formula

$A ::= P, Q, R, \ldots$ ← propositional letters

$\mid \neg A$

$\mid (A \wedge A)$

$\mid (A \vee A)$

$\mid (A \rightarrow A)$

$\mid (A \leftrightarrow A)$

$P \rightarrow Q \equiv \neg P \vee Q$

$P \leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$

$\equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

$\equiv (\neg P \vee Q) \wedge (\neg Q \vee P)$

$P \rightarrow Q \equiv \neg Q \rightarrow \neg P$

$\neg (P \vee Q) \equiv \neg P \wedge \neg Q$

$v : \underline{L} \longmapsto \{\underline{0}, \underline{1}\}$

$v \not\models A$

$v \models A$ ← sat

$A = P \vee Q$

$v(P) = 0$

$v(Q) = 1$

$v \models P \vee Q$

$\neg A$ unsat $\Longrightarrow$ A valid

$\neg A$ valid $\Longrightarrow$ A unsat

$v \models_\rho \underline{(x < y)}$

→ $\underline{\rho(x) = 7}$

$\rho(y) = 8$

$\underline{\rho}(x) = 7$

$\underline{v \models_\rho \exists x . x < 6}$

$+ \equiv \vee \qquad \cdot \equiv \wedge \qquad - \equiv \neg$

$$p \cdot (q + r) \equiv (p \cdot q) + (p \cdot r)$$

$$\bar{a} = -a$$

$B = \{0, 1\}$

$$(\underset{\uparrow \text{ non-empty set}}{B}, \underbrace{0, 1}_{\in B}, \underset{\uparrow \text{ binary func}}{+}, -)$$

$-$ ← unary

$(Pow(X), \phi, X, \cup, X\backslash)$

$\neg, \vee, \wedge$

$X = \{1, 2, 3\}$

$\neg, \vee \qquad \wedge, \vee$

$Pow(X) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \dots, \{1, 2, 3\}\}$

$\neg, \wedge \qquad \neg, \rightarrow$

P, Q, R predicate

$P(t_1, t_2, \dots, t_n)$ binary $P(t_1, t_2)$

$t \leftarrow$ term

$$t ::= t_1 \mid f(t_1, t_2, \dots t_n)$$

$$P(t_1, f(t_2, t_3))$$

$$A ::= P(t_1, t_2, \dots, t_n)$$
$$\mid \neg A$$
$$\mid (A \wedge A)$$
$$\mid (A \vee A)$$
$$\mid (A \rightarrow A)$$
$$\mid (A \leftrightarrow A)$$
$$\mid \forall x\, A$$
$$\mid \exists x\, A$$

$D = \mathbb{N}$

$$\forall x \exists y.\ x > y$$

$$\exists x \forall y.\ x > y$$

$$\forall x \forall y \equiv \forall y \forall x$$

$$\neg \forall x A \equiv \exists x \neg A$$

$$\neg \exists x A \equiv \forall x \neg A$$

$$\forall x (A \vee B) \equiv (\forall x A) \vee B$$ if x not in B free

$$\exists x (A \wedge B) \equiv (\exists x A) \wedge B$$

$$\forall x (A \wedge B) \equiv (\forall x A) \wedge (\forall x B)$$

$$\forall x (A \vee B) \not\equiv (\forall x A) \vee (\forall x B)$$

$$\exists x (A \vee B) \equiv (\exists x A) \vee (\exists x B)$$

$$\forall \underline{x} ((\underline{x} \rightarrow y) \wedge \boxed{(y \rightarrow z)})$$
$$\equiv (\forall x (x \rightarrow y)) \wedge (y \rightarrow z)$$

x & y are bound

$$\exists \underline{x} \forall \underline{y} ((\underline{x} \to \underline{y}) \wedge ((\underline{y} \to (\underline{z} \vee \underline{a})) \wedge \underline{x}))$$

free variables z & a

Sentence - a formula where every var is bound by some quantifier.

$$\exists \underline{y} \forall \underline{x} (\underline{x} \to \underline{y}) \wedge \forall \underline{z} (\neg \underline{z})$$

$$\Sigma_i = (\mathcal{F}, \mathcal{P})$$

FOL

set of function symbols

set of predicate symbols

$$\Sigma_i = (\{\underline{0}, \underline{s}, \underline{+}, \underline{\times}\}, \{=, <\})$$

functions - 0 - k ary function

predicates - relations

FO Structure

$$\mathcal{M} = (D, \mathcal{F}^{\mathcal{M}}, \mathcal{P}^{\mathcal{M}})$$

domain (non-empty)

functions

predicates

$$\mathcal{M} = (\mathbb{N}, \{0, s, +, \times\}, \{=, <\})$$

$\mathcal{M} \models A$

$\mathcal{M} \not\models_\rho \boxed{P(t_1, t_2)}$

$t_1 > t_2$

FO structure

bigger(x, y)

$$\mathcal{M} \models_\rho P(t_1, \dots, t_k) \iff (\rho(t_1), \dots, \rho(t_k)) \in P^{\mathcal{M}}$$

valuation

$\rho(x) = 7$

$\rho(y) = 8$

$$\mathcal{M} \models_\rho \neg A \iff \mathcal{M} \not\models_\rho A$$

$$\mathcal{M} \models_\rho A \land B \iff \mathcal{M} \models_\rho A \text{ and } \mathcal{M} \models_\rho B$$

$$\mathcal{M} \models_\rho A \lor B \iff \mathcal{M} \models_\rho A \text{ or } \mathcal{M} \models_\rho B$$

$$\mathcal{M} \models_\rho A \to B \iff \mathcal{M} \not\models_\rho A \text{ or } \mathcal{M} \models_\rho B$$

$$\mathcal{M} \models_\rho A \leftrightarrow B \iff (\mathcal{M} \models_\rho A \text{ and } \mathcal{M} \models_\rho B) \text{ or } (\mathcal{M} \not\models_\rho A \text{ and } \mathcal{M} \not\models_\rho B)$$

$$\mathcal{M} \models_\rho \forall x (A) \iff \mathcal{M} \models_{\rho[x \mapsto d]} A \text{ for all } d \in D$$

$$\mathcal{M} \models_\rho \exists x (A) \iff \mathcal{M} \models_{\rho[x \mapsto d]} A \text{ for some } d \in D$$