

Confidence Intervals

A confidence interval will give you a range of values for a given population parameter, within which the parameter falls in $100(1-\alpha)\%$ of the time.

In general a Confidence Interval (CI) is

$$\text{Statistic} \pm \text{margin of error at } 100(1 - \alpha)\% \text{ confidence level}$$

Confidence interval for mean and variance

The following table includes the standard errors of some statistics to help you in finding the confidence intervals for the respective population parameters.

Statistic	Standard Error (SE)
Sample mean, \bar{x}	s / \sqrt{n}
Sample proportion, p	$\sqrt{[p(1-p) / n]}$
Difference between means, $\bar{x}_1 - \bar{x}_2$	$\sqrt{[s_1^2 / n_1 + s_2^2 / n_2]}$
Difference between proportions, $p_1 - p_2$	$\sqrt{[p_1(1-p_1) / n_1 + p_2(1-p_2) / n_2]}$

$100(1 - \alpha)\%$ confidence interval for population mean (μ)

$$\bar{x} \pm Z_{\alpha/2}(SE)$$



Margin of Error

$100(1 - \alpha)\%$ confidence interval for population proportion of success (P)

$$\hat{p} \pm Z_{\alpha/2}(SE)$$

100(1 – α)% confidence interval for population variance(σ^2)

You know that;

$$\text{Chi-square statistic} = (n-1)s^2/\sigma^2 \text{ and } (n-1)s^2/\sigma^2 \sim \chi^2_{(n-1)}$$

$$\begin{aligned} \text{Thus, } & \left[\chi^2_{\alpha/2, (n-1)} \leq (n-1)s^2/\sigma^2 \leq \chi^2_{1-\alpha/2, (n-1)} \right] \\ = & \left[\chi^2_{\alpha/2, (n-1)} / (n-1)s^2 \leq 1/\sigma^2 \leq \chi^2_{1-\alpha/2, (n-1)} / (n-1)s^2 \right] \\ = & \left[(n-1)s^2 / \chi^2_{1-\alpha/2, (n-1)} \leq \sigma^2 \leq (n-1)s^2 / \chi^2_{\alpha/2, (n-1)} \right] \end{aligned}$$

Computational Exercise

Breakdown voltage is a characteristic of an insulator that defines the maximum voltage difference that can be applied across the material before the insulator collapses and conducts. In solid insulating materials, this usually creates a weakened path within the material by creating permanent molecular or physical changes by the sudden current. Within rarefied gases found in certain types of lamps, **breakdown voltage** is also sometimes called the "striking voltage". [Wikipedia]

The breakdown voltage of a material is observed on 17 experimental units as it is not a definite value because it is a form of failure. Thus we have $n = 17$ and $s^2 = 137324.3$. Find the 95% confidence interval for σ^2 , to describe more about the population variance (variance of the breakdown voltage of the material)?