Pricing an interest rate cap using Black Scholes

Interest rate cap is a complex OTC product which will only have an institutional audience. Companies with large floating rate loans or large interest incomes may use interest rate caps/floors as a hedge.

For eg. A company with a large floating-rate loan may want to hedge against any sudden increase in financing expenses. Such a company may buy a cap with a strike price well above the current floating rate. Also, if the company thinks that a normal cap is too expensive, it may go for a knock-out cap (a combination of cap and barrier options)[i].

Inputs

An interest rate cap is a combination of successive interest rate caplets, at every reset date. However, it can also be viewed as a [zero-coupon bond + a put option][ii], and can be easily priced using this approach.

To price an interest rate cap, we need

- **Term structure of yields**
  Black analytical solution requires a forward curve and a zero coupon curve.
  Alternatively, we need a risk free rate instead of a zero coupon curve to discount back the cash flows.
- **Volatility assumptions**
  Simplistically, vol may be held constant or we may use a calibrated volatility surface to efficiently price an interest rate cap
- Strike price
- Notional amount
- Day type convention

Market values

We are using the forward yield curve using USD LIBOR 1month rates from https://www.chathamfinancial.com/technology/us-forward-curves. For discounting the cash flows, we are using the average of the SOFR rate over the period from 02/09/2021 to 02/09/2022 as our interest rate cap is for that period.

We are using a flat volatility of 0.032 for the forward rates based on the standard deviation of the underlying during this period[iii].

Analytical solution

We are using the Black model for valuing the price of a caplet[iv]. Then, the sum of the price of the caplets will be the final Cap price.

Parameters

- d1 = $\frac{\ln(F/K) + 0.5\sigma^2 t}{\sigma\sqrt{t}}$
- d2 = d1 - $\sigma\sqrt{t}$

In Excel VBA, the parameters have been shown as follows:

```
d1 = ((Application.Ln(forward / strike)) + (volatility * volatility / 2) * T) / (volatility * Sqr(T))

d2 = ((Application.Ln(forward / strike)) - (volatility * volatility / 2) * T) / (volatility * Sqr(T))
```

Caplet formula

$V = \alpha\, P(0,t)\, (F\, N(d1) \;-\; K\, N(d2))$

In Excel VBA, the pricing formula has been shown as follows:

```
If (optionType = 1) Then 'Caplet

    Cap_Floor = ((faceValue * td) / (1 + (forward * td))) * Exp(-rf * T) * ((forward * CalD1(day_Convention, forward,
strike, volatility, start_Date, payment_Date)) - (strike * CalD2(day_Convention, forward, strike, volatility,
start_Date, payment_Date)))

Else

    Cap_Floor = ((faceValue * td) / (1 + (forward * td))) * Exp(-rf * T) * ((strike * (1 - CalD2(day_Convention,
forward, strike, volatility, start_Date, payment_Date)) - (forward * (1 - CalD1(day_Convention, forward, strike,
volatility, start_Date, payment_Date)))))

End If
```

Assumptions

- **Risk free rate:** Constant flat risk-free rate. average SOFR rate over the period from 02/09/2021 to 02/09/2022
- **Volatility:** Constant volatility
- **Forward rate curve:** USD LIBOR 1 month forward rate[v]
- **Strike price:** 0.0010
- **Notional:** 10,00,00,000

Inputs

Type	Cap
Day convention	365 days
Interest type	Semi-annual
Face value	10,00,00,000.00
Strike	0.0010
Volatility	0.0302
Risk-free rate	0.0600

Volatility of LIBOR
Average SOFR rate

Start date: September 1, 2021
Payment Dates

Date (MM/DD/YY)	USD LIBOR 1m Forward rate	Caplet at each time
September 2, 2021	0.0900	48,735.07
October 4, 2021	0.0800	7,07,138.74
November 2, 2021	0.0010	23.37
December 2, 2021	0.0013	2,528.26
January 3, 2022	0.0015	4,721.57
February 2, 2022	0.0015	4,115.15
March 2, 2022	0.0013	2,127.84
April 4, 2022	0.0011	737.53
May 2, 2022	0.0010	253.00
June 2, 2022	0.0012	1,300.12
July 4, 2022	0.0014	3,463.30
August 2, 2022	0.0016	5,193.04
September 2, 2022	0.0020	8,177.71

Source

Results

Cap value
7,88,514.70
Cap value is the sum of all the caplets

(See attached excel sheet[vi])

Cap value on 10,00,00,000.00 notional: 7,88,514

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[i] Barrier Interest Rate Options. www.kh.hu/documents/2074024/2729992/KH+treasury+eng+2013+IIIa+06.pdf/ccfff10c-8f12-4659-ba3ea309f169a5d7?t=1482245203894.

[ii] Brigo, Damiano, and Fabio Mercurio. Interest Rate Models: Theory and Practice. Springer Berlin Heidelberg, 2006.

[iii] Hidden Driver of Cap Cost, www.pensford.com/resources/the-hidden-driver-of-cap-cost-volatility

[iv] Caps, Floors and Collars. janroman.dhis.org/doc/AF2%20Interest%20Rate%20Caps,%20Floors%20and%20Collars.pdf

[v] US Forward Curves. www.chathamfinancial.com/technology/us-forward-curves

[vi] Caps and Floors using Black76, http://janroman.dhis.org/stud/II2010/CapFloor/AFII.pdf