Lookback options

Lookback options are quintessential path dependent options where the final payoff is extremely dependent on the path taken to reach that final price.

Lookback Call options

- Floating lookback call does not have a fixed strike price.
  The payoff is given by $S_T - S_{min}$
- Fixed lookback call has a fixed strike price.
  The payoff is given by $S_{max} \ - \ K$

where S_{max} and S_{min} are the asset's maximum and minimum price during the life of the option and and S_T is the price of the asset at maturity.

Lookback Put options

- Floating lookback put does not have a fixed strike price.
  The payoff is given by $S_{max} - S_T$
- Fixed lookback call has a fixed strike price.
  The payoff is given by $K - S_{min}$

where S_{max}, S_{min} are the asset's maximum and minimum price during the life of the option and and S_T is the price of the asset at maturity (time T).

Properties

- Given that Lookback options take the most favourable values as one of the parameters, they are very expensive when compared to regular options.
- Also, as with barrier options, lookbacks are liable to be sensitive to the frequency with which the asset price is observed for the purposes of computing the maximum or minimum.

Pricing

Since Lookback options are very path dependent, it is difficult to obtain a widely accepted analytical formula for pricing such an option. There are various approaches in which this problem can be handled.

- **Analytical:** Here we try to come with a model which can give an analytical solution for the price of lookback fixed and lookback floating options. Eg. Black-Scholes, various Stochastic volatility models etc.
- **Simulations:** We can conduct monte-carlo simulations under some parametric assumptions to determine the price of Lookback options.
- **Combination of Analytical and Simulation**: eg. Control Variate Monte-Carlo where we introduce a control variate into Monte Carlo simulations, and we price both Lookback and Vanilla options using the same simulations and then also calculate the analytical price of the Vanilla option using the Black-Scholes model.
  Then we estimate the price of the Lookback option by adjusting it for the error we observe between the simulated and analytical price of the Vanilla option. [i]

Black Scholes: Lookback option

In the Black-Scholes world, the price of a Lookback option is given as below [ii]:

Floating strike lookback call

where M is the realized minimum of the asset price, the fair value is given as:

$$Se^{-D(T-t)}N(d_1) - Me^{-r(T-t)}N(d_2) + Se^{-r(T-t)}\frac{\sigma^2}{2(r-D)}\left(\frac{S}{M}^{\frac{-2(r-D)}{\sigma^2}} N(-d_1 + \frac{2(r-D)\sqrt{T-t}}{\alpha}\right) - e^{(r-D)(T-t)}N(-d_1))$$

Where $d_1 = \frac{log\ (S/M) + (r-D+1/2\,\sigma^2)\ (T-t)}{\sigma\sqrt{T-t}}$ and $d_2 = d_1 - \sigma\sqrt{T-t}$

Floating strike lookback put

where M is the realized maximum of the asset price, the fair value is given as:

$$Me^{-r(T-t)}N(-d_2) - Se^{-D(T-t)}N(-d_2) + Se^{-r(T-t)}\frac{\sigma^2}{2(r-D)}\left(-\frac{S}{M}^{\frac{-2(r-D)}{\sigma^2}} N(d_1 - \frac{2(r-D)\sqrt{T-t}}{\alpha}\right) + e^{(r-D)(T-t)}N(d_1))$$

Where $d_1 = \frac{log\ (S/M) + (r-D+1/2\,\sigma^2)\ (T-t)}{\sigma\sqrt{T-t}}$ and $d_2 = d_1 - \sigma\sqrt{T-t}$

Fixed strike lookback call

- **For E > M** the fair value is:

$$Se^{-D(T-t)}N(d_1) - Ee^{-r(T-t)}N(d_2) + Se^{-r(T-t)}\frac{\sigma^2}{2(r-D)}\left(-\frac{S}{M}^{\frac{-2(r-D)}{\sigma^2}} N(d_1 - \frac{2(r-D)\sqrt{T-t}}{\alpha}\right) + e^{(r-D)(T-t)}N(d_1))$$

Where $d_1 = \frac{log\ (S/E) + (r-D+1/2\,\sigma^2)\ (T-t)}{\sigma\sqrt{T-t}}$ and $d_2 = d_1 - \sigma\sqrt{T-t}$

- **When E < M** the value is:

$$(M-E)e^{-r(T-t)}N(d_1) + Se^{-D(T-t)}N(d_1) - Me^{-r(T-t)}N(d_2) + Se^{-r(T-t)}\frac{\sigma^2}{2(r-D)}\left(-\frac{S}{M}^{\frac{-2(r-D)}{\sigma^2}} N(d_1 - \frac{2(r-D)\sqrt{T-t}}{\alpha}\right) + e^{(r-D)(T-t)}N(d_1))$$

Where $d_1 = \frac{log\ (S/E) + (r-D+1/2\,\sigma^2)\ (T-t)}{\sigma\sqrt{T-t}}$ $and\ d_2 = d_1 - \sigma\sqrt{T-t}$

Fixed strike lookback put

- **For E < M** the fair value is:

$$Ee^{-r(T-t)}N(-d_2) - Se^{-D(T-t)}N(-d_1) + Se^{-r(T-t)}\frac{\sigma^2}{2(r-D)}(\frac{S}{E}^{\frac{-2(r-D)}{\sigma^2}} N(-d_1 + \frac{2(r-D)\sqrt{T-t}}{\alpha}) - e^{(r-D)(T-t)}N(-d_1))$$

Where $d_1 = \frac{log\ (S/E) + (r-D+1/2\ \sigma^2)\ (T-t)}{\sigma\ \sqrt{T-t}}$ and $d_2 = d_1 - \sigma\sqrt{T-t}$

- **When E > M** the value is

$$E - M)e^{-r(T-t)} + Se^{-D(T-t)}N(-d_1) + Me^{-r(T-t)}N(-d_2) + Se^{-r(T-t)}\frac{\sigma^2}{2(r-D)}(\frac{S}{M}^{\frac{-2(r-D)}{\sigma^2}} N(-d_1 + \frac{2(r-D)\sqrt{T-t}}{\alpha}) - e^{(r-D)(T-t)}N(-d_1))$$

Where $d_1 = \frac{log\ (S/M) + (r-D+1/2\ \sigma^2)\ (T-t)}{\sigma\ \sqrt{T-t}}$ and $d_2 = d_1 - \sigma\sqrt{T-t}$

Example of usage of Lookback options

Lookback options can be used to maximize the risk/reward ratio of a trade assuming that the price being quoted for the option is reasonable enough given the probabilities of the trade.

Since lookback options are one of the only OTC products which offer such an exposure to 'best prices', thereby increasing the POP (probability of profit), which when combined with a logical betting size can be a profitable business.

Also, lookbacks can be used to benefit from the volatility of assets as increased asset volatility is likely to increase the profits for lookback options.

Pricing Lookback options on the VIX index

We will now use the above formulas to find out the price of a 1 month lookback option[iii] for the VIX index under the following assumptions

- **Date of pricing:** 01/01/2020 (~three months before the Coronavirus crash)
- **Data used for calculating volatility of VIX:** 01/01/2019 to 31/12/2019 i.e. the whole year of 2019
- **Convention for annualising volatility:** converted daily vol to annualised using $\sigma_{annualised} = \sigma_{daily} * \sqrt{252}$
- **Option days to expiry:** 1 month (30 days)
- **Risk free rate:** the T-bill yield for 1 month

Note: we are not conducting any simulation either for pricing or for calculating the volatility of VIX

Valuation parameters: John C Hull, "Options, Futures and Other Derivatives"

	Black-Scholes Inputs for VIX			**Vanilla Call pricing**		Floating lookback parameters		Fixed lookback parameters	
1	Stock (S0)	**$25.42**		d1	0.2077	a1	6.5544	a1a	-6.4326
	S(min)	**$11.54**	min value for 2019	N(d1)	0.5823	a2	6.4326	a2a	-6.5544
	S(max)	**$25.45**	max value for 2019	d2	0.0859	a3	6.5345	a3a	-6.5345
2	Strike (K)	**$25.00**		N(d2)	0.5342	Y1	0.6602	Y1Y	0.6602
3	Volatity, σ	**42.5%**	annualised vol for 2019	**Call Price, c**	**1.4619**				
	Variance, σ^2	18.05%				b1	0.0606	b1a	0.0612
4	Riskfree rate, r	**1.48%**	T-bill yield	**Vanilla Put pricing**		b2	-0.0612	b2a	-0.0606
5	Term, T	**0.08**	1 month option	-d1	-0.2077	b3	-0.0412	b3a	0.0412
6	Div Yield, q	**0.00%**		N(-d1)	0.4177	Y2	-0.0010	Y2Y	0.0010
				-d2	-0.0859	**Floating Lookback pricing**			
				N(-d2)	0.4658	lookback call	**13.89**	S_min_fix	**$11.54**
				Put Price, p	**1.0115**	lookback put	**2.55**	S_max_fix	**$25.45**
						Fixed Lookback pricing			
						lookback call	**18.99**		
						lookback put	**7.46**		

(See attached excel file[iv])

The valuation of the Lookback options is as follows:

- Floating Lookback Call: $ 13.89
- Floating Lookback Put: $ 2.55
- Fixed Lookback Call: $ 18.99
- Fixed Lookback Put: $ 7.46

It must be noted that Lookback options can have a negative price as nothing in the pricing formula prevents such from happening. It has therefore been suggested that this is not really an Option at all, although this is largely a matter of semantics[v].

Infact, when we take a larger valuation period in the attached excel sheet and use the volatility that VIX experienced during the Coronavirus crash in our sample period, we find that option prices for Lookback options do take on negative prices!

References

[i] Brooks, Chris. Introductory Econometrics for Finance by Chris Brooks, 3rd Edition. Content Technologies, 2016.

[ii] "Lookback Options." Paul Wilmott Introducing Quantitative Finance, Wiley, 2001.

[iii] "Chapter 26: Exotic Options." Options, Futures, and Other Derivatives, by John C. Hull, Pearson Educational Limited, 2018.

[iv] David Harper (Bionic Turtle), Exotic Options: Barrier Options. www.youtube.com/watch?v=CYA6LiXjSpE.

[v] "16.4 Lookback Options." Option Theory, by Peter James, Wiley, 2005.