# 1 Abstract

Mach-Zendher interferomter with phase shift used on diffusion and erosion in water. This interferomter have a higher resolution than the Michelson interferometer and gives a better picture of microfluidic devices. In this article it was tested on table salt diffused in water and erosion of calcit with running water. The diffusion was a test to see if the equipment still worked and that all the pieces could cooperate, and the calcite were to give an estimate of the precision of the equipment. The erosion showed that the setup that was used did not have enough resolution to get the full effect of the Mach-Zendher interferometer.

# 2 Introduction

The Mach-Zendher interferometer was first proposed in 1891 by Ludwig Zendher [1] and later on refined by Ludwig Mach [2]. In the later years it has been used for including understanding the kinetics of dense fluid transport properties [3, 4] and diffusion measurements [5]. Within high pressure experiments a classical showcase of agreement between experimental data and theory is the selfdiffusion data of methane [6, 7]. Exactly this interferometer were used by D. K. Dysthe in 1995 to measure interdiffusion in binary liquid mixtures, NaCl/water and 1-butanol/water. It was then found to have a precision of 0.6% for NaCl and 1.4% for 1-butanol [5].

We want to find out how accurate our setup is and what might be done with it to increase this accuracy. There is also the question of how well the phase-unwrapping tools available can handle these high-precision measurements. With this information the Mach-Zendher setup can be more permenantly constructed and be used for future experiments.

# 3 Method

## 3.1 Mach-Zenhder setup

This setup for the Mach-Zendher interferometer with phase shifting consists of a laser (Melles Griot, 05-LHP-111), which is chosen for its long coherens length. If we had used white light the components would have to been placed with micrometer precision. Using a laser gives a coherens length of about a meter. Though it will create noise in the measurements which needs to be weighed in. The laser beam goes through a beam expander, which due to the nature of lasers having a gaussian distribution of intensity, and us wanting a uniform distribution, we expand the laser and use only the middle of the beam. A beam splitter splits the laser beam in two, sending one to a piezo controlled mirror. The piezo is controlled by a computer sending signals to a Nidaq (National Instruments, NI USB-6211) which is connected to a high voltage DC OP amp (burleigh, PZ-70) converting a voltage in a range of -10 V to 10 V into 500 V to 1000 V. This gives high precision control of the movement of the mirror down to

about the order of 10 nm, thus making it possible to move the phase of the light wave with great accuracy. From the piezo mirror it is sent through a sample cell which contains what we are measuring. Then it is sent to a beam collector and the interference pattern is picked up by a CCD camera (Imaging Source, DMK 21BUC03) which is connected to a computer who stores the data. The second beam travels without hinderance to the collector.

## 3.2 Unwrapping

To analyze the data coming from the camera we used a method called unwrapping. As we only get an intensity from the camera we have to translate this into something we can use to analyze the motion of the phase picture. Most of this section is collected from Optical Measurement of Surface Topography. [8] The interference signal can be expressed as:

$$I(\phi) = I_{DC} + I_{AC}\cos[\theta + \phi] \tag{1}$$

where $I_{DC}$ and $I_{AC}$ are fixed coefficients and $\theta$ is the phase and $\phi$ is the phase shift in terms of the reference mirror displacement. This equation can then be expanded to

$$I(\phi) = I_{DC} + I_{AC}[\cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)] \tag{2}$$

Fitting the sine and cosine waves to the interference signal gives the $\sin(\theta)$ and $\cos(\theta)$ terms. To get the wanted $\theta$ out of this equation it is possible to integrate over a full cycle of phase $\phi$

$$N = -\int_{-\pi}^{\pi} I(\phi)\sin(\phi)d\phi \tag{3}$$

$$D = \int_{-\pi}^{\pi} I(\phi)\cos(\phi)d\phi \tag{4}$$

Then the phase $\theta$ can be extracted with

$$\tan(\theta) = N/D \tag{5}$$

The signal $I(\phi)$ is what we are measuring in the camera and thus by shifting $\phi$ with evenly placed discrete values $\Delta\phi = \pi/2$ the equation above becomes

$$N = I_0 + I_1 - I_2 + I_3 \tag{6}$$

$$D = -I_0 + I_1 + I_2 - I_3 \tag{7}$$

and we get

$$\tan(\theta) = \frac{I_0 - I_2}{-I_0 + 2I_1 - I_2} \tag{8}$$

This can be done for more intensity samples as well, and thus for 7, as we used, we get

$$\tan(\theta) = \frac{2I_1 - 2I_3}{-I_0 + 2I_2 - I_4} \tag{9}$$

By taking the arctan of this equation we are left with the shifted phase. But due to the nature of tan, the picture will only have a range of $2\pi$ and therefore have false drops. To correct these drops we used a 2 dimensional unwrapping

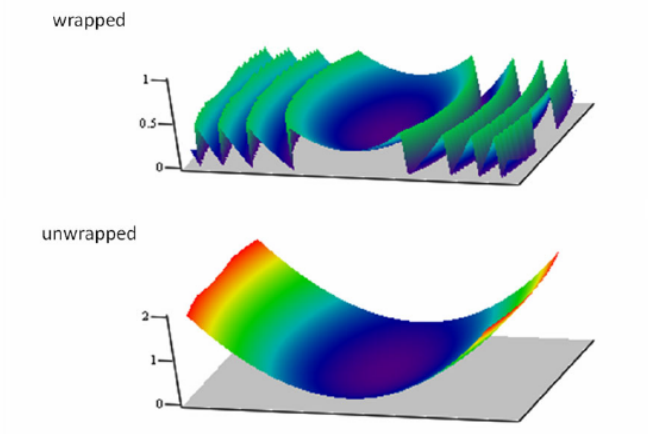


Figure 1: [8]An illustrative graph of a wrapped and unwrapped phase picture respectivly with unit length on the x and y axis, z in radians.

algorithm in Matlab called Constantini phase unwrapping which is based on an article written by M. Constantini [9]. This unwrapping of the phase gives a picture of how the phase $\theta$ is moving as $\phi$ is changed.

### 3.3 Diffusion of table salt in water

The reason for this experiment was to test the equipment to see if every component could work together and also get an approximation of how well the accuracy is. To be able to control the phase shift with the piezo we did a callibration to find which voltage represented one wavelength. By using the background it was measured to be a factor of 1.12 which we multiplied to the signal out to the Nidaq. Then we placed a rectangle sided vial with clear plastic in the middle in front of the beam.

The vial were filled with 2.16 g of water and a piece of 3 mg of salt was added to it, and a Matlab code took a picture about every 2 minutes depending on how long the unwrapping algorithm took. This went on overnight and we got around 300 measurements.

### 3.4 Calcit erosion by water

The goal of this experiment was to get an estimate of how accurate this setup can be. It was made a calcit sample by taking a plexi glass surface, bore four holes in it and two other plexi cubes. Then using a special sticker with glue

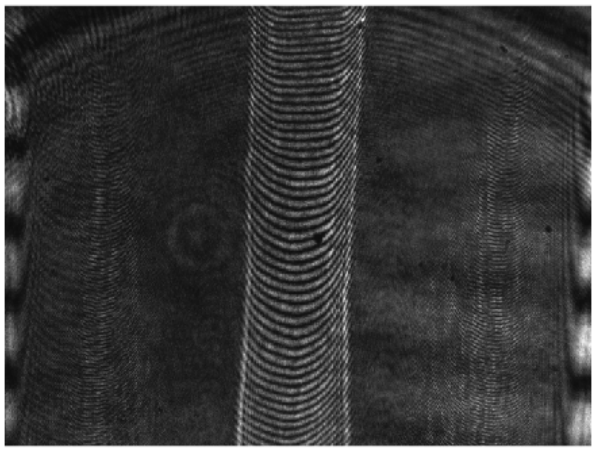

Figure 2: A vial with nothing in it. The black and white stripes are the inteference of the beam because of the difference in optical path length compared to the unhindered beam. At the far sides there is a background inteference pattern which arise from the camera not being completely aligned.

on it to fasten the calcit to the plexi surface and creating two canals, with a wdith of about 300 $\mu$m, between the glass and the calcit. Two plastic tubes are threaded down on each side of a canal through the plexi cubes so it can deliver a water stream through the canals. The water delivery system was a syringe put on an automated pump. We set the pump to 10 nl/s and started the same Matlab program as with the diffusion experiment only with about 5 minute spaces between the measurements.

### 3.5 Resultat

### 3.6 Diskusjon

## References


[1] L. Zendher. Ein neuer interferenzrefraktor. *Zeitschrift fr Instrumentenkunde*, 11:275–285, 1891.

[2] L. Mach. Ueber einen interferenzrefraktor. *Zeitschrift fr Instrumentenkunde*, 12:89–93, 1891.

[3] H. J. V. Tyrrell and K. R. Harris. Diffuson in liquids. 1984. Buttersworths: London.

[4] J. H. Dymond. *Chem. Soc. Rev.*, 14, 1984. and references therein.

[5] D. K. Dysthe, B. Hafskjold, J. Breer, and D. Cejka. Interferometric technique for measuring interdiffusion at high pressures. *J. Phys. Chem.*, 99, 1995. and references therein.

[6] K. R. Harris and N. J. Trappeniers. *Physica*, 104A:262, 1980.

[7] A. J. Eastrel and L. A. Woolf. *Physica*, 121A, 127A:286, 344, 1983, 1984.

[8] Richard Leach. *Optical Measurement of Surface Topography*. 2011.

[9] M. Constantini. *A Phase Unwrapping Method Based on Network Programming*. 1996.