TEXAS A&M UNIVERSITY
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Research Memo

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To: Distribution
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Subject: Precursors spatial treatment

1. Precursors spatial treatment

There is no need to expand the precursors as FEM solutions. Let us look at the term appearing in the flux equation

$$\dots = \dots + \lambda \mathcal{C}(\vec{r}) \tag{1}$$

When the flux equation is tested with $\varphi_i(\vec{r})$ and integrate over the whole computational domain, we obtain, for the delayed neutron source,

$$\int \lambda \mathcal{C}(\vec{r})\varphi_i(\vec{r}) = \lambda C_i \tag{2}$$

where the unknown C_i is

$$C_i = \int \mathcal{C}(\vec{r})\varphi_i(\vec{r}) \tag{3}$$

Now, let us take a look at the precursors equation:

$$\frac{d\mathcal{C}}{dt} = -\lambda \mathcal{C} + \beta\nu\Sigma_f \Phi \tag{4}$$

We only test this equation by φ_i and integrate over the whole computational domain (no need to expand $\mathcal{C}$ as a FEM solution)

$$\frac{dC_i}{dt} = -\lambda C_i + \int \left(\beta(\vec{r})\nu\Sigma_f(\vec{r})\Phi(\vec{r})\varphi_i(\vec{r})\right) \tag{5}$$

what is $\int (\beta\nu\Sigma_f\Phi\varphi_i)$? It is the i-th row of the $\underline{\underline{M}}\,\underline{\Phi}$ matrix-vector product, where the entries of matrix $\underline{\underline{M}}$ are

$$M_{ij} = \int \beta(\vec{r})\nu\Sigma_f(\vec{r})\varphi_i(\vec{r})\varphi_j(\vec{r}) \tag{6}$$

and the entries of $\underline{\Phi}$ are the flux nodal values: $\Phi(\vec{r}) = \sum_j \Phi_j\varphi_j(\vec{r})$.

So the initial values should be computed as

$$\begin{bmatrix} C_1 \\ \dots \\ C_i \\ \dots \\ C_n \end{bmatrix} = \frac{1}{\lambda}\underline{\underline{M}}\,\underline{\Phi} \tag{7}$$

If you want the concentration at a quadrature point, you say

$$\boxed{C(\vec{r}_q) = \sum_j C_i \varphi_j(\vec{r}_q)} \tag{8}$$

In Yak, the way the precursors are obtained at a quadrature point is as follows

$$\frac{\beta(\vec{r}_q)}{\lambda(\vec{r}_q)} \times Sf_q \tag{9}$$

where the fission source is expanded as

$$Sf(\vec{r}) = \sum_j Sf_j \varphi_j(\vec{r}) \quad \text{with } Sf_i = \int \nu\Sigma_f(\vec{r})\varphi_i(\vec{r})\Phi(\vec{r}) \tag{10}$$

and thus

$$\boxed{C(\vec{r}_q) = \frac{\beta(\vec{r}_q)}{\lambda(\vec{r}_q)} \sum_j Sf_j \varphi_j(\vec{r}_q)} \tag{11}$$

It is not the same thing.

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