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# Estimating fundamental Sharpe ratios: A Kalman filter approach

Hayette Gatfaoui

Associate Professor at NEOMA Business School,

Finance Department, 1 Rue du Maréchal Juin, BP 215, 76825 Mont-Saint-Aignan Cedex  
France, Phone: 00 33 2 3282 5821, Fax: 00 33 2 3282 58 34, hayette.gatfaoui@neoma-bs.fr

First draft: July 2009, Revised draft: February 2015

**Abstract:** *A wide community of practitioners still focuses on classic Sharpe ratio as a risk-adjusted performance measure due to its simplicity and easiness of implementation. Performance is computed as the excess return relative to the risk free rate whereas risk adjustment is provided by the asset return's volatility as a denominator. However, such risk-return representation is only relevant under a Gaussian world. Moreover, Sharpe ratio exhibits time variation and can also be biased by market trend and idiosyncratic risk. As an implementation, we propose to filter out classic Sharpe ratios (SR) so as to extract their fundamental component on a time series basis. Time-varying filtered Sharpe ratios are obtained while employing the Kalman filter methodology. In this light, fundamental/filtered Sharpe ratios (FSR) are free of previous reported biases, and reflect the pure performance of assets. A brief analysis shows that SR is strongly correlated with other well-known comparable risk-adjusted performance measures while FSR exhibits a low correlation. Moreover, FSR is a more efficient performance estimator than previous comparable risk-adjusted performance measures because it exhibits a lower standard deviation. Finally, a comparative analysis combines GARCH modeling, extreme value theory, multivariate copula representation and Monte Carlo simulations. Based on 10 000 trials and building equally-weighted portfolios with the 30 best performing stocks according to each considered performance measure, the top-30 FSR portfolio offers generally higher perspectives of expected gains as well as reduced Value-at-Risk forecasts (i.e. worst loss scenario) over one-week and one-month horizons as compared to other performing portfolios.*

**JEL Codes:** C15, C16, G12.

**Keywords:** Extreme Value Copula, Kalman Filter, GARCH, Latent factor, Pure Performance, Sharpe ratio, Value-at-Risk.

## 1. Introduction

A wide community of practitioners still focuses on classic Sharpe ratios as a tool to assess assets' performance (Bhargava et al. 2001; Elyasiani and Jia 2011; Ho et al. 2011; Robertson 2001; Scholz and Wilkens 2005b). Basically, portfolio managers use extensively such risk-adjusted performance measure due to its simplicity and easiness of implementation. Sharpe ratio is a performance measure whose assumptions come from the Capital Asset Pricing

Model (CAPM). The CAPM is an equilibrium relationship between security returns, which is derived under a basic and restrictive setting ([Lintner 1965a](#), [1965b](#); [Mossin 1966](#); [Sharpe 1963](#)). In particular, performance is computed as the excess return relative to the risk free rate (e.g. 1-month T-bill) whereas risk adjustment is provided by the asset return's volatility, or equivalently, the return's standard deviation as a denominator ([Sharpe 1964](#)). Hence, Sharpe ratio expresses the excess return, or equivalently, the investor's reward per unit of (total) risk. Such risk-return representation is only relevant under a Gaussian world while assuming the total risk to result exclusively from market risk (e.g. diversified and efficient portfolios, [Sharpe 1964](#)). However, Sharpe ratios can be biased because of existing idiosyncratic risk in considered financial assets and/or due to existing portfolio underdiversification ([Hwang et al., 2012](#); [Van Nieuwerburgh and Veldkamp, 2010](#)). In particular, idiosyncratic risk can contribute to increase volatility and exacerbate skewness and kurtosis effects in asset returns ([Angelidis and Tassaromatis, 2009](#); [Yan, 2011](#)). Therefore, deviations from normality as materialized by skewness and kurtosis patterns ([Black 2006](#); [Eling and Schuhmacher 2007](#)) generate biases in asset performance valuation ([Hodges 1998](#); [Klemkosky 1973](#); [Spurgin 2001](#); [Zakamouline and Koekebakker 2009](#)). For example, existing jumps in asset prices generate skewness in corresponding returns so as to invalidate CAPM-based relationships in their classic form, and therefore engender classic Sharpe ratio's misestimation ([Christensen and Platen 2007](#); [Platen 2006](#)). One statistically significant outlier return suffices to bias upward or downward Sharpe ratio<sup>1</sup> ([Gatfaoui 2012](#)) due to the impact of the outlier on both the average return level and corresponding standard deviation. Moreover, [Goetzmann et al. \(2007\)](#) and [Spurgin \(2001\)](#) show that managers can manipulate Sharpe ratio. Indeed, managers can bias upward Sharpe ratio estimates by taking well-chosen derivatives positions, which artificially lower standard deviation without really lowering the investments' risk.

Incidentally, [Sortino \(2004\)](#) shows that standard deviation underestimates risk during upward market trends while it overestimates risk during downward market trends. Thus, market trends distort Sharpe ratio, which should therefore account dynamically for market trend bias (e.g. time variation in returns' performance). Additionally, current research also advocates time variation in Sharpe ratio ([Tang and Whitelaw 2011](#); [Woehrmann et al. 2005](#)). Specifically, the equity premium and Sharpe ratio vary over the business cycle ([Kocherlakota 1996](#)). Such time variation may reflect changes in agents' risk aversion ([Raunig and Scharler 2010](#)) as well as cyclical/seasonal patterns ([Tang and Whitelaw 2011](#)). Therefore, if risk assessment is biased, the reward-to-risk assessment becomes a biased performance indicator ([Sortino 2004](#)). As a consequence, the stock or asset selection process resulting from such performance measure yields a flawed asset picking because it is based on a misestimated selection tool ([Klemkosky 1973](#)). As an improvement, we propose to filter out classic Sharpe ratios (SR) so as to extract their fundamental components on a time series basis. The obtained time-varying fundamental Sharpe ratios, or equivalently, filtered Sharpe ratios (FSR) are free of previous reported biases, and reflect the pure performance of assets. On a practical viewpoint, classic Sharpe ratios represent a noisy performance measure from which we extract the pure performance component, namely the FSR. In particular, the performance noise embedded in classic Sharpe ratios results from the market's impact and idiosyncratic risk among others. The employed Kalman filter model suggests that fundamental Sharpe ratios are obtained after removing directly the market's trend and volatility impact from

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<sup>1</sup> [Gatfaoui \(2012\)](#) assesses the impact of returns' asymmetry, as represented by skewness and kurtosis patterns, on Sharpe ratio through a simulation study. Introducing one outlier in normally distributed returns, the author quantifies the bias generated by such an outlier return on Sharpe ratio estimates. Such a bias is driven by the investment horizon, the frequency of the data, and the propensity of the outlier return to deviate from the average return level.

observed Sharpe ratios. Additionally, a comparison with six other well-known risk-adjusted performance measures highlights the clear discordances between the investment ranks resulting from those performance measures and the ones inferred from FSR. Those performance measures tend rather to track the performance classification, which is implied by classic Sharpe ratios to a large extent. Furthermore, FSR is a more efficient performance estimator as compared to such comparable risk-adjusted performance measures (RAPMs) because it exhibits a lower standard deviation. Finally, a comparative risk analysis (i.e. market risk exposure) accounts for the time-varying volatility and tail risk of stock returns as well as correlation risk across stock returns among others. Portfolios composed of FSR-based winning stocks offer higher expected gains and reduced Value-at-Risk levels over one-week and one-month horizons as compared to other RAPM-based performing portfolios.

Our paper is organized as follows. In the second section, we present the Kalman filter model. Filtering out observed Sharpe ratios, we extract unobserved fundamental Sharpe ratios after removing the noise resulting from the financial market's influence and existing idiosyncratic risk. In the third section, we then introduce the stock returns under consideration, namely 85 return time series so as to draw statistical inference and exhibit stylized facts. Further, section 4 leads a back-testing analysis proving the soundness of the model and related measurement's robustness. As an extension, section 5 proposes a comparative study relative to other risk-adjusted performance measures (RAPMs) such as Sortino, Omega, Kappa, and Upside potential ratios. Most RAPMs are strongly correlated with SR and low correlated with FSR, suggesting structural differences between SR-based and FSR-based stock picking processes. Finally, section 6 proposes a comparative analysis with respect to RAPMs' efficiency and relevance. Firstly, FSR is a more efficient performance estimator than other RAPMs because it exhibits a lower standard deviation. Secondly, the market risk analysis combines GARCH modeling, extreme value theory, multivariate copula representation and Monte Carlo simulations (i.e. GARCH-EVT-Copula model). Based on 10 000 trials and building equally-weighted portfolios with the 30 best performing stocks according to each considered performance measure, the top-30 FSR portfolio offers generally higher perspectives of expected gains as well as reduced Value-at-Risk forecasts (i.e. worst loss scenarios) over one-week and one-month horizons as compared to other performing portfolios. Then, major findings are summarized in section 7, which also introduces concluding remarks.

## **2. The model**

Classic Sharpe ratios are biased/noisy performance indicators whose biases result from market climate and idiosyncratic risk among others. In that way, they represent disturbed risk-adjusted performance measures, which require to be cleaned.

### *2.1. Motivations*

Sharpe ratios and then related performance assessment are subject to three types of bias, namely non normality, market climate and time variation. The latter bias requires a dynamic performance measurement.

The first bias results from deviations from normality as illustrated by stock returns' skewness and kurtosis patterns. Such deviations invalidate the appropriateness of both the mean return as a performance indicator and the standard deviation as a risk measure. Hence,

the Sharpe ratio of non-Gaussian stock returns provides an erroneous performance measure. The second bias arises from the trend of the financial market, which impacts the reliability of Sharpe performance measure ([Krimm et al. 2012](#); [Scholz and Wilkens 2005a](#); [Scholz 2007](#); [Sortino 2004](#)). As an example, Sharpe ratio overestimates the performance of poorly diversified portfolios or funds during bear markets while it underestimates the performance of such funds or portfolios during bull markets ([Krimm et al. 2012](#); [Scholz and Wilkens 2005a](#)). Market climate impacts then performance valuation and related investment rankings ([Sortino 2004](#)). As a result, economic variables represent key factors explaining the predictable time-variation of investment returns. Incidentally, [Ferson and Harvey \(1991\)](#) show the significance of the market risk premium for stock-specific investment returns. In particular, the risk of change (i.e. time-variation) in investment returns exhibits a common component according to [Alexander \(2005\)](#). Additionally, [Sharpe \(1963\)](#) exhibits the common systematic component in stock excess returns through the CAPM. Therefore, the market risk premium represents a systematic component of stock returns' risk premium and captures the market climate bias (e.g. time variation, market-based structural changes). Following [Fama and French \(1993\)](#), Sharpe ratios should therefore be linearly linked with the market risk premium, which is an explicit market bias indicator.

As regards the third bias, current research exhibits the cyclical pattern of Sharpe ratio. For example, [Lettau and Ludvigson \(2010\)](#), [Lustig and Verdelhan \(2012\)](#), [Whitelaw \(1997\)](#) as well as [Woehrmann et al. \(2005\)](#) exhibit countercyclical Sharpe ratios. Such countercyclical pattern can result from changes in investors' sentiment ([Doran et al. 2009](#); [Tang and Whitelaw 2011](#)) or in aggregate risk aversion over the business cycle ([Kamstra et al. 2003](#); [Tang and Whitelaw 2011](#)). Incidentally, stock returns' seasonality also supports Sharpe ratio's cyclical feature ([Fiore and Saha 2015](#)). Moreover, the implied volatility index (VIX) appears as a central factor for performance assessment since it represents a proxy of time varying volatility ([Brandt and Kang 2004](#)) as well as investors' fear gauge. In particular, [Brandt and Kang \(2004\)](#) show that time variation in volatility explains well-known deviations from the positive relationship between risk premium and volatility, which is advocated by the CAPM. Indeed, the arrival of new information as well as changes in the economic outlook or in investors' risk aversion can generate jumps in stock returns ([Raunig and Scharler 2010](#)). Hence, equity returns exhibit volatility regimes so that volatility is subject to shifts (i.e. jump risk) over the business cycle ([Cremers et al. 2015](#); [Hamilton and Lin 1996](#); [Santa-Clara and Yan 2010](#); [Schwert 1990](#)). Incidentally, [Whitelaw \(2000\)](#) advocates time-varying probabilities of regime switches while [Santa-Clara and Yan \(2010\)](#) exhibit time-varying jump and volatility risks. Following previous findings, we consider the impact of stock market volatility (as represented by VIX) on stock returns' level.<sup>2</sup> Hence, the impact of a change in market volatility on stock returns (i.e. volatility feedback) is linearly taken into account through VIX indicator in accordance with [Maheu et al. \(2013\)](#). Such feature is also supported by the fact that volatility feedback generates return asymmetry and therefore skewness ([Campbell and Hentschel 1992](#); [Gatfaoui 2013](#)). In this light, the impacts of structural changes in both the mean and variance of the stock market's return require to be taken into account (i.e. time variation at the return and volatility levels). Thus, biases resulting from the financial market's trend regimes and volatility regimes will be captured and quantified.

Consequently, classic Sharpe ratios need to be filtered out so as to correct them for their hidden biases. For this prospect, we select the linear Kalman filter methodology. In particular, classic Sharpe ratios are considered as noisy performance signals, which are disturbed/altered

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<sup>2</sup> Non-linearity in financial markets often arises from time-varying volatility patterns (i.e. volatility relationships, see [Nam et al. 2006](#)).

by specific biases. The linear Kalman filter helps clean the noisy observed signals in order to extract the true and unbiased corresponding signals, which are unobserved components (Durbin and Koopman 2001; Harvey 1989; Kalman 1960; Kalman and Bussy 1961). Once classic Sharpe ratios go through the Kalman filter, they yield their pure signal counterparts, which are freed from existing biases (i.e. fundamental Sharpe ratios or  $FSR_i$ ). Moreover, the linear Kalman filtering methodology is efficient in correcting observed Sharpe ratios because monthly Sharpe ratios are stable, and can be considered as approximately linear (see Table 1).

## 2.2. Specification

Applying an unobserved component methodology (Harvey et al. 2004; Harvey and Koopman 2009), we clean observed classic Sharpe ratios (i.e. the noisy signal) and extract the denoised signal corresponding to unbiased/fundamental Sharpe ratios. Specifically, a linear Kalman filter model extracts unobserved fundamental Sharpe ratios from observed stock-specific Sharpe ratios along with relevant market indicators and stock returns' stylized facts. Given that monthly  $SR$ s exhibit no first order autocorrelation (see section 3) and that they depend on the market climate, we specify the fundamental Sharpe ratios ( $FSR_i$ ) as a random cycle component with deterministic frequency, amplitude and phase. Moreover, the market premium and the natural logarithm of VIX are introduced in the dynamics of monthly Sharpe ratios (i.e. linear link between  $SR_i$  and factors).<sup>3</sup> Hence, fundamental Sharpe ratios are unobserved and free of market climate and time variation biases (i.e. unbiased). Previous considerations yield the following specification for any time  $t$  and any stock  $i$  with  $i \in \{1, \dots, 85\}$  and  $t \in \{1, \dots, 170\}$ :<sup>4</sup>

$$SR_{it} = FSR_{it} + a_i \times MktPremium_t + b_i \times \ln(VIX_t) + u_{it} \quad (1)$$

$$FSR_{it} = c_i \times \cos(d_i \times t) + f_i \times \sin(g_i \times t) + v_{it} \quad (2)$$

where equations (1) and (2) represent the dynamic and state equations respectively;  $SR_{it}$  represents the Sharpe ratio of stock  $i$  over time  $t$ ;  $MktPremium_t$  and  $VIX_t$  represent the market premium and VIX over time  $t$ ;  $(u_{it})$  and  $(v_{it})$  are serially independent and correlated Gaussian white noises with a zero mean (i.e. dynamic and state errors over time  $t$ );<sup>5</sup>  $FSR_{it}$  is the unobserved/latent component in  $SR_{it}$  over time  $t$ , the equation errors are assumed to follow a two-dimensional Gaussian variable with a covariance matrix  $\Omega_i$  defined as follows:

$$\Omega_i = \begin{pmatrix} e^{h_i} & k_i \\ k_i & e^{l_i} \end{pmatrix} \quad (3)$$

where  $a, b, c, d, f, g, h, k$  and  $l$  are constant parameters.<sup>6</sup> Hence, we consider a stationary representation, which is advocated by the stationary pattern of the data. The previous

<sup>3</sup> Model selection is based on information criteria (Akaike 1974; Hannan and Quinn 1979; Schwarz 1978).

<sup>4</sup> In unreported results, we tested for several specifications and concluded that either VIX or both SMB and HML factors can be used additionally to the market premium. However, most relevant results are obtained with VIX, which supports the findings of section 3.2. Hence, the market premium and VIX factors are incorporated to the model while dropping SMB and HML explanatory factors. Moreover, we also considered an additional unobserved trend component but it revealed also to be insignificant as compared to the cycle component in FSR.

<sup>5</sup> They represent residual idiosyncrasies.

<sup>6</sup> Gatafoui (2010) proposes a simulation study, which evaluates the impact of return asymmetry on Sharpe ratio. Applying asymmetric shocks to normally distributed returns, the author considers the distortion of the Sharpe



representation's consistency is twofold. First, such specification is relevant since classic Sharpe ratios are a special case of the previous representation under a Gaussian setting. Indeed, Gaussian returns imply at least that  $h$  tends towards minus infinity, and even that  $a=0$  and  $b=0$  under a neutral market trend and volatility assumption. Thus, the variance of errors ( $u_i$ ) is zero, and then  $SR_i = FSR_i$  (i.e. no dynamic error since the related mean and variance are zero). In such case, the presumed noisy signal ( $SR_i$ ) coincides with its filtered and unobserved signal counterpart ( $FSR_i$ ), which means that potential biases are not altering the observed signal such as the observed Sharpe ratios ( $SR_i$ ). Second, the possible correlation between state and dynamic errors accounts for possible remaining market-based commonalities (i.e. residual correlation risk) such as liquidity commonality, or equivalently, systematic liquidity risk in stock prices (Acharya and Pedersen 2005; Chordia et al. 2000; Hasbrouck & Seppi 2001; Keene and Peterson 2007; Kempf and Mayston 2008), and potentially unaccounted market volatility regime or jumps among others (Ammann and Verhofen 2009; Chang 2009; Chu, Santoni, and Tung 1996; Kim, Morley, and Nelson 2004).

### *Discussion about interest and benefits of FSR*

Sharpe ratio assumes risk symmetry and penalizes the average performance, as measured by the average excess return beyond the risk free rate, by the downside and upside variances which are embedded in stock returns' global variance. The upside and downside variances are simply the variances of positive and negative excess returns respectively. SR does not distinguish between the upside potential of stock returns and related downside potential. However, rational investors favor stocks exhibiting highly variable gains, or equivalently, high upside potential ([Zakamouline 2011](#)). In this light, the SR-based selection process favors a stock return exhibiting a low downside variance (higher SR) as compared to a stock return exhibiting a high upside variance (lower SR). Hence, inconsistencies about risk perception appear, generating then a biased stock selection process. Stock picking is also biased by the stock market's impact among which its trend. Such biases are handled within the proposed Kalman-based estimation. First, average computations on approximately one-month non-overlapping windows yield Gaussian SRs, which conforms to model assumptions. Second, the stock market's bias is handled through the stock market trend and volatility factors (e.g. trend and volatility regimes), releasing therefore FSR from the dependency on market climate and market volatility regimes. Finally, the cyclicity of FSR is acknowledged in line with financial markets' oscillatory pattern ([Dayri 2011](#)) and related sensitivity to business cycle ([Lettau and Ludvigson 2010](#); [Lustig and Verdelhan 2012](#); [Whitelaw 1997](#); [Woehrmann et al. 2005](#)) among others. As a result, proposed performance measurement accounts for the reported time variation, which results from market trend and volatility regimes as well as cyclical/seasonal patterns. Obtained FSR is dynamically adjusted to structural changes providing therefore a bias correction.

Moreover, corresponding model implications are threefold. First, estimating fundamental Sharpe ratios requires removing directly the market trend and volatility biases from observed monthly Sharpe ratios. Such representation follows the findings of Fama and French (1993) as

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ratio, which is induced by the resulting return skewness and kurtosis. Such a distortion allows for quantifying the bias in the Sharpe ratio, which results from return asymmetry. The author handles then a filtering process based on Kalman methodology in order to remove the bias in the Sharpe ratio. However, firm-specific and market-specific factors are not taken into account in the simulation study. Hence, our study improves and strengthens the work of Gatfaoui (2010) while illustrating the reality of financial markets and stocks' stylized features. In this light, we therefore propose a more sophisticated and more robust study.



well as stochastic volatility patterns (Maheu et al. 2013). Second, accounting for possibly remaining market commonalities through equation errors' covariance (i.e. correlation, volatility linkages) avoids model misspecification. Third, fundamental Sharpe ratios consist of a random cycle component, which conforms to the oscillatory (Mishchenko 2014), scaling (Dayri et al. 2011) and cyclical (Lettau and Ludvigson 2010; Woehrmann et al. 2005) patterns of financial markets. In particular, FSR encompasses a predictable cyclical trend and an unpredictable white noise component. The predictable cyclical trend is of interest to investors who target a market timing strategy and therefore bet on cycle reversals (i.e. active investment strategy as opposed to static buy and hold strategy). However, such predictable cyclical trend is balanced with an unpredictable shock, which illustrates prevailing uncertainty. Uncertainty materializes as either reinforcing or counteracting deviations from predictions. Thus, we are able to characterize the predictable variation in fundamental Sharpe ratios (Tang and Whitelaw 2011) and its uncertainty. Predictability is preserved when the unpredictable shock emphasizes the directional and cyclical trend so that a winning asset allocation is favored (when investors attempt to time cycle reversals from one month to another).<sup>7</sup> In the reverse case, predictability is compromised and asset allocation performs poorly because related market timing strategy fails. As a consequence, performing active investors consist of portfolio/fund managers who are able to mitigate uncertainty at the portfolio level (e.g. mitigating idiosyncratic shocks) so as to rely mainly on the predictable performance component conditionally on market climate (e.g. living over trend and volatility regimes).

### 3. Data

The data under consideration deal with stock prices and corresponding relevant fundamental factors on the U.S. financial market. Such factors bring in information about the performance of stocks. We first introduce the data and their properties, and then arrange the data in order to run the Kalman filter estimation on a time series basis.

#### 3.1. Description and properties

##### 3.1.1. Data description

The daily returns of 85 stocks are considered between 2000/01/04 and 2014/04/30, namely 3398 observations per series. Related risk premia versus the one-month T-Bill rate are computed over this investment horizon. Namely, yield differences such as  $R_{it} - R_{ft}$  are computed on each day  $t \in \{1, \dots, 3398\}$  for any stock  $i \in \{1, \dots, 85\}$  where  $(R_{ft})$  is the one-month T-Bill rate on day  $t$  and  $(R_{it})$  is stock  $i$ 's return on day  $t$ . The sample stocks are picked randomly on the U.S. market and also belong to specific Standard & Poor's indexes. Among the 85 stocks, 1 return is the S&P500 index return, 63 stocks belong to the S&P500 index (i.e. large-cap market), 9 stocks belong to the S&P MidCap 400 index (i.e. medium sized companies' equity market), and 12 stocks belong to the S&P SmallCap 600 index (i.e. small sized equity market segment).<sup>8</sup> Each stock is identified by a number  $i$  running from 1 to 85, and each stock  $i$  exhibits a specific daily Sharpe ratio ( $SR_i$ ) over the investment horizon.

<sup>7</sup> Following their predictions, aggressive investors focus on tactical asset allocation in order to maximize their investment returns in the short run. Successful active investors will enter the market after a favorable cycle reversal and surf on the upward performance trend. They will also be able to exit the market before the next cycle reversal so as to avoid any downward performance trend.

<sup>8</sup> The S&P SmallCap 600, S&P MidCap 400 and S&P500 indexes represent 3%, 7% and 75% of the U.S. equity market respectively.

Those stock-specific Sharpe ratios are computed as the ratios of average excess returns to corresponding excess returns' standard deviations over the sample horizon. Moreover, the three Fama and French (1993) factors are considered, namely the market portfolio's premium (*MktPremium*), the return of the Small minus Big portfolio<sup>9</sup> (*SMB*) and the return of the High minus Low portfolio<sup>10</sup> (*HML*). The daily implied volatility index level (*VIX*) is also considered as a market volatility indicator. All stock data as well as VIX values are extracted from Yahoo Finance website whereas Fama and French (1993) factors come from the authors' website.<sup>11</sup>

### 3.1.2. Properties

Stock returns exhibit generally a non-Gaussian behavior.<sup>12</sup> As a rough guide, Table 1 displays some descriptive statistics about the estimated stock-specific SR, mean, median, standard deviation, skewness and kurtosis across stocks. The risk profile of each stock return series results from the tradeoff between existing tails and their heaviness as represented by skewness and kurtosis (but also by the divergence between mean and median values).

[Insert Table 1 about here]

Results are mitigated and exhibit heterogeneous stock returns' behaviors. In particular, stock returns exhibit non-negligible skewness and kurtosis patterns on a daily basis.

### 3.2. Data transformation

In order to run a time series analysis based on Sharpe ratios, we operate a simple data transformation. Such transformation conforms to the financial practice while assessing investment or fund performance. In this light, we compute the monthly Sharpe ratios as well as corresponding average Fama and French (1993) factors and VIX levels. Monthly data are computed from daily data based on non-overlapping periods of 20 working days (i.e. approximately one month). Hence, our daily sample horizon comprises 170 monthly data, the first month (January 2000) encompassing 18 working days. In unreported results, we find that monthly SR series are Gaussian for all stock returns under consideration. Moreover, they are stationary, behave globally like white noises, and exhibit at least no first order autocorrelation.

As an extension, Table 2 displays the properties of Kendall tau b correlations between monthly Sharpe ratios and the four explanatory factors, namely the three Fama and French factors (market premium, *SMB*, *HML*) and the VIX.

[Insert Table 2 about here]

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<sup>9</sup> It comes from the difference between small-cap-specific portfolios and large-cap-specific portfolios (i.e. size indicators).

<sup>10</sup> It results from the difference between value stocks' portfolios and growth stocks' portfolios (i.e. book-to-market indicators).

<sup>11</sup> See [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

<sup>12</sup> Unreported Jarque Bera statistics confirm generally such a stylized fact (Jarque and Bera 1980, 1981, 1987). Indeed, the Jarque Bera statistics of our 85 stocks range from 371.4619 to 1473438, which invalidate the Gaussian distribution assumption at a 5% test level. Moreover, returns exhibit high excess kurtosis levels.

The market premium is non-negligibly linked with stocks' monthly Sharpe ratios whereas the linkages between remaining explanatory factors and monthly SRs are weaker. However, the Kendall correlations of market premium and VIX with monthly SRs exhibit the lowest coefficient of variations. Hence, former correlation coefficients display a low dispersion, which may suggest a more homogeneous link between monthly SRs and both the market premium and VIX factors. As a result, the market premium and VIX are the most relevant factors to describe stocks' monthly Sharpe Ratios as compared to the influence of SMB and HML factors, the latter influence being mitigated across stocks on a monthly basis.

## 4. Econometric results and robustness check

### 4.1. Estimations

The model is estimated separately for each of the 85 stock return series under consideration. Model parameters are estimated along with the log-likelihood maximization principle under the Gaussian joint multivariate distribution of state and dynamic errors. Hence, we obtain 85 series of FSRs, which are computed over 170 months. Moreover, unreported Cramer-Von-Mises statistics confirm the Gaussian behavior of monthly SRs and FSRs series for the 85 stock returns under consideration at a five percent test level (Cramer 1928; von Mises 1931).<sup>13</sup>

Given the Gaussian nature of obtained FSRs, we can consider the median FSR as a good performance proxy since it is an appropriate location indicator. Moreover, the Cramer-von Mises statistic of median FSR equals 0.0178, which validates its Gaussian behavior at a five percent test level. Differences between Sharpe ratios and their median fundamental counterparts result from market trend and volatility biases as well as idiosyncrasies. Such discrepancies highlight overestimation or under-estimation issues while assessing stock performance with classic Sharpe ratios. Conversely, any coincidence between SR and FSR indicates the goodness of Sharpe ratio as a performance measure in the light of possible bias factors. In such situation, either bias factors may not exist or the trade-off in between such factors eliminates potential biases. Moreover, Kendall's tau b correlation between SRs and median FSRs equals 0.2443 and is significant at a five percent test level. Hence, commonalities in SR- and FSR-based performance analyses arise in 24.43 percent of cases. However, such degree of commonality is still low. Investors can thus question the impact of SR and FSR differences on both related stock selection processes and resulting portfolios' performance.

Although results highlight the relevance of the employed Kalman filter model, the next step consists of investigating model's robustness. A robustness check is necessary so as to ensure a non-spurious relationship (i.e. ensuring an appropriate description of the phenomenon under consideration).

### 4.2. Robustness investigation

Checking for regression residuals' behavior, Fig. 1 plots Cramer-Von-Mises statistics as well as their five percent critical threshold. Cramer-Von-Mises normality test is validated as long as corresponding statistics avoid being higher than the critical threshold. Hence,

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<sup>13</sup> The critical value of the Cramer-Von-Mises statistic is 0.2200 for a sample size of 100 and a 5% test level.

estimated Cramer-Von-Mises statistics show the Gaussian behavior of dynamic ( $u_i$ ) and state ( $v_i$ ) errors at a 5% test level. Thus, dynamic and state errors conform to the theoretical Gaussian assumption, which supports Gaussian SRs and FRs.

[Insert Fig. 1 about here]

As a complementary diagnostic test, unreported Ljung-Box statistics ([Ljung and Box 1978](#)) underline generally the white noise property of dynamic and state errors. For example, the Ljung-Box statistics of order 1 range from 0.0013 to 3.6268, and from 0.0000 to 0.2301 for the dynamic and state errors respectively, and are below the 3.8415 critical value at a five percent test level. The same remark also holds for squared dynamic and state errors whose first order Ljung-Box statistics range from 0.0041 to 3.8051 and from 0.0011 to 3.7269 respectively. Hence, model residuals exhibit no heteroskedastic pattern (i.e. time-varying variance) as well as independency, which strengthen the robustness of our estimation method. Moreover, an unreported Phillips-Perron unit root test without trend and without constant term ([Phillips and Perron 1988](#)) supports stationary model residuals at a five percent level. As a conclusion, the Gaussian white noise assumption about dynamic and state errors is accepted.

Our previous study yields thus interesting results while providing a possible description of Sharpe ratio's biases. Moreover, the proposed filtering methodology is robust and in line with previous findings. The obtained fundamental Sharpe ratios can help building powerful performance assessment tools in a very simple framework.

## 5. Comparative study

We look for comparing our FSR with SR and existing RAPMs in the light of current literature review. We split the RAPMs into two distinct groups. The first group relies on normality assumptions about stock returns such as Sortino ratio whereas the second group relies on scale-independent RAPMs such as Omega, Kappa and Upside potential ratios.

### 5.1. Considering 6 other RAPMs

The first group of RAPMs is the Sortino ratio ([Sortino 1991, 2004](#); [Sortino and Price 1994](#); [Sortino and Forsey 1996](#)). Sortino ratio corresponds to the ratio of the excess return (relative to the risk free rate) to the standard deviation of losses as represented by the downside risk. The downside risk is defined as the lower partial moment of order one over the investment horizon. In particular, [Fishburn \(1977\)](#) defines the lower (LPM) and upper (UPM) partial moments of order  $n$  as follows:

$$\text{UPM}(i, r, n) = E[\text{Max}(R_i - r, 0)^n] \quad (4)$$

$$\text{LPM}(i, r, n) = E[\text{Max}(r - R_i, 0)^n] = (-1)^n E[\text{Min}(R_i - r, 0)^n] \quad (5)$$

where  $E[.]$  is the expectation operator,<sup>14</sup>  $R_i$  is the return on stock  $i$ ,  $n$  is the order of the moment, and  $r$  is the minimum targeted return of the investor. We'll set this latter variable to the risk free rate  $R_f$  for comparability reasons relative to SR and FSR. Hence, the Sortino ratio writes:

<sup>14</sup> In the simplest situation, it is simply the arithmetic mean of observed values over our investment period.

$$Sortino_i = \frac{E[R_i] - r}{\sqrt{LPM(i, r, 2)}} \quad (6)$$

Within the second group of RAPMs, the Omega ratio is the ratio of all gains to all losses over the investment horizon. In other words, it is the ratio of positive performance to negative performance in absolute terms (see Keating and Shadwick 2002). Considering the higher moments of the return distribution, it then writes:

$$Omega_i = \frac{UPM(i, r, 1)}{LPM(i, r, 1)} \quad (7)$$

In the same line, the Omega-Sharpe ratio corresponds to the Omega ratio minus 1, and illustrates a modified risk measure (Bacon 2008; Bernardo and Ledoit 2000; [Kazemi et al. 2004](#)). The risk measure in use focuses on all the losses over the investment horizon so that Omega-Sharpe ratio writes:

$$Omega - Sharpe_i = Omega_i - 1 = \frac{E[R_i] - r}{LPM(i, r, 1)} \quad (8)$$

Incidentally, Omega and Omega-Sharpe yield the same investment rankings. Therefore, their respective performance classifications should coincide. As a modification of Sortino ratio, the Kappa ratio accounts for higher moments ([Kaplan and Knowles 2004](#)) and focuses on losses over the investment horizon. It is the ratio of the excess return to a given lower partial moment and writes:

$$Kappa_{n,i} = \frac{E[R_i] - r}{\sqrt[n]{LPM(i, r, n)}} = \frac{E[R_i] - r}{[LPM(i, r, n)]^{1/n}} \quad (9)$$

When the targeted minimum return is the risk free rate, Omega-Sharpe and Sortino ratios are particular cases of Kappa ratio as follows:

$$Kappa_{1,i} = Omega - Sharpe_i \quad (10)$$

$$Kappa_{2,i} = Sortino_i \quad (11)$$

Caring about order 3 and 4 moments, we consider additionally Kappa<sub>3</sub> and Kappa<sub>4</sub> ratios. As a last performance measure, the Upside potential ratio (UPR) considers finally the ratio of all gains to the downside risk over the investment horizon (Sortino et al. 1999) and writes:

$$UPR_i = \frac{UPM(i, r, 1)}{\sqrt{LPM(i, r, 2)}} \quad (12)$$

We handle therefore a range of various risk-adjusted performance measures or RAPMs, which will be compared to our FSR as well as its classic SR counterpart.

## 5.2. Empirical results

As regards Sharpe ratios, Fig. 2 ranks classic Sharpe ratios by ascending order and plots ordered classic Sharpe ratios against their fundamental counterparts. As a rough guide, a regression line is plotted so as to check for a one-to-one correspondence between classic and

fundamental Sharpe ratio-based rankings. Obviously, the reported couples of classic and median fundamental Sharpe ratios do not follow the regression line (or at least gather homogeneously around it) but rather split widely around. Such a plot distribution highlights the wide ranking heterogeneity arising from classic and median fundamental Sharpe ratios. Switching from classic to median fundamental Sharpe ratios does not preserve performance ranks. Such discrepancies highlight the impact of previously reported market climate, time variation and idiosyncratic biases. As a result, accounting for structural biases modifies non-negligibly performance assessment, which should impact related investment selection and resulting portfolios' performance to a large extent. For example, ignoring market trend or volatility regime generates misestimation in SR performance measure and corresponding ranking. In this light, FSR proposes a correction for previous structural biases and therefore a bias-free performance ranking.

[Insert Fig. 2 about here]

As regards the 6 previous RAPMs, we rank corresponding stocks accordingly and propose then a comparative study in two steps. First, we investigate graphically the rank commonalities of those 6 RAPMs with our median FSR estimates as well as corresponding SRs. Then, we compare such rankings while testing for rank similarity and stability.

As a preliminary analysis, the panel (a) of Fig. 3 plots our 6 other RAPMs' ranks relative to SR ranks (i.e. excluding FSR ranks). A clear correlation appears for the 5 RAPMs above-mentioned since their respective relationships with SR ranks are almost perfectly linear. Conversely, a noticeable discordance between SR ranks and the ranks inferred from the Upside potential ratio is confirmed through the less linear correspondence between the two types of ranks. The significance of such discordance will be investigated in a forthcoming step. Panel (b) plots Upside potential ratio-based ranks against FSR-based ranks to check for rank commonalities. Unfortunately, no link does appear.

[Insert Fig. 3 about here]

As a conclusion, the obtained median FSR yields a performance classification, which is totally different from the RAPMs under consideration. Moreover, the other RAPMs' performance classification tracks well the performance ranking of SR to some extent. The latter result is indeed confirmed by a signed rank Wilcoxon test, which is displayed in Table 3 (see Bennett 1965; [McCornack 1965](#); [Wilcoxon 1945, 1947, 1949](#)). In order to check for the latter result, we performed a test of rank stability in between RAPMs' rankings and SR's benchmark ranking. The null hypothesis under consideration states that the median difference between the benchmark ranking and each ranking under consideration is zero. If the null assumption is confirmed, then SR and other RAPMs yield the same ranking on average. The first part of Table 3 displays corresponding results for a two-tailed signed Wilcoxon test based on paired samples (i.e. performance rankings are linked to some extent because RAPMs deal with the same stock returns). Of course, reported results confirm the acceptance of the null assumption at a five percent level for the two-tailed test. Apart from FSR, previous RAPMs yield therefore strongly correlated stock picking and performance-based investment strategies.

[Insert Table 3 about here]

The second part of Table 3 reports Kendall correlation coefficients between SR ranks and other RAPM ranks, on one side, and between FSR ranks and other RAPM ranks, on the other

side.<sup>15</sup> The reported statistic consists of Kendall's tau b, which is significant in the SR case and generally insignificant in the median FSR case at a five percent test level. Observed high levels of Kendall's tau highlight the ranking consistency and similarity between SR and other RAPMs. Conversely, Kendall's tau metric clearly underlines discrepancies between FSR ranking and other RAPM rankings. Moreover, the previous feature also applies to the comparison between SR and median FSR rankings since the observed significant correlation level between SR ranks and FSR ranks is 0.2443 only. However, such a metric mitigates the relationship between the rankings of both SR and the Upside potential ratio. Previous mitigation probably results from the presence of a few non-negligible "outliers" or extreme returns (e.g. extreme gains and/or losses). Such returns have a significant impact on the Upside potential ratio since it balances stock returns' upper tail (i.e. gains) with corresponding lower tail (i.e. losses) on an absolute value basis (e.g. magnitude and significance of distribution tails). Under UPR setting, gains are simply penalized by losses. Hence, stock returns exhibiting a strong risk asymmetry will exhibit higher UPRs when, for example, returns are more often over-performing than underperforming the benchmark return (i.e. right-skewed returns with a fatter right tail). Conversely, stock returns will exhibit lower UPRs when they are more often underperforming than over-performing the benchmark return (i.e. left-skewed returns with a fatter left tail, or equivalently, positive excess kurtosis). As a result, investors favor stock returns with a high upside potential and a low downside risk so that they exhibit a stronger risk-aversion under the UPR-driven selection process (as compared to a SR-driven selection process, Bacon 2008). Finally, UPR and SR differ because UPR handles risk asymmetry in stock returns whereas SR assumes risk symmetry, yielding then erroneous stock picking in the presence of highly skewed and fat tailed stock returns. Moreover, previous results confirm the findings of Eling and Schuhmacher (2007) according to which the choice of a performance measure has no impact on the performance ranking (of hedge funds). Apparently, this result applies to risk-measures, which are founded on excess risk premia relative to the risk free rate of interest (i.e. nature or structure of risk measure based on normalized excess returns).

## 6. Efficiency and stock picking ability of FSR

We assess the efficiency of median fundamental Sharpe ratio (FSR) as compared to other RAPMs such as Sharpe ratio. Incidentally, we investigate the stock picking and portfolio performance implications of FSR and remaining RAPMs. In this light, a value-at-risk analysis is proposed under various risk scenarios as a comparative study. Such analysis confirms the relevance of FSR while backtesting FSR-based and competing performance-based investment strategies.

### 6.1. Efficiency of performance measures

As a final investigation, we focus on the efficiency of FSR performance measure relative to the other risk-adjusted performance measures under consideration. In this light, the performance measures under consideration are envisioned as performance estimators. In statistics, the quality of an estimator and its efficiency in particular, is assessed through its variance. The more efficient the estimator is, the lower its variance should be. When the

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<sup>15</sup> Results remain the same when we compute the Kendall correlations of performance measures instead of their respective rankings.



variance of the estimator is low, the accuracy of such an estimator is therefore high (Kennedy 1998). In particular, accuracy refers to a reduced estimation bias, or equivalently, a lowered valuation error. For this purpose, Table 4 proposes the descriptive statistics relative to all the performance measures under consideration, namely the SR, median FSR, Sortino, Omega, Omega-Sharpe, Kappa 3, Kappa 4 and Upside potential ratios. Strikingly, the median FSR exhibits the lowest standard deviation as compared to SR and the 6 other RAPM performance measures. Hence, FSR is a more accurate performance measure as compared to SR and other RAPMs.

[Insert Table 4 about here]

## 6.2. Scenario analysis and value-at-Risk

For risk analysis prospects, we consider equally weighted portfolios composed of the 30 top-stock group and the 30 bottom-stock group. The portfolios under consideration are composed of the 30 best performing and the 30 worst performing stocks (i.e. over-performing/winning stocks versus underperforming stocks) in accordance with RAPMs such as the FSR, Sharpe ratio (SR), Omega ratio and Upside potential ratio (UPR).<sup>16</sup> The 30 top/bottom-stock portfolios differ from each other due to the benchmark performance measure, which is employed to select the 30 best/worst performing stocks. We label such portfolios the top/bottom (median) FSR, top/bottom SR, top/bottom Omega and top/bottom UPR portfolios respectively.

The risk analysis focuses on the market risk exposure of previous stock portfolios, which is measured with the Value-at-Risk (VaR) (Alexander, 2009; Dowd and Blake, 2006; Gouriéroux and Jasiak, 2010). The VaR measures downside risk while providing investors with the worst possible loss at a given confidence level (i.e. under a specified risk scenario). In this light, we apply a four-step methodology in line with Jondeau and Rockinger (2006), Kuester and Mittnik (2006), McNeil and Frey (2000), McNeil et al. (2005), Nyström and Skoglund (2005), Rockafellar and Uryasev (2002), Poon et al. (2004). More specifically, previous portfolios' VaRs are computed while combining Generalized Autoregressive Conditional Heteroskedastic (GARCH) modeling, copula models, extreme value theory (EVT) and Monte Carlo simulations (i.e. GARCH-EVT-Copula model, see Appendix). Based on GARCH modeling, the first step captures the time-varying volatility of stock returns and therefore portfolio returns. In accordance with EVT, the second step then handles stock returns' distributional asymmetries through the Generalized Pareto Distribution (GPD). As an extension and third step, the copula approach describes the joint dependence structure of constituting stocks within each portfolio under consideration. Finally, the fourth step use previous results, namely historical daily data's properties, to simulate stock returns and therefore portfolios' returns over a specified forecast horizon (e.g. one week, one month). Such simulation framework captures the time-varying nature of returns' volatility as well as the existing correlation between stock returns (i.e. correlation between underlying market risk

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<sup>16</sup> Given that SR is strongly correlated with Sortino, Omega, Omega-Sharpe, Kappa 3 and Kappa 4 ratios, we assimilate those five latter performance measures to SR performance measure. Hence, we disentangle three performance metrics corresponding to SR, median FSR and UPR.

factors). Then, VaR is inferred as the appropriate quantile of the probability distribution of portfolios' returns, which are rebuilt over the target horizon.

Based on 10 000 trials, Table 5 displays the maximum gain and loss while Table 6 displays the one-week and one-month VaR forecasts for various risk levels.

[Insert Table 5 about here]

Comparing the best performing portfolios selected in accordance with FSR, SR, Omega and UPR RAPMs, the top FSR portfolio exhibits the lowest loss potential over one-week and one-month horizons and the highest gain potential over one-month horizon (see Table 5). Comparing the worst performing portfolios selected in accordance with FSR, SR, Omega and UPR RAPMs, the bottom FSR portfolio exhibits the highest gain potential over one month and lowest loss potential over one week. Over the one-month horizon, bottom portfolios' results are mitigated. However, the FSR yields consistent results while building performing portfolios composed of winning stocks. Such performing portfolios should exhibit the lowest loss and highest gain potentials over the chosen target horizons.

[Insert Table 6 about here]

With respect to VaR forecasts in Table 6, the top FSR portfolio exhibits the highest one-month VaR forecasts (i.e. highest negative quantile values, or equivalently, lowest loss forecast or lowest absolute VaR) at the 5% and 1% risk levels (i.e. at a 95% and 99% confidence levels). Moreover, the bottom FSR portfolio exhibits the smallest one-week VaR (i.e. largest/strongest loss forecast) at all risk levels while it exhibits the smallest one-month VaR at 5% and 1% risk levels. Again, FSR yields generally consistent results for performing portfolios, which are composed of 30 winning stocks as compared to SR-, Omega- and UPR-based performing portfolios. Indeed, the top FSR portfolio, or equivalently the FSR-based performing portfolio exhibits higher VaR levels, which translate into lower loss forecasts.

Previous features ensure the appropriateness of FSR as a sound performance measure all the more that obtained FSR conforms to model assumptions. At the level of performing/top stock portfolios, simulation results support FSR as a stock selection and risk management tool. According to the profit and loss analysis, investors who rely on FSRs face a lesser degree of downside risk and higher upside potential. Such feature supports the preference of rational investors. Furthermore, the VaR study shows that FSR-based top portfolios face a lower risk exposure/a lower loss risk than SR-based top portfolios. Given that investors target their gains' maximization, they have interest in choosing FRS (rather than SR) for selecting stocks so that they access better gain possibilities with reduced loss risk and risk exposure.

## **7. Summary and conclusion**

In this article, we considered a risk-adjusted performance measure, which benefits from a large success among the portfolio management community. Namely, Sharpe ratio considers the ratio of a given stock's excess return to its corresponding standard deviation. However, such metric is relevant in a stable setting such as a Gaussian world. Unfortunately, Gaussian features are scarce in the real world, and Sharpe performance measure suffers from various biases. Such biases arise from returns' departure from normality, which often illustrates the non-negligible weights of large and/or extreme return values. Moreover, Sharpe ratios exhibit an upward bias during downward market trends and a downward bias during upward market

trends with respect to poorly diversified portfolios. Finally, Sharpe ratios also exhibit time-variation resulting from business cycle and volatility regimes among others.

To correct for potential biases, we apply a robust filtering method based on Kalman estimation technique. The Kalman approach helps extract fundamental Sharpe ratios from observed classic Sharpe ratios. Obtained fundamental Sharpe ratios are free of bias and exhibit a pure performance indicator. Indeed, removing market/systematic biases yields free-of-bias performance ratios, which are immediately comparable. Such ratios help therefore rank fairly investments on a pure performance basis because they belong to the same measurement scale. Corresponding results are interesting with regard to two findings. First, fundamental Sharpe ratios are obtained after removing directly the market trend and volatility impact. Second, fundamental Sharpe ratios exhibit a cyclical pattern in line with listed cyclical and oscillatory patterns of financial markets (Mishchenko 2014; [Tang and Whitelaw 2011](#); [Woehrmann et al. 2005](#)).

Our comparative study exhibited an obvious discordance between FSR performance classification and a set of well-known RAPMs' performance rankings. Conversely, SR performance classification on one side, and Sortino, Omega-type, Kappa-type, and Upside potential ratios' performance rankings on the other side exhibited a non-negligible correlation. Hence, the question about the impact of a RAPM choice on its corresponding performance ranking is still pending. The answer to such question probably depends on both the nature of the applied measure and the significance of reported biases. However, a simple robustness check highlights the consistency, effectiveness and efficiency of FSR in performance assessment. FSR is indeed a more accurate performance estimator than other RAPMs. Moreover, FSR-based winning portfolios offer lowest expected losses and reduced worst-case losses (i.e. VaR) over one-week and one-month forecast horizons as compared to other RAPM-based winning portfolios. The former portfolios offer rather highest expected gains over a one-month forecast horizon. Consequently, it is possible to extract reliable performance indicators, which are of primary importance for asset selection and performance ranking. Such concern is of huge significance to asset allocation policy, performance forecasts and cost of capital assessment, which are driven by performance indicators among others ([Farinelli et al. 2008](#); [Lien 2002](#); [Christensen and Platen 2007](#)).

## **Acknowledgements**

We thank participants at the AFBC conference (Sydney, Australia, December 2009), ISCEF conference (Sousse, Tunisia, February 2010), and SWFA annual conference (Houston, U.S.A., March 2011) whose questions helped improve the quality of this paper. We are also grateful to two anonymous referees. The usual disclaimer applies.

## **Appendix A. Describing the GARCH-EVT-Copula approach**

We describe the four steps building such simulation analysis, which yields VaR computations.

### *Step 1: Data filtering*

The first step consists of capturing the time-varying volatility of stock returns and therefore portfolio returns. For comparability prospects and complying with statistical assumptions, we filter the constitutive stock returns with a Threshold Generalized Autoregressive Conditional Heteroskedastic (TGARCH) model (Glosten et al., 1993). In line with Hansen and Lunde (2005), Nyström and Skoglund (2005) as well as Ashley and Patterson (2010), the applied representation combines GARCH and ARCH effects of order 1 with a threshold effect of order 1 so that a TGARCH(1,1,1) model is employed for the conditional variance while the mean equation satisfies an autoregressive dynamic of order 1 or AR(1). Hence, we consider an AR(1)-TGARCH(1,1,1) representation (see Appendix for explanatory details and justifications; Ling and McAleer, 2003). The standardized residuals in the mean equation are assumed to follow a Student t probability distribution, which conforms to the fat-tailed profile of stock returns. Such asymmetric behavior underlines the existence of frequent extreme return levels (i.e. fat tails in the probability distribution), which contradicts a Gaussian return behavior. Indeed, a Gaussian behavior assumes extreme return levels to be rare (i.e. thin tails) so that most of observed return values lie around the distribution's average level. Such GARCH representation is estimated for each of the 30 constitutive stocks within a given portfolio while applying the Maximum Likelihood Estimation (MLE) method (i.e. 240 GARCH-type models are estimated for the 240 stocks constituting the 8 portfolios under consideration). Moreover, the GARCH analysis disentangles a pre-, during- and post-crisis period over the sample horizon. Hence, estimations are obtained over three different volatility regimes of the stock market.

### *Step 2: Estimating the distribution of stock returns*

Given returns' distributional asymmetries and in line with EVT, we combine a Gaussian kernel estimation method with a Generalized Pareto Distribution (GPD) to estimate the cumulative distribution function of standardized residual series. The Gaussian kernel methodology captures the most frequent behavior of stock returns while the GPD focuses on their lower and upper tail behaviors. In particular, the GPD is calibrated to focus on the 10% extreme residuals belonging to the tails while the Gaussian kernel describes the empirical distribution of the remaining 90% of sample residuals. Namely, we consider extreme quantile levels so that 10% of the residuals lie beyond those extreme thresholds. Hence, the GPD describes that part of the residuals, which lie beyond the quantile thresholds (i.e. it describes the distribution of exceedances/peaks over thresholds; Davison and Smith, 1990; Embrechts et al., 1997; Smith, 1984). The GPD estimation process relies on MLE methodology. Thus, the frequent behavior and possible extreme quantiles of standardized residuals are both handled (see Appendix). Particularly, the interest of the GPD with respect to risk management relies on risk extrapolation perspectives (e.g. scenario analysis) because quantiles can be extrapolated to higher confidence level (i.e. stronger risk scenarios).

### *Step 3: Capturing the joint dependence structure of stock returns within a portfolio*

We estimate the joint dependence structure of the 30 stock components within each best/worst performing portfolio. Given stock returns' tail fatness and corresponding Student  $t$  distribution of standardized residuals, a multivariate Student  $t$  copula is selected and estimated with the canonical MLE (CMLE) method to describe joint dependencies within stock portfolios ([Cherubini et al., 2004](#); [Nelsen, 1999](#)). In particular, the Student copula is calibrated after transforming the standardized residuals with their respective empirical cumulative distributions function (CDF). In other words, standardized residuals are transformed while applying the CDF to them so that we obtain corresponding values, which lie between 0 and 1. Then, the multivariate Student copula is calibrated to the 30 transformed standardized residuals series with the MLE method for each portfolio under consideration. The estimation process yields estimates of the degree of freedom and the correlation matrix, which are the parameters of the Student copula. In this light, the Student copula captures the correlation structure of standardized residuals (i.e. assessing risk dependencies within stock portfolios).

#### *Step 4: Simulating portfolio returns and forecasting VaR*

We simulate equally weighted portfolios composed of the 30 best/worst performing stocks according to FSR, SR and Omega RAPMs (i.e. three performance measures and then 8 stock portfolios among which 3 best and 3 worst performing ones). In this light, we propose three stages. As a first stage, we simulate multivariate Student copula values based on previous parameter estimates. Hence, we obtain random variates from the Student copula, which correspond to the transformation of standardized residual series. Then, we invert the empirical CDF of standardized residuals from previous random variates (i.e. from simulated copula values) so as to obtain corresponding estimates of the standardized residual series, which describe the 30 constitutive stocks within each portfolio. The 30 standardized residual series, which are obtained, are independent and identically distributed (iid) processes, which also exhibit joint dependencies (i.e. correlated time series). The simulation procedure runs over a time horizon of 5 (i.e. one week of forecasts) and 22 (i.e. one month of forecasts) working days from the end of the sample horizon. Over such time windows, each time series under consideration is simulated 10 000 times, or equivalently, Monte Carlo simulations rely on a number of trials equal to 10 000. Hence, each standardized residual series is simulated/forecasted 10 000 times within a given portfolio over one week and over one month respectively (i.e. simulating 10 000 times a set of 180 residual series). As a second stage, the corresponding simulated returns are obtained while applying the GARCH representation, which was initially estimated and calibrated to empirical data. Incidentally, such GARCH simulation employs the last available value of previously estimated conditional variance as well as simulated residual series. As a third and final stage, we build equally weighted portfolios of 30 best and worst performing stocks and compute the corresponding logarithmic as well as cumulative logarithmic returns for each trial (i.e. the return series of each portfolio is simulated 10 000 times over a given forecast window). Recall that any portfolio returns result from the aggregation of the simulated returns of its stock components. Thus, the 10 000 trials describe the distribution of each portfolio's returns and cumulative returns from which relevant statistics can be inferred for risk management prospects. Recall that the Value-at-Risk is computed as follows for a given portfolio's cumulative return  $R_p$ :

$$\begin{aligned} VaR_{\alpha}(R_p) &= Q_{\alpha}(R_p) = -\min\{R/F_{R_p}(R) = \Pr(R_p \leq R) \geq \alpha\} \\ &= -\min\{R/R \geq F_{R_p}^{-1}(\alpha)\} \end{aligned} \quad (A. 1)$$

where  $F_{R_p}(\cdot)$  is the CDF of the portfolio's return  $R_p$  and  $F_{R_p}^{-1}(\cdot)$  is its inverse function,  $\Pr(\cdot)$  is the probability operator,  $\alpha$  is the risk level (so that  $1-\alpha$  is the confidence level) and  $Q_{\alpha}(R_p)$  is simply the cumulative return's quantile for risk level  $\alpha$  over the chosen investment horizon. As a result, we have:

$$\Pr(R_p < VaR_{\alpha}(R_p)) = \alpha \quad (A. 2)$$

Hence, the VaR informs the investor about possible thresholds of extreme loss risk over a forthcoming investment horizon. In other words, the worst possible portfolio's loss is  $VaR_{\alpha}(R_p)$  in  $(1 - \alpha)$  percent of cases. Equivalently, there is a  $\alpha$  percent probability (i.e. risk level, or equivalently, risk scenario) that the portfolio's loss exceeds the  $VaR_{\alpha}(R_p)$  level (i.e. extreme negative return scenarios, or risk of VaR violations) over the target horizon.

## Appendix B. Diagnosing stock returns

In present appendixes, we provide insightful explanations and details about the model and methodology (i.e. the GARCH-EVT-Copula model) employed to compute the VaR of portfolios' cumulative returns over one week and one month forecast horizons. As an example, we first plot the autocorrelation function of both the returns and squared returns of a given sample stock.

[Insert Fig. B.1 about here]

Fig. B.1 exhibits a significant first order autocorrelation in stock returns while squared returns exhibit stronger dependency over time (i.e. higher order autocorrelation). Hence, the first order autocorrelation of stock returns (i.e. the AR(1) dynamic) is captured in the GARCH model while adding a first order autoregressive or AR(1) component in the mean equation. Moreover, the dependency of squared residuals illustrates the time-dependency in stock returns' variance (i.e. heteroskedasticity, which means that returns' variance depends on time), which supports the use of a GARCH representation. Most of the 180 stocks composing the 3 top and 3 bottom performing portfolios under consideration exhibit such standard and well-known behavior.

## Appendix C. GARCH representation

The Generalized Autoregressive Conditional Heteroskedastic (GARCH) methodology specifies simultaneously one mean and one variance equations, which describe the conditional mean and the conditional variance of stock returns. Assume that  $R_t$  is the return of a given stock at time  $t \in [1, 1154]$  where  $T=1154$  is the end of the investment horizon (i.e. sample size). Then, the autoregressive asymmetric GARCH representation AR(1)-TGARCH(1,1,1) writes:

$$R_t = \theta + \psi R_{t-1} + \varepsilon_t \quad (\text{C. 3})$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma (\varepsilon_{t-1}^-)^2 \quad (\text{C. 4})$$

where  $\varepsilon_t$  is the regression error/residual,  $\varepsilon_t^- = \varepsilon_t$  when residual  $\varepsilon_t < 0$  and zero else,  $\sigma_t^2$  is the conditional variance of regression residuals, and the other parameters  $(\theta, \psi)$  are simply a constant and a factor loading (i.e. AR(1) term) in the mean equation. To account for tail heaviness in stock market returns, the standardized residuals  $(z_t)$ , so that  $\varepsilon_t = \sigma_t^2 z_t$  for  $t$  between 1 and  $T$ , are assumed to follow a Student t distribution and are independent and identically distributed by assumption. With respect to the variance equation,  $\omega$  illustrates the long-term average variance while  $(\varepsilon_{t-1}^2)$ ,  $(\sigma_{t-1}^2)$  and  $(\varepsilon_{t-1}^-)^2$  are respectively called the ARCH( $I$ ), GARCH( $I$ ) and Threshold( $I$ ) terms with their respective factor loadings  $\alpha$ ,  $\beta$  and  $\gamma$ . In particular,  $(\varepsilon_{t-1}^-)^2$  emphasizes the greater impact of negative residuals (i.e. bad news) on stock returns as compared to positive residuals' impact (i.e. good news). Stock returns' volatility simply correspond to the square root of their respective variance, namely  $\sigma_t$ . Moreover, the GARCH representation is estimated with the MLE methodology. As an example, Fig. C.1 plots the filtered residuals ( $\varepsilon_t$ ) as well as their corresponding conditional volatility ( $\sigma_t$ ) for a given stock over time (i.e. by observation number).

[Insert Fig. C.1 about here]

However, the time window under consideration encompasses the global financial crisis following the subprime mortgage market crash in August 2007. In this light, we split the sample into three sub-periods for the GARCH analysis according to the structural break dates introduced by Breitenfellner and Wagner (2012). Hence, we consider a pre-crisis, a crisis and a post-crisis period, which span from 2000/01/04 to 2007/07/02, from 2007/07/03 to 2009/05/01, and from 2009/05/02 to 2014/04/30 respectively. As a result, we consider three volatility regimes over the sample period, which are delimited by the stock market's structural changes (see Fig. C.2).

[Insert Fig. C.2 about here]

As a diagnostic of standardized residuals, Fig. C.3 displays the autocorrelation function of both standardized residuals  $(z_t)$  and squared standardized residuals  $(z_t^2)$  to check for their required white noise property. As expected, standardized residuals are independent and exhibit homoskedasticity (i.e. constant variance over time).

[Insert Fig. C.3 about here]

Accounting for the stock market's breaks and their corresponding volatility regimes reduces the conditional volatility, and strengthens the residuals' robustness.

## Appendix D. Extreme value theory and GPD

The Generalized Pareto Distribution (GPD) is used to describe return values, which exceed a given threshold such as a specified quantile level for example. It is designed to characterize the behavior of returns' distribution tails while modeling stock returns' exceedances (e.g. the difference between stock returns and a specified threshold such as the



corresponding 10%, 5% or 1% quantiles). Under such setting, the cumulative distribution function (CDF) of the GPD writes as follows for any exceedance  $y > 0$ :

$$F(y) = \begin{cases} 1 - \left(1 + \frac{\zeta y}{\eta}\right)^{-\frac{1}{\zeta}} & \text{if } \zeta \neq 0 \\ 1 - \exp\left(-\frac{y}{\eta}\right) & \text{if } \zeta = 0 \end{cases} \quad (\text{D. 1})$$

where  $\eta > 0$  is the scale parameter, and the shape parameter  $\zeta$  defines the tail fatness. When  $\zeta$  is negative, the distribution exhibits thin tails while the distribution exhibits fat tails when  $\zeta$  is positive. Distribution parameters are estimate with the MLE method.

Hence, the GPD is mixed with a Gaussian kernel method to estimate the empirical probability distribution of stock returns' standardized residuals. The Gaussian kernel illustrates 90% of frequently observed standardized residuals while the GPD describes the remaining 10% of observed standardized residuals (i.e. standardized residuals' lower and upper tail behaviors). As an example, Fig. D.1 plots the empirical CDF of a given stock return's standardized residuals while Fig. D.2 proposes a graphical assessment of the quality of the GPD fit for upper tail exceedances of standardized residuals.

[Insert Fig. D.1 about here]

[Insert Fig. D.2 about here]

## Appendix E. The Student t copula

The Student t copula illustrates the dependence structure of the standardized residuals peculiar to the stock components of each portfolio under consideration. Given that each portfolio encompasses 30 stocks, the copula under consideration has a dimension of 30 (i.e. multivariate case) and captures the tail fatness of stock returns.

Let  $\rho$  be a correlation matrix,  $\nu$  a degree of freedom and  $u_1, \dots, u_{30}$  in  $[0,1]$ , the *Student t* copula density writes:

$$c(u_1, \dots, u_{30}) = \frac{1}{|\rho|^{1/2}} \frac{\Gamma\left(\frac{\nu+30}{2}\right) \left\{ \Gamma\left(\frac{\nu}{2}\right) \right\}^{30} \left( 1 + \frac{1}{\nu} \xi^t \rho^{-1} \xi \right)^{-\frac{\nu+30}{2}}}{\left\{ \Gamma\left(\frac{\nu+1}{2}\right) \right\}^{30} \Gamma\left(\frac{\nu}{2}\right) \prod_{i=1}^{30} \left( 1 + \frac{\xi_i^2}{\nu} \right)^{-\frac{\nu+1}{2}}} \quad (E. 1)$$

where  $\rho$  and  $\rho^{-1}$  are a thirty-dimension matrix and its inverse respectively,  $|\rho|$  is the determinant of the correlation matrix,  $\Gamma$  is the Gamma function,  $\xi$  is the vector  $(\xi_1, \dots, \xi_{30})$  of the inverse univariate Student<sup>17</sup> cumulative distribution function, which applies to each element  $u_1, \dots, u_{30}$ , and finally  $\xi^t$  is the transposed vector of  $\xi$ .

As an example based on the top FSR portfolio, we have  $\nu = 25.3973$  and the estimated correlation matrix  $\rho$  for the 30 stock components is:

1.00	0.36	0.25	0.33	0.35	0.36	0.21	0.22	0.22	0.30	0.20	0.30	0.28	0.25	0.30	0.26	0.39	0.19	0.35	0.30	0.27	0.20	0.24	0.31	0.30	0.33	0.22	0.26	0.24	0.23
0.36	1.00	0.37	0.39	0.45	0.33	0.26	0.36	0.33	0.33	0.25	0.40	0.36	0.28	0.43	0.41	0.50	0.27	0.40	0.36	0.35	0.21	0.32	0.42	0.41	0.42	0.27	0.42	0.39	0.26
0.25	0.37	1.00	0.30	0.33	0.24	0.18	0.34	0.33	0.24	0.20	0.27	0.26	0.22	0.33	0.38	0.35	0.32	0.33	0.26	0.22	0.18	0.24	0.31	0.29	0.28	0.23	0.40	0.34	0.22
0.33	0.39	0.30	1.00	0.38	0.37	0.24	0.27	0.28	0.29	0.21	0.31	0.31	0.25	0.34	0.30	0.42	0.24	0.37	0.33	0.29	0.19	0.27	0.31	0.31	0.35	0.23	0.30	0.27	0.22
0.35	0.45	0.33	0.38	1.00	0.37	0.24	0.28	0.28	0.37	0.23	0.34	0.34	0.29	0.38	0.33	0.48	0.28	0.40	0.39	0.35	0.21	0.29	0.36	0.34	0.37	0.26	0.34	0.31	0.29
0.36	0.33	0.24	0.37	0.37	1.00	0.24	0.24	0.23	0.29	0.24	0.32	0.26	0.24	0.31	0.26	0.43	0.22	0.43	0.31	0.32	0.20	0.27	0.29	0.28	0.33	0.22	0.27	0.25	0.23
0.21	0.26	0.18	0.24	0.24	0.24	1.00	0.16	0.17	0.20	0.20	0.25	0.21	0.20	0.23	0.20	0.29	0.17	0.27	0.22	0.22	0.19	0.22	0.25	0.25	0.26	0.16	0.23	0.22	0.22
0.22	0.36	0.34	0.27	0.28	0.24	0.16	1.00	0.58	0.22	0.25	0.25	0.29	0.22	0.32	0.37	0.33	0.31	0.29	0.26	0.20	0.24	0.30	0.34	0.34	0.28	0.21	0.41	0.38	0.21
0.22	0.33	0.33	0.28	0.28	0.23	0.17	0.58	1.00	0.23	0.27	0.25	0.26	0.21	0.32	0.33	0.30	0.29	0.28	0.26	0.21	0.23	0.31	0.35	0.35	0.29	0.19	0.40	0.38	0.22
0.30	0.33	0.24	0.29	0.37	0.29	0.20	0.22	0.23	1.00	0.26	0.32	0.26	0.27	0.31	0.29	0.37	0.20	0.33	0.37	0.38	0.22	0.30	0.28	0.28	0.33	0.15	0.30	0.29	0.24
0.20	0.25	0.20	0.21	0.23	0.24	0.20	0.25	0.27	0.26	1.00	0.26	0.25	0.22	0.28	0.23	0.29	0.20	0.26	0.24	0.26	0.50	0.73	0.24	0.23	0.29	0.22	0.57	0.60	0.20
0.30	0.40	0.27	0.31	0.34	0.32	0.25	0.25	0.25	0.32	0.26	1.00	0.29	0.24	0.36	0.33	0.44	0.24	0.34	0.35	0.35	0.20	0.30	0.32	0.31	0.46	0.25	0.32	0.32	0.24
0.28	0.36	0.26	0.31	0.34	0.26	0.21	0.29	0.26	0.26	0.25	0.29	1.00	0.23	0.33	0.29	0.36	0.24	0.33	0.32	0.30	0.21	0.27	0.29	0.27	0.33	0.23	0.29	0.28	0.23
0.25	0.28	0.22	0.25	0.29	0.24	0.20	0.22	0.21	0.27	0.22	0.24	0.23	1.00	0.26	0.24	0.31	0.19	0.27	0.31	0.27	0.21	0.28	0.26	0.24	0.26	0.20	0.26	0.25	0.20
0.30	0.43	0.33	0.34	0.38	0.31	0.23	0.32	0.32	0.31	0.28	0.36	0.33	0.26	1.00	0.34	0.45	0.28	0.38	0.35	0.32	0.25	0.34	0.33	0.33	0.40	0.26	0.40	0.38	0.23
0.26	0.41	0.38	0.30	0.33	0.26	0.20	0.37	0.33	0.29	0.23	0.33	0.29	0.24	0.34	1.00	0.37	0.30	0.32	0.31	0.26	0.19	0.28	0.32	0.31	0.32	0.20	0.40	0.35	0.21
0.39	0.50	0.35	0.42	0.48	0.43	0.29	0.33	0.30	0.37	0.29	0.44	0.36	0.31	0.45	0.37	1.00	0.27	0.45	0.41	0.36	0.25	0.34	0.42	0.40	0.47	0.30	0.38	0.36	0.29
0.19	0.27	0.32	0.24	0.28	0.22	0.17	0.31	0.29	0.20	0.20	0.24	0.24	0.19	0.28	0.30	0.27	1.00	0.24	0.22	0.19	0.17	0.24	0.25	0.26	0.24	0.21	0.32	0.30	0.19
0.35	0.40	0.33	0.37	0.40	0.43	0.27	0.29	0.28	0.33	0.26	0.34	0.33	0.27	0.38	0.32	0.45	0.24	1.00	0.35	0.30	0.21	0.31	0.33	0.34	0.38	0.24	0.36	0.33	0.25
0.30	0.36	0.26	0.33	0.39	0.31	0.22	0.26	0.26	0.37	0.24	0.35	0.32	0.31	0.35	0.31	0.41	0.22	0.35	1.00	0.38	0.21	0.30	0.30	0.30	0.38	0.21	0.29	0.28	0.24
0.27	0.35	0.22	0.29	0.35	0.32	0.22	0.20	0.21	0.38	0.26	0.35	0.30	0.27	0.32	0.26	0.36	0.19	0.30	0.38	1.00	0.23	0.30	0.28	0.26	0.37	0.21	0.27	0.27	0.22
0.20	0.21	0.18	0.19	0.21	0.20	0.19	0.24	0.23	0.22	0.50	0.20	0.21	0.21	0.25	0.19	0.25	0.17	0.21	0.21	0.23	1.00	0.51	0.23	0.21	0.25	0.18	0.46	0.48	0.20
0.24	0.32	0.24	0.27	0.29	0.27	0.22	0.30	0.31	0.30	0.73	0.30	0.27	0.28	0.34	0.28	0.34	0.24	0.31	0.30	0.30	0.51	1.00	0.27	0.27	0.34	0.23	0.64	0.65	0.24
0.31	0.42	0.31	0.31	0.36	0.29	0.25	0.34	0.35	0.28	0.24	0.32	0.29	0.26	0.33	0.32	0.42	0.25	0.33	0.30	0.28	0.23	0.27	1.00	0.60	0.34	0.26	0.34	0.32	0.27
0.30	0.41	0.29	0.31	0.34	0.28	0.25	0.34	0.35	0.28	0.23	0.31	0.27	0.24	0.33	0.31	0.40	0.26	0.34	0.30	0.26	0.21	0.27	0.60	1.00	0.33	0.23	0.32	0.29	0.25
0.33	0.42	0.28	0.35	0.37	0.33	0.26	0.28	0.29	0.33	0.29	0.46	0.33	0.26	0.40	0.32	0.47	0.24	0.38	0.38	0.37	0.25	0.34	0.34	0.33	1.00	0.24	0.35	0.34	0.26
0.22	0.27	0.23	0.23	0.26	0.22	0.16	0.21	0.19	0.15	0.22	0.25	0.23	0.20	0.26	0.20	0.30	0.21	0.24	0.21	0.21	0.18	0.23	0.26	0.23	0.24	1.00	0.25	0.25	0.19
0.26	0.42	0.40	0.30	0.34	0.27	0.23	0.41	0.40	0.30	0.57	0.32	0.29	0.26	0.40	0.40	0.38	0.32	0.36	0.29	0.27	0.46	0.64	0.34	0.32	0.35	0.25	1.00	0.80	0.23
0.24	0.39	0.34	0.27	0.31	0.25	0.22	0.38	0.38	0.29	0.60	0.32	0.28	0.25	0.38	0.35	0.36	0.30	0.33	0.28	0.27	0.48	0.65	0.32	0.29	0.34	0.25	0.80	1.00	0.22
0.23	0.26	0.22	0.22	0.29	0.23	0.22	0.21	0.22	0.24	0.20	0.24	0.23	0.20	0.23	0.21	0.29	0.19	0.25	0.24	0.22	0.20	0.24	0.27	0.25	0.26	0.19	0.23	0.22	1.00

## References

- Acharya, V. V., & Pedersen, L. H. (2005). Asset pricing with liquidity risk. *Journal of Financial Economics*, 77(2), 375-410.
- Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control*, 19(6), 716-723.
- Alexander, C. (2005). The present and future of financial risk management. *Journal of Financial Econometrics*, 3(1), 3-25.
- Alexander, C. (2009). *Market risk analysis: Value at risk models*. Volume IV edition. Chichester, England: Wiley.
- Ammann, M., & Verhofen, M. (2009). The effect of market regimes on style allocation. *SSRN Working Paper*, [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1322278](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1322278)

<sup>17</sup> The Student t distribution has  $\nu$  degree(s) of freedom.

Angelidis, T., & Tassaromatis, N. (2009). Idiosyncratic risk matters! A regime switching approach. *International Review of Economics & Finance*, 18(1), 132-141.

Ashley, R. A., & Patterson, D. M. (2010). A test of the GARCH(1,1) specification for daily stock returns. *Macroeconomic Dynamics*, 14(S1), 137-144.

Bacon, C. R. (2008). *Practical portfolio performance measurement and attribution*. 2<sup>nd</sup> edn, Chapter 4-Risk (pp. 95-96). England: Wiley Finance.

Batagelj, V. (1995). Comparing resemblance measures. *Journal of Classification*, 12(1), 73-90.

Bennett, B. M. (1965). On multivariate signed rank tests. *Annals of the Institute of Statistical Mathematics*, 17(1), 55-61.

Bernardo, A. E., & Ledoit O. (2000). Gain, loss and asset pricing. *Journal of Political Economy*, 108(1), 144-172.

Bhargava, R., Gallo, J. G., & Swanson, P. E. (2001). The performance, asset allocation, and investment style of international equity managers. *Review of Quantitative Finance and Accounting*, 17(4), 377-395.

Black, K. H. (2006). Improving the hedge fund risk exposures by hedging equity market volatility, or how the VIX strategy ate my kurtosis. *Journal of Trading*, 1(2), 6-15.

Brandt, M.W. & Kang, Q. (2004). On the relationship between the conditional mean and volatility of stock returns: A latent VAR approach. *Journal of Financial Economics*, 72(2), 217-257.

Breitenfellner, B., & Wagner, N. (2012). Explaining aggregate credit default swap spreads. *International Review of Financial Analysis*, 22(1), 18-29.

Campbell, J. Y. & Hentschel, L. (1992). No news is good news: An asymmetric model of changing volatility in stock returns. *Journal of Financial Economics*, 31(3), 281-318

Chang, K.-L. (2009). Do macroeconomic variables have regime-dependent effects on stock return dynamics? Evidence from the Markov regime switching model. *Economic Modelling*, 26(6): 1283-1299.

Cherubini, U., Luciano, E., & Vecchiato, W. (2004 ). *Copula methods in finance*. Chichester: Wiley.

Chordia, T., Roll, R., & Subrahmanyam, A. (2000). Commonality in liquidity. *Journal of Financial Economics*, 56(1), 3-28.

Christensen, M. M., & Platen, E. (2007). Sharpe ratio maximization and expected utility when asset prices have jumps. *International Journal of Theoretical and Applied Finance*, 10(8), 1339-1364.

Chu, C.-S. J., Santoni, G. J., & Tung, L. (1996). Stock market volatility and regime shifts in returns. *Information Sciences*, 94(1), 179-190.

Cramer, H. (1928). On the composition of elementary errors: II. *Statistical Applications. Skandinavisk Aktuarietidskrift*, 11(1), 141-180.

- Cremers, M., Halling, M. & Weinbaum, D. (2015). Aggregate jump and volatility Risk in the cross-section of stock returns. Fortcoming in the *Journal of Finance*.
- Davison, A. C., & Smith R. L. (1990). Models for exceedances over high thresholds. *Journal of the Royal Statistical Society - Series B (Methodological)*, 52(3), 393-442.
- Dayri, K., Bacry, E. & Muzy, J. (2011). *Econophysics of Order-driven Markets*, Springer, Chapter: The nature of price returns during periods of high market activity. 155–172.
- Doran, J.S., Ronn, E.I. & Goldberg, R.S. (2009). A simple model for time-varying expected returns on the S&P 500 Index. *Journal of Investment Management*, 7(2), 47-72.
- Dowd, K., & Blake, D. (2006). After VaR: The theory, estimation, and insurance applications of quantile-based risk measures. *Journal of Risk and Insurance*, 73(2), 193-229.
- Durbin, J., & Koopman, S. J. (2001). *Time series analysis by state space methods*. Oxford: Oxford University Press.
- Elyasiani, E., & Jia, J. (2011). Performance persistence of closed-end funds. *Review of Quantitative Finance and Accounting*, 37(3), 381-408.
- Eling, M., & Schuhmacher, F. (2007). Does the choice of performance measure influence the evaluation of hedge funds? *Journal of Banking and Finance*, 31(9), 2632-2647.
- Embrechts, P., Kluppelberg, E., & Mikosch, T. (1997). *Modelling extremal events*. Berlin, Germany: Springer.
- Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1), 3-56.
- Farinelli, S., Ferreira, M., Rossello, D., Thoeny, M., & Tibiletti, L. (2008). Beyond Sharpe ratio: Optimal asset allocation using different performance ratios. *Journal of Banking and Finance*, 32(10), 2057-2063.
- Ferson, W. E., & Harvey, C. R. (1991). The variation of economic risk premiums. *Journal of Political Economy*, 99(2), 385-415.
- Fiore, C. & Saha, A. (2015). A tale of two anomalies: Higher returns of low-risk stocks and return seasonality. Forthcoming in the *Financial Review*.
- Fishburn, P. C. (1977). Mean-risk analysis with risk associated with below-target returns. *American Economic Review*, 67(2), 116-126.
- Gatfaoui, H. (2010). Deviation from normality and Sharpe ratio behavior: A brief simulation study. *Investment Management and Financial Innovations*, 7(4), 107-119.
- Gatfaoui, H. (2012). A correction for classic performance measures. *Chinese Business Review*, 11(1), 1-28.
- Gatfaoui, H. (2013). Translating financial integration into correlation risk: A weekly reporting's viewpoint for the volatility behavior of stock markets. *Economic Modelling*, 30, 776-791.

Glosten, L. R., Jaganathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance*, 48(5), 1779–1801.

Goetzmann, W., Ingersoll, J., Spiegel, M. & Welch, I. (2007) Portfolio performance manipulation and manipulation-proof performance measures. *Review of Financial Studies*, 20(5), 1503-1546.

Gourieroux, C., & Jasiak, J. (2010). Value at Risk. In Y. Aït-Sahalia, & L. P. Hansen (Eds.), *Vol. 1 of Handbook of Financial Econometrics, Tools and Techniques* (Chapter 10, pp. 553-615). North-Holland, U.S.A.: Elsevier.

Hamilton, J.D. & Lin, G. (1996). Stock market volatility and the business cycle. *Journal of Applied Econometrics*, 11(5), 573-593.

Hannan, E. J., & Quinn, B. G. (1979). The determination of the order of an autoregression. *Journal of the Royal Statistical Society, B*(41), 190–195.

Hansen, P. R., & Lunde, A. (2005). A forecast comparison of volatility models: Does anything beat a GARCH(1,1)? *Journal of Applied Econometrics*, 20(7), 873-889.

Harvey, A. (1989). *Forecasting, structural time series models and the Kalman filter*. Cambridge: Cambridge University Press.

Harvey, A. C., Ruiz, E., & Shephard, N. G. (1994). Multivariate stochastic variance models. *Review of Economic Studies*, 61(2), 247-264.

Harvey, A., Koopman, S. J., & Shephard, N. (2004). *State Space and unobserved component models-Theory and applications*. Cambridge: Cambridge University Press.

Harvey, A., & Koopman, S. J. (2009). Unobserved components models in economics and finance. *IEEE Control Systems Magazine*, 71-81.

Hasbrouck, J., & Seppi, D. J. (2001). Common factors in prices, order flows and liquidity. *Journal of Financial Economics*, 59(3), 383-411.

Ho, L.-C., Cadle, J., & Theobald, M. (2011). An analysis of risk-based asset allocation and portfolio insurance strategies. *Review of Quantitative Finance and Accounting*, 36(2), 247-267.

Hodges, S. D. (1998). A generalization of the Sharpe ratio and its applications to valuation bounds and risk measures. (Working Paper). FORC Preprint 98/88, Financial Options Research Centre, University of Warwick.

Hwang, T., Gao, S., & Owen, H. (2012). A two-pass model study of the CAPM: evidence from the UK stock market. *Studies in Economics and Finance*, 29(2), 89-104.

Jarque, C., & Bera, A. K. (1980). Efficient tests for normality, homoscedasticity and serial independence of regression residuals. *Economics Letters*, 6(3), 255-259.

Jarque, C., & Bera, A. K. (1981). Efficient tests for normality, homoscedasticity and serial independence of regression residuals: A Monte Carlo evidence. *Economics Letters*, 7(4), 313-318.

- Jarque, C., & Bera, A. K. (1987). A test for normality of observations and regression residuals. *International Statistical Review*, 55(2), 163-172.
- Jondeau, E., & Rockinger, M. (2006). The Copula-GARCH model of conditional dependencies: An international stock market application. *Journal of International Money and Finance*, 25(5), 827-853.
- Kalman, R. E. (1960). A new approach to linear filtering and prediction theory. *Transactions of the ASME—Journal of Basic Engineering*, 82(Series D), 35-45.
- Kalman, R. E., & Bucy, R. S. (1961). New results in linear filtering and prediction theory. *Transactions of the ASME—Journal of Basic Engineering*, 83(Series D), 95-108.
- Kamstra, M.J., Kramer, L.A. & Levi, M.D. (2003). Winter blues: A SAD stock market cycle. *American Economic Review*, 93(1), 324-343.
- Kaplan, P. D., & Knowles, J. A. (2004). Kappa: A generalized downside risk-adjusted performance measure. *Journal of Performance Measurement*, 8(3), 42-54.
- Kazemi, H., Schneweis, T., & Gupta, R. (2004). Omega as a performance measure. *Journal of Performance Measurement*, 8(3), 16-25.
- Keating, C., & Shadwick, W. F. (2002). A universal performance measure. *Journal of Performance Measurement*, 6(3), 59-84.
- Keene, M. A., & Peterson, D. R. (2007). The importance of liquidity as a factor in asset pricing. *Journal of Financial Research*, 30(1), 91-109.
- Kempf, A., & Mayston, D. (2008). Liquidity commonality beyond best prices. *Journal of Financial Research*, 31(1), 25-40.
- Kennedy, P. (1998). *A guide to econometrics*. Cambridge, Massachusetts: MIT Press.
- Kim, C.-J., Morley, J. C., & Nelson, C. R. (2004). Is there a positive relationship between stock market volatility and the equity premium? *Journal of Money, Credit, and Banking*, 36(3- Part 1), 339-360.
- Klemkosky, R. (1973). The bias in composite performance measures. *Journal of Financial and Quantitative Analysis*, 8(3), 505-514.
- Kocherlakota, N.R. (1996). The equity premium : It's still a puzzle. *Journal of Economic Literature*, 34(1), 42-71.
- Krimm, S., Scholz, H., & Wilkens, M. (2012). The Sharpe Ratio's Market Climate Bias: Theoretical and Empirical Evidence from US Equity Mutual Funds. *Journal of Asset Management*, 13(4), 272-242.
- Kuester, K., & Mittnik, S. (2006). Value-at-risk prediction: A comparison of alternative strategies. *Journal of Financial Econometrics*, 4(1), S53-S89.
- Lettau, M. & Ludvigson, S. C. (2010). Measuring and modeling variation in the risk-return tradeoff. In Yacine Ait-Sahalia and Lars Peter Hansen (eds.), *Handbook of Financial Econometrics*, vol. 1, Elsevier Science B.V., North Holland, Amsterdam, 617-690.

- Lien, D. (2002). A note on the relationships between some risk-adjusted performance measures. *Journal of Futures Markets*, 22(5), 483-495.
- Ling, S., & McAleer, M. (2003). Asymptotic theory for a vector ARMA-GARCH model. *Econometric Theory*, 19(2), 280-310.
- Lintner, J. (1965a). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics*, 47(1), 13-37.
- Lintner, J. (1965b). Security prices, risk and maximal gains from diversification. *Journal of Finance*, 20(4), 587-615.
- Ljung, G. M., & Box, G. (1978). On a measure of lack of fit in time series models. *Biometrika*, 65(2), 297-303.
- Lustig, H. & Verdelhan, A. (2012). Business cycle variation in the risk-return trade-off. *Journal of Monetary Economics*, 59(S), S35-S49.
- Maheu, J. M., McCurdy, T. H. & Zhao, X. (2013). Do jumps contribute to the dynamics of the equity premium? *Journal of Financial Economics*, 110 (2), 457-477.
- McCornack, R. L. (1965). Extended tables of the Wilcoxon matched pair signed rank statistic. *Journal of the American Statistical Association*, 60(311), 864-871.
- McNeil, A., & Frey, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance*, 7(3-4), 271-300.
- McNeil, A., Frey, R., & Embrechts, P. (2005). *Quantitative Risk Management*. Princeton, U.S.A.: Princeton University Press.
- Mishchenko, Y. (2014). Oscillations in rational economies. *PLoS ONE*, 9(2), 1-5.
- von Mises, R. (1931). *Wahrscheinlichkeitsrechnung und ihre anwendung in der statistik und theoretischen physik*. Deuticke, Leipzig: Germany.
- Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica*, 34(4), 768-783.
- Nam, K., Kim, S.-W., & Arize, A. C. (2006). Mean reversion of short-horizon stock returns: Asymmetry property. *Review of Quantitative Finance and Accounting*, 26(2), 137-163.
- Nelsen, R.B. (1999). *An Introduction to copulas. Lectures Notes in Statistics*, 139. New York: Springer.
- Nyström, K., & Skoglund, J. (2005). Efficient filtering of financial time series and extreme value theory. *Journal of Risk Finance*, 7(2), 63-84.
- Phillips, P. C. B., & Perron, P. (1988). Testing for unit roots in time series regression. *Biometrika*, 75(2), 335-346.
- Platen, E. (2006). Capital asset pricing for markets with intensity based jumps. In: A. N. Shiryaev, M. R. Grossinho, P. E. Oliveira, & M. L. Esquivel (Eds.), *Stochastic finance, Part I, Chapter 5* (pp. 157-182). Berlin: Springer.



Poon, S.-H., Rockinger, M., & Tawn, J. (2004). Extreme value dependence in financial markets: Diagnostics, models and financial implications. *Review of Financial Studies*, 17(2), 581–610.

Raunig, B. & Scharler, J. (2010). Stock market volatility and the business cycle. *Monetary Policy and the Economy*, Q2/10, 54-63.

Robertson, K. (2001). The Sharpe Ratio. *Panel Publishers 401(k) Advisor - Investment CORNER*, May 3.

Rockafellar, R.T., & Uryasev, S. (2002). Conditional value-at-risk for general loss distributions. *Journal of Banking & Finance*, 26(7), pp. 1443–1471.

Santa-Clara, P., & Yan, S. (2010). Crashes, volatility, and the equity premium: Lessons from S&P 500 options. *Review of Economics and Statistics*, 92(2), 435-451.

Scholz, H., & Wilkens, M. (2005a). Interpreting Sharpe ratios - The market climate bias. Forthcoming in *Finance Letters*.

Scholz, H., & Wilkens, M. (2005b). An investor-specific performance measurement: A justification of Sharpe ratio and Treynor ratio. *International Journal of Finance*, 17(4), 3671-3691.

Scholz, H. (2007). Refinements to the Sharpe ratio: Comparing alternatives for bear markets. *Journal of Asset Management*, 7(5), 347-357.

Schwarz, G. (1978). Estimating the dimension of a model. *Annals of Statistics*, 6(2), 461–464.

Schwert, G.W. (1990). Stock returns and real activity: A century of evidence. *Journal of Finance*, 45(4), 1237-1257.

Sharpe, W. F. (1963). A simplified model for portfolio analysis. *Management Science*, 9(2), 277-93.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, 19(3), 425-442.

Smith, R. L. (1984). Threshold methods for sample extremes, statistical extremes and applications. *NATO ASI Series, Series C*, 131, 621-638.

Sokal, R. R., & Michener, C. D. (1958). A statistical method for evaluating systematic relationships. *Scientific Bulletins*, 38(22), 1409-1438.

Sortino, F. A., Van Der Meer, R., & Plantinga, A. (1999). The Dutch triangle. *Journal of Portfolio Management*, 26(1), 50-58.

Sortino, F. A., & Price, L. N. (1994). Performance measurement in a downside risk framework. *Journal of Investing*, 3(3), 59-65.

Sortino, F. A., & Forsey, H. J. (1996). On the use and measure of downside risk. *Journal of Portfolio Management*, 22(1), 35-42.

Sortino, F. A. (2004). Historic performance measures distort current risk assessment. *Pensions & Investments*, 32(3), 20-21.

Spurgin, R. (2001). How to game your Sharpe ratio. *Journal of Alternative Investments*, 4(3), 38-46.

Tang, Y. & Whitelaw, R.F. (2011). Time varying Sharpe ratios and market timing. *Quarterly Journal of Finance*, 1(3), 465-493.

Van Nieuwerburgh, S., & Veldkamp, L. (2010). Information acquisition and under-diversification. *Review of Economic Studies*, 77(2), 779-805.

Whitelaw, R.F. (2000). Stock market risk and return: An equilibrium approach. *Review of Financial Studies*, 13(3), 521-547.

Wilcoxon, F. (1945). Individual comparison by ranking methods. *Biometrics Bulletin*, 1(6), 80-83.

Wilcoxon, F. (1947). Probability tables for individual comparisons by ranking methods. *Biometrics*, 3(3), 119-122.

Wilcoxon, F. (1949). *Some rapid approximate statistical procedures*. (pp. 5-8). New York: American Cyanamid Company.

Woehrmann, P., Semmler, W. & Lettau, M. (2005). Nonparametric estimation of the time varying Sharpe ratio in dynamic asset pricing models. IEW - Working Papers 225, Institute for Empirical Research in Economics - University of Zurich.

Yan, S. (2011). Jump risk, stock returns, and slope of implied volatility smile. *Journal of Financial Economics*, 99(1), 216-233.

Zakamouline, V., & Koekebakker, S. (2009). Portfolio performance evaluation with generalized Sharpe ratios: Beyond the mean and variance. *Journal of Banking and Finance*, 33(7), 1242-1254.

Zakamouline, V. (2011). The choice of performance measure does influence the evaluation of hedge funds. *Journal of Performance Measure*, 15(1), 48-64.

## TABLES

**Table 1** Descriptive statistics of cross-section statistics

Statistics	SR <sup>*</sup>	Mean	Median	Std. Dev.	Skewness	Kurtosis
Mean	0.0195	0.0608	0.0200	2.7039	0.2855	16.5332
Median	0.0197	0.0522	0.0146	2.6094	0.2334	12.0354
Max	0.0497	0.1782	0.1114	4.5768	4.1608	104.9361
Min	-0.0024	0.0017	-0.0830	1.3352	-2.5719	6.0898
Std. Dev.	0.0120	0.0374	0.0369	0.7715	0.7195	14.8127
Skewness	0.2474	0.8269	-0.0881	0.3635	1.1214	3.7393
Kurtosis	2.6772	3.6313	3.4368	2.4952	14.3464	19.5086
JB Test <sup>**</sup>	YES	NO	YES	YES	NO	NO

<sup>\*</sup> Stock-specific Sharpe ratio series computed over the whole sample horizon.

<sup>\*\*</sup> Jarque Bera normality test at a 5% level of significance.

Note: Sharpe ratio (SR) as well as stock-specific mean, median, standard deviation, skewness and kurtosis are computed for the 85 series of stock returns (i.e. 85 stocks are considered). We obtain then six series composed of 85 observations and for which we display corresponding descriptive statistics. For example, we have one series of stock-specific SRs and one series of stock-specific means. The abbreviation “Std. Dev.” stands for standard deviation.

**Table 2** Properties of Kendall tau b correlations between monthly Sharpe ratios and explanatory factors

Statistics	MktPremium	SMB	HML	VIX <sup>**</sup>
Mean	0.3306	0.0814	0.0175	-0.0732
Median	0.3264	0.0887	0.0328	-0.0669
Std. Dev.	0.0927	0.0805	0.0792	0.0448
Skewness	1.1751	-0.4008	-0.3524	-0.1303
Kurtosis	4.6585	-0.6787	0.0168	-0.6084
Coefficient of variation <sup>*</sup>	0.2806	0.9888	4.5272	-0.6127
# significant correlations <sup>***</sup>	85 (100%)	41 (48%)	21 (25%)	20 (24%)

<sup>\*</sup> It is computed as the ratio of standard deviation to the mean.

<sup>\*\*</sup> VIX and ln(VIX) yield the same results.

<sup>\*\*\*</sup> Number of significant correlation coefficients at a five percent bilateral test level.

Note: Monthly Sharpe ratios (SRs) are computed for the 85 stocks under consideration so as to obtain 85 series of stock-specific monthly SRs. Corresponding average monthly Fama and French (1993) factors (e.g. market premium [MktPremium], SMB and HML) as well as average implied volatility index (VIX) are also computed. Then, we compute Kendall correlation between each SR series and the four previous explanatory factors (i.e. four correlation coefficients per stock). Thus, we obtain four series of stock-specific correlations with the four explanatory factors, each series encompassing 85 observations. We report finally the statistical properties of those four correlation series.

**Table 3** Statistical measures for paired RAPM rankings

RAPM	Signed Wilcoxon test		Kendall's rank correlation <sup>□</sup>	
	Wilcoxon statistic	p-value	SR	Median FSR
Sortino	-0.0022	0.9983	-0.9759 <sup>*</sup>	-0.2370 <sup>*</sup>
Omega	-0.0223	0.9822	-0.9401 <sup>*</sup>	-0.2336
Omega-Sharpe	-0.0223	0.9822	-0.9401 <sup>*</sup>	-0.2336
Kappa <sub>3</sub>	-0.0112	0.9911	-0.9266 <sup>*</sup>	-0.2269
Kappa <sub>4</sub>	-0.0022	0.9983	-0.8689 <sup>*</sup>	-0.2196

Upside potential ratio	-0.0131	0.9895	-0.4426*	-0.0812
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<sup>a</sup> Kendall's tau b statistics are displayed.

\* Significant at the five percent level of a bilateral test.

Note: The table proposes diagnostic statistics to test for the similarity between the rankings obtained from the 7 risk-adjusted performance measures (RAPMs) on one side, and the Sharpe ratio (SR) and the median fundamental Sharpe ratio (FSR) on the other side. Basically, we rank SR, median FSR and the six other RAPMs by ascending order, which yields a performance ranking for the 85 considered stocks. Then, we compare the resulting SR-, median FSR- and other RAPMs-based performance rankings across stocks. In particular, the Wilcoxon test checks for ranking stability across performance measures. Differently, Kendall rank correlation tests for ranking commonalities between SR/median FSR and the RAPMs.

**Table 4** Descriptive statistics of cross-section risk-adjusted performance measures

	SR	Median FSR	Sortino	Omega	Omega-Sharpe	Kappa 3	Kappa 4	Upside potential ratio
Mean	0.0195	0.0011	0.0287	1.0605	0.0605	0.0182	0.0130	0.5032
Median	0.0197	0.0005	0.0289	1.0600	0.0600	0.0190	0.0136	0.5062
Maximum	-0.0024	-0.0202	-0.0036	0.9926	-0.0074	-0.0023	-0.0017	0.4013
Minimum	0.0497	0.0277	0.0776	1.1663	0.1663	0.0491	0.0351	0.5603
Std. Dev.	<b>0.0120</b>	<b>0.0095</b>	<b>0.0178</b>	<b>0.0380</b>	<b>0.0380</b>	<b>0.0113</b>	<b>0.0099</b>	<b>0.0320</b>
Skewness	0.2519	0.1709	0.3051	0.3490	0.3490	0.2839	0.3068	-0.8250
Kurtosis	2.8416	3.0624	2.9600	3.0196	3.0196	2.9542	3.0144	3.9746

Note: We consider SR, median FSR, Sortino, Omega, Omega-Sharpe, Kappa 3, Kappa 4 and Upside potential ratio (i.e. 8 performance metrics), which are computed for the 85 stocks under consideration. Thus, we get 8 series of performance measures, each series encompassing therefore 85 observations. This table displays the descriptive statistics of the 8 resulting risk-adjusted performance series.

The standard deviation is labeled "Std. Dev.".

**Table 5** Maximum profit and loss (P&L) of simulated portfolio cumulative returns

	Portfolios	One week maximum P&L		One month maximum P&L	
		Loss	Gain	Loss	Gain
Top Portfolios	Top FSR	10.6368%	9.7948%	18.6534%	25.7380%
	Top SR	11.0550%	11.9937%	22.8710%	21.1377%
	Top Omega	11.4982%	11.2303%	21.2303%	24.8223%
	Top UPR	11.0695%	12.6595%	18.9490%	22.5885%
Bottom portfolios	Bottom FSR	11.0722%	12.7898%	21.1547%	25.8513%
	Bottom SR	12.9585%	12.2338%	23.9003%	23.7436%
	Bottom Omega	12.1185%	13.0040%	24.5295%	24.8714%
	Bottom UPR	11.2627%	14.7606%	17.5297%	24.1297%

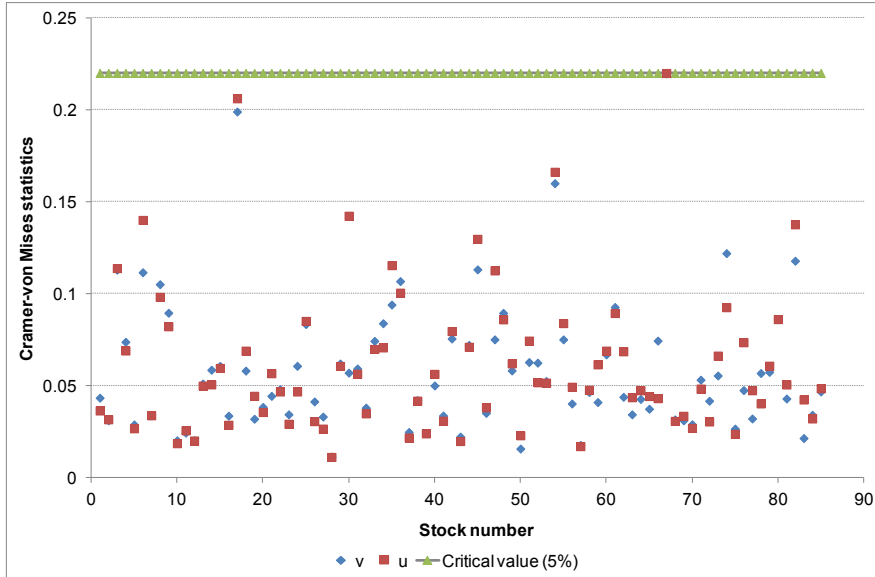
Note: Estimated profits and losses result from Monte Carlo simulations with 10 000 trials. The top/bottom portfolios are composed of the 30 best/worst performing stocks in accordance with the selected performance measures. P&L are computed at the portfolio return's level and displayed in absolute value.

**Table 6** One week and one month VaR forecasts for portfolios' cumulative returns at various risk levels

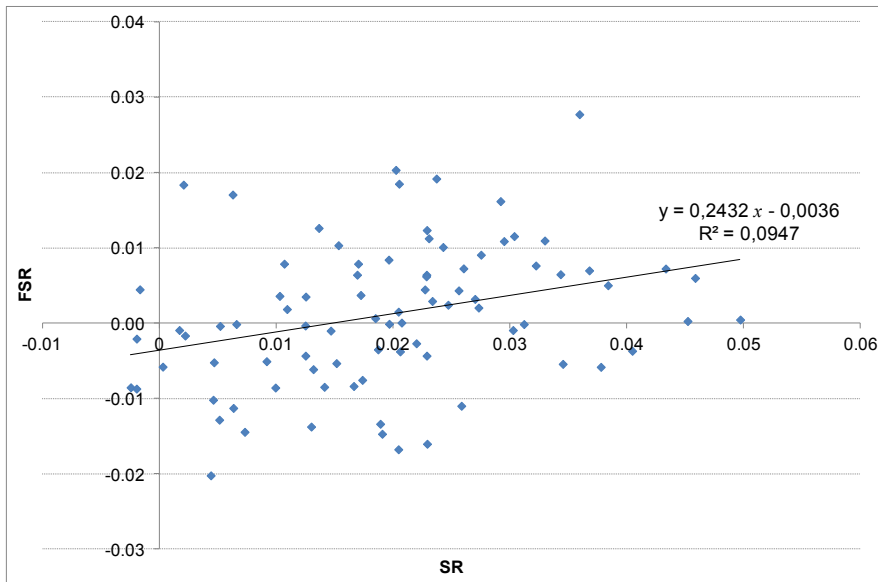
	Portfolios	One week VaR			One month VaR		
		10%	5%	1%	10%	5%	1%
Top Portfolios	Top FSR	-2.3271%	-3.2457%	-5.0585%	-3.9646%	-5.8087%	-9.6616%
	Top SR	-2.5486%	-3.4805%	-5.6394%	-3.8283%	-5.9828%	-10.0816%
	Top Omega	-2.4894%	-3.4739%	-5.3720%	-3.9259%	-6.1925%	-10.5474%
	Top UPR	-2.5531%	-3.4690%	-5.4950%	-4.4003%	-6.5233%	-10.5849%
Bottom portfolios	Bottom FSR	-2.5489%	-3.4821%	-5.4015%	-4.4874%	-6.3872%	-10.5160%
	Bottom SR	-2.9006%	-3.8632%	-6.2153%	-5.2734%	-7.3936%	-12.0561%
	Bottom Omega	-3.0204%	-4.0562%	-6.2925%	-5.1489%	-7.5758%	-12.5991%
	Bottom UPR	-2.7269%	-3.7217%	-6.0780%	-4.4728%	-6.7970%	-11.2336%

Note: Simulated results are based on a GARCH-EVT-Copula methodology. The top/bottom portfolios are composed of the 30 best/worst performing stocks in accordance with the selected performance measures. The VaR is computed at the portfolio return's level.

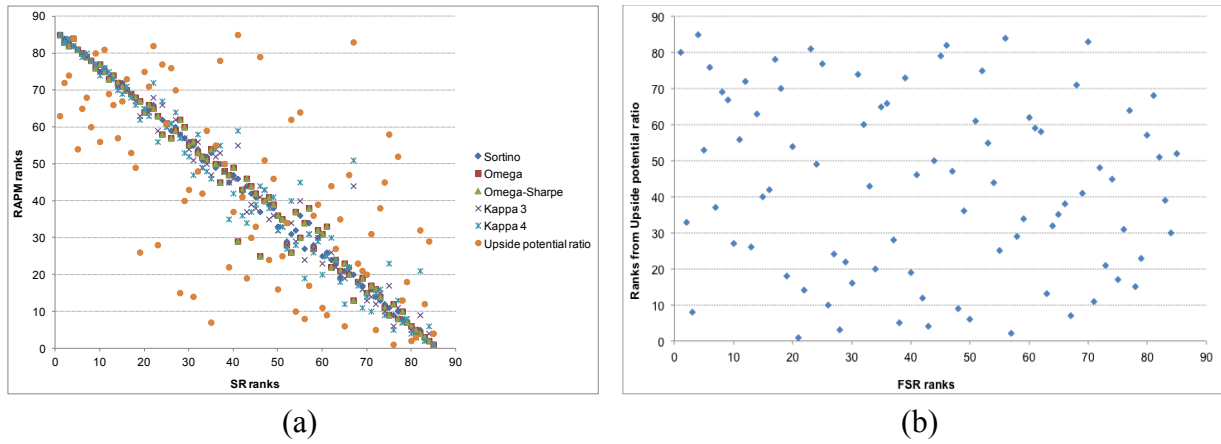
## FIGURES



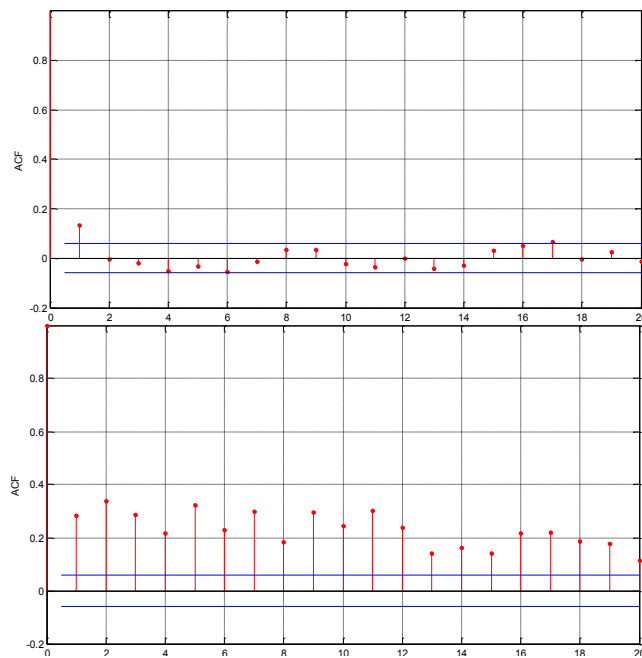
**Fig. 1.** Normality test of dynamic and state errors. This figure draws the Cramer-von Mises statistics of dynamic ( $u_i$ ) and state ( $v_i$ ) errors for each stock  $i$ .



**Fig. 2.** Plotting classic Sharpe ratios against fundamental ones. The figure draws classic Sharpe ratios (SR) against corresponding fundamental Sharpe ratios (Median FSR). A regression adjustment is proposed where  $x$  represents ranked Sharpe ratios while  $y$  represents their fundamental counterparts. Only around 10 percent of SR estimates coincide approximately with FSR estimates.

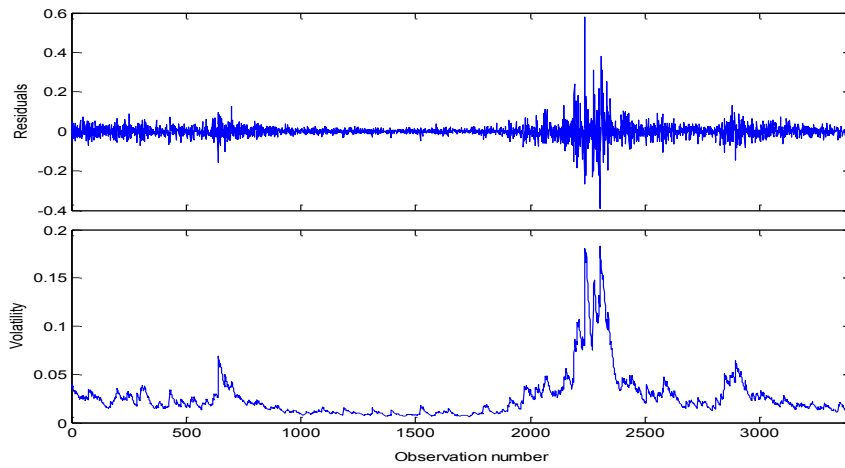


**Fig. 3.** RAPM ranks versus SR ranks. This figure plots the ranks of stocks according to the performance measures under consideration, namely the classic Sharpe ratio (SR), the fundamental Sharpe ratio (Median FSR) and the 6 other risk-adjusted performance measures (RAPMs). The panel (a) plots the SR-based ranks against the ranks induced by the other RAPMs. The panel (b) plots FSR-based ranks against the ranks induced by the Upside potential ratio.

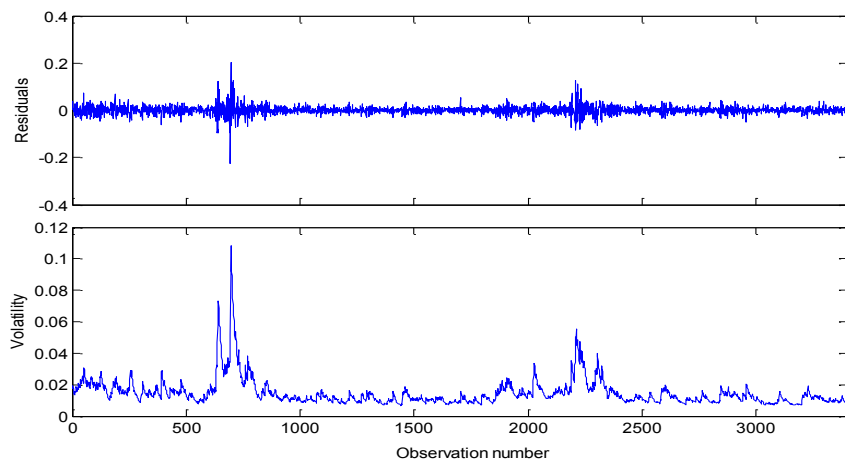


**Fig. B.1.** Autocorrelation function (ACF) of stock returns (upper panel) and squared stock returns (lower panel).

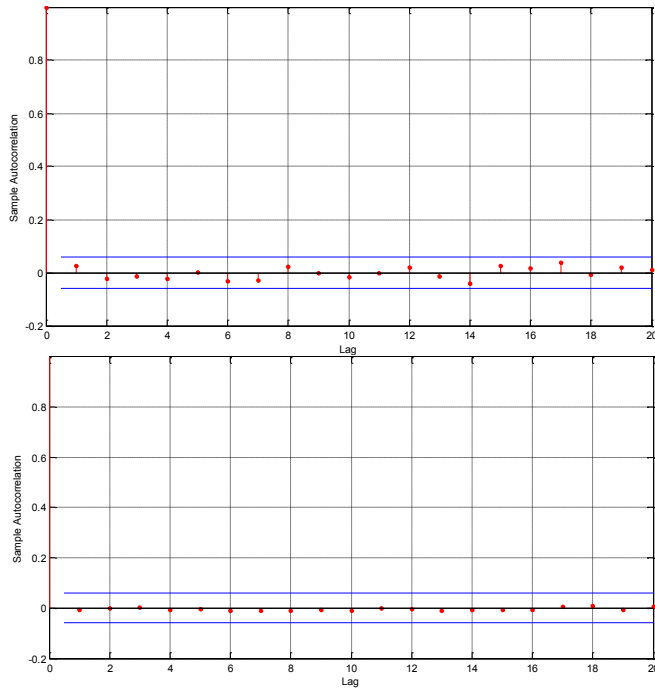




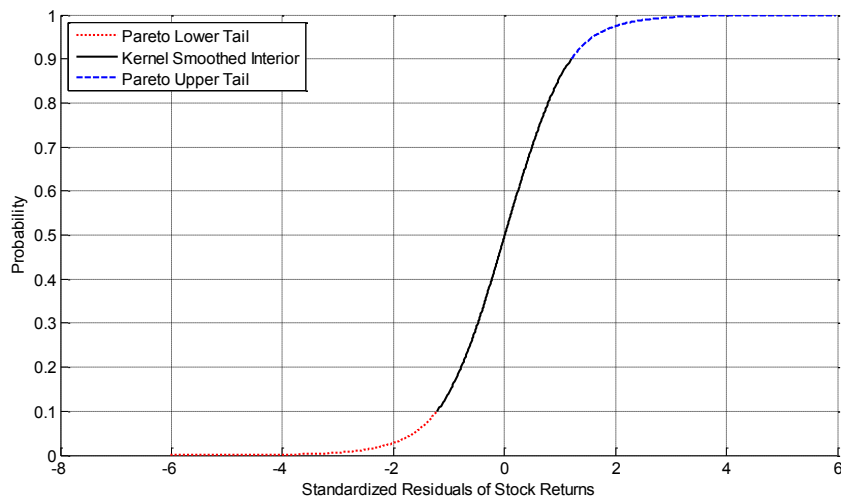
**Fig. C.1.** Residuals and conditional volatility of residuals in the AR(1)-TGARCH(1,1,1) without adjustment to volatility regimes (estimation over the sample horizon).



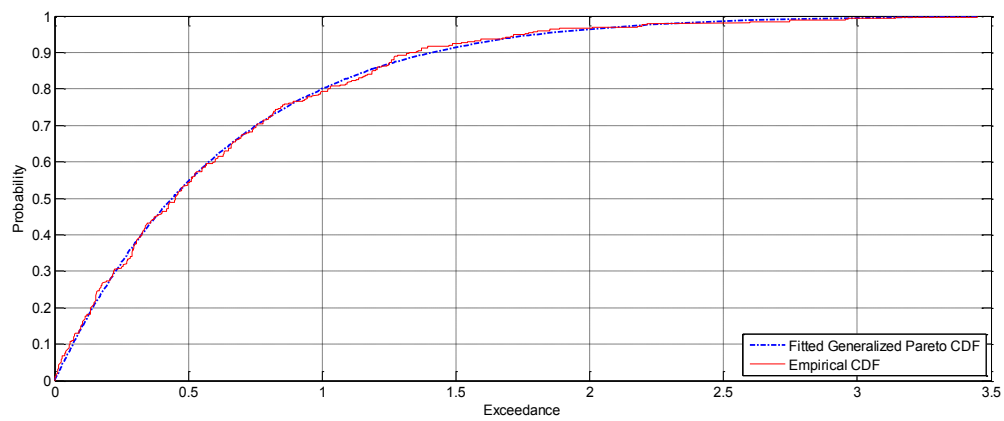
**Fig. C.2.** Residuals and conditional volatility of residuals in the AR(1)-TGARCH(1,1,1) after adjustment to volatility regimes (estimation after splitting the sample horizon into a pre-, during- and post-crisis period).



**Fig. C.3.** Autocorrelation function (ACF) of standardized residuals (upper panel) and squared standardized residuals (lower panel).



**Fig. D.1.** Empirical CDF of standardized residuals as a mixture of a Gaussian kernel estimate and a GPD.



**Fig. D.2.** Quality of the GPD's fit for upper tail exceedances of standardized residuals.