Class: 32A

FLUID MECHANICS

Tutorials

BY A GREAT PROFESSOR

1 Viscous fluid flows

1.1 Shear stress of olive oil

A large agrifood group is studying in its laboratory the steady, fully developed flow of a new olive oil on a flat plate. The fluid has a density d of 0.92 and a kinematic viscosity $\nu = 1.09 \times 10^{-4}\, m^2.s^{-1}$.

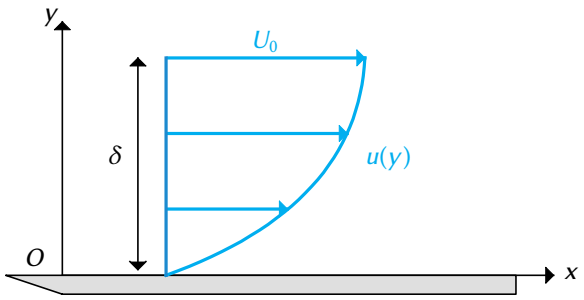

Figure 1. Boundary layer flow (sketch made with TEXMACS)

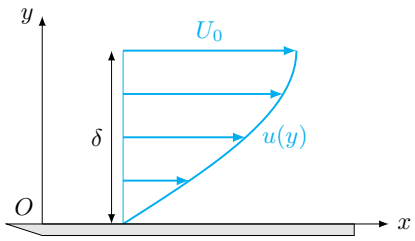

Figure 2. Boundary layer flow (sketch made with Tikz code rendered inside TEXMACS)

The velocity profile near the plate was measured in the laboratory and tests show that for $y \leqslant \delta$ the latter can be approximated by:

$$u(y) = U_0 \sin(\pi y / 2\,\delta) \qquad (1)$$

1. Determine the expression of the shear stress developed on the plate as a function of U_0 and d.

2. Compute the value of the viscous stress for $U_0 = 5\, m.s^{-1}$ at a distance $x = 1\, m$ from the leading edge of the plate.

   Laboratory tests show that the thickness δ is related to the local Reynolds number Re_x by: $\delta(x) = 4,92\, x/\sqrt{Re_x}$ with $Re_x = U_0\, x/\nu$.

1.2 Viscous friction in a cylindrical pipe

We consider here the steady and laminar flow of an incompressible viscous fluid in a cylindrical pipe of radius R.

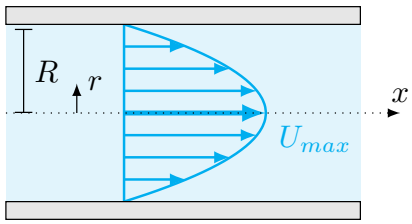

Figure 3. Velocity profile in the cylindrical pipe (Tikz code rendered in TEXMACS)

The velocity field is carried by the axis of the tube and the velocity profile is given by:

$$u(r) = U_{max}\left(1 - \frac{r^2}{R^2}\right)$$

where U_{max} is the maximum velocity at the center, r is the radial distance taken from the axis of the tube.

1. Write a relation giving the friction force exerted by the fluid on the wall per unit length of the tube.

2. Give the relation between the mean velocity U_0 of the flow in the tube and the maximum velocity U_{max} at the center.

2 Some laminar solutions of the Navier-Stokes equations