Dynamic Relationship Between Yield Curves and Commodity Futures Spreads: A Multi-Window DCC-GARCH Analysis

Abstract

This research examines the time-varying correlation between yield curve dynamics and commodity futures spreads using a sophisticated multi-window Dynamic Conditional Correlation GARCH approach. The analysis reveals significant temporal variation in these correlations, with distinct periods exhibiting both positive and negative relationships. These patterns depend on prevailing market conditions and the timeframes analyzed, suggesting a complex and evolving relationship between these financial variables.

The study specifically focuses on the relationship between the 10-year minus 2-year Treasury yield spread (T10Y2Y) and the 6-month futures contract spread across multiple window lengths (126, 256, and 512 trading days). The findings provide valuable insights for traders, portfolio managers, risk professionals, and macroeconomic analysts seeking to understand cross-asset relationships and develop more effective strategies.

Contents

1 Research Framework and Objectives

1.1 Primary Research Focus

This study investigates the dynamic relationship between two critical financial indicators:

- **Yield Curve Spreads**: 10-year minus 2-year Treasury yield spread (T10Y2Y), which serves as a key indicator of economic expectations and monetary policy outlook.
- **Commodity Futures Pricing Dynamics**: 6-month futures contract spread, which reflects market expectations for supply-demand balance, storage costs, and risk premiums in commodity markets.

The primary objective is to quantify the time-varying correlation between these variables across different market environments, providing a comprehensive understanding of cross-market dynamics that can inform trading strategies and risk management approaches.

1.2 Key Research Questions

1. What is the nature and direction of correlation between yield curve dynamics and commodity futures spreads?
2. How does this relationship evolve through time, especially during different economic cycles?
3. Are there identifiable market conditions or regimes where this relationship strengthens or weakens?
4. How do different analytical window lengths affect the observed correlation patterns?
5. Can persistent structural shifts in this relationship be identified and explained?

1.3 Research Scope

The analysis encompasses historical data from January 2014 through August 2022, capturing:

- Multiple economic cycles with varying growth and inflation expectations
- Yield curve steepening, flattening, and inversion periods
- COVID-19 pandemic disruptions
- Unprecedented monetary policy interventions
- Supply chain disruptions and commodity market volatility

This diverse set of market conditions allows for robust analysis of the relationship under different regimes.

2 Theoretical Framework

2.1 Foundational Theories

- **Cost-of-Carry Model**: $F_t = S_t e^{(r+c-y)(T-t)}$

  Interest rates form a fundamental component in futures pricing, with the theoretical price of a futures contract determined by the spot price adjusted for interest rates, storage costs, and convenience yield. As yield curves change shape, this should theoretically affect the pricing structure of futures contracts with different maturities.

- **Expectations Theory**: Interaction between yield curves and futures curves

  Yield curves reflect market expectations about future economic conditions and monetary policy. Similarly, commodity futures curves embody expectations about future supply-demand dynamics. The interaction between these expectation mechanisms provides insights into market efficiency and information transmission across asset classes.

- **Risk Premium Theory**: Joint risk pricing across asset classes

  Both yield curves and futures curves incorporate risk premiums that compensate investors for various uncertainties. The correlation between these risk premiums offers insights into how market participants price risk across different asset classes.

2.2 Research Hypotheses

- **Economic Regime Hypothesis**: Yield curve steepening, typically associated with expectations of economic expansion, leads to wider contango in commodities with high storage costs due to increased carrying charges.
- **Monetary Policy Hypothesis**: During periods of yield curve inversion, which often precedes economic contraction, commodities with supply constraints tend to exhibit increased backwardation as near-term scarcity concerns dominate.
- **Market Segmentation Hypothesis**: Different commodity classes (energy, precious metals, agriculture) exhibit varying sensitivity to yield curve dynamics based on their unique supply-demand characteristics and storage economics.

- **Regime Shift Hypothesis**: The relationship between yield curves and commodity futures spreads is not static but evolves through time as market structures and participant behaviors change.

3 Data and Methodology

3.1 Data Sources

Commodity Futures	**Yield Data**
MCX Bhavcopy	FRED (T10Y2Y)
https://www.mcxindia.com	https://fred.stlouisfed.org

Table 1: Data sources and collection periods

The research utilizes daily financial data spanning from January 2014 to August 2022, obtained from the following sources:

1. **Commodity Futures Data:**
   - Daily prices for nearest contract and 6-month forward contract
   - Sourced from commodity exchanges such as MCX (https://www.mcxindia.com/market-data/bhavcopy)
   - Includes complete price series with sufficient liquidity and continuous trading
2. **Yield Curve Data:**
   - Daily values of the 10-year minus 2-year Treasury yield spread (T10Y2Y)
   - Obtained from the Federal Reserve Economic Data (FRED) (https://fred.stlouisfed.org/series
   - Represents a widely monitored measure of yield curve steepness

3.2 Data Preprocessing

- **Spread Calculation**: Computation of the 6-month futures spread (6M_Spread), defined as the difference between the 6-month contract price and the nearest contract price, representing the term structure of futures prices.
- **Missing Value Treatment**: Application of the last observation carried forward (LOCF) method to address trading holidays and occasional missing observations, maintaining data continuity.
- **Return Transformation**: Conversion of price and yield levels to returns (day-to-day changes) to ensure stationarity, a critical assumption for GARCH modeling.
- **Outlier Management**: Winsorization of returns at the 1st and 99th percentiles to mitigate the influence of extreme observations while preserving the overall distributional characteristics.
- **Data Validation**: Comprehensive checks for data integrity, continuity, and statistical properties to ensure model reliability.

3.3 Multi-Window DCC-GARCH

The primary analytical technique employed is Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) modeling. This sophisticated approach was selected because it can simultaneously:

1. Capture time-varying volatility in each series
2. Model the dynamic correlation between series
3. Account for volatility clustering common in financial time series
4. Handle non-normal return distributions

Three rolling window sizes are implemented to capture correlation dynamics across different time horizons:

- **Short-term**: 126 days (6 months)
  - Captures short-term, rapidly changing market dynamics
  - Highly responsive to new information and market shifts
  - More susceptible to noise and transitory market movements
- **Medium-term**: 256 days (1 year)
  - Balances responsiveness and stability
  - Captures seasonal patterns while filtering some noise
  - Represents a medium-term perspective on the relationship
- **Long-term**: 512 days (2 years)
  - Emphasizes longer-term structural relationships
  - Filters out short-term noise and seasonal effects
  - Identifies persistent correlation regimes and major shifts

3.4 Model Specification

The DCC-GARCH model is specified in two stages for mathematical rigor:

3.4.1 Stage 1: Univariate GARCH Modeling

Univariate GARCH(1,1) models with Student's t-distribution are estimated for each series:

$$h_{i,t} = \omega_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i h_{i,t-1} \tag{1}$$

Where:

- $h_{i,t}$ represents the conditional variance of series i at time t
- ω_i is the constant term in the variance equation
- α_i captures the impact of recent shocks on current volatility
- β_i measures the persistence of volatility

- Student's t-distribution accommodates the fat tails commonly observed in financial returns

3.4.2 Stage 2: Dynamic Correlation Estimation

The standardized residuals from the first stage are used to estimate time-varying correlations:

$$Q_t = (1 - a - b)\bar{Q} + a(\epsilon_{t-1}\epsilon'_{t-1}) + bQ_{t-1} \tag{2}$$

$$R_t = diag(Q_t)^{-1/2}Q_t diag(Q_t)^{-1/2} \tag{3}$$

Where:

- Q_t is the conditional covariance matrix at time t
- $\bar{Q}$ is the unconditional covariance of standardized residuals
- a and b are parameters governing the dynamics of correlation
- R_t is the resulting time-varying correlation matrix

The estimation was implemented using the rmgarch package in R, with enhanced fitting controls to ensure convergence across all windows. The computational process leveraged parallel processing capabilities to handle the intensive calculations required across multiple rolling windows.

4 Empirical Results

4.1 Correlation Dynamics

The DCC-GARCH analysis revealed several significant patterns in the dynamic correlation between yield curve spreads and commodity futures spreads across the different window sizes.

Figure 1: 126-Day Window: High volatility (-0.8 to +0.4 range)

4.1.1 Short-Term Dynamics: 126-Day Window Results

The 126-day window analysis demonstrated the most volatile correlation pattern, with these key characteristics:

- **High Volatility**: Frequent shifts between positive and negative territory, reflecting rapidly changing market sentiment and conditions
- **Range**: Correlation values typically ranged from -0.2 to +0.2, with occasional excursions to more extreme values during periods of market stress
- **Sensitivity**: Highly responsive to new information and market developments, capturing immediate reactions rather than sustained relationships
- **Recent Positive Correlation**: The most pronounced positive correlation episodes occurred during late 2021 and early 2022, coinciding with significant yield curve flattening and heightened inflation concerns

- **Short-Term Signal Value**: Provides early indications of relationship changes that may later become more established in longer windows

This window captured immediate, short-term relationship changes between the variables, potentially serving as leading indicators for evolving market dynamics.

Figure 2: 256-Day Window: Persistent negative correlations (-0.75 to +0.5)

4.1.2 Medium-Term Perspective: 256-Day Window Results

The medium-term window (256 days) exhibited more stability while still capturing meaningful variation in the correlation structure:

- **Moderate Stability**: Less noise than the 126-day window but still responsive to significant market shifts
- **Persistent Negative Phase**: Extended periods of negative correlation during 2015-2017, with correlation values frequently reaching -0.1 to -0.2
- **Transition Period**: Gradual shift from negative toward neutral and occasionally positive territory in later periods
- **Notable Low Point**: Particularly negative correlations (around -0.26) in December 2015
- **Signal Consistency**: More reliable for identifying established correlation regimes lasting several months

This window effectively balanced responsiveness and stability, filtering out some noise while still capturing meaningful shifts in the relationship structure.

Figure 3: 512-Day Window: Structural regime shifts (-0.5 to +0.75)

4.1.3 Long-Term Structure: 512-Day Window Results

The longest window analysis (512 days) produced the smoothest correlation series, highlighting longer-term structural relationships:

- **Smooth Evolution**: Gradual changes in correlation with minimal short-term fluctuations
- **Persistent Regimes**: Predominantly negative correlation during 2016-2017, with values frequently between -0.1 and -0.2
- **Structural Shift**: Beginning in 2018, the correlation became less negative and occasionally turned positive
- **Recent Positive Phase**: During 2019-2020, several periods of stronger positive correlation emerged, exceeding +0.1
- **Regime Identification**: Clear distinction between the earlier negative correlation regime and the later neutral-to-positive regime

This window effectively filtered out short-term noise and seasonal effects, revealing the underlying structural relationship between yield curves and commodity futures spreads.

4.2 Key Findings

4.2.1 Cross-Window Comparative Analysis

The multi-window approach reveals several important insights when results are compared across different timeframes:

- **Short-term**: Rapid fluctuations with early signal detection
- **Medium-term**: Extended negative regime (2016-2018)
- **Long-term**: Structural shift post-2018 to neutral/positive
- **Convergence of Signals**: All three window sizes identified a notable shift from predominantly negative correlations in earlier years to more neutral or positive correlations in later periods, suggesting a fundamental change in market structure or behavior.
- **Signal Timing Differences**: The shorter 126-day window detected correlation changes earlier, while the 512-day window confirmed when these changes represented persistent regime shifts rather than transitory fluctuations.
- **Stability-Responsiveness Tradeoff**: Shorter windows captured more immediate market responses but exhibited higher volatility, while longer windows revealed more stable relationship structures but potentially obscured shorter-term signals.
- **Complementary Insights**: The three windows together provide a more comprehensive understanding than any single window in isolation, offering perspectives on short-term trading opportunities, medium-term positioning, and long-term structural relationships.
- **Consistency Effect**: The alignment of signals across multiple window sizes strengthens confidence in the observed regime shifts, reducing the probability that these patterns are merely statistical artifacts.

5 Interpretation and Applications

5.1 Correlation Regimes

The empirical findings reveal several important insights about the complex relationship between yield curves and commodity futures spreads.

5.1.1 Evolving Correlation Regimes

The most striking finding is that the correlation is not static but varies considerably over time. This temporal variation challenges the assumption of fixed relationships often embedded in traditional portfolio models and risk management frameworks.

The predominantly negative correlation observed in earlier periods (especially 2015-2017) suggests that during this time:

- Yield curve steepening (increasing T10Y2Y spread) was associated with narrowing commodity futures spreads
- Yield curve flattening coincided with widening futures spreads

This negative relationship can be explained through several potential mechanisms:

1. **Economic Growth Expectations**: A steepening yield curve typically signals improving economic growth prospects, which might lead to expectations of stronger near-term commodity demand relative to long-term demand, thus reducing contango in commodity futures.

2. **Monetary Policy Anticipation**: Yield curve changes often reflect expectations about future monetary policy, which affects the cost of carrying commodity positions differently across the futures curve.

3. **Risk Premium Dynamics**: During this period, changes in market risk perceptions may have affected both yield curves and commodity futures curves in opposite directions.

5.1.2 Significant Regime Shift

The shift toward more neutral and occasionally positive correlations in later periods (especially 2019-2022) represents a fundamental regime change. This transition period coincides with several major economic events:

- COVID-19 pandemic and its economic disruptions
- Unprecedented monetary policy responses including quantitative easing
- Supply chain disruptions affecting commodity markets
- Evolving inflation expectations and monetary policy uncertainty

In this changed environment, the positive correlation suggests that yield curve flattening was associated with narrowing commodity futures spreads, representing a fundamental shift in how these markets interact.

5.1.3 Window-Specific Interpretations

Each window size reveals different aspects of this evolving relationship:

1. **Short-Term Window (126 days)**: The high volatility in this window suggests that short-term trading relationships between these markets are highly context-dependent and can shift rapidly as new information emerges.

2. **Medium-Term Window (256 days)**: The more stable patterns in this window indicate that certain correlation regimes can persist for months, providing a basis for medium-term trading and hedging strategies.

3. **Long-Term Window (512 days)**: The clear regime shift visible in this window points to fundamental changes in market structure, possibly reflecting evolving monetary policy frameworks, market participant behavior, or global economic conditions.

5.1.4 Theoretical Implications

The findings have several implications for financial theory:

1. **Market Integration**: The time-varying correlation suggests that the degree of integration between fixed income and commodity markets fluctuates over time, challenging the assumption of consistently integrated markets.

2. **Information Transmission**: The lead-lag relationship between correlation shifts across different window sizes provides insights into how information is processed and transmitted between these markets.
3. **Regime-Dependent Pricing**: The existence of distinct correlation regimes suggests that pricing relationships between these assets are conditional on broader market environments rather than being fixed.

5.2 Trading Applications

The research findings have several direct applications for market participants in trading and portfolio management:

- **Cross-Asset Trading Strategies**: During periods of negative correlation, yield curve steepening might signal opportunities to position for narrowing commodity spreads. Conversely, during positive correlation regimes, yield curve movements might suggest parallel positioning in commodity spread markets.
- **Enhanced Diversification**: Understanding when these markets are positively versus negatively correlated allows for more effective portfolio diversification strategies that adapt to changing market conditions.
- **Tactical Asset Allocation**: The identification of correlation regime shifts provides signals for adjusting cross-asset allocations between fixed income and commodities.
- **Relative Value Opportunities**: The research highlights periods where the relationship between these markets may temporarily deviate from established patterns, potentially creating relative value opportunities.
- **Cross-asset relative value strategies**
- **Dynamic hedging approaches**
- **Term structure arbitrage**
- **Example**: Yield curve flattening signals during 2019-2020

5.3 Risk Management

Applications for risk management professionals include:

- **Dynamic Correlation Modeling**: Traditional risk models that assume static correlations may significantly underestimate or overestimate portfolio risk. The research demonstrates the importance of using dynamic correlation models when assessing cross-asset exposures.
- **Stress Testing Frameworks**: Identification of correlation regime shifts provides valuable inputs for developing more robust stress testing scenarios that account for changing market dynamics.
- **Tail Risk Management**: Understanding when these markets move together versus in opposition helps in managing tail risk exposures across asset classes.
- **Hedging Strategy Optimization**: The time-varying nature of the correlation suggests that optimal hedging strategies should be adaptive, changing with the prevailing correlation regime.

6 Extension Plan

6.1 Research Expansion

Component	Timeframe	Details
Multi-Spread Analysis	6 weeks	3/9-month futures spreads
Multi-Commodity	8 weeks	Natural gas, precious metals
Forecasting Models	10 weeks	VAR, RSM, ML approaches

Table 2: Extended research implementation plan

6.1.1 Expanding Spread Duration Analysis

3-Month and 9-Month Futures Spreads

The current research focuses on 6-month futures contract spreads. We propose extending this analysis to include:

- **3-Month Futures Spreads**: Capturing shorter-term market expectations and potentially greater sensitivity to immediate supply-demand dynamics
- **9-Month Futures Spreads**: Reflecting longer-term market expectations and potentially greater sensitivity to structural factors

Research Questions for Multiple Spread Durations

- How do correlations with yield curves differ across the futures curve (3-month vs. 6-month vs. 9-month spreads)?
- Do shorter-term spreads show different sensitivity to yield curve movements compared to longer-term spreads?
- Can we identify term structure patterns in how yield curve changes propagate through different points on the futures curve?
- Are there lead-lag relationships between shorter and longer-duration spreads in their response to yield curve changes?

6.1.2 Expanding Commodity Coverage

Additional Commodities

We propose extending the analysis to include:

- **Natural Gas**:
  - Highly seasonal commodity with unique storage economics
  - Potentially different relationship with interest rates due to storage constraints
  - Less global market compared to oil, possibly exhibiting more regional influences
- **Additional Commodity Classes**:
  - Precious Metals (Gold, Silver): Traditional inflation hedges with minimal storage costs

- Agricultural Commodities (Corn, Wheat, Soybeans): Seasonal production with moderate storage costs
- Base Metals (Copper, Aluminum): Industrial demand sensitivity with moderate storage costs

Cross-Commodity Research Questions

- Do different commodity classes exhibit varying sensitivities to yield curve changes?
- Is the natural gas futures curve more or less responsive to yield curve dynamics compared to oil?
- How do storage economics impact the relationship between yield curves and futures spreads across commodities?
- Are there cross-commodity patterns that provide additional signals about economic expectations?

6.1.3 Implementing Forecasting Models

Forecasting Approaches

We propose developing several forecasting models to predict future commodity futures spreads based on yield curve dynamics:

1. **Vector Autoregression (VAR) Models**:
   - Capture dynamic interactions between yield curve spreads and commodity futures spreads
   - Allow for both contemporaneous and lagged relationships
   - Enable impulse response analysis to quantify the impact of yield curve shocks
2. **Regime-Switching Models**:
   - Identify distinct correlation regimes (positive vs. negative correlation periods)
   - Develop regime-specific forecasting models
   - Incorporate regime probability predictions into the forecasting framework
3. **Machine Learning Approaches**:
   - Random Forest and Gradient Boosting models to capture non-linear relationships
   - Feature engineering incorporating yield curve levels, slopes, and changes
   - Appropriate cross-validation using time-series splits to prevent look-ahead bias
4. **Hybrid DCC-GARCH and Forecasting Models**:
   - Use estimated DCC-GARCH correlations as inputs for spread forecasting
   - Develop correlation-conditional forecasting models
   - Explore whether correlation regimes provide useful information for directional forecasts

Forecasting Evaluation Framework

- Out-of-sample testing with expanding window approach
- Multiple evaluation metrics: RMSE, MAE, directional accuracy
- Economic significance assessment through simulated trading strategies
- Comparison against benchmark models including random walk and autoregressive models

6.2 Methodological Enhancements

Advanced Modeling Approaches

1. **Wavelet Analysis**:
    - Decompose both yield curve and commodity futures data into different frequency components
    - Analyze correlation dynamics across different time horizons within a unified framework
    - Potentially identify frequency-specific relationships hidden in time-domain analysis
2. **Copula-Based Dependency Modeling**:
    - More flexible approach to modeling the joint distribution beyond correlation
    - Capture potential non-linear and tail dependencies between markets
    - Allow for asymmetric responses in different market environments
3. **Advanced DCC-GARCH Variants**:
    - Asymmetric DCC-GARCH to capture leverage effects
    - Markov-Switching DCC-GARCH for regime-dependent volatility dynamics
    - Rotated GARCH models for improved handling of skewed distributions

Additional Explanatory Variables

- Incorporate explicit measures of economic surprise indices
- Include measures of market sentiment and positioning
- Adjust for seasonality effects, particularly relevant for natural gas
- Control for inventory levels and storage capacity constraints

Implementation Plan

1. **Phase 1: Data Collection and Preprocessing (4 weeks)**
    - Collect 3-month and 9-month futures data for multiple commodities
    - Develop robust preprocessing pipeline including proper roll adjustment
    - Ensure consistent handling of missing values and outliers across datasets
    - Implement data quality checks and visualization for exploratory analysis
2. **Phase 2: Multi-Spread Analysis (6 weeks)**

- Extend DCC-GARCH analysis to include multiple spread durations
- Develop comparative framework for analyzing results across the futures curve
- Identify common patterns and divergences in correlation structures
- Document findings on term structure effects in correlation dynamics

3. **Phase 3: Multi-Commodity Analysis (8 weeks)**
   - Implement parallel analysis for natural gas and other selected commodities
   - Develop cross-commodity comparison framework
   - Identify commodity-specific and common factors in yield curve relationships
   - Document findings on how commodity characteristics influence correlation dynamics

4. **Phase 4: Forecasting Model Development (10 weeks)**
   - Implement multiple forecasting approaches
   - Develop robust evaluation framework
   - Conduct out-of-sample testing and model comparison
   - Refine models based on performance analysis

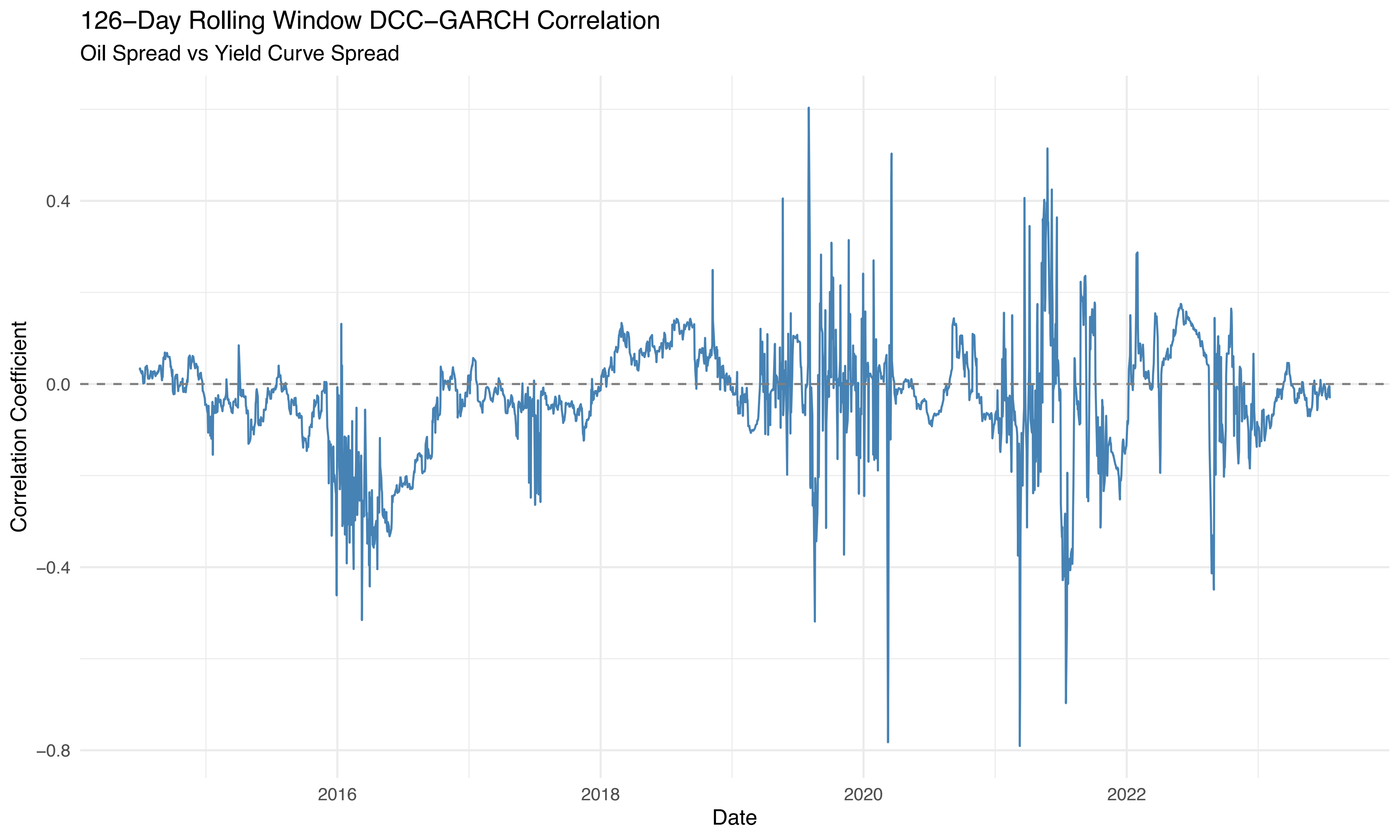
5. **Phase 5: Strategy Development and Validation (6 weeks)**
   - Translate analytical findings into practical trading and risk management strategies
   - Backtest strategies with realistic constraints and transaction costs
   - Develop implementation guidelines for practical applications
   - Document potential use cases and limitations

7 Conclusion

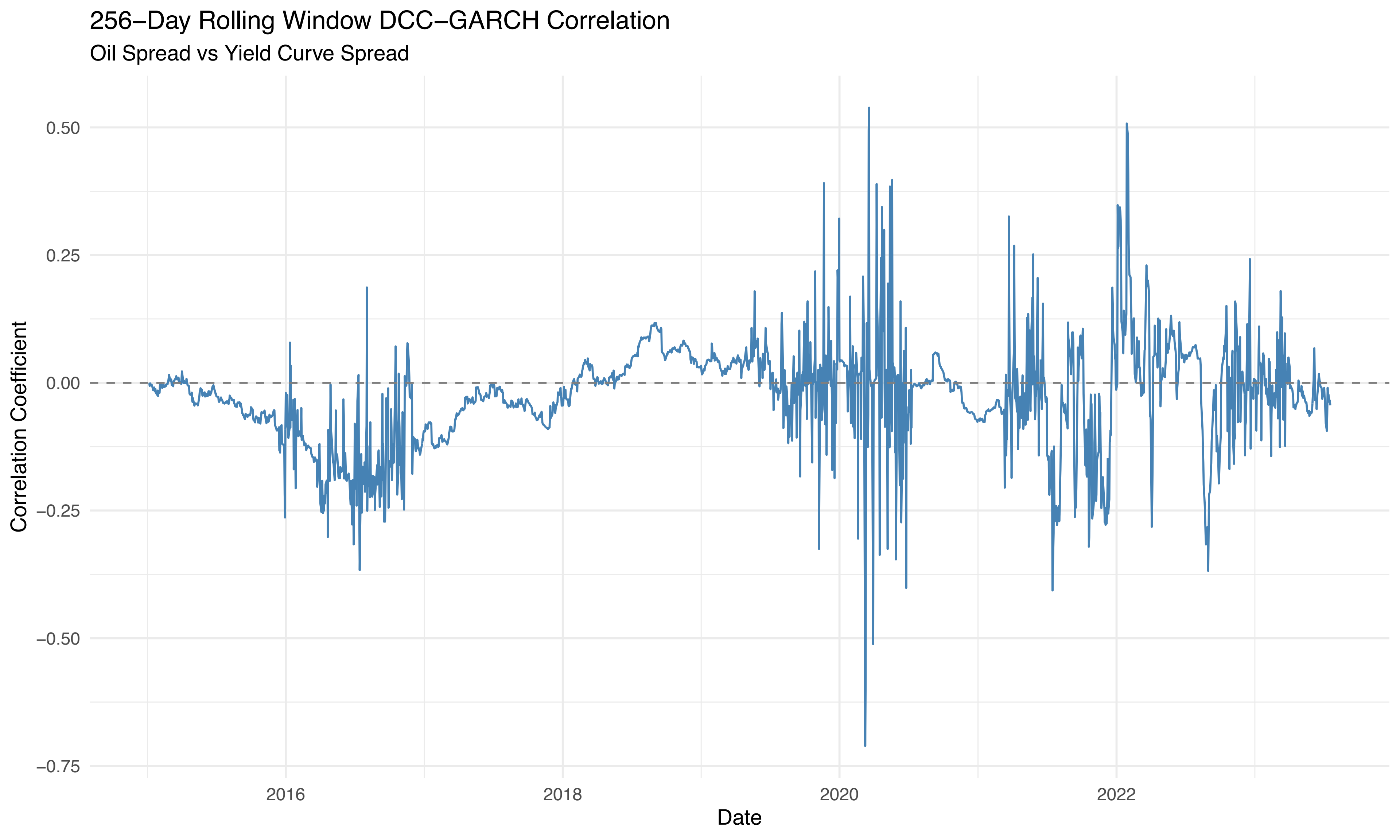
Key contributions of this research include:

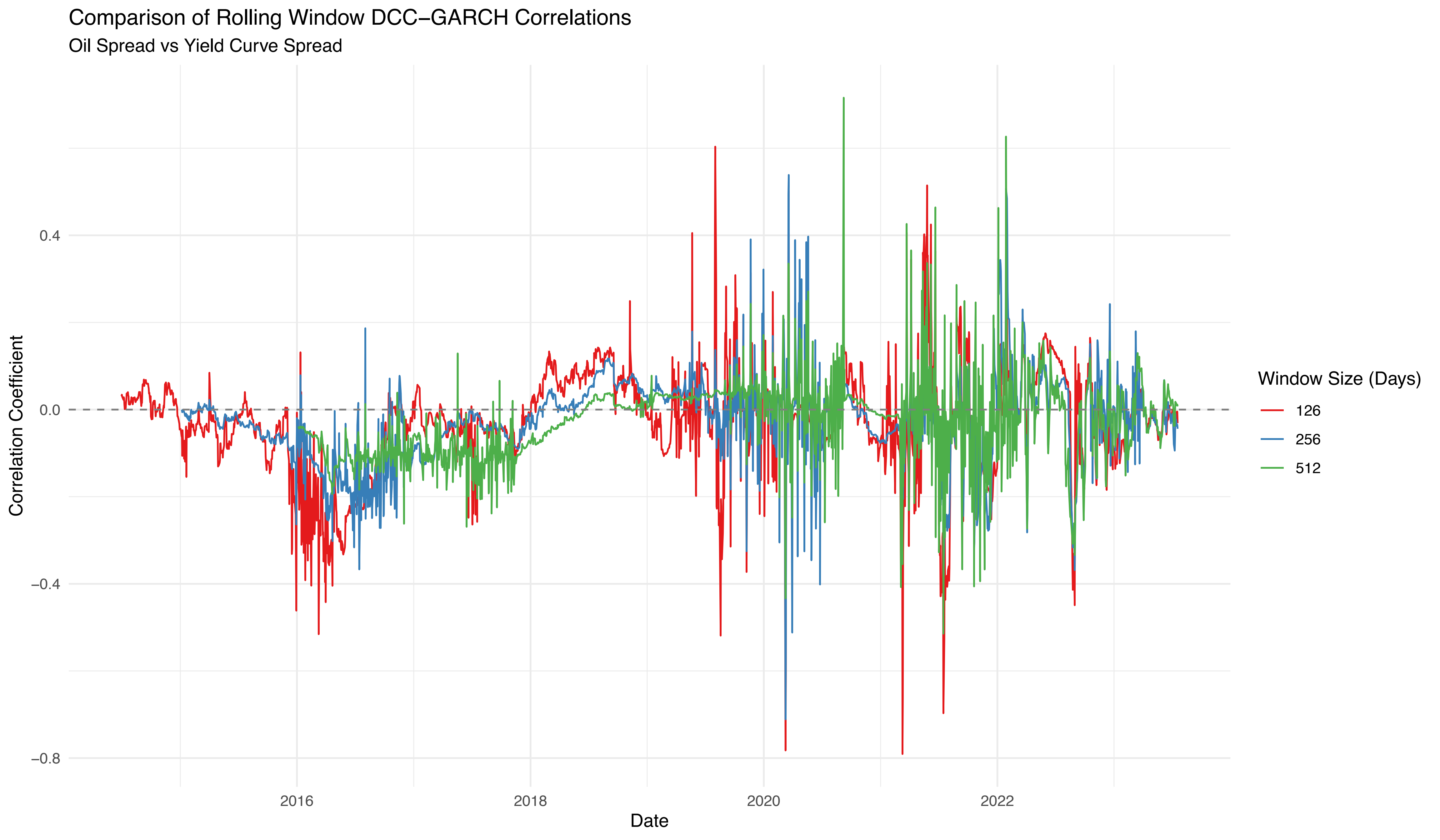
- **Demonstrated time-varying correlation dynamics**: The research clearly establishes that the relationship between yield curves and commodity futures spreads is not static but evolves significantly over time, with distinct positive and negative correlation regimes.
- **Identified structural regime shifts**: The analysis reveals a notable transition from predominantly negative correlations in 2015-2017 to more neutral and occasionally positive correlations in recent years, coinciding with significant economic events and policy changes.
- **Developed multi-window analytical framework**: The innovative three-window approach (126, 256, and 512 days) provides complementary insights into short-term fluctuations, medium-term trends, and long-term structural relationships, offering a more comprehensive understanding than single-window analysis.

- **Practical applications for trading/risk management**: The findings have direct implications for cross-asset trading strategies, dynamic hedging approaches, portfolio diversification, and risk management practices, enabling market participants to develop more effective approaches that account for time-varying correlations.

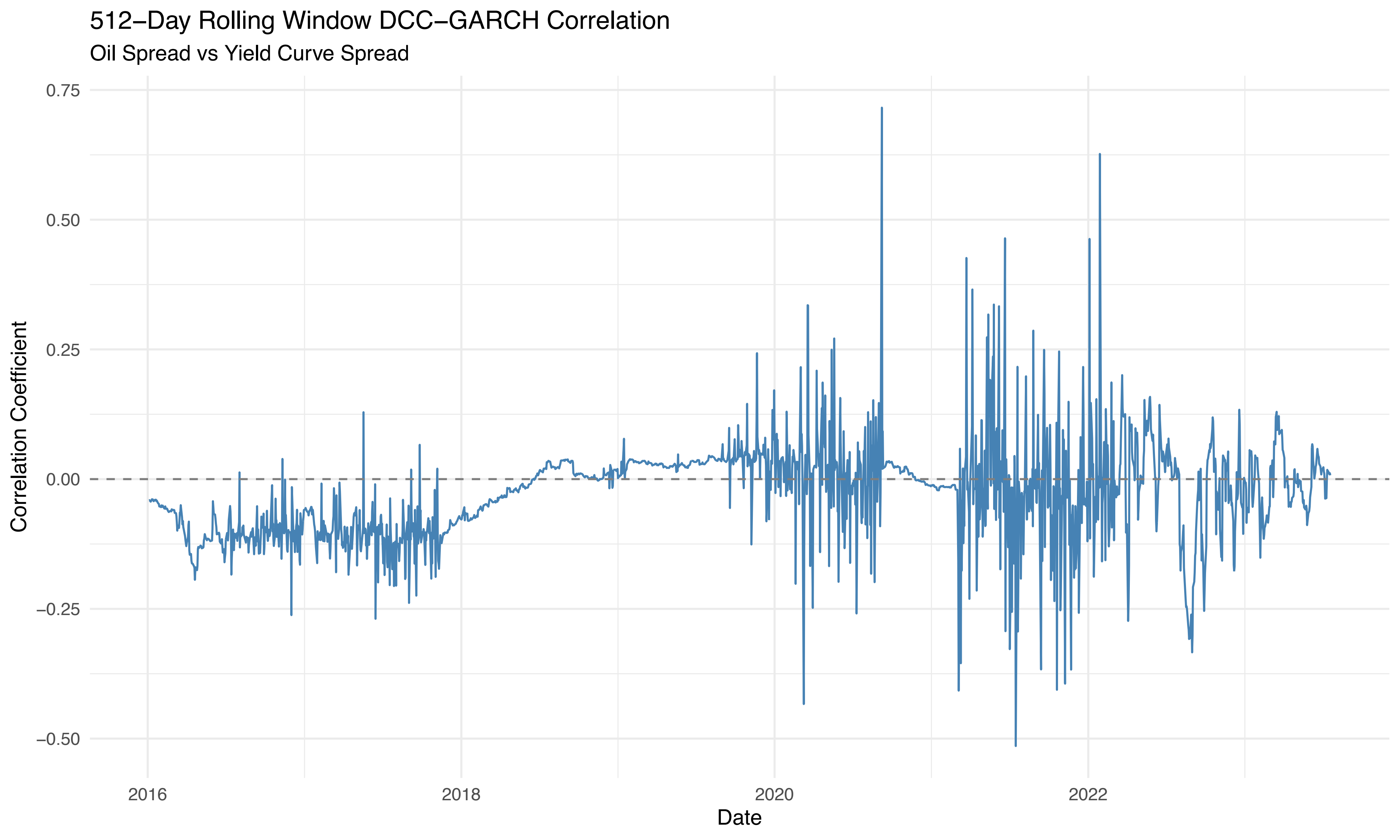


126−Day Rolling Window DCC−GARCH Correlation
Oil Spread vs Yield Curve Spread
Correlation Coefficient
0.4
0.0
−0.4
−0.8
2016
2018
2020
2022
Date


256−Day Rolling Window DCC−GARCH Correlation
Oil Spread vs Yield Curve Spread
0.50
0.25
0.00
−0.25
−0.50
−0.75
Correlation Coefficient
2016
2018
2020
2022
Date



Comparison of Rolling Window DCC−GARCH Correlations
Oil Spread vs Yield Curve Spread
Correlation Coefficient
0.4
0.0
−0.4
−0.8
2016
2018
2020
2022
Date
Window Size (Days)
126
256
512


512−Day Rolling Window DCC−GARCH Correlation
Oil Spread vs Yield Curve Spread
0.75
0.50
0.25
0.00
−0.25
−0.50
Correlation Coefficient
2016
2018
2020
2022
Date