FO Structure

$$M = (D, \mathcal{F}^M, \mathcal{P}^M)$$

domain (non-empty)

functions

predicates

$$M = (\mathbb{N}, \{0, S, +, \times\}, \{=, <\})$$

$M \vDash A$

FO structure

$M \nvDash_\rho \boxed{P(t_1, t_2)}$ $t_1 > t_2$

bigger(x, y)

$M \vDash_\rho P(t_1, \ldots, t_k) \iff (\rho(t_1), \ldots, \rho(t_k)) \in P^M$

valuation

$\rho(x) = 7$

$\rho(y) = 8$

$M \vDash_\rho \neg A \iff M \nvDash_\rho A$

$M \vDash_\rho A \wedge B \iff M \vDash_\rho A$ and $M \vDash_\rho B$

$M \vDash_\rho A \vee B \iff M \vDash_\rho A$ or $M \vDash_\rho B$

$A \to B \equiv \neg A \vee B$

$M \vDash_\rho A \to B \iff M \nvDash_\rho A$ or $M \vDash_\rho B$

$A \leftrightarrow B \equiv (A \wedge B) \vee (\neg A \wedge \neg B)$

$M \vDash_\rho A \leftrightarrow B \iff (M \vDash_\rho A$ and $M \vDash_\rho B)$ or $(M \nvDash_\rho A$ and $M \nvDash_\rho B)$

$M \vDash_\rho \forall x(A) \iff M \vDash_{\rho[x \mapsto d]} A$ for all $d \in D$

$M \vDash_\rho \exists x(A) \iff M \vDash_{\rho[x \mapsto d]} A$ for some $d \in D$

$$M = (\mathcal{F}, \mathcal{P})$$

$$\Sigma = (D, \mathcal{F}^M, \mathcal{P}^M)$$

non-ϕ domain

functions

relations / predicates

$$\Sigma = (\mathbb{N}, \{0, s, +, \times\}, \{=, >\})$$

$$\rho : Vars \longmapsto \{1, 0\} \leftarrow \text{prop}$$

$$\rho : Vars \longmapsto D$$

$$\rho(x) = 2 \qquad Vars = \{x, y\}$$

$$M \models_\rho A \leftarrow \text{formula}$$

$$A = s(x) > y$$

$$M \models A$$

$$\models A$$

n-ary

$$\underline{M} \models_{\underline{\rho}} \underline{P(t_1, t_2, \ldots, t_n)}$$

$$M \models P^M(\rho(t_1), \rho(t_2), \ldots, \rho(t_n))$$

$$\rightarrow \rho(t_1), \rho(t_2), \rho(t_n) \in P^M$$

$\exists$ $\forall$

$\rho[x \mapsto d]\ A$

replace all free x's in A w/ d

$A = (\exists x\ x = y) \wedge \neg x$

$\rho[x \mapsto d](A) = (\exists x\ x = y) \wedge \neg d$

$A[d/x]$

replace all free x's w/ d

Stevie

i) $\neg \exists x.\ S \rhd x \equiv \forall x.\ \neg(S \rhd x)$

ii) $\forall x \forall y.\ \neg(x = y) \wedge \neg(x \sim S) \wedge (y \sim S) \rightarrow x \rhd y$

iii) $\forall x \forall y.\ \neg(x = y) \wedge x \rhd y \rightarrow (x \rhd S \wedge y \rhd S)$

iv) $(\exists x. \neg(x = S) \wedge x \sim S) \rightarrow \forall x (x \sim S)$

v) $\exists x \forall y. \neg(x \sim y) \vee (x = y) \equiv \exists x \forall y.\ (x \sim y) \rightarrow (x = y)$

$(a+b)\cdot(b+c)\cdot(c+d)$

$(a+b)\cdot(\overline{b}\cdot\overline{c})\cdot(c+d)$

$=(a\cdot\overline{b}\cdot\overline{c}+\cancel{b\cdot\overline{b}\cdot\overline{c}})\cdot(c+d)$

$=a\cdot\overline{b}\cdot\overline{c}\cdot(c+d)$

$=\cancel{a\cdot\overline{b}\cdot\overline{c}\cdot c}+a\cdot\overline{b}\cdot\overline{c}\cdot d$

$=a\cdot\overline{b}\cdot\overline{c}\cdot d$

A	B	$\neg A$	$A \vee B$	$A \wedge B$
0	0	1	0	0
0	1		1	0
1	0	0	1	0
1	1		1	1

A	B	$A \sim B$
0	0	1
0	1	0
1	0	0
1	1	0

$\therefore \neg A = A \sim A$

$A \vee B = (A \sim B) \sim (A \sim B)$

$A \wedge B = \neg(\neg A \vee \neg B)$

$= \{(((A\sim A)\sim(B\sim B))\sim((A\sim A)\sim(B\sim B))) \sim$

$\{((A\sim A)\sim(B\sim B))\sim((A\sim A)\sim(B\sim B))$

$A \rightarrow B$:

$A \leftrightarrow B$: