6. The wavefunction

$$|\psi\rangle$$

Classical physics

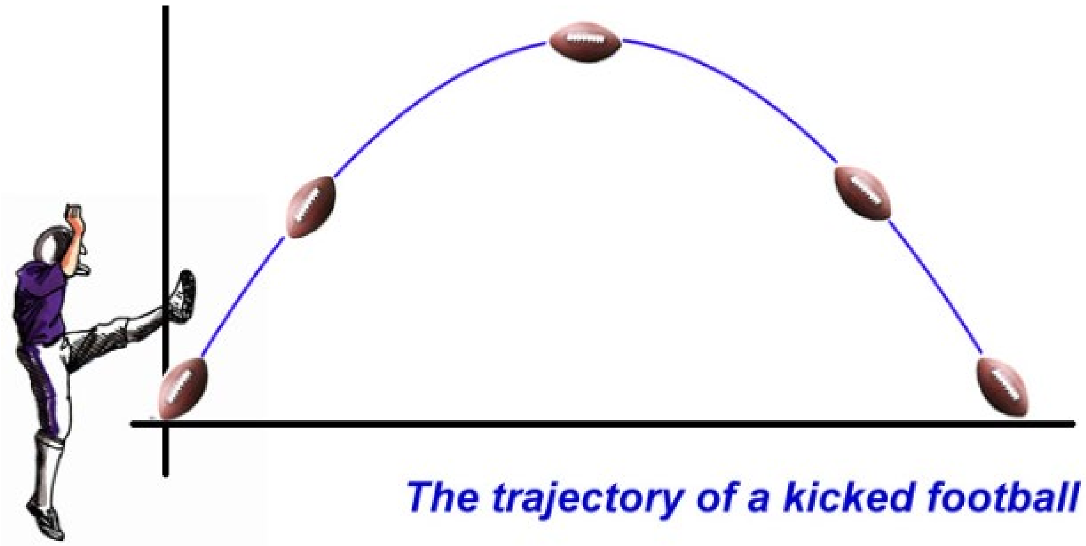
The typical problem in classical physics is projectile motion

We have given the

initial position $\vec{x}(t=0)=(x_0,y_0)$
velocity $\vec{v}(t=0)=(v_0^x,v_0^y)$

and from this we can figure out we the projectile will be for the rest of the trajectory.

$\vec{x}(t=0),\vec{v}(t=0)$ ⇒ Newton's laws etc. ⇒ $\vec{x}(t),\vec{v}(t)$

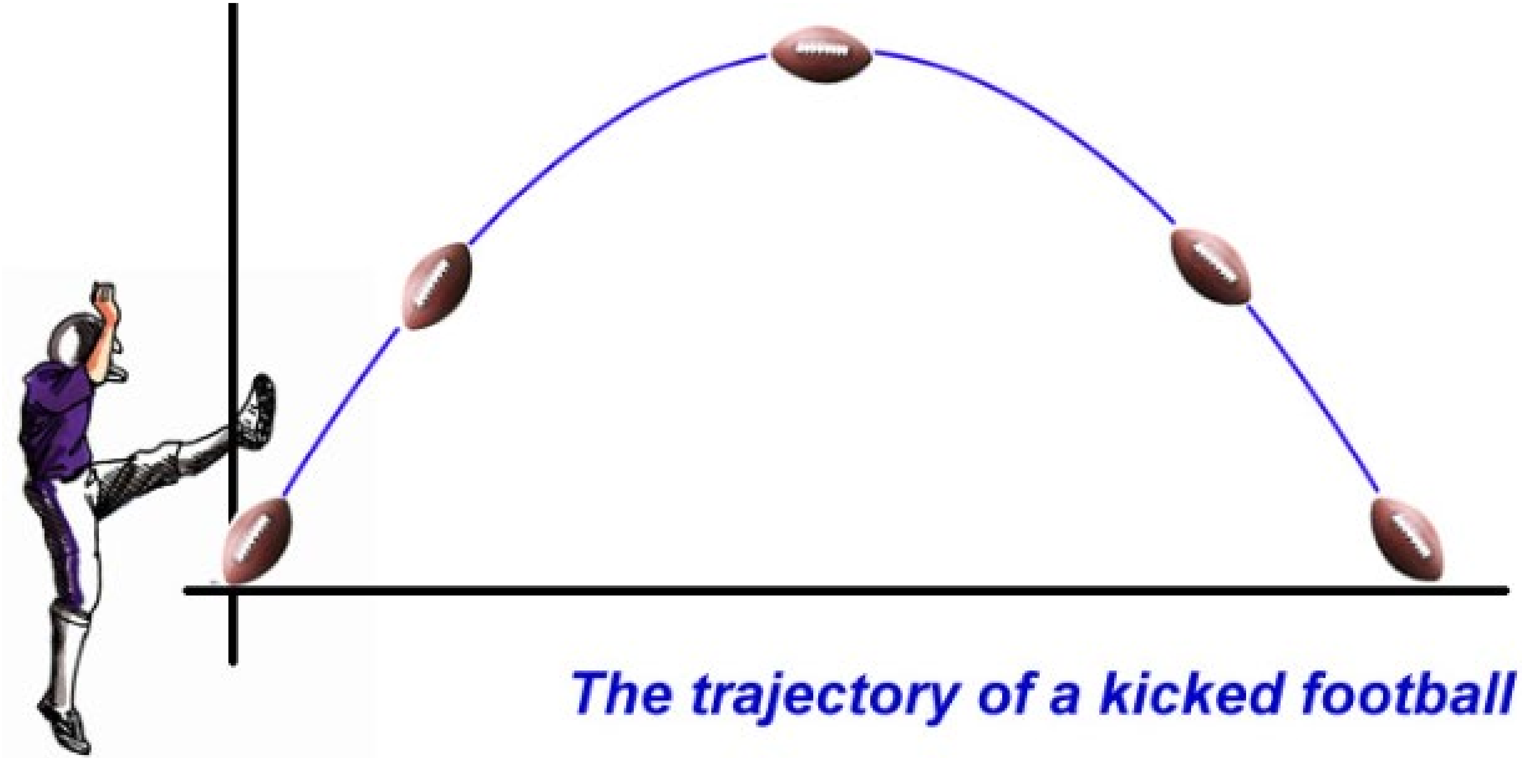


The trajectory of a kicked football

The state according to classical physics

At any point we can describe the "state" of the ball according to the position and velocity

$$\vec{x}(t), \vec{v}(t)$$



Even though the ball is a complicated object with a huge number of atoms we describe the ball by just 4 numbers:

$$\vec{x}(t) = (x, y)$$
$$\vec{v}(t) = (v^x, v^y)$$

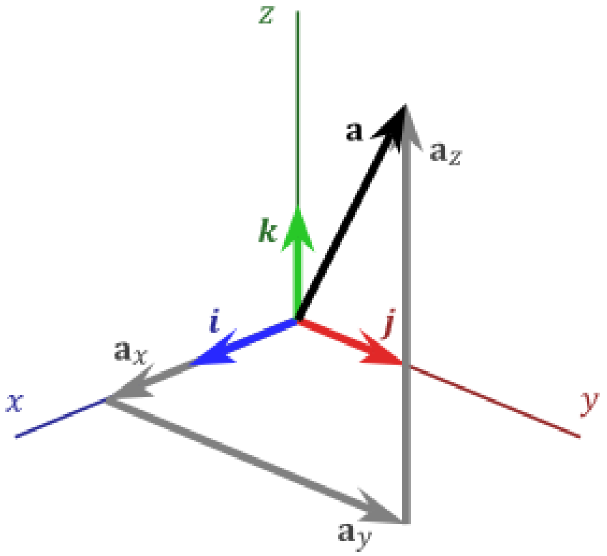
The state according to quantum mechanics

In quantum mechanics, we always describe the state of an object by a **vector**

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle + \ldots + a_N|N\rangle$$
$$= \sum_{n=0}^{N} a_n |n\rangle$$

Here, the a_n are complex numbers

The $|n\rangle$ are distinct states of the physical system.



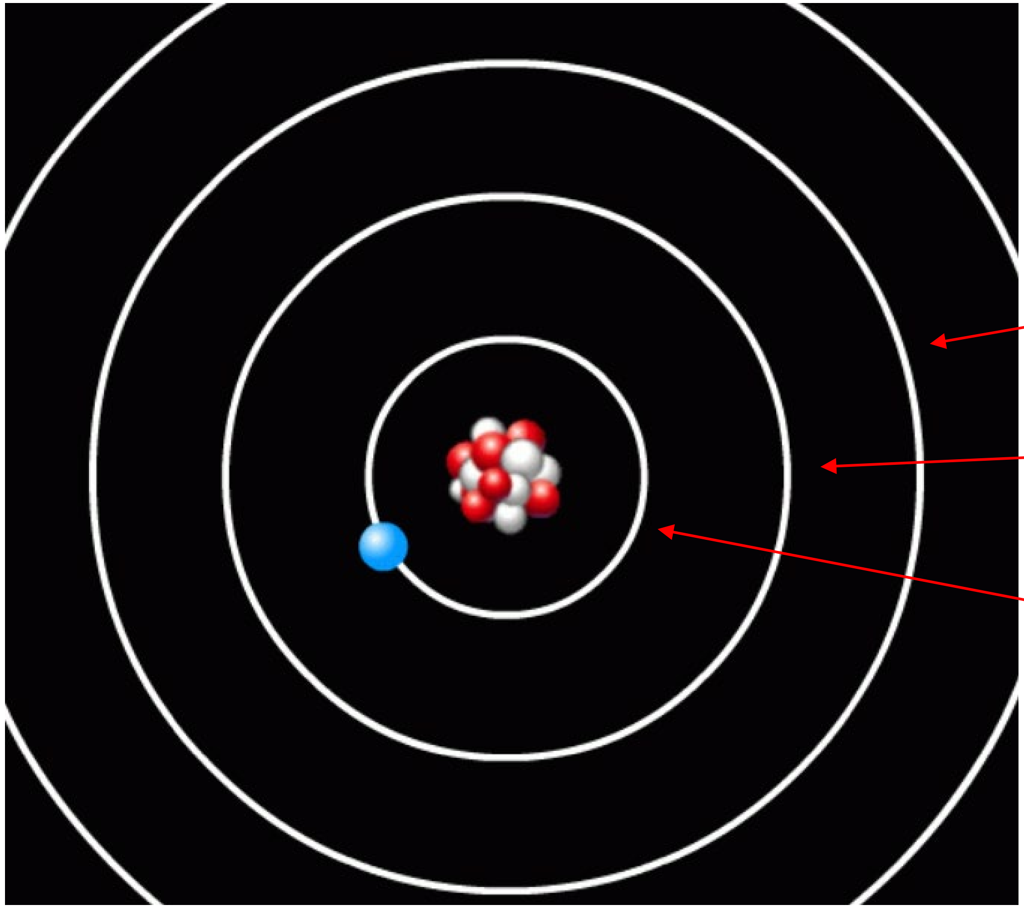
Examples of quantum states: atoms

$$|\psi\rangle = |0\rangle$$

Atom is in the lowest energy state

$$|\psi\rangle = |1\rangle$$

Atom is in the first excited state



$|2\rangle$

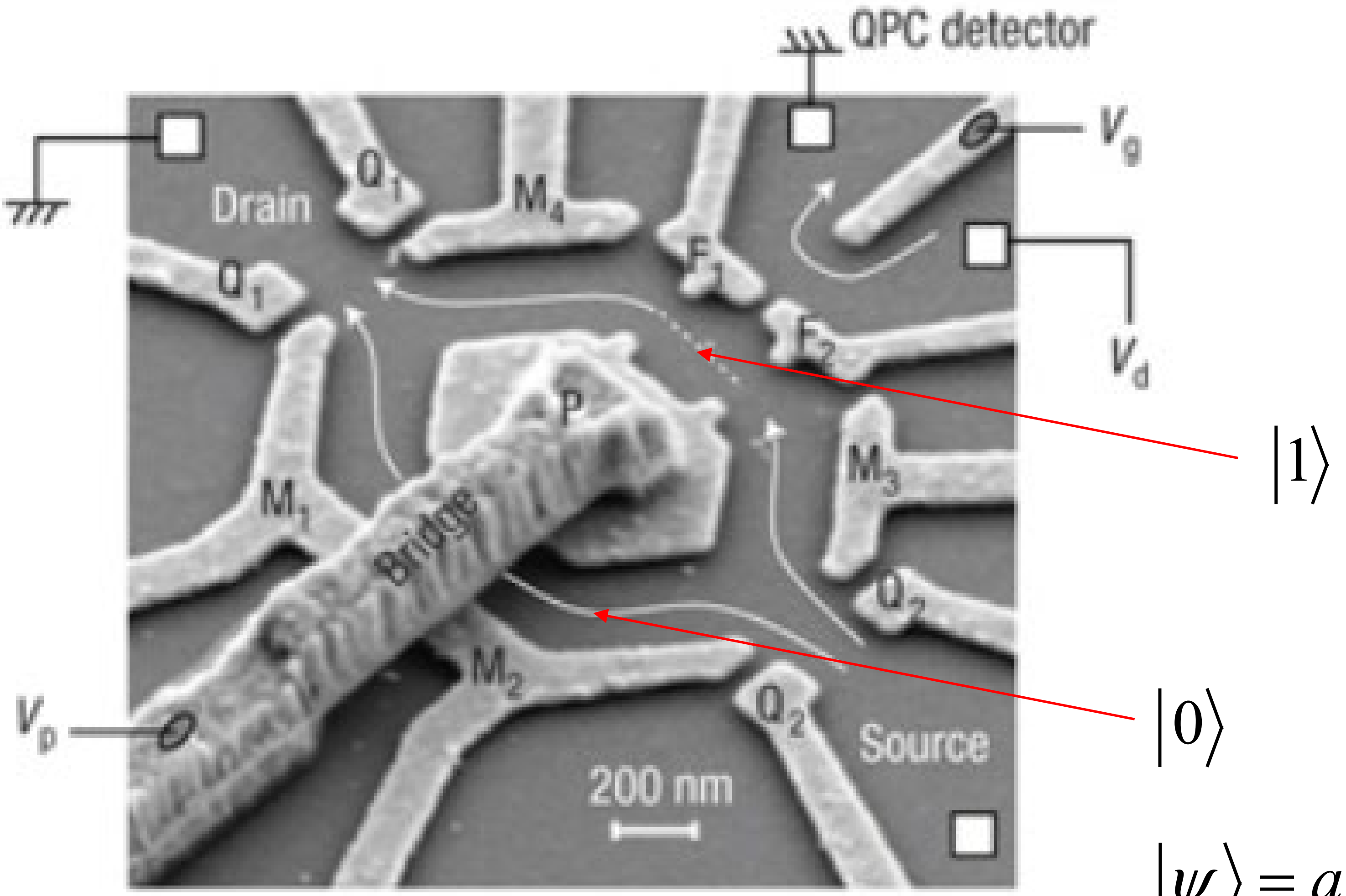
$|1\rangle$

$|0\rangle$

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle$$

Atom is partly in the lowest and first excited state

Examples of quantum states: split channel



$$|\psi\rangle = |0\rangle$$

Atom is in the left channel

$$|\psi\rangle = |1\rangle$$

Atom is in the right channel

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle$$

Atom is partly in the left and right channel

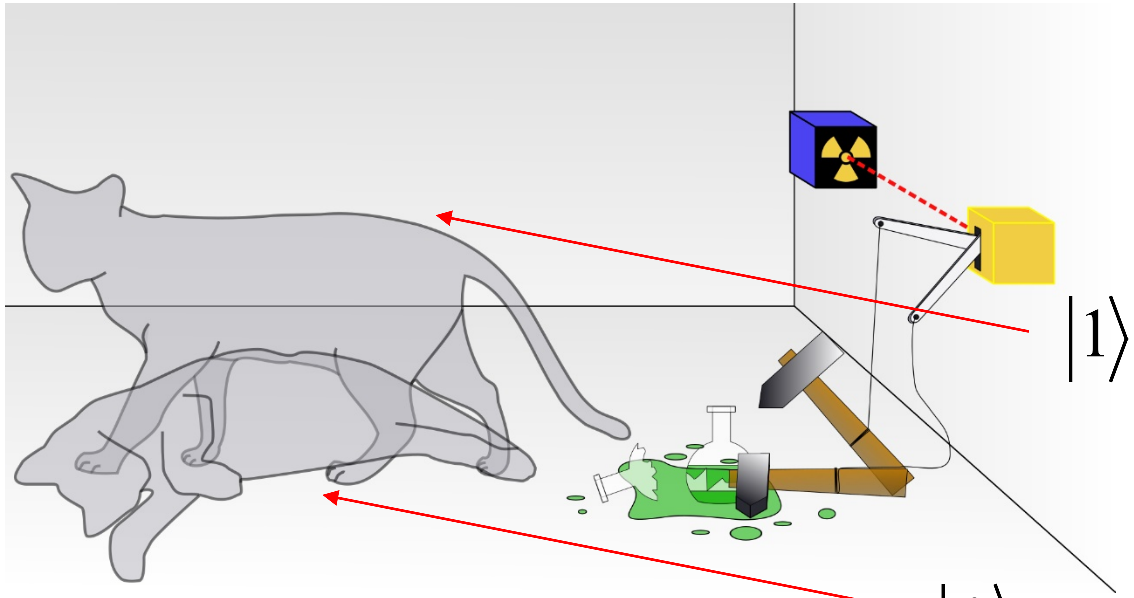
Examples of quantum states: Schrodinger's cat

$$|\psi\rangle = |0\rangle$$

Cat is dead

$$|\psi\rangle = |1\rangle$$

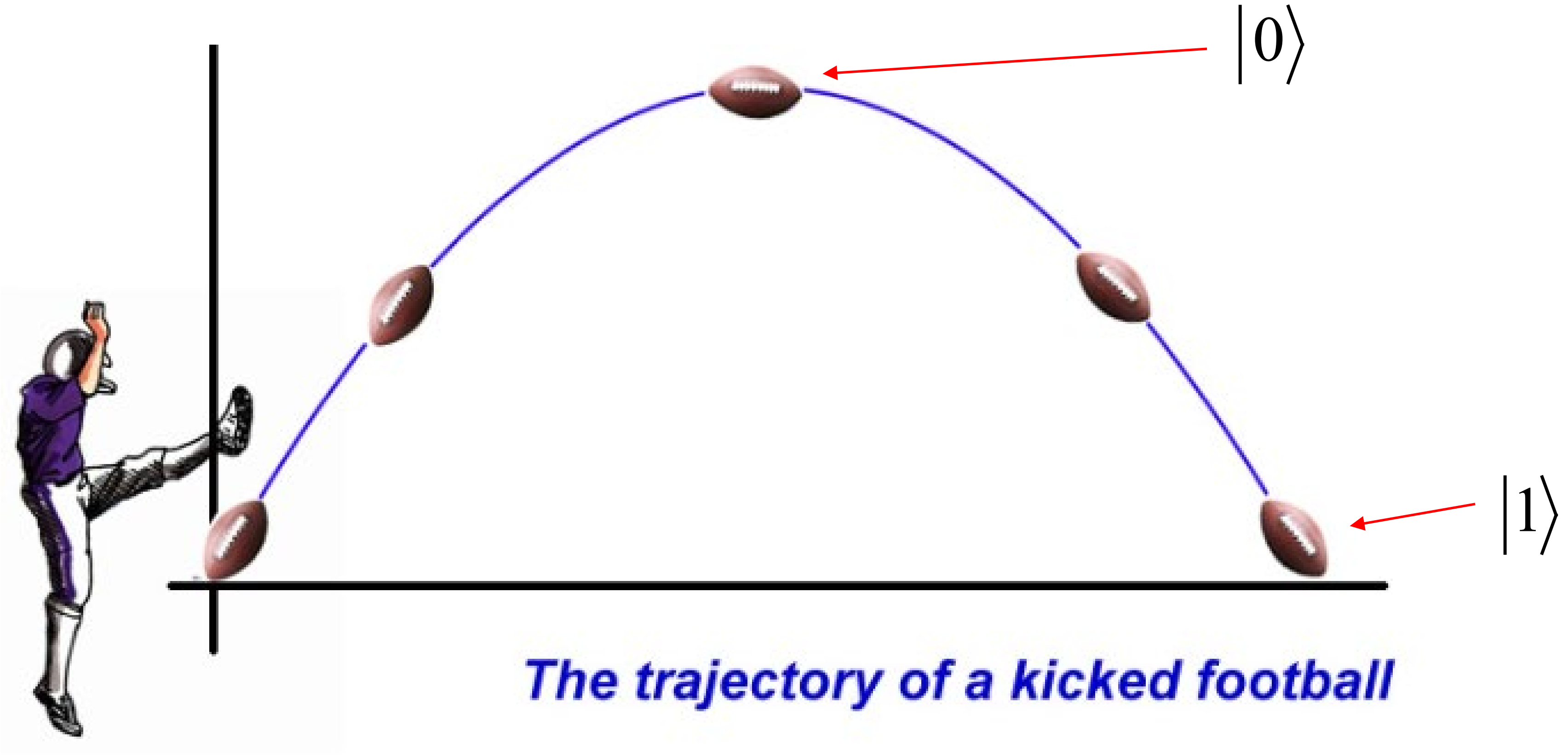
Cat is alive



$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle$$

Cat is alive and dead at the same time

Examples of quantum states: Projectile motion



$$|\psi\rangle = |0\rangle$$

Ball is at the top of the trajectory

$$|\psi\rangle = |1\rangle$$

Ball has just hit the ground

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle$$

Ball is at the top of the trajectory and hit the ground at the same time

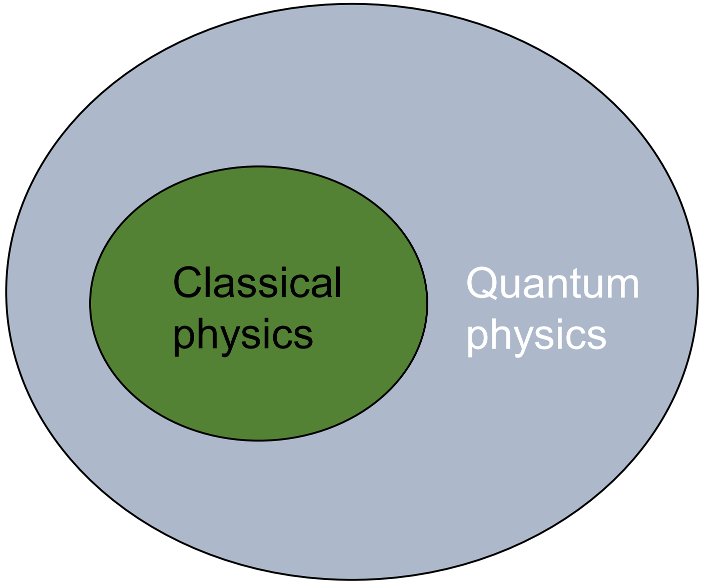
Comparison of classical vs quantum

The main difference between quantum physics and classical physics is that superpositions are allowed in quantum physics.

Classically, no superpositions are allowed, so it is impossible to have the ball both at the top of the trajectory and at the bottom at the same time. So the only vectors that are allowed classically are two cases

$$|\psi\rangle = |0\rangle \qquad \text{or} \qquad |\psi\rangle = |1\rangle$$

$$a_0 = 1, a_1 = 0 \qquad\qquad a_0 = 0, a_1 = 1$$



But in quantum physics you can have superpositions so any state like

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle$$

is also allowed. → Quantum physics is an EXTENSION of classical physics.

Normalization

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle + \ldots + a_N|N\rangle$$

$$= \sum_{n=0}^{N} a_n |n\rangle$$

The coefficients can be any complex numbers such that the length of the vector is 1

$$\langle\psi|\psi\rangle = 1$$

This implies

$$\sum_{n=0}^{N} |a_n|^2 = 1$$

Qubits

The simplest quantum system is a quantum two-level system, or a qubit.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Here the numbers a,b are complex and we need $|\alpha|^2 + |\beta|^2 = 1$

A neat way to write this so that the normalization is obeyed is $\alpha = \cos\frac{\theta}{2}, \beta = e^{i\phi}\sin\frac{\theta}{2}$

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$