ECE 270 (Spring 2022)

Homework 3 Solutions

Due on 02/04/2022 (Friday) by 11:59 pm sharp on BrightSpace.

1. As per consensus theorem:

   XY + X'Z + YZ = XY + X'Z

   XY + X'Z + YZ
   => XY + X'Z + YZ(X+X')
   => XY + XYZ + X'Z + YZX'
   => XY(1+Z) + X'Z(1+Y)
   => XY + X'Z

   (X+Y) . (X'+Z) . (Y+Z) = (X+Y) . (X'+Z)

   (X+Y) . (X'+Z) . (Y+Z)
   => (X+Y) . (X'+Z) . (Y+Z+(X.X'))
   => (X+Y) . (X+Y+Z) . (X'+Z) .(X'+Z+Y)
   => (X+Y) . (X'+Z)

2. 000, 011, 010, 001. To reduce the error, we can use gray code instead of binary-coded values since there is only one bit change between two consecutive gray code numbers.
3. Let a1, a2, a3, .. an be the inputs and b be the output of an n-input OR gate.

   Therefore, a1+a2+a3+...an = b
   => ((((a1+a2)+a3)+a4)+...an) = b

   If a1,a2 are inputs to the first 2-input OR gate. Its output and a3 are inputs to the 2nd OR-gate, and so on.
   Hence an n-input OR gate can be replaced by n-1 2-input OR gates.

   There are other ways to implement the same.
   Eg: ((a1+a2)+(a3+a4))+((a5+a6….) = b
   Any suitable implementation with proper justification is fine.

4. F = (A + B) . (B + C) . (C + A)
   = (A.B + B.B + A.C + B.C) . (C + A) (using Distributive law)

= (A.B.C + B.C + A.C + B.C + A.B + A.B + A.C + A.B.C) (using Distributive law)
= (A.B.C + B.C + A.C + A.B) (combining terms)
= (B.C + A.C + A.B) (combining terms)
= F^D
Hence, F is a self- dual logic function

5.

F = (X' . Y) + (X + Y')
F' = ((X' . Y) + (X + Y'))'
= (X' . Y) ' . (X + Y')'
= (X + Y') . (X' . Y)

6.

LHS = W . (X' + Y') + X' . (X' + Y') . (W' + X . Z)
= (W . X' + W. Y' + X' . X' + X'. Y') . (W' + X . Z) ---- (using Distributive law)
= (W . X' + W. Y' + X' + X'. Y') . (W' + X . Z) ---- (X' . X' = X')
= (W . X' + W. Y' + X') . (W' + X . Z) --- (X'. Y' + X' = X')
= (W. Y' + X') . (W' + X . Z) --- (W. X'+ X' = X')
= (W. Y'. W'+ X'.W' + W.X .Y'. Z + X' .X . Z) ---- (using Distributive law)
= W . X . Y' . Z + W' . X' ---- (X' .X . Z = W. Y'. W' = 0)

7. 2^(8) = 256 – since there are 8 rows in truth table with 3 variables, and each row has two options for output value.

8. Truth Table:

P	*T*	*Z*	*T'*	*PT'*	*P+Z*	*PT'(P+Z)*
0	0	0	1	0	0	0
0	0	1	1	0	1	0
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	1	1	1	1
1	0	1	1	1	1	1
1	1	0	0	0	1	0
1	1	1	0	0	1	0

9. Canonical Sum = $\sum(4,5)$ = PT'Z' + PT'Z
Canonical Product = $\pi(0,1,2,3,6,7)$

This can also be written as (P+T+Z).(P+T+Z').(P+T'+Z).(P+T'+Z').(P'+T'+Z).(P'+T'+Z')

10. Y = Π (2,3,6,8,10,11,13,14,15) – List of all maxterms corresponding to the truth table rows where the function is 0.