MECHANICAL DESIGN METHODS OF ROBOT

DESIGN OF A THREE DEGREES OF FREEDOM PLANAR PARALLEL MANIPULATOR

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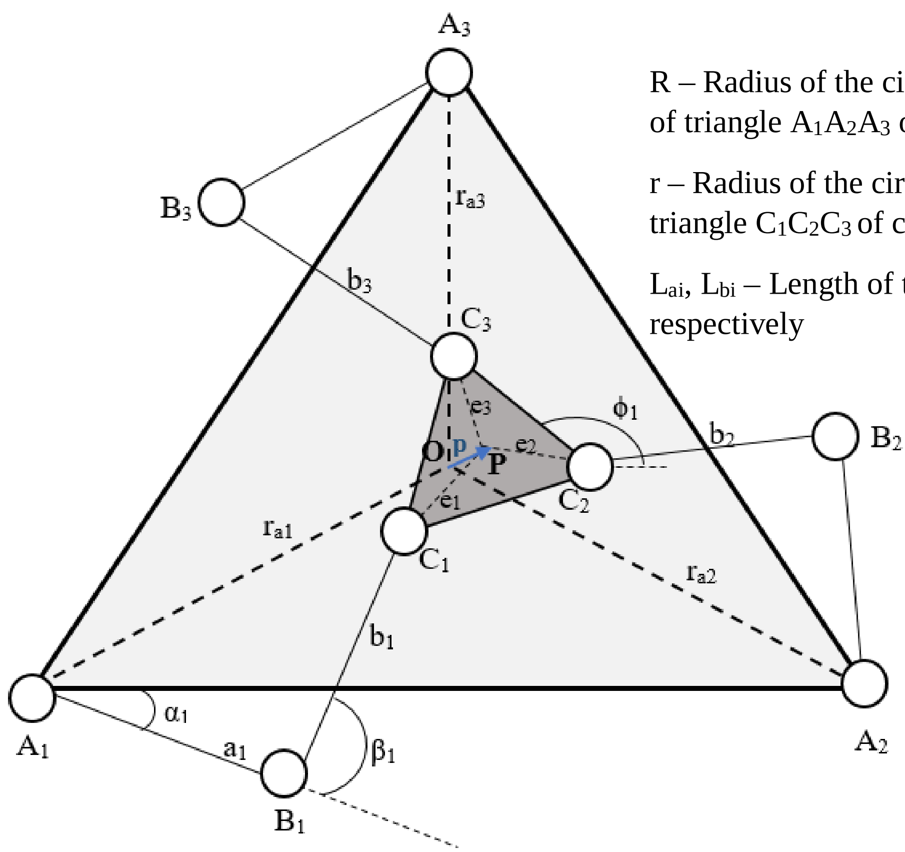
Aim : To design a three degrees of freedom planar parallel manipulator with a cylindrical regular workspace of diameter equal to 100 mm, its height corresponding to the range of rotation of the moving-platform.

1. Mechanism and Architecture

For a planar parallel manipulator all the component and motions, including the end effectors generate planar motions. There a several mechanisms for 3DOF PPMs like 3-RRR, 3-RPR, 3-PRR and 3-PPP. For the 3DOF PPM, we have chosen to be 3-RRR where 3 denotes three limbs of the manipulator and R stands for revolute joints. The schematics of the mechanism is shown in Figure 1. As seem in the figure each limb is comprised of three revolute joints out of which the first joint of each limb is actuated. The axis of rotation of all the joints are parallel to each other and are all on the same plane. The workspace of the manipulator is a cylinder with diameter equal to 100 mm.



R – Radius of the circumscribed circle of triangle $A_1A_2A_3$ of center O

r – Radius of the circumscribed circle of triangle $C_1C_2C_3$ of center P

L_{ai}, L_{bi} – Length of the link A_iB_i, B_iC_i respectively

Figure 1: Schematic of 3RRR Planar Parallel Manipulator

Figure 1 illustrates a 3DOF RRR Planar Parallel Manipulator with a cylindrical workspace. Each kinematic chain is RRR type, which has three revolute joints out of which the first one in each limb is the actuating joint. The intention of the manipulator is to move, position and orient the triangle $C_1C_2C_3$. P denotes the geometric center of the moving platform. The angle α_1, α_2 and α_3 are the angles made by the actuating joints. O denotes the center of the base triangle $A_1A_2A_3$ and it is the origin of the reference frame.

2. Kinematic Modelling of 3-RRR Planar Parallel Manipulator

2.1 Direct and Inverse Kinematic Models

The Direct Kinematic Model of this manipulator represents the position of the moving platform in terms of the joint values. This particular manipulator may have up to six solutions as each limb has two assembly modes so in total it has six different assembly modes.
The inverse Kinematic model of this manipulator is a representation of the joint values in terms of moving platform's pose. This manipulator may have up to eight solutions since there are two working modes for each limb.

2.2 Kinematic Jacobian

The relationship between the moving platform's velocity vectors and the actuators is defined by the Kinematic Jacobian matrix. The Kinematic Jacobian matrix is decomposed into Forward Kinematic Jacobian Matrix and Inverse Kinematic Jacobian Matrix.The Jacobian matrix is found using geometric analysis.

For the first kinematic chain, the closed loop vector equation can be written as :

$$\overrightarrow{OP} = \overrightarrow{OA_1} + \overrightarrow{A_1B_1} + \overrightarrow{B_1C_1} + \overrightarrow{C_1P} \tag{1}$$

This equations can extended to any of the limbs , So generally it can be formulated as :

$$\overrightarrow{OP} = \overrightarrow{OA_i} + \overrightarrow{A_iB_i} + \overrightarrow{B_iC_i} + \overrightarrow{C_iP} \tag{2}$$

where i = 1, 2, 3

Equation (2) can also be expressed as :

$$p = R\mathbf{r_{ai}} + L_{ai}\mathbf{a}_i + L_{bi}\mathbf{b_i} + r\mathbf{e_i} \tag{3}$$

where $\mathbf{r_{ai}}$,$\mathbf{a_i}$, $\mathbf{b_i}$ and $\mathbf{e_i}$ are the unit vectors along that direction.

Differentiating equation (3) we get ,

$$\dot{p} = L_{ai}\dot{\alpha}_i E\mathbf{a}_i + L_{bi}\,(\dot{\alpha}_i+\dot{\beta}_i)\,E\mathbf{b_i} + r\dot{\phi}_i E\mathbf{e_i} \tag{4}$$

where E is the right angle rotation matrix given by :

$$E = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Here in equation (4), α is active, whereas β is a passive angle. Hence, to get rid of β equation (4) is multiplied with $\mathbf{b_i}^{\mathbf{T}}$

$$\therefore \quad \mathbf{b_i}^{\mathbf{T}}\,\dot{p} = \mathbf{b_i}^{\mathbf{T}}\,L_{ai}\dot{\alpha}_i E\mathbf{a}_i + L_{bi}(\dot{\alpha}_i+\dot{\beta}_i)\,\mathbf{b_i}^{\mathbf{T}}\,E\mathbf{b_i} + \mathbf{b_i}^{\mathbf{T}}\,r\dot{\phi}_i E\mathbf{e_i} \tag{5}$$

Rearranging equation (5), we will get

$$\mathbf{b}_i^T \dot{p} - \mathbf{b}_i^T r\dot{\phi}E\mathbf{e}_i = L_{ai}\mathbf{b}_i^T \dot{\alpha}_i E\mathbf{a}_i + L_{bi}(\dot{\alpha}_i+\dot{\beta}_i)\, \mathbf{b}_i^T E\mathbf{b}_i \tag{6}$$

Equation (6) can be written in matrix form as :

$$\begin{bmatrix} \mathbf{b}_1^T & -r\, \mathbf{b}_1^T E\mathbf{e}_1 \\ \mathbf{b}_2^T & -r\, \mathbf{b}_2^T E\mathbf{e}_2 \\ \mathbf{b}_3^T & -r\, \mathbf{b}_3^T E\mathbf{e}_3 \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} L_{a1}\mathbf{b}_1^T E\mathbf{a}_1 & 0 & 0 \\ 0 & L_{a2}\mathbf{b}_2^T E\mathbf{a}_2 & 0 \\ 0 & 0 & L_{a3}\mathbf{b}_3^T E\mathbf{a}_3 \end{bmatrix} \dot{\boldsymbol{\alpha}} \tag{7}$$

where $\dot{\boldsymbol{\alpha}} = \begin{bmatrix} \dot{\alpha}1 \\ \dot{\alpha}2 \\ \dot{\alpha}3 \end{bmatrix}$ and $\dot{p} = \begin{bmatrix} \dot{p}x \\ \dot{p}y \end{bmatrix}$

In matrix form we know that

$$A\dot{p} = B\dot{\alpha} \tag{8}$$

Therefore by comparing equation (7) and equation (8) we find

$$A = \begin{bmatrix} \mathbf{b}_1^T & -r\, \mathbf{b}_1^T E\mathbf{e}_1 \\ \mathbf{b}_2^T & -r\, \mathbf{b}_2^T E\mathbf{e}_2 \\ \mathbf{b}_3^T & -r\, \mathbf{b}_3^T E\mathbf{e}_3 \end{bmatrix} = \begin{bmatrix} \cos(\alpha1+\beta1) & \sin(\alpha1+\beta1) & r\sin(\gamma1) \\ \cos(\alpha2+\beta2) & \sin(\alpha2+\beta2) & r\sin(\gamma2) \\ \cos(\alpha3+\beta3) & \sin(\alpha3+\beta3) & r\sin(\gamma3) \end{bmatrix} \tag{9}$$

and

$$B = \begin{bmatrix} L_{a1}\mathbf{b}_1^T E\mathbf{a}_1 & 0 & 0 \\ 0 & L_{a2}\mathbf{b}_2^T E\mathbf{a}_2 & 0 \\ 0 & 0 & L_{a3}\mathbf{b}_3^T E\mathbf{a}_3 \end{bmatrix} = \begin{bmatrix} L_{a1}\sin(\beta1) & 0 & 0 \\ 0 & L_{a2}\sin(\beta2) & 0 \\ 0 & 0 & L_{a3}\sin(\beta3) \end{bmatrix} \tag{10}$$

where A is the Forward Kinematic Jacobian and B is the Inverse Kinematic Jacobian of this 3-RRR PM.

The Kinematic Jacobian, **J** of this manipulator is given by:

$$J = B^{-1}A \tag{11}$$

2.3 Singularity Analysis

Kinematic Singularities are of two types, parallel singularity and serial singularity. If determinant of matrix A becomes equal to zero, it results in parallel singularity and if determinant of matrix B becomes equal to zero then it results in serial singularities.

Here if determinant of A becomes zero it causes b1//b2//b3, which results in the parallel singularity. If determinant of B becomes equal to zero it causes $b1 \perp b2 \perp b3$ which causes serial singularity.

3. Stiffness Matrix

In order to model an accurate model of the given mechanism, it is of great importance to precisely portray the stiffness of the manipulator, which depicts the relationship between the force applied and the displacement along the normal of the plane of movement. This relationship is mathematically represented with the use of stiffness matrix

Stiffness matrix of the actuator is given by

$$K_{actuator}^{-1} = \frac{1}{L/\ E \times (AL_{bi})} \tag{12}$$

where L is the characteristic length .

The stiffness matrix of actuator, leg and the moving platform is combined into a single diagonal matrix as :

$$K_{\theta}^{-1} = \begin{bmatrix} K_{act}^{-1} & 0_{1X6} & 0_{1X6} \\ 0_{1X6} & K_{leg}^{-1} & 0_{1X6} \\ 0_{1X6} & 0_{1X6} & K_{mp}^{-1} \end{bmatrix} \tag{13}$$

From the equation (13) the stiffness can be found using :

$$\therefore \text{Stiffness} = J^{T}_{reshaped} * K_{\theta}^{-1} * J$$

where $J^{T}_{reshaped}$ is the transpose of Jacobian, which is reshaped from 3 X 26 to 13 X 6.

Stiffness matrix for the intermediate legs are :

$$K_{leg}^{-1} = \begin{bmatrix} \frac{L}{EA_{Lb}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{L}{3I_z} & 0 & 0 & 0 & \frac{L^2}{2EI_z} \\ 0 & 0 & \frac{L^3}{3EI_y} & 0 & \frac{-L^2}{2EI_y} & 0 \\ 0 & 0 & 0 & \frac{L}{GI_x} & 0 & 0 \\ 0 & 0 & \frac{-L^2}{2EI_y} & 0 & \frac{L}{EI_y} & 0 \\ 0 & \frac{L^2}{2EI_z} & 0 & 0 & 0 & \frac{L}{EI_z} \end{bmatrix} \tag{14}$$

Similarly the stiffness matrix for the moving platform can be written as :

$$K_{mp}^{-1} = \begin{bmatrix} \frac{r}{EAr} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{r}{3I_z} & 0 & 0 & 0 & \frac{r^2}{2EI_z} \\ 0 & 0 & \frac{r^3}{3EI_y} & 0 & \frac{-r^2}{2EI_y} & 0 \\ 0 & 0 & 0 & \frac{r}{GI_x} & 0 & 0 \\ 0 & 0 & \frac{-r^2}{2EI_y} & 0 & \frac{r}{EI_y} & 0 \\ 0 & \frac{r^2}{2EI_z} & 0 & 0 & 0 & \frac{L}{EI_z} \end{bmatrix} \tag{15}$$

where E is the Young modulus of the material and G is the Shear Modulus.
A is the cross sectional area of the beam

4. Design Optimization

Objective: The aim of optimization problem is to understand, determine and modify the optimum geometric parameters a parallel kinematic chain so as to minimize the mass in motion of the mechanism. MATLAB is used for optimizing and the function *fmincon()* is used. The following constraints are to be validated throughout the workspace.

→The workspace of the given mechanism is described to be a cylinder with diameter equal to 100mm.

→The range of rotation should be greater than 60°.

4.1 Mass in Motion of the Mechanism

$$m_{total} = m_{pf} + 6m_b \tag{16}$$

where m_{pf} is the mass of moving platform

m_b is the mass of the three intermediate links

$m_{pf} = \pi r^2{}_p hd - \pi r_p{}^2 hd$

$m_b = L_b hd$

$$\therefore \quad f_1(x) = m_t \rightarrow \min \tag{17}$$

where x is the design variable vector of the mechanisms

4.2 Regular Workspace Size

The workspace defines the working volume of the manipulator and the area that could be reached by a reference frame located on the moving platform. Workspace based design optimization of parallel mechanisms can usually be solved with two different formulations.
The first formulation aims to design a manipulator whose workspace contains a prescribed workspace and the second approach aims to design a manipulator, of which the workspace is as large as possible.

For this manipulator, a cylindrical workspace defined with its radius R_w is considered. So in order to maximize the manipulator workspace, the objective of the optimization problem can be written as :

$$f_2(x) = R_w \rightarrow \max \tag{18}$$

For the manipulator designed R_w is chosen to be 50mm
At each point of the workspace, the range of rotation ($\Delta\phi$) of the moving platform should be higher than 60°. For this manipulator, it is designed in such a way that $\Delta\phi$ = 70°(about z axis).

4.3 Geometric Constraints

The first constraint, which is related to the mechanism assembly, is described by the expression as given:

$$g1 : \sqrt{2Lbi^2 - \left(\sqrt{\frac{3}{2}}(R-r)\right)^2} + \frac{1}{2}r - \frac{1}{2}R \ \leq \ 50 \tag{19}$$

4.5 Condition Numbers

The condition number is given by :

$$\kappa(J) = \left|\left|J\right|\right| \, \left|\left|J^{-1}\right|\right| \tag{20}$$

For design optimization of 3RRR PPM , it is ideal to chose the minimum of inverse of condition number to be greater that a optimum value, here 0.1 throughout its workspace. Hence,

$$g2 : \min(\kappa^{-1}(J)) > 0.1 \tag{21}$$

4.5 Accuracy Constraints

The position and orientation accuracy is measured based on different stiffness parameters of the mechanism.
Let (δx, δy,δz) and ($\delta\varphi x$, $\delta\varphi y$, $\delta\varphi z$) be the position and orientation errors of the moving platform subjected to external forces (Fx, Fy, Fz) and torques (τx,τy,τz).

$$\delta x \leq \delta x_{max}\,,\ \delta y \leq \delta y_{max}\,,\ \delta z \leq \delta z_{max}\,,$$
$$\delta\varphi x \leq \delta\varphi x_{max},\ \delta\varphi y \leq \delta\varphi y_{max},\ \delta\varphi z \leq \delta\varphi z_{max} \tag{22}$$

where (δx_{max}, δy_{max}, δz_{max}) being the maximum allowable position errors and ($\delta\varphi x_{max}$, $\delta\varphi y_{max}$, $\delta\varphi z_{max}$) the maximum allowable orientation errors of the moving platform.The accuracy constraints can be expressed in terms of the components of the mechanism stiffness matrix and the wrench applied to the moving platform.

Let the accuracy requirements be :

$$\delta z \leq 0.0001\text{m} \tag{23}$$

Consider the moving plate is exposed to a wrench, whose components are:

$$\|Fxy\| = Fz = 10 \text{ N} \tag{24}$$

Then the accuracy constraints will be,

$$g3 : k_z^{min} \geq \frac{Fz}{\delta z} = \frac{10}{0.0001} = 10^5 \text{ Nm}^{-1} \tag{25}$$

$$g4 : k\varphi z^{min} \geq \frac{\tau z}{\delta\varphi z} = \frac{10 \text{ x } 0.1 \text{ x } 10^{-3}}{\frac{\pi}{180} \text{ x } 60} = 9.549 \text{ x } 10^{-4} \text{ Nm rad}^{-1} \tag{26}$$

4.6 Design Variables

The design variables also known as decision variables also includes r_j which is the dimension of the circular cross section of the intermediate bars and r_p which is the circular cross section of the platform bars. It is assumed that the platform is made up three circular bars each of length r.

The design variable vector **x** is given by:

$$\mathbf{x} = \begin{bmatrix} L \\ r \\ r_p \\ h \\ d \end{bmatrix} \tag{27}$$

4.6 Multi Objective Optimization Problem

The multi objective optimization problem can be mathematically stated as :

Minimize → $f_1(x) = m_t$

Maximize → $f_2(x) = \Delta\phi > 60°$

Over $x = [\ R\ \ r\ \ L_b\ \ r_j\ \ r_p\ ]^T$

Subject to : $g1 \ : \sqrt{2\text{Lbi}^2 - \left(\sqrt{\frac{3}{2}}(R-r)\right)^2} + \frac{1}{2}r - \frac{1}{2}R \ \leq \ 50$

$g2 \ : \min(\kappa^{-1}(J)) > 0.1$

$g3 \ : k_z^{min} \geq \frac{Fz}{\delta z} = \frac{10}{0.0001} = 10^5 \text{ Nm}^{-1}$

$g4 : k\varphi z^{min} \geq \frac{\tau z}{\delta\varphi z} = \frac{10 \text{ x } 0.1 \text{ x } 10^{-3}}{\frac{\pi}{180} \text{ x } 60} = 9.549 \text{ x } 10^{-4} \text{ Nm rad}^{-1}$

$x_l \ \leq \ \mathbf{x} \ \leq \ x_u$

where x_l and x_u are lower and upper bound of **x** respectively.

5. Result

The given mechanism was modelled as a 3-RRR parallel planar manipulator in CATIA as shown in figure 2.The manipulator is modelled in x-y plane.

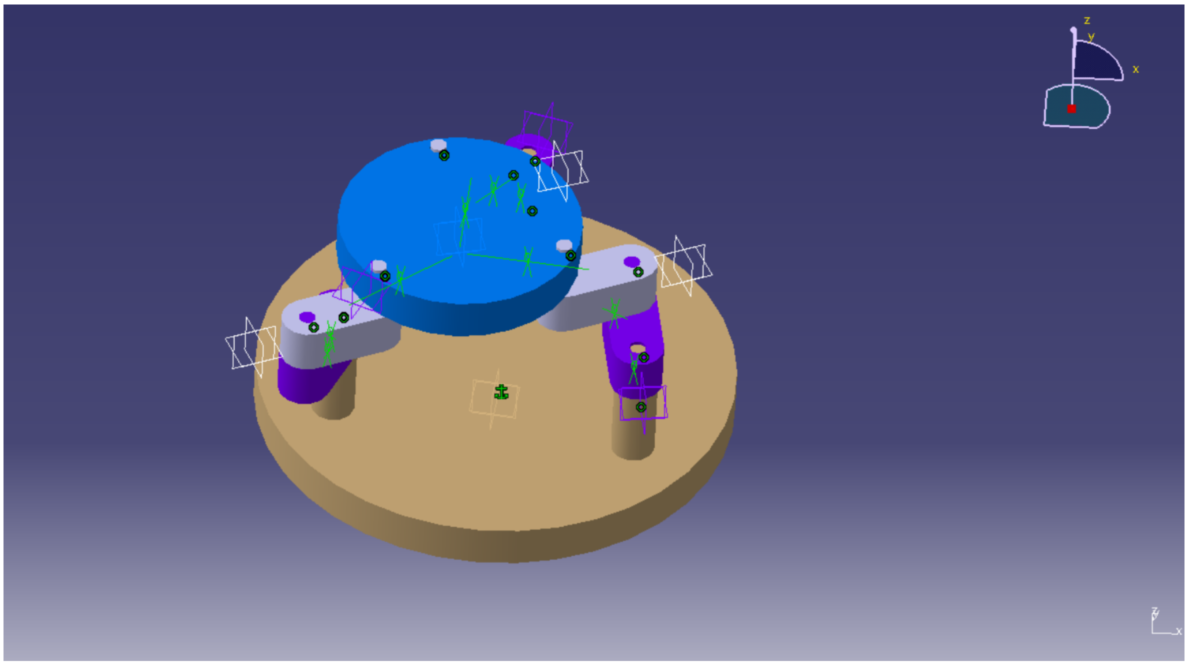


Figure 2: Simulation of 3RRR Planar Parallel Manipulator in CATIA

As shown in figure 2, the manipulator has a cylindrical workspace of base diameter equal to 100mm.The constraints are followed throughout the workspace.

The geometrical optimisation inmplemented in MATLAB aims in maximising the workspace volume as a function of design variabkes as presented in Table 1. Optimization were carried out on the initial parameters to match the constraints and workspace requirements . The optimization was performed using the function fmincon() in MATLAB. The resultant optimized values are given in Table 1.

DESIGN VARIABLES	INITIAL VALUE (in mm)	OPTIMISED VALUE (in mm)
LENGTH OF LINK	20	17.1
RADIUS OF MOVING PLATFORM	15	25
RADIUS OF HOLES ON MP	2	2
HEIGHT OF LINK	10	10.8
WIDTH OF LINK	10	12
BASE RADIUS	50	40

Table 1: Optimization Results