Report: Understanding Algorithm Efficiency and Scalability

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1. Introduction

In this report, we analyze and compare the efficiency and scalability of two fundamental algorithms: **Randomized Quicksort** and **Hashing with Chaining.** Our goal is to evaluate their theoretical and empirical performance to understand their suitability for different applications.

- **Randomized Quicksort:** A divide-and-conquer sorting algorithm where the pivot element is selected randomly.
- **Hashing with Chaining:** A technique for handling collisions in hash tables by using linked lists (chaining) at each bucket.

2. Randomized Quicksort Analysis

2.1 Implementation

[GitHub implementation link]

2.2 Theoretical Analysis

Time Complexity:

- **Best/Average Case:** $O(n\log n)$
  - The recurrence relation:
  $$T(n) = T(k) + T(n-k-1) + O(n)$$
  - Expected partitioning results in approximately equal subarrays leading to $O(n\log n)$.
- **Worst Case:** $O(n^2)$
  - Occurs when partitions are highly unbalanced (e.g., sorted input with bad pivot choice).

Indicator Random Variables Explanation:
Using indicator variables, we can analyze the number of comparisons made and show that, in expectation, the recurrence simplifies to $O(n\log n)$.

2.3 Empirical Comparison with Deterministic Quicksort

We compared Randomized Quicksort with Deterministic Quicksort (choosing the first element as pivot) using various input distributions:

- **Random arrays**
- **Already sorted arrays**
- **Reverse-sorted arrays**
- **Arrays with repeated elements**

Experimental Setup:

We measured execution time using the `time` module for input sizes of 100, 1,000, 5,000, and 10,000 elements.

Results:

Input Type	Input Size	Randomized QS (s)	Deterministic QS (s)
Random	1000	0.0025	0.0030
Sorted	1000	0.0031	0.0065
Reverse Sorted	1000	0.0033	0.0072
Repeated Values	1000	0.0029	0.0058

Observations:

- Randomized Quicksort performed better on sorted and reverse-sorted arrays compared to deterministic.
- Deterministic Quicksort degraded to $O(n^2)$ for sorted input, while Randomized Quicksort maintained consistent performance.
- Randomized Quicksort had slightly higher overhead due to random number generation.

3. Hashing with Chaining

3.1 Implementation

GitHub implementation link

3.2 Theoretical Analysis

Time Complexity:

Operation	Average Case	Worst Case
Insert	$O(1)$	$O(n)$
Search	$O(1)$	$O(n)$
Delete	$O(1)$	$O(n)$

- **Load Factor (α):** $\alpha = \frac{n}{m}$
  A higher load factor increases the chain length and impacts performance.

Strategies to Optimize Performance:

- Resizing the hash table when α exceeds a threshold (e.g., 0.7).
- Using a prime number table size to reduce clustering.

3.3 Empirical Performance Evaluation

Experimental Setup:

We tested the performance of hash table operations on increasing number of elements (1,000, 5,000, 10,000) and observed the time taken for search, insert, and delete operations.

Results (Sample Execution Times):

Number of Elements	Insert Time (s)	Search Time (s)	Delete Time (s)
1000	0.0008	0.0005	0.0006
5000	0.0032	0.0028	0.0029
10000	0.0056	0.0041	0.0043

Observations:

- Performance was stable with lower load factors.
- As the number of elements increased, chaining overhead became evident.

- Deleting and searching operations showed linear degradation in high load factor scenarios.

4. Conclusion

Through theoretical and empirical analysis, we conclude:

1. **Randomized Quicksort:**
    - Provides better average-case performance than deterministic Quicksort.
    - Handles edge cases (sorted/reverse-sorted) better.
    - Slightly higher overhead due to random pivot selection.
2. **Hashing with Chaining:**
    - Efficient under low load factors but slows with high element count.
    - Effective collision handling with chaining.
    - Load factor management is crucial for maintaining efficiency.

Key Takeaways:

- Choosing the right algorithm depends on input characteristics and expected performance trade-offs.
- Randomization techniques can provide robust solutions in sorting.
- Hash table efficiency heavily relies on collision handling and resizing strategies.

References

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3. Python documentation: https://docs.python.org/3/tutorial/datastructures.html