$$\frac{du}{dt} = au + b \qquad u_f - u_i = \int au + \int b$$

$$\frac{y_{i+1} + y_i}{2} \left(\cancel{y_{i+1} - y_i}\right)(t_{i+1} - t_i)$$

$$y = c\tau^2$$

$$\boxed{\frac{du}{dt} = 2u + t}$$

$$u_h(t) = e^{2t}$$

$$u_p = \alpha t + \beta$$

$$\alpha = 2(\alpha t + \beta) + t$$

$$\begin{cases} \alpha = 2\beta \\ 0 = 2\alpha + 1 \end{cases} \qquad \alpha = -\frac{1}{2} \qquad \beta = -\frac{1}{4}$$

$$u(t) = A e^{2t} - \frac{1}{4}(2t + 1)$$

$$u(0) = A - \frac{1}{4} = 1$$

$$u(t) = \frac{5}{4} e^{2t} - \frac{1}{4}(2t+1)$$

$$u(t) = \frac{1}{4}\left[5e^{2t} - 2t - 1\right]$$

$$\begin{array}{ccc} I & & \\ -(I+\frac{\tau}{2}A^1) & I-\frac{\tau}{2}A^2 & \\ & \ddots & \ddots \\ & -(I+\frac{\tau}{2}A^n) & I-\frac{\tau}{2}A^{n+1} \end{array} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{n+1} \end{bmatrix} \begin{array}{l} = u^{init} \\ = \frac{\tau}{2}(b^1+b^2) \\ \\ = \frac{\tau}{2}(b^n+b^{n+1}) \end{array}$$

$Qol =$

$\langle b, Kr \rangle$

$Qol = \sum$

$\boxed{Au = b}$

$\langle Au, Kr \rangle$

$\langle u, A^* K r \rangle$

$\langle u, K K^{-1} A^T K r \rangle$

$\underline{\langle Ku, K^{-1} A^T K r \rangle}$

$\langle u, Kr \rangle$

$= u_1 \frac{\tau}{2} r_1 + \sum_{i=2}^{N} \tau u_i r_i + u_{N+1} \frac{\tau}{2} r_{N+1}$

$=$ $\boxed{A^* u^* = r}$

$=$

Kr

$\langle b, A u^* \rangle = \langle Au, A u^* \rangle$

$= Au, Kr$

$= \langle u, K K^{-1} A^* K r \rangle$

$A u$

$\langle b, Kr \rangle$

$= \langle A^{-1} u, Kr \rangle$

$= u_1$

$u, K^{-1} A^* u^*$

K^{-1}

$u^*(Au = b)$

$u^* Au = \underline{u^* b} = u A^* u^* = \boxed{ur}$

$\langle u, Kr \rangle$

$Au = b$

$\mathcal{A} u^* = r$

$= \langle A^{-1} b, Kr \rangle$

$= \langle b, A^{-T} K r \rangle$

$= \langle b, K K^{-1} A^{-T} K r$

$= \langle K^T b, \underbrace{K^{-1} A^{-T} K r}_{\mathcal{A}^{-1}} \rangle$

$(AB)^{-1} = B^{-1} A^{-1}$

$$\mathcal{A}^{-1} = K^{-1} A^{-T} K = (K^{-1} A^T K)^{-1}$$

$$\boxed{\mathcal{A} = K^{-1} A^T K}$$

$\mathcal{A} u^* = K^{-1} A^T K u^* = r$

$$\boxed{\begin{matrix} I & -2\left(1+\frac{\tau}{2}A^1\right) & & \\ 0 & 1-\frac{\tau}{2}A^2 & -\left(1+\frac{\tau}{2}A^2\right) & \\ & \ddots & \ddots & \\ & & & -\frac{1}{2}\left(1+\frac{\tau}{2}A^{N+1}\right) \\ & & 0 & 1-\frac{\tau}{2}A^{N+1} \end{matrix}}$$

$$\boxed{\left(1-\frac{\tau}{2}A^{N+1}\right)U^{*N+1} = r_{N+1}}$$

$$U^{*N+1} = \Omega$$

$$-\frac{1}{2}\left(1+\frac{\tau}{2}A^{N+1}\right)U^{N+1} = r^{N} \qquad -\frac{1}{2}\left(1+\frac{\tau}{2}A^{N+1}\right)\left(\Omega-\left(1-\frac{\tau}{2}A^{N+1}\right)^{-1}r_{N+1}\right)$$

$$U^{N+1} = \Omega$$

$$u(t) = \left(y_0+\frac{1}{4}\right)e^{2t} - \frac{1}{4}(2t+1)$$

$$u^{N+1} = \Omega$$

$$\left(1-\frac{\tau}{2}A^{N+1}\right)U^{N+1} = \left(1-\frac{\tau}{2}A^{N+1}\right)\Omega = r^{N+1}$$

$$u^1 - 2\left(1+\frac{\tau}{2}A'\right)u^2 = r_1$$

$\boxed{\text{Explicit Solve}}$