MA 3014 - Problem Set on CI's and Hypothesis Testing

1. On the basis of the results obtained from a random sample of 100 men from a particular district, the 95% confidence interval for the mean height of the men in the district is found to be (177.22cm, 179.18cm).
   I. Find the value of the sample mean and the standard deviation of the normal population from which the sample is drawn.
   II. Calculate the 98% confidence interval for the mean height.
2. The result x of a stress test is known to be a normally distributed with mean μ and standard deviation $\sigma = 1.3$. It is required to have a 95% symmetrical confidence interval for μ with total width less than 2. Find the least number of test that should be carried out to achieve this.
3. Ten packets of a particular brand of biscuits are chosen at random and their weights are noted. The results are

397.3	399.6	401	392.9	396.8
400	397.6	392.1	400.8	400.6

   Assuming that sample is taken from a population with mean weight μ, calculate 99% confidence interval for μ.
4. Fifteen pupils experimented to find the value of g, the acceleration due to gravity. Their results were as follows.

9.806	9.807	9.810	9.802	9.805	9.806
9.804	9.811	9.801	9.804	9.805	9.808
9.803	9.809	9.807			

   I. Calculate the mean and standard deviation of these results.
   II. Give 95% confidence limits for the value of g based upon them.
   III. Estimate the number of experimenters needed to give a confidence interval of less than 0.001.
5. If 100 seeds are planted, and 83 seeds germinate, use the normal approximation to the binomial distribution to test the manufacture's claim of a 90% germination rate. (Use 5% level of significance).
6. The lengths of metal bars produced by a particular machine are normally distributed with mean length 420cm and standard deviation 12cm. The machine is serviced, after which a sample of 100 bars gives a mean length of 423cm. Is there evidence, at 5% significant level, of a change in the mean length of the bars produced by the machine, assuming that the standard deviation remains the same?
7. A machine produces elastic bands with breaking tension normally distributed with mean 45.00N and standard deviation 4.36N. On a certain day a sample of 50 was tested and found to have a mean breaking tension of 43.46N.
   I. Test at the 5% level of significance whether this indicates a change in the mean.
   II. Find the 95% confidence interval for the population mean based on the sample mean assuming an unchanged standard deviation.
   III. If the standard deviation has change to σ, find the least value of σ for a 95% confidence interval for the population mean to contain 45.00N.

8. The weights of components produced by a certain machine are normally distributed with mean weight 15.4g and standard deviation 2.3g. A random sample of 81 components have mean weight 15g. Test this, provide evidence at the 5% significant level to say that a deduction in a mean weight of component produce by the machine.

9. A certain political group maintains that girls reach a higher standard in single-sex classes than in mixed classes. To test this hypothesis 140 girls of similar ability are split into 2 groups with 68 attending classes containing only girls and 72 attending classes with boys. All the classes follow the same syllabus and after a specified time the girls are given the test. The test results are summarized,
   Girls in the mixed classes: $\sum x = 7920$ $\sum x^2 = 879912$
   Girls in single-sex classes: $\sum y = 7820$ $\sum y^2 = 904808$
   Testing both samples as large samples from normal distributions having the same variance, obtain a 2 sample pooled estimate of the common population variance. Test whether the results provide significant evidence, at the 1% level, that girls reach a higher standard in single-sex classes.

10. A manufacturer of the personal stereo can use batteries made by two different manufacturers. A standard deviation of life time for "never die" at least 3.1 hours and for "everlasting" battery at least 2.9 hours. Random sample of 90 everlasting batteries and 80 never die batteries tested and the mean life time 7.9 hours 8.2 hours respectively. Test at 5% significance level whether or not there is any evidence of difference between the mean life time of the two made of batteries.

11. The height (to the nearest cm) of a random sample of policemen from a certain force in USA firm to be 176, 180, 179, 181, 183 and 179. The height of the random sample of 11 policemen from a certain force in UK, the mean height is 181cm and standard deviation 5.4cm. Test at the 5% significance level the hypothesis that USA policemen are shorter than UK policemen. Assume that the heights of a policemen on both forces are normally distributed and have a common population variance.

12. The probability that the oyster larva will develop in unpolluted water is 0.9, while in polluted water this probability is less than 0.9.
    I. Given that 20 oyster larva are placed in unpolluted water, find the probability that the number that will develop is at least 17.
    II. An oyster breeder put 20 larva in a sample of water and observed that only 16 of them developed. Use a 10% significance level to determine whether the breeder would be justified in concluding that the water is polluted.
