

# Control of Machining Instability and Cutting Forces in the Grinding Process

Submitted by

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Under the Guidance of

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## Introduction

Material removal – as the most significant operation in manufacturing industry – is facing the ever-increasing challenge of increasing proficiency at the micro and nano scale levels of high-speed manufacturing. Fabrication of submicron size three-dimensional features and free-form surfaces demands unprecedented performance in accuracy, precision, and productivity. Meeting the requirements for significantly improved quality and efficiency, however, are contingent upon the optimal design of the machine-tools on which machining is performed. As dynamic instability is inherently prominent and particularly damaging in high-speed precision cutting, *design for dynamics* is favored for the design of precision machine tool systems. It is largely driven by a critical piece of information – the vibration of the machine-tool. Due to the large set of parameters that affect cutting vibrations, such as regenerative effects, tool nonlinearity, cutting intermittency, discontinuous frictional excitation, and environmental noise, among many others, the effectiveness of the approach commands that the dynamics of machining be completely established throughout the entire process.

## Chatter Instability and Control

In any machining process, the objective is to increase the Material Removal Rate(MRR) while keeping a check on quality at the same time. So, we increase the MRR either by increasing the depth of cut, the spindle speed or by the traverse rate. This leads to increase in the internal forces caused by the normal reaction and results in a self-excited vibration which is termed as chatter. The vibration occurs due to the regenerative effect of the tool path. Tobias first developed a regenerative machine tool chatter theory by using the concept that the cutting force is a function of not only the current cut but also the previous cuts. This theory has been acknowledged to be the most suitable to describe the regenerative-type chattering phenomenon. This theory has become the foundation of both theoretical and experimental studies concerning machining processes such as turning, milling, drilling and grinding. In turning, the work-piece comes with stiffness but the tool remains rigid and hence retraction of the work-piece during turning causes the regenerative effect. In milling, it is the other way round. The work-piece is kept and supported rigidly by the bed and the tool has a stiffness associated with it and hence contributes to the regenerative effect. In grinding on the other hand, both tool and work-piece have stiffness and both can undergo retraction and hence it is a double regenerative system. Hence the grinding model has two delays associated with it.

Chatter is an undesirable phenomenon. Although it is bound in the time domain, it could become unstably broadband in the frequency domain, inadvertently causing poor tolerance, substandard surface finish and tool damage. If we view the same in frequency domain, the Frequency Response Function will keep changing with time (Non-Stationary Process) and this leads to Bifurcation in which a periodic response ultimately deteriorates to aperiodic and broadband response. There are two ways of dealing with chatter- control or avoid. The strategy of avoiding the unstable chatter regions is adopted by the stability lobe diagrams in which the system is linearized at the equilibrium points and the characteristic equation is derived and analyzed for stability by ensuring that all the poles are on the left half plane. The strategy of control is done in either time or frequency domain. In frequency domain, the system is linearized and the controller is developed. In the time domain, strategies like OGY control, Lyapunov Control and Delayed Feedback Control all are implemented in non-linear systems in which the dynamics is known beforehand as it is time invariant. But the manufacturing processes are time varying as the interaction between the tool and work-piece will change the nature of the system. Since it is a non-linear non-stationary system, these control strategies fail. Hence, the requirement for a controller which operates in both time and frequency domain is required to keep the amplitude of vibration under control in time domain and the frequency broadband under control in frequency domain.

Here we try to bring out the importance of retaining the non-linear characteristics of the system and bring out clearly what we lose out on by linearizing the system. Then later introduce Instantaneous Frequency as a method to analyze the process as it undergoes bifurcation from periodicity to chaos and then bring out the need for a controller which works in both time and frequency domain.

## Impact of Linearization

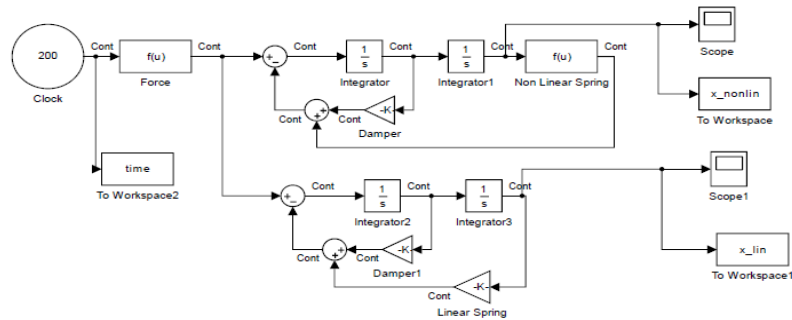
To examine the impact of linearization, the response of a nonlinear Duffing oscillator and its linearized version under stationary excitation are investigated by FFT, time-frequency analysis tools and Instantaneous Frequency. The general form of the non-dimensional Duffing Oscillator is

$$\ddot{x} + 2\mu\dot{x} + \beta x + \alpha x^3 = a.\cos(\omega t)$$

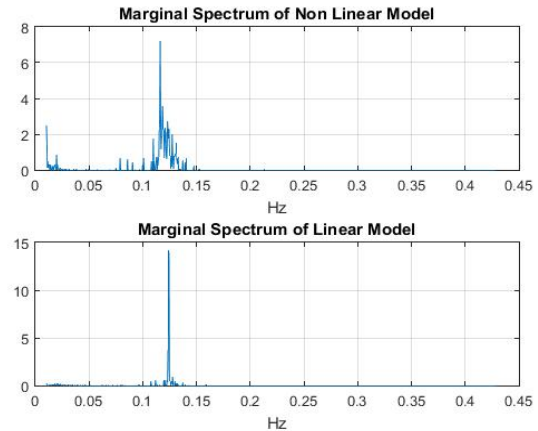
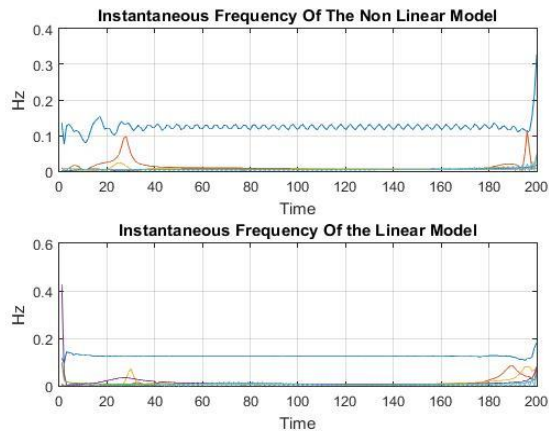
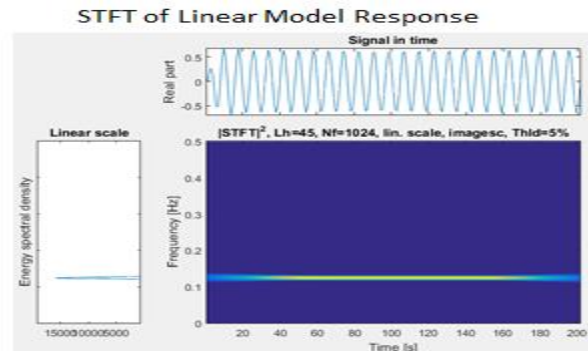
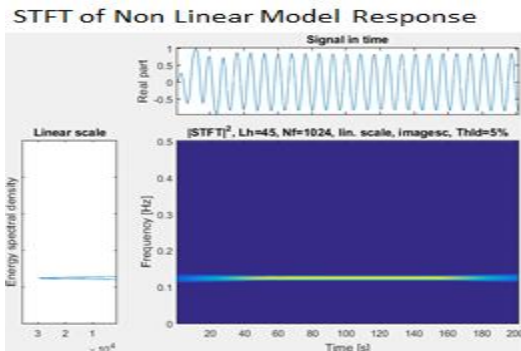
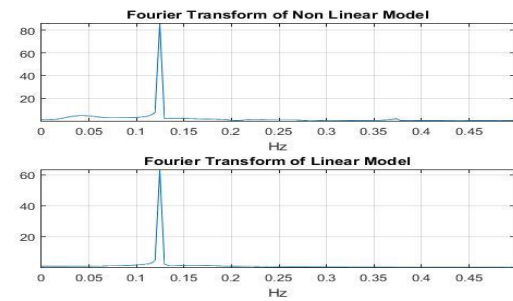
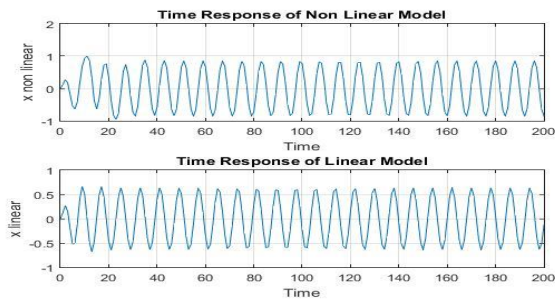
Where  $\mu, \beta, \alpha, a$  and  $\omega$  are constants. When the motion is small, the cubic term can be linearized with respect to equilibrium point zero and be ignored as

$$\ddot{x} + 2\mu\dot{x} + \beta x = a.\cos(\omega t)$$

Now this is done in Simulink where the same input is supplied to both systems and the signal  $x$  is analyzed and the results are presented.



SIMULINK DIGRAM WITH NON LINEAR MODEL ON TOP AND LINEAR MODEL ON THE BOTTOM

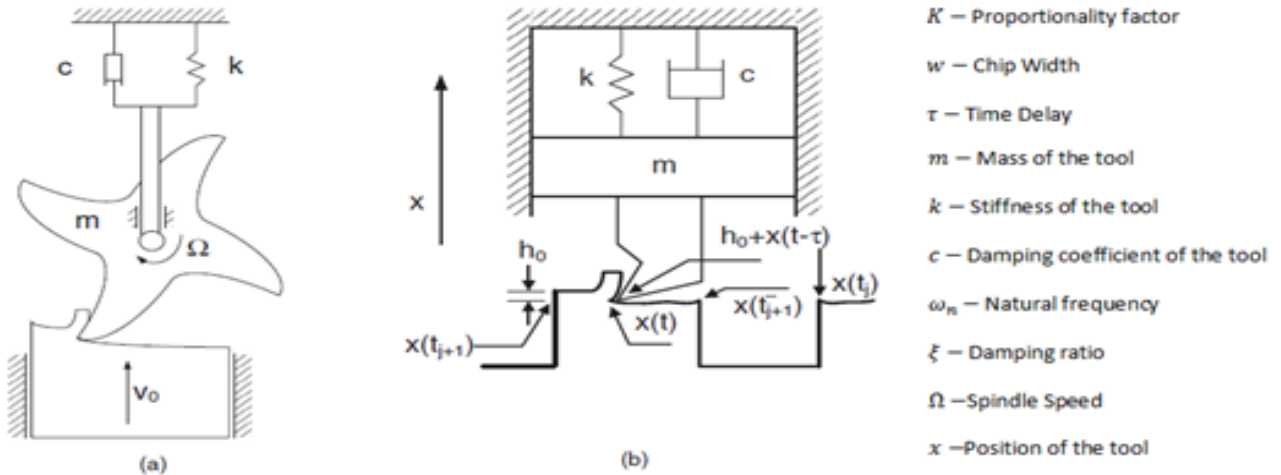


From the observed results, it can be clearly seen that visual inspection, Fourier Transform and time-frequency analysis like Short Time Fourier Transform fail to bring out any significant difference between the Linear and the Non Linear Response. The Instantaneous frequency brings out the clear picture. The linearized model suggests that the response is purely periodic but the non-linear model suggests a fluctuation in frequency about a mean frequency which is not a periodic response. There are these band of additional frequencies which come into play leading to vibrations that are undesirable. In a machining process, the process is chaotic when there is a broadband of frequencies present and this is clearly captured by the Instantaneous Frequency. The marginal spectrum reemphasizes the same point that the linear response is periodic and the non-linear response is not periodic but has a range of frequencies.

From this discussion, we can conclude that Fourier Spectrum is unable to reveal the true characteristic of the Non Linear Response. By comparing the marginal spectra of the non-linear and linearized responses, it is observed that linearization misinterprets non-linear features, replacing multiple frequencies with a single frequency. This implies that the frequency-domain based controllers designed using linearization would misinterpret the frequency response as it can't realize the evolution of bifurcation. Moreover, route-to-chaos is a transient process in which the spectral response deteriorates from being periodic to aperiodic and broadband, linearization and Fourier-based controller design would fail to identify the inception of bifurcation and chaos, and the stability bound of the system.

## Machining Instability in Milling

We take the model of milling and without compromising on the non-linearity; we study the response of the process



$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2x = \frac{\delta(t)}{m}F_c(h(t))$$

$$h(t) = h_0 + x(t - \tau) - x(t)$$

$$F_c(h(t)) = Kw[h(t)]^{\frac{3}{4}}$$

$$\omega_n = \sqrt{\frac{k}{m}} \quad ; \quad \xi = \frac{c}{2m\omega_n} \quad ; \quad \tau = \frac{2\pi}{N\Omega}$$

$$\delta(t) = \begin{cases} 0 & \text{if } \exists j \in \mathbb{Z} : t_j \leq t \leq t_{j+1}^- \\ 1 & \text{if } \exists j \in \mathbb{Z} : t_{j+1}^- \leq t \leq t_{j+1} \end{cases}$$

$K$  – Proportionality factor

$w$  – Chip Width

$\tau$  – Time Delay

$m$  – Mass of the tool

$k$  – Stiffness of the tool

$c$  – Damping coefficient of the tool

$\omega_n$  – Natural frequency

$\xi$  – Damping ratio

$\Omega$  – Spindle Speed

$x$  – Position of the tool

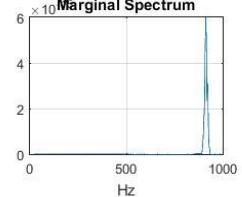
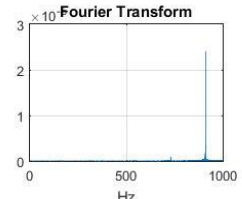
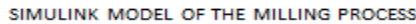
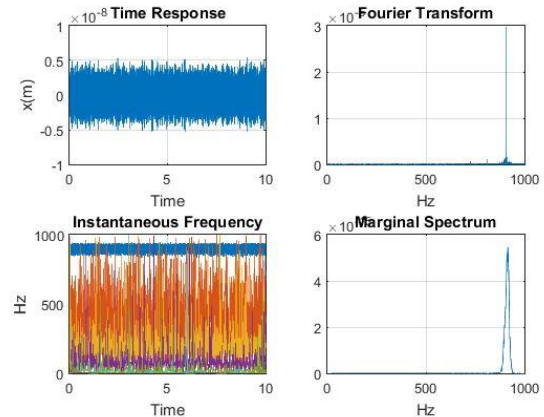
$F_c$  – Cutting Force

$N$  – Number of Cutting Edges

$h_0$  – Theoretical Depth of Cut

$h$  – Actual Depth of Cut

and axial depth of cut (ADOC)

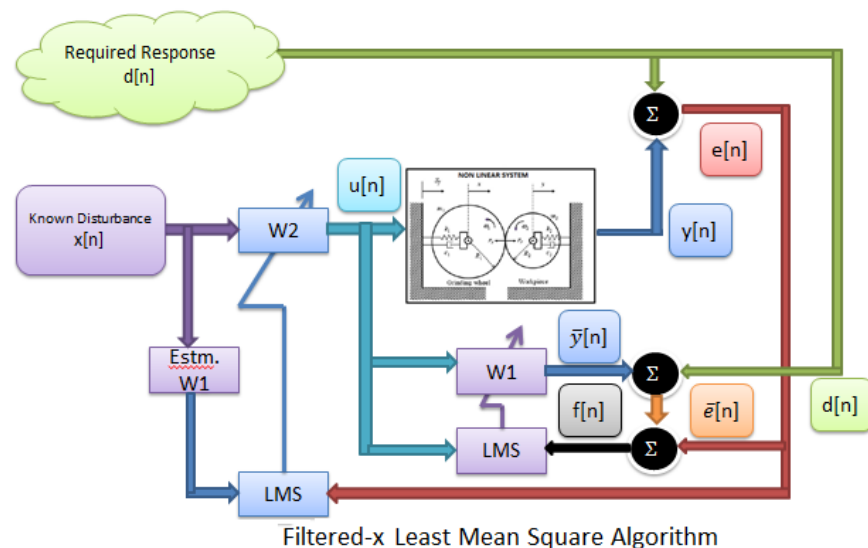

$$\Omega = 14,000rpm \quad ADOC = 1mm$$

$$\Omega = 12,000rpm \quad ADOC = 5mm$$

This response is stable in both time and frequency domain. Since we can see a frequency around 900Hz and a frequency around 450Hz and another one around 225Hz, this process clearly brings out the period

doubling bifurcation where the periods that are  $\frac{1}{2}$  and  $\frac{1}{4}$  of the actual period come into the picture. Now as we reduce the RPM or increase the ADOC, the response tends towards chaos as the frequency becomes broadband. Now time domain controllers have no idea of what goes on in the frequency domain. Hence we need controller which has one foot in the time domain and other in the frequency domain.

## Control Strategy

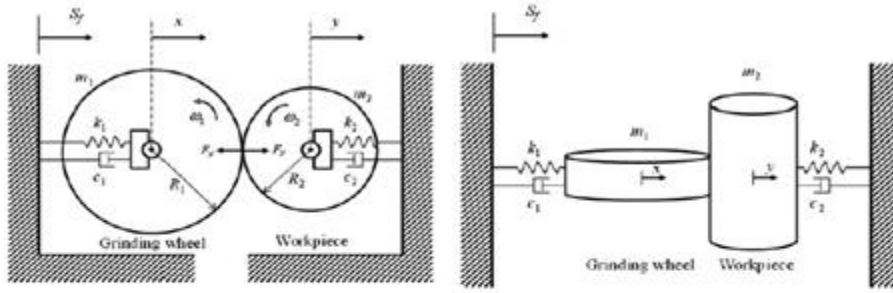
The control strategy adopted is an Adaptive Feed Forward Controller which takes inspiration from the Active Noise control where the estimate of the noise is found at one location and the filter produces an output which cancels out the noise at the destination. Similarly in a machining operation, with one rotation of the spindle, the displacement of the tool can be captured and this can be used as an estimate of the disturbance that we are bound to face in the next cycle. Now the objective is to generate an input to the process such that this vibration is at a minimal. The idea of the proposed controller is to use two filters- one to identify the system as a Finite Impulse Response Model based on the input force and the output displacement of the tool. Another filter is implemented to find the required cutting force which is the input to the process. Now based on the deviation from the required response, the filters are updated.



To make the controller more efficient, we bring in the discrete wavelet transform, where we break the signal into coefficients in wavelet domain which gives information about the contribution of a band of frequencies to the original signal. The discretization in frequency is such that lower frequencies have higher resolution and higher frequencies have lower resolutions in the frequency domain. This is effective in bringing about the convergence. This is yet to be implemented in the milling process.



## Grinding Process



$$m_1 \ddot{x} + c_1 \dot{x} + k_1 x = -K_v d^{z_0} = -K_v (\delta_s + \delta_d)^{z_0} = -F_N$$

$$m_2 \ddot{y} + c_2 \dot{y} + k_2 y = K_v d^{z_0} = K_v (\delta_s + \delta_d)^{z_0} = F_N$$

$$K_v = K s_1^{\gamma} \left( \frac{v_2}{v_1} \right)^{2z_0-1} D^{1-z_0}$$

$$\delta_s = \beta_v s_f^{z_0} = \beta \left( \frac{k_1 k_2 D^{z_0-1} \left( \frac{v_2}{v_1} \right)^{2z_0-1}}{K s_1^{\gamma} (k_1 + k_2)} \right) s_f^{\frac{1}{z_0}}$$

$$\delta_d = x(t) - x(t - \tau_1) - y(t) + \alpha y(t - \tau_2)$$

$$\tau_1 = \frac{2\pi}{\omega_1}; \tau_2 = \frac{2\pi}{\omega_2};$$

$K$  — Proportionality factor

$s_1$  — Cutting edge density of the grinding wheel

$\gamma, z_0$  — Exponential Parameters

$D$  — Equivalent Grinding Wheel Diameter

$\beta$  — Empirical value, constant value ( $0 < \beta < 1$ )

$\alpha$  — Overlapping factor ( $\alpha = 1$  for plunge grinding)

$\tau_1, \tau_2$  — The time delays for displacements of the grinding wheel and work-piece caused by their material loss

$\delta_s$  — Static depth of cut

$\delta_d$  — Dynamic depth of cut

The Grinding process brings into the picture a whole new set of complications. The grinding wheel is not something which is in our hand. The cutting edges are randomly distributed and so nothing can be inferred about the periodicity of the impact which is not the case in milling. In milling, the cutting edges are evenly distributed in the tool and we know when the impact is felt by the work-piece. Further, we cannot measure the cutting force as both the work-piece and the cutting tool are always in motion and we could only hope for an estimate. In addition to that, there is the double regenerative effect where the present cutting force is going to depend on the past position of the cutting tool and the past position of the work-piece. The strategy is to use the displacement measurement in the spindle and compare it with the desired response. The work ahead is to implement the control strategy in the milling process and then later adapt and extend it to the grinding process.

## References

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