ECE 27000 Introduction to Digital System Design
Spring 2022
Exam 1, February 24, 2022

SOLUTION

Last Name: _____ First Name: _____

As a boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together - we are Purdue.

I certify that I have neither given nor received unauthorized aid on this exam.

Signed: _____

Solve the following problems. The number of points for each problem is shown in the table below.

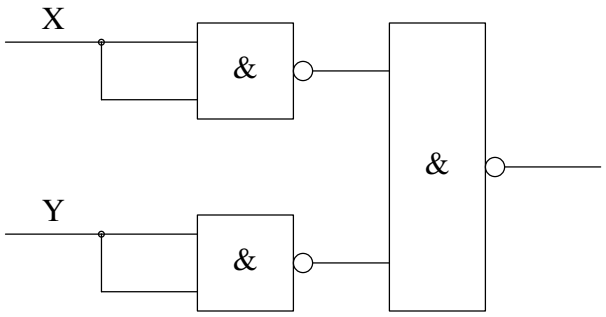
Use only the space provided to solve each problem, and copy the answers to the space marked "Answer:..."

Problem	Outcome	Points
1	2	/ 20
2	1	/ 20
3		/ 20
4	2	/ 20
5	2	/ 20
Total		/ 100

Problem 1

Show that it is possible to implement a two-input OR function $Z = X + Y$ using only two-input NAND gates.
Use only X and Y as inputs. If you need X' or Y' implement it using a two-input NAND gate.

Answer (a circuit made up of two-input NAND gates):



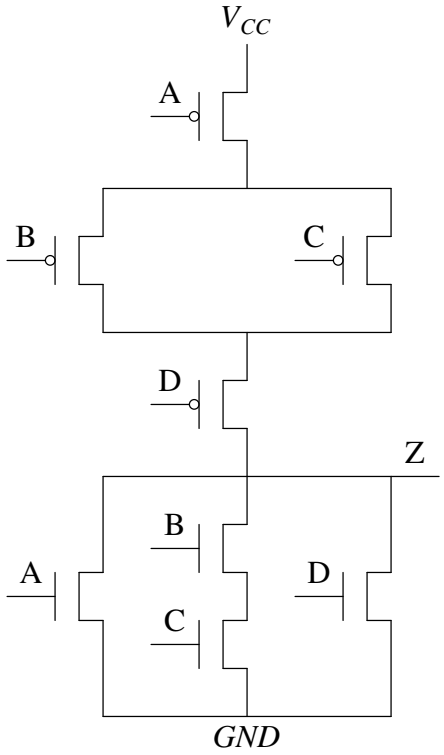
Explanation:

$Z = X + Y$
$Z' = (X + Y)' = X' \cdot Y'$
$Z = (X' \cdot Y')'$

Problem 2

Draw a complex CMOS gate for the function $Z = (A + B \cdot C + D)'$.

Answer (a circuit consisting of CMOS transistors):



Explanation:

$Z = (A + B \cdot C + D)' = A' \cdot (B' + C') \cdot D'$
The N network is designed based on $Z' = A + B \cdot C + D$.
The P network is designed based on $Z = A' \cdot (B' + C') \cdot D'$.

Problem 3

A 4-bit two's complement number $b_3b_2b_1b_0$ was shifted to the right by one bit position. Its leftmost bit was not changed (sign extension was used).

Write an equation for computing the decimal value of the number before and after it was shifted, and find a relationship between the two values.

Answer:

Value before shifting:
$B_0 = -8 \cdot b_3 + 4 \cdot b_2 + 2 \cdot b_1 + b_0$

Value after shifting:
$B_1 = -4 \cdot b_3 + 2 \cdot b_2 + b_1$

Relationship: $B_1 = B_0/2$ (integer division)

Explanation:

$B_0 = (b_3b_2b_1b_0)_2 = -8 \cdot b_3 + 4 \cdot b_2 + 2 \cdot b_1 + b_0.$

$B_1 = (b_3b_3b_2b_1)_2 = -8 \cdot b_3 + 4 \cdot b_3 + 2 \cdot b_2 + b_1 = -4 \cdot b_3 + 2 \cdot b_2 + b_1$

$B_1 = B_0/2$ (integer division)

Problem 4

Find all the prime implicants of the function
$F(W, X, Y, Z) = \sum(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$.
List the prime implicants as product terms using the variables W, X, Y and Z.

Answer (all the prime implicants):
$W' \cdot X \cdot Z$, $X' \cdot Z'$, $X' \cdot Y$, $Y \cdot Z$, $W \cdot Y$.

Explanation: Using the Quine-McCluskey method.

0	0000	0000+	00-0+	-0-0
2	0010		-000+	
3	0011	0010+		-01-
5	0101	1000+	001-+	
7	0111		-010+	--11
8	1000	0011+	10-0+	1-1-
10	1010	0101+		
11	1011	1010+	0-11+	
14	1110		-011+	
15	1111	0111+	01-1	
		1011+	101-+	
		1110+	1-10+	
		1111+	-111+	
			1-11+	
			111-+	

Prime implicants:

W	X	Y	Z	
0	1	-	1	$W' \cdot X \cdot Z$
-	0	-	0	$X' \cdot Z'$
-	0	1	-	$X' \cdot Y$
-	-	1	1	$Y \cdot Z$
1	-	1	-	$W \cdot Y$

Problem 5

The function $F(W, X, Y, Z)$ has the following prime implicant table. Find a minimal sum for $F(W, X, Y, Z)$.
Write the minimal sum in terms of the variables W, X, Y and Z.
Note that you can derive a product term for a prime implicant from the minterms where the prime implicant is equal to 1.

	1	3	4	5	9	11	12	13	14	15
1	x	x			x	x				
2			x	x			x	x		
3					x	x		x		x
4	x			x	x			x		
5							x	x	x	x

Answer (minimal sum as a function of W, X, Y and Z):
$X' \cdot Z + X \cdot Y' + W \cdot X$

Explanation:

Minterms 3, 4 and 14 are distinguished (only one prime implicant covers them). These minterms make prime implicants 1, 2 and 5 essential.
Prime implicants 1, 2 and 5 cover all the minterms. No additional prime implicants are selected.

1		2		5	
1	0001	4	0100	12	1100
3	0011	5	0101	13	1101
9	1001	12	1100	14	1110
11	1011	13	1101	15	1111
	-0-1		-10-		11--
	$X' \cdot Z$		$X \cdot Y'$		$W \cdot X$