Numerical Methods

Assignment 1 (To be done during the Lab)

1. Write a program to evaluate the function exp(-x) using the series approximation (Note that $e^x = 1+x+x^2/2! +x^3/3! + \ldots\ldots\ldots\ldots$). The evaluation is considered complete when two successive summations do not differ by more than 10^{-5}. Use your program to evaluate e^{-x} for x=1, 5, 10, 100. Compute the values using single and double precision.
2. The floating point arithmetic is characterized by a machine epsilon, the smallest floating point number ε that $1.+ \varepsilon > 1$. Write a program to determine epsilon for your machine. The most common way to do this is to initialize a variable to 1, and keep halving it and adding it to 1 and checking if the result is greater than 1.
3. Consider open-channel flow of water in a circular pipe of diameter D. The water level is at a height h from the lowest point in the cross-section of the pipe. Compute the value of the ratio h/D using the bisection method, when (a) 20 % (b) 40 % (c) 60 %, and (d) 80 % of the cross-sectional area is occupied by liquid.
4. Consider the generalised equation of state given by Redlich-Kwong:

$$p = \frac{RT}{(v-b)} - \frac{a}{\sqrt{T}v(v+b)}$$

where $a = 0.4275\ (R^2 T_c^{2.5})/p_c$ and $b = 0.08664\ RT_c/p_c$, T_c and P_c being the critical temperature and pressure respectively in Pa and K, R is gas constant in J/kg-k. Using this equation of state for water at a pressure of 1 bar, compute the value of the specific volume of water vapour using the Secant and the Newton methods, at temperatures varying from 100 ^oC to 300 ^oC insteps of 50^oC. Compare your result with the values given in the steam table. State the number of iterations each of the method takes to get a non-dimensional (or scaled) tolerance value of 10^{-8}.

This will be submitted as home work.

5. Use Newton's method to find the roots of (a) $f(x) = x^2 - 2x + 1$ and (b) $f(x) = x^2 - 3x + 2$. Both the functions have a root x = 1. For both cases, start with an initial guess x(0) = 1.1. Use double precision variables in your program. Terminate your iterations when the absolute value of f(x) is less than 10^{-12}. Tabulate the values of x(k), e(k) = x(k)-1 and f(x(k)) for each iteration. Print out the number of iterations required for convergence for each case. What is the ratio e(k+1)/e(k) for the two cases? Comment on the rates of convergence for the two cases.