PCNF to PDNF

If the principle disjunctive (conjunctive) normal form of a given formula S consisting of n variables is known, then the principle disjunctive normal form of $\neg S$ will consist the disjunction of the remaining minterms (maxterms) which do not appear in the principle disjunctive or conjunctive normal form of S

From $S \rightleftharpoons \neg(\neg S)$ one can obtain the principle conjunctive or disjunctive normal form of S

① Without constructing the truth table obtain the product of sums (PCNF) canonical form of the formula, $(\neg P \to R) \wedge (Q \rightleftarrows P)$. Hence find the sum of product PDNF canonical form

Sol

$S \Leftrightarrow (\neg P \to R) \wedge (Q \rightleftarrows P)$

$(P \vee R) \wedge [(Q \to P) \wedge (P \to Q)]$

$(P \vee R) \wedge [(\neg Q \vee P) \wedge (\neg P \vee Q)]$

$\Rightarrow [P \vee R \vee F] \wedge [\neg Q \vee P \vee F] \wedge [\neg P \vee Q \vee F]$

$\Rightarrow [P \vee R \vee (Q \wedge \neg Q)] \wedge [\neg Q \vee P \vee (R \wedge \neg R)] \wedge$
$[\neg P \vee Q \vee (R \wedge \neg R)]$

$\Rightarrow [(P \vee R \vee Q) \wedge (P \vee R \vee \neg Q) \wedge (\neg Q \vee P \vee R)$
$\wedge (\neg Q \vee P \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)]$

$\Rightarrow [(P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge$
$(P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)]$

$S \Rightarrow [(P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge$
$(\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)]$

Max terms $\Rightarrow$ $^{*}(P \vee Q \vee R)$ $\quad$ $^{*}(P \vee \neg Q \vee R)$
$^{*}(\neg P \vee Q \vee R)$ $\quad$ $(P \vee Q \vee \neg R)$
$(\neg P \vee \neg Q \vee R)$ $\quad$ $^{*}(\neg P \vee Q \vee \neg R)$
$(\neg P \vee \neg Q \vee \neg R)$ $\quad$ $^{*}(P \vee \neg Q \vee \neg R)$

$\neg S \Rightarrow$ Remaining maxterms of P, Q, R

$\neg S \Leftrightarrow (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee \neg R)$

$\neg\neg S \Leftrightarrow \neg(\neg P \vee \neg Q \vee R) \vee \neg(\neg P \vee \neg Q \vee \neg R) \vee \neg(P \vee Q \vee \neg R)$

$(P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

2 Without using truthtable find the PCNF then find PDNF of

$P \to (Q \wedge P) \wedge [\neg P \to (\neg Q \wedge \neg R)]$

Sol:

$\neg P \vee (Q \wedge P) \wedge [P \vee (\neg Q \wedge \neg R)]$

$(\neg P \vee Q) \wedge (\neg P \vee P) \wedge [(P \vee \neg Q) \wedge (P \vee \neg R)]$

maxterm minterm

PCNF & PDNF

using truth table

using truth table we can easily find PDNF and PCNF of given statement formula

working rule to find PDNF

construct truth table for the given statement formula

choose each & every row in which the final column value is true

In the selected row, if the truth value of each individual variable value is true select that variable and truth value is false then select the negation of that variable

In such a way collect all possible minterms

Sum of all minterms gives the required PDNF working rule to find PCNF

construct truth table for the given statement formula

choose each & every row in which the final column value is false.

In the selected row of the truth value of each individual variable value is false, select that variable & truth value is true. then select the negation of that variable.

In such a way collect all possible maxterms

Product of all maxterms gives the required PCNF.

① obtain PDNF and PCNF $(\neg P \to R) \wedge (Q \leftrightarrow P)$

Soln

P	Q	R	$\neg P$	$\neg P \to R$	$Q \leftrightarrow P$	$(\neg P \to R) \wedge (Q \leftrightarrow P)$	PDNF (Min) PCNF Max
T	T	T	F	T	T	T	$\boxed{P \wedge Q \wedge R}$
T	T	F	F	T	T	T	$\boxed{P \wedge Q \wedge \neg R}$
T	F	T	F	T	F	F	$\neg P \vee Q \vee \neg R$
T	F	F	F	T	F	F	$\neg P \vee Q \vee R$
F	T	T	T	T	F	F	$P \vee \neg Q \vee \neg R$
F	T	F	T	F	F	F	$P \vee \neg Q \vee R$
F	F	T	T	T	T	T	$\boxed{\neg P \wedge \neg Q \wedge R}$
F	F	F	T	F	T	F	$P \vee Q \vee R$

PDNF => Sum of Products

PDNF = $(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R)$

PCNF => Product of sums

PCNF => $(\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge$
$(P \vee \neg Q \vee R) \wedge (P \vee Q \vee R)$

② Find the PCNF and PDNF of the proposition $P \wedge (Q \rightarrow R)$

P	Q	R	$Q \rightarrow R$	$P \wedge (Q \rightarrow R)$	(PDNF) Minterms	(PCNF) Maxterms
T	T	T	T	T	$P \wedge Q \wedge R$	
T	T	F	F	F		$\neg P \vee \neg Q \vee R$
T	F	T	T	T	$P \wedge \neg Q \wedge R$	
T	F	F	T	T	$P \wedge \neg Q \wedge \neg R$	
F	T	T	T	F		$P \vee \neg Q \vee \neg R$
F	T	F	F	F		$P \vee \neg Q \vee R$
F	F	T	T	F		$P \vee Q \vee \neg R$
F	F	F	T	F		$P \vee Q \vee R$

PDNF = $(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$

PCNF = $(\neg P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee \neg Q \vee R)$
$\wedge (P \vee Q \vee \neg R) \wedge (P \vee Q \vee R)$

3, Find the PCNF and PDNF of $(P \wedge R) \vee (P \wedge \neg Q)$

P	Q	R	$\neg Q$	$P \wedge R$	$P \wedge \neg Q$	$(P \wedge R) \vee (P \wedge \neg Q)$	terms
T	T	T	F	T	F	T	$P \wedge Q \wedge R$
T	T	F	F	false	F	F	$\neg P \vee \neg Q \vee R$
T	F	T	T	True	T	T	$P \wedge \neg Q \wedge R$
T	F	F	T	F	T	T	$P \wedge \neg Q \wedge \neg R$
F	T	T	F	F	F	F	$P \vee \neg Q \vee \neg Q$
F	T	F	F	F	F	F	$P \vee \neg Q \vee R$
F	F	T	F	F	F	F	$P \vee Q \vee \neg R$
F	F	F	T	F	F	F	$P \vee Q \vee R$

4, Obtain the PDNF & PCNF of $(P \wedge Q) \vee (\neg P \wedge R)$
i) using truth table
ii) without using truth table

5, Find the PCNF of $(P \wedge Q) \vee (\neg P \wedge R)$

PDNF $= (P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$

PCNF $= (\neg P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee \neg R) \wedge (P \vee Q \vee R)$.

Theory of Inference.

The main aim of logic is to provide the rules of inference or principles of reasoning. Here we are concerned with the infering of a conclusion from given premises, we are going to check the logical validity of the conclusion from the given set of premises by making use of equalence rule and implication rule, the theory associated with such things is called inference theory.

Rules of inference:

1) Rule P:

A premise may be introduced at any point in the derivation.

2) Rule T:

A formula S may be introduced at any point in a derivation if S is tautologically implied by any one or more of the preceding formulas.

3) Rule CP:

If S can be derived from R and set of premises, then $R \to S$ can be derived from the set of premises alone

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Implication rules:

1) a) $P, P \to Q \Rightarrow Q$ (modus Phones)

b) $\neg Q, P \to Q \Rightarrow \neg P$ (modus tollens)

c) $\neg P, P \vee Q \Rightarrow Q$ (disjunctive syllogism)

2) $P \to Q, Q \to R \Rightarrow P \to R$ (chain rule)

3) $P, Q \Rightarrow P \wedge Q$ (Simplification rule)

4) $P \wedge Q \Rightarrow P, Q$ (Simplification rule)

5) $P, Q \Rightarrow P \vee Q$ (Addition rule)

6) $P \wedge \neg Q \Rightarrow \neg(P \to Q)$ Equivalence rule)

① Demonstate that R is a valid inference from the premises $P \to Q$, $Q \to R$ & P

sol: Given the premises are $P \to Q$,
$Q \to R$,
P

conclusion: R

{1}	1) $P \to Q$	Rule P	
{2}	2) $Q \to R$	Rule P	
{1,2}	3) $P \to R$	Rule T	$[P \to Q, Q \to R \equiv P \to R]$
{3}	4) P	Rule P	
{1,2,3}	5) R	Rule T	$[P, P \to Q \equiv Q]$

② Show that $\neg P$ follows logically from the premises, $\neg(P \wedge \neg Q)$, $(\neg Q \vee P)$ & $\neg R$

Soln-

Premises are $\neg(P \wedge \neg Q)$

$\neg Q \vee P$

$\neg R$

conclusion $\neg P$

{1}	1) $\neg(P \wedge \neg Q)$	Rule P
{1}	2) $\neg P \vee Q$	Rule T ⇒ Demorgan's law
{1}	3) $P \to Q$	Rule T ⇒ $P \to Q \equiv \neg P \vee Q$
{2}	4) $\neg Q \vee R$	Rule P
{2}	5) $Q \to R$	Rule T ⇒ $P \to Q \equiv \neg P \vee Q$
{1,2}	6) $P \to R$	Rule T ⇒ chain rule $P \to Q, Q \to R \Rightarrow P \to R$
{3}	7) $\neg R$	Rule P
{1,2,3}	8) $\neg P$	Rule T $[\neg Q, P \to Q \equiv \neg P]$

③ Show that $R \vee S$ follows logically from the premises:

$C \vee D$, $(C \vee D) \to \neg H$, $\neg H \to (A \wedge \neg B)$ &

$(A \wedge \neg B) \to (R \vee S)$

Solution

Premises are $C \vee D$, $(C \vee D) \to \neg H$,

$\neg H \to (A \wedge \neg B)$, $(A \wedge \neg B) \to (R \vee S)$

Conclusion: $R \vee S$

$\{1\}$	1) $C \vee D$	Rule P
$\{2\}$	2) $(C \vee D) \to \neg H$	Rule P
$\{1,2\}$	3) $\neg H$	Rule T $[P, P \to Q \Rightarrow Q]$
$\{3\}$	4) $\neg H \to (A \wedge \neg B)$	Rule P
$\{1,2,3\}$	5) $(A \wedge \neg B)$	Rule T $[P, P \to Q \Rightarrow Q]$
$\{4\}$	6) $(A \wedge \neg B) \to (R \vee S)$	Rule P
$\{1,2,3,4\}$	7) $R \vee S$	Rule T $[P, P \to Q \Rightarrow Q]$