

Fundamentals of Graph Theory

An Introduction

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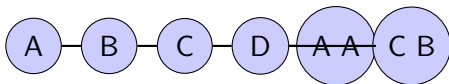
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Simple Undirected Graph

Diagonal Edge: A-C



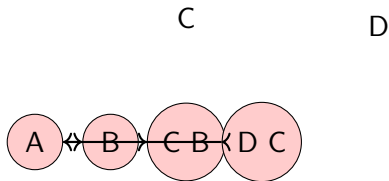
Cycle: A-B-C-D-A
Cross Edge: B-D

Explanation:

An undirected graph where edges have no direction. Each edge is bidirectional, and the graph contains a cycle, diagonal, and cross edges.



Directed Graph



Cycle: A \rightarrow B \rightarrow C \rightarrow D \rightarrow A
Path: A \rightarrow B \rightarrow C \rightarrow D

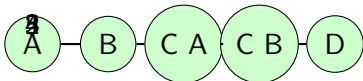
Explanation:

A directed graph where edges have a specific direction, indicated by arrows. This graph includes a directed cycle and a directed path.



Weighted Graph

5: B-C C-D



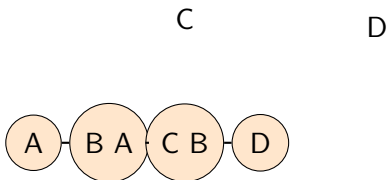
Weights on Edges: A-C 4: B-D

Explanation:

A graph where edges have weights. Weights represent the cost or distance between nodes, providing additional information on the graph's structure.



Bipartite Graph



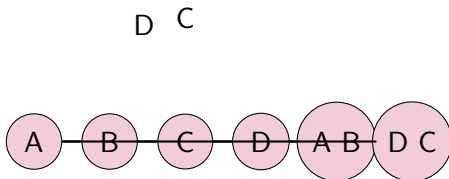
Two Disjoint Sets: $\{A, B\}$, $\{C, D\}$

Explanation:

A bipartite graph where nodes can be divided into two disjoint sets. Every edge connects a node from one set to a node from the other set.



Planar Graph



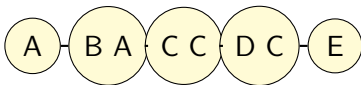
Planar Graph: No Edge Crossings

Explanation:

A planar graph can be drawn on a plane without any edges crossing. This property is important in graph theory for various applications.



Tree Graph



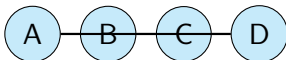
Tree: A Root B Root C D E **Explanation:**

A tree graph is a connected acyclic graph with a single root. Each node has exactly one parent, except the root, which has none.



Cycle Graph

D C



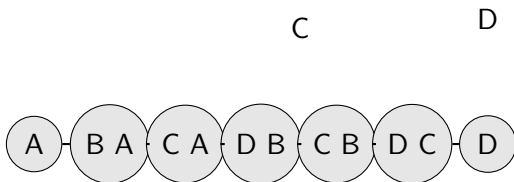
A_{Cycle}: A-B-C-D-A

Explanation:

A cycle graph where nodes are connected in a circular fashion. Each node is connected to two other nodes, forming a closed loop.



Complete Graph



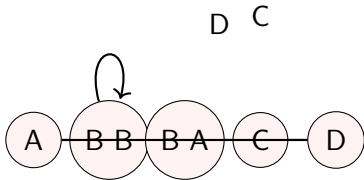
Complete Graph: Every Pair Connected

Explanation:

A complete graph where every pair of nodes is connected by an edge. It is denoted as K_n , where n is the number of nodes.



Graph with Loops



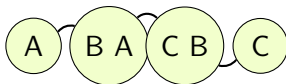
A Loop: B-BB Cycle: A-C-D-A **Explanation:**

A graph that includes loops (edges connected to the same node) and cycles. Loops are edges that start and end at the same node.



Graph with Multiple Edges

C



Multiple Edges Between Multiple Paths: A-C

Explanation:

A graph with multiple edges between the same pair of nodes and multiple paths between nodes. This allows for redundancy and alternative routes.