Model Predictive Control

State Estimation
Using
Kalman Filter and its Variants

PROJECT

SUBMITTED BY

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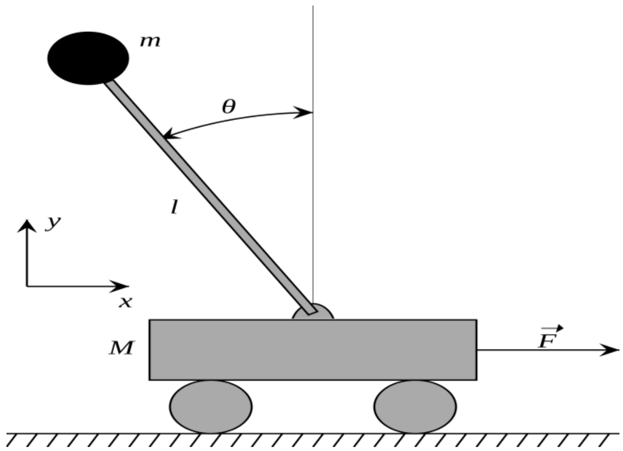
Process Model – Inverted Pendulum



Process Input: $u = F$

States of the Process: $Displacement(x), Velocity(\dot{x}), Angle\ of\ Rotation(\theta)\ and\ Angular\ Velocity(\ \dot{\theta})$

$$x_1 = x \quad x_2 = \dot{x} \quad x_3 = \theta \quad x_4 = \dot{\theta}$$

Measured Outputs :

$$y_1 = x = x_1 \qquad y_2 = \theta = x_3$$

Assumptions in the model:

1. There is no friction anywhere
2. The whole mass of the pendulum is at the bob and the beam is massless

Process Model:

$$\dot{x_1} = f_1(x) = x_2$$

$$\dot{x_2} = f_2(x) = \frac{1}{M + m.\sin^2 x_3}[-m.l.x_4^2.\sin(x_3) + m.g.\sin(x_3).\cos(x_3) + u]$$

$$\dot{x_3} = f_3(x) = x_4$$

$$\dot{x_4} = f_4(x) = \frac{1}{M.l + m.l.\sin^2 x_3}[m.l.x_4^2.\sin(x_3).\cos(x_3) - (M + m).g.\sin(x_3) - u\cos(x_3)]$$

Measurement Model:

$$y_1(x) = g_1(x) = x_1$$

$$y_2(x) = g_2(x) = x_2$$

State Estimation

The process is observed at time instant $t = t_0$ and we are interested in predicting the dynamics of the process. From first principles we can generate a model but the issue remains that we are unaware of the states of the system under consideration. If we can uniquely identify the state at some instant of time we can get an estimate of the state at any other time past or future. This is where the domain of state estimation comes to our aid.

The objective of State Estimation is to use the model and the measurement of the outputs optimally to estimate the state at a given time instant. The Linear Kalman Filter is the best filter available in the estimation of linear processes and it is a recursive filter. But many engineering systems are non-linear and non-Gaussian. Hence we use the Kalman Filter as a base and use its variants in the non-linear process. In this project, we have implemented Extened Kalman Filter, Unscented Kalman Filter, Ensemble Kalman Filter and Particle Filter.

BASIC KALMAN FILTER

This filter involved two steps- Prediction and Correction. In the prediction step, we use the linear model to make a one step-ahead prediction of the state. Use this to get an estimate of the output. Once we get the measurement of the output from the process, we take a linear combination of the error in the output and the prediction of the state to get the optimal estimate of the state.

EXTENDED KALMAN FILTER

This is the very basic of the non-linear filter where it just approximates the non-linear function by a linear one using the Taylor's Series Expansion. It just procees the same way as the basic Kalman Filter. Hence it is just a linearized Kalman Filter.

UNSCENTED KALMAN FILTER

The unscented kalman filter is an improvement over the Extended Kalman Filter while dealing with non-linear functiond. The format followed is almost the same. It differs from the EKF in obtaining a Gaussian approximation in the transformed domain. It uses the Unscented Transform process.

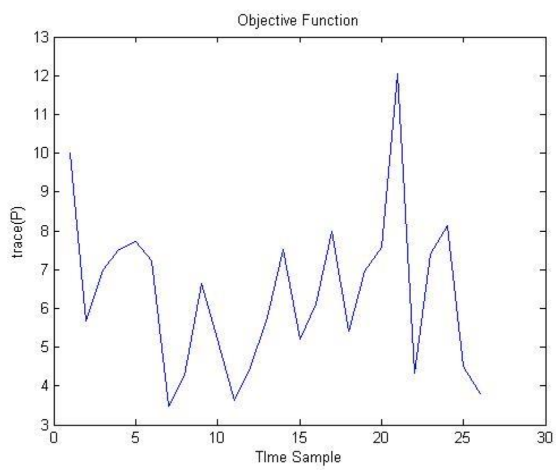
ENSEMBLE KALMAN FILTER

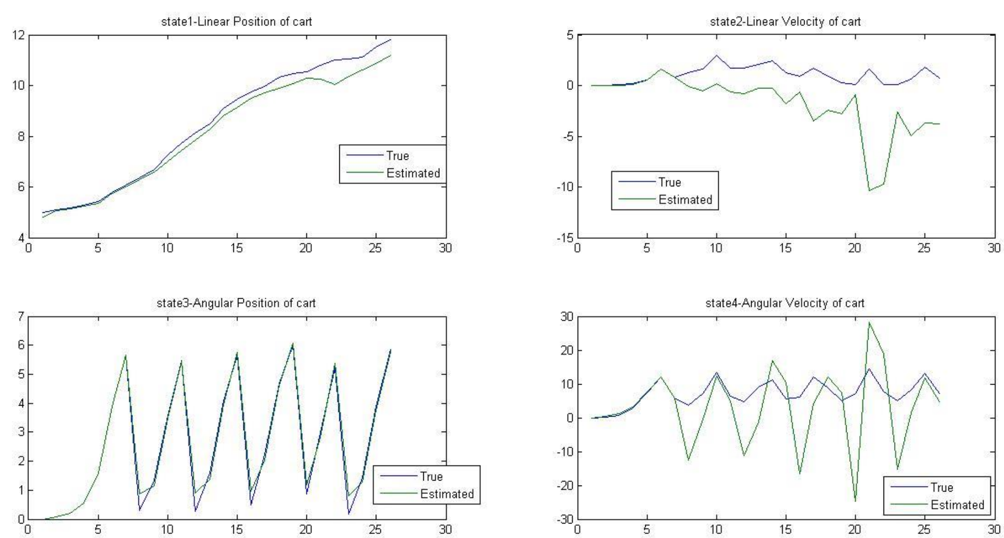
This assumes a Gaussian Distribution of the initial states and generates many samples to create the ensemble space. All the realizations are passed through the model and updated by using realizations of the output. Here the model is also linearized since the Gaussian distribution of the state will hold only when we pass is through a linear transformation. The estimates at any instant are calculated using the sample mean and sample variance from the realizations.

PARTICLE FILTER

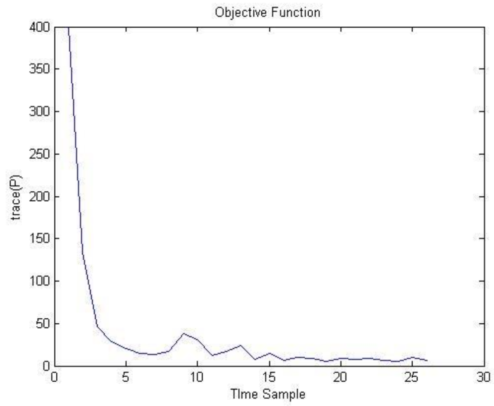
This relaxes the assumption that the distribution is Gaussian. Here also we define a posterior distribution and generate samples from it. Then weights are assigned to each sample. Both the samples and their weights together constitute the particle set. All the samples are propagated through the model. The updating step is just the modification of the weights. Then we use the Roulette's Wheel to resample the particles since we have only finite samples. Hence this works on the principle of the survival of the fittest. The estimate of state at any instant is given by the weighted combination of all the realizations.

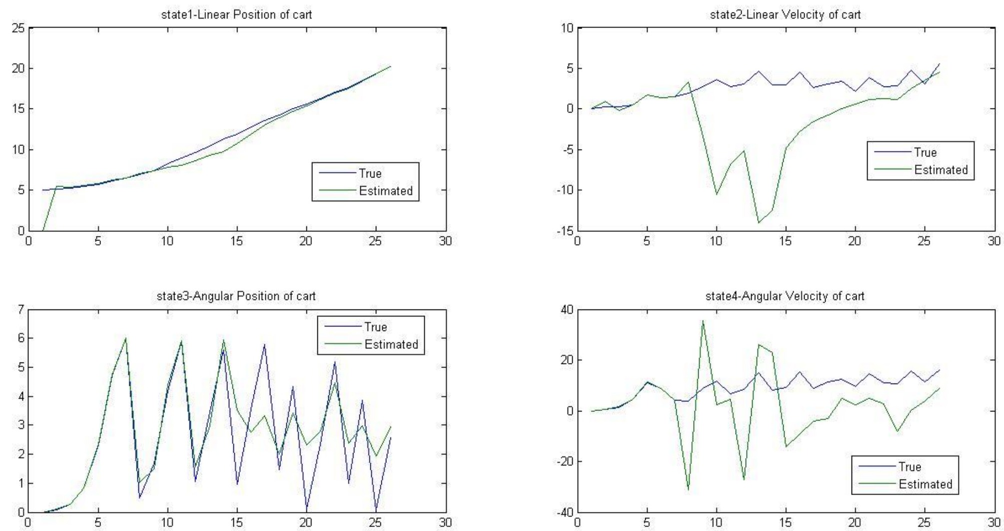
Implementation Of Extended Kalman Filter





Situation: Reasonably Good Initial Guess, Lesser Noise in both process and measurement.





Situation: Very Bad Initial Guess, very high variance, High process and Measurement Noise Errors

Observations and Inferences:

The observed states are estimated better. This is because in the estimation of the other unobserved states, the noise propagation occurs and estimates are poor. But as time progresses, the estimate do converge to the actual though it doesn’t coincide.

The trace of the covariance matrix falls down in the second case rapidly showing the effectiveness of the filter. In the first case it fluctuates about a minor value.

If the state is observed, the estimate of the state converges very quickly to the actual state.

Implementation of Unscented Kalman Filter

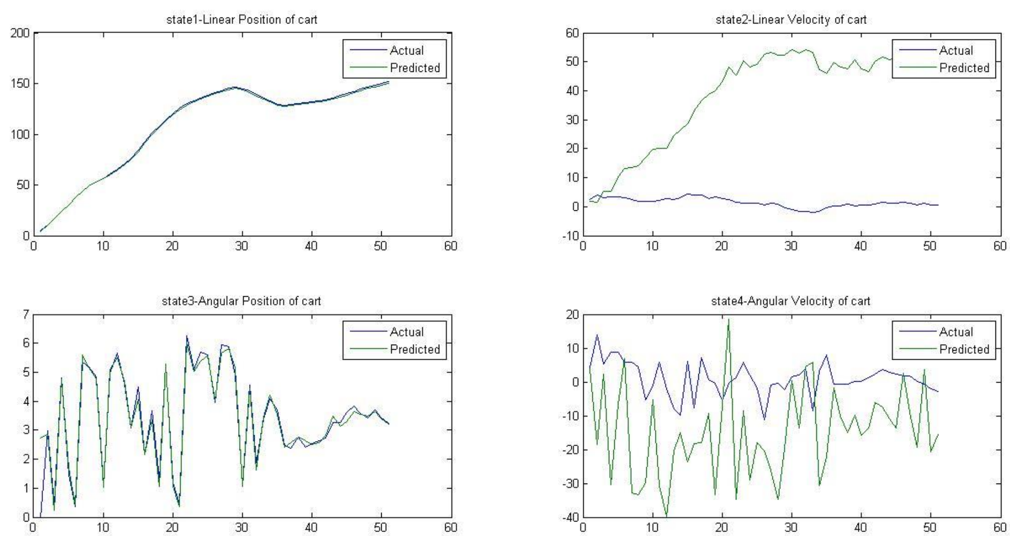
I sample a set of sigma points (in a special way) and assign weights to each to it. I then take the non-linear transform and obtain the corresponding sigma-points in the transformed domain, to which I fit a normal distribution. In this process, we get the best possible Gaussian transform.

For the inverted pendulum process, the UKF was unsuccessfully implemented. There was an issue that the variance-covariance matrix of X not being positive definite after a few iterations.

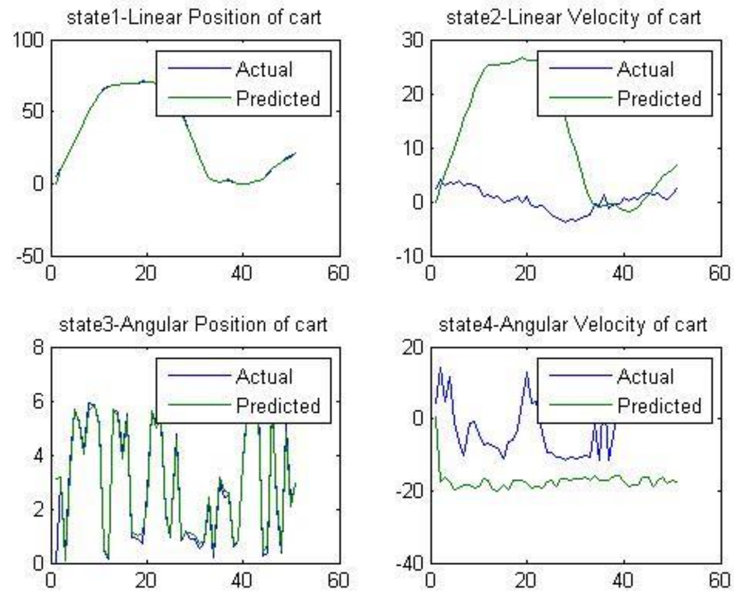
Comparison with EKF:

1. The EKF is suboptimal when either the function is highly non-linear or the variance is high. The UKF gets a much better approximation.
2. The EKF is computationally much lighter than the UKF.
3. The EKF can have an issue with the first order approximation. There is no such problem with the UKF since it doesn't deal with jacobians.

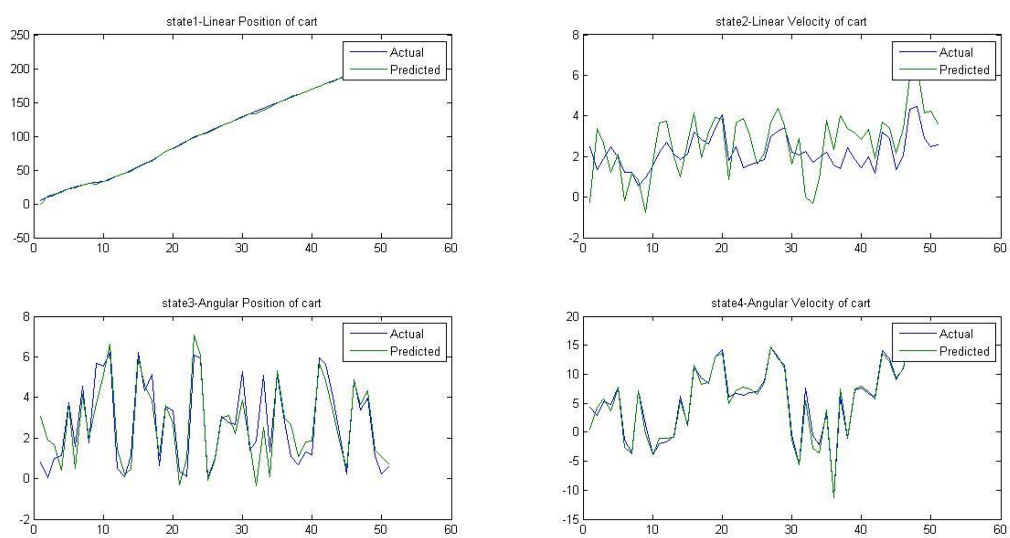
Implementation of Ensemble Kalman Filter



Situation: The initial guess is close to the true value. The process and measurement model errors coincide with the actual. The displacement and angle are observed.



Situation: Very bad initial Guess with high variance. Erroneous process and measurement model error. The displacement and the angle is observed.



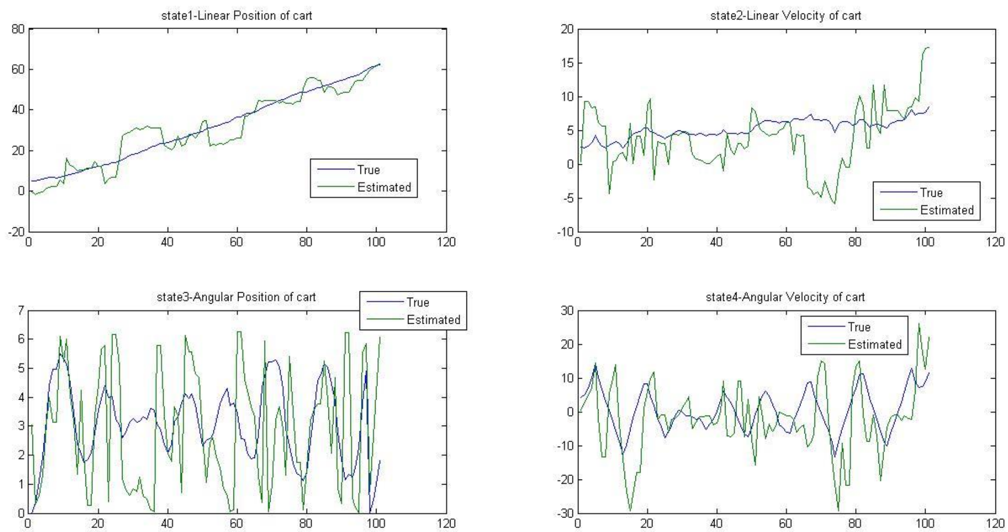
Situation: All the four states are measured, bad initial guess and high variance, bad estimates of model and process error.

Observations and Inferences:

The unobserved states cannot be estimated properly. This is because when we use realizations, the errors propagate and may lead to very erroneous estimates.

When all the states are measured, no matter if the measurement noise is high or the initial guess is bad, the states are estimated properly to a very high extent.

Implementation Of Particle Filter



Situation: a very bad initial guess and very high variance and 1000 particles. The particles are equally weighted initially and based on the prediction and the observation the weights are modified and resampling is done by Roulette’s wheel approach where the particles are selected by using random numbers to select from the cumulative probability distribution of the particles.

The reason why the estimates do not converge is that sometimes the weights become NaN when the particles go way off from the observed output. Then we start generating new samples base on the previous estimate and then the Particle Filter initiates from the beginning

The advantages of it are that it is a recursive Bayesian’s Filter, non-parametric, accommodates non-Gaussian distributions, resampling which takes only the fittest of the samples.

Conclusion

We have done estimation of states of an inverted pendulum process by four different variants of the Basic Linear Kalman Filter and the results have been summarized.