Machine Learning
(CS-601)

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Introduction

Neural Network

A Neural Network is a parallel distributed system made up of simple processing units, known as neurons, which has a natural tendency of storing experiential knowledge and making it available for use.

It resembles the brain in two respects:

1. Knowledge is acquired through the learning process.
2. Inter-neuron connection weights, are used to store the acquired knowledge.

Source: Neural Networks,'' by Simon Haykin

Introduction

Example Neural Networks

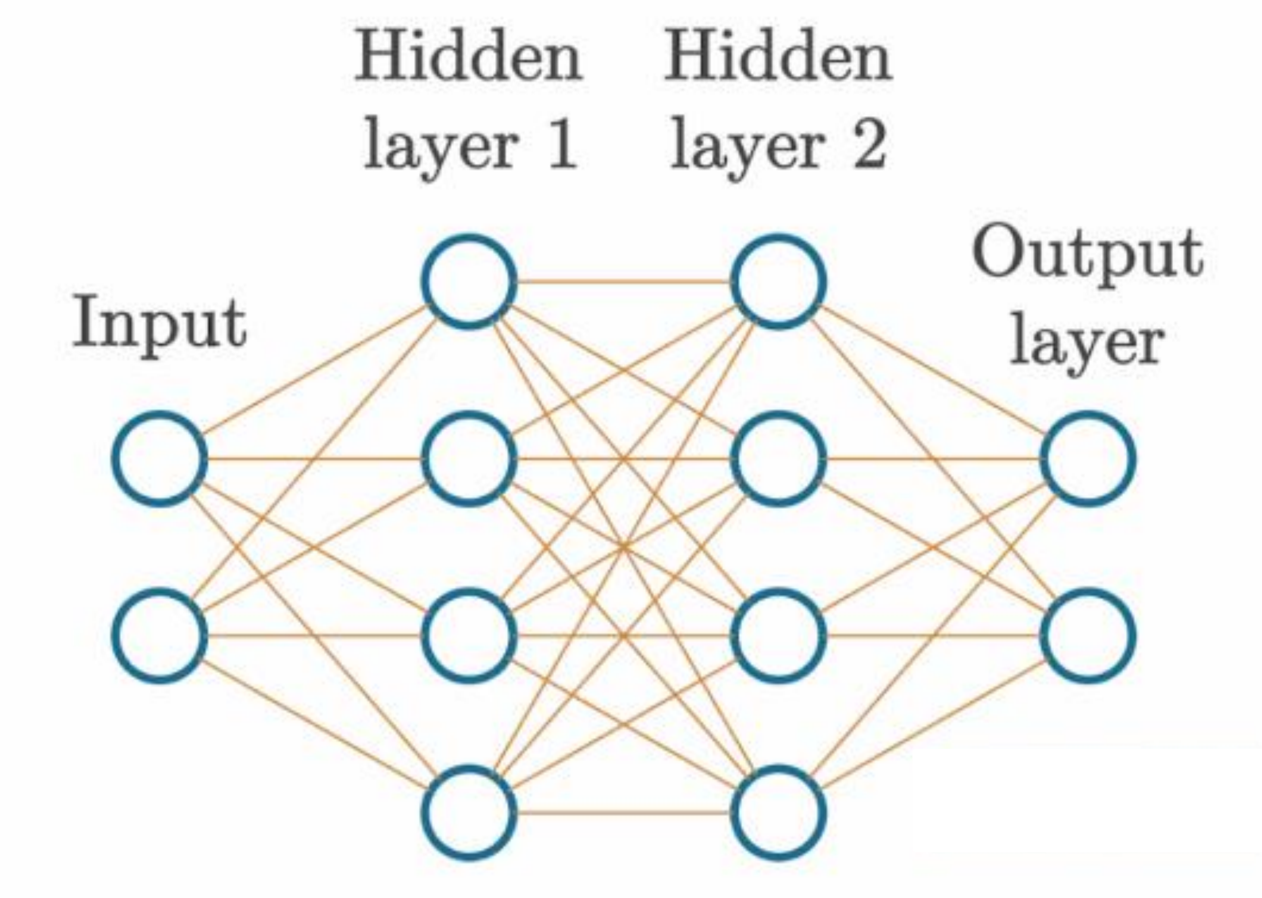


Fig : Example basic neural network.

Source: Google Images

Activation Functions

- The Activation Function is applied to the output of a neuron.
- The Activation Function modifies the neuron output.
- We use Activation Functions to introduce nonlinearity or desired mapping in the model.
- In general, neural networks use two types of activation functions.
- The first will be the activation function used in hidden layers, and the second will be used in the output layer.

Loss Function

- In neural network training, a loss function/cost function/objective function is a mathematical function.
- The goal of training a neural network is to minimize this loss function, to improving the accuracy of the model's predictions.
- Different types of loss functions are used depending on the task, such as regression, or classification.
- During training, the neural network uses optimization algorithms to minimize the loss function.
- The gradients of the loss function with respect to the network's parameters are computed through backpropagation, and these gradients guide the updates to the parameters in each training iteration.

Backpropagation

Forward Pass

- Let’s start with a simplified forward pass with just one neuron.
- Let’s backpropagate the ReLU function for a single neuron.
- We’re first doing this only as a demonstration to simplify the explanation.
- We’ll start by showing how we can understand the chain rule with derivatives and partial derivatives to calculate the impact of each variable on the ReLU activated output.

Backpropagation

Forward Pass

- We'll use an example neuron with 3 inputs, which means that it also has 3 weights and a bias.

```python
x = [1.0, -2.0, 3.0]  # input values
w = [-3.0, -1.0, 2.0]  # weights
b = 1.0  # bias
```

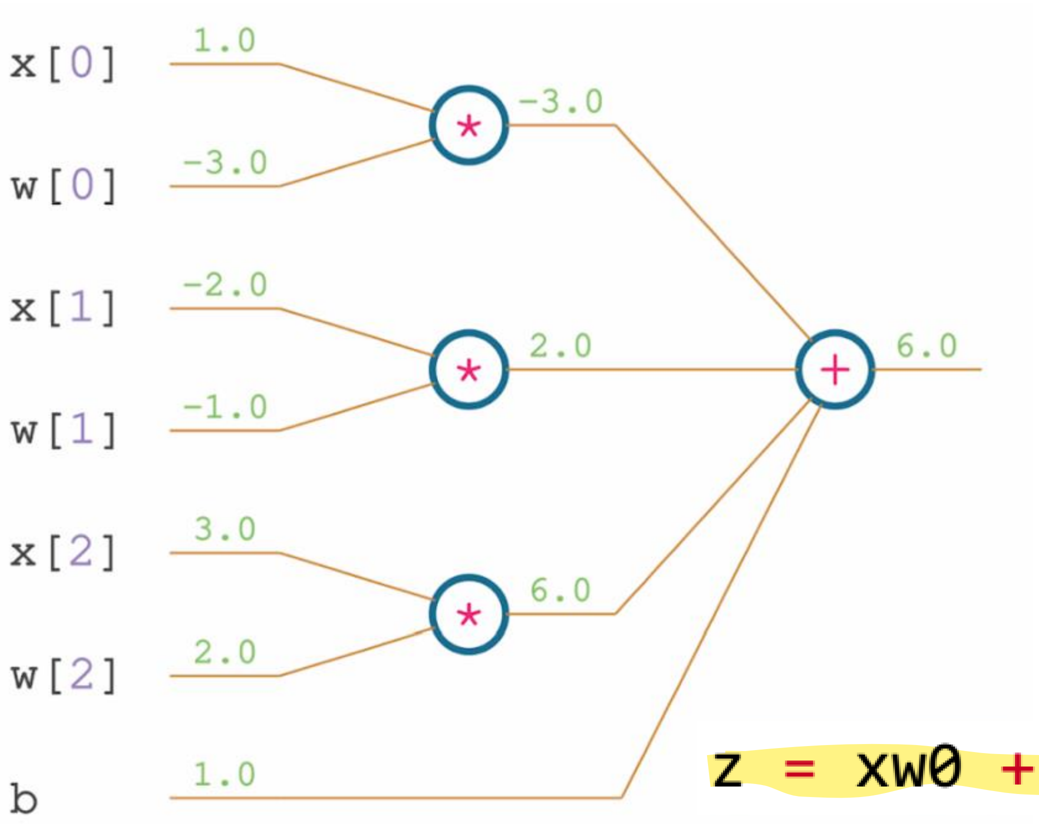
- We have to multiply the input by the weight:


```python
xw0 = x[0] * w[0]
```

Backpropagation

Forward Pass

- The next operation to perform is a sum of all weighted inputs with a bias:

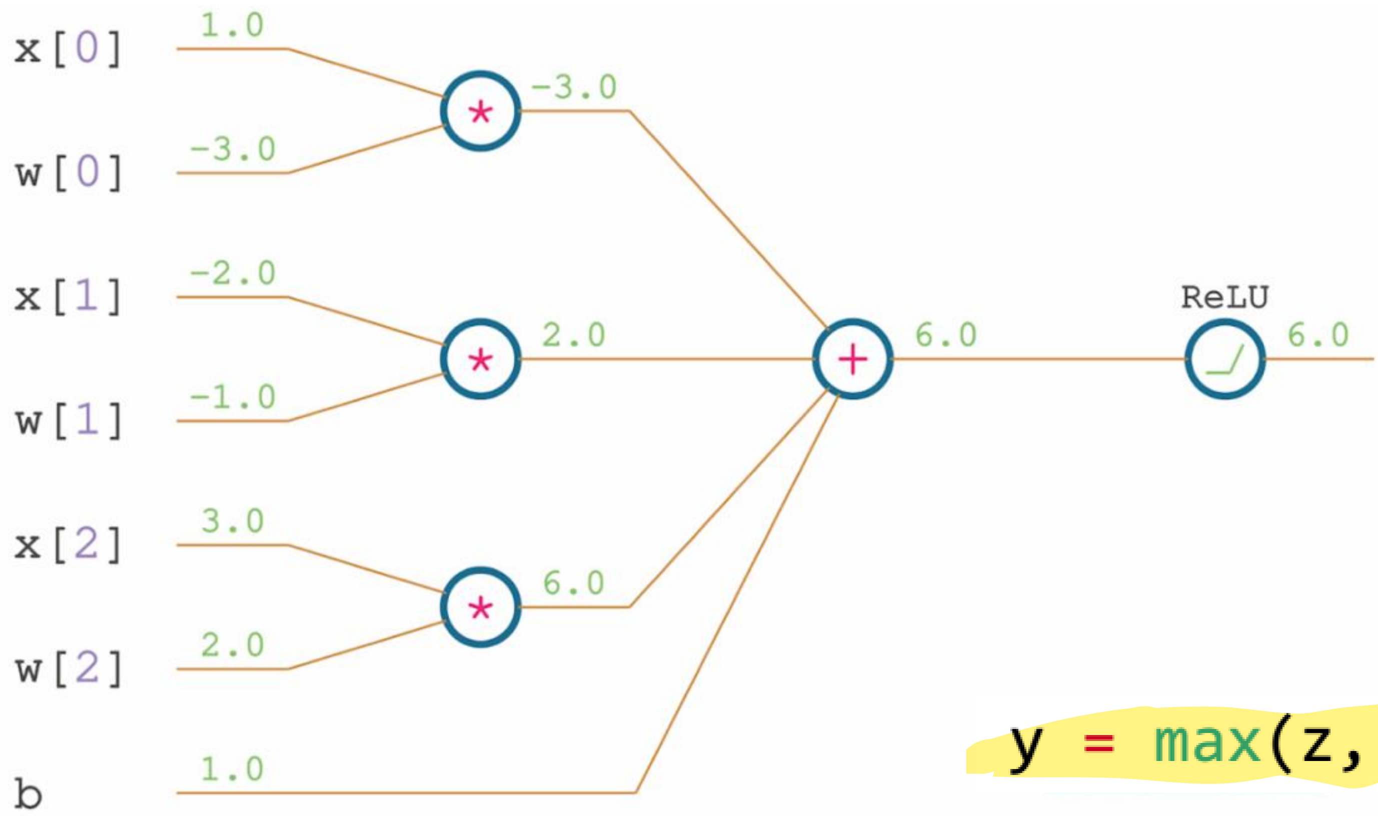


z = xw0 + xw1 + xw2 + b

Backpropagation

Forward Pass

- The last step is to apply the ReLU activation function on this output:




```
y = max(z, 0)
```

Backpropagation

The Chain Rule

- We want to know the impact of a given weight or bias on the output's loss function.
- During the backward pass, we'll calculate the derivative of the loss function.
- Then derivative of the activation function of the output layer.
- Then derivative of the output layer, and so on, through all of the hidden layers and activation functions.
- The derivative with respect to the weights and biases will form the gradients that we'll use to update the weights and biases.

Backpropagation

The Chain Rule

- Recall that the derivative of ReLU() with respect to its input is 1, if the input is greater than 0, and 0 otherwise as:

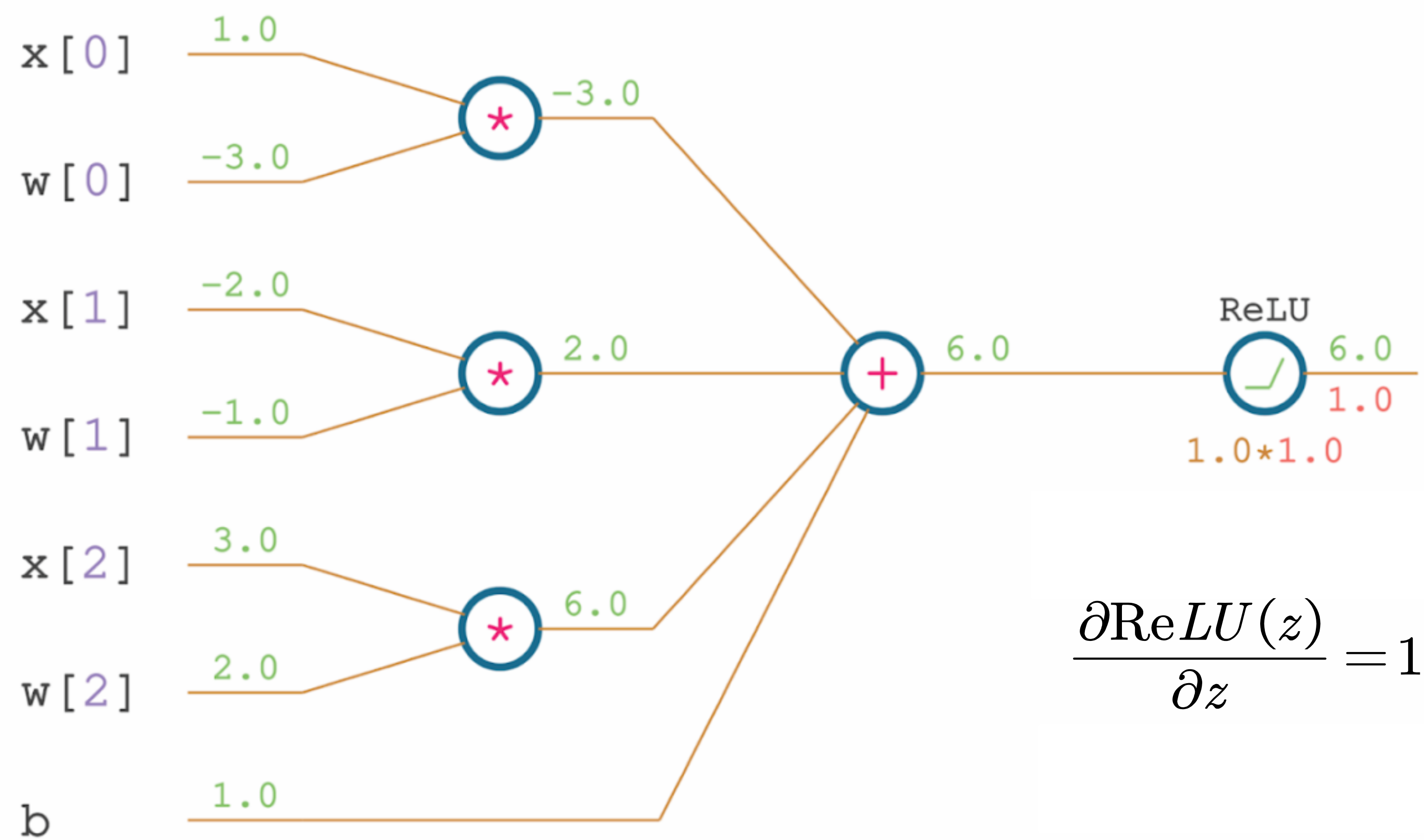
$$\frac{\partial \mathrm{Re}LU(sum(\ ))}{\partial sum(\ )} = \frac{\partial \mathrm{Re}LU(z)}{\partial z} = \frac{\partial}{\partial z}\max(z, 0) = 1(z > 0)$$

- The input value to the ReLU function is 6 , so the derivative equals 1.

Backpropagation

The Chain Rule

- Use the chain rule and multiply this derivative with the derivative received from the next layer



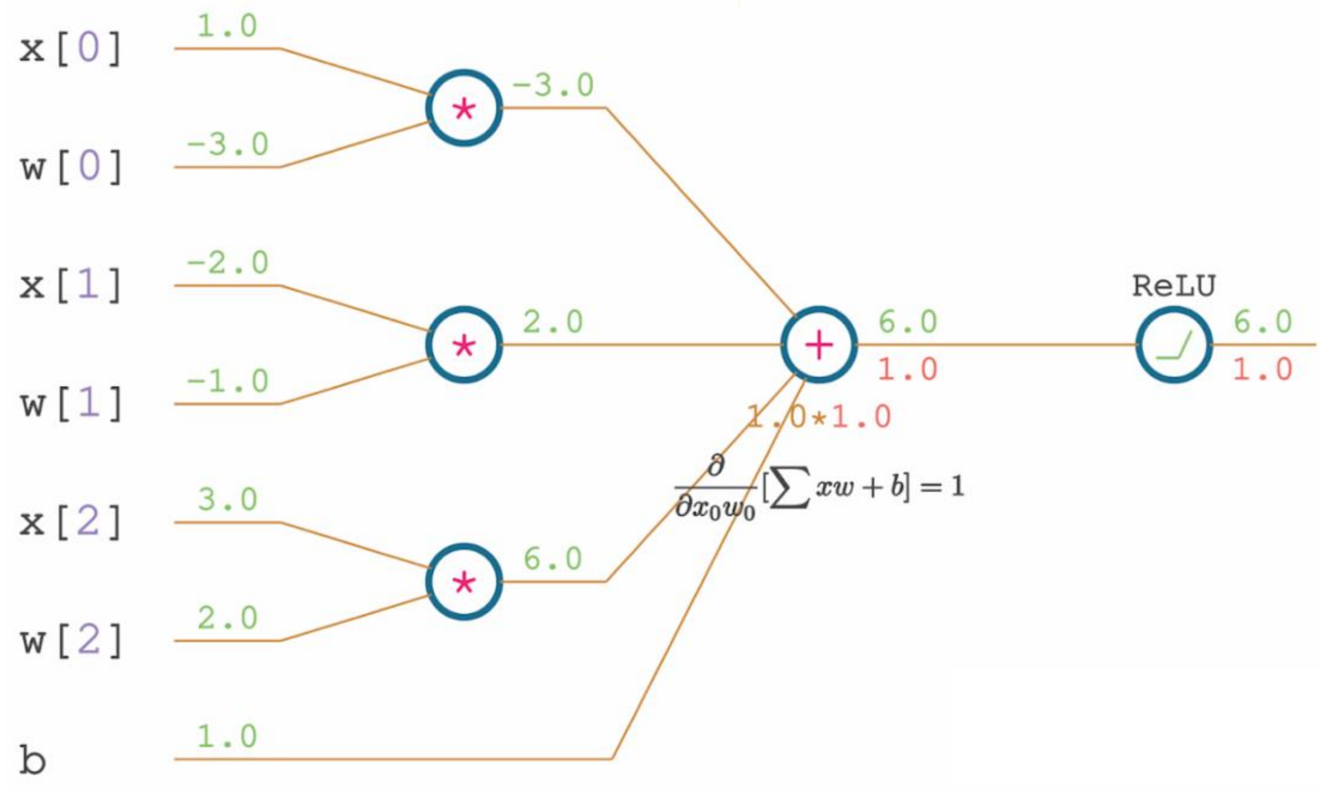
$$\frac{\partial \mathrm{Re}LU(z)}{\partial z} = 1$$

Backpropagation

The Chain Rule

- The partial derivative of the sum operation

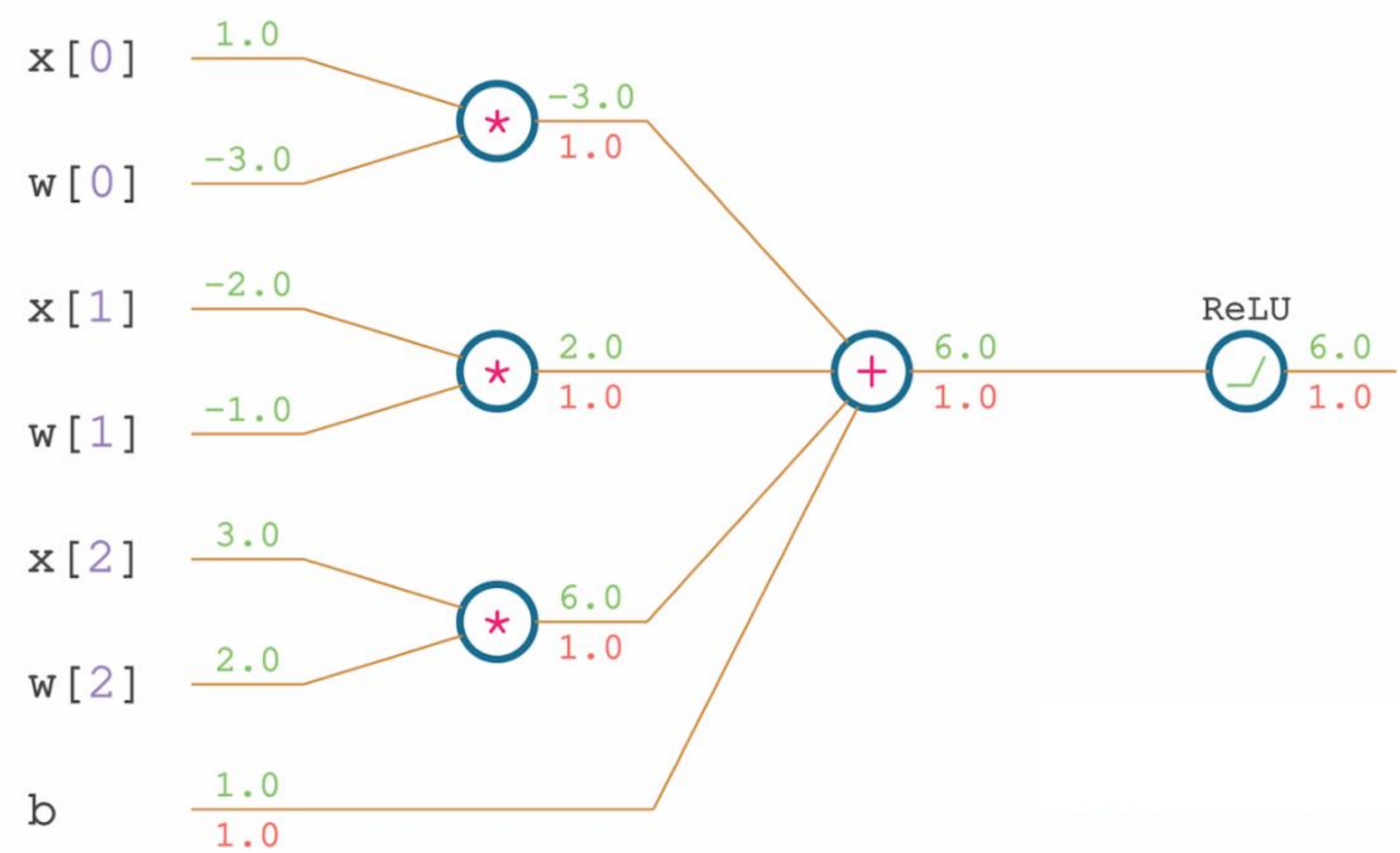
$$\frac{\partial sum(\ )}{\partial mul(x_0, w_0)} = \frac{\partial(x_0 w_0 + x_1 w_1 + x_2 w_2 + b)}{\partial x_0 w_0} = 1$$



Backpropagation

The Chain Rule

➤ The partial derivative of the sum operation for all input & weight pairs and bias



Backpropagation

The Chain Rule

- The overall gradient terms are:

$$\frac{\partial \mathrm{Re}LU(sum(\ ))}{\partial w_0} = \frac{\partial \mathrm{Re}LU(sum(\ ))}{\partial sum(\ )} \times \frac{\partial sum(\ )}{\partial mul(x_0, w_0)} \times \frac{\partial mul(x_0, w_0)}{\partial w_0} = 1 \times 1 \times x_0 = x_0$$

$$\frac{\partial \mathrm{Re}LU(sum(\ ))}{\partial w_1} = \frac{\partial \mathrm{Re}LU(sum(\ ))}{\partial sum(\ )} \times \frac{\partial sum(\ )}{\partial mul(x_1, w_1)} \times \frac{\partial mul(x_1, w_1)}{\partial w_1} = 1 \times 1 \times x_1 = x_1$$

$$\frac{\partial \mathrm{Re}LU(sum(\ ))}{\partial w_2} = \frac{\partial \mathrm{Re}LU(sum(\ ))}{\partial sum(\ )} \times \frac{\partial sum(\ )}{\partial mul(x_2, w_2)} \times \frac{\partial mul(x_2, w_2)}{\partial w_2} = 1 \times 1 \times x_2 = x_2$$

$$\frac{\partial \mathrm{Re}LU(sum(\ ))}{\partial b} = \frac{\partial \mathrm{Re}LU(sum(\ ))}{\partial sum(\ )} \times \frac{\partial sum(\ )}{\partial b} = 1 \times 1 = 1$$

Backpropagation

The Parameter Update Rule

- The i^{th} weight (w_i) is updated by the rule:

$$w_i(new) = w_i(old) - \alpha \frac{\partial L}{\partial w_i}$$

- α is known as "learning rate"

- L is known as the "loss function"

$$b_j(new) = b_j(old) - \alpha \frac{\partial L}{\partial b_j}$$

Backpropagation

The Parameter Update Rule

- For our example:

```
x = [1.0, -2.0, 3.0]  # input values
w = [-3.0, -1.0, 2.0]  # weights
b = 1.0  # bias
```

- If $\alpha = 0.01$, then

$$w_0(new) = -3.01$$

$$w_1(new) = -0.98$$

$$w_2(new) = 1.97$$

$$b(new) = 0.99$$

Backpropagation

The Parameter Update Rule

- For our example:

```
x = [1.0, -2.0, 3.0]  # input values
w = [-3.0, -1.0, 2.0]  # weights
b = 1.0  # bias
```

- The new output

$$w_0(new) = -3.01, \; w_1(new) = -0.98, \; w_2(new) = 1.97$$

$$b(new) = 0.99$$

$$out(new) = \mathrm{Re}LU(-3.01 \times 1.0 - 0.98 \times -2.0 + 1.97 \times 3.0 + 0.99)$$

$$= 5.85$$

Parameters Optimization

Weights and bias Optimization in Neural Networks

- Optimizing weights and biases in neural networks is crucial for training models effectively and achieving high performance.
- There are several optimization methods used to adjust the weights and biases during training.

- Gradient Descent (GD)
- Gradient Descent with Momentum (GDM)
- Adaptive Gradient Algorithm (AdaGrad)
- Root Mean Square Propagation (RMSProp)
- Adaptive Moment Estimation (Adam)

Parameters Optimization

Weights and bias Optimization in Neural Networks

- The choice of optimization method depends on several factors, including the dataset size, model architecture, and specific problem.
- Experimentation is key to finding the best optimizer for a particular task.

- Gradient Descent (GD)
- Gradient Descent with Momentum (GDM)
- Adaptive Gradient Algorithm (AdaGrad)
- Root Mean Square Propagation (RMSProp)
- Adaptive Moment Estimation (Adam)

Parameters Optimization

Gradient Descent (GD)

- Update Rule: The parameters are updated iteratively using the formula:

$$\theta_{t+1} = \theta_t - \alpha \nabla L(\theta_t)$$

where:

- θ_t are the parameters at iteration(epoch) t,
- α is the learning rate, a hyperparameter that controls the size of the steps taken toward the minimum,
- $\nabla L(\theta_t)$ is the gradient of the loss function with respect to θ_t .

Parameters Optimization

Gradient Descent with Momentum (GDM)

- The momentum update rules are:

$$v_{t+1} = \gamma v_t + \alpha \nabla L(\theta_t)$$

$$\theta_{t+1} = \theta_t - v_{t+1}$$

$$v_t = \alpha \sum_{i=1}^{t} \gamma^{t-i} \nabla L(\theta_{i-1})$$

where:

- v_t is the velocity vector at iteration t,
- γ (gamma) is the momentum coefficient, typically a value between 0 and 1 (e.g., 0.9),
- α (alpha) is the learning rate,
- $\nabla L(\theta_t)$ is the gradient of the loss function with respect to the parameters θ_t.

Parameters Optimization

Adaptive Gradient Algorithm (AdaGrad)

- The AdaGrad update rule for each parameter θ_i at iteration t is given by:

$$\theta_{i,t+1} = \theta_{i,t} - \frac{\alpha}{\sqrt{G_{i,t} + \epsilon}} \nabla_{\theta_i} L(\theta_t)$$

where:

- $G_{i,t}$ is the cumulative sum of the squares of the gradients for parameter θ_i up to time step t:

$$G_{i,t} = \sum_{j=0}^{t} (\nabla_{\theta_i} L(\theta_j))^2$$

- ϵ is a small constant (e.g., 10^{-8}) added to prevent division by zero.
- $\nabla_{\theta i} L(\theta_t)$ is the gradient of the loss function with respect to parameter θ_i at iteration t.

Parameters Optimization

Root Mean Square Propagation (RMSProp)

- The update rule for a parameter θ_i at iteration t is given by:

$$\theta_{i,t+1} = \theta_{i,t} - \frac{\alpha}{\sqrt{E[g_{i,t}^2] + \epsilon}} \nabla_{\theta_i} L(\theta_t)$$

where:

- $g_{i,t}=\nabla\theta_i L(\theta_t)$ is the gradient of the loss function with respect to parameter θ_i at iteration t.
- $E[g_{i,t}^2]$ is the exponentially decaying average of past squared gradients for parameter θ_i .
- The $E[g_{i,t}^2]$ is updated as:

$$E[g_{i,t}^2] = \gamma E[g_{i,t-1}^2] + (1-\gamma) g_{i,t}^2$$

- γ is the decay rate (a hyperparameter, typically set to 0.9).

Parameters Optimization

Adaptive Moment Estimation (Adam)

- **Adam** is an optimization algorithm that combines the benefits of AdaGrad and RMSProp methods.
- Adam maintains an adaptive learning rate for each parameter by computing first (mean) and second moments (variance) of the gradients.
- The first moment is an exponentially decaying average of past gradients, while the second moment is an exponentially decaying average of past squared gradients.

Parameters Optimization

Adaptive Moment Estimation (Adam)

- The update rule for a parameter θ_i at iteration t is given by:

$$\theta_{i,t+1} = \theta_{i,t} - \frac{\alpha}{\sqrt{\hat{v}_{i,t} + \epsilon}} \hat{m}_{i,t}$$

where:

- $\hat{v}_{i,t}$ is the bias-corrected first moment estimate (mean of the gradients).

- $\hat{m}_{i,t}$ is the bias-corrected second moment estimate (variance of the gradients).

Parameters Optimization

Adaptive Moment Estimation (Adam)

- First Moment (Mean) Estimate:

$$m_{i,t} = \beta_1 m_{i,t-1} + (1 - \beta_1) g_{i,t}$$

where:

- $m_{i,t}$ is the moving average of the gradient.

- β_1 is the exponential decay rate for the first moment (typically β_1=0.9).

- $g_{i,t}=\nabla\theta_i L(\theta_t)$ is the gradient of the loss function with respect to parameter θ_i at iteration t.

Parameters Optimization

Adaptive Moment Estimation (Adam)

- Second Moment (Variance) Estimate:

$$v_{i,t} = \beta_2 v_{i,t-1} + (1 - \beta_2) g_{i,t}^2$$

where:

- $v_{i,t}$ is the moving average of the squared gradient.

- β_2 is the exponential decay rate for the second moment (typically β_2=0.999).

Parameters Optimization

Adaptive Moment Estimation (Adam)

- Since $m_{i,\mathrm{t}}$ and $v_{i,\mathrm{t}}$ are initialized as zeros, they are biased toward zero, therefore, bias-corrected estimates:

$$\hat{m}_{i,t} = \frac{m_{i,t}}{1-\beta_1}, \ \hat{v}_{i,t} = \frac{v_{i,t}}{1-\beta_2}$$

Coding Parameters Optimization

Gradient Descent (GD)

- Update Rule: The parameters are updated iteratively using the formula:

$$\theta_{t+1} = \theta_t - \alpha \nabla L(\theta_t)$$

where:

- θ_t are the parameters at iteration(epoch) t,
- α is the learning rate, a hyperparameter that controls the size of the steps taken toward the minimum,
- $\nabla L(\theta_t)$ is the gradient of the loss function with respect to θ_t .

Coding Parameters Optimization

Gradient Descent (GD)

- Example: Let the function to be minimized is:

$$L(\theta) = \theta^2$$

Iteration 1:

$$\theta_{t+1} = \theta_t - \alpha \nabla L(\theta_t) \rightarrow \theta_1 = \theta_0 - \alpha \nabla L(\theta_0)$$

Let

- θ_0 = 10
- α = 0.1
- $\nabla L(\theta_0)$ = 2*θ_0 = 20.

$$\theta_1 = \theta_0 - \alpha \nabla L(\theta_0) \rightarrow \theta_1 = 10 - 0.1 * 20 = 8$$

Coding Parameters Optimization

Gradient Descent (GD)

- Example: Let the function to be minimized is:

$$L(\theta) = \theta^2$$

Iteration 2:

$$\theta_{t+1} = \theta_t - \alpha \nabla L(\theta_t) \rightarrow \theta_2 = \theta_1 - \alpha \nabla L(\theta_1)$$

Now

- $\theta_1 = 8$
- $\alpha = 0.1$
- $\nabla L(\theta_1) = 2*\theta_1 = 16.$

$$\theta_2 = \theta_1 - \alpha \nabla L(\theta_1) \rightarrow \theta_2 = 8 - 0.1*16 = 6.4$$

Coding Parameters Optimization

Gradient Descent (GD)

```
import numpy as np
import matplotlib.pyplot as plt

# Define the function f(x) = x^2
def f(x):
    return x**2

# Derivative of f(x), which is 2*x
def gradient(x):
    return 2 * x
```

Coding Parameters Optimization

Gradient Descent (GD)

```
# Gradient Descent function
def gradient_descent(starting_x, learning_rate, num_iterations):
    x = starting_x  # Initial value of x
    x_values = []   # To store the values of x during the iterations
    f_values = []   # To store the values of f(x)

    for i in range(num_iterations):
        x_values.append(x)
        f_values.append(f(x))

        grad = gradient(x)
        x = x - learning_rate * grad  # Update x using gradient

        # Print the iteration details
        print(f"Iteration {i+1}: x = {x}, f(x) = {f(x)}")

    return x_values, f_values
```

Coding Parameters Optimization

Gradient Descent (GD)

```
# Parameters for gradient descent
starting_x = 10    # Start far from the minimum
learning_rate = 0.1
num_iterations = 25

# Perform gradient descent
x_values, f_values = gradient_descent(starting_x, learning_rate, num_iterations)

x_var = np.linspace(-10,10,num_iterations)
y_var = f(x_var)
plt.plot(x_var,y_var)

# Plot the results
plt.plot(x_values,f_values, '-*', label='Optimized Parameter')
plt.plot(x_var,y_var, label = 'Loss Function')
plt.xlabel('x')
plt.ylabel('f(x)')
plt.title('Gradient Descent Optimization of f(x) = x^2')
plt.legend()
plt.show()
```

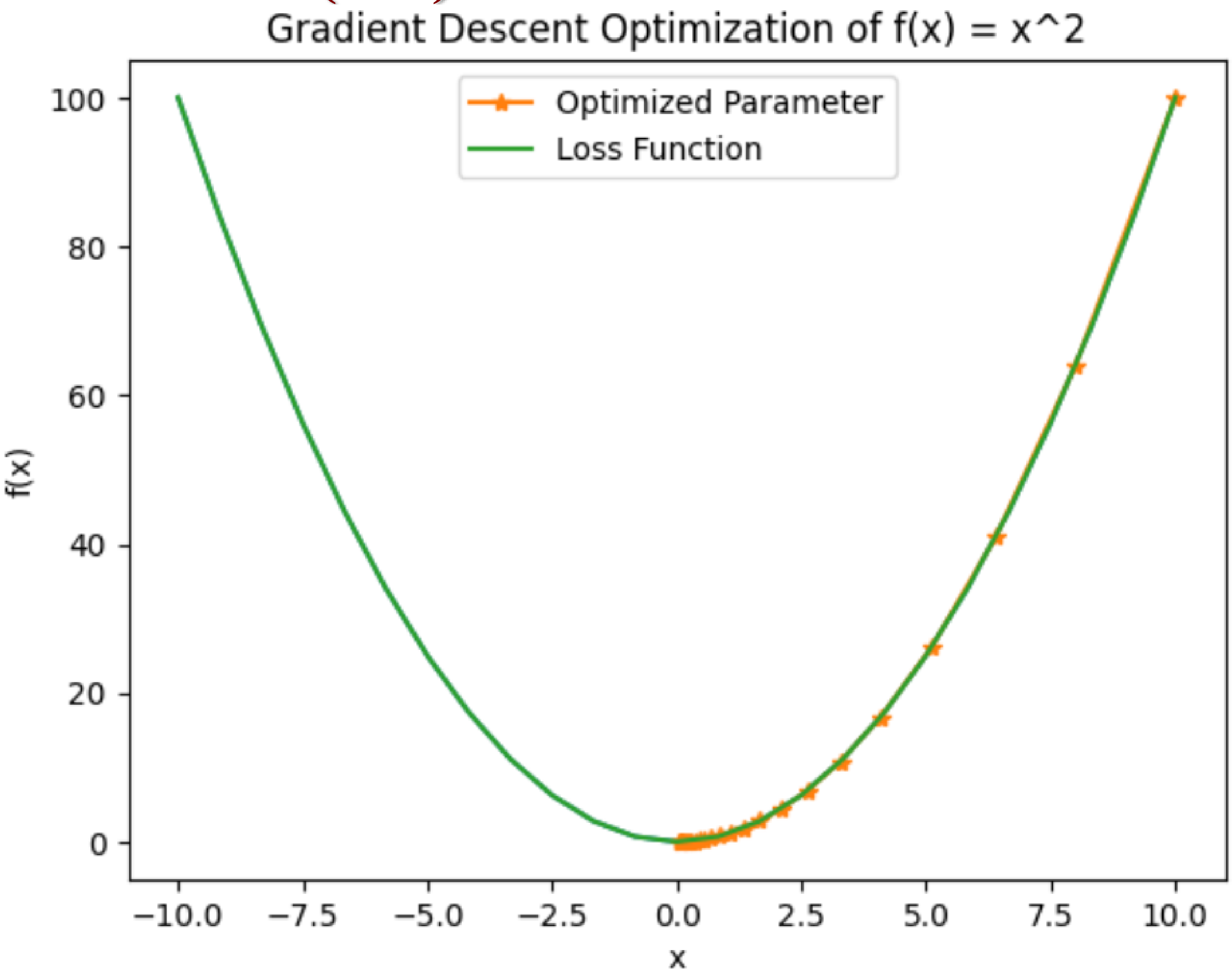
Coding Parameters Optimization

Gradient Descent (GD)

```
Iteration 1: x = 8.0, f(x) = 64.0
Iteration 2: x = 6.4, f(x) = 40.96000000000001
Iteration 3: x = 5.12, f(x) = 26.2144
Iteration 4: x = 4.096, f(x) = 16.777216
Iteration 5: x = 3.2768, f(x) = 10.73741824
Iteration 6: x = 2.62144, f(x) = 6.871947673600001
Iteration 7: x = 2.0971520000000003, f(x) = 4.398046511104002
Iteration 8: x = 1.6777216000000004, f(x) = 2.8147497671065613
Iteration 9: x = 1.3421772800000003, f(x) = 1.801439850948199
Iteration 10: x = 1.0737418240000003, f(x) = 1.1529215046068475
Iteration 11: x = 0.8589934592000003, f(x) = 0.7378697629483825
Iteration 12: x = 0.6871947673600002, f(x) = 0.4722366482869647
Iteration 13: x = 0.5497558138880001, f(x) = 0.3022314549036574
Iteration 14: x = 0.43980465111040007, f(x) = 0.19342813113834073
Iteration 15: x = 0.35184372088832006, f(x) = 0.12379400392853807
Iteration 16: x = 0.281474976710656, f(x) = 0.07922816251426434
Iteration 17: x = 0.22517998136852482, f(x) = 0.050706024009129186
Iteration 18: x = 0.18014398509481985, f(x) = 0.03245185536584268
Iteration 19: x = 0.14411518807585588, f(x) = 0.020769187434139313
Iteration 20: x = 0.11529215046068471, f(x) = 0.013292279957849162
Iteration 21: x = 0.09223372036854777, f(x) = 0.008507059173023463
Iteration 22: x = 0.07378697629483821, f(x) = 0.0054445178707350165
Iteration 23: x = 0.05902958103587057, f(x) = 0.0034844914372704
Iteration 24: x = 0.04722366482869646, f(x) = 0.0022300745198530063
Iteration 25: x = 0.03777893186295716, f(x) = 0.0014272476927059603
```

Coding Parameters Optimization

Gradient Descent (GD)



Coding Parameters Optimization

Gradient Descent with Momentum (GDM)

- The momentum update rules are:

$$v_{t+1} = \gamma v_t + \alpha \nabla L(\theta_t)$$

$$\theta_{t+1} = \theta_t - v_{t+1}$$

$$v_t = \alpha \sum_{i=1}^{t} \gamma^{t-i} \nabla L(\theta_{i-1})$$

where:

- v_t is the velocity vector at iteration t,
- γ (gamma) is the momentum coefficient, typically a value between 0 and 1 (e.g., 0.9),
- α (alpha) is the learning rate,
- $\nabla L(\theta_t)$ is the gradient of the loss function with respect to the parameters θ_t.

Coding Parameters Optimization

Gradient Descent with Momentum (GDM)

- Example: For the same function

Iteration 1

$$v_{t+1} = \gamma v_t + \alpha \nabla L(\theta_t) \rightarrow v_1 = \gamma v_0 + \alpha \nabla L(\theta_0)$$

$$\theta_{t+1} = \theta_t - v_{t+1} \rightarrow \theta_1 = \theta_0 - v_1$$

Let:

- v_0 is 0, θ_0 is 10,
- γ (gamma) is 0.9,
- α (alpha) is 0.1,
- $\nabla L(\theta_0)$ is $2^*\theta_0$.

$$v_1 = \gamma v_0 + \alpha \nabla L(\theta_0) \rightarrow v_1 = 0.9 * 0 + 0.1 * 20 = 2$$

$$\theta_1 = \theta_0 - v_1 \rightarrow \theta_1 = 10 - 2 = 8$$

Coding Parameters Optimization

Gradient Descent with Momentum (GDM)

- Example: For the same function

Iteration 2

$$v_{t+1} = \gamma v_t + \alpha \nabla L(\theta_t) \rightarrow v_2 = \gamma v_1 + \alpha \nabla L(\theta_1)$$

$$\theta_{t+1} = \theta_t - v_{t+1} \rightarrow \theta_2 = \theta_1 - v_2$$

Let:

- v_1 is 2, θ_0 is 8,
- γ (gamma) is 0.9,
- α (alpha) is 0.1,
- $\nabla L(\theta_1)$ is $2*\theta_1$.

$$v_2 = \gamma v_1 + \alpha \nabla L(\theta_1) \rightarrow v_2 = 0.9*2 + 0.1*16 = 3.4$$

$$\theta_2 = \theta_1 - v_2 \rightarrow \theta_2 = 8 - 3.4 = 4.6$$

Coding Parameters Optimization

Gradient Descent with Momentum (GDM)

```
import numpy as np
import matplotlib.pyplot as plt

# Define the function f(x) = x^2
def f(x):
    return x**2

# Derivative of f(x), which is 2*x
def gradient(x):
    return 2 * x
```

Coding Parameters Optimization

Gradient Descent with Momentum (GDM)

```python
# Gradient Descent with Momentum function
def gradient_descent_with_momentum(starting_x, learning_rate, momentum_factor, num_iterations):
    x = starting_x  # Initial value of x
    velocity = 0    # Initialize velocity to 0
    x_values = []   # To store the values of x during the iterations
    f_values = []   # To store the values of f(x)

    for i in range(num_iterations):
        x_values.append(x)
        f_values.append(f(x))

        grad = gradient(x)
        velocity = momentum_factor * velocity - learning_rate * grad  # Update velocity
        x = x + velocity  # Update x using the velocity

        # Print the iteration details
        print(f"Iteration {i+1}: x = {x}, f(x) = {f(x)}, velocity = {velocity}")

    return x_values, f_values
```

Coding Parameters Optimization

Gradient Descent with Momentum (GDM)

```
# Parameters for gradient descent with momentum
starting_x = 10    # Start far from the minimum
learning_rate = 0.1
momentum_factor = 0.9  # Momentum factor (between 0 and 1)
num_iterations = 25

# Perform gradient descent with momentum
x_values_momentum, f_values_momentum = gradient_descent_with_momentum \
                    (starting_x, learning_rate, momentum_factor, num_iterations)



x_var = np.linspace(-10,10,num_iterations)
y_var = f(x_var)
plt.plot(x_var,y_var)

# Plot the results
plt.plot(x_values_momentum,f_values_momentum, '-*', label='Optimized Parameter')
plt.plot(x_var,y_var, label = 'Loss Function')
plt.xlabel('x')
plt.ylabel('f(x)')
plt.title('Gradient Descent with Momentum for f(x) = x^2')
plt.legend()
plt.show()
```

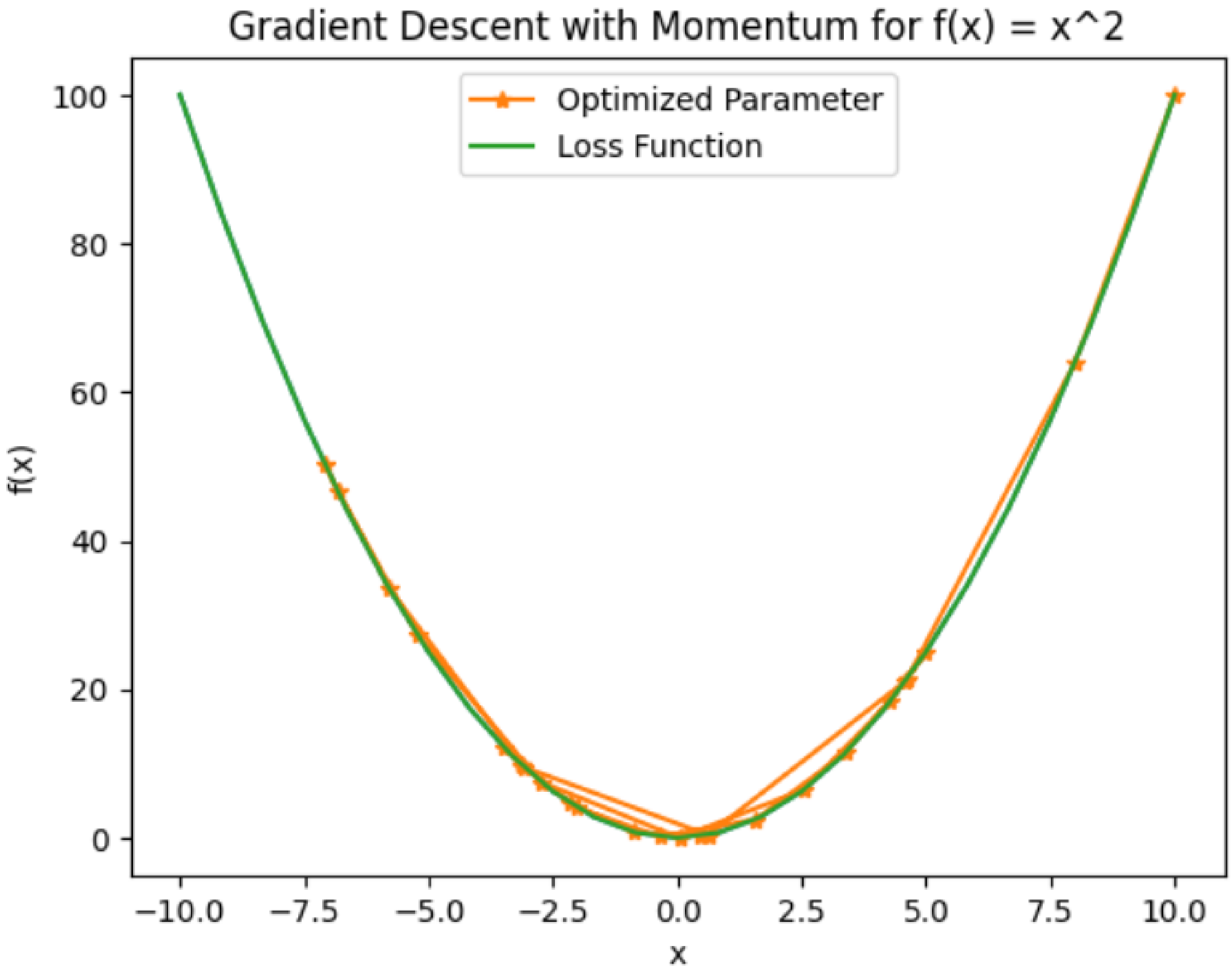
Coding Parameters Optimization

Gradient Descent with Momentum (GDM)

```
Iteration 1: x = 8.0, f(x) = 64.0, velocity = -2.0
Iteration 2: x = 4.6, f(x) = 21.159999999999997, velocity = -3.4000000000000004
Iteration 3: x = 0.6199999999999992, f(x) = 0.384399999999999, velocity = -3.9800000000000004
Iteration 4: x = -3.0860000000000007, f(x) = 9.523396000000005, velocity = -3.706
Iteration 5: x = -5.8042, f(x) = 33.68873764, velocity = -2.7181999999999995
Iteration 6: x = -7.089739999999999, f(x) = 50.264413267599984, velocity = -1.2855399999999995
Iteration 7: x = -6.828777999999999, f(x) = 46.63220897328399, velocity = 0.2609620000000036
Iteration 8: x = -5.228156599999998, f(x) = 27.333621434123543, velocity = 1.6006214000000003
Iteration 9: x = -2.7419660199999982, f(x) = 7.518377654834631, velocity = 2.48619058
Iteration 10: x = 0.04399870600000133, f(x) = 0.0019358861296745532, velocity = 2.785964725999996
Iteration 11: x = 2.542567218200008, f(x) = 6.46464805906529, velocity = 2.498568512199994
Iteration 12: x = 4.28276543554, f(x) = 18.342079775856124, velocity = 1.740198217339997
Iteration 13: x = 4.992390744037995, f(x) = 24.92396534115629, velocity = 0.709625308497998
Iteration 14: x = 4.632575372878599, f(x) = 21.460754585401293, velocity = -0.3598153711594002
Iteration 15: x = 3.382226464259419, f(x) = 11.439455855536771, velocity = -1.2503489086191801
Iteration 16: x = 1.580467153650273, f(x) = 2.4978764237673956, velocity = -1.801759310609146
Iteration 17: x = -0.3572096566280132, f(x) = 0.12759873878830308, velocity = -1.9376768102782862
Iteration 18: x = -2.029676854552868, f(x) = 4.119588133907623, velocity = -1.6724671979248549
Iteration 19: x = -3.128961961774664, f(x) = 9.790402958232752, velocity = -1.099285107221796
Iteration 20: x = -3.4925261659193474, f(x) = 12.197739019631296, velocity = -0.3635642041446836
Iteration 21: x = -3.1212287164656933, f(x) = 9.74206870049008, velocity = 0.3712974494536542
Iteration 22: x = -2.1628152686642657, f(x) = 4.67776988636728, velocity = 0.9584134478014276
Iteration 23: x = -0.8676801119101276, f(x) = 0.7528687766043716, velocity = 1.295135156754138
Iteration 24: x = 0.4714775515506222, f(x) = 0.22229108161616962, velocity = 1.3391576634607498
Iteration 25: x = 1.5824239383551726, f(x) = 2.504065520679495, velocity = 1.110946368045504
```

Coding Parameters Optimization

Gradient Descent with Momentum (GDM)



Coding Parameters Optimization

Adaptive Gradient Algorithm (AdaGrad)

- The AdaGrad update rule for each parameter θ_i at iteration t is given by:

$$\theta_{i,t+1} = \theta_{i,t} - \frac{\alpha}{\sqrt{G_{i,t}} + \epsilon} \nabla_{\theta_i} L(\theta_t)$$

where:

- $G_{i,t}$ is the cumulative sum of the squares of the gradients for parameter θ_i up to time step t:

$$G_{i,t} = \sum_{j=0}^{t} (\nabla_{\theta_i} L(\theta_j))^2$$

- ϵ is a small constant (e.g., 10^{-8}) added to prevent division by zero.
- $\nabla_{\theta i} L(\theta_t)$ is the gradient of the loss function with respect to parameter θ_i at iteration t.

Coding Parameters Optimization

Adaptive Gradient Algorithm (AdaGrad)

- For the same example: Iteration 1:

$$\theta_{i,t+1} = \theta_{i,t} - \frac{\alpha}{\sqrt{G_{i,t} + \epsilon}} \nabla_{\theta_i} L(\theta_t) \rightarrow \theta_{i,1} = \theta_{i,0} - \frac{\alpha}{\sqrt{G_{i,0} + \epsilon}} \nabla_{\theta_i} L(\theta_0)$$

- $G_{i,0}$ is:

$$G_{i,t} = \sum_{j=0}^{t} \left(\nabla_{\theta_i} L(\theta_j)\right)^2 \rightarrow G_{i,0} = 400$$

- ϵ is 10^{-8}，$\nabla_{\theta i} L(\theta_0)$ is 20, $\alpha = 1.0$.

$$\theta_{i,1} = \theta_{i,0} - \frac{\alpha}{\sqrt{G_{i,0} + \epsilon}} \nabla_{\theta_i} L(\theta_0)$$

$$\theta_{i,1} = 10 - \frac{1}{\sqrt{400 + 10^{-8}}} * 20 = 9$$

Coding Parameters Optimization

Adaptive Gradient Algorithm (AdaGrad)

- For the same example: Iteration 2:

$$\theta_{i,t+1} = \theta_{i,t} - \frac{\alpha}{\sqrt{G_{i,t}+\epsilon}} \nabla_{\theta_i} L(\theta_t) \rightarrow \theta_{i,2} = \theta_{i,1} - \frac{\alpha}{\sqrt{G_{i,1}+\epsilon}} \nabla_{\theta_i} L(\theta_1)$$

- $G_{i,1}$ is:

$$G_{i,1} = \sum_{j=0}^{1} (\nabla_{\theta_i} L(\theta_j))^2 \rightarrow G_{i,1} = 20*20 + 18*18 = 724$$

- ϵ is 10^{-8} ， $\nabla_{\theta i} L(\theta_1)$ is 18, α = 1.0.

$$\theta_{i,2} = \theta_{i,1} - \frac{\alpha}{\sqrt{G_{i,0}+\epsilon}} \nabla_{\theta_i} L(\theta_0)$$

$$\theta_{i,1} = 9 - \frac{1}{\sqrt{724}+10^{-8}} * 18 = 8.33104$$

Coding Parameters Optimization

Adaptive Gradient Algorithm (AdaGrad)

```
import numpy as np
import matplotlib.pyplot as plt

# Define the function f(x) = x^2
def f(x):
    return x**2

# Derivative of f(x), which is 2*x
def gradient(x):
    return 2 * x
```

Coding Parameters Optimization

Adaptive Gradient Algorithm (AdaGrad)

```python
# AdaGrad function
def adagrad(starting_x, learning_rate, num_iterations, epsilon=1e-8):
    x = starting_x  # Initial value of x
    grad_squared_sum = 0  # Initialize the sum of squared gradients
    x_values = []   # To store the values of x during the iterations
    f_values = []   # To store the values of f(x)

    for i in range(num_iterations):
        x_values.append(x)
        f_values.append(f(x))

        grad = gradient(x)
        grad_squared_sum += grad**2  # Accumulate the sum of squared gradients

        # Update rule for AdaGrad
        adjusted_learning_rate = learning_rate / (np.sqrt(grad_squared_sum) + epsilon)
        x = x - adjusted_learning_rate * grad  # Update x

        # Print the iteration details
        print(f"Iteration {i+1}: x = {x}, f(x) = {f(x)}, learning rate = {adjusted_learning_rate}")

    return x_values, f_values
```

Coding Parameters Optimization

Adaptive Gradient Algorithm (AdaGrad)

```
# Parameters for AdaGrad
starting_x = 10    # Start far from the minimum
learning_rate = 1  # Initial learning rate (larger than usual)
num_iterations = 25
epsilon = 1e-8  # Small value to prevent division by zero

# Perform AdaGrad
x_values_adagrad, f_values_adagrad = adagrad(starting_x, learning_rate, num_iterations, epsilon)

x_var = np.linspace(-10,10,num_iterations)
y_var = f(x_var)
plt.plot(x_var,y_var)


# Plot the results
plt.plot(x_values_adagrad,f_values_adagrad, '-*', label='Optimized Parameter')
plt.plot(x_var,y_var, label = 'Loss Function')
plt.xlabel('x')
plt.ylabel('f(x)')
plt.title('AdaGrad for f(x) = x^2')
plt.legend()
plt.show()
```

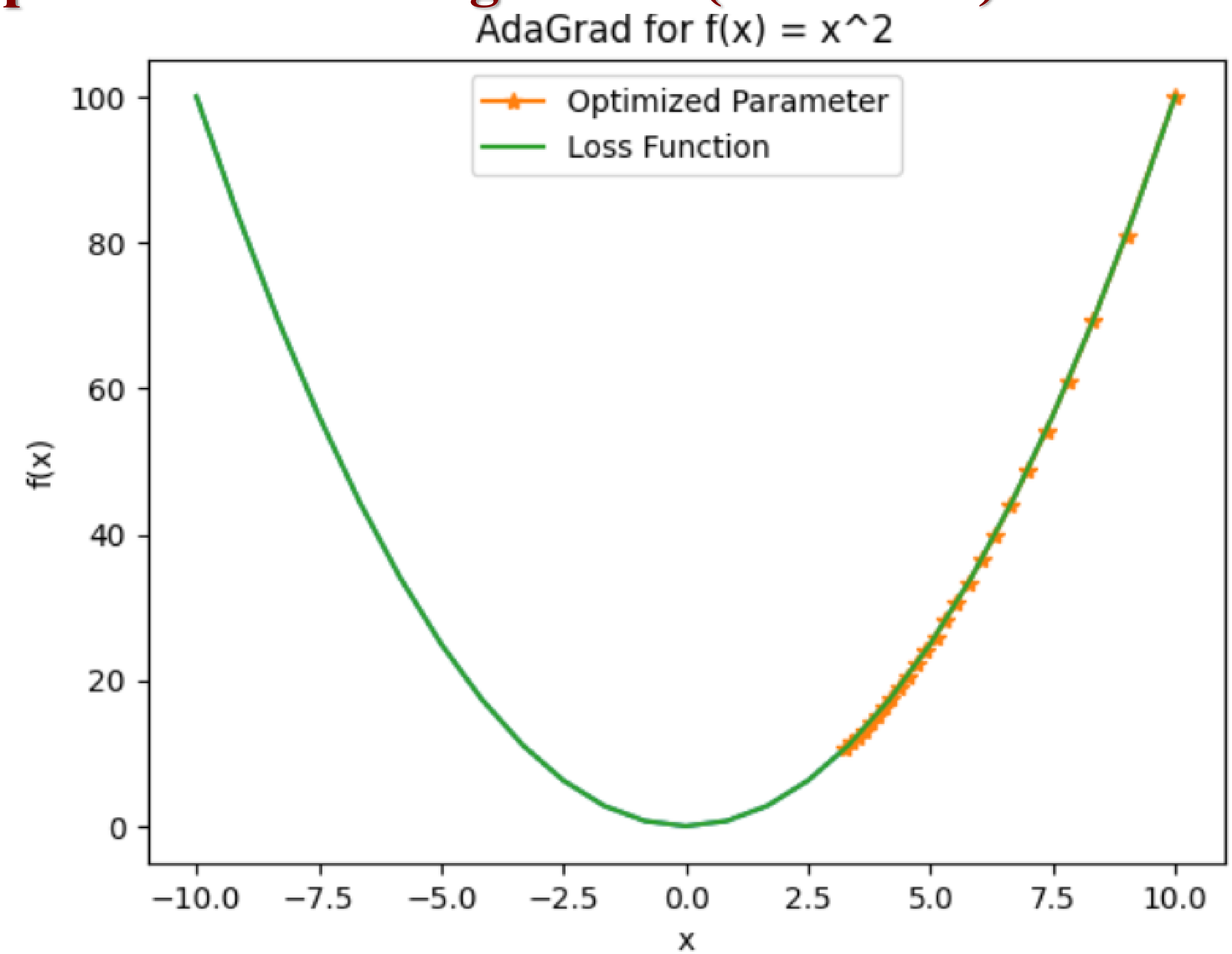
Coding Parameters Optimization

Adaptive Gradient Algorithm (AdaGrad)

```
Iteration 1: x = 9.0000000005, f(x) = 81.000000009, learning rate = 0.049999999975
Iteration 2: x = 8.331035269105636, f(x) = 69.406148655082, learning rate = 0.03716470729762175
Iteration 3: x = 7.804561814351674, f(x) = 60.911185114036286, learning rate = 0.031597120750785204
Iteration 4: x = 7.362231327484282, f(x) = 54.202450119390974, learning rate = 0.028337944998654396
Iteration 5: x = 6.977148621485869, f(x) = 48.68060288630216, learning rate = 0.026152581253515023
Iteration 6: x = 6.634323432648377, f(x) = 44.014247408987345, learning rate = 0.02456771436556293
Iteration 7: x = 6.32439447017695, f(x) = 39.99796541440478, learning rate = 0.023357993141111116
Iteration 8: x = 6.041052051430944, f(x) = 36.49430988809801, learning rate = 0.022400754734870177
Iteration 9: x = 5.779803025647223, f(x) = 33.4061230152808, learning rate = 0.021622808706129093
Iteration 10: x = 5.537312004775913, f(x) = 30.66182423823544, learning rate = 0.020977446791463652
Iteration 11: x = 5.311021038418302, f(x) = 28.206944470521815, learning rate = 0.020433286598482796
Iteration 12: x = 5.0989162104553465, f(x) = 25.99894652124431, learning rate = 0.019968366386487038
Iteration 13: x = 4.899377258503165, f(x) = 24.00389752113799, learning rate = 0.0195668004450658
Iteration 14: x = 4.711076758751052, f(x) = 22.194244226844315, learning rate = 0.019216778971787356
Iteration 15: x = 4.532910267695452, f(x) = 20.54727549497885, learning rate = 0.018909317357719394
Iteration 16: x = 4.363946542413079, f(x) = 19.04402942503907, learning rate = 0.018637444302231274
Iteration 17: x = 4.203391208154996, f(x) = 17.66849764879472, learning rate = 0.018395657771887196
Iteration 18: x = 4.050559684115051, f(x) = 16.40703375457822, learning rate = 0.018179550328724727
Iteration 19: x = 3.9048566381226113, f(x) = 15.247905364290222, learning rate = 0.017985544882086084
Iteration 20: x = 3.7657601436683796, f(x) = 14.180949459641296, learning rate = 0.017810704379803668
Iteration 21: x = 3.632809287493172, f(x) = 13.197303319296648, learning rate = 0.017652592186300917
Iteration 22: x = 3.5055943516746795, f(x) = 12.289191758493416, learning rate = 0.017509167940147702
Iteration 23: x = 3.3837489454617797, f(x) = 11.449756925913706, learning rate = 0.01737870871378658
Iteration 24: x = 3.2669436337362607, f(x) = 10.672920706009883, learning rate = 0.017259748522740728
Iteration 25: x = 3.154880728403642, f(x) = 9.953272410452696, learning rate = 0.017151031345535826
```

Coding Parameters Optimization

Adaptive Gradient Algorithm (AdaGrad)



Coding Parameters Optimization

Root Mean Square Propagation (RMSProp)

- The update rule for a parameter θ_i at iteration t is given by:
$$\theta_{i,t+1} = \theta_{i,t} - \frac{\alpha}{\sqrt{E[g_{i,t}^2] + \epsilon}} \nabla_{\theta_i} L(\theta_t)$$

where:

- $g_{i,t} = \nabla\theta_i L(\theta_t)$ is the gradient of the loss function with respect to parameter θ_i at iteration t.
- $E[g_{i,t}^2]$ is the exponentially decaying average of past squared gradients for parameter θ_i .
- The $E[g_{i,t}^2]$ is updated as:
$$E[g_{i,t}^2] = \gamma E[g_{i,t-1}^2] + (1 - \gamma) g_{i,t}^2$$
- γ is the decay rate (a hyperparameter, typically set to 0.9).

Coding Parameters Optimization

Root Mean Square Propagation (RMSProp)

- For the same example: Iteration 1

$$\theta_{i,t+1} = \theta_{i,t} - \frac{\alpha}{\sqrt{E[g_{i,t}^2] + \epsilon}} \nabla_{\theta_i} L(\theta_t) \rightarrow \theta_{i,1} = \theta_{i,0} - \frac{\alpha}{\sqrt{E[g_{i,0}^2] + \epsilon}} \nabla_{\theta_i} L(\theta_0)$$

where:

- $g_{i,0}$=$\nabla \theta_i L(\theta_0)$ is 20, γ is set to 0.9.
- The $E[g_{i,0}^2]$ is calculated as:

$$E[g_{i,t}^2] = \gamma E[g_{i,t-1}^2] + (1-\gamma) g_{i,t}^2$$

$$E[g_{i,0}^2] = 0.9 * 0.0 + (1 - 0.9) * 400 = 40$$

$$\theta_{i,1} = \theta_{i,0} - \frac{\alpha}{\sqrt{E[g_{i,0}^2] + \epsilon}} \nabla_{\theta_i} L(\theta_0)$$

$$\theta_{i,1} = 10 - \frac{1}{\sqrt{40 + 10^{-8}}} * 20 = 6.83772$$

Coding Parameters Optimization

Root Mean Square Propagation (RMSProp)

- For the same example: Iteration 2

$$\theta_{i,t+1} = \theta_{i,t} - \frac{\alpha}{\sqrt{E[g_{i,t}^2] + \epsilon}} \nabla_{\theta_i} L(\theta_t) \rightarrow \theta_{i,2} = \theta_{i,1} - \frac{\alpha}{\sqrt{E[g_{i,1}^2] + \epsilon}} \nabla_{\theta_i} L(\theta_1)$$

where:

- $g_{i,1}$=$\nabla\theta_i L(\theta_1)$ is 13.68, γ is set to 0.9.
- The $E[g_{i,1}^2]$ is calculated as:

$$E[g_{i,1}^2] = \gamma E[g_{i,0}^2] + (1-\gamma) g_{i,0}^2$$

$$E[g_{i,0}^2] = 0.9 * 40.0 + (1-0.9) * 13.68 * 13.68 = 54.7142$$

$$\theta_{i,2} = \theta_{i,1} - \frac{\alpha}{\sqrt{E[g_{i,1}^2] + \epsilon}} \nabla_{\theta_i} L(\theta_1)$$

$$\theta_{i,1} = 6.83772 - \frac{1}{\sqrt{54.71 + 10^{-8}}} * 13.68 = 4.98823$$

Coding Parameters Optimization

Root Mean Square Propagation (RMSProp)

```python
# RMSProp function
def rmsprop(starting_x, learning_rate, num_iterations, decay_rate=0.9, epsilon=1e-8):
    x = starting_x  # Initial value of x
    grad_squared_avg = 0  # Initialize the moving average of squared gradients
    x_values = []   # To store the values of x during the iterations
    f_values = []   # To store the values of f(x)

    for i in range(num_iterations):
        x_values.append(x)
        f_values.append(f(x))

        grad = gradient(x)

        # Update the moving average of squared gradients
        grad_squared_avg = decay_rate * grad_squared_avg + (1 - decay_rate) * grad**2

        # Update rule for RMSProp
        adjusted_learning_rate = learning_rate / (np.sqrt(grad_squared_avg) + epsilon)
        x = x - adjusted_learning_rate * grad  # Update x

        # Print the iteration details
        print(f"Iteration {i+1}: x = {x}, f(x) = {f(x)}, learning rate = {adjusted_learning_rate}")

    return x_values, f_values
```

Coding Parameters Optimization

Root Mean Square Propagation (RMSProp)

```
# Parameters for RMSProp
starting_x = 10    # Start far from the minimum
learning_rate = 1  # Initial learning rate
decay_rate = 0.9  # Decay rate for the moving average
num_iterations = 25
epsilon = 1e-8  # Small value to prevent division by zero

# Perform RMSProp
x_values_rmsprop, f_values_rmsprop = \
           rmsprop(starting_x, learning_rate, num_iterations, decay_rate, epsilon)

# Plot the results
plt.plot(x_values_rmsprop,f_values_rmsprop, '-*', label='Optimized Parameter')
plt.plot(x_var,y_var, label = 'Loss Function')
plt.xlabel('x')
plt.ylabel('f(x)')
plt.title('RMSProp for f(x) = x^2')
plt.legend()
plt.show()
```

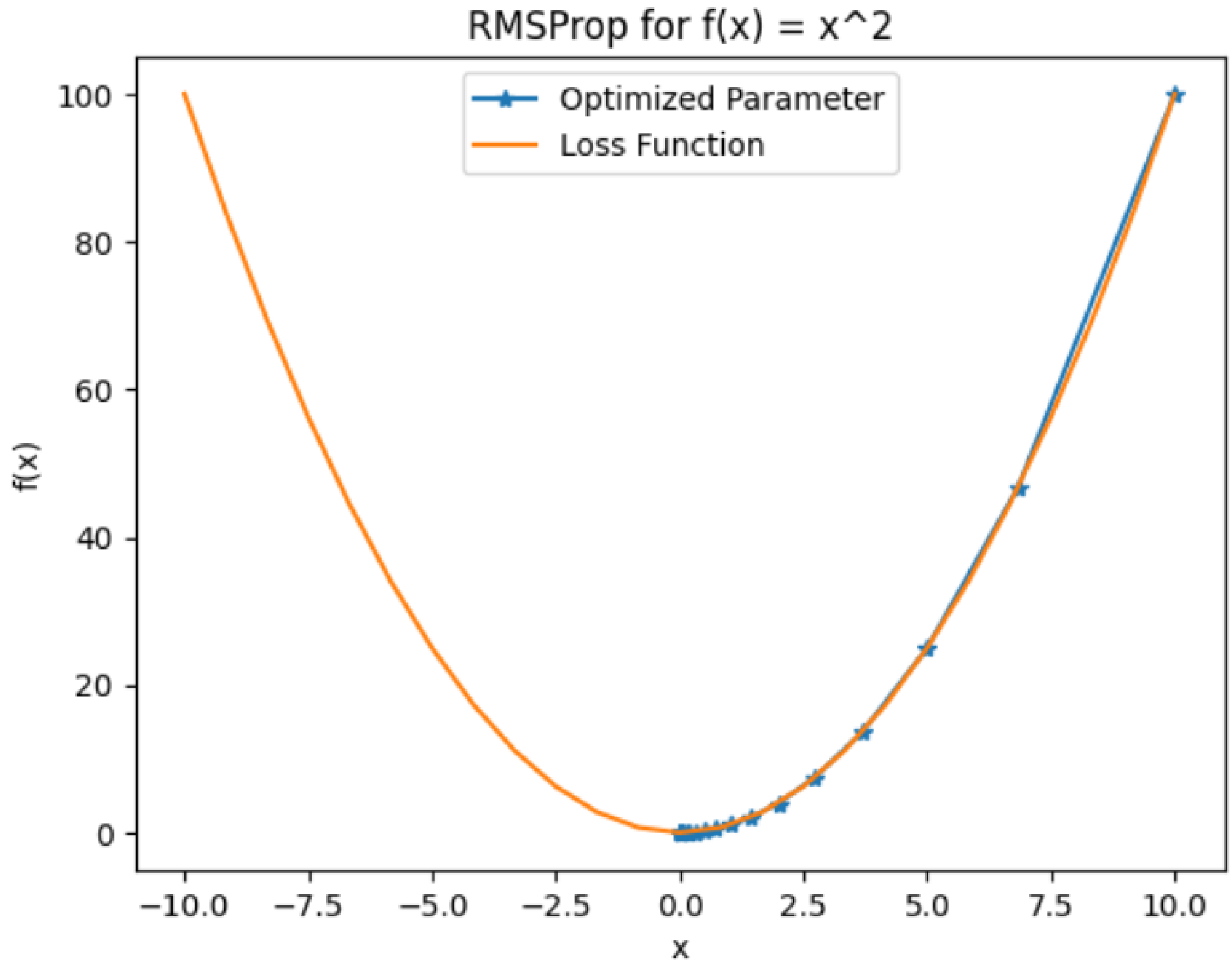
Coding Parameters Optimization

Root Mean Square Propagation (RMSProp)

```
Iteration 1: x = 6.8377223448316204, f(x) = 46.75444686500963, learning rate = 0.15811388275841898
Iteration 2: x = 4.988706075578886, f(x) = 24.88718830851769, learning rate = 0.1352070306459823
Iteration 3: x = 3.6918055386894855, f(x) = 13.629428135498362, learning rate = 0.12998365881266208
Iteration 4: x = 2.728248854368664, f(x) = 7.443341811363928, learning rate = 0.13049938224303984
Iteration 5: x = 1.9979515428511783, f(x) = 3.9918103675814036, learning rate = 0.1338399373554363
Iteration 6: x = 1.4429607379737774, f(x) = 2.0821356913338285, learning rate = 0.1388899562812721
Iteration 7: x = 1.0241749420328934, f(x) = 1.0489343118880805, learning rate = 0.1451133717362774
Iteration 8: x = 0.7123800671598245, f(x) = 0.507485360086636, learning rate = 0.1522175861157980
Iteration 9: x = 0.4843702896369644, f(x) = 0.23461457748299677, learning rate = 0.160033799395811
Iteration 10: x = 0.3211707872482439, f(x) = 0.10315067458165673, learning rate = 0.1684656407301927
Iteration 11: x = 0.20717894627728273, f(x) = 0.0429231157805652, learning rate = 0.1774629659621766
Iteration 12: x = 0.12969144164975607, f(x) = 0.016819870037192083, learning rate = 0.1870062234118602
Iteration 13: x = 0.07856808539588585, f(x) = 0.006172944042775211, learning rate = 0.1970961059710233
Iteration 14: x = 0.04592360102576324, f(x) = 0.002108977131173482, learning rate = 0.2077464673196173
Iteration 15: x = 0.02581093975880276, f(x) = 0.0006662046112365466, learning rate = 0.2189795749640769
Iteration 16: x = 0.013895417466702722, f(x) = 0.0001930826265739471, learning rate = 0.2308231006598279
Iteration 17: x = 0.007133675126461505, f(x) = 5.088932080989557e-05, learning rate = 0.2433083553064968
Iteration 18: x = 0.003474537027013623, f(x) = 1.2072407552088668e-05, learning rate = 0.2564693537749393
Iteration 19: x = 0.001595907751163574, f(x) = 2.546921550223976e-06, learning rate = 0.270342388243986
Iteration 20: x = 0.0006863492170486867, f(x) = 4.710752477433452e-07, learning rate = 0.2849658864842687
Iteration 21: x = 0.0002740174899419085, f(x) = 7.508558479406392e-08, learning rate = 0.3003804163132957
Iteration 22: x = 0.00010049385410321911, f(x) = 1.009901471251908e-08, learning rate = 0.3166287594917322
Iteration 23: x = 3.341299705846426e-05, f(x) = 1.1164283724289412e-09, learning rate = 0.3337560174369213
Iteration 24: x = 9.902961940402537e-06, f(x) = 9.806865519306119e-11, learning rate = 0.3518097325559441
Iteration 25: x = 2.558132747664291e-06, f(x) = 6.544043154672456e-12, learning rate = 0.3708400192255859
```

Coding Parameters Optimization

Root Mean Square Propagation (RMSProp)



Coding Parameters Optimization

Adaptive Moment Estimation (Adam)

- The update rule for a parameter θ_i at iteration t is given by:

$$\theta_{i,t+1} = \theta_{i,t} - \frac{\alpha}{\sqrt{\hat{v}_{i,t} + \epsilon}} \hat{m}_{i,t}$$

where:

- $\hat{v}_{i,t}$ is the bias-corrected first moment estimate (mean of the gradients).

- $\hat{m}_{i,t}$ is the bias-corrected second moment estimate (variance of the gradients).

Coding Parameters Optimization

Adaptive Moment Estimation (Adam)

- First Moment (Mean) Estimate:

$$m_{i,t} = \beta_1 m_{i,t-1} + (1 - \beta_1) g_{i,t}$$

where:

- $m_{i,\mathrm{t}}$ is the moving average of the gradient.

- β_1 is the exponential decay rate for the first moment (typically β_1=0.9).

- $g_{i,\mathrm{t}}$=$\nabla\theta_i L(\theta_t)$ is the gradient of the loss function with respect to parameter θ_i at iteration t.

Coding Parameters Optimization

Adaptive Moment Estimation (Adam)

- Second Moment (Variance) Estimate:

$$v_{i,t} = \beta_2 v_{i,t-1} + (1-\beta_2) g_{i,t}^2$$

where:

- $v_{i,t}$ is the moving average of the squared gradient.
- β_2 is the exponential decay rate for the second moment (typically β_2=0.999).
- Since $m_{i,t}$ and $v_{i,t}$ are initialized as zeros, they are biased toward zero, therefore, bias-corrected estimates:

$$\hat{m}_{i,t} = \frac{m_{i,t}}{1-\beta_1}, \ \hat{v}_{i,t} = \frac{v_{i,t}}{1-\beta_2}$$

Coding Parameters Optimization

Adaptive Moment Estimation (Adam)

- Iteration 1:

$$\theta_{i,t+1} = \theta_{i,t} - \frac{\alpha}{\sqrt{\hat{v}_{i,t} + \epsilon}} \hat{m}_{i,t} \rightarrow \theta_{i,1} = \theta_{i,0} - \frac{\alpha}{\sqrt{\hat{v}_{i,0} + \epsilon}} \hat{m}_{i,0}$$

$$\theta_{i,0} = 10,\ \alpha = 0.5, \beta_1 = 0.9,\ \beta_2 = 0.999,\ \epsilon = 10^{-8}$$

$$v_{i,-1} = 0,\ m_{i,-1} = 0,\ g_{i.0} = 20$$

$$m_{i,t} = \beta_1 m_{i,t-1} + (1 - \beta_1) g_{i,t} \rightarrow m_{i,0} = (1 - 0.9) * 20 = 2$$

$$v_{i,t} = \beta_2 v_{i,t-1} + (1 - \beta_2) g_{i,t}^2 \rightarrow v_{i,0} = (1 - 0.999) * 400 = 0.4$$

$$\hat{m}_{i,0} = \frac{m_{i,0}}{1 - \beta_1} = \frac{2}{1 - 0.9} = 20,\ \hat{v}_{i,0} = \frac{v_{i,0}}{1 - \beta_2} = \frac{0.4}{1 - 0.999} = 400$$

$$\theta_{i,1} = \theta_{i,0} - \frac{\alpha}{\sqrt{\hat{v}_{i,0} + \epsilon}} \hat{m}_{i,0} \rightarrow \theta_{i,1} = 10 - \frac{0.5}{\sqrt{400}} * 20 = 9.5$$

Coding Parameters Optimization

Adaptive Moment Estimation (Adam)

```python
# Adam function
def adam(starting_x, learning_rate, num_iterations, beta1=0.9, beta2=0.999, epsilon=1e-8):
    x = starting_x  # Initial value of x
    m = 0  # Initialize the first moment (mean of gradients)
    v = 0  # Initialize the second moment (mean of squared gradients)
    t = 0  # Time step
    x_values = []   # To store the values of x during the iterations
    f_values = []   # To store the values of f(x)
    for i in range(num_iterations):
        t += 1
        x_values.append(x)
        f_values.append(f(x))
        grad = gradient(x)
        # Update biased first moment estimate (m) and second moment estimate (v)
        m = beta1 * m + (1 - beta1) * grad
        v = beta2 * v + (1 - beta2) * grad**2
        # Compute bias-corrected first and second moment estimates
        m_hat = m / (1 - beta1**t)
        v_hat = v / (1 - beta2**t)
        # Update rule for Adam
        x = x - learning_rate * m_hat / (np.sqrt(v_hat) + epsilon)
        # Print the iteration details
        print(f"Iteration {i+1}: x = {x}, f(x) = {f(x)}, m = {m}, v = {v}")
    return x_values, f_values
```

Coding Parameters Optimization

Adaptive Moment Estimation (Adam)

```
# Parameters for Adam
starting_x = 10    # Start far from the minimum
learning_rate = 0.5  # Initial learning rate
beta1 = 0.9  # Decay rate for first moment
beta2 = 0.999  # Decay rate for second moment
num_iterations = 25
epsilon = 1e-8  # Small value to prevent division by zero

# Perform Adam optimization
x_values_adam, f_values_adam = \
    adam(starting_x, learning_rate, num_iterations, beta1, beta2, epsilon)

# Plot the results
plt.plot(x_values_adam,f_values_adam, '-*', label='Optimized Parameter')
plt.plot(x_var,y_var, label = 'Loss Function')
plt.xlabel('x')
plt.ylabel('f(x)')
plt.title('Adam Optimization for f(x) = x^2')
plt.legend()
plt.show()
```

Coding Parameters Optimization

Adaptive Moment Estimation (Adam)

```
Iteration 1: x = 9.500000000250001, f(x) = 90.25000000475002, m = 1.9999999999999996, v = 0.4000000000000036
Iteration 2: x = 9.00083242855694, f(x) = 81.01498440696223, m = 3.7000000000499993, v = 0.7606000000190007
Iteration 3: x = 8.503116957169732, f(x) = 72.30299798730745, m = 5.130166485756387, v = 1.0838993376468309
Iteration 4: x = 8.007524472326658, f(x) = 64.12044817491032, m = 6.317773228614694, v = 1.3720274302584141
Iteration 5: x = 7.514778901283166, f(x) = 56.47190193517063, m = 7.287500800218556, v = 1.6271371955277971
Iteration 6: x = 7.025658644527531, f(x) = 49.35987938942442, m = 8.061706500453333, v = 1.8513976660729519
Iteration 7: x = 6.5409974707103276, f(x) = 42.7846479118389, m = 8.660667579313506, v = 2.046985785964577
Iteration 8: x = 6.061684707194276, f(x) = 36.74402148943296, m = 9.10280031552422, v = 2.216077391825968
Iteration 9: x = 5.588664535297112, f(x) = 31.233171288087686, m = 9.404857225410653, v = 2.360837400391874
Iteration 10: x = 5.122934178536807, f(x) = 26.244454597620592, m = 9.58210440992901, v = 2.4834092481438326
Iteration 11: x = 4.665540757532327, f(x) = 21.767270560195318, m = 9.64848080464347, v = 2.585903657286171
Iteration 12: x = 4.217576580430411, f(x) = 17.787952211795076, m = 9.616740875685588, v = 2.670386835869666
Iteration 13: x = 3.780172647157486, f(x) = 14.289705242317636, m = 9.49858210420311, v = 2.7388682578809767
Iteration 14: x = 3.3544901739954085, f(x) = 11.252604327431746, m = 9.304758423214297, v = 2.793288210592366
Iteration 15: x = 2.9417099961522313, f(x) = 8.653657701461961, m = 9.04518061569195, v = 2.835505339691501
Iteration 16: x = 2.5430197831736097, f(x) = 6.466949617612353, m = 8.7290045533532, v = 2.8672844651576574
Iteration 17: x = 2.1595991060771524, f(x) = 4.663868298969236, m = 8.364708054652601, v = 2.8902849791629492
Iteration 18: x = 1.7926025236453471, f(x) = 3.2134238077796673, m = 7.960157070402771, v = 2.9060501673796635
Iteration 19: x = 1.4431410019801139, f(x) = 2.082655951596167, m = 7.522661868091564, v = 2.9159978124434023
Iteration 20: x = 1.112262135376103, f(x) = 1.2371270577914086, m = 7.0590238816784305, v = 2.9214124384373434
Iteration 21: x = 0.8009297829721906, f(x) = 0.6414885172518804, m = 6.575573920585808, v = 2.923439534230072
Iteration 22: x = 0.5100038570394293, f(x) = 0.26010393419509464, m = 6.078202485121665, v = 2.9230820487648494
Iteration 23: x = 0.24022107746654114, f(x) = 0.057706166059185965, m = 5.5723830080173835, v = 2.921199382452865
Iteration 24: x = -0.007822471879864429, f(x) = 6.119106631126973e-05, m = 5.063188922708954, v = 2.9185090077346487
Iteration 25: x = -0.2336861946874083, f(x) = 0.0546092375874813, m = 4.555305536062085, v = 2.915590743491179
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Coding Parameters Optimization

Adaptive Moment Estimation (Adam)