ECE 270 (Spring 2022)

Homework 4 Solutions

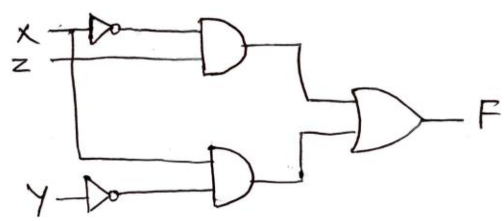
1. $F = X'Y'Z + X'YZ + Y'X + Y'Z$

   $= X'Z(Y' + Y) + Y'X + Y'Z$

   $= X'Z + XY' + Y'Z$

   $= X'Z + XY'$

   Implementation:



2. Proof can be obtained by solving as below:

   $F1 = (X + Y) . (Y' + Z) . (X' + Z')$
   $= (X + Y + (Z . Z')) . ((X . X') + Y' + Z) . (X' + (Y . Y') + Z')$
   $= ((X + Y + Z) . (X + Y + Z')) . ((X' + Y' + Z) (X + Y' + Z)) . ((X' + Y + Z'). (X' + Y + Z'))$
   $= \pi (0,1,2,6,5,7)$

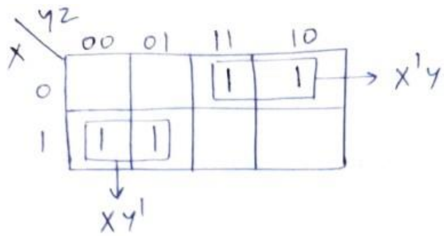
   $F2 = (X + Z) . (X' + Y') . (Y + Z')$
   $= (X + (Y . Y') + Z) . (X' + Y' + (Z . Z')) . ((X . X') + Y + Z')$
   $= ((X + Y + Z) . (X + Y' + Z')) . ((X' + Y' + Z) (X' + Y' + Z')) . ((X + Y + Z'). (X' + Y + Z'))$
   $= \pi (0,2,6,7,1,5)$

   F1 = F2
   Hence Proved.

3. The canonical sum of a function is the sum of all minterms that produce a value of 1. If those minterms cannot be grouped in any manner in the K-map, then the minimal sum is the canonical sum. The no. of literals in the product term of such a function is n since canonical sums always have n literals.

   Ex: Even parity detector function.

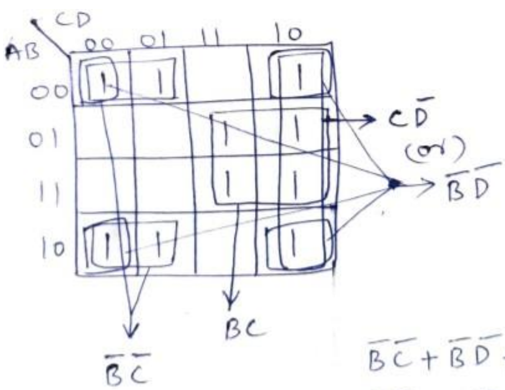
4. K-Map can be drawn as below:



xy' and $x'y$ are the prime implicants

So, minimal sum $= xy' + x'y$

5. K-Map can be drawn as below:



$\bar{B}\bar{C} + \bar{B}\bar{D} + BC$ (or)

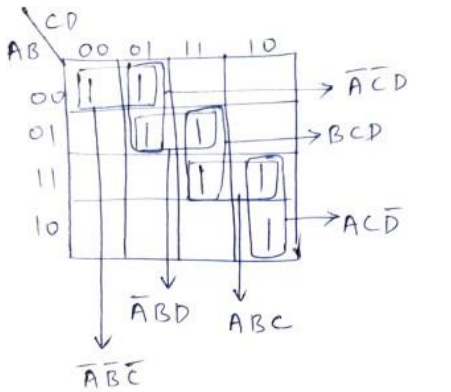
So, minimal sum $= \bar{B}\bar{C} + C\bar{D} + BC$

Essential prime implicants $= \{BC, \bar{B}\bar{C}, C\bar{D}\}$

(or)

$\{BC, \bar{B}\bar{C}, \bar{B}\bar{D}\}$

6. K-Map can be drawn as below:

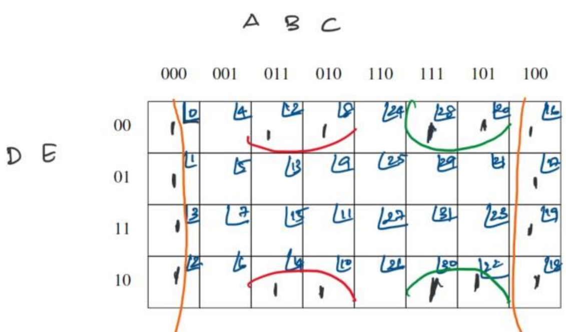


So, above 6 terms are the Prime implicants

→ Essential prime implicants

$= AC\bar{D}, \bar{A}\bar{B}\bar{C}$

7. K-Map can be drawn as below:



$F(A,B,C,D,E) = \Sigma(0,1,2,3,8,10,12,14,16,17,18,19,20,23,28,30)$

$= \bar{B}\bar{C} + \bar{A}B\bar{E} + AC\bar{E}$

FINDING PRIME IMPLICANTS USING QUINE McCLUSKEY METHOD

Group	Decimal	Binary
0	0	00000
1	1	00001
1	2	00010
1	8	01000

1	16	10000
2	3	00011
2	10	01010
2	12	01100
2	17	10001
2	18	10010
2	20	10100
3	14	01110
3	19	10011
3	22	10110
3	28	11100
4	30	11110

Group	Decimal	Binary
0	0, 16	-0000
0	0, 8	0-000
0	0, 2	000-0
0	0, 1	0000-
1	1, 17	-0001
1	1, 3	000-1
1	18, 2	-0010
1	10, 2	0-010
1	2, 3	0001-
1	12, 8	01-00

1	10, 8	010-0
1	16, 20	10-00
1	16, 18	100-0
1	16, 17	1000-
2	19, 3	-0011
2	10, 14	01-10
2	12, 28	-1100
2	12, 14	011-0
2	17, 19	100-1
2	18, 22	10-10
2	18, 19	1001-
2	20, 28	1-100
2	20, 22	101-0
3	14, 30	-1110
3	22, 30	1-110
3	28, 30	111-0

Group	Decimal	Binary
0	0, 16, 18, 2	-00-0
0	0, 1, 16, 17	-000-
0	0, 10, 2, 8	0-0-0 ✷
0	0, 1, 2, 3	000--
1	1, 17, 19, 3	-00-1
1	18, 19, 2, 3	-001-

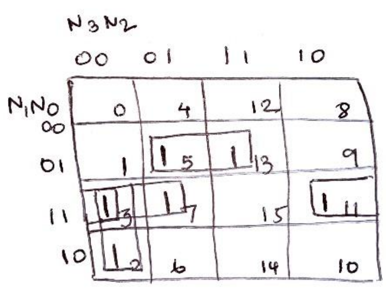
1	10, 12, 14, 8	01--0 ✷
1	16, 18, 20, 22	10--0 ✷
1	16, 17, 18, 19	100--
2	12, 14, 28, 30	-11-0 ✷
2	20, 22, 28, 30	1-1-0 ✷

Group	Decimal	Binary
0	0, 1, 16, 17, 18, 19, 2, 3	-00-- ✷

Prime Implicants: A'C'E', A'BE', AB'E', BCE', ACE', B'C'

8. A prime number detector provides an output of 1 if the inputs correspond to a prime no.
   Eg: If $N_1N_2N_3N_4$ is 0010, then output F is 1 since, 2 is a prime number.

   K-Map can be drawn and simplified as below.

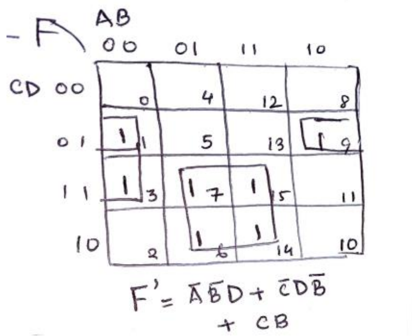


$F = N_2\overline{N_1}N_0 + \overline{N_2}N_1N_0 + \overline{N_3}N_1N_0 + \overline{N_3}\overline{N_2}N_1$

9. $F = \Pi_{ABCD}(1,3,6,7,9,15,14)$

   In other words,

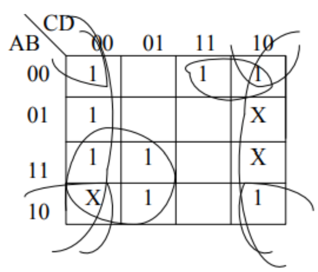
   $F' = \sum_{ABCD}(1,3,6,7,9,15,14)$



Hence, Simplified solution is

F = F’’ = (A’B’D + C’DB’ + CB)’ = (A+B+D’).(C+D’+B).(C’+B’)

10. The K-Map can be drawn as below. Don’t cares are denoted by ‘X’.



The prime implicants are : D’, B’D’, AC’, A’B’C
The essential prime implicants are : D’, AC’, A’B’C
The simplified expression is : Y = D’ + AC’+ A’B’C