Practice Problems 3 Solutions

1. Given function F (W, X, Y, Z) = W’. X + X. Y + W. (X’ + Z’)

F’ (W, X, Y, Z) = (W’.X + X.Y + W. (X’ + Z’))’ ------- equation 1

→ We know that according to De Morgan’s law,

(A+B)’ = A’. B’ and (A.B)’ = A’ + B’.

Using these, we can write, the equation 1 as (W+X’). (X’+Y’). (W’ + (X. Z)) = F’ (W, X, Y, Z).

(If needed, this can be further multiplied and simplified) -> next question.

2. Given F’ (W, X, Y, Z) to be simplified and show as W. X. Y’. Z + W’. X’

Actual answer we got for F’ is

F’ (W, X, Y, Z)

= (W+X’). (X’+Y’). (W’ + (X. Z)) → Using switching algebra theorems to simplify, we get,

= (WX’ + WY’ + X’X’ + X’Y’). (W’ + XZ)

= WX’W’ + WY’W’ + X’X’W’ + X’Y’W’ + WX’XZ + WY’XZ + X’X’XZ + X’Y’XZ

= 0 + 0 + X’W’ + X’Y’W’ + 0 + WY’XZ + 0 + 0 (Using X. X’ = 0 as applicable)

= X’W’ + X’Y’W’ + WXY’Z

= WXY’Z + X’W’(1+Y’)

= W. X. Y’. Z + W’. X’ (Using 1+A = 1)

Hence Proved.

3. Let G (W, X, Y, Z) = F’(W, X, Y, Z) = W. X. Y’. Z + W’. X’

$$G'(W,X,Y,Z) = \left(W\cdot X\cdot Y'\cdot Z + X'\cdot W'\right)'$$

Applying DeMorgan's law,

$$= \left[(W\cdot X\cdot Y'\cdot Z)'\cdot(X'\cdot W')'\right] = \left[(W'+X'+Y+Z')\cdot(X'\cdot W')'\right]$$

$$= \left[(W'+X'+Y+Z')\cdot(X+W)\right]$$

$$= W'X + \overset{0}{\cancel{W'W}} + \overset{0}{\cancel{X'X}} + X'W + YX + YW + Z'X + Z'W$$

$= W'X + X'W + XY + WY + Z'X + Z'W$

Applying consensus law $(XY + X'Z + YZ = XY + X'Z)$

with $(X = X, Y = Y \;\&\; Z = W)$, we get

$= WZ' + XW' + XZ' + [(XY + X'W)]$

Again apply consensus law as above, but with

$(X = X, Y = Z', Z = W)$, we get

$= XW' + XY + XZ' + X'W$

$= X'.W + X.Y + X(W' + Z')$

$= F(W, X, Y, Z)$

Hence proved.