Short rate models

Short rate models are broadly of two types[i]

- Arbitrage free models
- Equilibrium models

Arb free models have a deterministic part which allows the model to be fit to market prices. In contrast, equilibrium models have a long term mean reversion level around which the rates are expected to evolve. Arb free models have great utility in terms of enabling market makers to quote price for securities that are illiquid (after calibration).

We will now briefly describe the other models discussed in Module 7

Ho-Lee model

$$dr = \lambda_t dt + \sigma dw$$

Ho-Lee Model is a one-factor model with λ_t being a time dependent deterministic part instead of a constant variable.

Vasicek model

$$dr = k(\theta - r)dt + \sigma dw$$

With θ being a constant variable instead of time dependent as in Hull-White, it brings stability at the cost of accurateness. Although a variable θ in Hull-White is indeed more difficult to handle computationally.

Cox Ingersoll Ross (CIR) model

$$dr = k(\theta - r)dt + \sigma\sqrt{r}dw$$

CIR avoids negative rates[ii]. This may be seen as an advantange by some, although current economic conditions say that -ve rates may be very plausible.

One factor Ross (CIR) model

$$dr = k(\theta(t) - \alpha r_t)dt + \sigma dw$$

The One-Factor Hull-White model does not calibrate to current market structures that well, esp when compared to the two factor Hull-White. This has been empirically observed, esp as the two factor has additional parameters .

Below is a summary of the major short rate models and the category that they belong to:

Model		Type	Description
Vasicek	$dr = k(\theta - r)dt + \sigma dw$	Equilibrium	Mean reverting drift, constant vol
Ho-Lee	$dr = \lambda_t dt + \sigma dw$	No-arbitrage	Time dependent drift, constant vol
CIR	$dr = k(\theta - r)dt + \sigma\sqrt{r}dw$	Equilibrium	Mean reverting drift, vol function of rate

			level (square root of rate level)
Hull-White (single factor)	$dr = k(\theta(t) - \alpha r_t)dt + \sigma dw$	No arbitrage	Time and rate level dependent drift, constant vol
Hull-White (two factor)	$dr(t) = (\theta(t) + u(t) - a(t))dt + \sigma_1(t)dW_1(t)$ $du(t) = (-bu(t)\,dt + \sigma_2(t)dW_2(t)$	No arbitrage	Two factor, time dependent deterministic function and drift and also time dependent vol

[i] "The Art of Term Structure Model", Analyst Prep, https://www.youtube.com/watch?v=1MjogfwFINI

[ii] Interest Rate Forecasting Using CIR Model. www.financetrainingcourse.com/education/2010/11/interest-rate-forecasting-using-cir-cox-ingersoll-ross-model