Project 1 – Linear Programming



Submitted By:

Group 6

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Problem Description

Marketing budgets now comprise 11 percent of total company budgets, based on a CMO survey sponsored by the Fuqua School of Business at Duke University, Deloitte LLP, and the American Marketing Association. However, the effectiveness of marketing varies significantly: on the one hand, P&G cut more than \$100 million in digital marketing spending because their digital ads were largely ineffective; on the other hand, Netflix plans a 54% boost in ad spending because they got very positive feedback in international markets.

One potential reason for such variation is the way of making marketing budget allocations. Namely, how much to invest in each advertisement platform. As stated in the Handbook of Marketing Analytics:

...budget decisions are often based on gut feelings or on the negotiation skills of individual managers. Consequently, politics and individual opinions tend to shape the decision process instead of fact-based discussions. Obviously, these rules and practices bear the risk of results far away from the optimal, profit-maximizing budget.

Indeed, the marketing strategy of Netflix seems to be steered by data.

In this project, we use linear programming to build a simple marketing budget allocation strategy.

The marketing department is tasked with recommending the allocation of a \$10 million budget across various marketing mediums based on estimated Return on Investment (ROI) data provided by two consulting firms. The goal is to find an optimal budget allocation while adhering to specific constraints set by the Chief Marketing Officer.

The problem involves formulating this budget allocation as a linear program and using Gurobi to find the optimal solution based on both sets of ROI data.

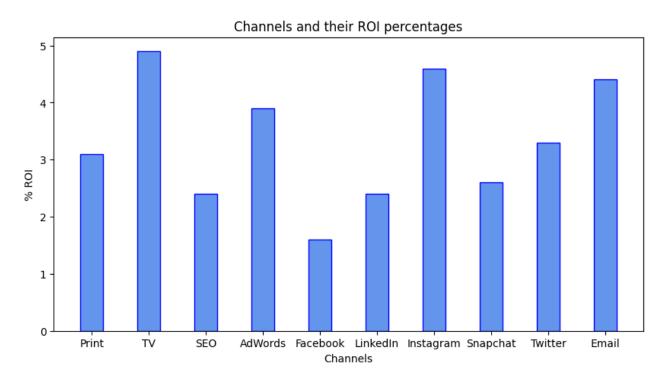
Solution

Q1-Q3

We begin by importing the various packages that would be utilized in our project. These include:

- NumPy: Package for handling arrays and other scientific calculations.
- Pandas: Package for data manipulation and analysis.
- Gurobipy: Package for optimizing linear and quadratic problems.
- Matplotlib: Package for plotting and data visualization.

The first consultancy firm provides us with a list of channels and their respective ROI percentages as shown here:



As we can see- TV, Instagram and surprisingly Email have the best ROI among all the available channels. We read all these in into a pandas data frame and extract the list of Channels and their respective ROIs as shown.

Our job is to allocate a budget of \$10 million among all these channels so that we can get the maximum ROI possible. Additionally, we have the following constraints set:

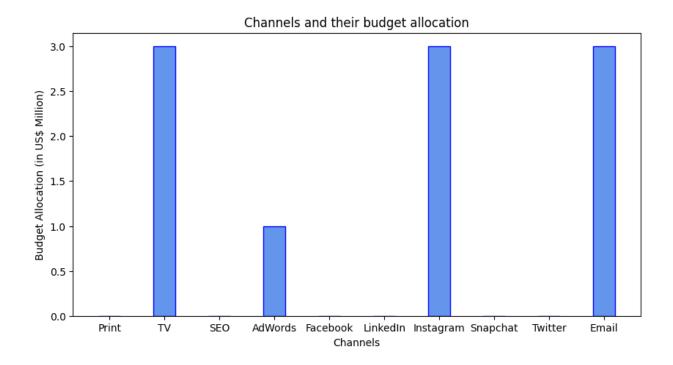
- The amount invested in print and TV should be no more than the amount spent on Facebook and Email. Surprisingly, email seems to be a great channel for reaching real people.
- The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice that of SEO and AdWords.
- For each platform, the amount invested should be no more than \$3M.

We then proceed to build a model using Gurobi and factor in the above constraints.

```
# Create model
mktMod = gp.Model()
# Add variables, with constraint_c
mktModX = mktMod.addVars(10, lb=0, ub=3)
# Set objective
mktMod.setObjective(sum(rois_1st[i] * mktModX[i] for i in range(10)), sense=gp.GRB.MAXIMIZE)
# Add constraints
conlist = [0]*3
# Total budget constraint
conlist[0] = mktMod.addConstr(sum(mktModX[i] for i in range(10)) <= 10)
# constraint_a
conlist[1] = mktMod.addConstr(mktModX[0] + mktModX[1] <= mktModX[4] + mktModX[9])
# constraint_b
conlist[2] = mktMod.addConstr(mktModX[4] + mktModX[5] + mktModX[6] + mktModX[7] + mktModX[8] >= 2*(mktModX[2] + mktModX[3]))
mktMod.Params.OutputFlag = 0
# solve the model
mktMod.optimize()
```

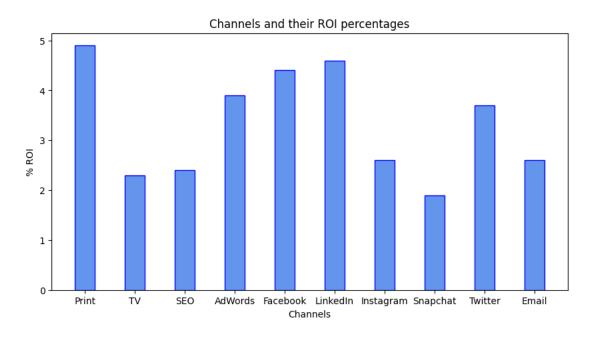
We end up with the following budget distribution which satisfies the above conditions and maximizes the ROI:

We see that TV, Instagram and Email have the highest allocation as they provide the biggest ROI. As the budget allocated to any one of them couldn't exceed \$ 3 million due to our constraints, the remaining \$ 1 million is allocated to the channel with the next best ROI – AdWords. Using these channels, our optimal ROI comes out to be \$ 0.456 million or \$456,000.

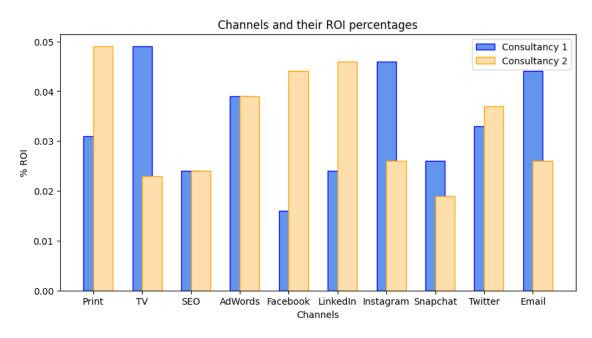


Q4-Q5

Now due to the cautious nature of the CFO, the firm has decided to take ROI figures from another consultancy. Here are their estimates:

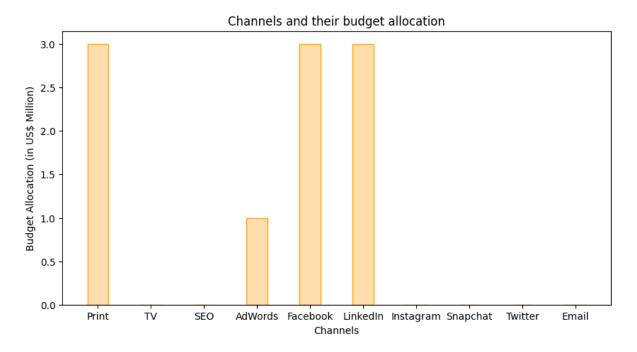


As we can see, these new estimates vary quite a bit from the first consultancy's estimates. This can be highlighted from the following graph:

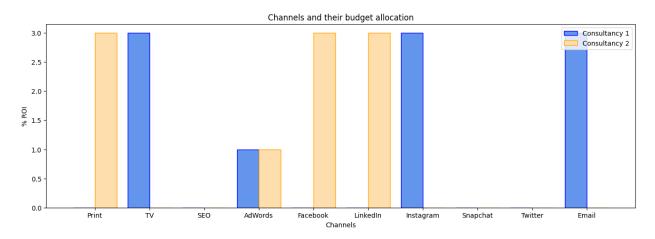


Keeping the constraints, we build a model with the help of Gurobi taking these new values.

The budget allocation comes out as:



The best channels to allocate money to come out as Print, Facebook and LinkedIn, completely different from the results we get from the first model. Only the budget allocated to AdWords remains the same. The ROI for this allocation comes out to be \$ 0.456 million or \$456,000- exactly the same as the previous model. Hence making the difference between both ROIs 0. A comparison of both the consultancies is shown here:

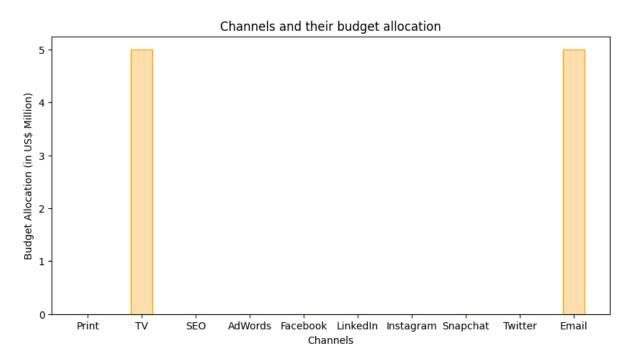


Now assuming the ROI data from the first consultancy is correct, if we use the allocation from the second consultancy, the total ROI comes out to be \$252,000 – \$204,000 less than the optimal budget calculated. After switching up the roles of the consultancy, the allocation comes out to be \$264,000- \$192,000 less than the optimal.

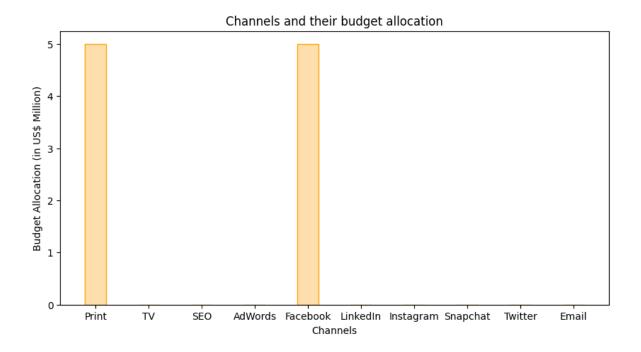
At first, the third constraint set by the boss doesn't seem useful if we just look at the numbers. Ideally, we just invest all our budget in a couple of channels that seem to have the highest ROI. But further looking into the problem, we understand it's always better to diversify where a company allocates its marketing budget because we never know when a channel will become inefficient or when another untapped channel's potential to attract customers would increase.

But perhaps the answer would be clearer if we removed the third constraint altogether and then tried the calculations.

The new allocation for the first consultancy's prediction if we remove the third constraint comes out as:



The allocation for second consultancy' prediction if we remove the third constraint:



As we can see, the number of channels in which we invest falls from 4 to 2 for both cases. The total ROI for the first case comes out to be \$456,000- the same as previously. However, the ROI for the second consultancy's figures increases slightly with this newer approach to \$465,000.

Q6

To explore this further, we performed some analysis of how changing the ROI percentages would still result in the optimal solution. We have summarized each medium's ROI ranges so that we get the optimal allocation:

| | Platforms | Min_roi | Actual_roi | Max_roi |
|---|-----------|---------|------------|---------|
| 0 | Print | 0.0 | 3.1 | 4.9 |
| 1 | TV | 3.9 | 4.9 | 6.2 |
| 2 | SEO | 0.0 | 2.4 | 3.9 |
| 3 | AdWords | 3.3 | 3.9 | 4.6 |
| 4 | Facebook | 0.0 | 1.6 | 2.9 |
| 5 | LinkedIn | 0.0 | 2.4 | 3.9 |
| 6 | Instagram | 3.9 | 4.6 | inf |
| 7 | Snapchat | 0.0 | 2.6 | 3.9 |
| 8 | Twitter | 0.0 | 3.3 | 3.9 |
| 9 | Email | 2.9 | 4.4 | inf |

Where Min_roi is the minimum ROI possible, Actual_roi is the actual given ROI and Max roi is the maximum ROI, for the optimal solution.

Q7

The expected ROI for each channel for the first company is available. Additionally provided is the actual monthly ROI for each channel. We will optimise the model to deliver the best financing allocation based on the ROI projections of the first firm in order to obtain the optimal allocation for each month. The real monthly ROI will then be calculated using these allocations. Based on the criteria given above, this will provide us with the budget for the upcoming month.

The table shows the optimal budget allocation for each marketing channel in each month, along with the total budget available for that month. These allocations are based on the reinvestment rules and constraints, and they maximize the overall ROI. These values represent the monthly budget allocation for each marketing

channel, maximizing the overall ROI while adhering to the constraints and reinvestment rules. The "Total budget" column shows the available budget for each month, which includes the reinvestment from the previous month.

| | Print | TV | SEO | AdWords | Facebook | LinkedIn | Instagram | Snapchat | Twitter | Email | Total budget |
|-----------|----------|----------|-----|----------|----------|----------|-----------|----------|----------|----------|--------------|
| January | 3.000000 | 0.000000 | 0.0 | 1.406929 | 0.000000 | 0.0 | 2.813859 | 0.0 | 0.000000 | 3.000000 | 10.000000 |
| February | 3.000000 | 0.000000 | 0.0 | 2.333333 | 3.000000 | 0.0 | 0.000000 | 0.0 | 1.666667 | 0.000000 | 10.186468 |
| March | 0.000000 | 0.000000 | 0.0 | 3.000000 | 0.000000 | 3.0 | 1.186468 | 0.0 | 3.000000 | 0.000000 | 10.203390 |
| April | 0.000000 | 0.000000 | 0.0 | 3.000000 | 0.000000 | 3.0 | 3.000000 | 0.0 | 1.203390 | 0.000000 | 10.203788 |
| May | 1.203788 | 0.000000 | 0.0 | 0.000000 | 0.000000 | 0.0 | 3.000000 | 0.0 | 3.000000 | 3.000000 | 10.200169 |
| June | 3.000000 | 0.000000 | 0.0 | 0.000000 | 0.000000 | 0.0 | 3.000000 | 0.0 | 1.200169 | 3.000000 | 10.205493 |
| July | 0.000000 | 0.000000 | 0.0 | 3.000000 | 1.205493 | 0.0 | 3.000000 | 0.0 | 3.000000 | 0.000000 | 10.211914 |
| August | 2.711914 | 0.000000 | 0.0 | 1.500000 | 0.000000 | 0.0 | 0.000000 | 0.0 | 3.000000 | 3.000000 | 10.214142 |
| September | 0.607071 | 0.000000 | 0.0 | 3.000000 | 0.000000 | 3.0 | 0.000000 | 0.0 | 3.000000 | 0.607071 | 10.218998 |
| October | 0.000000 | 0.000000 | 0.0 | 3.000000 | 0.000000 | 3.0 | 3.000000 | 0.0 | 0.000000 | 1.218998 | 10.201361 |
| November | 3.000000 | 0.000000 | 0.0 | 1.201361 | 0.000000 | 0.0 | 3.000000 | 0.0 | 0.000000 | 3.000000 | 10.185683 |
| December | 3.000000 | 2.092841 | 0.0 | 0.000000 | 3.000000 | 0.0 | 0.000000 | 0.0 | 0.000000 | 2.092841 | 10.220788 |

The optimal budget allocation at the end of the year: \$0.43M

Based on the provided monthly budget allocations and the ROI optimization, we can observe the following insights regarding the importance of each marketing channel per month based on ROI:

January:

Print and email maintain high allocations in January.

February:

- "Print" "Facebook" and "Email" receive the highest budget allocations, indicating their importance for generating ROI in February.
- "Adwords" and "Twitter" receive moderate allocations.

March:

- "Facebook" "Instagram" and "Email" receive the highest budget allocations, suggesting their importance for generating ROI in March.
- "TV" and "AdWords" receive moderate allocations.
- "LinkedIn" "Snapchat" "Twitter" and "SEO" receive lower allocations.

April:

• "Adwords" "Instagram" maintain their high allocations, indicating their importance for generating ROI in April.

May:

• "Twitter" "Email" and "Instagram" receive higher budget allocations, suggesting their importance for generating ROI in May.

June:

• "Print" "Email" and "Instagram" maintain their high allocations, indicating their importance for generating ROI in June.

July:

• "AdWords" "Instagram" and "Twitter" receive higher budget allocations, suggesting their importance for generating ROI in July.

August:

- "Twitter" and "Email" receive higher budget allocations, indicating their importance for generating ROI in August.
- "Print" receive moderate allocations.

September:

• "AdWords" "LinkedIn" and "Twitter" receive higher budget allocations, suggesting their importance for generating ROI in September.

October:

- "AdWords" "LinkedIn" and "Instagram" maintain their high allocations, indicating their importance for generating ROI in October.
- "Email" receives a moderate allocation.

November:

• "Print" "Facebook" and "Email" receive higher budget allocations, suggesting their importance for generating ROI in November.

December:

- "Print" "Facebook" receive higher budget allocations, indicating their importance for generating ROI in December.
- "TV" and "Email" receive moderate allocations.

These monthly budget allocations have been adjusted based on ROI optimization and are reflective of the expected returns on investment for each marketing channel during each month.

These observations are based on the monthly optimization of ROI while considering budget constraints and reinvestment. The importance of each marketing channel can vary from month to month, reflecting the dynamic nature of marketing campaigns and the expected returns on investment.

Q8

For each channel, the first business has given us the ROI data. The monthly ROI for each channel is also available. To provide the best allocation of money based on the first firm's ROI estimates, we will enhance the model. We will then apply these allocations to the actual monthly ROI to produce the best allocation each month. In accordance with the aforementioned parameters, this will provide us with the budget for the following month.

The table below shows the optimal budget allocation for each marketing channel in each month, along with the total budget available for that month. These allocations are based on the reinvestment rules and constraints, and they maximize the overall ROI. These values represent the monthly budget allocation for each marketing channel, maximizing the overall ROI while adhering to the constraints and reinvestment rules. The "Total budget" column shows the available budget for each month, which includes the reinvestment from the previous month.

| | Print | TV | SEO | AdWords | Facebook | LinkedIn | Instagram | Snapchat | Twitter | Email | Total budget |
|---|--|----------------------------|----------------------------|--|---|-------------------------------------|--|----------|--|--|--------------|
| January | 0.000000 | 0.000000 | 0.0 | 0.000000 | 0.000000 | 0.0 | 0.000000 | 0.0 | 0.000000 | 0.000000 | 0.000000 |
| February | 0.000000 | 0.000000 | 0.0 | 0.926404 | 3.000000 | 0.0 | -2.813859 | 0.0 | 1.666667 | -3.000000 | 0.186468 |
| March | -3.000000 | 0.000000 | 0.0 | 0.666667 | -3.000000 | 3.0 | 1.186468 | 0.0 | 1.333333 | 0.000000 | 0.016922 |
| April | 0.000000 | 0.000000 | 0.0 | 0.000000 | 0.000000 | 0.0 | 1.813532 | 0.0 | -1.796610 | 0.000000 | 0.000398 |
| May | 1.203788 | 0.000000 | 0.0 | -3.000000 | 0.000000 | -3.0 | 0.000000 | 0.0 | 1.796610 | 3.000000 | -0.003619 |
| June | 1.796212 | 0.000000 | 0.0 | 0.000000 | 0.000000 | 0.0 | 0.000000 | 0.0 | -1.799831 | 0.000000 | 0.005325 |
| July | -3.000000 | 0.000000 | 0.0 | 3.000000 | 1.205493 | 0.0 | 0.000000 | 0.0 | 1.799831 | -3.000000 | 0.006420 |
| August | 2.711914 | 0.000000 | 0.0 | -1.500000 | -1.205493 | 0.0 | -3.000000 | 0.0 | 0.000000 | 3.000000 | 0.002228 |
| September | -2.104843 | 0.000000 | 0.0 | 1.500000 | 0.000000 | 3.0 | 0.000000 | 0.0 | 0.000000 | -2.392929 | 0.004856 |
| October | -0.607071 | 0.000000 | 0.0 | 0.000000 | 0.000000 | 0.0 | 3.000000 | 0.0 | -3.000000 | 0.611927 | -0.017637 |
| November | 3.000000 | 0.000000 | 0.0 | -1.798639 | 0.000000 | -3.0 | 0.000000 | 0.0 | 0.000000 | 1.781002 | -0.015678 |
| December | 0.000000 | 2.092841 | 0.0 | -1.201361 | 3.000000 | 0.0 | -3.000000 | 0.0 | 0.000000 | -0.907159 | 0.035105 |
| | | | | | | | | | | | |
| | | | | | | | | | | | |
| | Print | TV | SEO | AdWords | Facebook | LinkedIn | Instagram | Snapcha | t Twitte | Email | Total budget |
| January | Print - | TV | SEO - | AdWords - | Facebook | LinkedIn - | Instagram - | Snapcha | t Twitter | Email | Total budget |
| January February | Print - | TV - | SEO - | AdWords - Unstable | Facebook - Unstable | LinkedIn - - | Instagram - Unstable | | Twitter | - | Total budget |
| February | Print Unstable | TV | SEO | - | - Unstable | LinkedIn - Unstable | - Unstable | • | | - Unstable | Total budget |
| February | - | TV | - - - - | - Unstable | - Unstable | - | - Unstable Unstable | | | - Unstable | Total budget |
| February March April | - | TV | | - Unstable | - Unstable Unstable | - Unstable | Unstable Unstable Unstable | | - Unstable | Unstable | Total budget |
| February March April May | - Unstable | - - - | - - - | - Unstable - | - Unstable Unstable | - Unstable | Unstable Unstable Unstable | | UnstableUnstableUnstable | Unstable Unstable Unstable | Total budget |
| February March April May June | - Unstable - Unstable | - | - - - | - Unstable - - Unstable | - Unstable Unstable - | - Unstable | Unstable Unstable Unstable | | UnstableUnstableUnstableUnstable | Unstable - Unstable Unstable | Total budget |
| February March April May June July | Unstable Unstable Unstable | - - - - | - - - | Unstable Unstable Unstable | - Unstable Unstable - - | Unstable Unstable | Unstable Unstable Unstable | | UnstableUnstableUnstableUnstableUnstableUnstable | Unstable - Unstable Unstable | Total budget |
| February March April May June July | Unstable Unstable Unstable Unstable Unstable | - - - - - | - - - | Unstable Unstable Unstable Unstable | Unstable Unstable Unstable Unstable Unstable | Unstable Unstable | Unstable Unstable Unstable | | Unstable Unstable Unstable Unstable Unstable Unstable | Unstable Unstable Unstable Unstable Unstable | Total budget |
| February March April May June July August September | Unstable Unstable Unstable Unstable Unstable Unstable | - - - - - - | - - - - | Unstable Unstable Unstable Unstable | Unstable Unstable Unstable Unstable Unstable | Unstable Unstable Unstable | Unstable Unstable Unstable Unstable | | - Unstable | Unstable Unstable Unstable Unstable Unstable Unstable Unstable | Total budget |
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Conclusion

In conclusion, our project has successfully demonstrated the practical application of linear programming in the realm of marketing budget allocation. We were tasked with the challenge of optimizing a \$10 million budget across various marketing channels, all while considering specific constraints set by the Chief Marketing Officer. Through this endeavor, we recognized the pivotal role of accurate and dependable ROI data, as variations between two consultancy firms' estimates significantly impacted our optimal budget allocations.

One noteworthy aspect of our analysis was the exploration of constraints. By removing the limitation on the number of channels to invest in, we gained insights into the value of diversification. Our findings underscored the importance of spreading the budget across multiple channels to mitigate risk and adapt to changing dynamics within the marketing landscape.

Furthermore, our project showcased the flexibility of our model in handling diverse ROI scenarios. By introducing a monthly optimization approach, we provided valuable insights into the evolving significance of marketing channels over the course of a year. In today's ever-changing business environment, data-driven decision-making and a strategic approach to budget allocation remain essential for achieving long-term marketing success.