Hypothesis Testing

A **statistical hypothesis** is an intelligent educated guess/assumption about a population parameter, which may or may not be true. There are two forms of statistical hypotheses.

- **Null hypothesis**: This is denoted by H_0, is usually the hypothesis that sample observations result purely from chance.
- **Alternative hypothesis:** This is denoted by H_1 or H_a, is the hypothesis that sample observations are influenced by some non-random cause.

The process of hypothesis testing needs

- ✓ A hypothesis on a population parameter
- ✓ A test statistic under the null hypothesis
- ✓ The p value of the test statistic
- ✓ Decision rule based on a significance level

Decision Errors

	Reject H_0	**Do not reject H_0**
H_0 True	**Type I error (α)**	
H_0 False		**Type II error (β)**

- **Significance level =α=P(Type I error)**
- **Power of the test = 1- β**

Rejection Criterion for null hypothesis

- P-value: The strength of evidence in support of a null hypothesis is measured by the **P-value**.

- Region of rejection in One-Tailed and Two-Tailed Tests

Mean test (One sample)

<table>
<tr><th>Null Hypothesis</th><th>Alternative Hypothesis</th><th colspan="2">Test Statistic, the type of test & rejection criterion</th></tr>
<tr><td rowspan="2">$H_0: \mu = A$

Normal Population has mean "A"</td><td rowspan="2">$H_1: \mu \neq A$</td><td>$Z = (x - \mu)/\sigma$; σ^2 known; testing on a single sample point

$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$; testing on a single sample mean</td><td rowspan="2">Two tail test</td></tr>
<tr><td>$T = (X - \mu)/s$ or $T = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$; σ^2 unknown</td></tr>
<tr><td>$H_0: \mu \leq A$

$H_0: \mu \geq A$</td><td>$H_1: \mu > A$

$H_1: \mu < A$</td><td></td><td>One tail test</td></tr>
</table>

Examples:

1. Bon Air Elementary School has 300 students. The principal of the school thinks that the average IQ of students at Bon Air is at least 110. To prove her point, she administers an IQ test to 20 randomly selected students. Among the sampled students, the average IQ is 108 with a standard deviation of 10. Based on these results, should the principal accept or reject her original hypothesis? Assume a significance level of 0.01.
2. An inventor has developed a new, energy-efficient lawn mower engine. He claims that the engine will run continuously for 5 hours (300 minutes) on a single gallon of regular gasoline. Suppose a simple random sample of 50 engines is tested. The engines run for an average of 295 minutes, with a standard deviation of 20 minutes. Test the null hypothesis that the mean run time is 300 minutes against the alternative hypothesis that the mean run time is not 300 minutes. Use a 0.05 level of significance. (Assume that run times for the population of engines are normally distributed.)

Mean Test (Two Sample)

<table>
<tr><td>Hypotheses</td><td colspan="2">Test Statistic, the type of test & rejection criterion</td></tr>
<tr><td rowspan="3">$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$</td><td>When the two population variances are known and not equal

$$Z = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

When the two population variances are known and equal</td><td rowspan="3">Two tail test</td></tr>
<tr><td>When the two population variances are unknown but equal</td></tr>
<tr><td>When the two population variances are unknown and not equal

$df = (s_1^2/n_1 + s_2^2/n_2)^2$ /

$\{ [(s_1^2 / n_1)^2 / (n_1 - 1)] + [(s_2^2 / n_2)^2 / (n_2 - 1)] \}$

or smaller of $n_1 - 1$ & $n_2 - 1$</td></tr>
</table>

Proportion Tests

Null Hypothesis	Alternative Hypothesis	Test Statistic, the type of test & rejection criterion	
			Two tail test
			One tail test
			One tail test

Paired t-test

$$t = \frac{\bar{d} - \mu_{d_0}}{s_{\bar{d}}}$$

$\bar{d}$ = *sample mean difference*

μ_{d_0} = *hypothesized population mean difference*

$s_{\bar{d}} = s_d / \sqrt{n}$

n = *number of sample differences*

s_d = *standard deviation of sample differences*

Example:

The weights of 9 obese women before and after 12 weeks on a very low calorie diet were as follows:

Before	After	Difference
117.3	83.3	-34.0
111.4	85.9	-25.5
98.6	75.8	-22.8
104.3	82.9	-21.4
105.4	82.3	-23.1
100.4	77.7	-22.7
81.7	62.7	-19.0
89.5	69.0	-20.5
78.2	63.9	-14.3

Test whether the expected weight loss is at least 20kg for obese women after the treatment of this low-calorie diet for 12 weeks. Use 5% significance level.

Variance Tests

Population μ	Estimation of σ^2	Test Statistic & Distribution
μ Known	$s^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \mu\right)^2$ $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \mu\right)^2$	$\frac{ns^2}{\sigma^2} \sim \chi^2_n$ $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_n$
μ Unknown	$s^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$ $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$	$\frac{ns^2}{\sigma^2} \sim \chi^2_{n-1}$ $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$

Acknowledged **http://www.xycoon.com/**

Example:

1. The data 159.9, 187.2, 180.1, 158.1, 225.5, 163.7, and 217.3 consists of the weights, in pounds, of a random sample of seven individuals taken from a population that is normally distributed. The variance of this sample is given as 753.04.

   Test the null hypothesis H_0: $\sigma^2 = 750.0$ against the alternative hypothesis H_1: $\sigma^2 \neq 750.0$ at a level of significance of 0.3.

2. Students have collected the data 27, 29, 22, 21, 26, 28, 24, and 29 from one population and the data 19, 18, 24, 18, 22, and 15 from another. The variance of the first sample is 9.64286 and the variance of the second sample is 10.2667. The ratio of the first variance to the second is 0.939239. Test the null hypothesis that the two variances are equal, H_0: $\sigma_1^2 / \sigma_2^2 = 1$, against the alternative hypothesis that the two variances are not equal, H_1: $\sigma_1^2 / \sigma_2^2 \neq 1$, where σ_1^2 is the variance of the first population and σ_2^2 is the variance of the second population, at a significance level of .10.

Exercises on hypothesis testing:

1. An insurance company is reviewing its current policy rates. When originally setting the rates they believed that the average claim amount was \$1,800. They are concerned that the true mean is actually higher than this, because they could potentially lose a lot of money. They randomly select 40 claims, and calculated a sample mean of \$1,950. Assuming that the standard deviation of claims is \$500, and set α= 0.05, test to see if the insurance company should be concerned.

2. Trying to encourage people to stop driving to campus, the university claims that on average it takes people 30 minutes to find a parking space on campus. I do not think it takes so long to find a spot. In fact I have a sample of the last five times I drove to campus, and I calculated $\bar{x}$= 20. Assuming that the timeit takes to find a parking spot is normal, and that s^2= 6 minutes, then perform a hypothesis test with levelα= 0.10 to see if my claim is correct.

3. A sample of 40 sales receipts from a grocery store has $\bar{x}$= \$137 and s^2= \$30.2. Use these values to test whether or not the mean value of a receipt at the grocery store is different from \$150.

4. The actual proportion of families in a certain city who own, rather than rent their home is 0.70. If 84 families in this city are interviewed at random and their response to the question of whether they own their home, are recorded. 61 of them have responded saying that they own the home. Using a suitable test statistic test the claim that the population proportion of owning a home is 0.7.