**Overall research thrust**

Adjoint formulations are often preferred to determine Quantities of Interests (QoI) and their sensitivities in response to perturbed inputs when the number of uncertain parameter is large. The method requires two unperturbed transport solves, one for the forward flux, one for the adjoint flux; the remainder of the calculations involve inner products employing these fluxes to quickly find the response and its sensitivity to a large number of perturbation scenarios. However, for time-dependent systems, using an adjoint-based method requires storing the full unperturbed forward angular flux at each time step. Only storing the converged scattering source, as is often done in steady state to save up on memory requirements, is not an applicable approach for transient problems. A potential workaround the high-dimensionality of the transport solution is to use the Variable Eddington Tensor formulation (VET) in which the angular dependence has been of the solution has been ``factored’’ out, reducing the dimensionality of the problem by 2 dimensions. This formulation relies on an Eddington factor, which relates the 2nd angular moment of the flux to the scalar flux, which would greatly reduce the memory burden by requiring only the scalar flux and Eddington factor be stored at each time step. The general idea is that a transient forward Sn problem would still be required (to generate and store the Eddington tensor at various moments in time: a memory cost of 6 real values per mesh vertex, as opposed to the memory cost of the transport solution of the number of directions per vertex). One would then perform adjoint-based sensitivity studies using the adjoint VET formulation instead of the adjoint Sn formulation. The resulting sensitivity coefficients will be missing the first-order perturbation effect on the Eddington tensor (which is only computed using the nominal parameter values).

The key to using the VET equation for finding the response to perturbations is that the Eddington factor remains relatively unperturbed. A discontinuous FEM solver was written to construct the forward and adjoint Sn-transport solutions in 1D, which can also be used to find an Eddington factor for use in the VET equation. Similarly, another discontinuous FEM solver was constructed to find the forward and adjoint VET equations. The necessary inner product functions to determine sensitivity of the system under changes to material cross sections, source strengths, and incident flux using the adjoint solution were derived for both Sn and VET formulations, and implemented alongside the solvers. We have verified that the solvers and inner products return the expected sensitivity values in theoretically simple 1D systems.

**Updates for the past quarter**

**-** Material boundaries were identified as a particularly strong region for Eddington perturbation,

which generally results in the Eddington model failing with respect to Sn sensitivity models.

* First order Eddington perturbation terms implemented in solver. These reconcile reasonably well the difference between the Eddington model and Sn models. Some difference may still exist due to higher order perturbations, as well as errors in the numerical solves, since the solution methods for VET and Sn differ.

**Goals for the next quarter**

* Source perturbation scenarios are a system in which Sn adjoint methods can provide an exact solution. However, source perturbation scenarios can be constructed in which the VET method fails to obtain the exact answer. In response, consider a “blended” method that can utilize sensitivity techniques from both the Sn and VET methods in a way that is computationally friendly, while providing better sensitivity calculations than VET alone.
* Explore the effects system perturbations have on the Eddington Tensor. If the perturbed Eddington can be predicted in some way, then the previously mentioned Eddington perturbation terms may be used to close the gap in some scenarios between VET and Sn sensitivity calculations. Consider methods to interpolate a perturbed Eddington using a few perturbation data points, which may be useful for situations when numerous sensitivity scenarios must be tested on a single system.