ME 543 Assignment 3

Computational Fluid Dynamics: Flow Inside Lid Driven Cavity

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GUWAHATI, 2022

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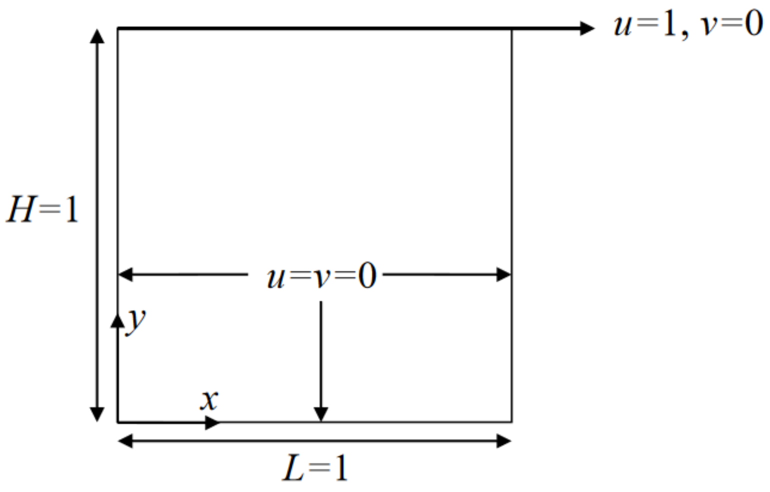
1 Introduction

This assignment aims to solve flow inside a lid driven cavity using finite difference discretization. The figure below describes the problem and its boundary conditions.



$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \tag{1}$$

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) \tag{2}$$

$$u = \frac{\partial \psi}{\partial y} \tag{3}$$

$$v = -\frac{\partial \psi}{\partial x} \tag{4}$$

2 Method

The flow is defined by two main quantities - vorticity and streamfunction. The governing equations for these two quantities are are discretized to arrive at the following algebraic equation for vorticity.

$$\begin{aligned}\omega_{i,j} = \frac{1}{2(1+\beta^2)}\Bigg[&\left(1 - (\psi_{i,j+1} - \psi_{i,j-1})\frac{\beta}{4\gamma}\right)\omega_{i+1,j} \\ &+ \left(1 + (\psi_{i,j+1} - \psi_{i,j-1})\frac{\beta}{4\gamma}\right)\omega_{i-1,j} \\ &+ \left(1 + (\psi_{i+1,j} - \psi_{i-1,j})\frac{\beta}{4\gamma}\right)\omega_{i,j+1} \\ &+ \left(1 - (\psi_{i+1,j} - \psi_{i-1,j})\frac{\beta}{4\gamma}\right)\omega_{i,j-1}\Bigg]\end{aligned}$$

The discretized governing equation for streamfunction is written as follows,

$$\psi_{i,j} = \frac{1}{2(1+\beta^2)}\left[\psi_{i+1,j} + \psi_{i-1,j} + \beta^2(\psi_{i,j+1} + \psi_{i,j-1}) + (\Delta x)^2\omega_{i,j}\right]$$

The boundary conditions for stream function and velocity are defined as,

$$\begin{aligned}
\psi_{0,j} &= 0 \\
\psi_{m-1,j} &= 0 \\
\psi_{i,0} &= 0 \\
\psi_{i,n-1} &= 0 \\
u_{0,j} &= 0 \\
u_{m-1,j} &= 0 \\
u_{i,0} &= 0 \\
u_{i,n-1} &= 1 \\
v_{0,j} &= 0 \\
v_{m-1,j} &= 0 \\
v_{i,0} &= 0 \\
v_{i,n-1} &= 0
\end{aligned}$$

Finally, the boundary conditions for vorticity can be written as,

$$\begin{aligned}
\omega_{0,j} &= 0 \\
\omega_{m-1,j} &= 0 \\
\omega_{i,0} &= 0 \\
\omega_{i,n-1} &= -\frac{2u_{i,n-1}}{dy}
\end{aligned}$$

3 Procedure

A square grid of m x n (100 x 100) size is defined to solve the problem for two cases - Re=100 and Re=400. The boundary conditions defined in the previous section are applied and the discretized equations are used to simultaneously solve for both vorticity and streamfunction. Convergence of solution is achieved based on error. When error goes below 10^{-6} for both streamfunction and vorticity, iterations are halted. Error is calculated as,

$$\epsilon_\psi = \sqrt{\frac{\sum_{i,j}(\psi^{k+1} - \psi^k)^2}{(M-2)\times(N-2)}} \tag{5}$$

$$\epsilon_\omega = \sqrt{\frac{\sum_{i,j}(\omega^{k+1} - \omega^k)^2}{(M-2)\times(N-2)}} \tag{6}$$

where M and N are the number of horizontal and vertical grid points. Finally, known values of ψ is used to calculate the horizontal and vertical velocity at the grid points.

4 Results and Discussion

4.1 Streamfunction

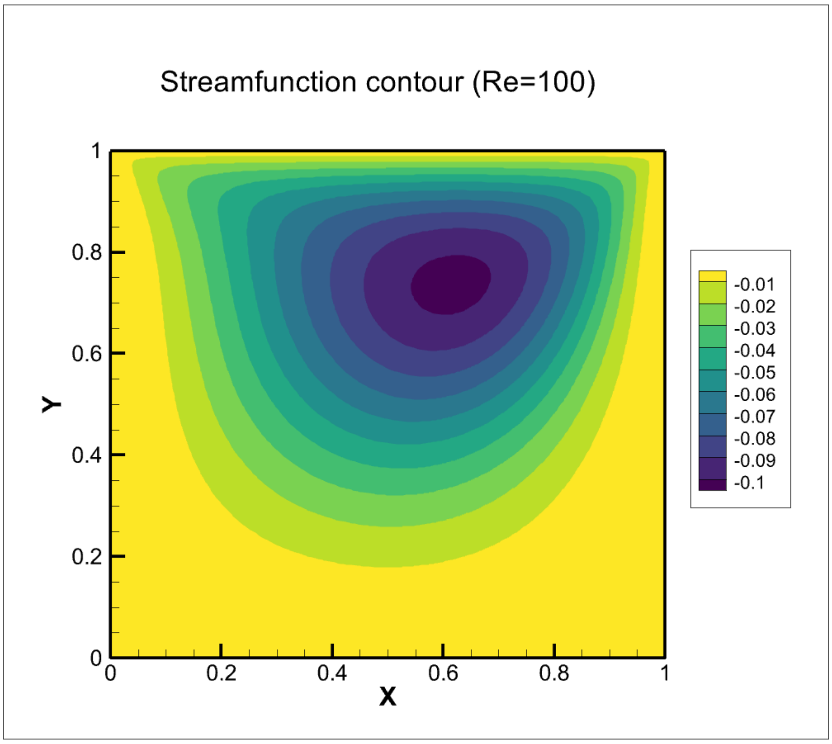


Figure 1: Streamfunction contour for Re=100

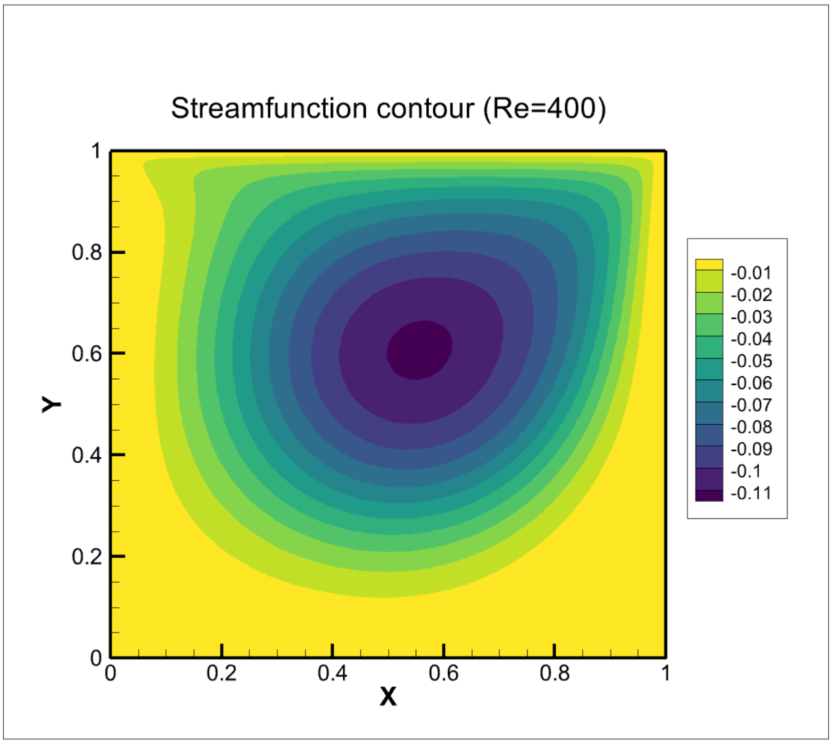


Figure 2: Streamfunction contour for Re=400

From the contour plots for both cases of Re, it is observed that the flow induced due to the lid is transmitted farther from the top boundary for Re=400 case.

4.2 Velocity Vectors

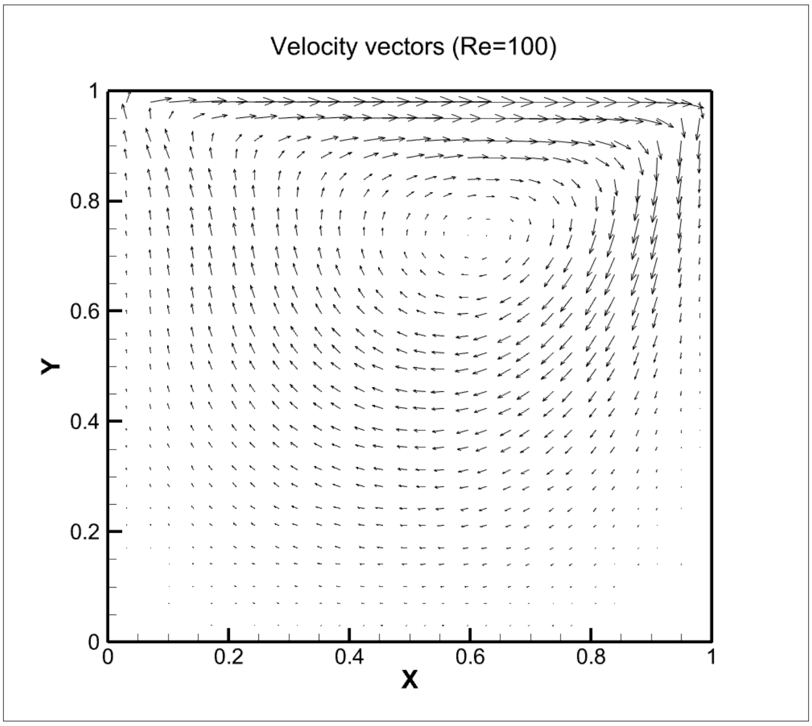


Figure 3: Velocity vectors inside cavity for Re=100

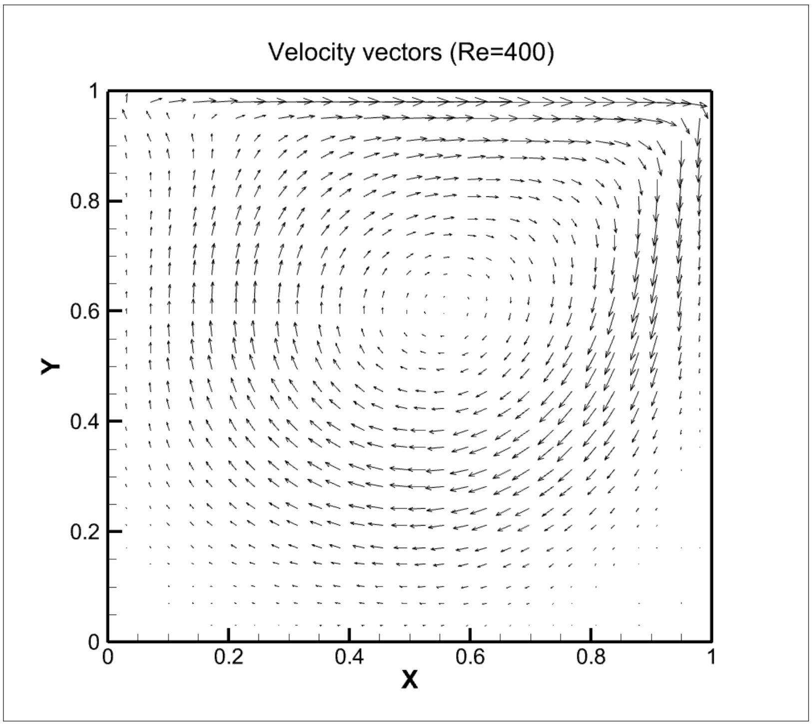


Figure 4: Velocity vectors inside cavity for Re=400

Velocity vectors are plotted above for both Re=100 and Re=400 cases. An interesting observation is the position of the center of the vortex flow, which is farther from the lid for Re=400 case when compared to Re=100.

4.3 Horizontal Flow Velocity Along Centerline

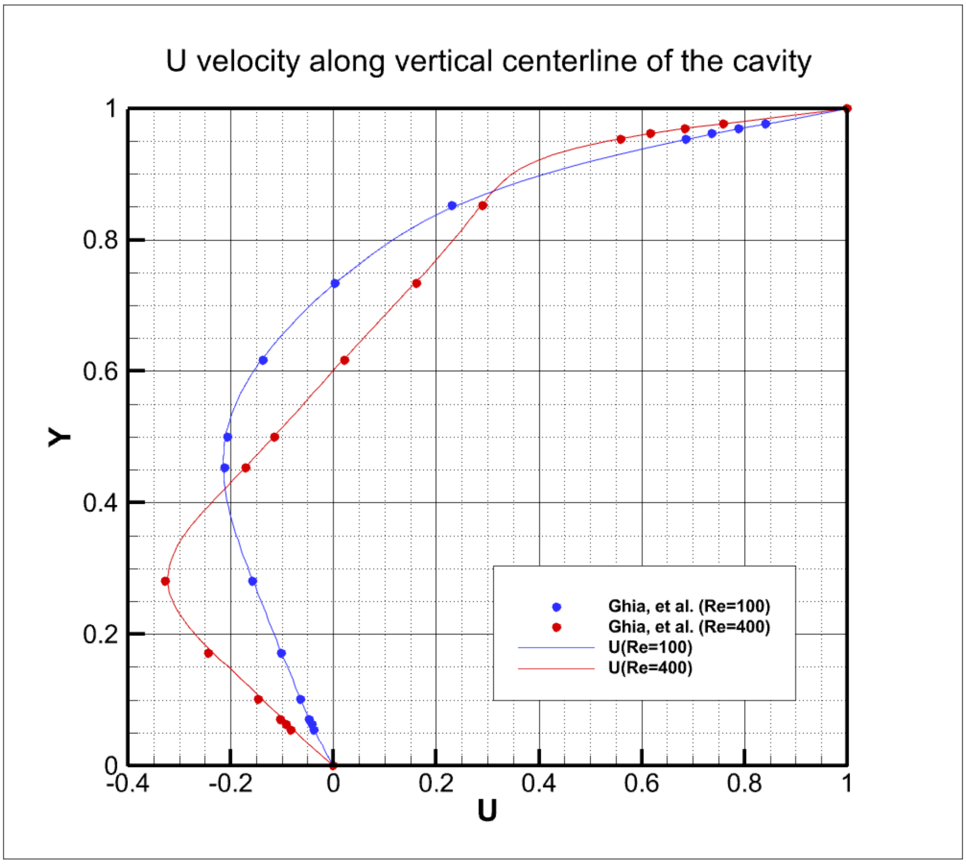


Figure 5: Horizontal velocity profile along centerline of cavity

In the plot above, horizontal flow velocity along the centerline of the cavity is plotted vs y coordinate. The resultant curve is compared to values from Ghia, et al. for both Re=100 and Re=400 cases. It can be observed that the computational results conform well to the literature data points.

4.4 Vertical Flow Velocity Along Centerline

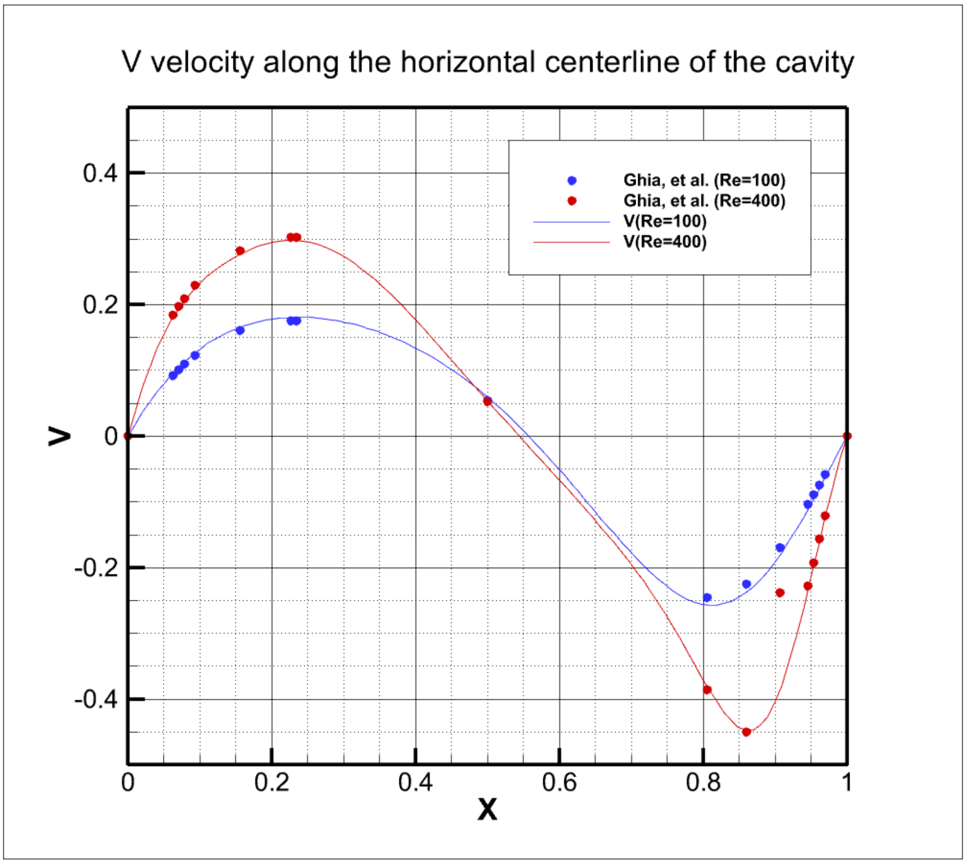


Figure 6: Vertical velocity profile along centerline of cavity

In the plot above, vertical flow velocity along the centerline of the cavity is plotted vs x coordinate. The resultant curve is compared to values from Ghia, et al. for both Re=100 and Re=400 cases. It can be observed that the computational results conform well to the literature data points.