Indian Institute of Space Science and Technology – Thiruvananthapuram

MA622 Pattern Recognition and Machine Learning

Assignment-11

Date: 01-03-2019

1. Give an example of a RKHS.

2. Is the space of square integrable functions $L_2[0,1]$ a RKHS? Justify your answer.

3. Apply SVM classification on Data1, Data2, Data3 and Data4. Choose the kernel as follows: for Data 1 & Data 2, $k(x,y) = \langle x,y \rangle$ and for Data 3 & Data 4, $k(x,y) = \langle x,y \rangle^2$.
   (a) Apply Lagrangian optimization techniques to solve the dual.
   (b) Plot the decision boundary and the SVM points.
   (c) Report the value of C and slack variables η_i.
   (d) Find the value of primal and dual objective function. Report the value of the duality gap.
   (e) Report the values of α, w and b.
   (f) Assess the performance of the model.

4. Consider the mapping $\phi(x_i) = (x_{i1}^2, x_{i2}^2, \sqrt{2}x_{i1}x_{i2}), x_i \in \mathscr{X}$. Let $\tilde{\mathscr{X}} = \{((1,1),-1),(1,-1),1),(-1,-1),-1),(-1,1),1)\}$. Let $\tilde{\mathscr{X}}_{new} = \{(\phi(x_i),y_i)/(x_i,y_i) \in \tilde{\mathscr{X}}_{new}\}$, Data3$_{new}$ $= \{(\phi(x_i),y_i)/(x_i,y_i) \in$ Data3$\}$ and Data4$_{new}$ $= \{(\phi(x_i),y_i)/(x_i,y_i) \in$ Data4$\}$.
   (a) Plot the nonlinear boundary in the input space for the data set $\mathscr{X}$.

(b) Apply SVM classification on $\tilde{\mathscr{X}}_{new}, Data3_{new}$ and $Data4_{new}$ by choosing the linear kernel.

i. Plot the decision boundary and the SVM points.

ii. Report the values of the parameters of the model.

(c) Let $\tilde{f}$, $\tilde{f}_3$,$\tilde{f}_4$ are the functions generated using $\mathscr{X}$, Data 3 and Data 4 respectively. Let $\tilde{f}^{new}$, $\tilde{f}_3^{new}$ and $\tilde{f}_4^{new}$ are the functions generated using $\tilde{\mathscr{X}}_{new}$, $Data3_{new}$ and $Data4_{new}$ respectively. Check whether

i. $\tilde{f}(x_i) = \tilde{f}^{new}(\phi(x_i)), \forall i$

ii. $\tilde{f}_3(x_i) = \tilde{f}_3^{new}(\phi(x_i)), \forall i$

iii. $\tilde{f}_4(x_i) = \tilde{f}_4^{new}(\phi(x_i)), \forall i$

5. Consider a data modeling problem of finding a hyperplane $\tilde{f}$ using $\{x_i, y_i)\}, i = 1,2,\ldots N, x_i \in \mathbb{R}^n, y_i \in \mathbb{R}$. By kernel methods, $\tilde{f}(x) = \sum_{i=1}^{N} \alpha_i \langle x_i, x\rangle + b, \alpha_i, b \in \mathbb{R}$. The equation of the hyperplane in R^n is given by $h(x) = \alpha_1 x^1 + \alpha_2 x^2 + \ldots \alpha_n x^n + b$ where $x = (x^1, x^2, \ldots x^n)^T$. Are the exressions $\tilde{f}$ and h the same? Justify your answer.

6. Apply SVM on Data 5.

(a) Apply Lagrangian optimization.

(b) Discuss the technique used to find the optimal kernel and other hyperparameters.

(c) Plot the decision boundary and support vectors.

(d) Report the values of the parameters and hyperparameters of the model.

(e) Assess the performance of the model.

(f) Check whether the KKT complimentary conditions have been satified by all the data points.

Notes

- Assignment has to be written in latex.
- All the files related with the assignment should be saved in a single folder and send to sumitra@iist.ac.in.
- Last date of submission: 08-03-2019.
- **As far as assignments are concerned, students are expected to observe academic honesty and integrity. Though the students can collaborate and discuss, copying directly other students' assignment or allowing your own assignment to be copied constitute academic dishonesty and is highly discouraged.**