Section A (1 mark each) - Total 5 Marks

1. **Find the 15th term of an arithmetic sequence where the 7th term is 17, and the 10th term is 26.**

   Solution:
   Given $a_7 = 17$ and $a_{10} = 26$.
   Using the formula for the n-th term of an arithmetic sequence:
   $a_n = a + (n-1)d$,
   we have two equations:

$$a + 6d = 17$$

$$a + 9d = 26$$

   Subtracting the first from the second, we get $3d = 9$, so $d = 3$.
   Substitute $d = 3$ into the first equation:

$$a + 6 \cdot 3 = 17 \Rightarrow a = -1$$

   Now, $a_{15} = -1 + 14 \cdot 3 = 41$.

2. **If $\tan\theta = \frac{3}{4}$, find $\sin\theta$ and $\cos\theta$.**

   Solution:
   Since $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$, let the opposite side be 3 and the adjacent side be 4.
   By the Pythagorean theorem:

   $$\text{Hypotenuse} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

   Then, $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$
   and $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$.

3. **The top of a lighthouse, 100 meters above sea level, is observed from a boat at an angle of elevation of 30°. How far is the boat from the base of the lighthouse?**

   Solution:
   Using the tangent of the angle:

   $$\tan 30^\circ = \frac{100}{d}$$

   Since $\tan 30^\circ = \frac{1}{\sqrt{3}}$, we get:

   $$d = 100 \cdot \sqrt{3} \approx 173.2 \text{ meters}$$

4. **A cone has a slant height of 13 cm and a radius of 5 cm. Find its total surface area.**

   Solution:

   Total surface area of a cone $= \pi r(r + l)$:

   $$= \pi \cdot 5(5 + 13) = 90\pi \approx 282.6 \text{ cm}^2$$

5. **Determine the nature of roots for the equation $2x^2 - 4x + 3 = 0$.**

   Solution:

   Discriminant $D = b^2 - 4ac$:

   Here, $a = 2$, $b = -4$, $c = 3$.

   $$D = (-4)^2 - 4 \cdot 2 \cdot 3 = 16 - 24 = -8$$

   Since $D < 0$, the roots are imaginary.

Section B (2 marks each) - Total 10 Marks

6. **If the first term of an arithmetic sequence is 4 and the sum of its first 10 terms is 145, find the common difference.**

   Solution:
   The formula for the sum of the first n terms of an arithmetic sequence is:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

   Given $S_{10} = 145$, $a = 4$, and $n = 10$, we substitute:

$$145 = \frac{10}{2}(2 \cdot 4 + 9d)$$

$$145 = 5(8 + 9d)$$

$$145 = 40 + 45d$$

$$105 = 45d \Rightarrow d = \frac{105}{45} = \frac{7}{3} = 2.33$$

7. **Prove that** $\cot A \cdot \csc A = \frac{1}{\sin A}$.

   Solution:
   We know:

$$\cot A = \frac{\cos A}{\sin A} \quad \text{and} \quad \csc A = \frac{1}{\sin A}$$

   Therefore:

$$\cot A \cdot \csc A = \frac{\cos A}{\sin A} \cdot \frac{1}{\sin A} = \frac{\cos A}{\sin^2 A} = \frac{1}{\sin A}$$

8. **The angle of elevation of the top of a tower from a point on the ground is 45°. Moving 20 meters closer to the tower, the angle of elevation becomes 60°. Find the height of the tower.**

   Solution:
   Let the height of the tower be h meters and the initial distance from the point to the tower be x meters.
   From the first observation (45° angle):

$$\tan 45^\circ = \frac{h}{x} \Rightarrow h = x$$

   From the second observation (60° angle):

$$\tan 60^\circ = \frac{h}{x-20} \Rightarrow \sqrt{3} = \frac{h}{x-20}$$

   Substituting $h = x$ in the second equation:

$$\sqrt{3} = \frac{x}{x-20}$$

   Solving for x:

$$x\sqrt{3} = x - 20\sqrt{3} = 20 + 20$$

9. **A solid cylinder has a height of 14 cm and a radius of 3.5 cm. Find its total surface area.**

   Solution:
   The total surface area A of a cylinder is given by:

$$A = 2\pi r(h + r)$$

   where $r = 3.5$ cm and $h = 14$ cm.

   Substituting the values:

$$A = 2 \times \pi \times 3.5 \times (14 + 3.5)$$

$$= 2 \times \pi \times 3.5 \times 17.5$$

$$= 2 \times 3.1416 \times 3.5 \times 17.5$$

$$= 385\,\mathrm{cm}^2$$

10. **Solve for x: $2x^2 - 5x + 3 = 0$ using the quadratic formula.**

Solution:

The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the equation $2x^2 - 5x + 3 = 0$, we have $a = 2$, $b = -5$, and $c = 3$.

Substitute these values into the formula:

$$\begin{aligned} x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2} \\ &= \frac{5 \pm \sqrt{25 - 24}}{4} \\ &= \frac{5 \pm \sqrt{1}}{4} \\ &= \frac{5 \pm 1}{4} \end{aligned}$$

So, the solutions are:

$$x = \frac{5+1}{4} = \frac{6}{4} = 1.5$$

$$x = \frac{5-1}{4} = \frac{4}{4} = 1$$

Therefore, $x = 1.5$ and $x = 1$.

Section C (3 marks each) - Total 15 Marks

11. **In an arithmetic progression, the sum of the first n terms is given by $S_n = 3n^2 + 5n$. Find the n-th term and the first term.**

Solution:

Given $S_n = 3n^2 + 5n$.

The n-th term a_n of an arithmetic progression is given by:

$$a_n = S_n - S_{n-1}$$

where S_n is the sum of the first n terms and S_{n-1} is the sum of the first $(n-1)$ terms.

1. Substitute $S_n = 3n^2 + 5n$ and find S_{n-1}:

$$S_{n-1} = 3(n-1)^2 + 5(n-1)$$

   Expanding S_{n-1}:

$$S_{n-1} = 3(n^2 - 2n + 1) + 5(n-1) = 3n^2 - 6n + 3 + 5n - 5$$

$$S_{n-1} = 3n^2 - n - 2$$

2. Calculate $a_n = S_n - S_{n-1}$:

$$a_n = (3n^2 + 5n) - (3n^2 - n - 2)$$

$$a_n = 6n + 2$$

3. For the first term a when $n = 1$:

$$a = 6 \cdot 1 + 2 = 8$$

So, the first term $a = 8$ and the n-th term $a_n = 6n + 2$.

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12. **Prove that** $\sec^4 A - \tan^4 A = 1 + 2\tan^2 A$.

Solution:
Start with the left side:

$$\sec^4 A - \tan^4 A$$

We can factor this as a difference of squares:

$$(\sec^2 A + \tan^2 A)(\sec^2 A - \tan^2 A)$$

Using the identity $\sec^2 A - \tan^2 A = 1$:

$$= (\sec^2 A + \tan^2 A) \cdot 1$$

Since $\sec^2 A = 1 + \tan^2 A$:

$$= (1 + \tan^2 A + \tan^2 A) = 1 + 2\tan^2 A$$

Therefore,

$$\sec^4 A - \tan^4 A = 1 + 2\tan^2 A$$

13. **From a point 50 m away from the base of a building, the angle of elevation of the top of the building is observed to be 60°. Find the height of the building.**

Solution:
Let the height of the building be h.
From the point 50 m away, we use the tangent ratio:

$$\tan 60^\circ = \frac{h}{50}$$

Since $\tan 60^\circ = \sqrt{3}$:

$$\sqrt{3} = \frac{h}{50}$$

$$h = 50\sqrt{3} \approx 86.6 \text{ meters}$$

Therefore, the height of the building is approximately 86.6 meters.

14. **A hollow cylindrical pipe has an outer radius of 10 cm, an inner radius of 8 cm, and a height of 15 cm. Calculate its volume.**

Solution:
The volume V of a hollow cylinder is given by the difference between the outer and inner cylinders:

$$V = \pi h(R^2 - r^2)$$

where $R = 10$ cm, $r = 8$ cm, and $h = 15$ cm.

Substitute the values:

$$V = \pi \cdot 15 \cdot (10^2 - 8^2)$$

$$= \pi \cdot 15 \cdot (100 - 64)$$

$$= \pi \cdot 15 \cdot 36$$

$$= 540\pi \approx 1696.5 \, \text{cm}^3$$

So, the volume of the hollow cylindrical pipe is approximately $1696.5 \, \text{cm}^3$.

15. **A man saves a certain amount every month such that he is able to save Rs. 2700 in the first 12 months and Rs. 5800 in the next 8 months. Assuming that he saves in an increasing arithmetic pattern, find his savings in the 20th month.**

Solution:

Since he saves in an arithmetic pattern, let his first month's saving be a and the common difference be d.

Step 1: Use the information about his first 12 months' savings:

$$S_{12} = 2700 = \frac{12}{2}(2a + 11d)$$

Simplify:

$$2700 = 6(2a + 11d) \Rightarrow 450 = 2a + 11d \Rightarrow 2a + 11d = 450 \quad \text{(Equation 1)}$$

Step 2: Use the information about his next 8 months (13th to 20th months):

$$S_8 = 5800 = \frac{8}{2}(2a + 15d)$$

Simplify:

$$5800 = 4(2a + 15d) \Rightarrow 1450 = 2a + 15d \Rightarrow 2a + 15d = 1450 \quad \text{(Equation 2)}$$

Step 3: Subtract Equation 1 from Equation 2:

$$4d = 1000 \Rightarrow d = 250$$

Substitute $d = 250$ into Equation 1:

$$2a + 11 \cdot 250 = 450 \Rightarrow 2a + 2750 = 450$$

$$2a = -2300 \Rightarrow a = -1150$$

Step 4: Find his savings in the 20th month (a_{20}):

$$a_{20} = a + 19d = -1150 + 19 \cdot 250$$

$$= -1150 + 4750 = 3600$$

His savings in the 20th month are Rs. 3600.

Section D (5 marks each) - Total 20 Marks

16. **The sum of the first 25 terms of an arithmetic progression is 525, and the 10th term is 25. Find the first term and common difference, and hence, determine the sum of the first 50 terms of this sequence.**

Solution:

Given:

- Sum of the first 25 terms $S_{25} = 525$
- 10th term $a_{10} = 25$

Let the first term be a and the common difference be d.

Step 1: Use the formula for the n-th term of an AP:

$$a_{10} = a + 9d$$

Since $a_{10} = 25$:

$$a + 9d = 25 \quad \text{(Equation 1)}$$

Step 2: Use the sum formula for the first 25 terms:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Substitute $S_{25} = 525$:

$$525 = \frac{25}{2}(2a + 24d)$$

$$1050 = 25(2a + 24d)$$

$$2a + 24d = 42 \quad \text{(Equation 2)}$$

Step 3: Solve Equation 1 and Equation 2 simultaneously: From Equation 1: $a = 25 - 9d$. Substitute $a = 25 - 9d$ into Equation 2:

$$2(25 - 9d) + 24d = 42$$

$$50 - 18d + 24d = 42$$

$$6d = -8 \Rightarrow d = -\frac{4}{3}$$

Substitute $d = -\frac{4}{3}$ into Equation 1:

$$a + 9\left(-\frac{4}{3}\right) = 25$$

$$a - 12 = 25 \Rightarrow a = 37$$

Step 4: Find the sum of the first 50 terms:

$$S_{50} = \frac{50}{2}(2a + 49d)$$

Substitute $a = 37$ and $d = -\frac{4}{3}$:

$$S_{50} = 25(2 \times 37 + 49 \times -\frac{4}{3})$$

$$= 25\left(74 - \frac{196}{3}\right)$$

$$= 25 \times \frac{26}{3} = \frac{650}{3} \approx 216.67$$

So, the sum of the first 50 terms is approximately 216.67.

17. **A man standing on top of a building 50 meters high observes the top of a tower at an angle of elevation of 30° and the foot of the tower at an angle of depression of 45°. Find the height of the tower and the distance between the building and the tower.**

Solution:

Let the height of the tower be h and the horizontal distance between the building and the tower be d.

Step 1: Find d using the angle of depression: Since the angle of depression is 45°, the height of the building is equal to the horizontal distance d:

$$\tan 45^\circ = \frac{50}{d} \Rightarrow d = 50 \text{ meters}$$

Step 2: Use the angle of elevation to find h: With the angle of elevation at 30°, the additional height above the building to the top of the tower is $h - 50$:

$$\tan 30^\circ = \frac{h-50}{50}$$

Since $\tan 30^\circ = \frac{1}{\sqrt{3}}$, we get:

$$\frac{1}{\sqrt{3}} = \frac{h-50}{50}$$

$$h - 50 = \frac{50}{\sqrt{3}} \Rightarrow h = 50 + \frac{50}{\sqrt{3}}$$

$$h = 50 + \frac{50\sqrt{3}}{3} \approx 50 + 28.87 = 78.87 \text{ meters}$$

So, the height of the tower is approximately 78.87 meters, and the distance between the building and the tower is 50 meters.

18. **A hemispherical bowl of radius 10 cm is full of water. A solid metal sphere of radius 5 cm is completely immersed in the water. Calculate the volume of water that will spill out.**

Solution:

The volume of water that will spill out is equal to the volume of the metal sphere that is immersed.

Step 1: Calculate the volume of the metal sphere: The volume V of a sphere is given by:

$$V = \frac{4}{3}\pi r^3$$

where $r = 5$ cm.

Substitute $r = 5$:

$$V = \frac{4}{3}\pi(5)^3$$

$$= \frac{4}{3}\pi \times 125 = \frac{500}{3}\pi \approx 523.6\,\mathrm{cm}^3$$

So, the volume of water that will spill out is approximately $523.6\,\mathrm{cm}^3$.

19. **A school wants to build a conical tent with a radius of 7 m and a height of 24 m. Calculate the area of the canvas required to build this tent, and also find the cost if the canvas costs Rs. 150 per square meter.**

Solution:

Step 1: Find the slant height l of the cone:

$$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2}$$

$$= \sqrt{49 + 576} = \sqrt{625} = 25\,\text{m}$$

Step 2: Calculate the curved surface area of the cone (canvas area required):

$$\text{Curved Surface Area} = \pi r l$$

Substitute $r = 7$ and $l = 25$:

$$= \pi \cdot 7 \cdot 25 = 175\pi \approx 549.78\,\text{m}^2$$

Step 3: Calculate the cost:

$$\text{Cost} = 549.78 \times 150 = 82467\,\text{Rs.}$$

So, the area of the canvas required is approximately $549.78\,\text{m}^2$, and the cost is Rs. 82,467.