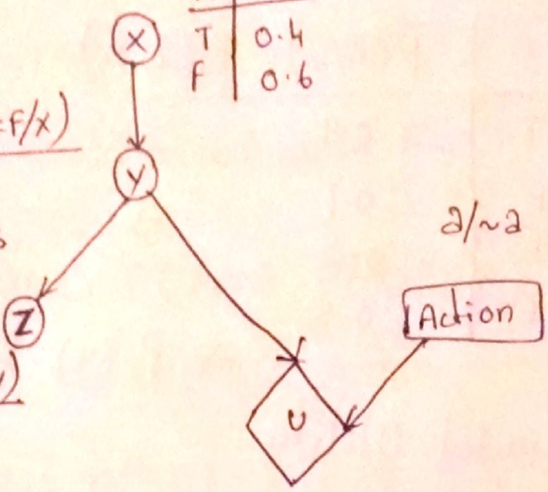
PROBLEM - I

Given Decision Network.

X	P(X)
T	0.4
F	0.6



X	P(Y=T/X)	P(Y=F/X)
T	0.2	0.8
F	0.7	0.3

Y	P(Z=T/Y)	P(Z=F/Y)
T	0.9	0.1
F	0.2	0.8

Y	Action	U(Y, Action)
T	a	800
T	~a	400
F	a	200
F	~a	1000

a> Action to be taken ??

This can be acheived using Expected Utility of a particular action.

Joint Distribution for the above network is

$$P(X)\ P(Y/X)\ P(Z/Y)$$

So, We have to eliminate X and Z to find the proper action

Eliminate X

X	Y	$f_1(X,Y) = P(X)\ P(Y/X)$
T	T	$0.4 \times 0.2 = 0.08$
F	T	$0.6 \times 0.7 = 0.42$
T	F	$0.4 \times 0.8 = 0.32$
F	F	$0.6 \times 0.3 = 0.18$

Y	$f_2(Y) = \sum_X f_1(X,Y)$
T	0.5
F	0.5

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$\Rightarrow f_2(y)\, P(Z/Y)$

Eliminate Z

Y	Z	$P(Z/Y) = f_3(Z, Y)$
T	T	0.9
T	F	0.1
F	T	0.2
F	F	0.8

$\Rightarrow$

Y	$f_4(Y) = \sum_Z f_3(Z\, Y)$
T	1
F	1

$\Rightarrow f_2(y)\, f_4(Y)$

Y	$f_2(y)\, f_4(y)$
T	$0.5 \times 1 = 0.5$
F	$0.5 \times 1 = 0.5$

Hence, Expected Utility

When action = a

$EU = 800 \times 0.5 + 200 \times 0.5 = 0.5(800 + 200)$
$= 0.5(1000) = 500$

When action = $\sim a$

$EU = 400 \times 0.5 + 1000 \times 0.5 = 0.5(1000 + 400)$
$= 0.5(1400) = 700$

As $EU(a) < EU(\sim a)$ the action to be performed is $\sim a$

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b> Information of Z??

This can be calculated using.

$$\Rightarrow P(Z=T)\,EU(Z) + P(Z=F)\,EU(Z) - EU.$$

When $Z=T$ We need to calculate EU.

$$P(Y/A=a,\ Z=T) = P(X)\,P(Y/X)\,P(Z=T/Y)$$

Eliminate x.

Y	X	$P(X)\ P(Y/X) = f_1(X,Y)$
T	T	$0.4 \times 0.2 = 0.08$
T	F	$0.6 \times 0.7 = 0.42$
F	T	$0.4 \times 0.8 = 0.32$
F	F	$0.6 \times 0.3 = 0.18$

$\Rightarrow$

Y	$f_2(Y) = \sum_x f_1(X,Y)$
T	0.5
F	0.5

$$P(Y/A=a,\ Z=T) \approx f_2(Y)\,P(Z=T/Y)$$

Y	$f_2(Y)\ P(Z=T/Y)$	
T	$0.5 \times 0.9 = 0.45$	normalize $\Rightarrow 0.81$
F	$0.5 \times 0.2 = 0.1$	$\Rightarrow 0.19$

EU When action is a

$$= 0.81 \times 800 + 0.19 \times 200 = 648 + 38 = 686$$

EU When action is $\sim a$

$$= 0.81 \times 400 + 0.19 \times 1000 = 324 + 180 = 504$$

When Z = F

$\Rightarrow P(Y/A=a, Z=F) = P(x)\ P(Y/X)\ P(Z=F/Y)$

$= f_2(Y)\ P(Z=F/y)$

Eliminate X

Y	$f_2(Y) = \sum_X f_1(X,Y)$
T	0.5
F	0.5

Y	$f_2(Y)\ P(Z=F/y) = f_3(Y)$
T	$0.5 \times 0.1 = 0.05$
F	$0.5 \times 0.8 = 0.40$

$\Rightarrow$ Normalize 0.11, 0.89

Expected Utility When action = a $\Rightarrow 800 \times 0.11 + 200 \times 0.89 = 88 + 178 = 266$

When action = $\sim a \Rightarrow 400 \times 0.11 + 1000 \times 0.89 = 44 + 890 = 934$

~~Expected Utility~~ Probability of (Z)

$P(Z) = P(X)\ P(Y/X)\ P(Z/Y)$ Eliminate X & Y

$= f_2(Y)\ P(Z/y)$

Eliminate X:

Y	X	$f_1(X,Y) = P(X)\ P(Y/X)$
T	T	$0.4 \times 0.2 = 0.08$
T	F	$0.6 \times 0.7 = 0.42$
F	T	$0.4 \times 0.8 = 0.32$
F	F	$0.6 \times 0.3 = 0.18$

Y	$f_2(Y) = \sum_X$
T	0.5
F	0.5

Eliminate Z

Y	Z	$f_2(Y)\ P(Z/y) = f_3(Z)$
T	T	$0.5 \times 0.9 = 0.45$
T	F	$0.5 \times 0.1 = 0.05$
F	T	$0.5 \times 0.2 = 0.10$
F	F	$0.5 \times 0.8 = 0.40$

$\Rightarrow$

Z	$f_3(Z) =$
T	0.55
F	0.45

Hence, Value at Z.

$= P(Z=T)\ EU(Z) + P(Z=F)\ EU(Z) - EU$

$\Rightarrow 0.55 \times 684 + 0.45 \times 934 - 700$

$= 96.5$

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c) Value of Information of X??

This can be calculated using

$P(X=T)\ EU(X) + P(X=F)\ EU(X) - EU$

When $X=T$

$P(Y/A=a, X=T) = P(X=T)\ P(Y/X=T)\ P(Z/Y)$

Eliminate Z

Y	Z	$P(Z/Y) = f_1(Z,Y)$
T	T	0.9
T	F	0.1
F	T	0.2
F	F	0.8

$\Rightarrow$

Y	$f_2(Y)$
T	1
F	1

$\Rightarrow$ $P(Y/A=a, X=T) = P(X=T)\ P(Y/X=T)\ f_2(Y)$

Y	$P(X=T)\ P(Y/X=T)\ f_2(Y)$		
T	$0.4 \times 0.2 \times 1 = 0.08$	normalize $\Rightarrow$	0.2
F	$0.4 \times 0.8 \times 1 = 0.32$		0.8 //

When action $= a$

$EU = 800 \times 0.2 + 200 \times 0.8 = 160 + 160 = 320$

action $= \sim a$

$EU = 400 \times 0.2 + 1000 \times 0.8 = 80 + 800 = 880$

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When $x = f$

$$P(X/A=a, X=f) = P(X=f)\ P(Y/X=f)\ P(Z/Y)$$

Eliminate 'Z' $\Rightarrow f_2(Y)$ from above

$$= P(X=f)\ P(Y/X=f)\ f_2(Y)$$

Y	P(X=f) P(Y/X=f) f_2(Y)			
T	0.6 × 0.7 × 1	0.42	⇒	0.7
F	0.6 × 0.3 × 1	0.18		0.3

$\left.\begin{matrix}0.7\\0.3\end{matrix}\right\}$ Normalize

When action = a

$$EU = 800 \times 0.7 + 200 \times 0.3 = 560 + 60 = 620$$

When action = ~a

$$EU = 400 \times 0.7 + 1000 \times 0.3 = 280 + 300 = 580$$

Value of Information at X.

$$\Rightarrow P(X=T)\ EU(x) + P(X=F)\ EU(x) - EU$$

$$= 0.4 \times 880 + 0.6 \times 620 - 700$$

$$= 352 + 372 - 700$$

$$= 24$$

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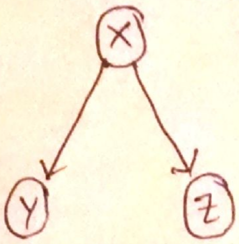
PROBLEM - II

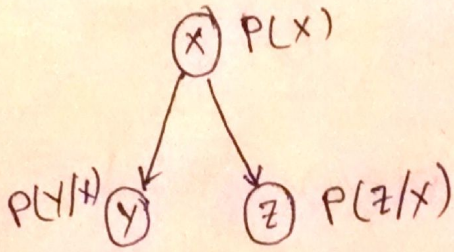
Given Bayesian Network.



Sample Data

X	Y	Z	Counts
T	T	R	56
T	T	G	24
T	F	R	84
T	F	G	36
F	T	R	400
F	T	G	240
F	T	B	160

a) Maximum Likelihood Estimation.



<u>P(X)</u>

$$P(X=T) = \frac{200}{1000} = 0.2$$

$$P(X=F) = \frac{800}{1000} = 0.8$$

P(Y/X)

$P(Y=T/X=T) = 80/200 = 0.4$

$P(Y=F/X=T) = 120/200 = 0.6$

$P(Y=T/X=F) = \frac{800}{800} = 1$

$P(Y=F/X=F) = 0/800 = 0$

P(Z/X)

$P(Z=R/X=T) = 140/200 = 0.7$

$P(Z=B/X=T) = 0/200 = 0$

$P(Z=G/X=T) = 60/200 = 0.3$

$P(Z=R/X=F) = \frac{400}{800} = 0.5$

$P(Z=G/X=F) = \frac{240}{800} = 0.3$

$P(Z=B/X=F) = 160/800 = 0.2$

Counts	P(X) P(Y/X) P(Z/X)	Product	1000x Product
56	$0.2 \times 0.4 \times 0.7$	$= 0.056$	$= 56$
24	$0.2 \times 0.4 \times 0.3$	$= 0.024$	$= 24$
84	$0.2 \times 0.6 \times 0.7$	$= 0.084$	$= 84$
36	$0.2 \times 0.6 \times 0.3$	$= 0.036$	$= 36$
400	$0.8 \times 1 \times 0.5$	$= 0.40$	$= 400$
240	$0.8 \times 1 \times 0.3$	$= 0.24$	$= 240$
160	$0.8 \times 1 \times 0.2$	$= 0.16$	$= 160$

b) Laplace Smoothing. (5)

$\Rightarrow$ $\alpha = 1$, $\beta = 1$ & $\alpha_1, \alpha_2, \alpha_3 = 1$ for multinomial.

P(X)

X	P(X)
T	$200+1 / 1000+2 = 0.2005$
F	$\frac{800+1}{1000+2} = 0.799$

P(Y/X)

X	P(Y=T)	P(Y=F)
T	$\frac{80+1}{200+2} = 0.40$	$\frac{120+1}{200+2} = 0.59$
F	$\frac{800+1}{800+2} = 0.99$	$\frac{0+1}{800+2} = 0.001$

P(Z/X)

X	P(Z=R)	P(Z=B)	P(Z=G)
T	$\frac{140+1}{200+3} = 0.69$	$\frac{0+1}{0+3} = 0.049$	$\frac{60+1}{200+3} = 0.300$
F	$\frac{400+1}{800+3} = 0.499$	$\frac{160+1}{800+3} = 0.200$	$\frac{240+1}{800+3} = 0.300$

Counts	P(X) P(Y/X) P(Z/X)	Product	Product * 1000
56	0.2005 × 0.40 × 0.69	0.055	55
24	0.2005 × 0.40 × 0.30	0.024	24
84	0.2005 × 0.59 × 0.69	0.081	81
36	0.2005 × 0.59 × 0.30	0.035	35
400	0.799 × 0.99 × 0.499	0.394	394
240	0.799 × 0.99 × 0.300	0.237	237
160	0.799 × 0.99 × 0.200	0.158	158

Problem III

Given data

F_1	F_2	C	Counts
T	T	T	9
T	T	F	3
T	F	T	27
T	F	F	9
F	T	T	3
F	T	F	21
F	F	T	1
F	F	F	7

P(C) T F
<0.5, 0.5>

a) Maximum Likelihood Estimation

$P(C) = \langle 0.5, 0.5 \rangle$

$P(F_1/C)$ & $P(F_2/C)$

$P(F_1/C)$

C	$P(F_1=T/C)$	$P(F_1=F/C)$
T	$36/40 = 0.9$	$4/40 = 0.1$
F	$12/40 = 0.3$	$28/40 = 0.7$

$P(F_2/C)$

C	$P(F_2=T/C)$	$P(F_2=F/C)$
T	$12/40 = 0.3$	$28/40 = 0.7$
F	$24/40 = 0.6$	$16/40 = 0.4$

b) Using the Naives Baye's

$P(C \mid F_1 = F, F_2 = T)$

$$= \frac{P(C) \; P(F_1 = F/C) \; P(F_2 = T/C)}{P(F_1 = F, F_2 = T)}$$

When $C = T$//

$$= \frac{P(C=T) \; P(F_1 = F/C=T) \; P(F_2 = T/C=T)}{P(F_1 = F, F_2 = T)}$$

$$= \frac{0.5 \times 0.1 \times 0.3}{P(F_1 = F, F_2 = T)} = \frac{0.015}{P(F_1 = F, F_2 = T)}$$

$C = F$//

$$= \frac{P(C=F) \; P(F_1 = F/C=F) \; P(F_2 = T/C=F)}{P(F_1 = F, F_2 = T)}$$

$$= \frac{0.5 \times 0.7 \times 0.6}{P(F_1 = F, F_2 = T)} = \frac{0.21}{P(F_1 = F, F_2 = T)}$$

$\Rightarrow P(F_1 = F, F_2 = T) =$ Sum of the numerators.

as Sum of $C = T$ & $C = F$ equals 1//

Hence, $P(C=T/F_1=F, F_2=T) = \frac{0.015}{0.015 + 0.21} = \frac{0.015}{0.225}$

$P(C=F/F_1=F, F_2=T) = \frac{0.21}{0.015+0.21} = \frac{0.21}{0.225}$

c) Naive's Bayes using Laplace Smoothing.

$\alpha = \beta = 1$

$P(C=T) = \frac{40+1}{80+2} = 41/82 = 0.5$

$P(C=F) = \frac{40+1}{80+2} = 41/82 = 0.5$

$P(F_1/C)$

C	$P(F_1=T/C)$	$P(F_1=F/C)$
T	$\frac{36+1}{40+2} = 37/42 = 0.88$	$\frac{4+1}{40+2} = 5/42 = 0.11$
F	$\frac{12+1}{40+2} = 13/42 = 0.309$	$\frac{28+1}{40+2} = 29/42 = 0.69$

$P(F_2/C)$

C	$P(F_2=T/C)$	$P(F_2=F/C)$
T	$\frac{12+1}{40+2} = 13/42 = 0.309$	$\frac{28+1}{40+2} = \frac{29}{42} = 0.69$
F	$\frac{24+1}{40+2} = 25/42 = 0.59$	$\frac{16+1}{40+2} = 17/42 = 0.40$

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Problem $\underline{IV}$

Given Inputs.

F_1	F_2	C
-3	32	False
-2	16	False
-1	8	False
1	4	True
2	2	True
3	1	True

$$P(C=\text{False} \mid F_1, F_2) = \frac{1}{1+e^{W_0+\sum_{i=1}^{n} W_i F_i}}$$

Note

$W_0 = W_b = 0$

$W_1 = 0$

$W_2 = 0$

$$P(C=\text{true} \mid F_1, F_2) = \frac{e^{W_0+\sum_{i=1}^{n} W_i F_i}}{1+e^{W_0+\sum_{i=1}^{n} W_i F_i}}$$

$$P(C=\text{false} \mid -3, 32) = \frac{1}{1+e^{0+\sum_{i=1}^{n} 0\times F_i}} = \frac{1}{1+e^{0+0}} = \frac{1}{1+1} = 1/2 = 0.5$$

$P(C=\text{false} \mid -2, 16) = 0.5$

$P(C=\text{false} \mid -1, 8) = 0.5$

$$P(C=\text{true} \mid 1, 4) = \frac{e^{0+0}}{1+e^{0+1}} = 0.5$$

$P(C=\text{true} \mid 2, 2) = 0.5$

$P(C=\text{true} \mid 3, 1) = 0.5$

a) $W_b = 0$, $W_1 = 0$, $W_2 = 0$

Conditional Log Likelihood:

$$(CLL) = \sum_{t} \ln P(c = true / F_1, F_2) + \sum_{f} \ln P(c = false / F_1, F_2)$$

From the given & retrieved database $\Rightarrow \sum_{t} \ln(0.5) + \sum_{f} \ln(0.5)$

We are give 3 elements for c=true

$\Rightarrow 3 \ln(0.5) + 3 \ln(0.5)$

$\Rightarrow 3(-0.69) + 3(-0.69) = -4.14$

b) $W_b = 0$, $W_1 = 0$, $W_2 = 0$ Gradient With respect to W_b

$$\frac{d(CLL)}{dW_b} = \sum_{t} \left(1 - P(c = true / F_1, F_2)\right) - \sum_{f} \left[P(c = true / F_1, F_2)\right]$$

$$= 3 \times [1 - 0.5] - 3 \times [0.5]$$

$$= 3 \times [0.5] - 3 \times [0.5] = 0$$

c) $W_b = 0$, $W_1 = 0$, $W_2 = 0$ We have to calculate gradient with respect to W_i

$$\frac{d\, CLL}{dW_1} = \sum_{t} F_1 \left[1 - P(c = true / F_1, F_2)\right] - \sum_{f} F_1 \left[1 - P(c = false / F_1, F_2)\right]$$

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$\Rightarrow \quad 1\times0.5 + 2\times0.5 + 3\times0.5$

$- (-3\times0.5 + -2\times0.5 + -1\times0.5)$

$\Rightarrow \quad 3 - (-3) = 6$

d) $W_b = 0,\ W_1 = 0,\ W_2 = 0$ Gradient with respect to W_2.

$$\frac{d\ CLL}{dW_2} = \sum_t F_2\left[1 - P(c = true \mid F_1, F_2)\right]$$

$$- \sum_f F_1\left[1 - P(c = false \mid F_1, F_2)\right]$$

$$= 4\times[1-0.5] + 2[1-0.5] + 1\cdot[1-0.5]$$

$$- [32\times0.5 + 16\times0.5 + 8\times0.5]$$

$$= 3.5 - 28 = 24.5$$