Project Report

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Modeling and Simulation



Problem 1

# Problem Formulation

## Environment

The environment in this problem is a Multiple-channel Queue represented in a banking system that serves customers in two queues. The first queue, is the drive-in teller queue which serves customers in their cars, this queue has a maximum capacity of 2 customers. When full, newly arriving customers are served instead in a queue inside the bank (Inside queue) this queue has no maximum capacity. It is also assumed that the servers of both queues have the same performance.

## Objectives

**Estimate the system performance for the following**

* Estimate the average serving times of both queues.
* Estimate the average waiting time of both queues.
* Estimate the maximum queue in the inside queue.
* Estimate how often will a customer go to the inside queue.
* Estimate the idle time of the inside queue server.
* Determine if the Theoretical average of the service time and interarrival times match the practical ones.
* Estimate the average waiting times of both queues if the Drive-in has a maximum capacity of 3.

# Model Conceptualization

## System Components

|  |  |  |
| --- | --- | --- |
| Entity | Attribute | Event |
| Customer | Interarrival time | Arrival, Departure |
| Teller | Time of serving customer | Begin serving customer, End serving customer |

## System Analysis

**Inter-arrival Time Probability Table**

|  |  |  |  |
| --- | --- | --- | --- |
| Arrival time | Probability | Cumulative | Range |
| 0 | 0.09 | 0.09 | 1-09 |
| 1 | 0.17 | 0.26 | 10-26 |
| 2 | 0.27 | 0.53 | 27-53 |
| 3 | 0.20 | 0.73 | 54-73 |
| 4 | 0.15 | 0.88 | 74-88 |
| 5 | 0.12 | 1 | 89-00 |

Theoretical Average: 2.51  
**Service Time Probability Table**

|  |  |  |  |
| --- | --- | --- | --- |
| Service Time | Probability | Cumulative | Range |
| 1 | 0.2 | 0.20 | 1-20 |
| 2 | 0.4 | 0.60 | 21-60 |
| 3 | 0.28 | 0.88 | 61-88 |
| 4 | 0.12 | 1 | 89-00 |

Theoretical Average: 2.32  
**Calendar Table**(D) = Drive-in queue, (I) = Inside bank queue

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Customer | Rand Interarrival Time | Rand Service Time | Interarrival Time | Arrival Time | Service Time | Service Begin | Waiting | Service End | Time spent | Idle Time |
| 1 (D) | 19 | 18 | - | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 2 (D) | 88 | 7 | 4 | 4 | 1 | 4 | 0 | 5 | 3 | 3 |
| 3 (D) | 64 | 63 | 3 | 7 | 3 | 7 | 0 | 10 | 2 | 2 |
| 4 (D) | 34 | 25 | 2 | 9 | 2 | 10 | 1 | 12 | 0 | 0 |
| 5 (D) | 13 | 80 | 1 | 10 | 3 | 12 | 2 | 15 | 0 | 10 |
| 6 (I) | 5 | 92 | 0 | 10 | 4 | 10 | 0 | 14 | 10 | 0 |
| 7 (D) | 44 | 27 | 2 | 12 | 2 | 15 | 3 | 17 | 0 | 0 |
| 8 (D) | 77 | 16 | 4 | 16 | 1 | 17 | 1 | 18 | 0 | 2 |
| 9 (D) | 40 | 4 | 2 | 18 | 1 | 18 | 0 | 19 | 0 | 1 |
| 10 (D) | 74 | 12 | 4 | 22 | 1 | 22 | 0 | 23 | 3 | 3 |

# Experimental Design

## Parameters

The simulation is done with 30 Trials with 100 Customers.

## Justification

Due to some statistical studies (1,2), at 30 samples we start to see the data approach a normal distribution which is further proven with the Central Limit Theorem (3). Given these reasons it was reasonable to do the simulation with 30 Trials to take benefit of the CLT.

# Result analysis and Conclusion

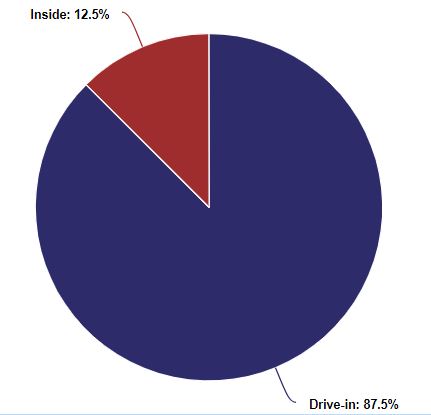
## Results

|  |  |  |
| --- | --- | --- |
| Criteria | Value | Comment |
| Avg Service Time (ALL) | 2.33 | This result matches to the theoretical average (2.32) since all the customers have a probability which is normally distributed­ |
| Avg Interarrival Time (ALL) | 2.35 | This result is strongly close to the theoretical average (2.5) since all the customers have a probability which is normally distributed |
| Avg Service Time Drive-in | 2.34 | This result is close to the theoretical average (2.32) since all the customers have a probability which is normally distributed |
| Avg Service Time Inside | 2.35 | This result is close to the theoretical average (2.32) since all the customers have a probability which is normally distributed |
| Avg Waiting Time Drive-in | 0.87 | A relatively small waiting time ‘since the gap between service times and arrival times to the queue is relatively low. |
| Avg Waiting Time Inside | 0.40 | Relatively small waiting time since the gap between service times and arrival times is low and also due to the low number of customers inside concurrently |
| Maximum Inside Queue Length | 2.2 | low maximum queue length since the probability to go inside is low (0.17) |
| Probability to Wait Inside | 0.19 | Low probability to wait inside since the inside concurrent customers is relatively low (probability to go in the inside queue is 0.17) |
| Portion of Idle Time Inside | 87% | Relatively High portion of idle time inside due to the low probability to go inside (0.17) |
| Avg Waiting Drive-in (Two Cars) | 1.81 | A higher waiting time than the normal queue since more customers will be served concurrently in it. |
| Avg Waiting Inside (Two Cars) | 0.37 | A lower waiting time than the normal queue since less customers will be served concurrently in it. |

**Conclusions:**

* Close to theoretical service and inter arrival times which coincide with the Central Limit Theorem (3)
* Low Waiting times due to the dual channel nature of the system
* Low maximum Queue Lengths since the customers are divided among two queues
* Low probability to go to the inside queue since the average numbers of the service and interarrival times make it unlikely for two customers to arrive before one of the customers in the drive-in queue finish serving.
* High idle time in the inside queue due to the low probability for customers to go in it.

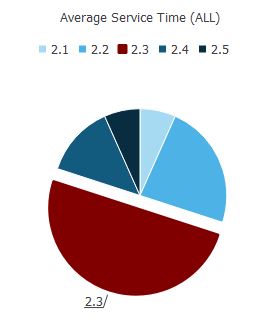
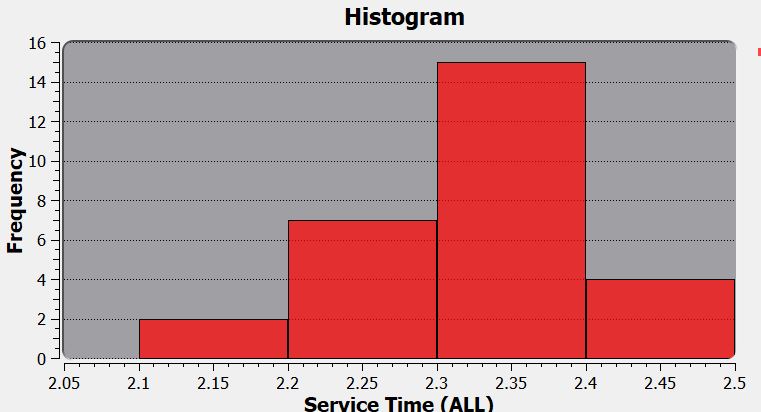
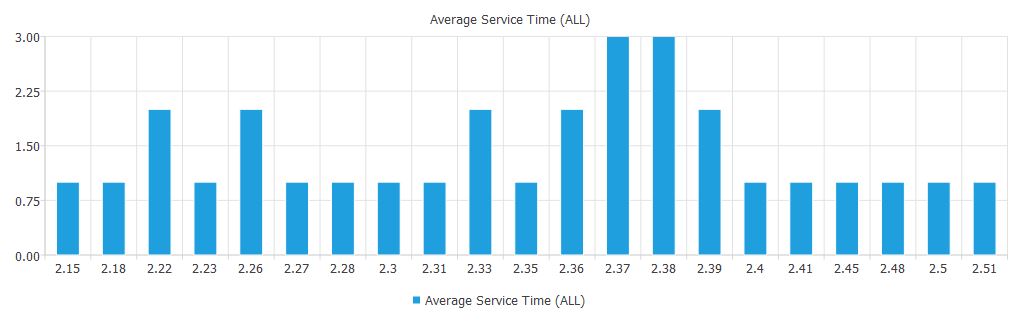
**Observations and extra statistics:**

* Customers have a probability of 0.125 to go in the inside bank queue
* The Average time spent by a customer in the system is about 3.1 seconds.

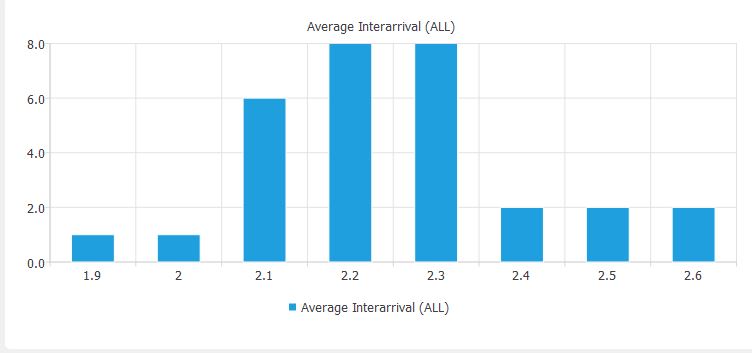
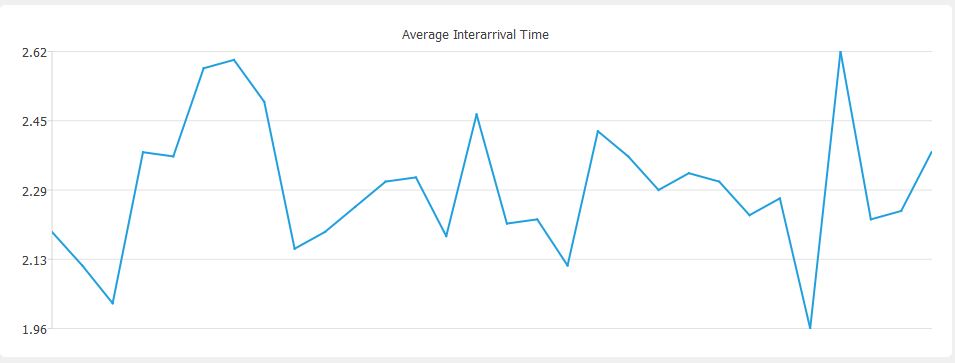
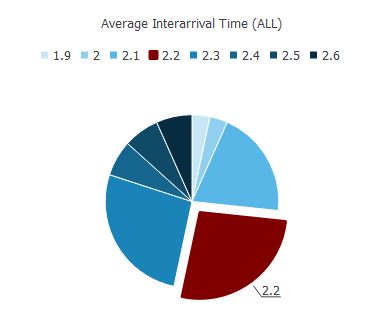
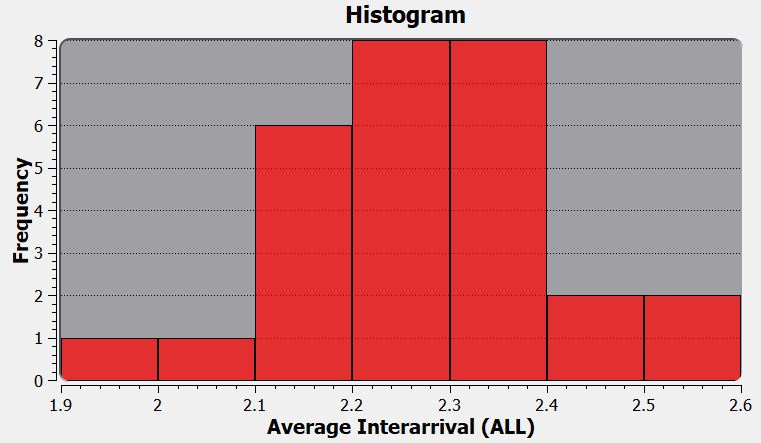
## Charts

For each estimate four charts are calculated.  
**Histogram :** To represent the distributions of the run data   
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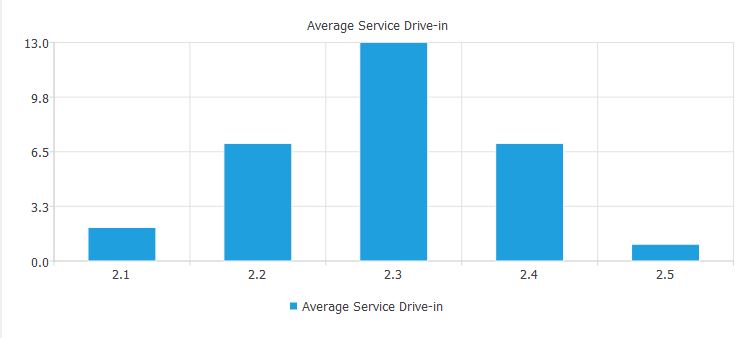
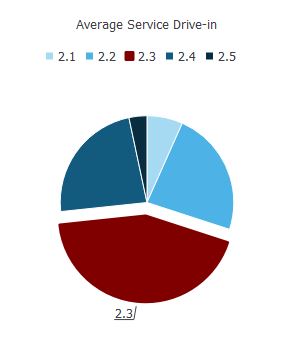
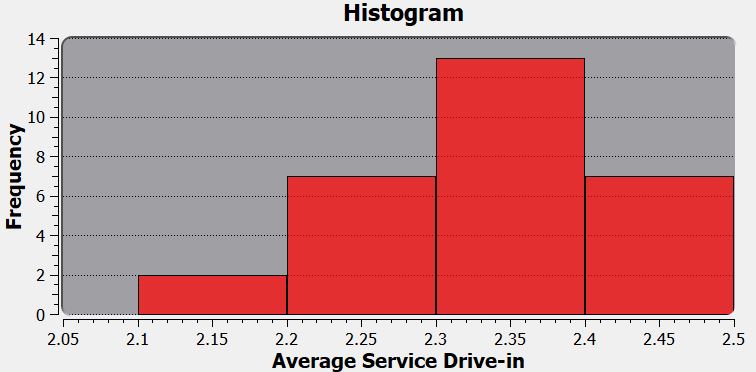
Avg Service Time (ALL)

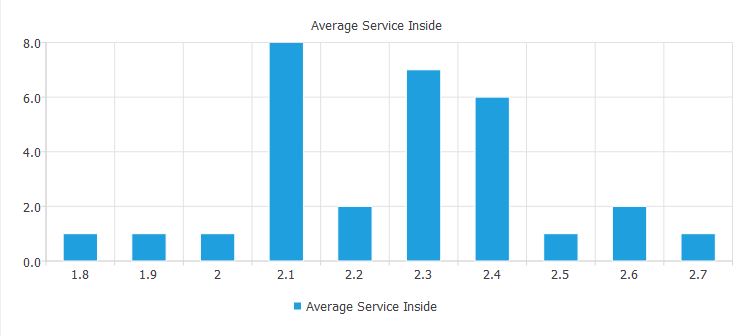
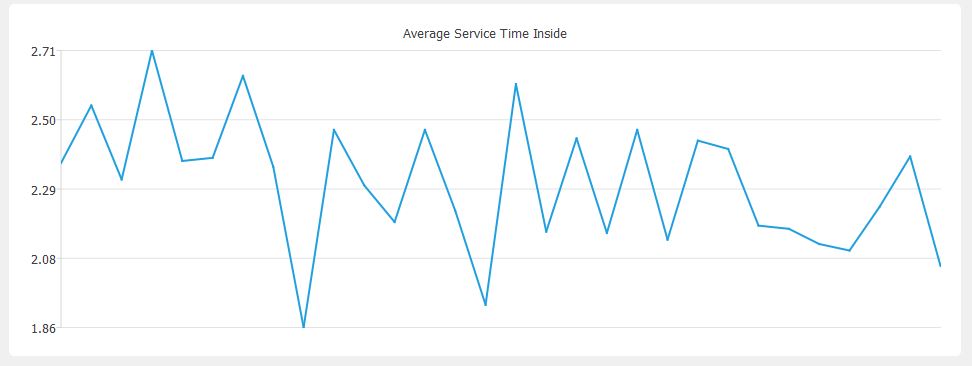
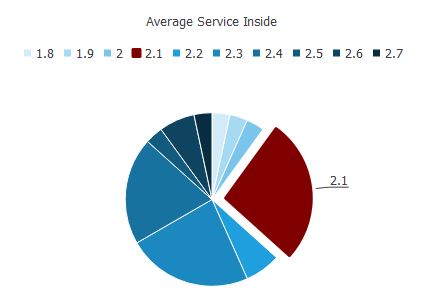
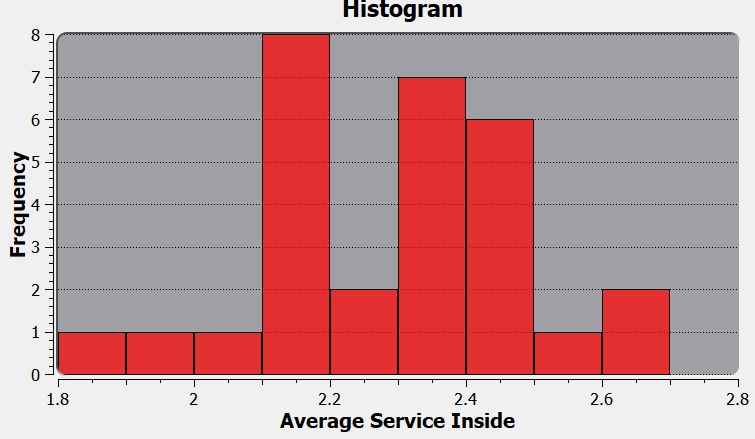
Avg Interarrival Time (ALL)



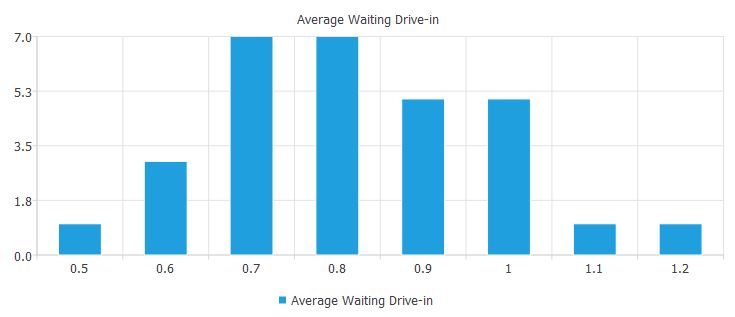
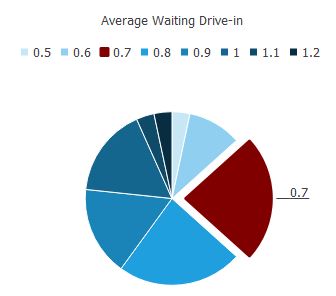
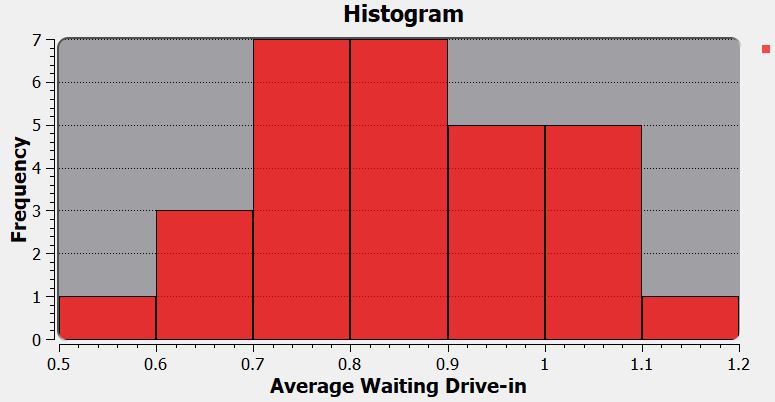
Avg Service Time Drive-in



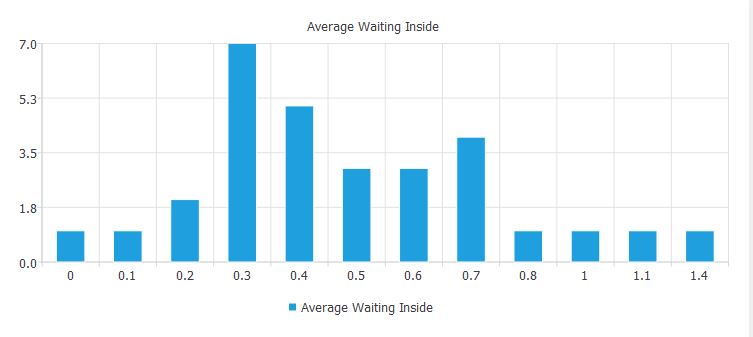
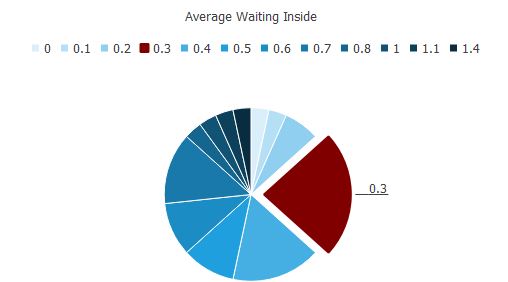
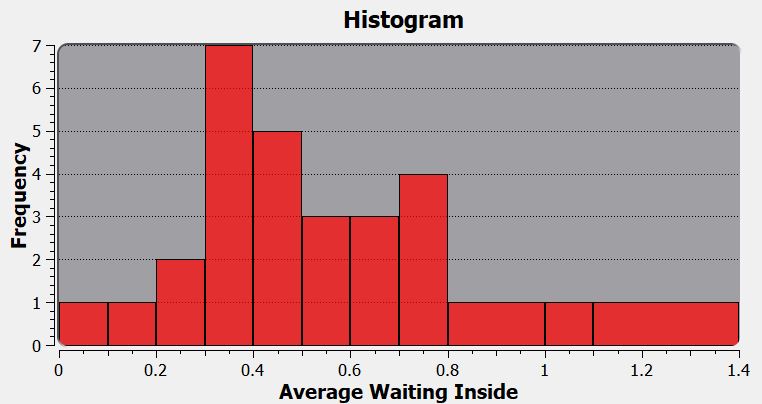
Avg Service Time Inside



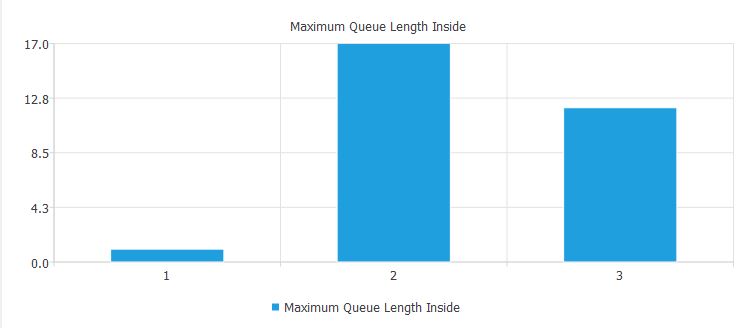
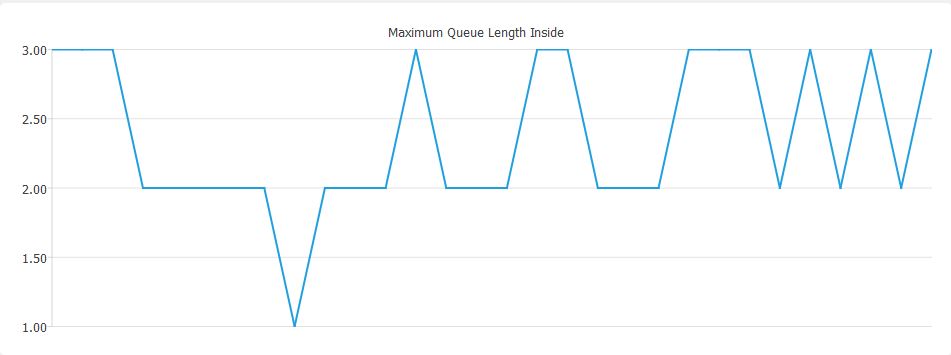
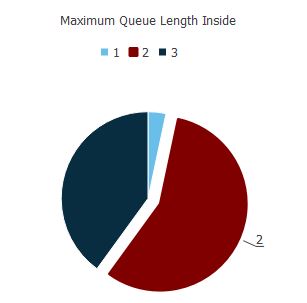
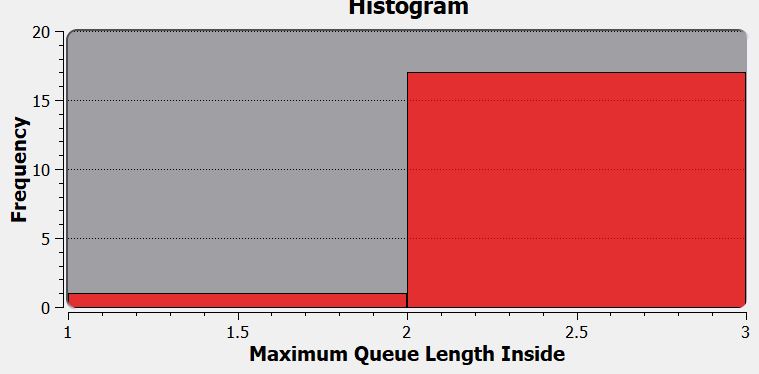
Avg Waiting Time Drive-in



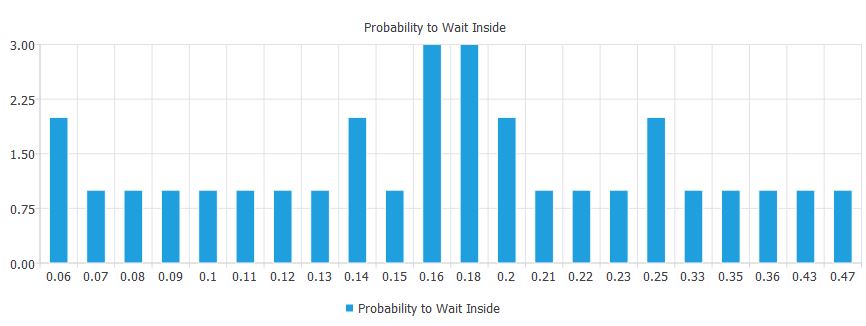
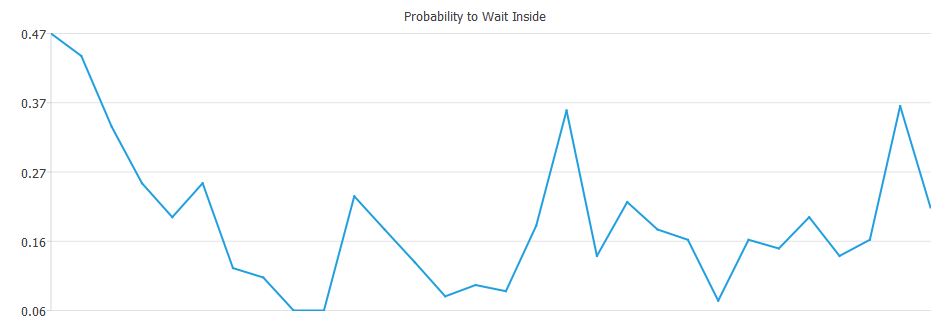
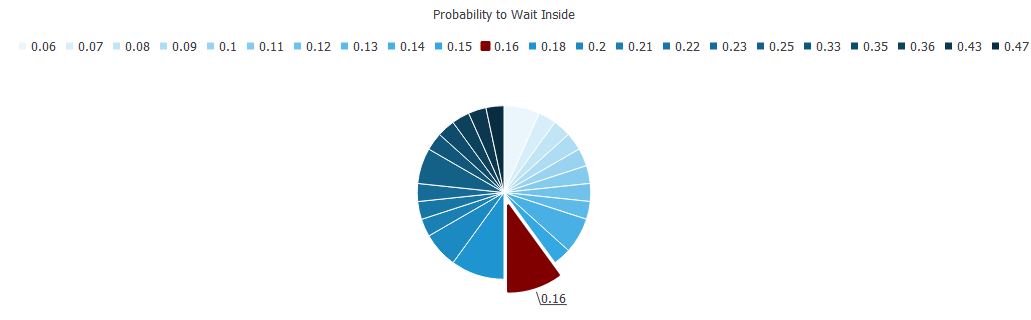
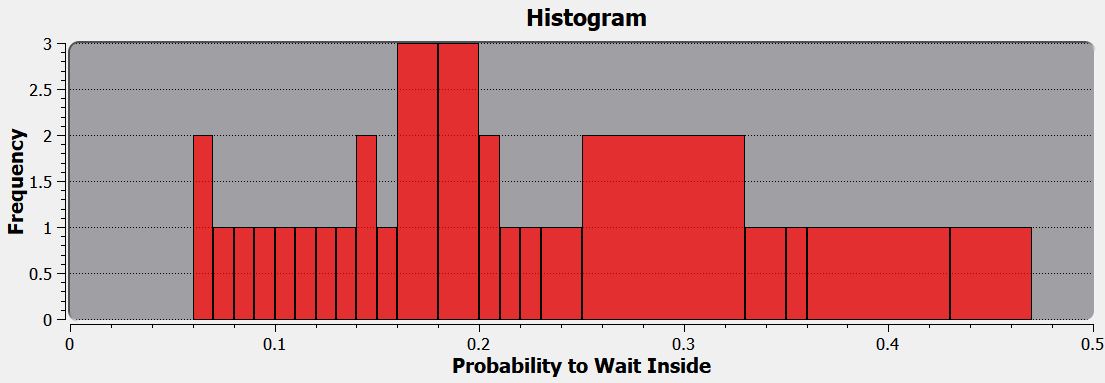
Avg Waiting Time Inside

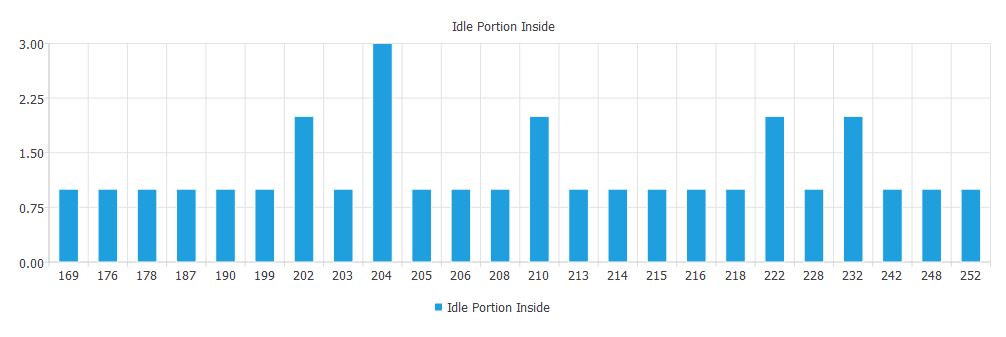
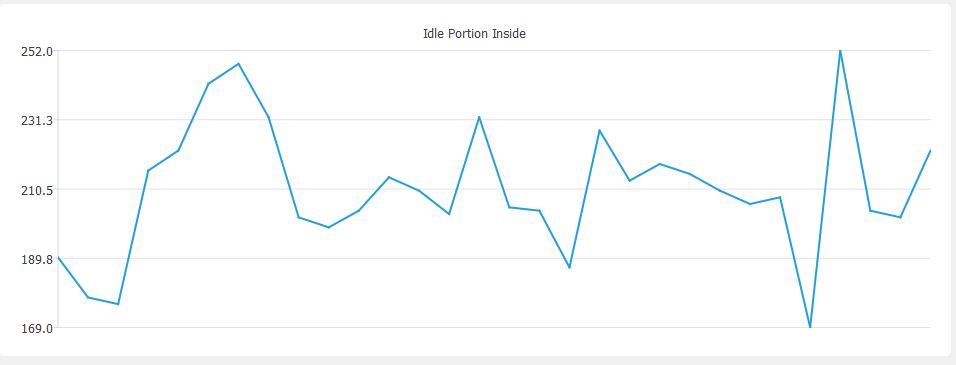
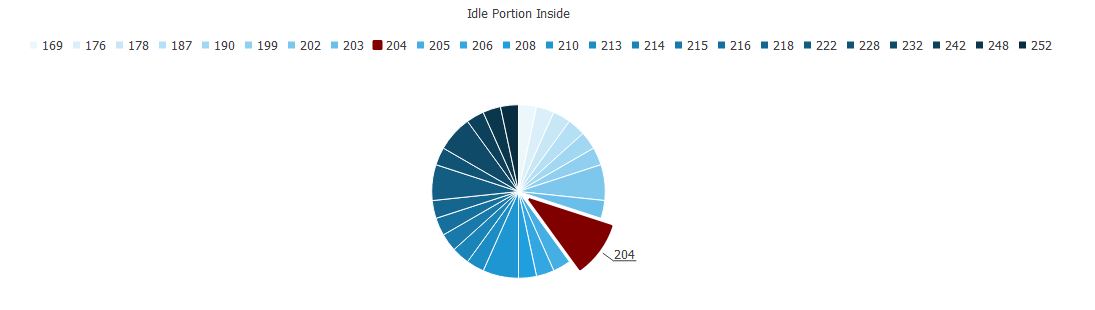
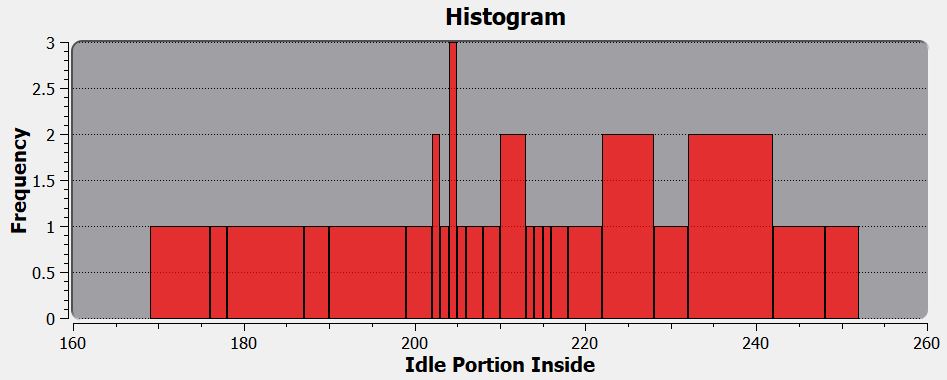


Maximum Inside Queue Length



Probability to Wait Inside



Portion of Idle Time Inside

Problem 2

Problem Formulation

**Environment**

The environment in this problem is a car shop that consists of a showroom and an inventory. A certain random purchase demand is made every business day, cars from the show room are sold from the inventory first, after the inventory runs out, cars start to get sold from the showroom. The showroom’s maximum capacity is 4 cars while that of the inventory is 8 cars.   
Each defined period (review period) of time if the inventory has less than a certain number of cars (minimum threshold), the owner makes an order to restock the showroom and the inventory to their maximum capacity. This order has a random lead time (delay before arrival) to the shop.

**Assumptions**

**-** It is assumed that the review period is 2 days and the minimum inventory threshold to make an order is 4 cars.

- It is assumed the car shop starts with 4 cars in the show room and 2 cars in the inventory. Also, there is an already pending order scheduled to arrive after 2 days with 5 cars.

- It is assumed that orders are made at the end of a business day and are received on the start of a business day. For example, an order made at the end of day 1 with a lead time of 1 will arrive at the start of day 3.   
  
- It is assumed that no orders are placed before receiving already pending orders. For example, if an order is made on day 1 with a lead time of 1, the owner can’t place any other orders before day 3 (since he will receive this pending order on day 3).

- It is assumed that if the demand of the cars is more than the current stock, the day is considered a shortage day. All the cars get sold and the remaining demanded cars are dismissed.

**Objectives**

**Estimate the system performance for the following**

* Estimate the average demand of the customers.
* Estimate the average order lead time.
* Estimate the average ending cars in the show room.
* Estimate the average ending cars in the inventory.
* Estimate the average number of days a shortage occurs.
* Determine if the Theoretical average of the demand and order lead time match the practical ones.
* Determine if there is a better review period than the assumed one (2).
* Determine the optimal combination of period time and minimum threshold.

Model Conceptualization

**System Components**

|  |  |  |
| --- | --- | --- |
| Entity | Attribute | Event |
| Customer | Demand cars | Purchase a car |
| Owner | Order Lead Time | Make orders |
| Car distributer | Order delivery lead time | Deliver orders |

**System Analysis**

**Demand Probability Table**

|  |  |  |  |
| --- | --- | --- | --- |
| Demand | Probability | Cumulative | Range |
| 0 | 0.05 | 0.05 | 1-5 |
| 1 | 0.28 | 0.33 | 6-33 |
| 2 | 0.37 | 0.70 | 34-70 |
| 3 | 0.20 | 0.90 | 71-90 |
| 4 | 0.10 | 1 | 91-00 |

Theoretical Average: 2  
**Order Lead Time Probability Table**

|  |  |  |  |
| --- | --- | --- | --- |
| Service Time | Probability | Cumulative | Range |
| 1 | 0.55 | 0.55 | 1-20 |
| 2 | 0.35 | 0.90 | 56-90 |
| 3 | 0.10 | 1 | 91-00 |

Theoretical Average: 1.55

**Calendar Table**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Day | Demand | Lead Time | Start Showroom | Start Inventory | End Showroom | End Inventory | Shortage |
| 1 | 1 | - | 4 | 2 | 4 | 1 | No |
| 2 | 2 | - | 4 | 1 | 3 | 0 | No |
| 3 | 1 | - | 4 | 4 | 4 | 3 | No |
| 4 | 1 | 1 | 4 | 3 | 4 | 2 | No |
| 5 | 1 | - | 4 | 2 | 4 | 1 | No |
| 6 | 1 | - | 4 | 7 | 4 | 6 | No |
| 7 | 2 | - | 4 | 6 | 4 | 4 | No |
| 8 | 3 | 2 | 4 | 4 | 4 | 1 | No |
| 9 | 2 | - | 4 | 1 | 3 | 0 | No |
| 10 | 3 | - | 3 | 0 | 0 | 0 | No |

Experimental Design

**Parameters**

The simulation is done with 30 Trials with 100 Days.

**Justification**

Due to some statistical studies (1,2), at 30 samples we start to see the data approach a normal distribution which is further proven with the Central Limit Theorem (3). Given these reasons it was reasonable to do the simulation with 30 Trials to take benefit of the CLT.

Result analysis and Conclusion

**Results**

|  |  |  |
| --- | --- | --- |
| Criteria | Value | Comment |
| Average Demand | 2 | This result matches the theoretical average (2) since all the demands have a probability which is normally distributed­ |
| Average Lead Time | 1.5 | This result matches the theoretical average (1.55) since all the lead times have a probability which is normally distributed­ |
| Average Ending Showroom | 3.1 | Relatively high to the maximum capacity which indicated a low shortage because the show room on average is still full which means that it is rare that an event will happen that will make it go to 0 |
| Average Ending Inventory | 1.5 | Relatively low to the maximum capacity, indicates that the inventory is close to be empty most of the days. |
| Shortage Days | 5.3 | A low value indicating that most of the time no shortage conditions are met. |

**Conclusions:**

* Close to theoretical demand and lead time which coincide with the Central Limit Theorem (3)
* The low average ending in the inventory indicates that it is close to being empty most of the days.

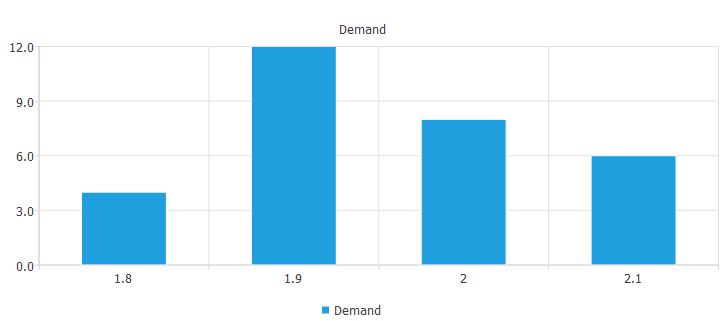
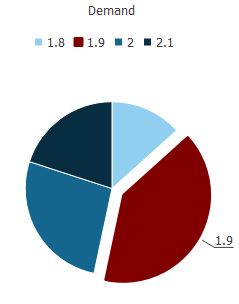
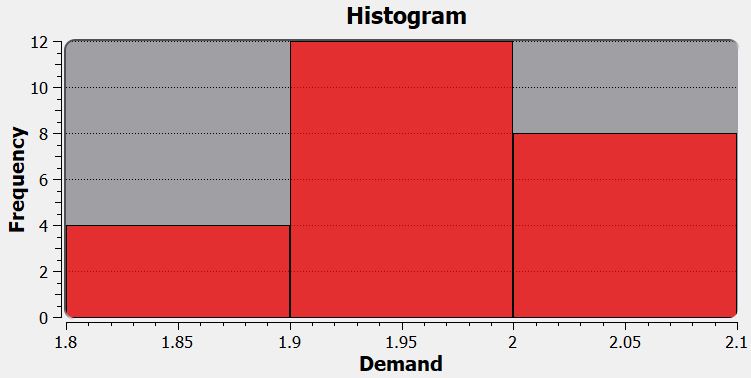
**Observations and Extra Statistics**

* As review period increases, the shortage increases directly
* Higher minimum threshold doesn’t necessarily mean lower shortage days
* From the test results, a minimum threshold of 4 with a review period of 1 produced the least shortage of 2.9 days tested over 30 review periods and displaying only five in the following table.

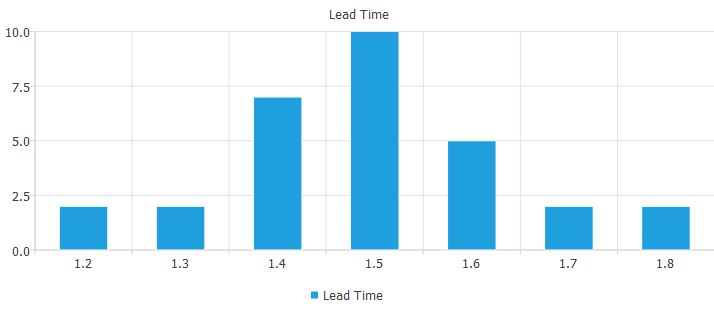
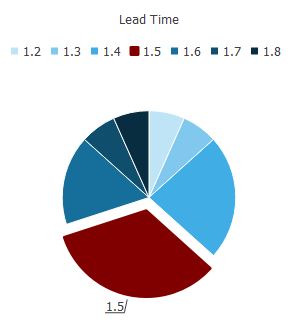
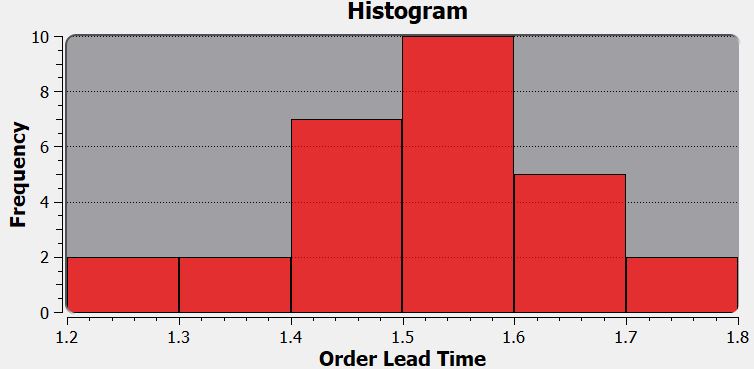
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| --- | --- | --- | --- | --- | --- |
| Minimum Threshold | Review Period | | | | |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 6.7 | 10 | 16.2 | 18.8 | 20.2 |
| 2 | 3.9 | 8.5 | 11.9 | 13.1 | 16.1 |
| 3 | 3.3 | 6.4 | 8.9 | 10.6 | 15.3 |
| 4 | 2.9 | 5.3 | 7 | 9.2 | 14.4 |
| 5 | 3.4 | 5.7 | 6.5 | 8.9 | 13.4 |
| 6 | 3.7 | 5.1 | 5.9 | 9 | 13.6 |
| 7 | 3 | 5.6 | 7 | 7.6 | 14 |
| 8 | 3.2 | 5.2 | 6.9 | 7.7 | 12.8 |

**Charts**

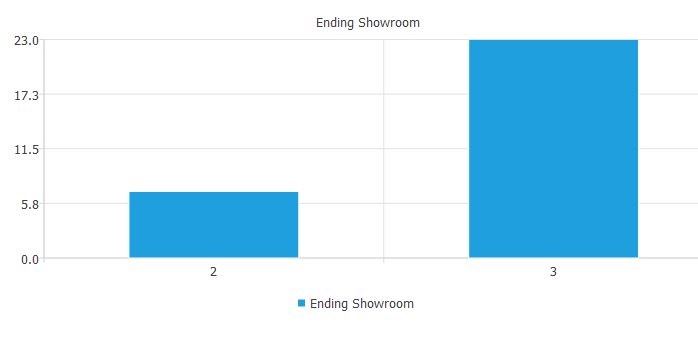
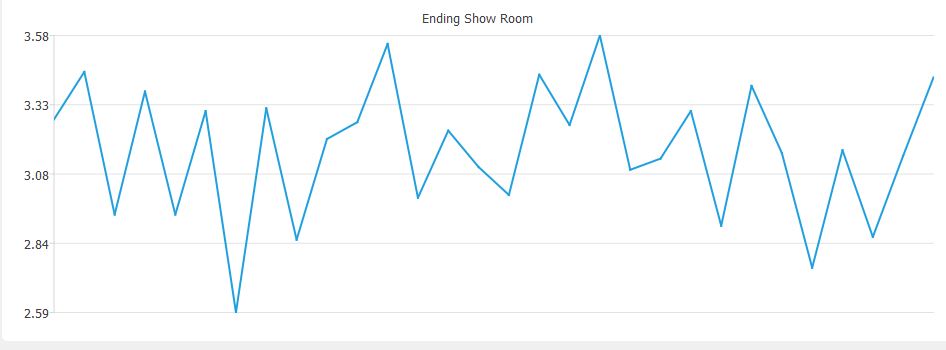
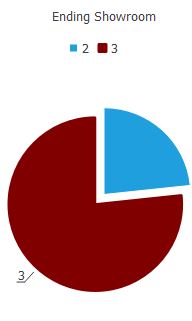
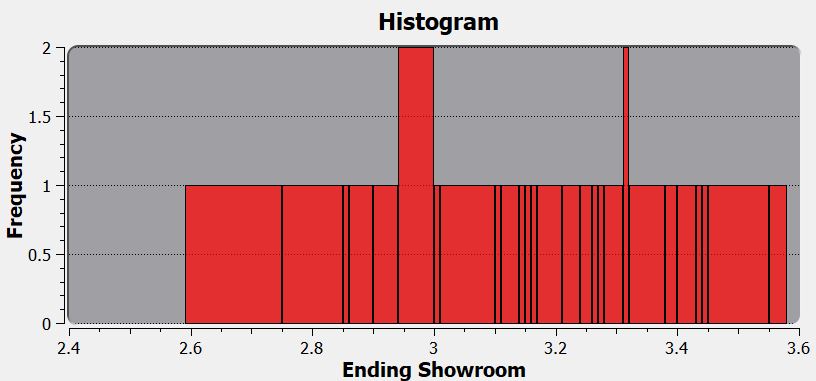
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Average Demand  


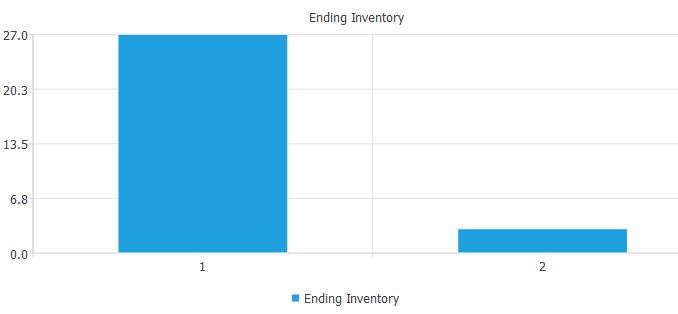
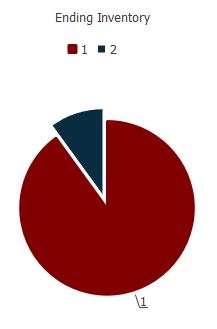
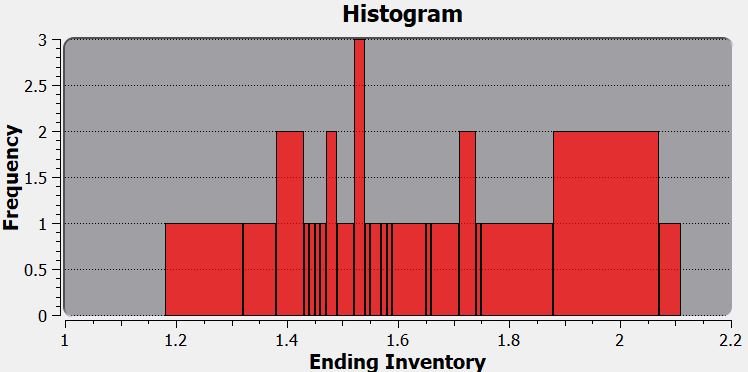
Average Lead Time



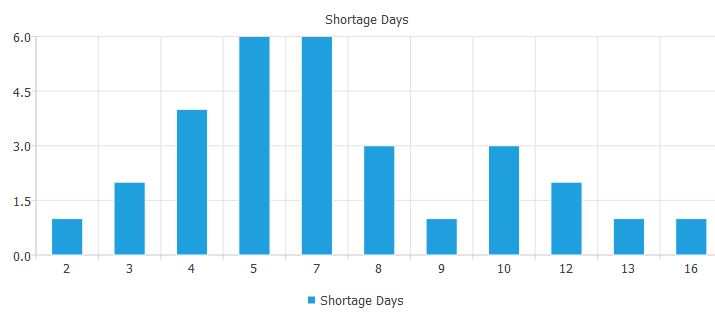
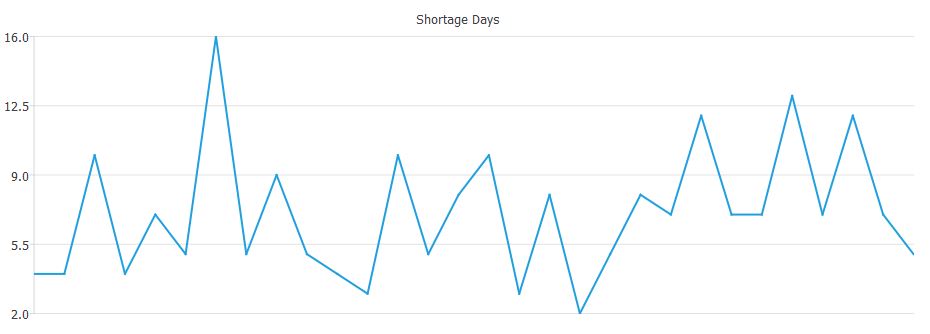
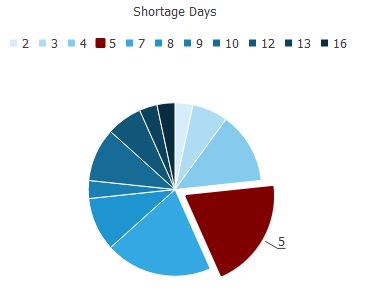
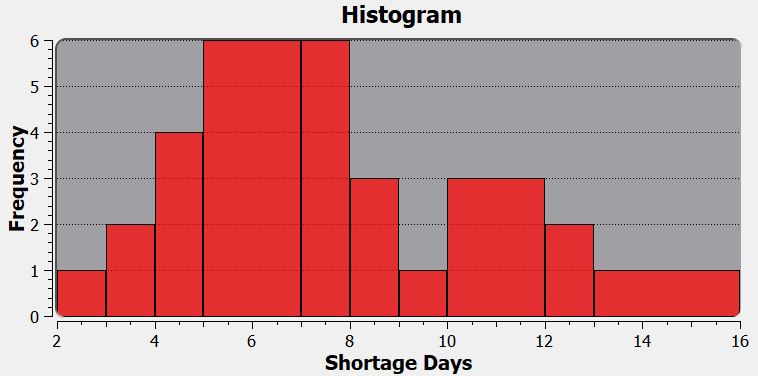
Average Ending Showroom



Average Ending Inventory



Average Shortage Days



References

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