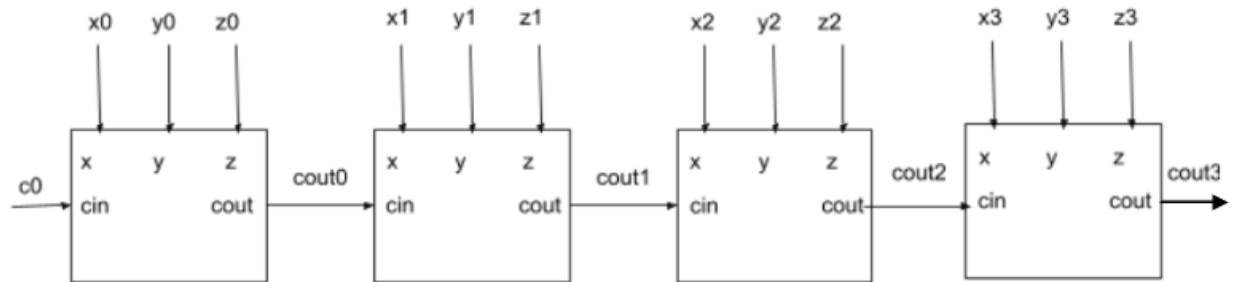


HOMEWORK 13-14 SOLUTIONS

1)



Final output = Cout3

Initialize C0 to 1

$$Cout = (X' \cdot Y' \cdot Z' + X \cdot Y \cdot Z) \cdot Cin$$

2)

Truth table for Half-Subtractor:

A	B	Bout	Diff
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

From the above truth table we arrive at the following expressions for borrow and difference:

$$\text{Borrow} = \text{Bout} = A'B$$

$$\text{Difference} = \text{Diff} = A \text{ XOR } B$$

3)

Truth table for 3-input majority function:

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$Y = AB + BC + AC$$

= Cout of a full-adder

We observe that the carry-out of a full-adder circuit equals the output of a majority function.

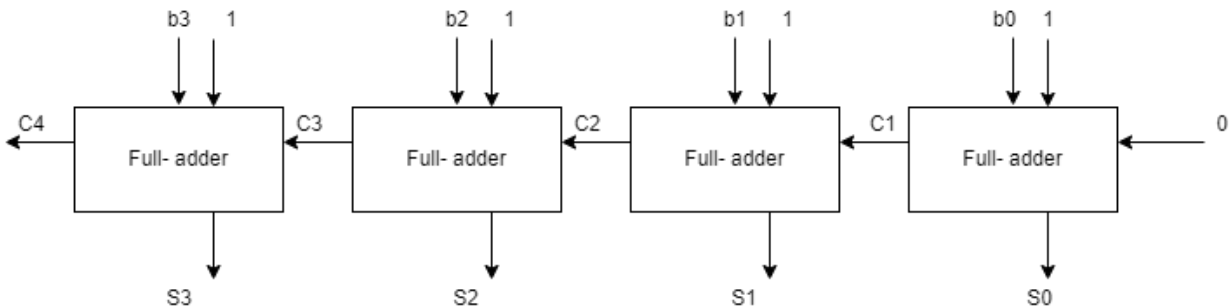
4)

A four-bit decrementer subtracts 1 from a 4-bit number.

Subtracting 1 from 4-bit number = 4-bit number + (-1)

(-1) in 4-bit 2's complement form = 1111.

Let the 4-bit number be $b_3 b_2 b_1 b_0$.



Carry = $C_4 C_3 C_2 C_1$

Sum = $S_3 S_2 S_1 S_0$

5)

From the truth table, we observe that binary representation of number of ones in input = binary number with Cout of full-adder as MSB and Sum as LSB.

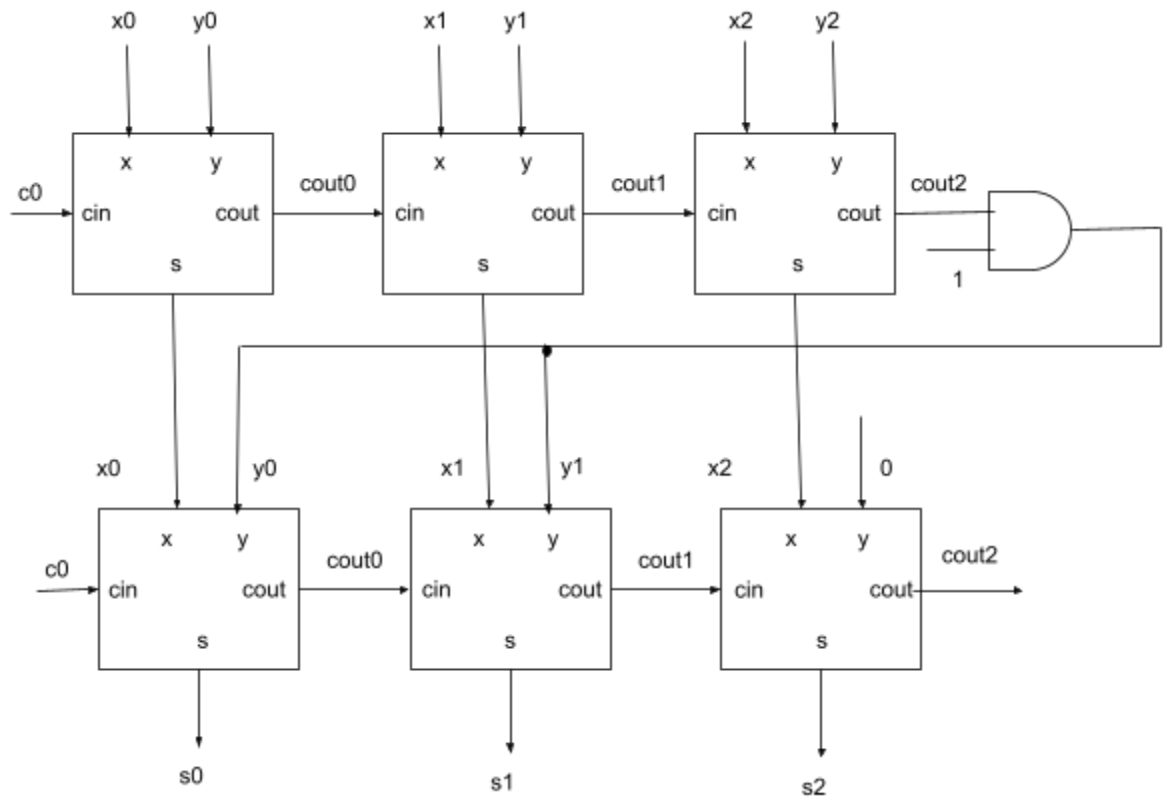
A	B	C	Cout	Sum	Cout-Sum	No. of 1s in input
0	0	0	0	0	00(0)	0
0	0	1	0	1	01(1)	1
0	1	0	0	1	01(1)	1
0	1	1	1	0	10(2)	2
1	0	0	0	1	01(1)	1
1	0	1	1	0	10(2)	2
1	1	0	1	0	10(2)	2
1	1	1	1	1	11(3)	3

6) Worst case delay = delay for the carries to propagate from least significant adder to most-significant adder + delay for the sum to be generated in the most significant adder

$$= (16-1) \cdot 12 + 15 \text{ ns}$$

$$= 195 \text{ ns}$$

7)



Each block represents a full adder.

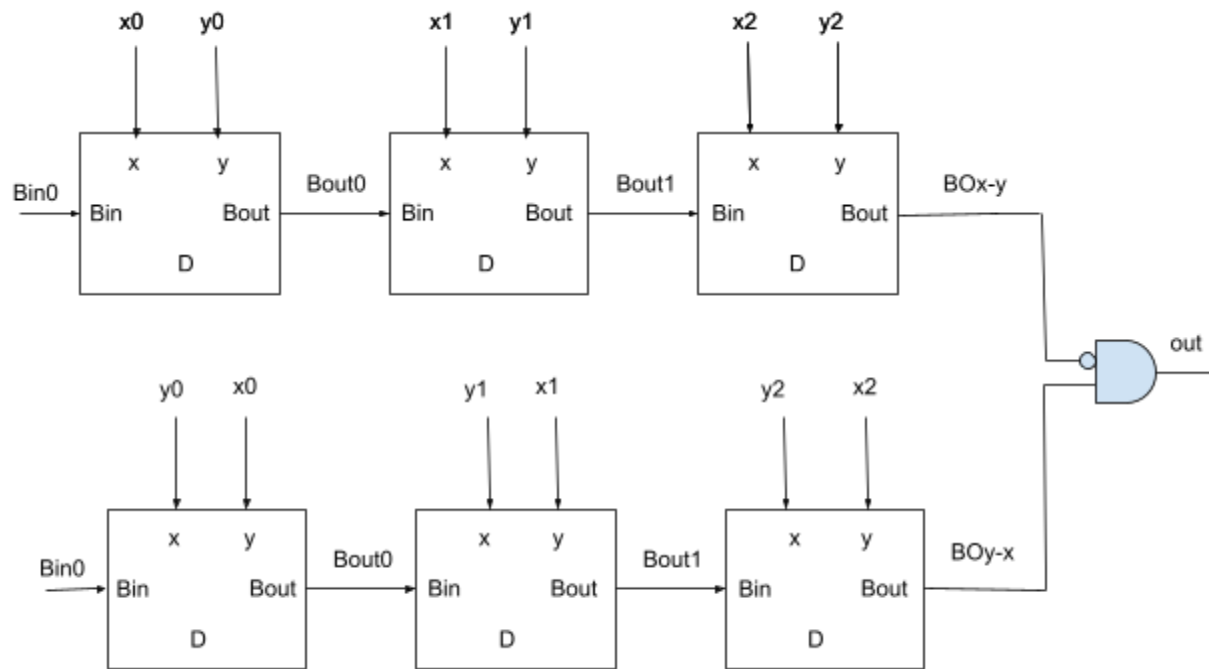
If the sum is greater than 7, then Cout2 in the upper ripple carry adder is 1. To subtract 5 from the sum, we use two's complement representation.

$$\text{sum} - 5 = \text{sum} + (-5)$$

-5 in two's complement = 1011

Adding the less significant 3 bits of -5 to the sum bits, we get the new sum and cout2 bits, which gives us the required result.

8)



Each block is a full subtractor with $D = X \oplus Y \oplus \text{Bin}$

$$\text{Bout} = X' \cdot Y + X' \cdot \text{Bin} + Y \cdot \text{Bin}$$

If $\text{BO}_{x-y} = 0$ and $\text{BO}_{y-x} = 1$, then the output is 1 signifying $X > Y$

9) Carry lookahead adder does not wait for the carry to be propagated from one adder to another. Instead, it takes advantage of the parallelly fed input to obtain the generate carry and propagate carry functions that can produce the result instantaneously.

Cell i of an adder is said to generate a carry if $C_{i+1} = 1$ based only on X_i and Y_i , and independent of $X_{i-1} \dots X_0, Y_{i-1} \dots Y_0, C_0$. The carry-generate function is denoted by

$$G_i = X_i \cdot Y_i$$

Cell i of an adder is said to propagate a carry if $C_i = 1$ results in $C_{i+1} = 1$ based only on X_i and Y_i , and independent of $X_{i-1} \dots X_0, Y_{i-1} \dots Y_0$ and C_0 . The carry-propagate function is denoted by

$$P_i = X_i + Y_i$$

For a 4-bit carry lookahead adder:

$$C_1 = G_0 + P_0 \cdot C_0$$

$$C2 = G1 + P1.C1 = G1 + P1. (G0 + P0.C0) = G1 + P1.G0 + P1.P0.C0$$

$$C3 = G2 + P2.C2 = G2 + P2. (G1 + P1.G0 + P1.P0.C0) = G2 + P2.G1 + P2.P1.G0 + P2.P1.P0.C0$$

$$C4 = G3 + P3.C3 = G3 + P3. (G2 + P2.G1 + P2.P1.G0 + P2.P1.P0.C0) \\ = G3 + P3.G2 + P3.P2.G1 + P3.P2.P1.G0 + P3.P2.P1.P0.C0$$

$$S0 = X0 \text{ xor } Y0 \text{ xor } C0$$

$$S1 = X1 \text{ xor } Y1 \text{ xor } C1$$

$$S2 = X2 \text{ xor } Y2 \text{ xor } C2$$

$$S3 = X3 \text{ xor } Y3 \text{ xor } C3$$

10)

Let the input value = $A = A_3 A_2 A_1 A_0$.

Output = $Y = 3A = 2A + A$.

$2A$ can be obtained by left shifting A by 1 bit.

A bit-binary adder can be used to add A and the left shifted value of A .

