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COE-3

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# Assignment-2

1) (a) $S = \frac{1}{n-1}\sum_{i=1}^{n} x_i \qquad E(S) = \frac{1}{n-1}\sum_{i=1}^{n} E(x_i)$

using chebyshev inequality

$P(|S-p| \geq E) \leq \frac{Var(E)}{E^2}$ for small E

when $n \to \infty$, $var(S) \to 0$

$\Rightarrow P(|S-p| \geq E) \to 0 \Rightarrow S$ is a consistent estimate of p

b) sample is $x_1, x_2, \ldots x_n$

for rectangular distribution, $E(x) = \frac{a+b}{2}$

$\frac{a+b}{2} = \frac{x_1 + x_2 \ldots + x_n}{n} = m$

$var(x) = E(x^2) - E(x)^2 = \frac{(b-a)^2}{12}$

$E(x)^2 = \frac{(b-a)^2}{12} + \frac{(a+b)^2}{12} = \frac{a^2+b^2+ab}{3}$

$\frac{a^2+b^2+ab}{3} = \frac{x_1^2 + x_2^2 + \ldots x_n^2}{n} = p$

$a^2+b^2+ab = 3p$ — (1)

$a+b = 2m$ — (2)

$\Rightarrow b = 2m - a$

$\Rightarrow a^2 + (2m-a)^2 + a(2m-a) = 3P$

$a^2 + 4m^2 - 4am + 2am - \cancel{a^2} = 3P$

$(a-m)^2 = 3P - 3m^2$

$a = m \pm \sqrt{3P - 3m^2}$

$b = m \mp \sqrt{3P - 3m^2}$

(3)

(2) $n_1 = 50$, $\mu_1 = 173.3$, $S_1 = 6.4$ cm.
$n_2 = 50$, $\mu_2 = 171.5$, $S_2 = 7.1$ cm.
$\alpha = 5\%$ 10

$DF = n + m - 2 = 98$

$H_0 = \mu_1 = \mu_2$

$H_1 = \mu_1 > \mu_2$

$$t = \frac{\mu_1 - \mu_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{173.3 - 171.5}{\sqrt{\frac{40.96 + 50.43}{50}}} = 1.33315$$

Range is $(-8, 1.6601)$ for diff $= 98$
As. St lies in range, null hypothesis is accepted.
so no difference in height of students.

(4)

$$\frac{1.8}{\sqrt{91.37}} \times \sqrt{n_2} > 1.6661$$

$n_2 > 77.7$

$n_2 \geq 78$ sample size be increased by 28 in ~~which~~ ~~to~~ each group.

③ $n_1 = 8$ $\quad n_2 = 10$

$(\sigma_1^2) = 20 \quad (\sigma_2)^2 = 36$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$P(S_1^2 > 2S_2^2) = P\left(\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} > \frac{2\sigma_2^2}{\sigma_1^2}\right)$$

using F distribution,

$$P\left(F_{7,9} > \frac{2 \times 36}{20}\right) = P(F_{7,9} > 3.6)$$

$\Rightarrow$ prabability lies b/w 0.01 and 0.05

④ $P(\mu - 5 < \bar{x}\ \mu + 5) = 0.954$

samplesize $= n$

$$z = \frac{\bar{x} - \mu}{\sqrt{\frac{var(x)}{n}}}$$

~~$P(-5 < \bar{x} < \mu$~~

$P(-5 < \bar{x} - \mu < 5) = 0.954$

$P\left(-\frac{5\sqrt{n}}{10} < z < \frac{5\sqrt{n}}{10}\right) = 0.954$

$\Rightarrow P\left(-\frac{\sqrt{n}}{2} < z < \frac{\sqrt{n}}{2}\right) = 0.954$

from distribution table,

$P(-2 < z < 2) = 0.954$

$= \frac{\sqrt{n}}{2} = 2 \Rightarrow n = 16$ Ans

(5) $n = 100$, $\mu = 1600$, $x = 1570$, $\sigma = 120$

null hypothesis ($H_0$): $\mu = 1600$ hrs

Alternative hypothesis ($H_0$): $\mu \neq 1600$ hr

$\alpha = 0.05 \Rightarrow z_{\alpha/2} = 1.96$

$z = \frac{x - \mu}{\sigma/\sqrt{n}} = \frac{1570 - 1600}{120/\sqrt{100}} = -2.5$

z is outside the range of $-1.96$ to $1.96$

$\Rightarrow$ ~~we reject the~~ we reject the hypothesis $H_0$ at 0.05 level of significance.

P value $= 2P(z \leq 2.5) = 2 * 0.0062 = 0.0124 <$ 0.05

$\rightarrow$ we reject the hypothesis $H_0$