Electrical Machines Design Assignment

Andreas Carlsson
Raghuram Shankar
Rohit Puthukode Ramakrishnan

Group 7

2019/11/07

Contents

Introduction

Permanent Magnet Synchronous Machines (PMSM) are one of the most efficient machines for high power and high torque density applications. They are thus widely used in Electric Vehicles and other traction applications.

A Permanent Magnet (PM) motor uses rare earth permanent magnets, usually based on Neodymium, to produce a magnetic field in the rotor. The stator is powered by Three-Phase AC, which produces a rotating magnetic field in the stator. This rotating magnetic field in the stator and static magnetic field in the rotor interact to create a torque on the rotor and also induce a back EMF in the stator.

For the application discussed in this assignment, an insert-mounted type of PMSM with V-shaped rotor magnets is designed to meet the output requirements of power, efficiency, power factor, torque, etc.

Table 1 shows the design parameters of the PMSM.

Table 1: Values of Parameters of our design

Parameter	Name	Value
Outer Diameter of Stator $[mm]$	OD_stator	176
Inner Diameter of Stator $[mm]$	ID_stator	124
Outer Diameter of Rotor $[mm]$	OD_rotor	122
Inner Diameter of Rotor $[mm]$	ID_rotor	60
Length of Stack $[mm]$	L_stack	100
Slot Opening Height $[mm]$	Hs_0	0.5
Slot Wedge Height $[mm]$	Hs_1	0.5
Slot Body Height $[mm]$	Hs_2	14
Width of Tooth $[mm]$	w_tooth	4.4
Radius of Slot Bottom Fillet $[mm]$	Rs	0.5
Slot Opening $[mm]$	Bs_0	2
Number of Poles	N_pole	8
Number of Slots	N_slot	48
Number of Winding Layers	w_layer	4
Coil Pitch	k_p	5
Diameter of Wire $[mm]$	D_wire	0.72
DC Link Voltage $[V]$	V_DC	400
Base Speed $[RPM]$	$Omega\_base$	4000
Maximum Speed $[RPM]$	$Omega\_max$	12000
Number of Parallel Branches	$N\_parallel$	4
Number of Strands	N_strand	2
Resistivity of Copper$[\frac{ohm}{m}]$	rho_Cu	$1.72 \cdot 10^{-8}$
Thickness of Magnet $[mm]$	t_mag	5
Width of Magnet $[mm]$	w_mag	20

Table 2 shows the output parameters of the PMSM.

Table 2: Performance and output parameters of our design

Parameter	**Value**
Total number of turns per phase	76
Total number of coils in series per phase	2
Total number of turns per coil	19
Total number of conductors per slot	76
Total Slot area $[mm^2]$	69.4
Total Copper Area $[mm^2]$	30.9
Fill Factor	44.58%
Maximum per phase induced voltage at no-load $[V]$	155.5
Maximum line-to-line induced voltage at no-load $[V]$	269.3
Maximum line-to-line induced voltage at maximum speed $[V]$	807.8
Ratio of maximum line to line induced and DC link voltage at max speed	2.0196
Resistance per phase $[ohm]$	144.5
Reluctance of d-flux path $[H^{-1}]$	4879126.4
Reluctance of q-flux path $[H^{-1}]$	1063649.4
Inductance of D-axis $[mH]$	1.0
Inductance of Q-axis $[mH]$	4.7
Saliency Ratio	4.6

1 Magnetic Circuit Design

Question 1

We only study $\frac{1}{8}^{th}$ of the because the motor is symmetrical and we have to consider 2.5 poles to obtain enough information from just one fraction to be able to apply it to the rest of the motor.

Question 2

The magnetic flux density in the air gap is more uniform and predictable than the flux density in other parts of the motor. Hence, by assuming values from $0.01T$ to $1T$, the other parameters computed based on this assumption will be closer to the correct values.

Question 3

Amperes law is given by:

$$\mathcal{F} = \oint_l H\, dl = \int_s JdS = \sum i(t) + \frac{\partial \phi_e}{\partial t}$$

When this equation is applied to electrical machines, the displacement current can be neglected since the machine operates at low frequencies. That gives the following equation:

$$\oint_l H\, dl = \int_s JdS = \sum i(t) = \mathcal{F}$$

In magnetic circuits the magnetomotive force is calculated by multiplying the flux with the reluctance which is analogous to ohms law in electric circuit theory.

$$\mathcal{F} = \phi \cdot \Re$$

When calculating the MMF, the flux is known and the reluctance for the different parts of the machine is calculated.

Question 4

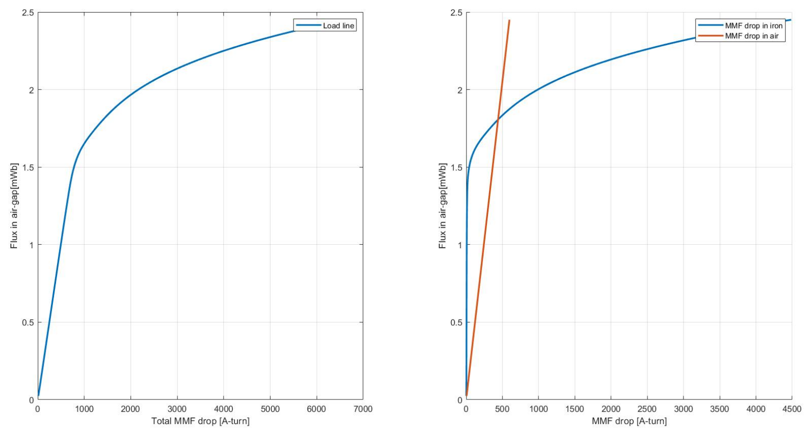

Figure 1: Load line and MMF drop in air gap and iron core separately

The slope of the load line represent the permeance which is the inverse of reluctance. Permeance is analogous to electric conductance and can be calculated using the following equation.

$$\frac{d\phi}{dF} = \frac{1}{\Re} = P = Permeance$$

Question 5

For low levels of flux the core has not been saturated. In figure 1 it becomes clear that the linear MMF drop at low levels of flux is caused by the MMF drop in the air gap. When the flux increases the core reaches saturation and the flux stops increasing. The reason for this behaviour of the core is because the permeability is not constant.

Question 6

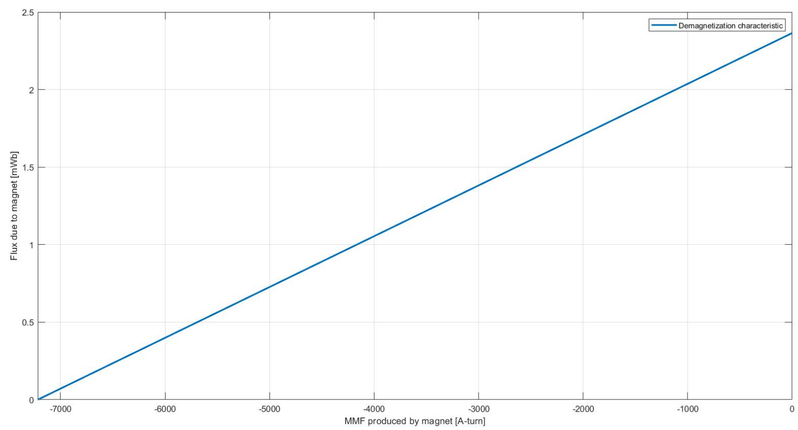

Figure 2: Demagnetization characteristic

The open circuit MMF is -9022.85A-turns and the short circuit flux is 2.3648mWb.

Question 7

Magnet thickness $= 5.00mm$ Magnet width $= 20.00mm$

Question 8

No load flux density in stator tooth $= 1.56T$. No load flux density in stator yoke $= 1.64T$. No load flux density in rotor $= 0.86T$.

Question 9

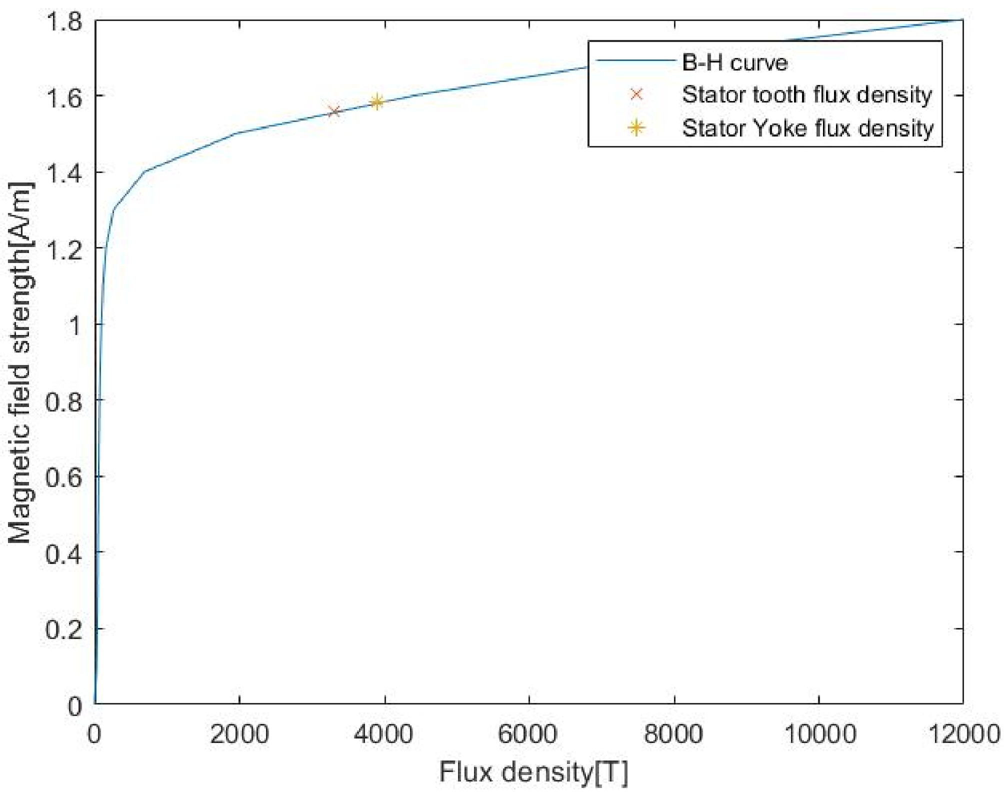

Figure 3: BH-curve of the core material with marked stator tooth and yoke flux densities

The flux density is related to the flux and the cross-section area that the flux passes through. Given that the area of the stator yoke is smaler that the stator tooth the flux density will be higher in the stator yoke since the flux is uniform in the magnetic circuit. Having a larger flux density will lead to a higher magnetic field strength since both parts are of the same material.

Question 10

$t = 4.4mm$; 1.1% change in flux and 6.35% change in $\mathcal{F}$.
$w = 22mm$; 1.99% change in flux and 21.998% change in $\mathcal{F}$. When this question was done, the magnet thickness chosen to 4 mm but this was later changed to 5 mm when optimizing and we decided against redoing this part due to time constraints. When increasing the magnet thickness by 10% the length of the

flux path through the magnet increases which leads to a higher MMF from the magnet. When changing the width the magnet the flux that is produced by the magnet will increase.

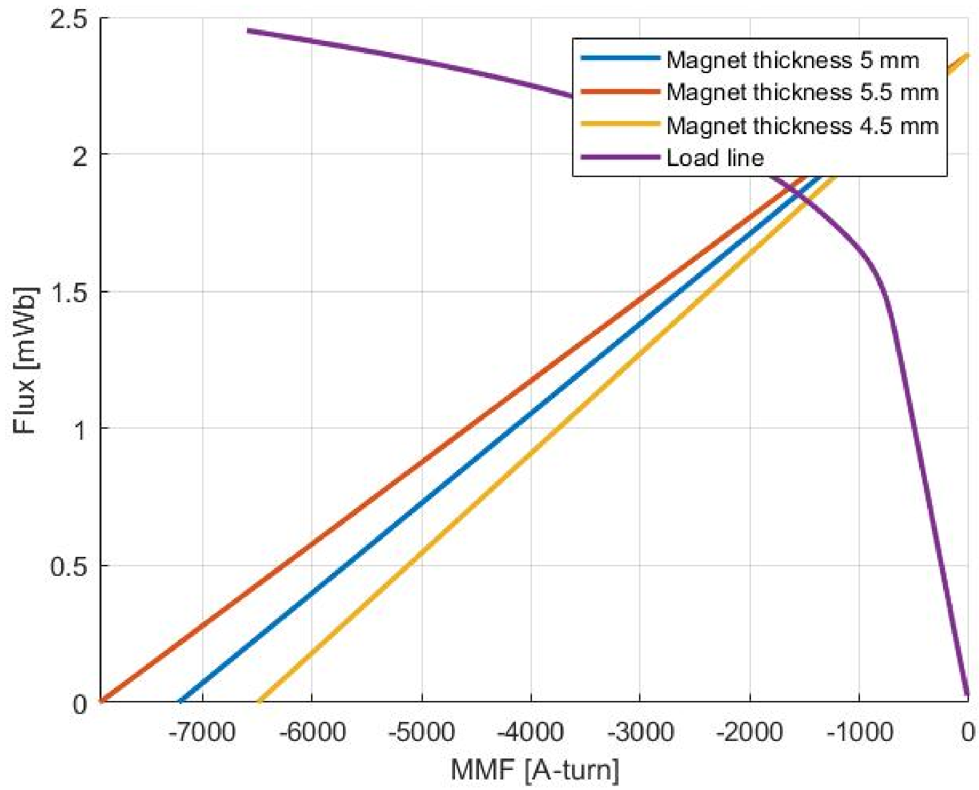

Figure 4: Load line and source line using different magnet thicknesses

2 Stator Winding Design

Question 1

The number of coils per phase under each pole pair is $\frac{Q}{2 \cdot m \cdot p} = 4$.

Question 2

We know that the total number of connected coils is given by $N^2 = 2 \cdot p$ which can also be written as $N_{series} \cdot N_{parallel} = 2 \cdot p$. We know that the parallel connected coils is equal to 4 which means the number of series connected coils is 2. This shows that 4 parallel coils under each pole pair are connected to the 2 series branches which gives a total of 8 half-repeatable groups.

Question 3

$\alpha_{elec} = \frac{360°}{2 \cdot m \cdot p} = 30°$ is the electrical slot angle.
$\alpha_{mech} = \frac{360°}{Q} = 7.5°$ is the mechanical slot angle.

Question 4

If the induced voltage of each coil is E, the induced voltage per phase under one pole pair can be given by:

$$E = 2 \cdot E_m \cdot 1 + cos\alpha$$

where E_m is the maximum induced voltage. Substituting $\alpha = 30°$, we get $E = 3.782 \cdot E_m$.

Question 5

Base speed is defined as the speed of the motor speed at the intersection of the current limit circle and voltage limit ellipse. Until base speed is reached, Maximum Torque per Ampere (MTPA) can be applied. Once base speed is reached, the back-emf of the machine touches the maximum value and requires field weakening control to be applied.

Question 6

The tooth area can be calculated by:

$$A_{tooth} = (H_{s \cdot 2} + R_s^2) \cdot w_{tooth}$$

It is found to be $68.2mm^2$
The slot area can be calculated by:

$$A_{slot} = \frac{(B_{s \cdot 1} + B_{s \cdot 2})}{2} \cdot H_{s \cdot 2} + (B_{s \cdot 2} - 2 \cdot R_s) \cdot R_s + \frac{\pi}{2} \cdot R_s^2$$

where $B_{s \cdot 2}$ is the upper base of the slot, $B_{s \cdot 1}$ is the lower base of the slot, $H_{s \cdot 2}$ is the height of the slot body and R_s is the radius of the half-circle at the corners of the slot.

It is found to be $69.414mm^2$.

Question 7

When the machine is at base speed, the EMF cannot be higher than the maximum line terminal voltage, $U_{line \cdot max}$ and is given by the following equation:

$$U_{line \cdot max} \geq E_{line} = 400V$$

To get an integral number of turns, $N_{turn \cdot coil}$ is given by:

$$N_{turn \cdot coil} = Floor(\frac{U_{line \cdot max}}{\sqrt{3} \cdot (p \cdot \Omega_r) \cdot (\phi_{pole} \cdot q \cdot k_w \cdot \frac{2 \cdot p}{N_{parallel}})}) \tag{1}$$

We have $\Omega_r = \frac{4000 \cdot \pi}{30}$ which gives $\frac{400 \cdot \pi}{3}$ rad/s. $p = 4$, $q = 2$ and $r = 2$. Winding factor, k_w is given by:

$$k_w = k_p \cdot k_d = sin(\frac{y_1}{\tau} \cdot \frac{\pi}{2}) \cdot sin(\frac{\frac{q \cdot \alpha}{2}}{q \cdot sin\frac{\alpha}{2}})$$

Pole pitch $\tau = \frac{Q}{2 \cdot p} = 6$ and the coil pitch $y_1 = \tau - 1 = 5$, then the winding factor $k_w = 0.933$.

Substituting all the obtained values in 1, we get that the number of turns per coil, $N_{turn \cdot coil}$ is 20.

The total number of turns per phase $N_{turn \cdot ph} = N_{turn \cdot coil} \cdot N_{coil \cdot ph}$.

$$N_{coil \cdot ph} = \frac{q \cdot r}{2 \cdot N_{series}}$$

where,

$$N_{series} = \frac{p}{N_{parallel}}$$

gives the $N_{coil \cdot ph} = 4$. Therefore, number of turns per phase is $4 \cdot 19 = 76$.

$N_{cond \cdot slot} = N_{turn \cdot coil} \cdot r \cdot N_{strand}$. We consider that $N_{strand} = 2$ since we have two layers.
Therefore, the number of conductors per slot $N_{cond \cdot slot} = 19 \cdot 2 \cdot 2 = 76$.

Question 8

The given speed at no-load is 12000rpm. So,$\Omega_r = 400 \cdot \pi$ rad/s.

We know that the induced voltage is given by:

$$E_{line} = \sqrt{3} \cdot E_{phase} = \sqrt{3} \cdot p \cdot \Omega_r \cdot \frac{\phi_{pole} \cdot N_{turn} \cdot q \cdot k_w \cdot r \cdot p}{N_{parallel}}$$

By solving, we get $E_{line,new} = 807.8445V$.

Question 9

The DC link voltage is $400V$. The ratio of line-line induced voltage at $12000RPM$ and DC link voltage is given by:

$$\frac{E_{line}}{V_{DC}} = \frac{807.8447}{400} = 2.0196$$

E_{line} is almost twice the DC link voltage at $12000rpm$. This can cause winding insulation failure if its withstand voltage is low. So, a negative d-axis current must be injected to decrease the airgap flux.

Question 10

The given maximum speed is $12000rpm$. We know that for a PMSM traction motor,

$$\frac{E_{line \cdot old}}{E_{line \cdot new}} = \frac{n_{max \cdot old}}{n_{max \cdot new}}$$

The given ratio is $\frac{E_{line \cdot old}}{V max} = 2$.

Using $E_{line \cdot old} = 800V$, $\frac{E_{line \cdot old}}{E_{line \cdot new}} = \frac{800}{807.8447}$ which gives the $n_{max \cdot new} = 11883.472rpm$.

The given ratio $\frac{E_{line \cdot old}}{V max} = 3$.

Using $E_{line \cdot old} = 1200V$, $\frac{E_{line \cdot old}}{E_{line \cdot new}} = \frac{1200}{807.8447}$ which gives the $n_{max \cdot new} = 17825.208rpm$.

The rough sketch for torque-speed boundary of the machine can be indicated as follows:

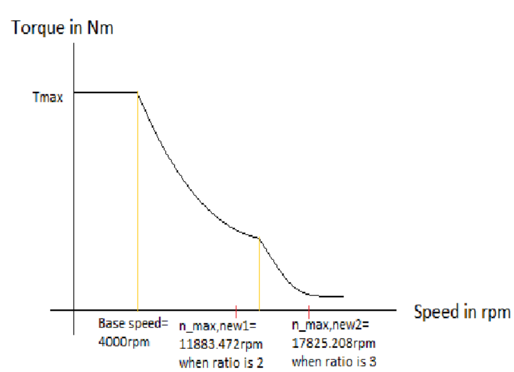

Figure 5: Torque-Speed boundary of the machine

Question 11

When field weakening is not applied i.e. when a negative d-axis current is not injected, the machine flux will be the same as the value at base speed. But the DC link voltage is the same as voltage of the battery and cannot be altered by field weakening. We know that:

$$\frac{\Omega_r.new \cdot \psi_s}{\Omega_r.old \cdot \psi_s} = \frac{E_{line \cdot old}}{E_{line \cdot new}} = \frac{n_{max \cdot old}}{n_{max \cdot new}}$$

To change the EMF at a certain speed, the flux linkage should be increased. By increasing the flux of the machine or by changing the PWM frequency of the inverter, the ratio $\frac{E_{line}}{V_{DC}}$ can be changed. But increasing the speed beyond the constant power region of the torque-speed plot, the PMSM performance decreases.

Question 12

The copper fill factor, k_{fill} is assumed as 45%. The slot copper area is obtained by:

$$SlotCopperArea, A_{Cu} = \frac{\pi}{4} \cdot d_{strand}^2 \cdot N_{strand} \cdot N_{turn} \cdot r$$

Taking N_{strand}=2 and we get A_{Cu}=30.9mm^2

Question 13

N_{strand} is given by:

$$N_{strand} = Floor(\frac{\frac{A_{slot} \cdot k_{fill \cdot max}}{r \cdot N_{turn}}}{\frac{\pi}{4} \cdot d_{strand}^2}) = 2$$

K_{fill} is given by:

$$k_{fill} = \frac{\frac{\pi}{4} \cdot d_{strand}^2 \cdot N_{strand} \cdot N_{turn} \cdot r}{A_{slot}} \cdot 100$$

It is found to be 44.58%. The difference between the maximum copper fill factor and the obtained fill factor is 45-44.58=0.42%. The copper fill factor can be improved by increasing the diameter of the strand such that N_{strand} is not below 1.

Question 14

We changed the wire diameter to 0.72mm, which gives us $K_{fill} = 44.58\%$. We chose not to go with a more precise diameter due to manufacturing difficulties that may arise with a higher precision of the d_{strand}.

3 Resistance and Inductance Calculations

Question 1

To calculate the per-phase resistance the following equation is used:

$$R_s = \rho \cdot \frac{l_{coil} \cdot N_{turn}}{\frac{\pi}{4} \cdot d_{Strand}^2} \cdot \frac{r \cdot q \cdot p}{N_{Strand} \cdot N_{parallel}^2}$$

Where L_{coil} is calculated according to:

$$l_{coil} = 2.8 \cdot L_{stack}$$

It is assumed that the end winding length is equal to 80% of the stack length. The length of the end windings is mainly determined by the slot width and the tooth width as well as the coil pitch.

The per-phase resistance is 144.5mΩ.

The end winding length is determined by the slot distribution. If the coils are short-pitched, the advantage is that the end windings will be shorter and give rise to lower leakage flux and copper losses.

Question 2

The permeability of the magnet is calculated using the demagnetization curve of the magnet. The relation between flux density and magnetic field intensity is given by the following equation.

$$B_{mag} = \mu_0 \cdot \mu_r \cdot H_{mag}$$

The relative permeability is then obtained by calculating the following equation:

$$\mu_r = \frac{B_{mag}}{\mu_0 \cdot H_{mag}}$$

The magnetic permeability of the magnet is $1.0428H/m$ and the magnetic permeability of air is $4\pi \cdot 10^-7H/m$. The magnetic permeability is much higher than the permeability of air.

Question 3

The reluctance in q- and d-axis is calculated by the following equations:

$$\Re_d = \frac{\mathcal{F}_{Total}}{\phi} + \Re_{mag}$$

$$\Re_q = \frac{\mathcal{F}_{Total}}{\phi}$$

Where the magnet reluctance is calculated using:

$$\Re_{mag} = \frac{\mathcal{F}_{mag}}{\phi_{mag}} = \frac{2 \cdot H_c \cdot t_{mag}}{B_r \cdot A_{mag}} = \frac{2 \cdot H_c \cdot t_{mag}}{B_r \cdot w_{mag} \cdot L_{stack}}$$

The reluctance of the d-axis flux path is $4.0300 \cdot 10^6$ Ω and the reluctance of the q-axis path is $9.7765 \cdot 10^5$ Ω.

Question 4

The inductance in d- and q-axis is calculated using the following equations:

$$L_d = \frac{N_d^2}{\Re_d} \cdot \frac{p}{N_{parallel}}$$

$$L_q = \frac{N_q^2}{\Re_q} \cdot \frac{p}{N_{parallel}}$$

To calculate N_d and N_q the following equation is used

$$N_d = N_q = N_{turn} \cdot k_w \cdot q \cdot r$$

The d-axis inductance is $0.3mH$ and the q-axis inductance is $1.4mH$.

Question 5

The saliency is defined as the ratio between L_q and L_d.

$$Saliency = \frac{L_q}{L_d}$$

The inductances affect the flux-linkage which is used to calculate the torque. The torque equation is given as:

$$T_{em} = \frac{3}{2} \cdot p \cdot (\psi_d \cdot I_d - \psi_q \cdot I_q)$$

And the flux linkages as:

$$\psi_d = L_d \cdot I_d + \psi_{mag}$$

$$\psi_q = L_q \cdot I_q$$

From these equations, the effect saliency has on torque is shown. A lower saliency will give a lower torque since L_q will be smaller in relation to L_d. To increase the saliency, one can increase the inductance by having lower reluctance in the magnet. This will increase the reluctance in d-axis but not in q-axis. Hence the inductance in d-axis will decrease and the inductance in q-axis is unchanged. In order to decrease the magnet reluctance the dimensions of the magnet should be changed. One could either increase the magnet thickness or decrease the stack length or the magnet width.

4 Load Calculations

Question 1

To calculate the d and q axis currents, the following equations are used:

$$I_d = I_s \cdot \cos\theta$$

$$I_q = I_s \cdot \cos\theta$$

We are given that:

$$I_s = 100A$$

$$\theta = 90°$$

Here, I_s is the stator current and θ is the current angle.

The d-axis current is calculated to be $0A$ and the q-axis current is calculated to be $100A$ at the given current angle of $90°$.

Question 2

To calculate the d and q axis flux linkages, the following equations are used:

$$\psi_d = L_d \cdot I_d + \psi_{mag}$$

$$\psi_q = L_q \cdot I_q$$

where,

$$\psi_{mag} = \frac{\phi_{pole} \cdot N_{turn} \cdot q \cdot k_w \cdot r \cdot p}{N_{parallel}}$$

The d-axis flux linkage is calculated to be $0.1312Wb$. The q-axis flux linkage is calculated to be $0.4727Wb$.

Question 3

To calculate the d and q axis inductance, the following equations are used:

$$L_d = \frac{N^2}{\Re_d} \cdot \frac{p}{N_{parallel}}$$

$$L_q = \frac{N^2}{\Re_q} \cdot \frac{p}{N_{parallel}}$$

where,

$$\Re_d = \frac{\mathcal{F}}{\phi} + \Re_{mag}$$

c

$$\Re_q = \frac{\mathcal{F}}{\phi}$$

Here, N is the number of turns. It is assumed to be the same for the d and q axes.

The d-axis inductance is calculated to be $1.0mH$. The q-axis inductance is calculated to be $4.7mH$.

Question 4

The electromagnetic torque is given by the equation:

$$T_{em} = \frac{3}{2} \cdot p \cdot (\psi_d \cdot I_q + \psi_q \cdot I_d)$$

where T_{em}, the electromagnetic torque is calculated to be $78.7342Nm$.

$$U_s = \sqrt{U_d^2 + U_q^2}$$

Where U_s, the terminal voltage is calculated to be $395.088V$.

Question 5

The frequency of the magnetic field in all the parts of the machine is given by the formula:

$$f_{elec} = \frac{\omega_{elec}}{2 \cdot \pi}$$

The frequency was calculated to be $200Hz$.
The Magnetic field density in different parts of the machine remain the same. Please refer to Part 1 Question 8 for the values.

Question 6

Copper losses are given by the formula:

$$P_{Cu} = 3 \cdot I_s^2 \cdot R_s$$

The Copper losses are calculated to be $2167.2W$.
Iron losses are given by the formula:

$$P_{Fe} = P_{Fe \cdot yoke} + P_{Fe \cdot tooth}$$

where,

$$P_{Fe\cdot yoke} = K_h \cdot B^2_{yoke\cdot noload} \cdot f_{elec} + K_c \cdot B^2_{yoke\cdot noload} \cdot f^2_{elec} \cdot Volume_{yoke}$$

$$P_{Fe\cdot tooth} = K_h \cdot B^2_{tooth\cdot noload} \cdot f_{elec} + K_c \cdot B^2_{tooth\cdot noload} \cdot f^2_{elec} \cdot Volume_{tooth}$$

$$Volume_{yoke} = (OD^2_{stator} - ID^2_{stator}) \cdot \frac{\pi}{4} - (A_{slot} + A_{tooth}) \cdot N_{slot}) \cdot L_{stack}$$

$$Volume_{tooth} = A_{tooth} \cdot L_{stack} \cdot N_{slot}$$

The Iron losses are calculated to be $80.8106W$.

Question 7

Shaft Power is given by the formula:

$$P_{em} = T_{em} \cdot \omega_r$$

The shaft power is calculated to be $24735.6W$. The Input Power is given by the formula:

$$P_{in} = P_{em} + P_{Fe} + P_{Cu}$$

The input power is calculated to be $26988W$.

Question 8

Efficiency is given by the formula:

$$\eta = \frac{P_{em}}{P_{in}}$$

Efficiency is calculated to be 91.65%.
Power Factor is given by the formula:

$$PF = \frac{P_{in}}{S_{in}}$$

where,

$$S_{in} = \sqrt{3} \cdot U_{s\cdot line} \cdot I_{s\cdot phase}$$

The Power Factor is calculated to be 0.29.

Question 9

The plots showing the variation of the terminal voltage and electromagnetic torque with the current angle is shown below:

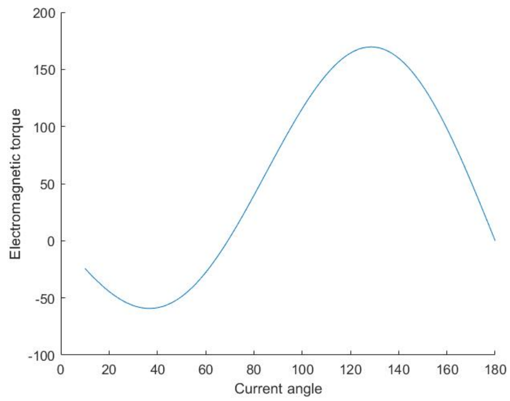

Figure 6: Electromagnetic Torque VS Current Angle

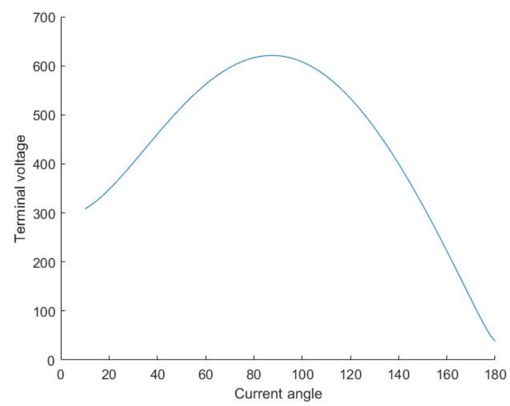

Figure 7: Terminal Voltage VS Current Angle

Conclusion

In this Design Assignment, we took the reference values of a given Permanent Magnet Synchronous Machine, analyzed the performance of the machine and made changes to optimize the output parameters to the given target values.

During this process, we learnt how to design an electrical machine for the given requirements using a design procedure that is very similar to how it is done in the industry currently.