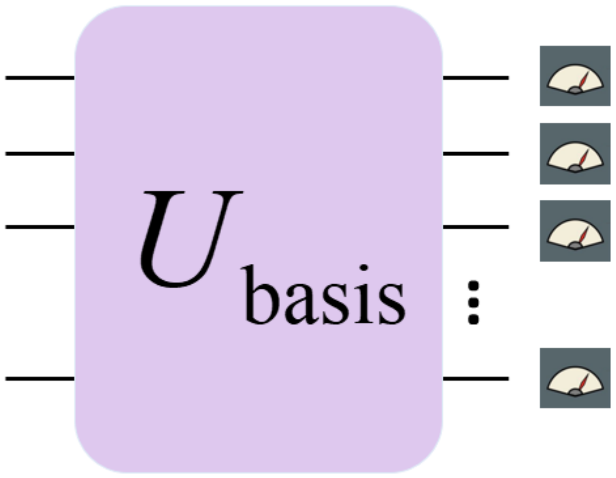


Part 2: Measurement in Bell basis

The next part of the circuit is an example of what was seen in Lecture 14.



A unitary in front of a measurement makes a measurement in a different basis



$$M_n^{\text{gen}} = |n\rangle\langle b_n|$$

The basis is in this case

$$|b_n\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

Part 2: Measurement in Bell basis

We can also just work this out step by step as before

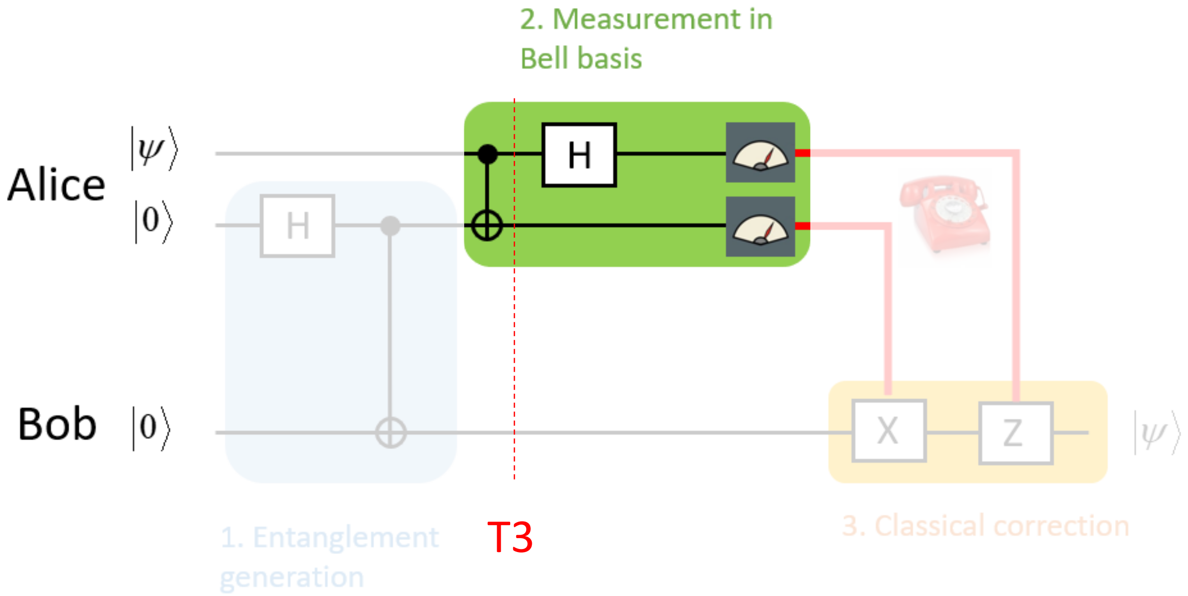
$$|T3\rangle = U_{CNOTq2}^{(ctrl=q1)}|T2\rangle = U_{CNOTq2}^{(ctrl=q1)}|\psi\rangle\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$

$$= U_{CNOTq2}^{(ctrl=q1)}\left(\alpha|0\rangle+\beta|1\rangle\right)\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$

$$= U_{CNOTq2}^{(ctrl=q1)}\frac{1}{\sqrt{2}}(\alpha|000\rangle+\alpha|011\rangle+\beta|100\rangle+\beta|111\rangle)$$

$$= \frac{1}{\sqrt{2}}(\alpha|000\rangle+\alpha|011\rangle+\beta|110\rangle+\beta|101\rangle)$$

$$= \alpha|0\rangle\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)+\beta|1\rangle\frac{1}{\sqrt{2}}(|10\rangle+|01\rangle)$$



Part 2: Measurement in Bell basis

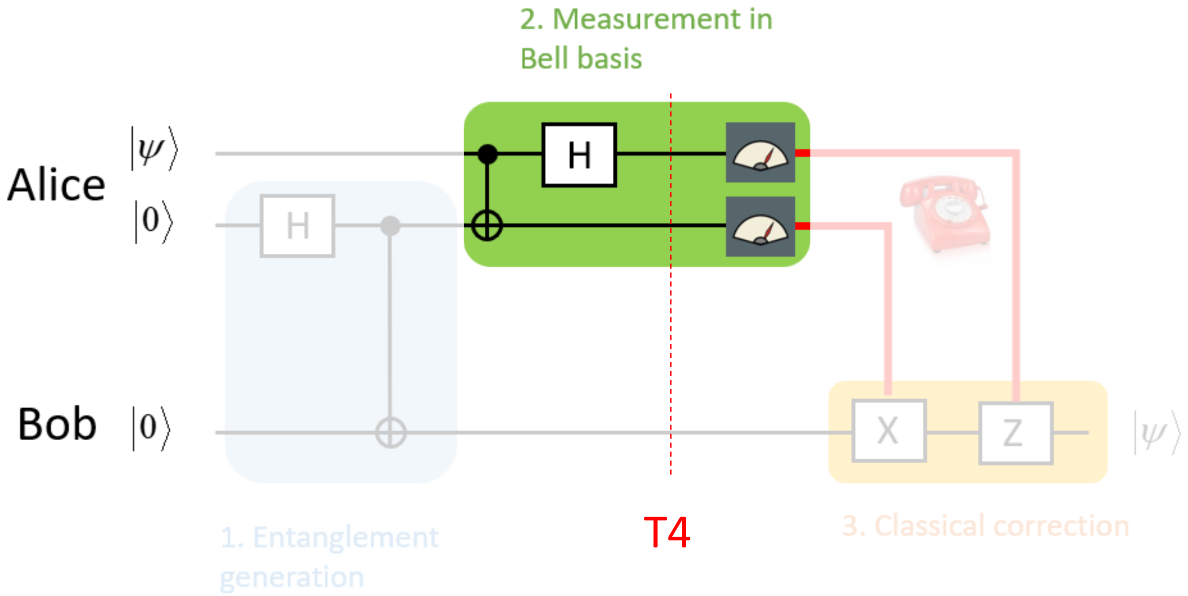
Applying the Hadmard

$$|T4\rangle = H_1|T3\rangle$$

$$= H_1\left[\alpha|0\rangle\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)+\beta|1\rangle\frac{1}{\sqrt{2}}(|10\rangle+|01\rangle)\right]$$

$$= \frac{\alpha}{2}(|0\rangle+|1\rangle)(|00\rangle+|11\rangle)+\frac{\beta}{2}(|0\rangle-|1\rangle)(|10\rangle+|01\rangle)$$

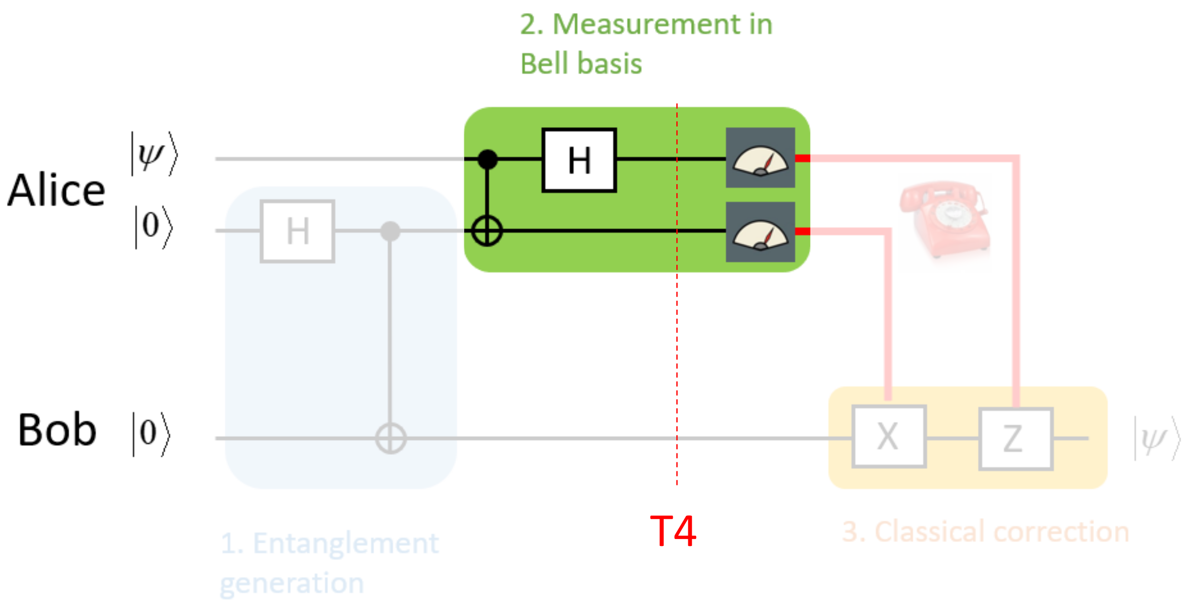
$$= \frac{\alpha}{2}(|000\rangle+|011\rangle+|100\rangle+|111\rangle)+\frac{\beta}{2}(|010\rangle+|001\rangle-|110\rangle-|101\rangle)$$



Part 2: Measurement in Bell basis

Now we measure the qubits in the $|0\rangle, |1\rangle$ basis

Here it is easiest to write the state before the measurement in terms of the four measurement outcomes $|00\rangle, |01\rangle, |10\rangle, |11\rangle$

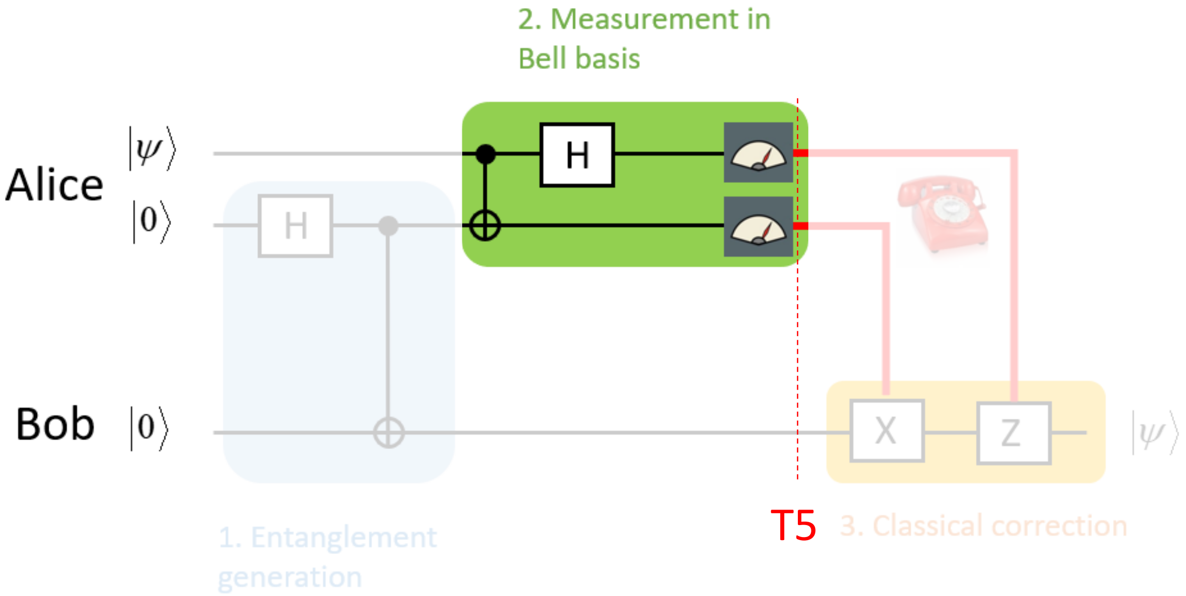


$$|T4\rangle = \frac{\alpha}{2}\left(|000\rangle + |011\rangle + |100\rangle + |111\rangle\right) + \frac{\beta}{2}\left(|010\rangle + |001\rangle - |110\rangle - |101\rangle\right)$$

$$= \frac{1}{2}|00\rangle\left(\alpha|0\rangle + \beta|1\rangle\right) + \frac{1}{2}|01\rangle\left(\alpha|1\rangle + \beta|0\rangle\right) + \frac{1}{2}|10\rangle\left(\alpha|0\rangle - \beta|1\rangle\right) + \frac{1}{2}|11\rangle\left(\alpha|1\rangle - \beta|0\rangle\right)$$

Part 2: Measurement in Bell basis

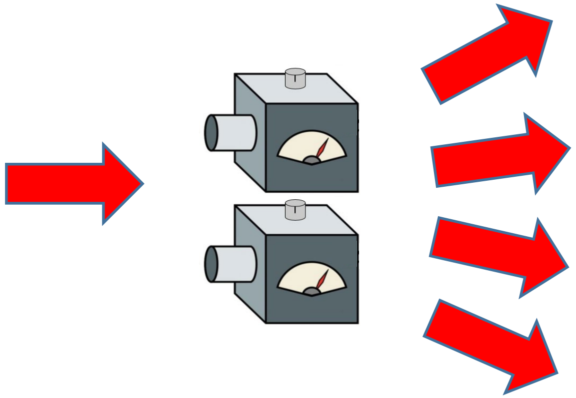
$$|T4\rangle = \frac{1}{2}|00\rangle(\alpha|0\rangle+\beta|1\rangle)+\frac{1}{2}|01\rangle(\alpha|1\rangle+\beta|0\rangle)$$
$$+\frac{1}{2}|10\rangle(\alpha|0\rangle-\beta|1\rangle)+\frac{1}{2}|11\rangle(\alpha|1\rangle-\beta|0\rangle)$$



Measuring the state randomly collapses the state to

$$|T5\rangle$$

$|T4\rangle$



$$|00\rangle(\alpha|0\rangle+\beta|1\rangle) \qquad p_{00}=1/4$$

$$|01\rangle(\alpha|1\rangle+\beta|0\rangle) \qquad p_{01}=1/4$$

$$|10\rangle(\alpha|0\rangle-\beta|1\rangle) \qquad p_{10}=1/4$$

$$|11\rangle(\alpha|1\rangle-\beta|0\rangle) \qquad p_{11}=1/4$$

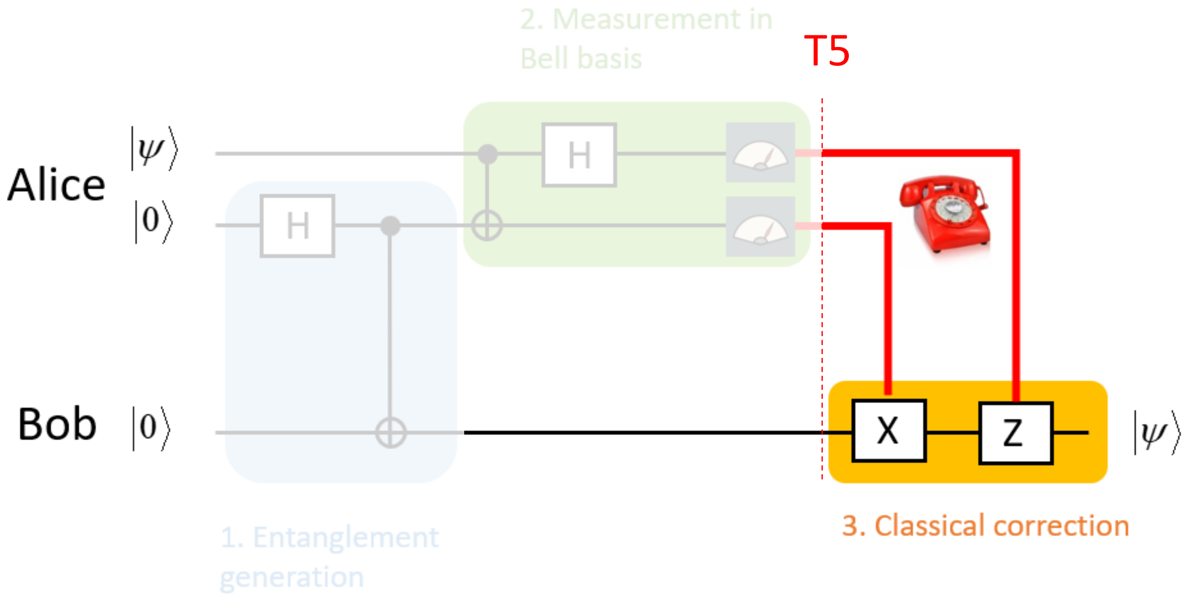
Can work out probabilities using the measurement operators

$$M_n = |00\rangle\langle 00|, |01\rangle\langle 01|, |10\rangle\langle 10|, |11\rangle\langle 11|$$

Part 3: Classical correction

After the measurement we have

$$|T5\rangle = \begin{cases} |00\rangle(\alpha|0\rangle+\beta|1\rangle) & p_{00}=1/4 \\ |01\rangle(\alpha|1\rangle+\beta|0\rangle) & p_{01}=1/4 \\ |10\rangle(\alpha|0\rangle-\beta|1\rangle) & p_{10}=1/4 \\ |11\rangle(\alpha|1\rangle-\beta|0\rangle) & p_{11}=1/4 \end{cases}$$



We can see already that the teleportation is nearly there.

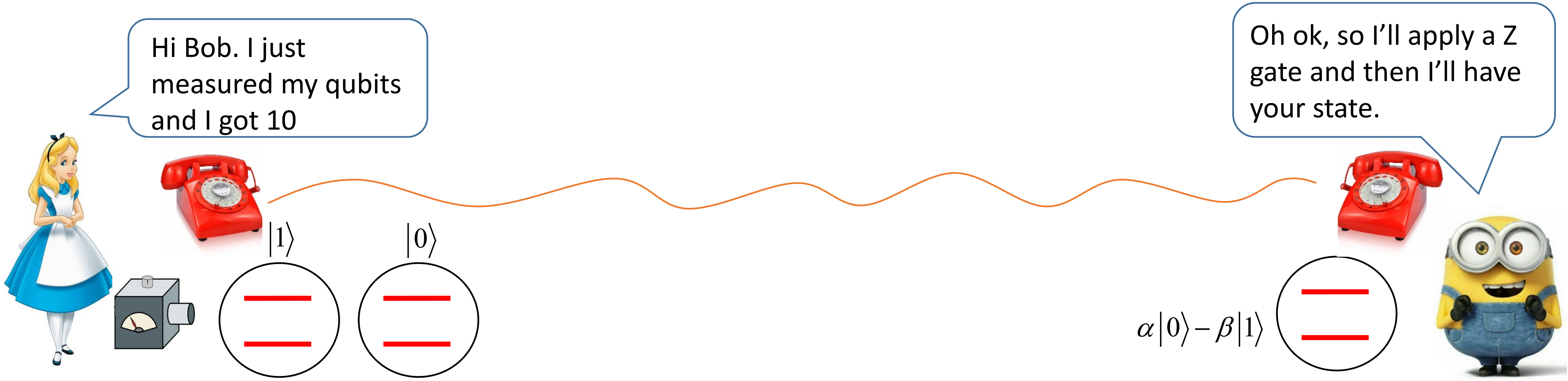
For the 00 outcome it actually is already working, but for the other cases Bob gets a state that is almost the state, but with an extra bit flip (01 case) or a phase flip (10 case), or both (11 case).

We can "fix up" the state by removing the extra bit flip or phase flip! Since Alice has in her possession the outcomes, she just needs to tell Bob which measurement she got, and the teleportation will work for all cases!

Part 3: Classical correction

So for example, if Alice gets the state $|01\rangle$

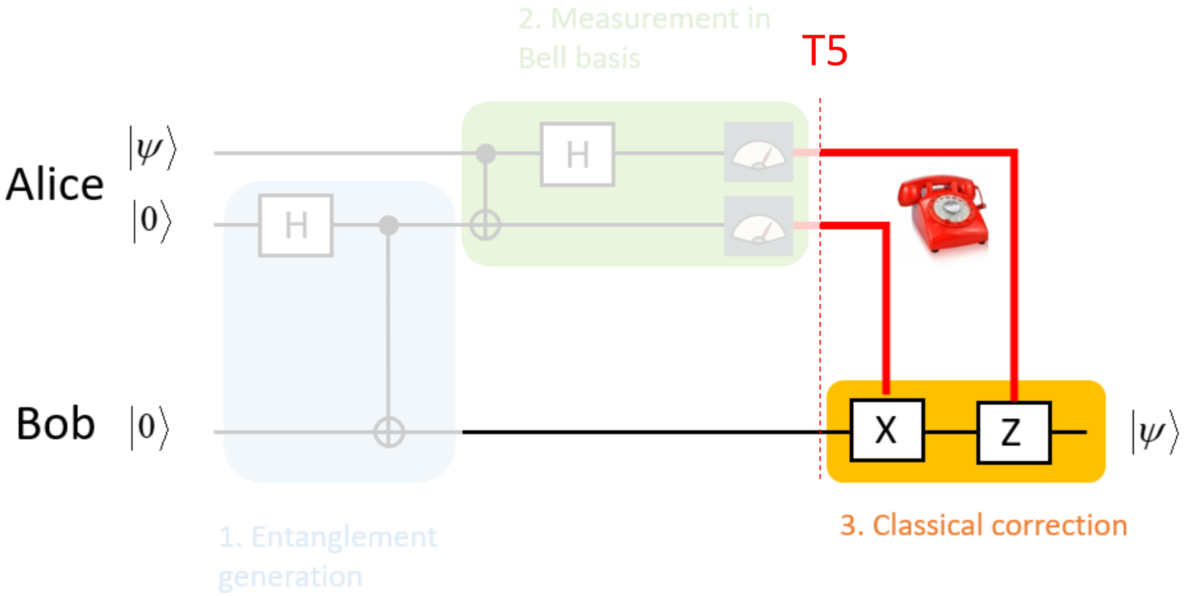
$$|T5\rangle = \begin{cases} |00\rangle(\alpha|0\rangle+\beta|1\rangle) & p_{00}=1/4 \\ |01\rangle(\alpha|1\rangle+\beta|0\rangle) & p_{01}=1/4 \\ |10\rangle(\alpha|0\rangle-\beta|1\rangle) & p_{10}=1/4 \\ |11\rangle(\alpha|1\rangle-\beta|0\rangle) & p_{11}=1/4 \end{cases}$$



Part 3: Classical correction

After the measurement we have

$$|T5\rangle = \begin{cases} |00\rangle(\alpha|0\rangle+\beta|1\rangle) & p_{00}=1/4 \\ |01\rangle(\alpha|1\rangle+\beta|0\rangle) & p_{01}=1/4 \\ |10\rangle(\alpha|0\rangle-\beta|1\rangle) & p_{10}=1/4 \\ |11\rangle(\alpha|1\rangle-\beta|0\rangle) & p_{11}=1/4 \end{cases}$$



Alice's outcome qubit 1	Alice's outcome qubit 2	Bob's state	Classical correction	Bob's state after correction
0	0	$\alpha\|0\rangle+\beta\|1\rangle$	I (do nothing)	$\alpha\|0\rangle+\beta\|1\rangle$
0	1	$\alpha\|1\rangle+\beta\|0\rangle$	X	$\alpha\|0\rangle+\beta\|1\rangle$
1	0	$\alpha\|0\rangle-\beta\|1\rangle$	Z	$\alpha\|0\rangle+\beta\|1\rangle$
1	1	$\alpha\|1\rangle-\beta\|0\rangle$	ZX	$\alpha\|0\rangle+\beta\|1\rangle$

Bob gets Alice's state for every case!

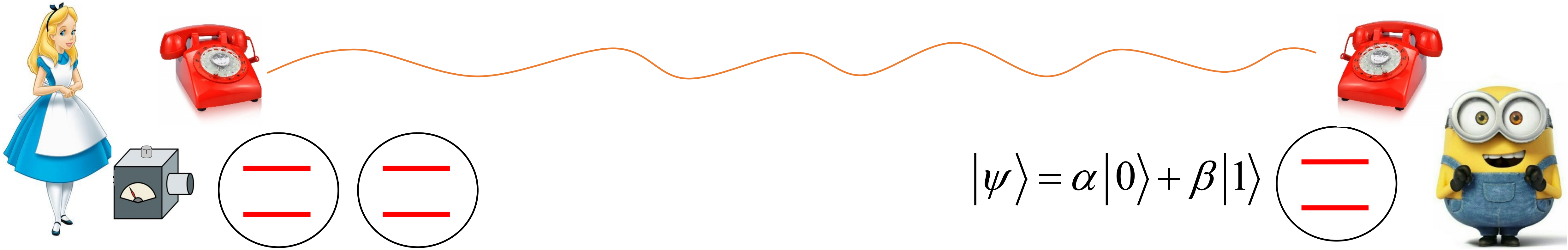
What have we achieved?

Bob has received Alice's state without any knowledge of what the state is. Neither Alice nor Bob needs to know what the state is. (of course its fine if they know what the state is, they just do not need to know)

The main things that are needed are:

1) Entangled qubits between Alice and Bob
2) Alice needs to tell Bob what measurement outcomes she got (classical communication)

The fidelity of the transfer is (ideally) perfect, F=1. This is better than any state estimation method.



Superluminal teleportation?

Provided that the entanglement can be set up before, there is no reason that this could not work over arbitrary distances.

Then couldn't we teleport states between different stars separated across the universe?

The thing that spoils this is that Alice and Bob need to do the classical correction step.

This cannot travel faster than the speed of light, so it is not really possible to complete the teleportation faster than the speed of light.

Generally entanglement is viewed as a resource that can be used to transfer information and perform other quantum information tasks.