Slonczewski windmill with dissipation and asymmetry

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J. Slonczewski invented spin-transfer effect in layered systems in 1996. Among his first predictions was the regime of "windmill motion" of a perfectly symmetric spin valve where the magnetizations of the layers rotate in a fixed plane keeping the angle between them constant. Since "windmill" was predicted to happen in the case of zero magnetic anisotropy, while in most experimental setups the anisotropy is significant, the phenomenon was not a subject of much research. However, the behavior of the magnetically isotropic device is related to the interesting question of current induced ferromagnetism and is worth more attention. Here we study the windmill regime in the presence of dissipation, exchange interaction, and layer asymmetry. It is shown that the windmill rotation is almost always destroyed by those effects, except for a single special value of electric current, determined by the parameters of the device.

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Spin-transfer effect as a method of controlling magnetic dynamics by electric current was suggested by Berger[1] for domain wall motion and by Slonczewski[2] for spin-valves and multilayer structures. The unusual property of spin-transfer interaction found in Ref. 2 was the tendency of current induced torques to rotate magnetic moments of both spin valve layers in the same direction, much like the oncoming wind rotates the wings of a windmill (Fig. 1). If one assumes that layers have no magnetic anisotropy (crystalline or shape), are identical, and there is no RKKY exchange or dipole-dipole interaction between them, the resulting motion is a perpetual rotation of magnetic moments $\mathbf{m}_1$ and $\mathbf{m}_2$ in clockwise or counterclockwise direction, depending on the direction of electric current I passing through the spin valve. We will call this type of motion a Slonczewski "windmill regime".

Since actual spin-transfer devices have significant magnetic shape anisotropy, normally the windmill regime is not realized. Instead, switching between different preferred magnetic configurations was predicted[2] and is intensively studied since then both experimentally and theoretically. However, the windmill regime still constitutes an interesting problem due to the following. Spin-transfer effect can be viewed as reciprocal to the giant magnetoresistance effect.[3] The resistance of a spin valve is minimal in the parallel configuration $\mathbf{m}_1 \uparrow\uparrow \mathbf{m}_2$. Thus one can hypothesize, that in response to a current pumped through the valve the magnetizations will tend to assume this minimal resistance configuration in order to make electron flow easier. More generally, an idea arises that a current passing through a metal with paramagnetic impurities will tend to orient them parallel and create some sort of current-induced ferromagnetism.[4,5] The two-magnet device is the minimal model where the validity of this idea can be tested. We study the behavior of such a device with arbitrary parameters, except for the restriction of zero magnetic anisotropy. The results give a generalized picture of the Slonczewski windmill regime, and shed some light on the possibility of current-induced ferromagnetism.

We use the single domain approximation. The magnetic moments $\mathbf{m}_i$ ($i = 1, 2$) of the layers have time-independent absolute values m_i and variable directions defined by a unit vector $\mathbf{n}_i(t)$. The LLG equations in terms of $\mathbf{m}_i$ read[2,6]

$$\dot{\mathbf{m}}_1 = \gamma(\mathbf{T}_{ex} + \boldsymbol{\tau}_1) + \frac{\alpha_1}{m_1}[\mathbf{m}_1 \times \dot{\mathbf{m}}_1] , \qquad (1)$$

$$\dot{\mathbf{m}}_2 = \gamma(-\mathbf{T}_{ex} + \boldsymbol{\tau}_2) + \frac{\alpha_2}{m_2}[\mathbf{m}_2 \times \dot{\mathbf{m}}_2] , \qquad (2)$$

where $\mathbf{T}_{ex}$ is the exchange torque, $\boldsymbol{\tau}_{1,2}$ are spin-transfer torques, γ is the gyromagnetic ratio, and $\alpha_{1,2}$ are Gilbert damping constants of the magnets. Note that in conventional experiments $\mathbf{m}_2$ is fixed by magnetic anisotropy, while $\mathbf{m}_1$ can rotate under the influence of spin-transfer torque. Magnet number one is then called a "free layer" and magnet number two is called a "fixed layer", or spin polarizer. In the present investigation no restrictions are imposed on $\mathbf{m}_2$ and both magnetic moments are treated on equal footing.

The exchange torque acting on $\mathbf{m}_1$ is given by $\mathbf{T}_{ex} = J[\mathbf{m}_2 \times \mathbf{m}_1]$ ($J > 0$ corresponds to ferromagnetic coupling
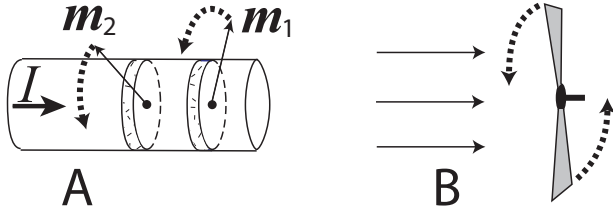

FIG. 1: (A) spin-transfer torques acting on two magnetizations tend to rotate them in the same direction. Magnetic layers are shaded. (B) "windmill" mechanical analogy.

between the moments). The exchange torque acting on $\mathbf{m}_2$ is $-\mathbf{T}_{ex}$ since we are dealing with an internal interaction between two moments.

The spin-transfer torques $\boldsymbol{\tau}_{1,2}$ are given by

$$\dot{\boldsymbol{\tau}}_1 = u_1 \cdot [\mathbf{n}_1 \times [\mathbf{n}_2 \times \mathbf{n}_1]] , \qquad (3)$$
$$\dot{\boldsymbol{\tau}}_2 = -u_2 \cdot [\mathbf{n}_2 \times [\mathbf{n}_1 \times \mathbf{n}_2]] , \qquad (4)$$

with torque strengths

$$u_i = \frac{\hbar}{2} \frac{I}{e} g_i[(\mathbf{n}_1 \cdot \mathbf{n}_2)] . \qquad (5)$$

Here I is the electric current flowing form magnet 2 to magnet 1, e is the (negative) electron charge, and $g_i[(\mathbf{n}_1 \cdot \mathbf{n}_2)]$ are material and device specific spin-polarization factors. For negligible spin-relaxation in the non-magnetic spacer between the magnets one has $g_1 = g_2$ (see Ref. 2). Note that both $u_{1,2}$ are positive when electrons flow from magnet 2 to magnet 1. The minus sign in front of the right hand side of Eq. (4) reflects the symmetry of spin-transfer torque.[2]

First, we rewrite Eqs. (1) and (2) so that time derivatives are on the left hand side only. Defining $\mathbf{T}_1 = \mathbf{T}_{ex} + \boldsymbol{\tau}_1$, $\mathbf{T}_2 = -\mathbf{T}_{ex} + \boldsymbol{\tau}_2$, we get for $i = 1, 2$

$$(1 + \alpha_i^2)\dot{\mathbf{m}}_i = \gamma \left(\mathbf{T}_i + \frac{\alpha_i}{m_i}[\mathbf{m_i} \times \mathbf{T_i}] \right) . \qquad (6)$$

It is convenient to introduce vectors $\boldsymbol{\nu} = [\mathbf{m}_2 \times \mathbf{m}_1]$, $\mathbf{l}_1 = [\mathbf{m}_1 \times [\mathbf{m}_2 \times \mathbf{m}_1]]$, $\mathbf{l}_2 = [\mathbf{m}_2 \times [\mathbf{m}_1 \times \mathbf{m}_2]]$. Then

$$\dot{\mathbf{m}}_1 = A_1\boldsymbol{\nu} + B_1\mathbf{l}_1 ,$$
$$\dot{\mathbf{m}}_2 = -A_2\boldsymbol{\nu} + B_2\mathbf{l}_2 , \qquad (7)$$

with

$$A_1 = \frac{\gamma}{1 + \alpha_1^2} \left(J - \frac{\alpha_1 u_1}{m_1 m_2} \right) ,$$
$$A_2 = \frac{\gamma}{1 + \alpha_2^2} \left(J + \frac{\alpha_2 u_2}{m_1 m_2} \right) ,$$
$$B_1 = \frac{\gamma}{(1 + \alpha_1^2)m_1} \left(\alpha_1 J + \frac{u_1}{m_1 m_2} \right) ,$$
$$B_2 = \frac{\gamma}{(1 + \alpha_2^2)m_2} \left(\alpha_2 J - \frac{u_2}{m_1 m_2} \right) . \qquad (8)$$

Consider now the total magnetic moment of the system $\mathbf{M} = \mathbf{m}_1 + \mathbf{m}_2$ and calculate the derivative $d\mathbf{M}^2/dt$. Using Eq. (6) and the properties $(\mathbf{m}_i \cdot \mathbf{l}_i) = 0$, $((\mathbf{m}_1 + \mathbf{m}_2) \cdot \mathbf{l}_i) = \boldsymbol{\nu}^2$, we find

$$\frac{d\mathbf{M}^2}{dt} = 2C[\mathbf{m}_1 \times \mathbf{m}_2]^2 \qquad (9)$$

with constant coefficient C that depends on material parameters and spin-transfer strengths

$$C = B_1 + B_2 = \gamma \left(\frac{\alpha_1}{(1 + \alpha_1^2)m_1} + \frac{\alpha_2}{(1 + \alpha_2^2)m_2} \right) J$$
$$+ \frac{\gamma}{m_1 m_2} \left(\frac{u_1}{(1 + \alpha_1^2)m_1} - \frac{u_2}{(1 + \alpha_2^2)m_2} \right) . \qquad (10)$$

Since $[\mathbf{m}_1 \times \mathbf{m}_2]^2$ is always positive, except in parallel or antiparallel configurations, we can conclude that after a transient period the magnetic configuration will reach either the state of maximal M (i.e., parallel state) for $C > 0$, or the state of minimal M (i.e, antiparallel state) for $C < 0$. Since in both collinear states $\mathbf{T}_{ex} = 0$ and $\boldsymbol{\tau}_{1,2} = 0$, the system will come to rest and no "windmill" motion will happen. For small spin transfer torques $u_{1,2}$ the final state will be determined by the sign of J and, as expected, the device will end up in a configuration corresponding to the minimum of exchange energy.

The marginal case $C = 0$ is the only situation when the "windmill" is possible. According to Eq. (9), the value of C linearly depends on electric current I through $u_{1,2}$. The only exception is the singular case when device parameters satisfy $g_1/[(1 + \alpha_1^2)m_1] = g_2/[(1 + \alpha_2^2)m_2]$, and C is current-independent. Thus in general one can achieve the windmill regime by tuning the current exactly to the "marginal" value I_w, such that $C(I_w) = 0$. Note that this value corresponds to a spin transfer strength of $u_w \sim \alpha J m_1 m_2$, and since $\alpha \ll 1$ the required spin torque is much smaller than the exchange torque. The situation is similar to the switching regime, where spin transfer effect works against the magnetic anisotropy. In both cases critical values of spin torque are proportional to the small Gilbert damping coefficient.

The original discussion of the windmill regime in Ref. 2 assumed $J = 0$, $\alpha_{1,2} = 0$, and $m_1 = m_2$. It was found that magnetic moments rotate in the plane spanned by vectors $\mathbf{m}_1$ and $\mathbf{m}_2$ at the initial moment, and the angle θ between them remains constant. How will the windmill motion look in the general situation? At $C = 0$ the total magnetic moment is conserved, $\mathbf{M}^2 = \mathbf{m}_1^2 + \mathbf{m}_2^2 + 2(\mathbf{m}_1 \cdot \mathbf{m}_2) = \text{const}$, thus θ is constant in general case as well. Since magnitudes of $\mathbf{m}_1$ and $\mathbf{m}_2$ are also fixed, constant θ implies that both vectors will rotate with the same angular velocity,

$$\dot{\mathbf{m}}_i = [\boldsymbol{\omega} \times \mathbf{m}_i] \qquad (11)$$

(cf. the theorem on the motion of a rigid body with a fixed point). To find $\boldsymbol{\omega}$ we expand it in the basis of vectors $(\boldsymbol{\nu}, \mathbf{m}_1, \mathbf{m}_2)$ as $\boldsymbol{\omega} = a\boldsymbol{\nu} + b_1\mathbf{m}_1 + b_2\mathbf{m}_2$ with unknown coefficients a and $b_{1,2}$. Substituting this form of $\boldsymbol{\omega}$ into Eqs. (11), using expressions (7) for $\dot{\mathbf{m}}_i$, the fact that for the marginal value of current one has $B_1 = -B_2 \equiv B_w$, and properties $[\boldsymbol{\nu} \times \mathbf{m}_1] = -\mathbf{l}_1$, $[\boldsymbol{\nu} \times \mathbf{m}_2] = \mathbf{l}_2$, we find $a = -B_w$, $b_1 = A_2$, and $b_2 = A_1$

$$\boldsymbol{\omega} = B_w[\mathbf{m}_1 \times \mathbf{m}_2] + A_2\mathbf{m}_1 + A_1\mathbf{m}_2 . \qquad (12)$$

Since $\dot{\boldsymbol{\omega}} = [\boldsymbol{\omega} \times \boldsymbol{\omega}] = 0$, $\boldsymbol{\omega}$ is an invariant of motion, determined by the initial conditions.

Since $\alpha_{1,2} \ll 1$ and $u_w \sim \alpha J$ at the marginal point, we can make approximations in expressions (8) and use $A_1 \approx A_2 \approx \gamma J$, $B_1 \approx \gamma(\alpha_1 J m_1 m_2 + u_1)/m_1^2 m_2$, $B_2 \approx \gamma(\alpha_2 J m_1 m_2 - u_2)/m_2^2 m_1$. Equation $C = B_1 + B_2 = 0$

then gives the following values at the marginal point

$$I_w \approx \frac{2e}{\hbar} \left(\frac{m_1\alpha_2 + m_2\alpha_1}{m_1g_2 - m_2g_1} \right) Jm_1m_2 , \qquad (13)$$

$$B_w \approx \gamma \frac{\alpha_1g_2 + \alpha_2g_1}{m_1g_2 - m_2g_1} J .$$

Note that approximation $u_w \sim \alpha J$ is violated when parameters are close to the degenerate situation $m_1g_2 - m_2g_1 = 0$. This is the situation when C is independent of the current and the windmill regime cannot be achieved. Far away from the degenerate situation one has

$$\boldsymbol{\omega} \approx \gamma J \left(\frac{\alpha_1g_2 + \alpha_2g_1}{m_1g_2 - m_2g_1} [\mathbf{m}_1 \times \mathbf{m}_2] + (\mathbf{m}_1 + \mathbf{m}_2) \right) .$$
$$(14)$$

The first term in parentheses is smaller than the second one by a factor of $\alpha \ll 1$. Thus vectors $\mathbf{m}_i$ precess approximately around the total magnetic moment $\mathbf{M} = \mathbf{m}_1 + \mathbf{m}_2$. In the presence of damping and spin-transfer $\mathbf{M}$ is not conserved and performs a small angle rotation around the constant vector $\boldsymbol{\omega}$. In this respect the motion is very different from Slonczewski's situation at $J = 0$, where $\mathbf{M}$ was performing 360° rotations around $[\mathbf{m}_1 \times \mathbf{m}_2]$. The $J = 0$ rotations, however, require the fulfillment of a condition $g_1/[(1+\alpha_1^2)m_1] = g_2/[(1+\alpha_2^2)m_2]$, which is, in particular, satisfied in a completely symmetric valve considered in Ref. 2.

Finally, we return to Eq. (9) and investigate the $C \neq 0$ case. It is convenient to rewrite (9) in terms of $x = \cos\theta$

$$\dot{x} = Cm_1m_2(1 - x^2) .$$

The solution reads

$$x(t) = \cos\theta(t) = \tanh\left(\frac{t + t_0}{\mathrm{sgn}[C] \; T_*} \right) ,$$

with

$$T_* = \frac{1}{|C(I)|m_1m_2} , \qquad (15)$$

and parameter t_0 determined by the initial angle, $\cos\theta_0 = \tanh(t_0/T_*)$. We conclude that as $t \to \infty$ the system approaches a collinear configuration with a current dependent characteristic time $T_*(I)$. The latter diverges in the vicinity of the marginal current I_w.

In conclusion, we studied the motion of a two layer spin-transfer device with zero magnetic anisotropy. We show that in the presence of damping, layer asymmetry, and exchange interaction between the layers the windmill rotation decays with characteristic time constant T_*. The decay time depends on the current pumped through the device and diverges at a "marginal" current I_w. For $I \neq I_w$ the system reaches either a parallel or an antiparallel state after a transient period. Exactly at the marginal point $I = I_w$ the system performs a perpetual generalized windmill motion.

Interestingly, precession motion analogous to the windmill regime was also found in multilayers and bilayers with magnetic anisotropy.[7,8] In those systems it exists not at a singular point, but in the whole range of current values. Thus, rather unexpectedly, anisotropy can be advantageous for the windmill regime.

Finally, coming to the discussion of the current induced ferromagnetism, we see that in a two magnet device current can induce both ferromagnetic and antiferromagnetic order. However, the situation with only two magnets can be special, and it is necessary to consider devices with three and more magnets to predict what happens in the system of many isotropic paramagnetic impurities under the influence of spin-transfer torques.

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