MANIPAL UNIVERSITY JAIPUR

(University under Section 2(f) of the UGC Act)

B.TECH SECOND YEAR

ACADEMIC YEAR: 2022-2023

COURSE NAME: ENGINEERING MATHEMATICS-III

COURSE CODE : MA 2101

LECTURE SERIES NO :

CREDITS : 3

MODE OF DELIVERY : ONLINE (POWER POINT PRESENTATION)

FACULTY : DR. BHOOPENDRA PACHAURI

EMAIL-ID : Bhoopendra.pachauri@jaipur.manipal.edu

PROPOSED DATE OF DELIVERY:

MANIPAL UNIVERSITY JAIPUR

VISION

Global Leadership in Higher Education and Human Development

MISSION

- Be the most preferred University for innovative and interdisciplinary learning
- Foster academic, research and professional excellence in all domains
- Transform young minds into competent professionals with good human values

VALUES

Integrity, Transparency, Quality,
Team Work, Execution with Passion, Humane Touch

SESSION OUTCOME

"APPLICATION OF GRAPH THEORY USING TWO IMPORTANT GRAPHS "

ASSESSMENT CRITERIA'S

ASSIGNMENT

QUIZ

MID TERM EXAMINATION –I & II

END TERM EXAMINATION

PROGRAM OUTCOMES MAPPING WITH CO2

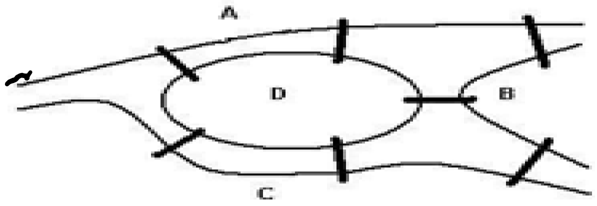
[PO1]

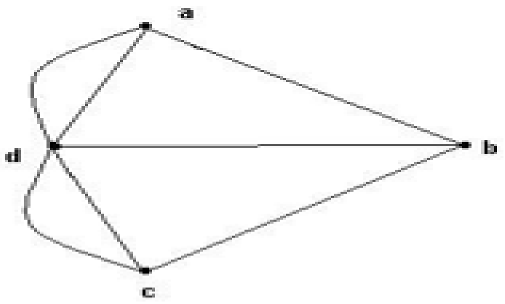
ENGINEERING KNOWLEDGE: APPLY THE KNOWLEDGE OF MATHEMATICS, SCIENCE, ENGINEERING FUNDAMENTALS, AND AN ENGINEERING SPECIALIZATION TO THE SOLUTION OF COMPLEX ENGINEERING PROBLEMS.

Euler cycles

- An *Euler cycle* in a graph G is a simple cycle that passes through every edge of G only once.
- The Konigsberg bridge problem:

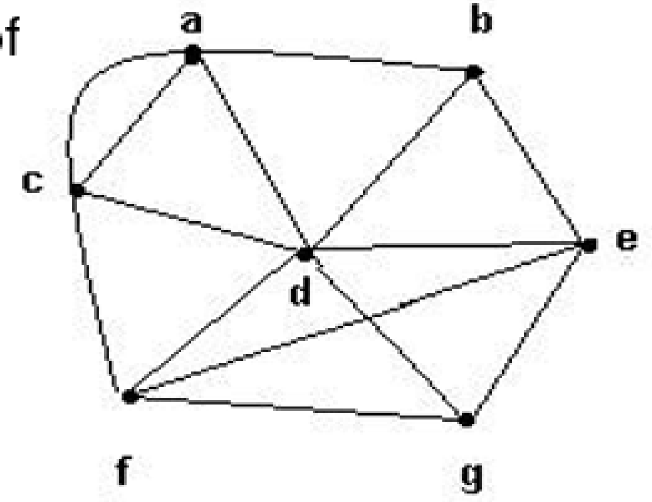
  Starting and ending at the same point, is it possible to cross all seven bridges just once and return to the starting point?
  - This problem can be represented by a graph
- Edges represent bridges and each vertex represents a region.





Degree of a vertex

- The *degree* of a vertex v, denoted by δ(v), is the number of edges incident on v
- Example:
  - δ(a) = 4, δ(b) = 3,
  - δ(c) = 4, δ(d) = 6,
  - δ(e) = 4, δ(f) = 4,
  - δ(g) = 3.



Sum of the degrees of a graph

Theorem : If G is a graph with m edges and n vertices $v_1, v_2, \ldots, v_n$, then

$$\sum_{i=1}^{n} \delta(v_i) = 2m$$

In particular, the sum of the degrees of all the vertices of a graph is even.

Euler's formula

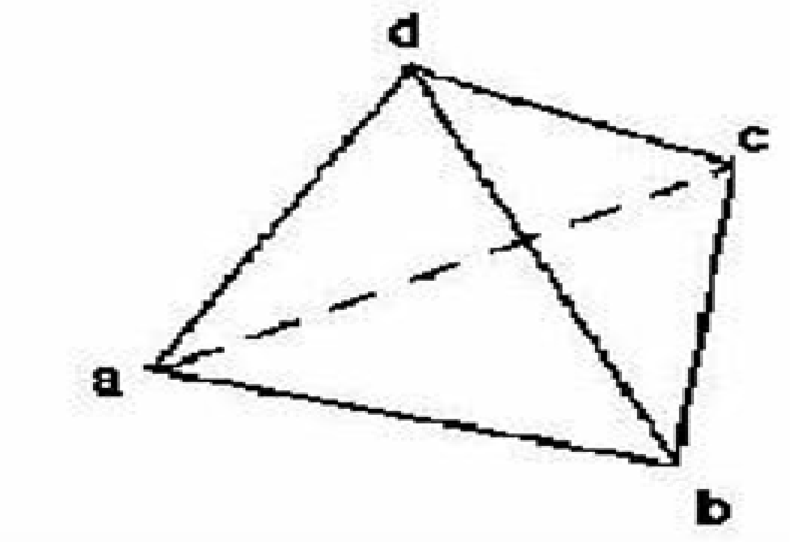
If G is *planar* graph,

v = number of vertices

e = number of edges

f = number of faces, including the exterior face

Then: $v - e + f = 2$

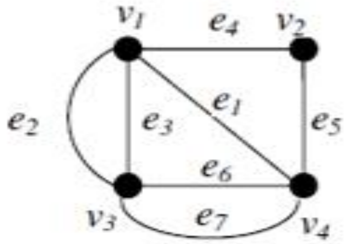


Euler Graph

A path in a graph G is called Euler path if it includes every edges exactly once. Since the path

contains every edge exactly once, it is also called Euler trail / Euler line.

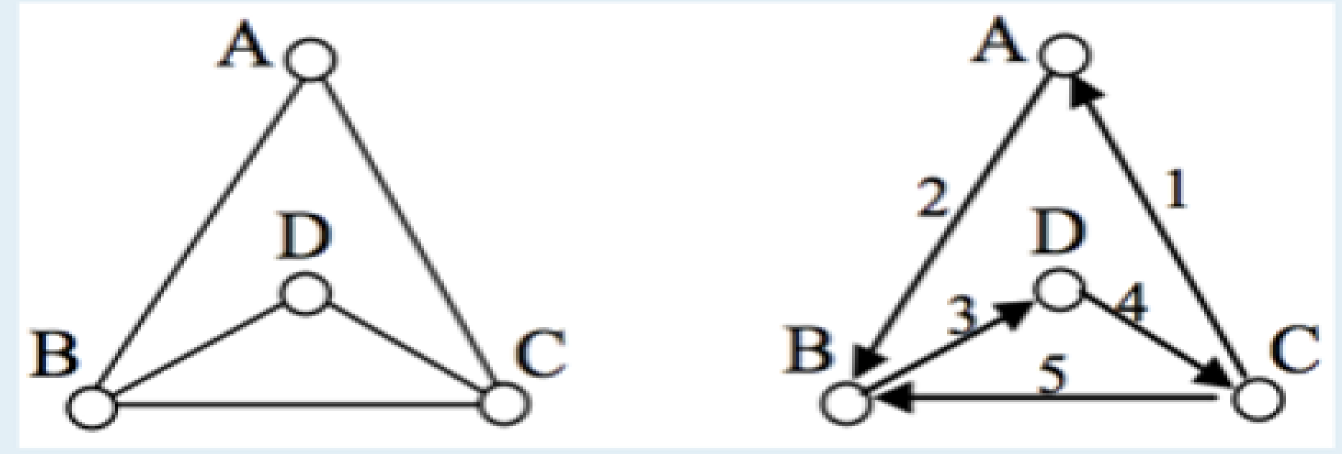
A closed Euler path is called Euler circuit. A graph which contains an Eulerian circuit is called an Eulerian graph.



v_4 e_1 v_1 e_2 v_3 e_3 v_1 e_4 v_2 e_5 v_4 e_6 v_3 e_7 v_4 is an Euler circuit. So the above graph is Euler graph.

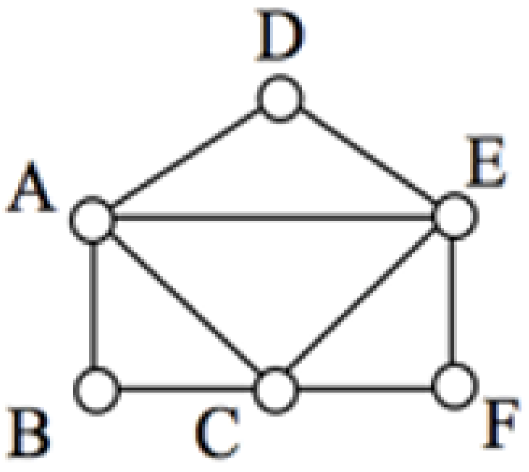
Example:

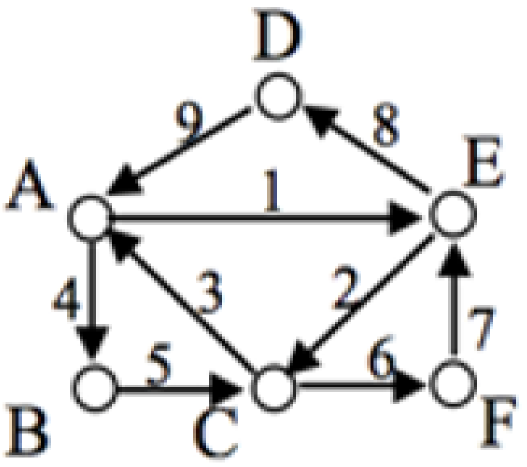
In the graph shown below, there are several Euler paths.
One such path is CABDCB.
The path is shown in arrows to the right,
with the order of edges numbered.



Example:

The graph below has several possible Euler circuits.
Here's a couple, starting and ending at vertex A: ADEACEFCBA and AECABCFEDA.
The second is shown in arrows.

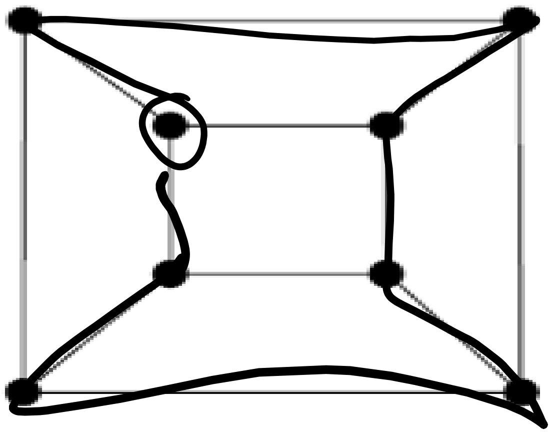


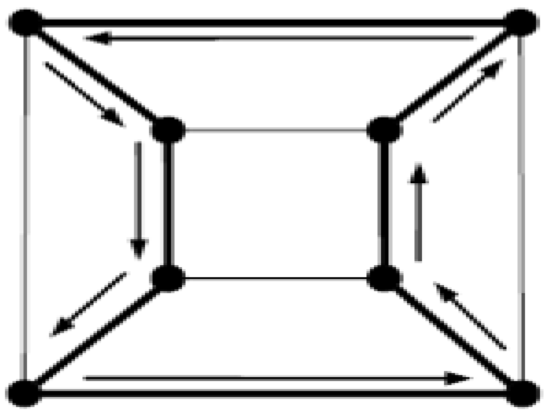


Hamiltonian Path and Circuit

A **Hamiltonian circuit** in a connected graph is defined as a closed walk that traverses every vertex of graph G exactly once except starting and terminal vertex.

Removal of any one edge from a Hamiltonian circuit generates a path. This path is called **Hamiltonian path**.





Hamiltonian Graph

A Hamiltonian graph may be defined as-

If there exists a closed walk in the connected graph that visits every vertex of the graph exactly once (except starting vertex) without repeating the edges,

then such a graph is called as a Hamiltonian graph.

OR

Any connected graph that contains a Hamiltonian circuit is called as a Hamiltonian Graph.

Example

Which of the following is / are Hamiltonian graphs?

A)

The graph neither contains a Hamiltonian path nor it contains a Hamiltonian circuit.

Since graph does not contain a Hamiltonian circuit, therefore **It is not a Hamiltonian Graph**.

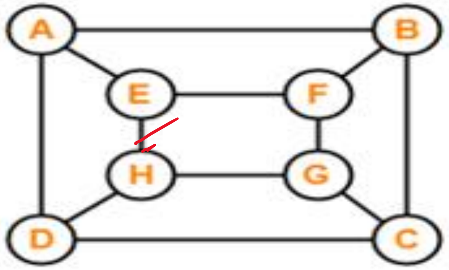
B)

The graph neither contains a Hamiltonian path nor it contains a Hamiltonian circuit.

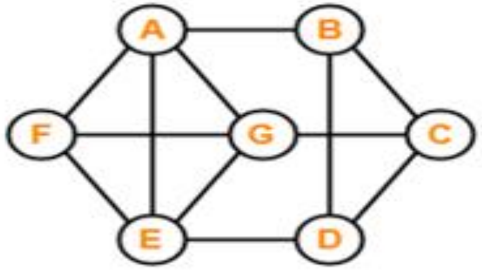
Since graph does not contain a Hamiltonian circuit, therefore **It is not a Hamiltonian Graph**.

Example

C)



D)



C)

The graph contains both a Hamiltonian path (ABCDHGFE) and a Hamiltonian circuit (ABCDHGFEA).

Since graph contains a Hamiltonian circuit, therefore **It is a Hamiltonian Graph**.

D)

The graph contains both a Hamiltonian path (ABCDEFG) and a Hamiltonian circuit (ABCDEFGA).

Since graph contains a Hamiltonian circuit, therefore **It is a Hamiltonian Graph**.