Improved Quasi-Static Methods in Rattlesnake

Development and Implementation of Improved Quasi-Static (IQS) methods for time-dependent neutron diffusion and neutron transport solvers in Rattlesnake

Zachary M. Prince[†], **Jean C. Ragusa**[†], **Yaqi Wang**[⋆], **Mark D. DeHart**[⋆]

[†]Department of Nuclear Engineering

Texas A&M University, College Station, TX, USA

[⋆]Idaho National Laboratory

zachmprince@tamu.edu, jean.ragusa@tamu.edu, yaqi.wang@inl.gov, mark.dehart@inl.gov

September 2, 2016

Abstract This report reviews uncollided flux techniques (first and last collision methods) to be implemented in the Rattlesnake S_N code in order to mitigate ray effects in modeling the TREAT reactor+hodoscope system. Angular discretization techniques (S_N and P_N) for the transport equation are notoriously poor at capturing effectively streaming effects.

1 Introduction

This report reviews uncollided flux techniques (first and last collision methods) to be implemented in the Rattlesnake S_N code in order to mitigate ray effects in modeling the TREAT reactor+hodoscope system. Angular discretization techniques (S_N and P_N) for the transport equation are notoriously prop at capturing accurately streaming effects. The uncollided component of the angular flux solution is the most anisotropic part and it can be difficult for discrete-ordinate methods to accurately represent it; this phenomena is well-known and termed “ray effects”. However, it has long been recognized that an analytical or semi-analytical treatment of the uncollided flux, coupled with a discrete-ordinate treatment of the collided flux, can yield dramatic improvements in accuracy and computational efficiency. In this report, we present an algorithm for semi-analytical calculation of the uncollided flux. The algorithm seeks to compute

1. the existing uncollided flux at each point in a collect of point on the surface of the problem domain, with the angular flux reported for each “source point”. The direction is fro the source point to the surface point. The source points are chosen to be spatial quadrature points that integrate the source volume.

2. Spatial and angular distributions of the uncollided flux in every cell in the problem. These provide all of the information necessary to form the first-collision source in each cell.

The proposed algorithm is tailored for parallel efficiency given a spatial domain decomposition.

1.1 Basic Approach

For simplicity, we describe the technique using a single-speed description. Generalization to multigroup is straightforward. We recall the transport equation below:

$$\vec{\Omega} \cdot \vec{\nabla}\Psi(\vec{r},\vec{\Omega}) + \sigma_t(\vec{r})\Psi(\vec{r},\vec{\Omega}) = \sum_{\ell} \frac{2\ell+1}{4\pi}\sigma_{s,\ell}(\vec{r}) \sum_{m=-\ell}^{\ell} \Phi_{\ell,m}(\vec{r})Y_{\ell,m}(\vec{\Omega}) + q(\vec{r},\vec{\Omega}) \,, \qquad (1)$$

where $\Psi(\vec{r},\vec{\Omega})$ is the angular flux at position $\vec{r}$ and in direction $\vec{\Omega}$, $Y_{\ell,m}(\vec{\Omega})$ is the spherical harmonic function of degree ℓ and order m, $\Phi_{\ell,m}$ is the flux moment of degree ℓ and order m

$$\Phi_{\ell,m}(\vec{r}) = \int_{4\pi} d\Omega\Psi(\vec{r},\vec{\Omega})Y_{\ell,m}(\vec{\Omega}) \,.$$

It helps to introduce an operator notation for brevity:

$$L\Psi = H\Psi + q \,. \qquad (2)$$

where L is the streaming and total interaction operator, H the scattering operator. Typically, the transport equation is solved iteratively (index k)

$$L\Psi^{(k+1)} = H\Psi^{(k)} + q \,. \qquad (3)$$

Let us now introduce a decomposition into the collided and uncollided components of the angular flux:

$$\Psi = \Psi^u + \Psi^c$$

Then, Equation 2 can be re-cast as

$$\begin{cases} L^{RT}\Psi^u = q \\ L^{S_N}\Psi^c = H\Psi^c + H\Psi^u \end{cases} \tag{4}$$

where we have emphasized how the transort operator L will be solved in each case. RT denotes ray-tracing. S_N stands for discrete-ordinate method.
The first equation,

$$L^{RT}\Psi^u = q \quad \text{or} \quad \vec{\Omega}\cdot\vec{\nabla}\Psi^u(\vec{r},\vec{\Omega}) + \sigma_t(\vec{r})\Psi^u(\vec{r},\vec{\Omega}) = q(\vec{r},\vec{\Omega})\,,$$

can be re-written, along a given direction $\vec{\Omega}$ as

$$L^{RT}\Psi^u = q \quad \text{or} \quad \frac{d\Psi^u(\vec{r},\vec{\Omega})}{ds} + \sigma_t(\vec{r})\Psi^u(\vec{r},\vec{\Omega}) = q(\vec{r},\vec{\Omega})\,, \tag{5}$$

where $\vec{r} = \vec{r}_0 + s\vec{\Omega}$ and $\vec{r}_0$ is an origin point. Equation 5 can be solved analytical for simple geometries or semi-analytical, using ray-tracing, for more complicated geometries. The uncollided flux solution is thus available throughout the domain.
Once Ψ^u is known, the first collision source q^{1st}, can be computed:

$$q^{1st} = H\Psi^u = \sum_\ell \frac{2\ell+1}{4\pi}\sigma_{s,\ell}(\vec{r}) \sum_{m=-\ell}^{\ell} \Phi^u_{\ell,m}(\vec{r})Y_{\ell,m}(\vec{\Omega})$$

where Φ^u are the flux moments due to the uncollided flux. Then, one simply needs to solve the equation for the collided component Ψ^c

$$L^{S_N}\Psi^c = H\Psi^c + H\Psi^u = L^{S_N}\Psi^c = H\Psi^c + q^{1st}\,. \tag{6}$$

Note that Equation 6 is very similar to Equation 1. An iterative technique (Source Iteration) is often employed:

$$L^{S_N}\Psi^{c,(k+1)} = H\Psi^{c,(k)} + q^{1st}\,.$$

Equation 6 gives the collided component of the angular flux. Standard discrete-ordinates methods are used to solve Equation 6.
The above uncollided flux treatment addresses ray effect issues in the uncollided flux solution. However, ray effects can also occur in the collision flux solution, for instance, when there is a large distance from a small scatter source to a detector (e.g., neutrons streaming in a duct). TREATs fuel motion monitoring systems clearly falls into this category. In a manner analogous to the uncollided flux treatment, a last collided flux treatment can be employed to remove ray effects in a detector response. The last collision source treatment is simply expressed as the original transport equation, where the first-collided source due to the uncollided angular flux and the collided source are known (they come from the ray-tracing and the discrete-ordinate solution, previously described):

$$L^{RT}\Psi = H\Psi^c + H\Psi^u + q = q^{last}\,, \tag{7}$$

that is, given a first-collided and a multiple collided source (and the external source), one ray-traces the angular flux solution, from any point in the domain to the detector surface.

References