Midterm
Name :

Note that some of this has been cut and pasted from web resources — so there may be arbitrary numbering, please ignore. Also paper will be marked out of 100 pts, 20 pts are bonus

Q1. Show that parallel lines converge at a point in perspective projection (20 pts)

3D lines. Lines in 3D are less elegant than either lines in 2D or planes in 3D. One possible representation is to use two points on the line, $(\boldsymbol{p}, \boldsymbol{q})$. Any other point on the line can be expressed as a linear combination of these two points

$$\boldsymbol{r} = (1 - \lambda)\boldsymbol{p} + \lambda\boldsymbol{q}, \tag{2.9}$$

as shown in Figure 2.3. If we restrict $0 \leq \lambda \leq 1$, we get the *line segment* joining $\boldsymbol{p}$ and $\boldsymbol{q}$.

Use this to show two lines r1 and r2 meet at a point, and find point

Q2. “We found that zebras and horses received a similar number of approaches from horseflies, probably attracted by their smell—but zebras experienced far fewer landings. Around horses, flies hover, spiral and turn before touching down again and again. In contrast, around zebras flies either flew right past them or made a single quick landing and flew off again.” (20 pts)

A. In the context of this course, can you propose an explanation for these observations? (10)

B. What properties, parameters, and measurements of zebras, horses, horse flies and tse-tse flies would be helpful in supporting your proposed answer? (5)

C. How can we test your proposed explanation — and eliminate alternatives? (5)

Q3. Cameras have mechanisms to adapt to the illumination in the environment so that neutral (white or gray) objects look neutral. (20 pts)

Incorrect white balance

Correct white balance

Before encoding the sensed RGB values, most cameras perform some kind of *color balancing* operation in an attempt to move the white point of a given image closer to pure white (equal RGB values). A common (in-camera or post-processing) tech- nique for performing white point adjustment is to take a picture of a white piece of paper and to adjust the RGB values of this image to make this a neutral color.

a. Describe how you would adjust the RGB values in any image given a sample "white color" of (R_w , G_w , B_w) to make the color neutral (without changing the exposure too much). (10)

b. If (Rw, Gw, Bw) of a white image are not known how would you guess what is the transform needed for correct white balance (10)

Q4. *Ignore Problem 3 label* (30 pts)

Problem 3: Vector notation can reduce the size of expressions encountered when dealing with motion vision problems. Let us revisit the recovery of camera motion and/or the orientation of a planar surface from an image sequence. The equation $aX + bY + cZ = d$ applies to a planar surface.

(a) Show that the equation of the plane can be written in the form

$$\mathbf{R} \cdot \mathbf{n} = 1,$$

for some vector $\mathbf{n}$, where $\mathbf{R} = (X, Y, Z)^T$. Give an expression for the unit surface normal (in terms of a, b, c, and d). What is the perpendicular distance of the plane from the origin?

(b) Suppose the plane is in front of an imaging system. Show that under perspective projection

$$\frac{1}{f}\mathbf{r} \cdot \mathbf{n} = \frac{1}{\mathbf{R} \cdot \hat{\mathbf{z}}}$$

where f is the principal distance of the imaging system, and $\mathbf{r} = (x, y, f)^T$ is the image of the point $\mathbf{R} = (X, Y, Z)^T$ in the world.

(c) Suppose now that the plane is moving with velocity $\mathbf{t} = (U, V, W)^T$ with respect to the camera (or equivalently that the camera is moving with velocity $-\mathbf{t}$ w.r.t to plane). Differentiate the perspective projection equations

$$x/f = X/Z \quad \text{and} \quad y/f = Y/Z$$

to obtain expressions for the motion field components $u = dx/dt$ and $v = dy/dt$ in terms of the components of the translational motion vector $U = dX/dt$, $V = dY/dt$ and $W = dZ/dt$.

Q5. (10 pts)

Orthogonal transforms and l_∞ norm: Orthogonal transforms conserve the l_2 norm, but not others, in general. The l_∞ norm of a vector is defined as (assume $v \in R^n$):

$$l_\infty[v] \;=\; \max_{i=0,\ldots,n-1} |v_i|.$$

(a) Consider $n = 2$ and the set of real orthogonal transforms T_2, that is, plane rotations. Given the set of vectors v with unit l_2 norm (that is, vectors on the unit circle), give lower and upper bounds such that

$$a_2 \;\leq\; l_\infty[T_2 \cdot v] \;\leq\; b_2.$$

Q6 (20 pts)

Consider the space of square-integrable real functions on the interval $[-\pi, \pi]$, $L_2([-\pi, \pi])$, and the associated orthonormal basis given by

$$\left\{ \frac{1}{\sqrt{2\pi}}, \frac{\cos nx}{\sqrt{\pi}}, \frac{\sin nx}{\sqrt{\pi}} \right\}, \qquad n = 1, 2, \ldots$$

Consider the following two subspaces: S – space of symmetric functions, that is, $f(x) = f(-x)$, on $[-\pi, \pi]$, and A – space of antisymmetric functions, $f(x) = -f(-x)$, on $[-\pi, \pi]$.

(a) Show how any function $f(x)$ from $L_2([-\pi, \pi])$ can be written as $f(x) = f_s(x) + f_a(x)$, where $f_s(x) \in S$ and $f_a(x) \in A$.

(b) Give orthonormal bases for S and A.