TEXAS A&M UNIVERSITY
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Research Memo

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To: Distribution
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Subject: Coupled ODE-PDE in MOOSE

1. Governing laws

Assume I have a coupled PDE-ODE system (for simplicity, the PDE is a time-dependent diffusion)

$$\frac{\partial \mathcal{U}}{\partial t} - \nabla^2 \mathcal{U} = \mathcal{V} \tag{1}$$

$$\frac{d\mathcal{V}}{dt} = -\mathcal{V} + \mathcal{U} \tag{2}$$

$\mathcal{V}$ depends on space but there are no spatial operators in the ODE.

2. FEM formulation

We seek $\mathcal{U}$ as a FEM solution

$$\mathcal{U}(\vec{r}) \approx \sum_j u_j \varphi_j(\vec{r}) \tag{3}$$

We test the diffusion equation against all basis functions $\varphi_i(\vec{r})$ and get (ignore boundary terms for simplicity):

$$M\frac{dU}{dt} + KU = V \tag{4}$$

where the entries of the mass and stiffness matrices are

$$M_{ij} = \int \varphi_i(\vec{r})\varphi_j(\vec{r}) \quad K_{ij} = \int \vec{\nabla}\varphi_i(\vec{r}) \cdot \vec{\nabla}\varphi_j(\vec{r}) \tag{5}$$

Vector U is simply

$$U = \begin{bmatrix} u_1 \\ \dots \\ u_i \\ \dots \\ u_N \end{bmatrix} \tag{6}$$

while the entries of V are

$$V = \begin{bmatrix} v_1 \\ \dots \\ v_i \\ \dots \\ v_N \end{bmatrix} \qquad \text{where } v_i = \int \mathcal{V}(\vec{r})\varphi_i(\vec{r}) \tag{7}$$

I stress that we do not need to expand $\mathcal{V}$ as a FEM solution. What are the equations satisfied by the v_i's ???? See below.

We simply test the ODE against each of the $\varphi_i(\vec{r})$ are integrate over space.

$$\int \varphi_i(\vec{r}) \frac{d\mathcal{V}}{dt} = -\int \varphi_i(\vec{r})\mathcal{V} + \int \varphi_i(\vec{r})\mathcal{U} \tag{8}$$

Some simple algebra yields ($\mathcal{U}$ is a FEM solution, hence the mass matrix):

$$\frac{dV}{dt} = -V + MU \tag{9}$$

Clearly, the only thing that matters are the v_i's, the entries of V.

3. How does this work in MOOSE?

I would like to have a vector V and to be able to add it to the residual of the diffusion equation.

There are as many v_i's are there are DOFs in the vector U. I never need to think of $\mathcal{V}$ at quadrature points as with a material or an auxvar. I just want a vector V. Is this possible?

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