Project Report

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Modeling and Simulation



Problem 1

# Problem Formulation

## Environment

The environment in this problem is a Multiple-channel Queue represented in a banking system that serves customers in two queues. The first queue, is the drive-in teller queue which serves customers in their cars, this queue has a maximum capacity of 2 customers. When full, newly arriving customers are served instead in a queue inside the bank (Inside queue) this queue has no maximum capacity. It is also assumed that the servers of both queues have the same performance.

## Objectives

**Estimate the system performance for the following**

Estimate the average serving times of both queues.  
Estimate the average waiting time of both queues.  
Estimate the maximum queue in the inside queue.  
Estimate how often will a customer go to the inside queue.  
Estimate the idle time of the inside queue server.  
Determine if the Theoretical average of the service time and interarrival times match the practical ones.  
Estimate the average waiting times of both queues if the Drive-in has a maximum capacity of 3.

# Model Conceptualization

## System Components

|  |  |  |
| --- | --- | --- |
| Entity | Attribute | Event |
| Customer | Interarrival time | Arrival, Departure |
| Teller | Time of serving customer | Begin serving customer, End serving customer, |

## System Analysis

**Inter-arrival Time Probability Table**

|  |  |  |  |
| --- | --- | --- | --- |
| Arrival time | Probability | Cumulative | Range |
| 0 | 0.09 | 0.09 | 01-09 |
| 1 | 0.17 | 0.26 | 10-26 |
| 2 | 0.27 | 0.53 | 27-53 |
| 3 | 0.20 | 0.73 | 54-73 |
| 4 | 0.15 | 0.88 | 74-88 |
| 5 | 0.12 | 1 | 89-00 |

Theoretical Average: 2.5  
**Service Time Probability Table**

|  |  |  |  |
| --- | --- | --- | --- |
| Service Time | Probability | Cumulative | Range |
| 1 | 0.2 | 0.20 | 01-20 |
| 2 | 0.4 | 0.60 | 21-60 |
| 3 | 0.28 | 0.88 | 61-88 |
| 4 | 0.12 | 1 | 89-00 |

Theoretical Average: 2.5  
**Calendar Table**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Number and Queue | Rand Interarrival Time | Rand Service Time | Interarrival Time | Arrival Time | Service Time | Service Begin | Waiting | Service End | Time spent | Idle Time |
| 1 (D) | 19 | 18 | - | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 2 (D) | 88 | 7 | 4 | 4 | 1 | 4 | 0 | 5 | 1 | 3 |
| 3 (D) | 64 | 63 | 3 | 7 | 3 | 7 | 0 | 10 | 3 | 2 |
| 4 (D) | 34 | 25 | 2 | 9 | 2 | 10 | 1 | 12 | 3 | 0 |
| 5 (I) | 13 | 80 | 1 | 10 | 3 | 10 | 0 | 13 | 3 | 10 |
| 6 (I) | 5 | 92 | 0 | 10 | 4 | 13 | 1 | 17 | 7 | 0 |
| 7 (D) | 44 | 27 | 2 | 12 | 2 | 12 | 0 | 14 | 2 | 0 |
| 8 (D) | 77 | 16 | 4 | 16 | 1 | 16 | 0 | 17 | 1 | 2 |
| 9 (D) | 40 | 4 | 2 | 18 | 1 | 18 | 0 | 19 | 1 | 1 |
| 10 (D) | 74 | 12 | 4 | 22 | 1 | 22 | 0 | 23 | 1 | 3 |

# Experimental Design

## Parameters

The simulation is done with 30 Trials with 100 Customers.

## Justification

Due to some statistical studies (1,2), at 30 samples we start to see the data approach a normal distribution which is further proven with the Central Limit Theorem (3). Given these reasons it was reasonable to do the simulation with 30 Trials to take benefit of the CLT.

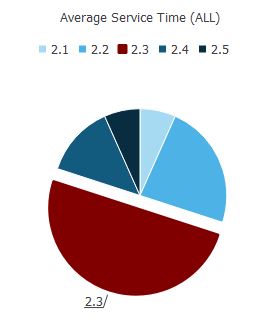
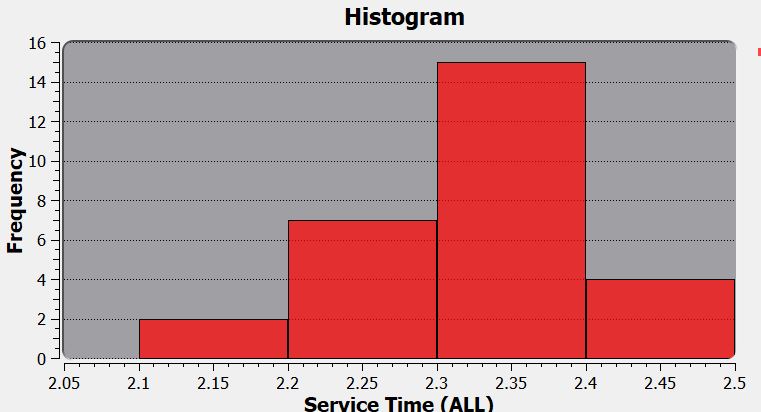
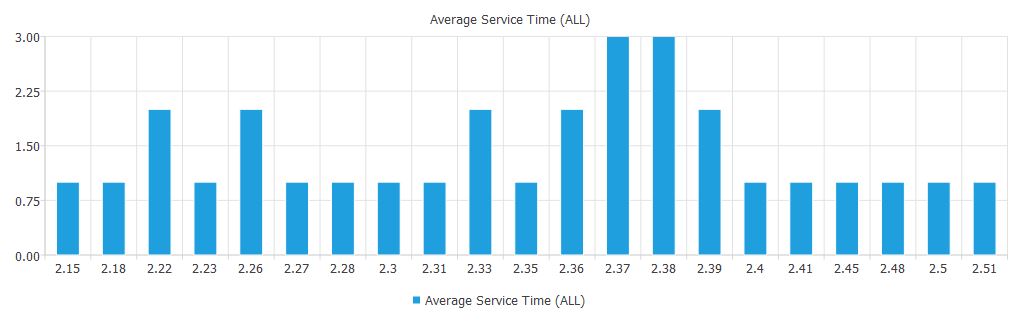
# Result analysis and Conclusion

## Results

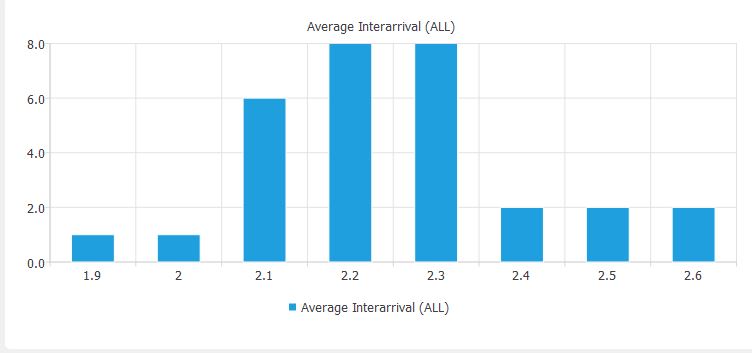
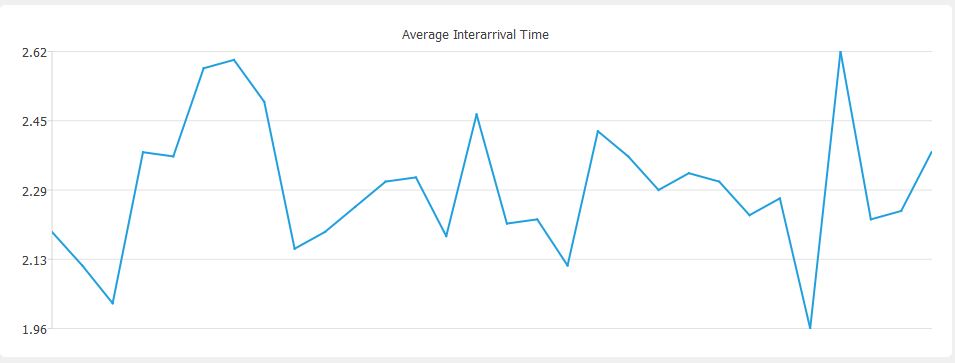
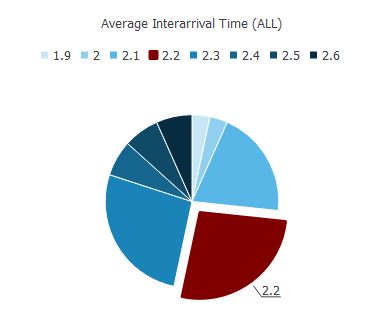
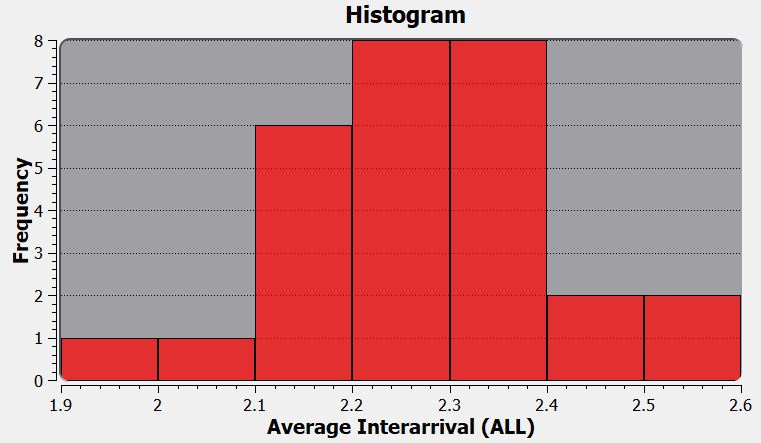
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| --- | --- | --- |
| Criteria | Value | Discussion |
| Avg Service Time (ALL) | 2.33 | This result is close to the theoretical average (2.5) |
| Avg Interarrival Time (ALL) | 2.29 | This result is close to the theoretical average (2.5) |
| Avg Service Time Drive-in | 2.34 | This result is close to the theoretical average (2.5) |
| Avg Service Time Inside | 2.30 | This result is close to the theoretical average (2.5) |
| Avg Waiting Time Drive-in | 0.57 | A relatively small waiting time |
| Avg Waiting Time Inside | 0.53 | Relatively small waiting time |
| Maximum Inside Queue Length | 2 | Very low maximum queue length |
| Probability to Go Inside | 0.17 | Very low probability to go inside |
| Portion of Idle Time Inside | 210 | Relatively High portion of idle time inside |
| Avg Waiting Drive-in (Two Cars) | 1.49 | A higher waiting time than the normal queue |
| Avg Waiting Inside (Two Cars) | 0.36 | A lower waiting time than the normal queue |

## Charts

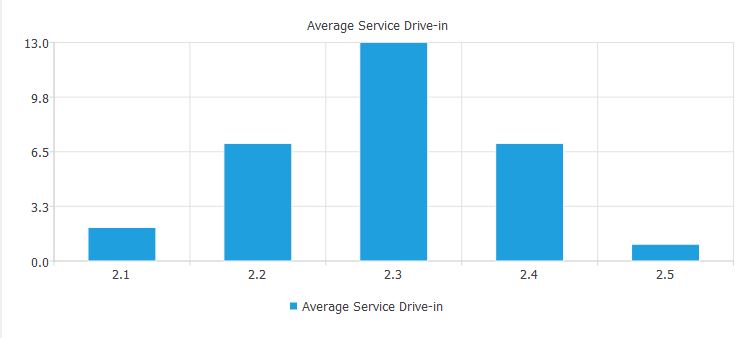
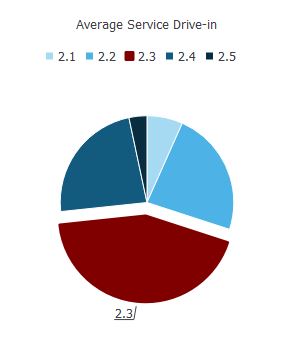
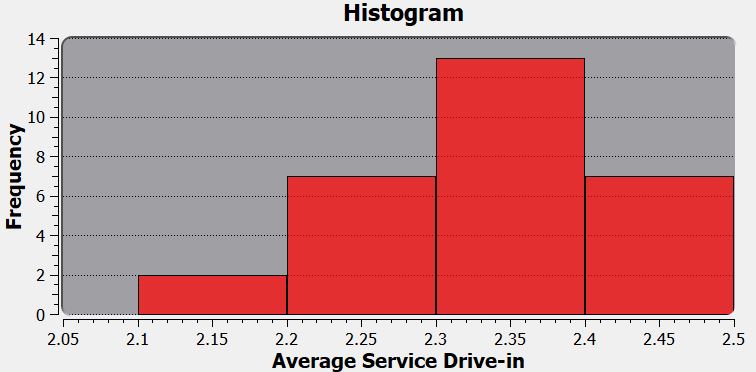
Avg Service Time (ALL)

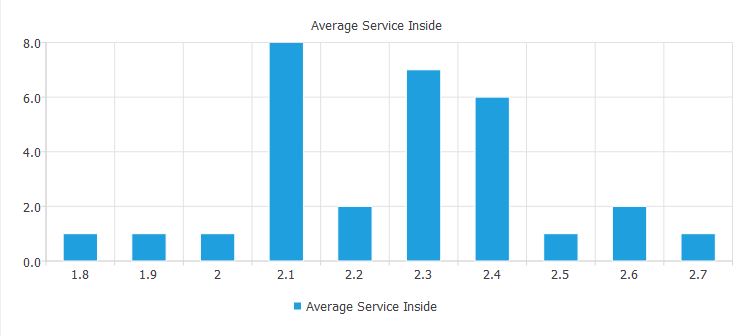
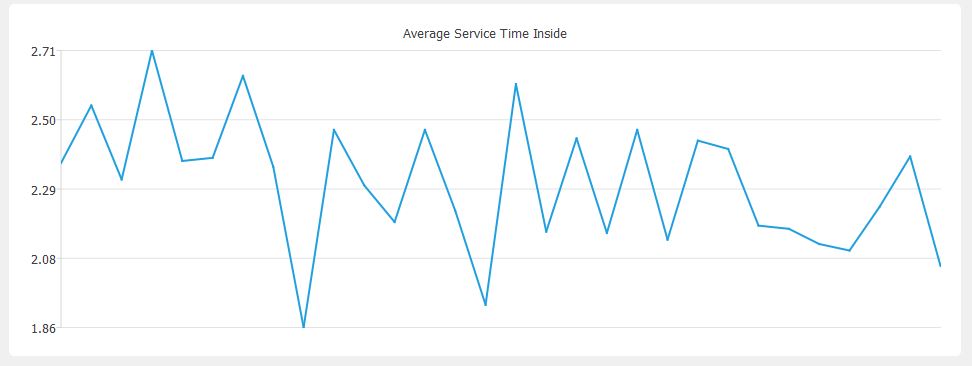
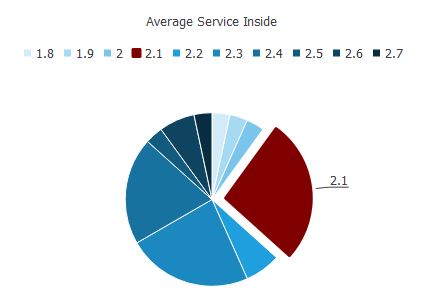
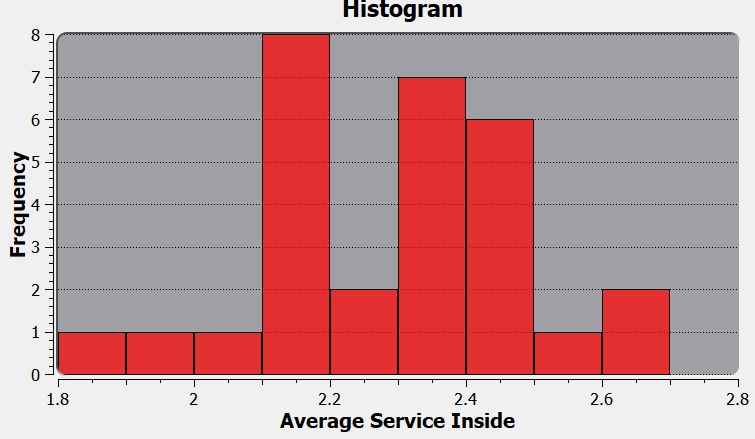
Avg Interarrival Time (ALL)



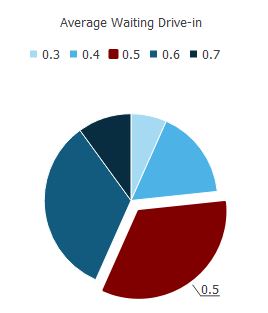
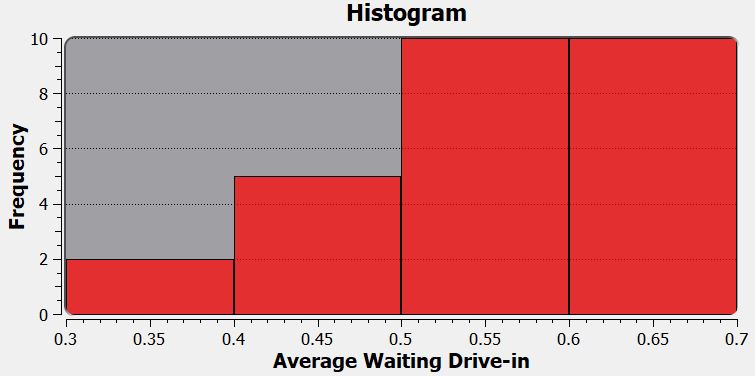
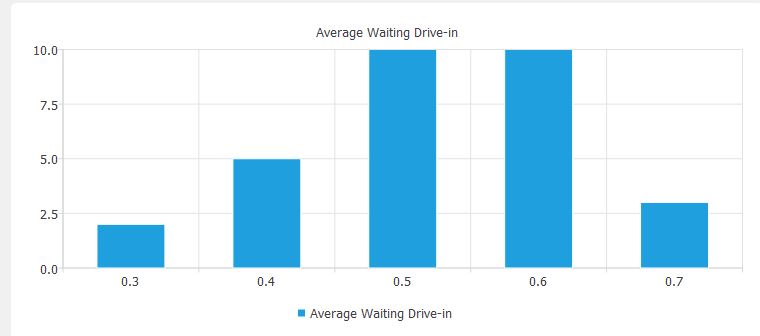
Avg Service Time Drive-in



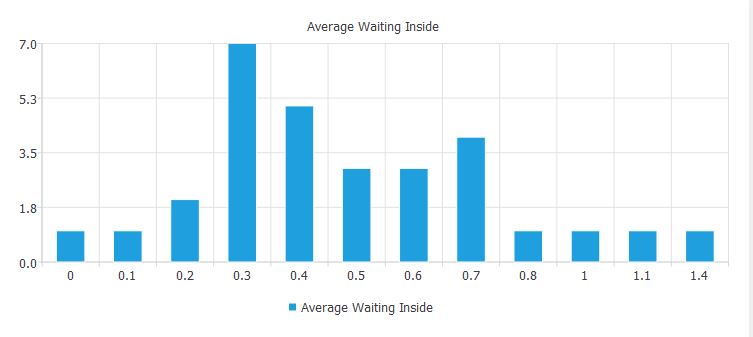
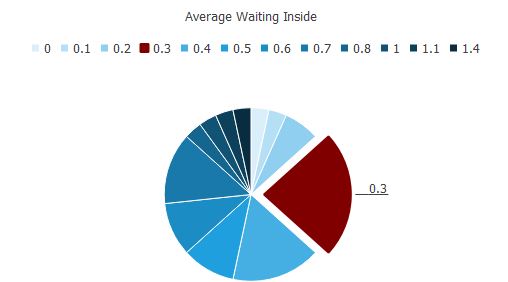
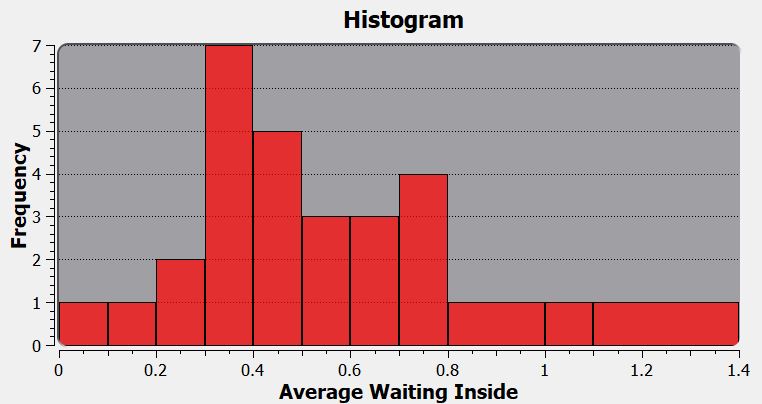
Avg Service Time Inside



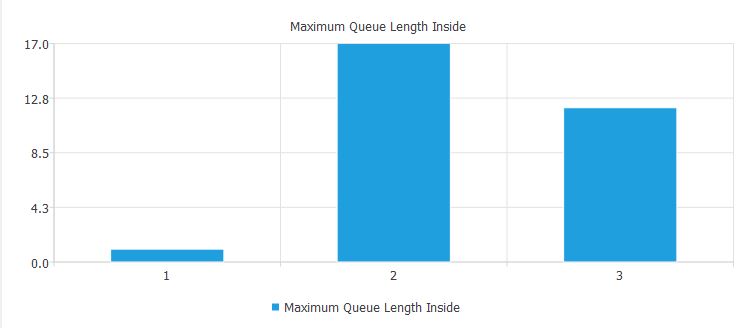
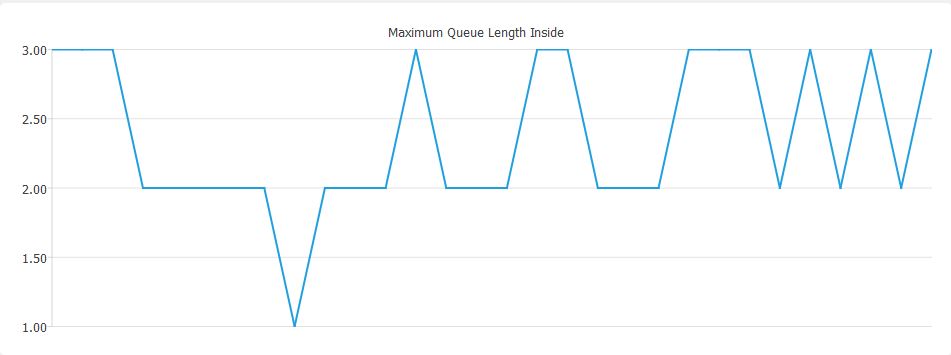
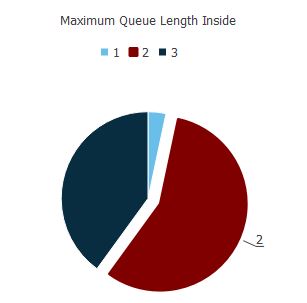
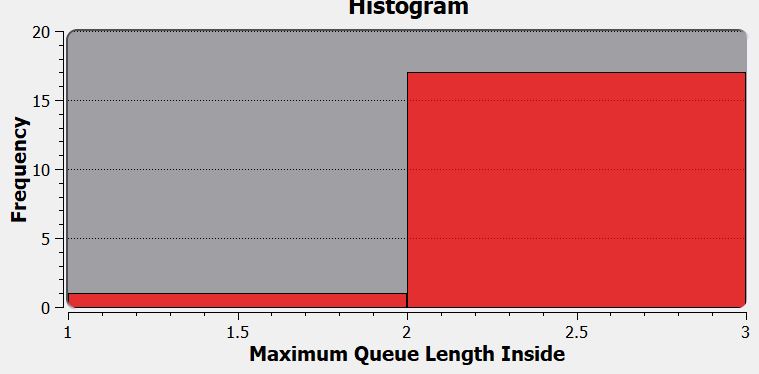
Avg Waiting Time Drive-in



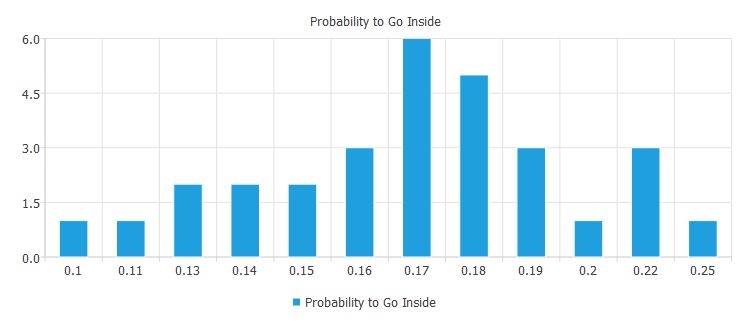
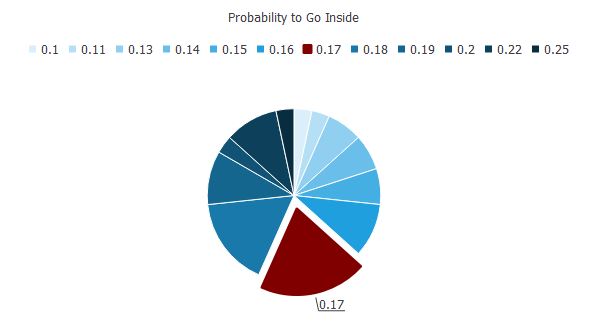
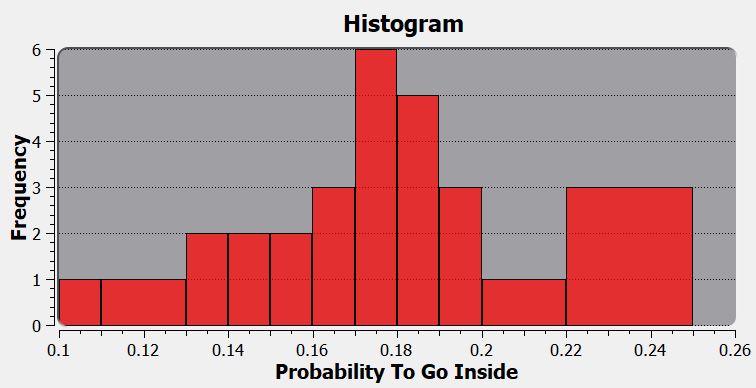
Avg Waiting Time Inside



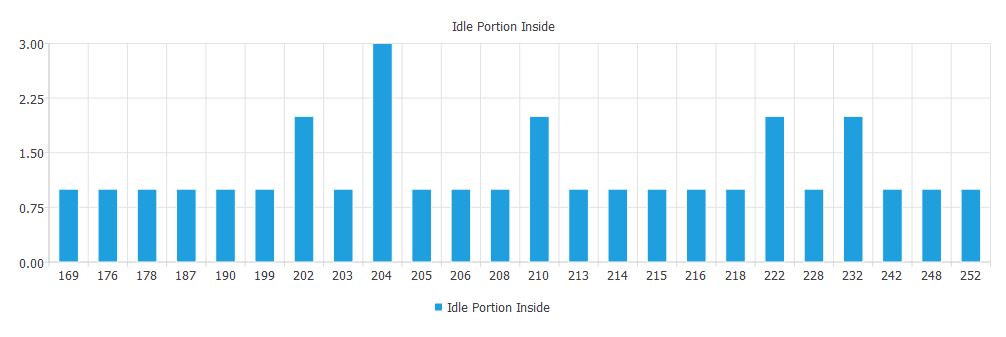
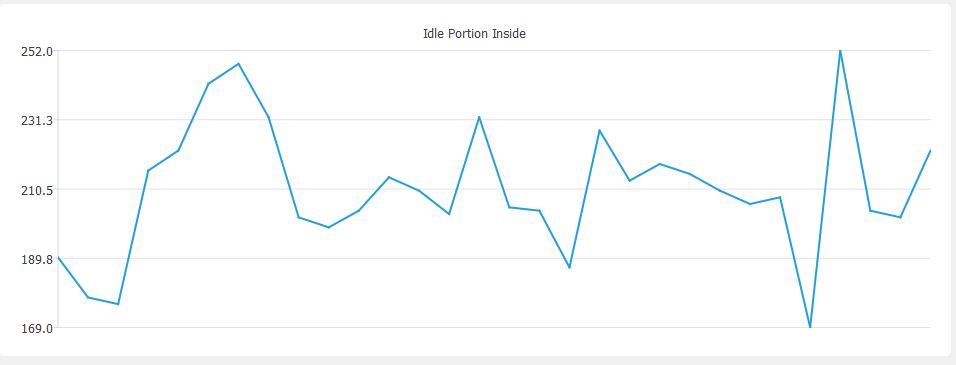
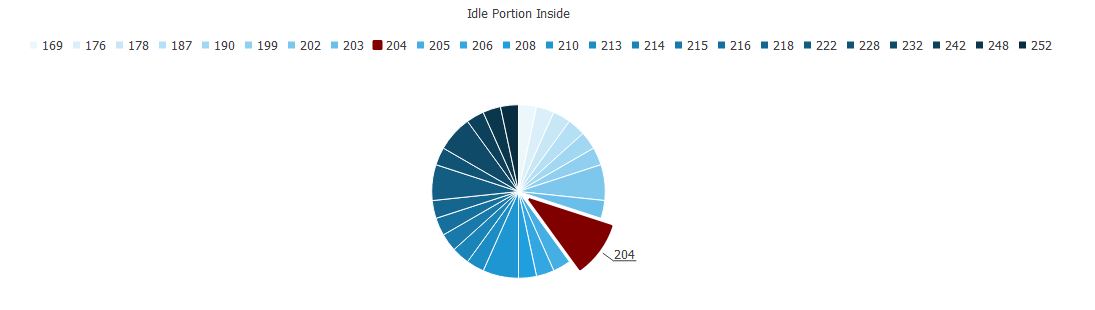
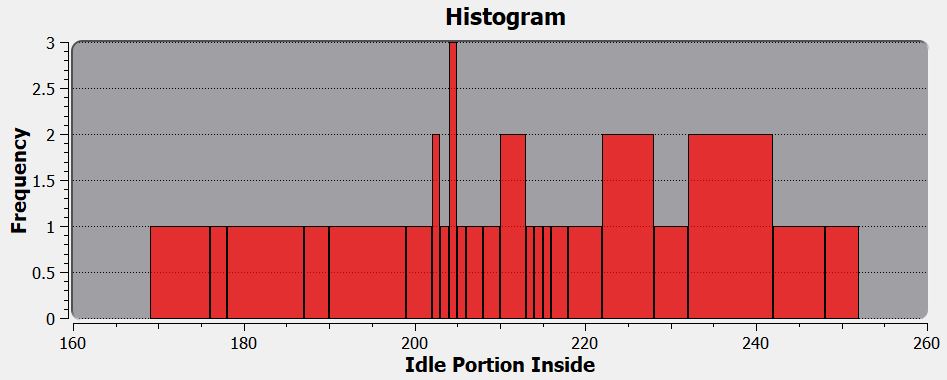
Maximum Inside Queue Length



Probability to Go Inside



Portion of Idle Time Inside



**Conclusions:**

* Close to theoretical service and inter arrival times which coincide with the Central Limit Theorem (3)
* Low Waiting times due to the dual channel nature of the system
* Low maximum Queue Lengths since the customers are divided among two queues
* Low probability to go to the inside queue since the average numbers of the service and interarrival times make it unlikely for two customers to arrive before one of the customers in the drive-in queue finish serving.
* High idle time in the inside queue due to the low probability for customers to go in it.

References

1. Westland, J. Christopher (2010). "Lower bounds on sample size in structural equation modeling". Electron. Comm. Res. Appl. 9 (6): 476–487.
2. Yamane, Taro. 1967. Statistics: An Introductory Analysis, 2nd Ed., New York: Harper and Row
3. Peter Brown,2011, Measure Theory and the Central Limit Theorem