

Chapter 11. Counters and Shift Registers

Read Chapter 11 for additional information.

11.1. Counters

A *counter* is a finite-state machine whose state diagram contains a single cycle.

An n – *bit binary counter* has n flip-flops and 2^n states , which are visited in the order
 $0, 1, \dots, 2^n - 1, 0, 1, \dots, 2^n - 1, \dots$

We saw an implementation of a 3-bit binary counter using T flip-flops.

Characteristic equation:

$$Q^* = TQ' + T'Q = T \oplus Q$$

$$T = Q \oplus Q^*$$

Q2	Q1	Q0	Q2*	Q1*	Q0*	T2	T1	T0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1

$$T_0 = 1$$

$$T_1 = Q_0$$

$$T_2 = Q_1 \cdot Q_0$$

For an n -bit counter, Q_i changes one clock cycle after $Q_{i-1} \cdots Q_1 Q_0 = 1 \cdots 11$. Therefore,

$$T_i = Q_{i-1} \cdots Q_1 \cdot Q_0$$

$$T_0 = 1$$

$$T_1 = Q_0$$

$$T_2 = Q_1 \cdot Q_0$$

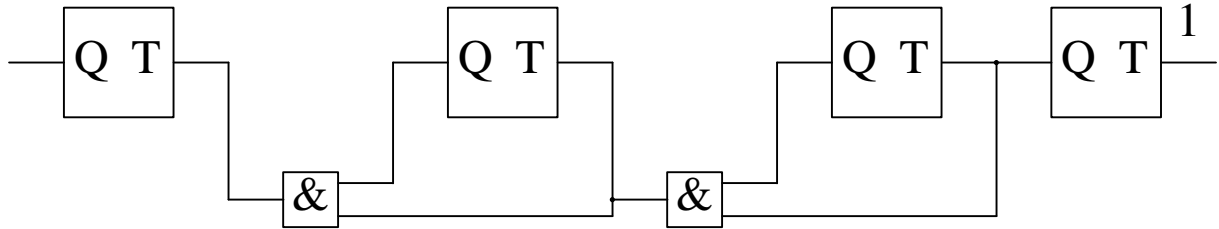
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$$T_i = Q_{i-1} \cdots Q_1 \cdot Q_0$$

...

$$T_n = Q_{n-1} \cdots Q_1 \cdot Q_0$$

Implementation:



To count only when CNT=1:

$$T_0 = CNT$$

$$T_1 = CNT \cdot Q_0$$

$$T_2 = CNT \cdot Q_1 \cdot Q_0$$

...

$$T_i = CNT \cdot Q_{i-1} \cdot \dots \cdot Q_1 \cdot Q_0$$

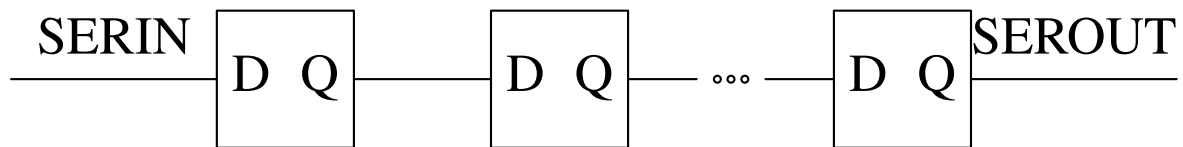
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$$T_n = CNT \cdot Q_{n-1} \cdot \dots \cdot Q_1 \cdot Q_0$$

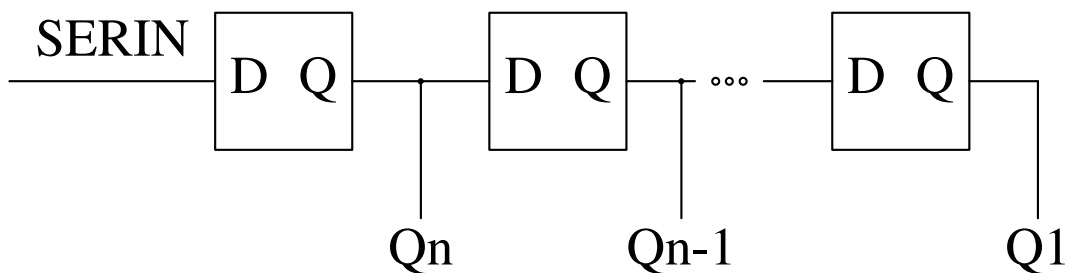
11.2. Shift Registers

A *shift register* is an n -bit register with a provision for shifting its data by one position at every clock cycle.

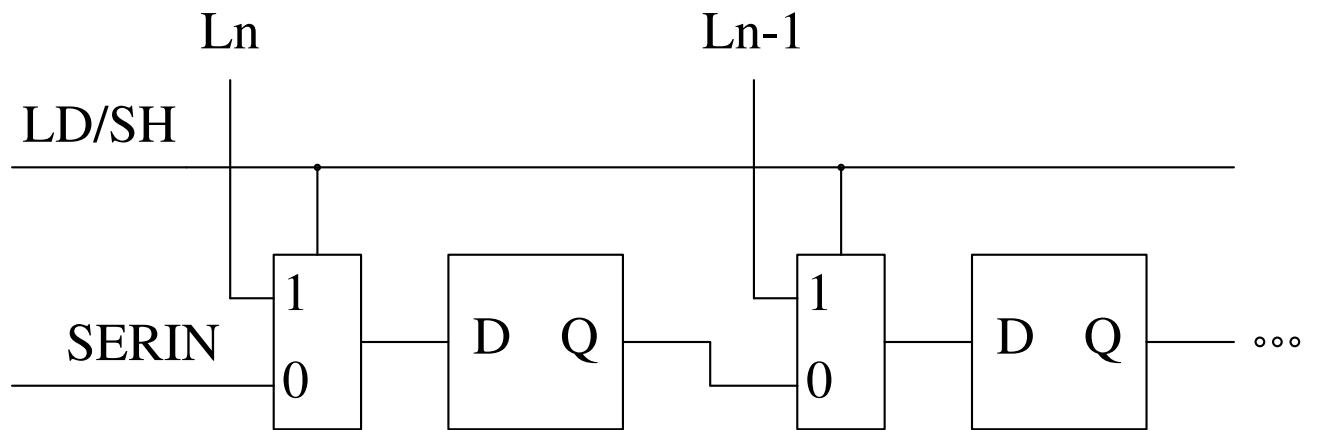
A *serial – in serial – out* shift register:



A *serial – in parallel – out* shift register:



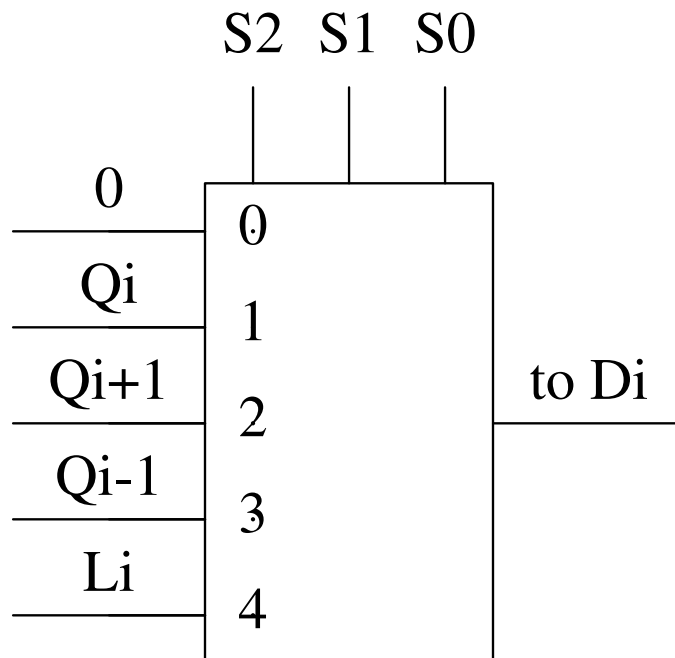
A parallel – in serial – out shift register:



With larger MUXes and more control inputs (also called select inputs) it is possible to design a shift-register with more functions.

Example:

S2	S1	S0	Operation
0	0	0	Clear
0	0	1	Hold
0	1	0	Shift right
0	1	1	Shift left
1	0	0	Load



An 8-bit *ring counter* goes through the sequence of states

00000001

00000010

00000100

...

01000000

10000000

00000001

00000010

...

This sequence can be implemented by an n -bit shift-register as follows.

Connect Q_n to LEFTIN (on the righthand side, driving Q_1).

Use parallel load to initialize the counter to state 00...0001

Then use left shift for counting.

An 8-bit *twisted – ring* counter goes through the sequence of states

00000000

00000001

00000011

00000111

...

11111111

11111110

11111100

...

11000000

10000000

00000000

00000001

...

This sequence can be implemented by an n -bit shift-register as follows.

Connect Q_n' to LEFTIN (on the righthand side, driving Q_1).

Use clear to initialize the shift-register.

Then use left shift for counting.