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## Human arm mechanism

The goal of this thesis was to design and analyse a tensegrity mechanism which mimics human arm. The proposed model has two 4-SPS-S tensegrity structures, as shown in Figure 1, base and the top platforms which mimic shoulder and wrist and both the structures are connected by a 2-SPS-R structure which mimics the elbow part of an arm. As you can see two types of mechanisms have been shown one of which has a 4-SPS-S structure at both ends and as for the other one. A tetrahedral structure is placed with 4-SPS limbs as shown in the Figure 6. The total dof of the structure is 7, where 3 is from moving platform, 3 is from fixed platform and 1 from the 2-SPS-R structure.

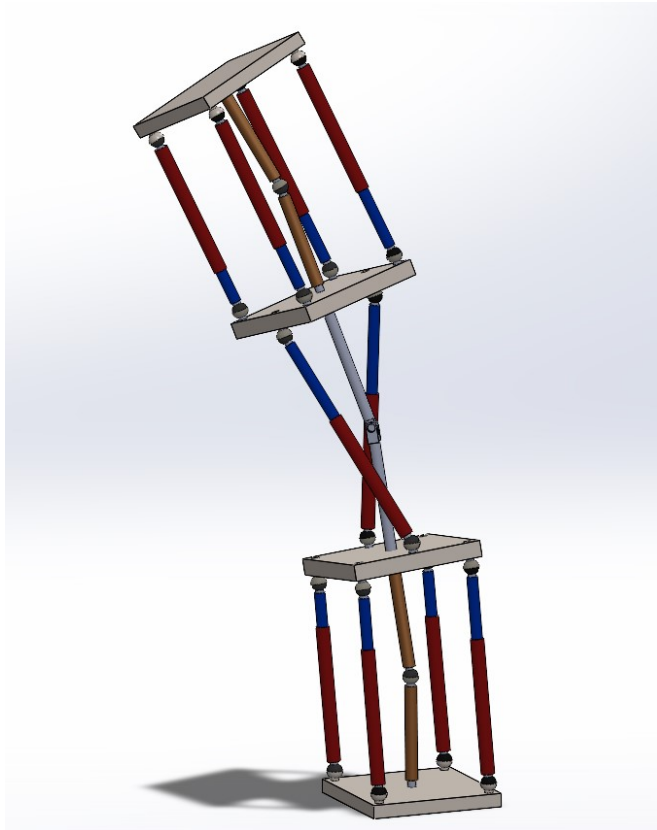


Figure 1: Human arm mimic mechanism 1

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## Kinematics Analysis of the Shoulder Joint

To derive the relationship between the lengths of the joint, a kinematics model of shoulder joint is established. As shown in Figure, fixed coordinate frame  $B_{O-X_b Y_b Z_b}$  and moving coordinate frame  $C_{O-X_c Y_c Z_c}$  are located in the center of the base frame and the end frame of the shoulder section, and frame C also acts as base frame for the elbow joint. Note that the Base frame B is connected to frame C by 4 leg joints which each constitutes of a spherical joint connected to base frame B and a prismatic joint and a spherical joint connected to the frame C, along with a central joint which connects the pyramid structure to the base frame B with a spherical joint.

For both the coordinate frames B and C the x and y axis are parallel to the sides of the base structure  $B_1, B_2, B_3, B_4$  and  $C_1, C_2, C_3, C_4$  respectively and z axis normal to the frame. The shoulder joint has the following 3 rotational DOFs: rotating the yaw angle  $\eta_x$  around the X-axis, rotating the pitching angle  $\phi_y$  around the Y-axis, and rotating the roll angle  $\theta_z$  around the Z-axis. The rotating Euler angle of roll-pitch-yaw is used to describe the transform matrix Rotz and rotXY as shown in the equations, for the moving coordinate frame C relative to the fixed coordinate frame B.

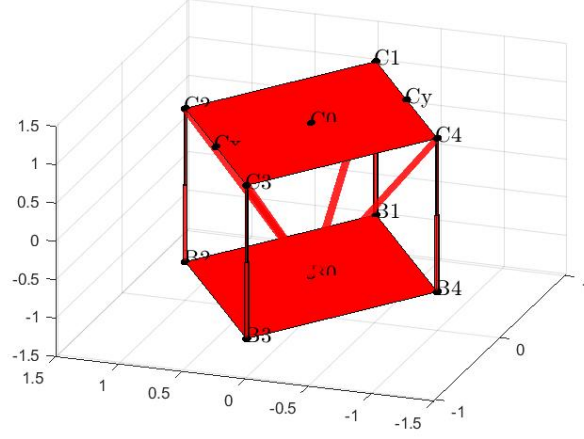


Figure 2: Shoulder Joint simulation

The coordinate system of the frame B is given as follows :

$$\begin{aligned}
 b0 &= [0; 0; -h] * r \\
 b1 &= [\cos(0 * \pi/2); \sin(0 * \pi/2); -h] * r \\
 b2 &= [\cos(1 * \pi/2); \sin(1 * \pi/2); -h] * r \\
 b3 &= [\cos(2 * \pi/2); \sin(2 * \pi/2); -h] * r \\
 b4 &= [\cos(3 * \pi/2); \sin(3 * \pi/2); -h] * r
 \end{aligned} \tag{1}$$

$$Ori = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Rot_z = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$B_i = Rot_z * b_i \tag{3}$$

where  $i = 0 - 4$

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The coordinates for the second plane , that is the next surface from the base  $C_0$  to  $C_4$  and as for  $C_x$  and  $C_y$  are the coordinates for the mid structure base as shown in the Equation 8 and Figure

$$\begin{aligned}
x &= \cos(pi/4) * r \\
A1 &= [\cos(0 * pi/2); \sin(0 * pi/2); h] * r \\
A2 &= [\cos(1 * pi/2); \sin(1 * pi/2); h] * r \\
A3 &= [\cos(2 * pi/2); \sin(2 * pi/2); h] * r \\
A4 &= [\cos(3 * pi/2); \sin(3 * pi/2); h] * r \\
A0 &= [0; 0; h] * r \\
Ax &= [\cos(1 * pi/2 + pi/4) * x; \sin(1 * pi/2 + pi/4) * x; h * r] \\
Ay &= [\cos(3 * pi/2 + pi/4) * x; \sin(3 * pi/2 + pi/4) * x; h * r]
\end{aligned} \tag{4}$$

$$rotX = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(eta) & -\sin(eta) \\ 0 & \sin(eta) & \cos(eta) \end{bmatrix} \tag{5}$$

$$rotY = \begin{bmatrix} \cos(phi) & 0 & \sin(phi) \\ 0 & 1 & 0 \\ -\sin(phi) & 0 & \cos(phi) \end{bmatrix} \tag{6}$$

$$rotXY = rotX rotY \tag{7}$$

$$\begin{aligned}
C_i &= Rotz * rotXY * A_i \\
i &= 0 \text{ to } 4 \text{ and } x, y
\end{aligned} \tag{8}$$

The length of each leg is determined and developed by using matlab simulations as shown below

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$$L_1 = \sqrt{l_{11} + l_{12} + l_{13}} \quad (9)$$

where

$$l_{11} = (hr - r \cos(\eta) \sin(\phi) + hr \cos(\eta) \cos(\phi))^2$$

$$l_{12} = (r(c(\phi)c(\theta) - s(\theta)s(\eta)s(\phi)) - r * c(\theta) + hr(s(\phi)c(\theta) + s(\theta)s(\eta)c(\phi)))^2$$

$$l_{13} = (r(c(\phi)s(\theta) + c(\theta)s(\eta)s(\phi)) - r * s(\theta) + hr(s(\phi)s(\theta) - c(\theta)s(\eta)c(\phi)))^2$$

$$L_2 = \sqrt{l_{21} + l_{22} + l_{23}} \quad (10)$$

$$l_{21} = \left( hr + r \sin(\eta) - \frac{var1 r \cos(\eta) \sin(\phi)}{var2} + hr \cos(\eta) \cos(\phi) \right)^2$$

$$l_{22} = \left( \frac{var6}{var2} - r \cos(\theta) - \frac{var1 r \sin(\theta)}{var2} + r \cos(\eta) \cos(\theta) + hr var7 \right)^2$$

$$l_{23} = \left( \frac{var8}{var2} - \frac{var1 r \cos(\theta)}{var2} + r \sin(\theta) - r \cos(\eta) \sin(\theta) + hr var9 \right)^2$$

$$L_3 = \sqrt{l_{31} + l_{32} + l_{33}} \quad (11)$$

$$l_{31} = \left( hr + \frac{var1 r \sin(\eta)}{var3} + r \cos(\eta) \sin(\phi) + hr \cos(\eta) \cos(\phi) \right)^2$$

$$l_{32} = \left( var11 + \frac{var1 r \cos(\eta) \cos(\theta)}{var3} + hr (\sin(\phi) \sin(\theta) - \cos(\phi) \cos(\theta) \sin(\eta)) \right)^2$$

$$l_{33} = (var12 + hr (\cos(\theta) \sin(\phi) + \cos(\phi) \sin(\eta) \sin(\theta)))^2$$

$$L_4 = \sqrt{l_{41} + l_{42} + l_{43}} \quad (12)$$

$$l_{41} = \left( hr - r \sin(\eta) + \frac{var4 r \cos(\eta) \sin(\phi)}{var5} + hr \cos(\eta) \cos(\phi) \right)^2$$

$$l_{42} = (var13 - r \cos(\eta) \cos(\theta) + hr (\sin(\phi) \sin(\theta) - \cos(\phi) \cos(\theta) \sin(\eta)))^2$$

$$l_{43} = (var14 - r \sin(\theta) + r \cos(\eta) \sin(\theta) + hr (\cos(\theta) \sin(\phi) + \cos(\phi) \sin(\eta) \sin(\theta)))^2$$

The variables used to shorten the equations above mentioned are as follows

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$$\text{var1} = 4967757600021511$$

$$\text{var2} = 81129638414606681695789005144064$$

$$\text{var6} = \text{var1} r (\cos(\phi) \sin(\theta) + \cos(\theta) \sin(\eta) \sin(\phi))$$

$$\text{var8} = \text{var1} r (\cos(\phi) \cos(\theta) - \sin(\eta) \sin(\phi) \sin(\theta))$$

$$\text{var11} = r s(\theta) - \frac{\text{var1} r c(\theta)}{\text{var3}} - r (c(\phi) s(\theta) + c(\theta) s(\eta) s(\phi))$$

$$\text{var12} = r c(\theta) - r (c(\phi) c(\theta) - s(\eta) s(\phi) s(\theta)) + \frac{\text{var1} r s(\theta)}{\text{var3}} - \frac{\text{var1} r c(\eta) s(\theta)}{\text{var3}}$$

$$\text{var13} = r c(\theta) - \frac{\text{var4} r (c(\phi) s(\theta) + c(\theta) s(\eta) s(\phi))}{\text{var5}} + \frac{\text{var4} r s(\theta)}{\text{var5}}$$

$$\text{var14} = \frac{\text{var4} r c(\theta)}{\text{var5}} - \frac{\text{var4} r (c(\phi) c(\theta) - s(\eta) s(\phi) s(\theta))}{\text{var5}}$$

## Kinematics Analysis of the Elbow Joint

The elbow joint is a single-DOF structure, and its kinematics model is established as shown in Figure. Fixed coordinate frame  $C_{OX_cY_cZ_c}$  and moving coordinate frame  $C_{OX_qY_qZ_q}$  are located in the center of the two coordinate frames .

For both the coordinate frames Q and C the x and y axis are parallel to the sides of the base structure  $Q_1, Q_2, Q_3, Q_4$  and  $C_1, C_2, C_3, C_4$  respectively and z axis normal to the frame. The Elbow joint has the following 1 DOF: rotating the angle  $\beta$ .

Transform matrix R1 for the moving coordinate frame Q relative to the fixed coordinate frame C is given as  $rot_{y2}$ .

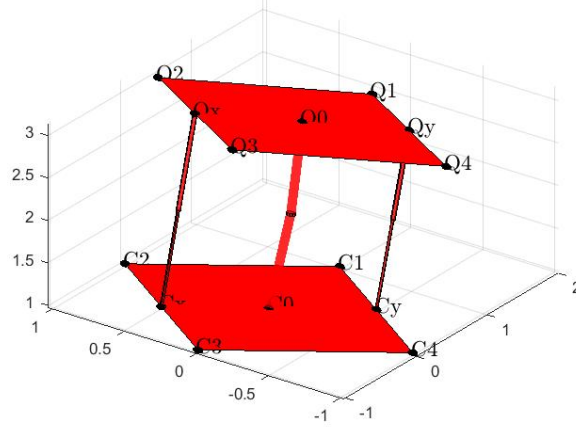


Figure 3: Elbow Joint simulation

The coordinate system of the frame Q is given as follows :

$$\begin{aligned}
 P1 &= [\cos(0 * \pi/2); \sin(0 * \pi/2); 3 * h] * r \\
 P2 &= [\cos(1 * \pi/2); \sin(1 * \pi/2); 3 * h] * r \\
 P3 &= [\cos(2 * \pi/2); \sin(2 * \pi/2); 3 * h] * r \\
 P4 &= [\cos(3 * \pi/2); \sin(3 * \pi/2); 3 * h] * r \\
 P0 &= [0; 0; 3 * h] * r \\
 Ori_2 &= [0; 0; 2 * h] * r \\
 Px &= [\cos(1 * \pi/2 + \pi/4) * x; \sin(1 * \pi/2 + \pi/4) * x; 3 * h * r] \\
 Py &= [\cos(3 * \pi/2 + \pi/4) * x; \sin(3 * \pi/2 + \pi/4) * x; 3 * h * r]
 \end{aligned} \tag{13}$$

$$rot_{z2} = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

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$$rot_{x2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix} \quad (15)$$

$$rot_{y2} = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \quad (16)$$

$$Q_i = rotY2 * P_i; i = 0 \text{ to } 4 \text{ and } x, y \quad (17)$$

The length of each leg is determined and developed by using matlab simulations as shown below

$$L_5 = \sqrt{l_{51} + l_{52} + l_{53}} \quad (18)$$

$$\begin{aligned} l_{51} &= \left( \frac{r \sin(\eta)}{2} - \frac{r \sin(\beta)}{2} + \frac{r \cos(\eta) \sin(\phi)}{2} - 3hr \cos(\beta) + hr \cos(\eta) \cos(\phi) \right)^2 \\ l_{52} &= \left( var15 + \frac{r \cos(\eta) \sin(\theta)}{2} - hr (\cos(\theta) \sin(\phi) + \cos(\phi) \sin(\eta) \sin(\theta)) \right)^2 \\ l_{53} &= (var16 - hr (\sin(\phi) \sin(\theta) - \cos(\phi) \cos(\theta) \sin(\eta)))^2 \end{aligned}$$

$$L_6 = \sqrt{l_{61} + l_{62} + l_{63}} \quad (19)$$

$$\begin{aligned} l_{61} &= \left( \frac{r \sin(\eta)}{2} - \frac{r \sin(\beta)}{2} + \frac{r \cos(\eta) \sin(\phi)}{2} + 3hr \cos(\beta) - hr \cos(\eta) \cos(\phi) \right)^2 \\ l_{62} &= \left( var17 - 3hr \sin(\beta) + \frac{r \cos(\eta) \sin(\theta)}{2} + hr (\cos(\theta) \sin(\phi) + \cos(\phi) \sin(\eta) \sin(\theta)) \right)^2 \\ l_{63} &= \left( var18 - \frac{r \cos(\eta) \cos(\theta)}{2} + hr (\sin(\phi) \sin(\theta) - \cos(\phi) \cos(\theta) \sin(\eta)) \right)^2 \end{aligned}$$

The variables used to shorten the equations above mentioned are as follows

$$\begin{aligned} var15 &= \frac{r (\cos(\phi) \cos(\theta) - \sin(\eta) \sin(\phi) \sin(\theta))}{2} - \frac{r \cos(\beta)}{2} + 3hr \sin(\beta) \\ var16 &= \frac{r}{2} + \frac{r (\cos(\phi) \sin(\theta) + \cos(\theta) \sin(\eta) \sin(\phi))}{2} - \frac{r \cos(\eta) \cos(\theta)}{2} \end{aligned}$$



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$$\begin{aligned} \text{var17} &= \frac{r (c(\phi) c(\theta) - s(\eta) s(\phi) s(\theta))}{2} - \frac{r c(\beta)}{2} \\ \text{var18} &= \frac{r}{2} + \frac{r (c(\phi) s(\theta) + c(\theta) s(\eta) s(\phi))}{2} \end{aligned}$$

## Kinematics Analysis of the wrist Joint

To derive the relationship between the lengths of the joint, a kinematics model of shoulder joint is established. As shown in Figure, fixed coordinate frame  $Q_{OX_qY_qZ_q}$  and moving coordinate frame  $S_{OX_sY_sZ_s}$  are located in the center of the base frame and the end frame of the shoulder section, and frame Q also acts as base frame for the wrist joint. Note that the Base frame S is connected to frame Q by 4 leg joints which each constitutes of a spherical joint connected to base frame S and a prismatic joint and a spherical joint connected to the frame Q, along with a central joint which connects the pyramid structure to the base frame S with a spherical joint.

For both the coordinate frames S and Q the x and y axis are parallel to the sides of the base structure  $S_1, S_2, S_3, S_4$  and  $Q_1, Q_2, Q_3, Q_4$  respectively and z axis normal to the frame. The shoulder joint has the following 3 rotational DOFs: rotating the yaw angle  $\eta_x$  around the X-axis, rotating the pitching angle  $\phi_y$  around the Y-axis, and rotating the roll angle  $\theta_z$  around the Z-axis. The rotating Euler angle of roll-pitch-yaw is used to describe the transform matrix Rotz and rotXY as shown in the equations, for the moving coordinate frame Q relative to the fixed coordinate frame S.

The coordinate system of the frame S is given as follows :

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$$\begin{aligned}
R1 &= [\cos(0 * \pi/2); \sin(0 * \pi/2); 5 * h] * r \\
R2 &= [\cos(1 * \pi/2); \sin(1 * \pi/2); 5 * h] * r \\
R3 &= [\cos(2 * \pi/2); \sin(2 * \pi/2); 5 * h] * r \\
R4 &= [\cos(3 * \pi/2); \sin(3 * \pi/2); 5 * h] * r \\
R0 &= [0; 0; 5 * h] * r \\
R00 &= [0; 0; 7 * h] * r \\
Ori_3 &= [0; 0; 4 * h] * r \\
Ori_4 &= [0; 0; 6 * h] * r
\end{aligned} \tag{20}$$

$$\begin{aligned}
R11 &= [\cos(0 * \pi/2) * (r/3); \sin(0 * \pi/2) * (r/3); 5 * h * r] \\
R22 &= [\cos(1 * \pi/2) * (r/3); \sin(1 * \pi/2) * (r/3); 5 * h * r] \\
R33 &= [\cos(2 * \pi/2) * (r/3); \sin(2 * \pi/2) * (r/3); 5 * h * r] \\
R44 &= [\cos(3 * \pi/2) * (r/3); \sin(3 * \pi/2) * (r/3); 5 * h * r]
\end{aligned} \tag{21}$$

$$rot_{z3} = \begin{bmatrix} \cos(\gamma_3) & -\sin(\gamma_3) & 0 \\ \sin(\gamma_3) & \cos(\gamma_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{22}$$

$$rot_{x3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha_3) & -\sin(\alpha_3) \\ 0 & \sin(\alpha_3) & \cos(\alpha_3) \end{bmatrix} \tag{23}$$

$$rot_{y3} = \begin{bmatrix} \cos(\beta_3) & 0 & \sin(\beta_3) \\ 0 & 1 & 0 \\ -\sin(\beta_3) & 0 & \cos(\beta_3) \end{bmatrix} \tag{24}$$

$$rot_{XY_3} = rot_{x3} rot_{y3} \tag{25}$$

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$$S_i = Rotz3 * rotXY3 * R_i; i = 0to4 \quad (26)$$

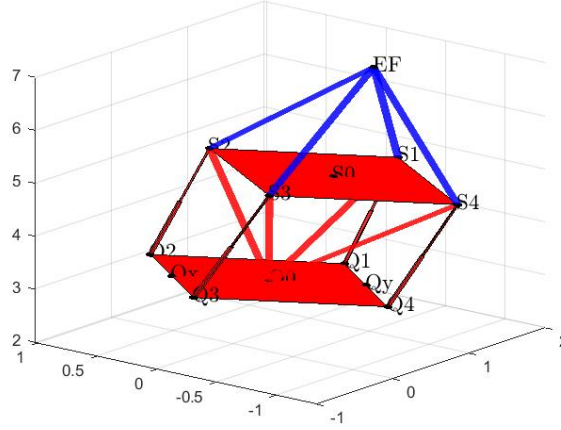


Figure 4: wrist Joint simulation

The length of each leg is determined and developed by using matlab simulations as shown below

$$L_9 = \sqrt{l_{91} + l_{92} + l_{93}} \quad (27)$$

$$\begin{aligned} l_{91} &= (r \text{ var19} + 5 h r (s(\beta_3) s(\gamma_3) - c(\beta_3) c(\gamma_3) s(\alpha_3)))^2 \\ l_{92} &= (r s(\beta) - r c(\alpha_3) s(\beta_3) - 3 h r c(\beta) + 5 h r c(\alpha_3) c(\beta_3))^2 \\ l_{93} &= (r \text{ var20} - r c(\beta) - 3 h r s(\beta) + 5 h r (c(\gamma_3) s(\beta_3) + c(\beta_3) s(\alpha_3) s(\gamma_3)))^2 \end{aligned}$$

$$L_{10} = \sqrt{l_{101} + l_{102} + l_{103}} \quad (28)$$

$$\begin{aligned} l_{101} &= (\text{var21} + r c(\alpha_3) c(\gamma_3) + 5 h r (s(\beta_3) s(\gamma_3) - c(\beta_3) c(\gamma_3) s(\alpha_3)))^2 \\ l_{102} &= \left( r s(\alpha_3) + \frac{\text{var1} r s(\beta)}{\text{var2}} - \frac{\text{var1} r c(\alpha_3) s(\beta_3)}{\text{var2}} - 3 h r c(\beta) + 5 h r c(\alpha_3) c(\beta_3) \right)^2 \\ l_{103} &= (\text{var22} + 3 h r s(\beta) - 5 h r (c(\gamma_3) s(\beta_3) + c(\beta_3) s(\alpha_3) s(\gamma_3)) + r c(\alpha_3) s(\gamma_3))^2 \end{aligned}$$

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$$L_{11} = \sqrt{l_{111} + l_{112} + l_{113}} \quad (29)$$

$$\begin{aligned} l_{111} &= \left( \frac{var1 r s(\alpha_3)}{var3} - r s(\beta) + r c(\alpha_3) s(\beta_3) - 3 h r c(\beta) + 5 h r c(\alpha_3) c(\beta_3) \right)^2 \\ l_{112} &= \left( r var23 - 5 h r (c(\gamma_3) s(\beta_3) + c(\beta_3) s(\alpha_3) s(\gamma_3)) + \frac{var1 r c(\alpha_3) s(\gamma_3)}{var3} \right)^2 \\ l_{113} &= (var24 - 5 h r (s(\beta_3) s(\gamma_3) - c(\beta_3) c(\gamma_3) s(\alpha_3)))^2 \end{aligned}$$

$$L_{12} = \sqrt{l_{121} + l_{122} + l_{123}} \quad (30)$$

$$\begin{aligned} l_{121} &= \left( r s(\alpha_3) + \frac{var4 r s(\beta)}{var5} - \frac{var4 r c(\alpha_3) s(\beta_3)}{var5} + 3 h r c(\beta) - 5 h r c(\alpha_3) c(\beta_3) \right)^2 \\ l_{122} &= (var25 - 3 h r s(\beta) + 5 h r (c(\gamma_3) s(\beta_3) + c(\beta_3) s(\alpha_3) s(\gamma_3)) + r c(\alpha_3) s(\gamma_3))^2 \\ l_{123} &= (var26 + 5 h r (s(\beta_3) s(\gamma_3) - c(\beta_3) c(\gamma_3) s(\alpha_3)))^2 \end{aligned}$$

The variables used to shorten the equations above mentioned are as follows

$$var4 = 40564819207303340847894502572032$$

$$var3 = 40564819207303340847894502572032$$

$$var5 = 20282409603651670423947251286016$$

$$var7 = (\sin(\phi) \sin(\theta) - \cos(\phi) \cos(\theta) \sin(\eta))$$

$$var9 = (\cos(\theta) \sin(\phi) + \cos(\phi) \sin(\eta) \sin(\theta))$$

$$var19 = (c(\beta_3) s(\gamma_3) + c(\gamma_3) s(\alpha_3) s(\beta_3))$$

$$var20 = (c(\beta_3) c(\gamma_3) - s(\alpha_3) s(\beta_3) s(\gamma_3))$$

$$var21 = \frac{var1 r (c(\beta_3) s(\gamma_3) + c(\gamma_3) s(\alpha_3) s(\beta_3))}{var2} - r$$

$$var22 = \frac{var1 r c(\beta)}{var2} - \frac{var1 r (c(\beta_3) c(\gamma_3) - s(\alpha_3) s(\beta_3) s(\gamma_3))}{var2}$$

$$var23 = (c(\beta_3) c(\gamma_3) - s(\alpha_3) s(\beta_3) s(\gamma_3)) - r c(\beta) + 3 h r s(\beta)$$

$$var24 = \frac{var1 r}{var3} + r (c(\beta_3) s(\gamma_3) + c(\gamma_3) s(\alpha_3) s(\beta_3)) - \frac{var1 r c(\alpha_3) c(\gamma_3)}{var3}$$

$$var25 = \frac{var4 r c(\beta)}{var5} - \frac{var4 r (c(\beta_3) c(\gamma_3) - s(\alpha_3) s(\beta_3) s(\gamma_3))}{var5}$$

$$var26 = r - \frac{var5}{var4 r (c(\beta_3) s(\gamma_3) + c(\gamma_3) s(\alpha_3) s(\beta_3))} - r c(\alpha_3) c(\gamma_3)$$

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The total body simulation of Human arm mechanism is shown in the figure below

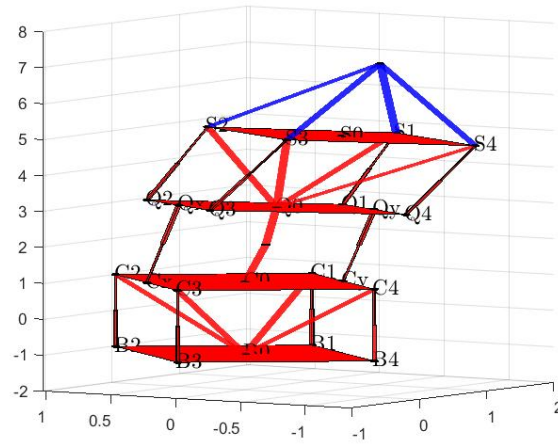


Figure 5: wrist Joint simulation

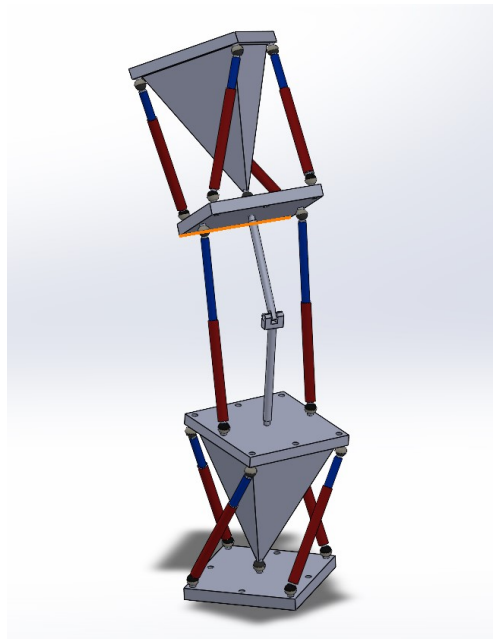


Figure 6: Human arm mimic mechanism