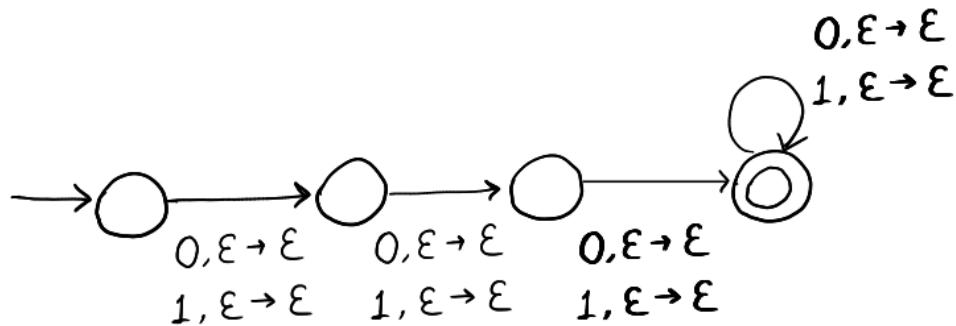
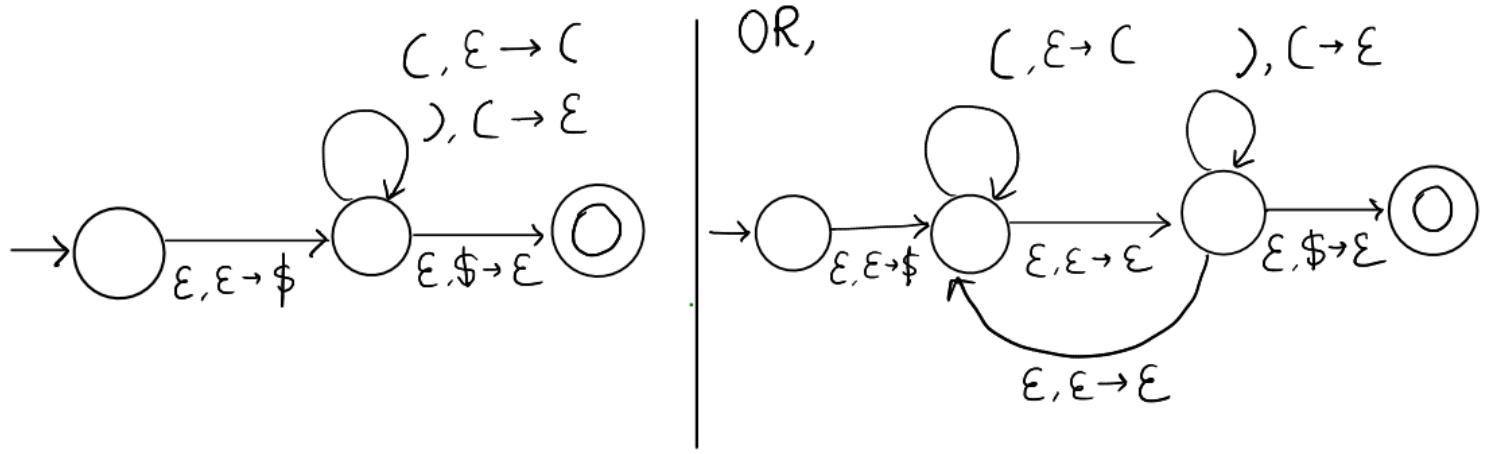


Construct Pushdown Automata for the following languages.

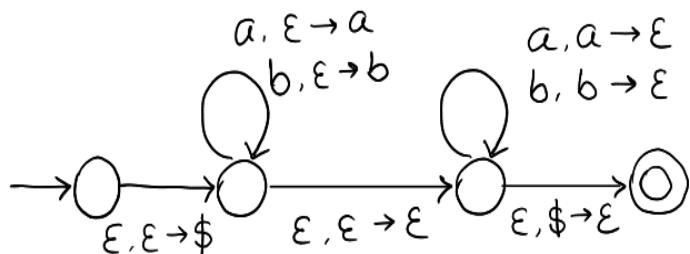
- a) $L = \{w \in \{0,1\}^*: \text{length of } w \text{ is at least three.}\}$ [Hint: Recall what kind of language L is.]



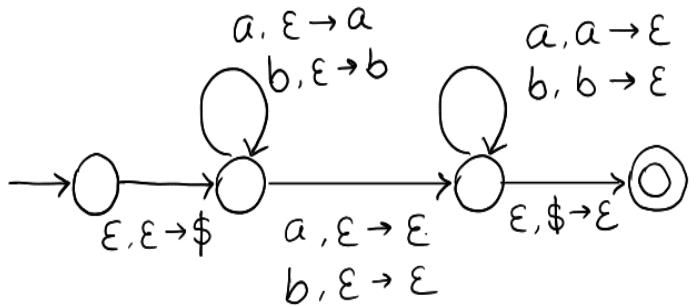
- b) $L = \{w \in \{(), *\}^*: w \text{ is a valid parenthesis}\}$



c) $L = \{w \in \{a, b\}^*: w \text{ is an even length palindrome}\} / L = \{w \in \{a, b\}^*: w \text{ is an odd length palindrome}\}$



even Length Palindrome

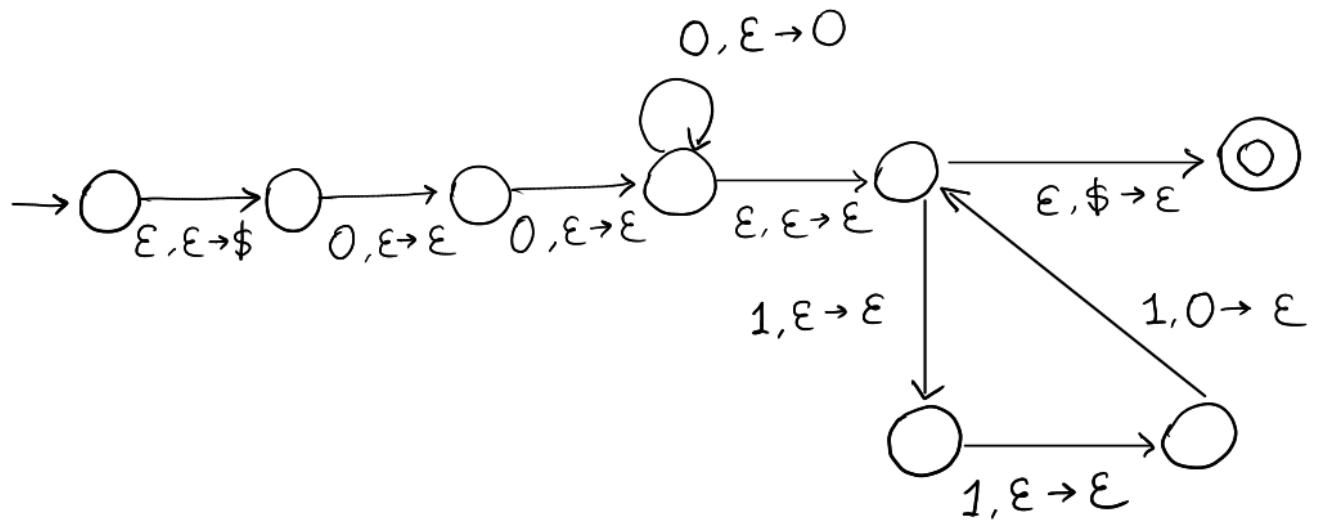


odd Length Palindrome

d) $L = \{w \in \{0, 1\}^*: 0^{n+2}1^{3n}, \text{ where } n \geq 0\}$

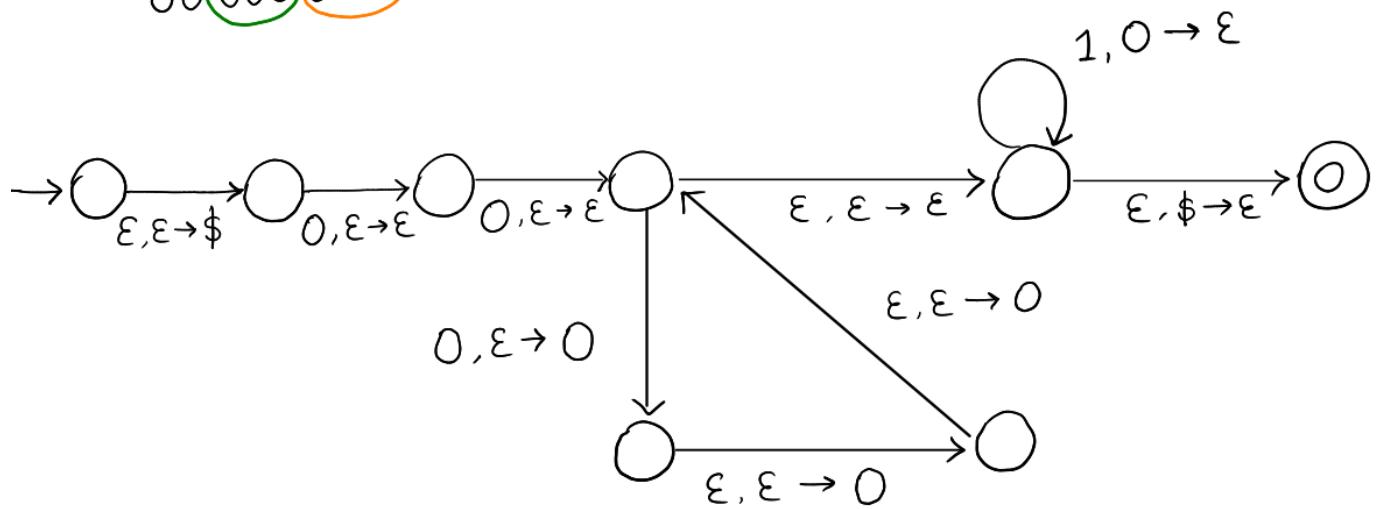
$$0^{n+2}1^{3n} \Rightarrow 0^2 0^n 1^{3n}$$

$n=0 : 00$
 $n=1 : 000111$
 $n=2 : 00\underline{00}\underline{111}\underline{111}$



e) $L = \{w \in \{0, 1\}^*: 0^{n+2}1^{3n}, \text{ where } n \geq 0\}$ [Alternate Solution Idea]

$n=2, \quad \begin{matrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ \text{green circle} & \text{orange circle} & & & & & & & & \end{matrix}$



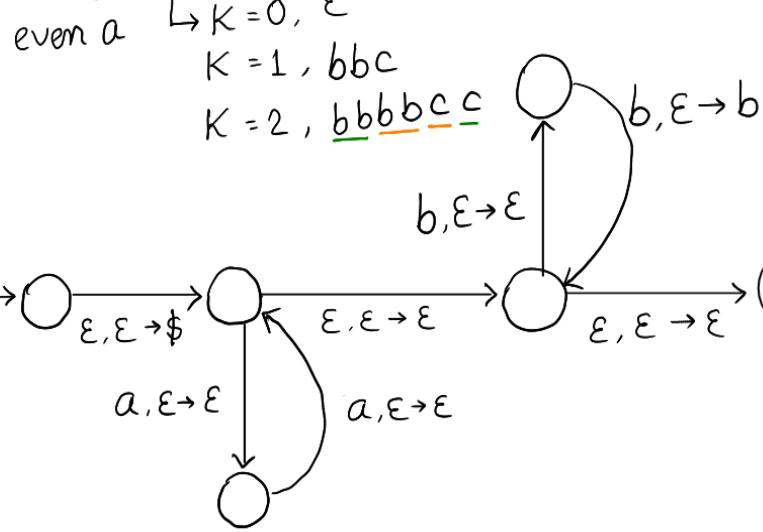
f) $L = \{w \in \{a, b, c\}^*: a^i b^j c^k, \text{ where } i \text{ is even, } j = 2k \text{ and } i, j, k \geq 0\}$

$$a^i b^j c^k, \quad j=2k$$

$$\Rightarrow \begin{array}{c} a^i \\ \downarrow \\ \text{even } a \end{array} \quad \begin{array}{c} b^j \\ \downarrow \\ K=0, \epsilon \end{array} \quad \begin{array}{c} c^k \\ \downarrow \\ K=1, \epsilon \end{array}$$

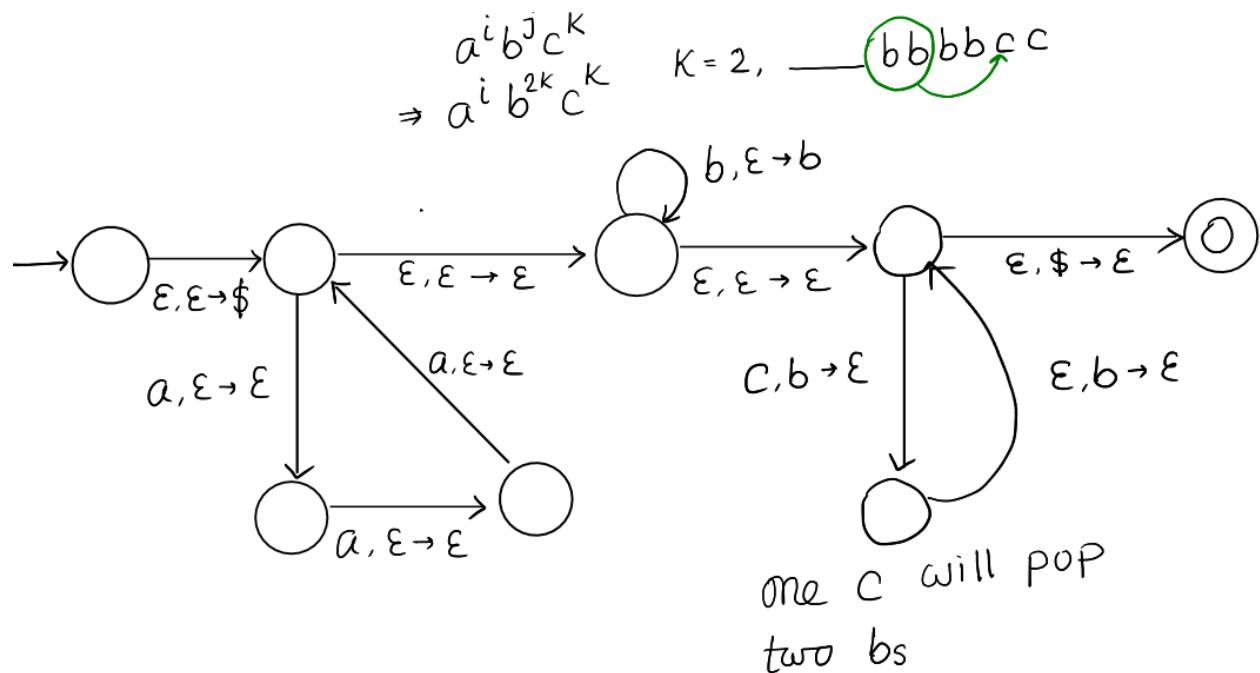
$$K=1, bbc$$

$$K=2, bbbbc$$

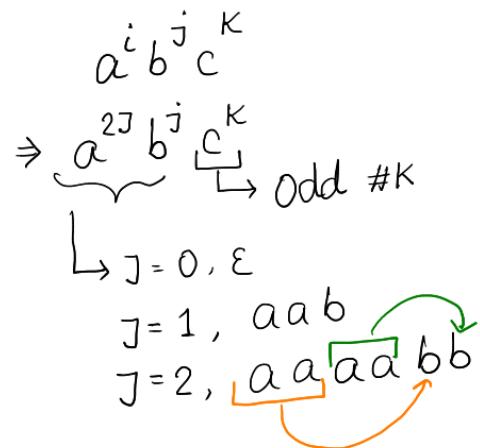


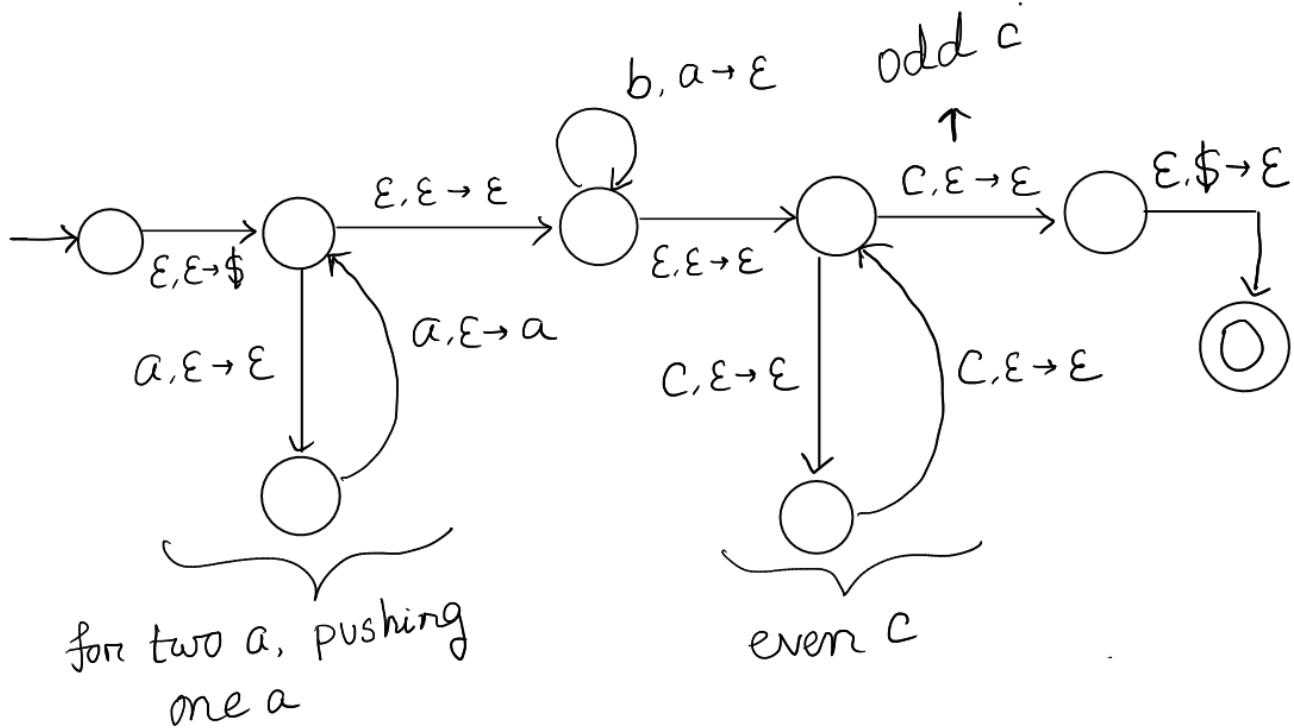
g) $L = \{w \in \{a, b, c\}^*: a^i b^j c^k, \text{ where } i \text{ is multiple of three, } j = 2k \text{ and } i, j, k \geq 0\}$.

Same as the previous question, an alternate way to handle $j=2k$

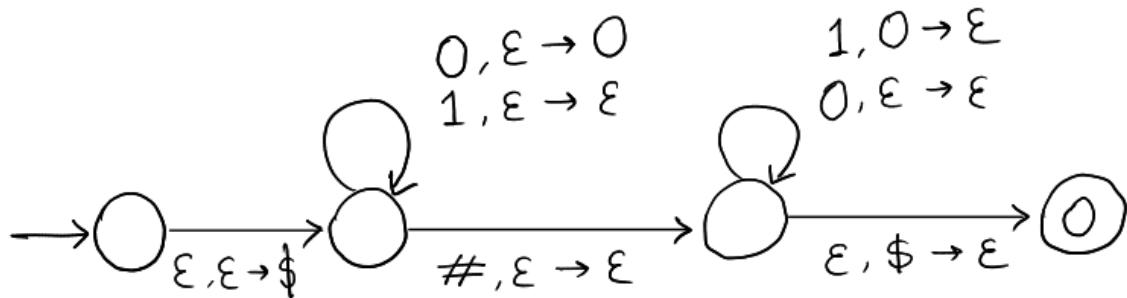


h) $L = \{w \in \{a, b, c\}^*: a^i b^j c^k, \text{ where } k \text{ is odd, } i = 2j \text{ and } i, j, k \geq 0\}$.

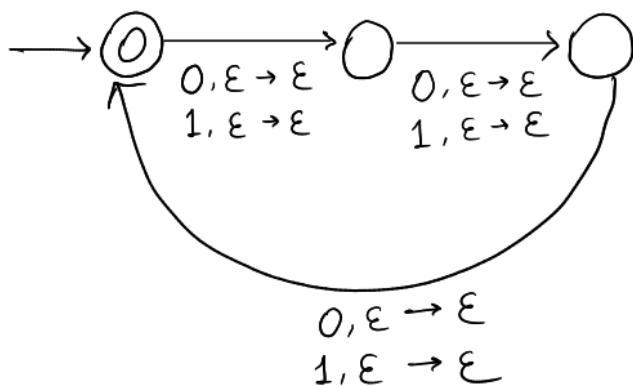




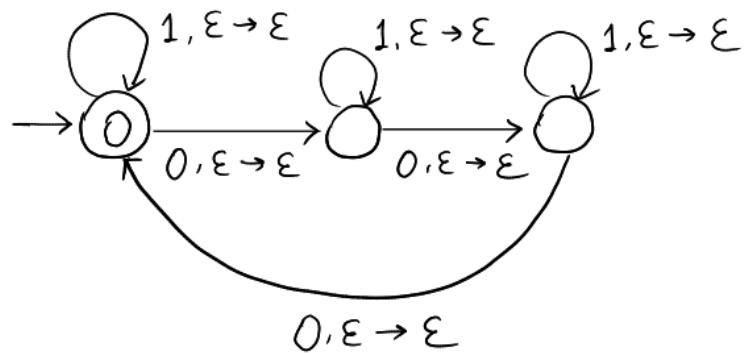
- i) Let $\Sigma = \{0, 1, \#\}$. $L = \{ w_1 \# w_2 \mid \text{number of } 0\text{s in } w_1 \text{ is equal to number of } 1\text{s in } w_2 \}$



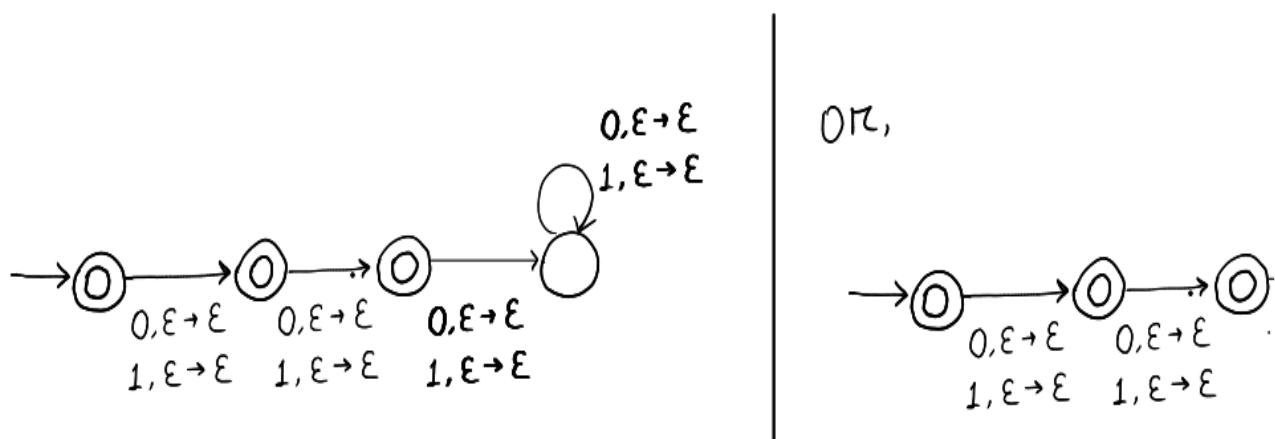
- j) $L = \{w \in \{0,1\}^*: \text{length of } w \text{ is a multiple of three}\}$ [Hint: Recall what kind of language L is.]



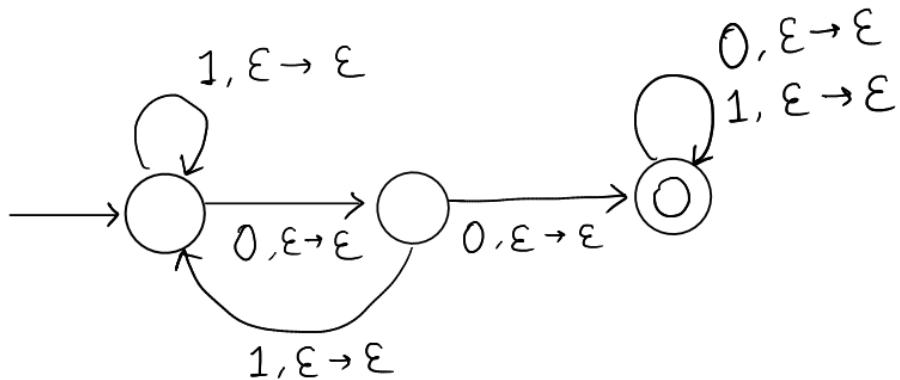
k) $L = \{w \in \{0,1\}^*: \text{number of } 0\text{s in } w \text{ is a multiple of three}\}$ [Hint: Recall what kind of language L is.]



l) $L = \{w \in \{0,1\}^*: \text{length of } w \text{ is at most two.}\}$ [Hint: Recall what kind of language L is.]



m) $L = \{w \in \{0,1\}^*: w \text{ contains } 00 \text{ as a substring}\}$. Construct a PDA for L. [Hint: Recall what kind of language L is.]



- n) $L = \{ w\#x : w, x \in \{a, b\}^* \text{ and } x \text{ contains } w^R \text{ as a substring}\}$. [Recall: For a string w , w^R denotes w in reverse order.]

