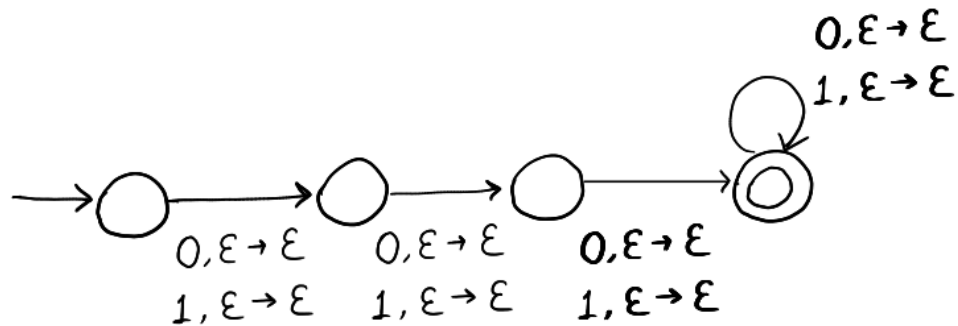
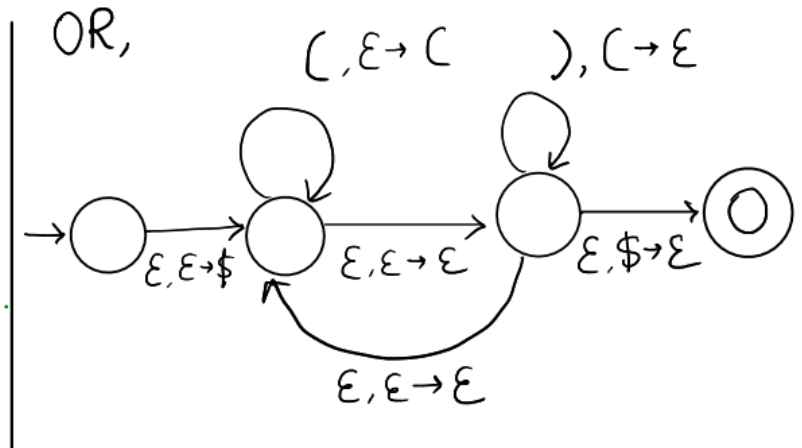
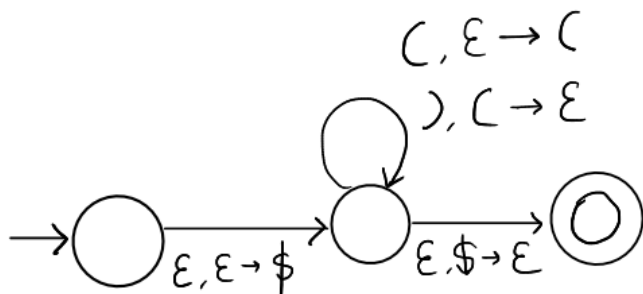


Construct Pushdown Automata for the following languages.

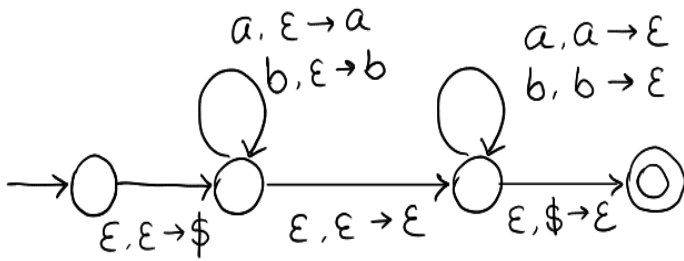
a) $L = \{w \in \{0,1\}^* : \text{length of } w \text{ is at least three.}\}$ [Hint: Recall what kind of language L is.]



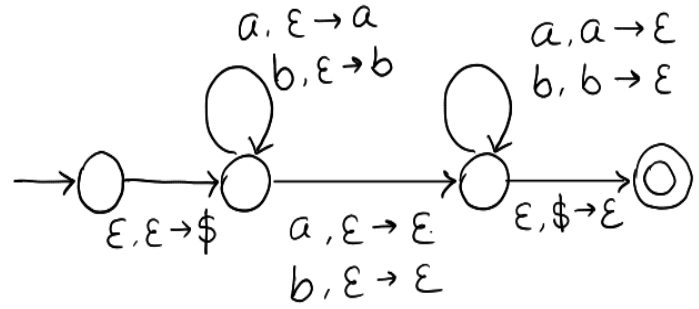
b) $L = \{w \in \{ (,) \}^* : w \text{ is a valid parenthesis}\}$



c) $L = \{w \in \{a, b\}^*: w \text{ is a even length palindrome}\} / L = \{w \in \{a, b\}^*: w \text{ is a odd length palindrome}\}$



even Length Palindrome



odd Length Palindrome

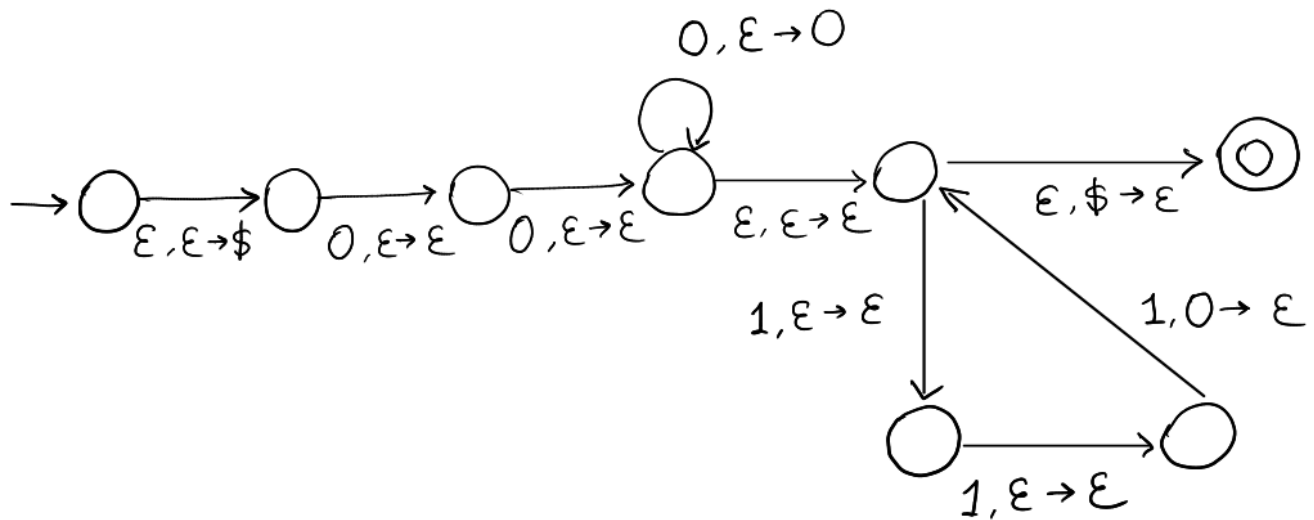
d) $L = \{w \in \{0, 1\}^*: 0^{n+2}1^{3n}, \text{ where } n \geq 0\}$

$$0^{n+2}1^{3n} \Rightarrow 0^2 0^n 1^{3n}$$

$$n = 0 : 00$$

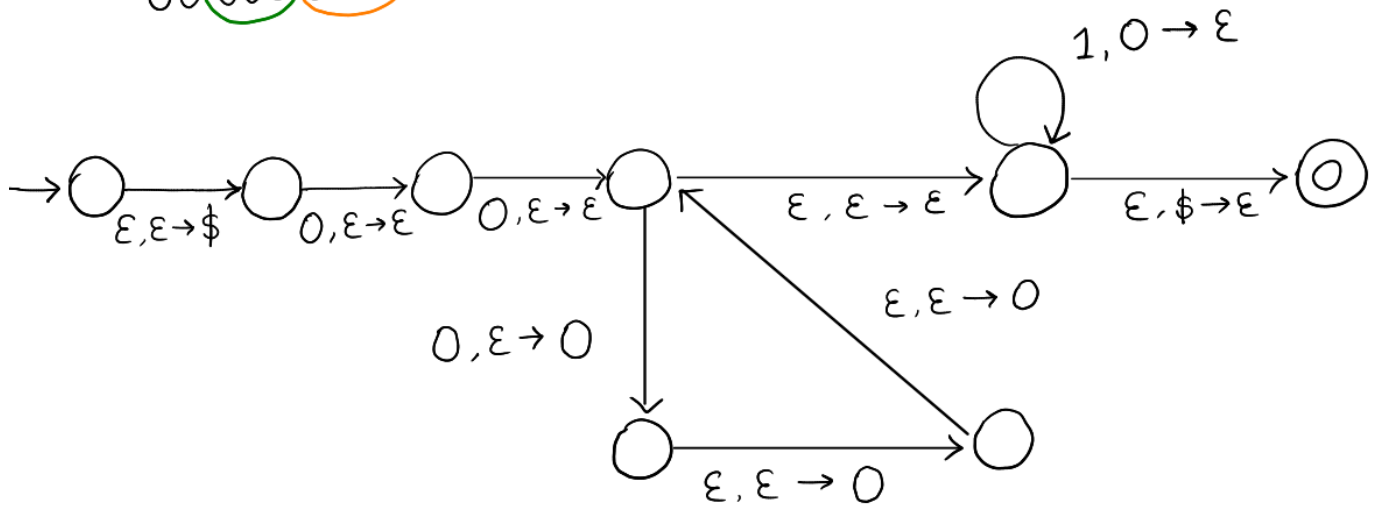
$$n = 1 : 000111$$

$$n = 2 : 00 \underline{00} \underline{111} \underline{111}$$



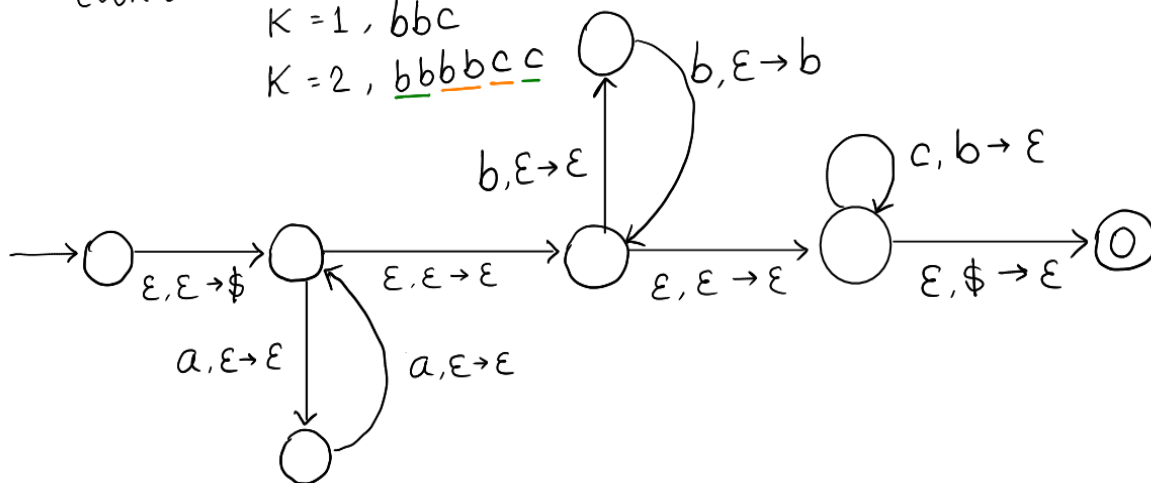
e) $L = \{w \in \{0, 1\}^*: 0^{n+2}1^{3n}, \text{ where } n \geq 0\}$ [Alternate Solution Idea]

$n=2$, 0000111111
 00(000)(000)111111



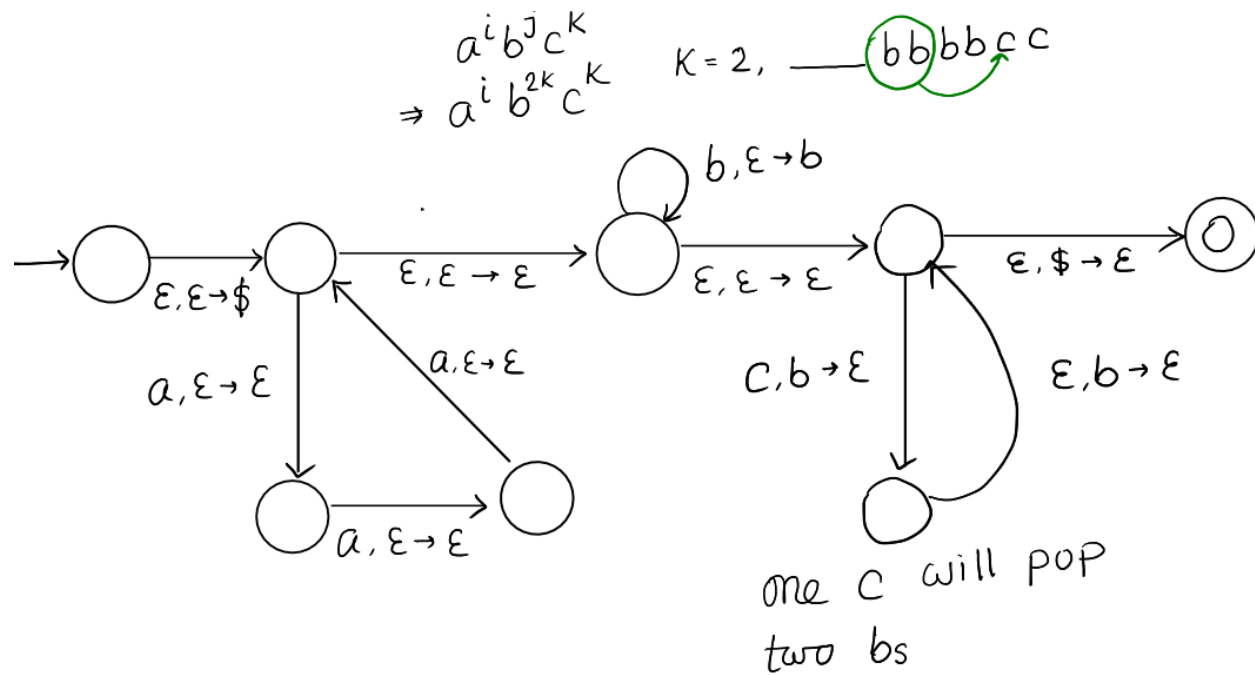
f) $L = \{w \in \{a, b, c\}^*: a^i b^j c^k, \text{ where } i \text{ is even, } j = 2k \text{ and } i, j, k \geq 0\}$

$a^i b^j c^k, j=2k$
 $\Rightarrow a^i b^{2k} c^k$
 even a \downarrow \downarrow
 \downarrow \downarrow
 $K=0, \epsilon$
 $K=1, bbc$
 $K=2, \underline{bb} \underline{bb} \underline{cc}$



g) $L = \{w \in \{a, b, c\}^*: a^i b^j c^k, \text{ where } i \text{ is multiple of three, } j = 2k \text{ and } i, j, k \geq 0\}.$

Same as the previous question, an alternate way to handle $j=2k$



h) $L = \{w \in \{a, b, c\}^*: a^i b^j c^k, \text{ where } k \text{ is odd, } i = 2j \text{ and } i, j, k \geq 0\}.$

Handwritten notes for part h):

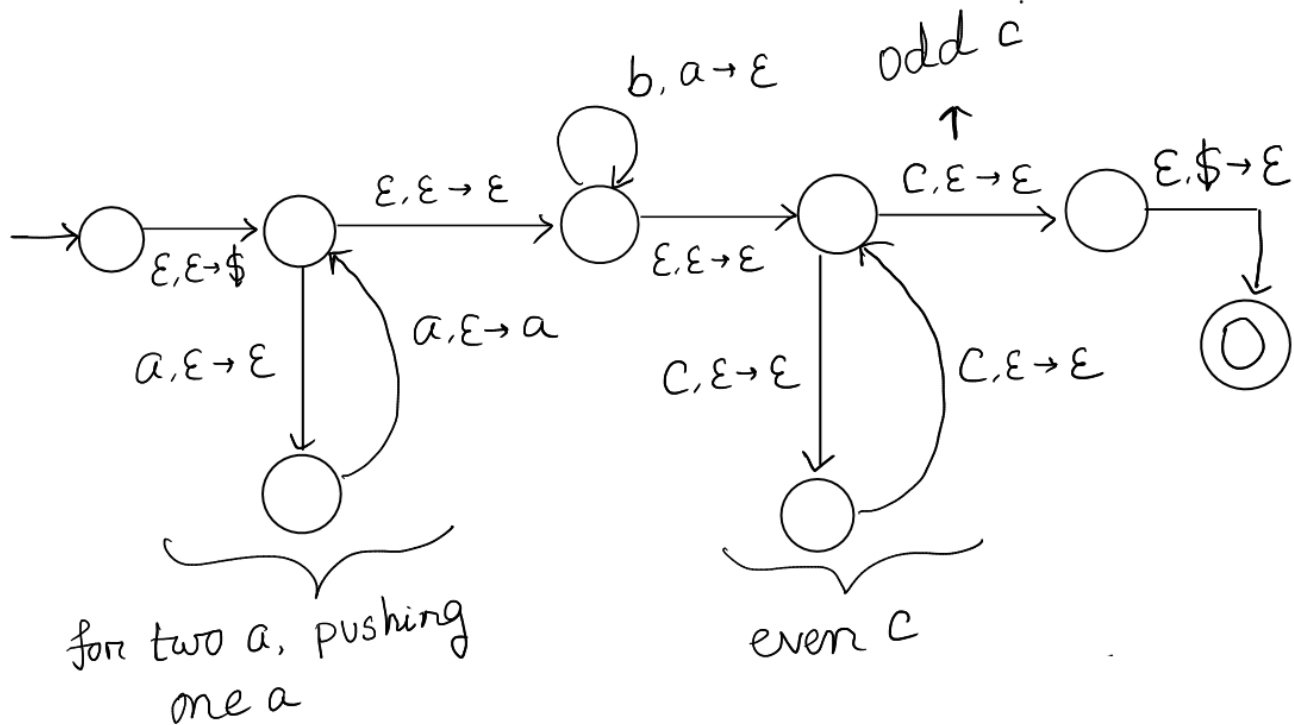
$$a^i b^j c^k \Rightarrow a^{2j} b^j c^k$$

Odd #k

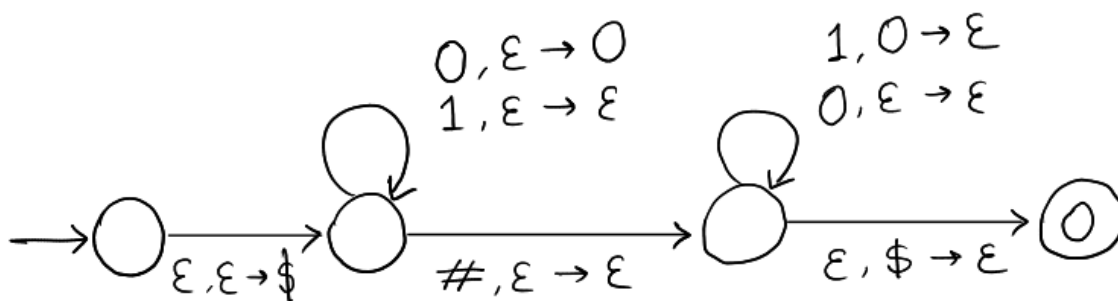
$j=0, \epsilon$

$j=1, aab$

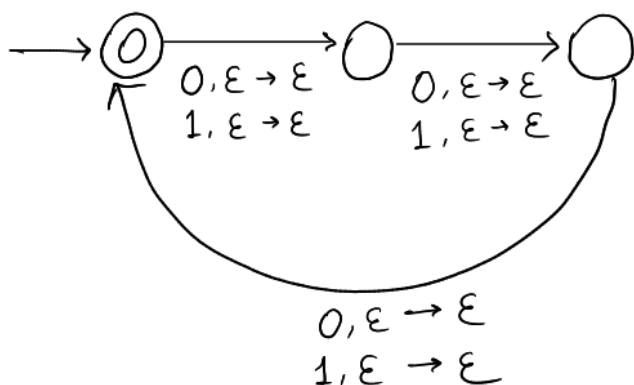
$j=2, \text{ } \boxed{aa} \boxed{aa} bb$



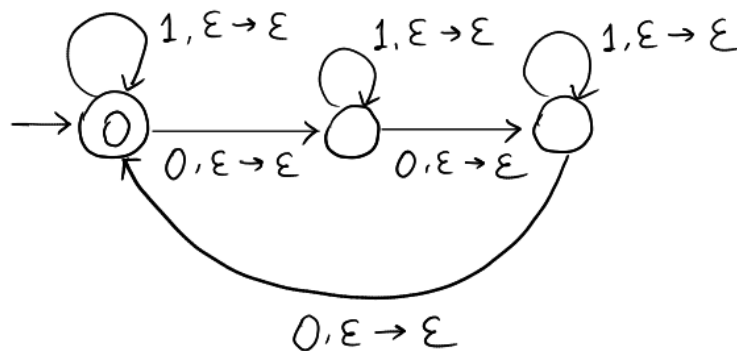
i) Let $\Sigma = \{0, 1, \#\}$. $L = \{w_1\#w_2 \mid \text{number of 0s in } w_1 \text{ is equal to number of 1s in } w_2\}$



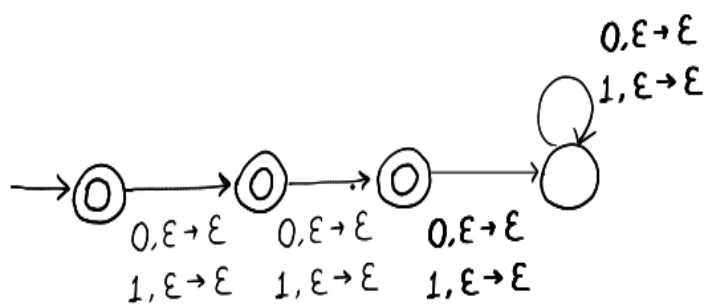
j) $L = \{w \in \{0,1\}^* : \text{length of } w \text{ is a multiple of three}\}$ [Hint: Recall what kind of language L is.]



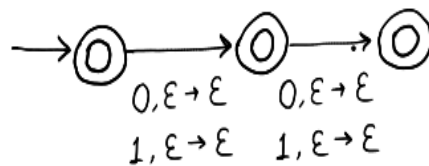
k) $L = \{w \in \{0,1\}^* : \text{number of 0s in } w \text{ is a multiple of three}\}$ [Hint: Recall what kind of language L is.]



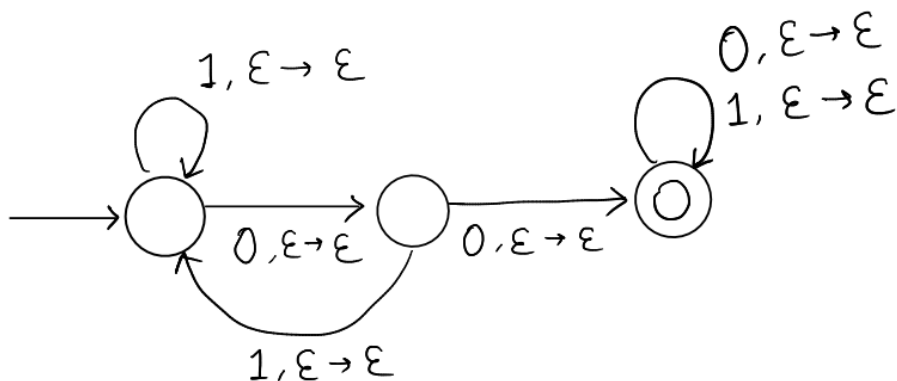
l) $L = \{w \in \{0,1\}^* : \text{length of } w \text{ is at most two.}\}$ [Hint: Recall what kind of language L is.]



OR,



m) $L = \{w \in \{0,1\}^* : w \text{ contains } 00 \text{ as a substring}\}$. Construct a PDA for L . [Hint: Recall what kind of language L is.]



n) $L = \{ w\#x : w, x \in \{a, b\}^* \text{ and } x \text{ contains } w^R \text{ as a substring} \}$. [Recall: For a string w , w^R denotes w in reverse order.]

