PROBLEM-I

Pl. Sentences into CNF Form.

a) $(P \wedge Q) \Rightarrow (R \wedge S)$ Conjunction(∧)of Disjunctions(∨)

$\neg(P \wedge Q) \vee (R \wedge S)$

$(\neg P \vee \neg Q) \vee (R \wedge S)$

$(\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee S)$

b) $(P \wedge Q) \Rightarrow (R \vee S)$

$\neg(P \wedge Q) \vee (R \vee S)$

$\neg P \vee \neg Q \vee R \vee S$

c) $(P \vee Q) \Rightarrow (R \wedge S)$

$\neg(P \vee Q) \vee (R \wedge S)$

$(\neg P \wedge \neg Q) \vee (R \wedge S)$

$[\neg P \vee (R \wedge S)] \wedge [\neg Q \vee (R \wedge S)]$
     ××××   ××××

$(\neg P \vee R) \wedge (\neg P \vee S) \wedge (\neg Q \vee R) \wedge (\neg Q \vee S)$

d) $(P \vee Q) \Rightarrow (R \vee S)$

$\neg(P \vee Q) \vee (R \vee S)$

$(\neg P \wedge \neg Q) \vee (R \vee S)$

$(\neg P \vee R \vee S) \wedge (\neg Q \vee R \vee S)$

e) $(\neg P \wedge Q) \Leftrightarrow (R \vee \neg S) \rightarrow [(\neg P \wedge Q) \Rightarrow (R \vee \neg S)] \wedge [(R \vee \neg S) \Rightarrow (\neg P \wedge Q)]$

$[\neg(\neg P \wedge Q) \vee (R \vee \neg S)] \wedge [\neg(R \vee \neg S) \vee (\neg P \wedge Q)]$

$[(P \vee \neg Q) \vee (R \vee \neg S)] \wedge [(\neg R \wedge S) \vee (\neg P \wedge Q)]$

$[(P \vee \neg Q) \vee (R \vee \neg S)] \wedge [\neg R \vee (\neg P \wedge Q) \wedge S \vee (\neg P \wedge Q)]$

$[(P \vee \neg Q) \vee (R \vee \neg S)] \wedge (\neg R \vee \neg P) \wedge (\neg R \vee Q) \wedge (S \vee \neg P) \wedge (S \vee Q)$

PROBLEM-II

a) KB Entail $(\neg A \wedge \neg B)$?

Given KB.

$R_1:$ $\neg A \vee B \vee E$

$R_2:$ $\neg B \vee A$

$R_3:$ $\neg E \vee A$

$R_4:$ $\neg E \vee D$

$R_5:$ $\neg C \vee \neg F \vee \neg B$

$R_6:$ $\neg E \vee B$

$R_7:$ $\neg B \vee F$

$R_8:$ $\neg B \vee C.$

$R_9:$ $\neg(\neg A \wedge \neg B) \approx (A \vee B)$ (Proof By Contradiction)

$R_{10}:$ A $\quad (R_2 \& R_9)$

$R_{11}:$ $(B \vee E)$ $\quad (R_1 \& R_{10})$

$R_{12}:$ B $\quad (R_6 \& R_{11})$

$R_{13}:$ F $\quad (R_7 \& R_{12})$

$R_{14}:$ C $\quad (R_8 \& R_{12})$

$R_{15}:$ $\neg C \vee \neg F$ $\quad (R_5 \& R_{12})$

$R_{17}:$ $\neg F$ $\quad (R_{14} \& R_{15})$

$R_{18}:$ $\emptyset$ $\quad (R_{14} \& R_{17})$

Hence KB Entails $(\neg A \wedge \neg B)!!$

$KB \vDash (\neg A \wedge \neg B)$

PROBLEM-II   b> KB Entails E??   ①

Given KB:

R₁ : ¬A ∨ B ∨ E

R₂: ¬B ∨ A

R₃: ¬E ∨ A

R₄: ¬E ∨ D

R₅: ¬C ∨ ¬F ∨ ¬B

R₆: ¬E ∨ B

R₇: ¬B ∨ F

R₈: ¬B ∨ C

R₉: ¬E   Proof By Contradiction.

R₁₀: ¬A ∨ B   · R₁ & R₉

R₁₁: ¬B ∨ B ≈ True   R₂ & R₁₀

R₁₂: ¬C ∨ ¬A ∨ ¬F   R₅ & R₁₀

R₁₃: ¬A ∨ F   R₇ & R₁₀

R₁₄: ¬A ∨ C   R₈ & R₁₀

R₁₅: ¬B ∨ ¬C ∨ ¬F   R₂ & R₁₂

R₁₆: ¬E ∨ ¬C ∨ ¬F   R₃ & R₁₂

R₁₇: ¬A ∨ ¬B ∨ ¬C   R₇ & R₁₂

R₁₈: ¬A ∨ ¬F ∨ ¬B   R₈ & R₁₂

R₁₉: ¬E ∨ F   (R₃ & R₁₃)

R₂₀: ¬A ∨ ¬C ∨ ¬B   R₅ & R₁₃

R₂₁: ¬E ∨ C   R₃ & R₁₄

R₂₂: ¬A ∨ E ∨ ¬C ∨ ¬F   R₁ & R₁₅

R₂₃: ¬B ∨ ¬C   R₇ & R₁₅

R₂₄: ¬B ∨ ¬F   R₈ & R₁₅

R₂₅: ¬A ∨ B ∨ ¬C ∨ ¬F   R₁ & R₁₆

R₂₆: ¬B ∨ ¬E ∨ ¬F   R₈ & R₁₆
                                      8

b) KB Entails E??   ②

$R_{27}:$ $\neg A \vee \neg E \vee \neg F$ R_{14} & R_{16}

$R_{28}:$ $\neg A \vee \neg E \vee \neg C$ R_{13} & R_{16}

$R_{29}:$ $\neg E \vee \neg C$ R_{16} & R_{19}

$R_{30}:$ $\neg E \vee \neg F$ R_{16} & R_{21}

$R_{31}:$ $\neg A \vee \neg C$ R_{19} & R_{22}

PROBLEM III

English Sentences Using FOL

Given Predicates

Occupation $(P, 0)$

Customer (P_1, P_2)

Boss (P_1, P_2)

Given Constants.

Doctor, Surgeon, Lawyer, Actor.

Emily, Joe.

a) Emily is either a surgeon or a lawyer.

Occupation (Emily, Surgeon) $\vee$ Occupation (Emily, Lawyer).

b) Joe is an actor but he holds another Job.

Occupation (Joe, Actor) $\wedge$ ~~Occupation (Joe, other)~~.

$\exists_x$ Occupation (Joe, x).

c) All Surgeons are Doctors.

$\forall_x \left[\text{Occupation} (x, \text{Surgeon}) \Rightarrow \text{Occupation} (x, \text{doctor}) \right]$

D) Joe doesn't have a Lawyer.

$\exists_x \left[\text{Occupation} (x, \text{Lawyer}) \wedge \neg \text{Customer} (\text{Joe}, x) \right]$

E) Every Surgeon has a Lawyer.

$\forall_x \left[\text{Occupation} (x, \text{Surgeon}) \Rightarrow \right.$ ~~Customer~~

$\exists_y$ Occupation $(y, \text{Lawyer}) \wedge$

$\left. \text{Customer} (x, y) \right]$

a) $\forall x \; P(x) \Rightarrow Q(x)$

$\forall x \; \neg P(x) \lor Q(x)$

$\neg P(x) \lor Q(x)$

b) $\forall x \forall y \; P(x,y) \Rightarrow Q(x)$

$\forall x \; \neg \left[\forall y \; P(x,y) \right] \lor Q(x)$

$\forall x \; \exists y \; \neg P(x,y) \lor Q(x)$

$\forall x \; P(x, F(x))) \lor Q(x)$

$P(x, F(x))) \lor Q(x)$ //

c) $\exists x \; P(x) \land Q(x)$

$\exists x \; P(x) \land Q(x)$

$P(A) \land Q(A)$

D) $\exists x \exists y \; P(x,y) \land Q(y,x)$

$\exists x \exists y \; P(x,y) \land Q(y,x)$

$P(A, B) \land Q(B,A)$

E) $\exists x \forall y \; P(x,y)$

$\exists x \forall y \; P(x,y)$

$\forall y \; P(A,y)$

$P(A, y)$

F) $\forall x \exists y \; P(x,y)$

$\forall x \exists y \; P(x,y)$

$\forall x \; P(x, F(x)))$

$P(x, F(x))$

G> $\forall x \, \forall y \, \exists z \, P(x, y, z)$

$\forall x \, \forall y \, \exists_z \, P(x, y, z)$

$\forall x \, \forall x \, P(x, y, F(x,y))$

$P(x, y, F(x,y))$

H· $\exists x \forall y \, \forall_z \, P(x, y, z)$

$\exists_x \, \forall y \, \forall_z \, P(x, y, z)$

$\forall y \, \forall_z \, P(A, y, z)$

$P(A, y, z)$

I· $\forall x \, (\exists y \, P(x,y) \land Q(y)) \Rightarrow R(x)$

$\forall x \, \neg [\exists y \, P(x,y) \land Q(y)] \lor R(x)$

$\forall x \, \forall y \, \neg P(x,y) \lor \neg Q(y) \lor R(x)$

$\neg P(x,y) \lor \neg Q(y) \lor R(x)$.

J· $\forall x \, (\forall y \, P(x,y) \Rightarrow Q(y)) \Rightarrow R(x)$

$\forall x \, \neg (\forall y \, \neg P(x,y) \lor Q(y)) \lor R(x)$

$\forall x \, \exists y \, (P(x,y) \land Q(y)) \lor R(x)$

$\forall x \, [P(x, F(x)) \land Q(F(x))] \lor R(x)$

$[P(x,F(x)) \land Q(F(x))] \lor R(x)$

$[P(x,F(x)) \lor R(x)] \land [Q(F(x)) \lor R(x)]$

PROBLEM V

Substantation to Unify Sentences.

a. $P(x)$
b. $P(A)$ $\{x/A\}$ a. $P(A)$
                                b. $P(A)$

c. $P(x) \lor Q(x,A)$
d. $P(B) \lor Q(x,A)$ $\{x/B\}$ c: $P(B) \lor Q(B,A)$
                                    d: $P(B) \lor Q(B,A)$

e. $P(x) \lor Q(A,x)$
f. $P(x) \lor Q(A,B)$ $\{x/B\}$ e: $P(B) \lor Q(A,B)$
                                    f: $P(B) \lor Q(A,B)$

g. $P(x,A) \lor Q(A,x)$
h. $P(B,y) \lor Q(y,B)$ $\{x/B, y/A\}$ g: $P(B,A) \lor Q(A,B)$
                                    h: $P(B,A) \lor Q(A,B)$

i: $P(x) \lor Q(F(x))$
j: $P(A) \lor Q(F(A))$ $\{x/A\}$ i: $P(A) \lor Q(F(A))$
                                    j: $P(A) \lor Q(F(A))$

k: $P(x,A) \lor Q(F(x),x)$
l: $P(B,y) \lor Q(F(B),B)$ $\{x/B, y/A\}$ k: $P(B,A) \lor Q(F(B),B)$
                                    l: $P(B,A) \lor Q(F(B),B)$.

m: $P(x,A) \lor Q(F(x),x)$
n: $P(B,y) \lor Q(F(A),A)$ No Substantation to Unify these sentences.

o: $P(x,y) \lor Q(F(A),B)$ $\{x/F(A), y/B\}$ o: $P(F(A),B) \lor Q(F(A),B)$
p: $P(x,y) \lor Q(x,y)$ p: $P(F(A),B) \lor Q(F(A),B)$

q: $P(x,y) \lor Q(F(A),A)$ $\{x/F(A), y/A\}$ q: $P(F(A),A) \lor Q(F(A),A)$
r: $P(x,y) \lor Q(x,y)$ r: $P(F(A),A) \lor Q(F(A),A)$

s: $P(x,y) \lor Q(F(x),y)$ No Substantation to
t: $P(z,y) \lor Q(z,y)$ Unify these sentences.