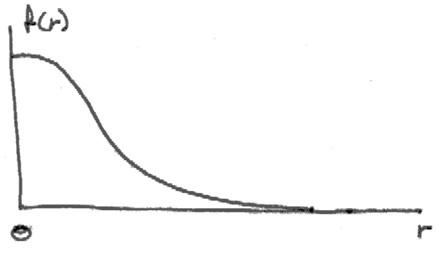
Expected Distribution of Target

If a target is known to be more likely at a certain position and the probability of being distance r from that position decays rapidly then $f(r)\,dA$ is the probability that the target is in area dA, r miles from origin O



May approximate situation with circular normal distⁿ

so $f(r) \propto e^{-r^2/2\sigma^2}$

and $$f(r) = (1/2\pi\sigma^2)\, e^{-r^2/2\sigma^2}$$

If target moves, distⁿ will change with t. Assuming speed is known but not direction ie u but not ϕ, and all directions are equally likely. $\therefore$ problem is to find new distⁿ $f(r,t)$ after time t.

$$f(r,t) = \frac{1}{2\pi\sigma^2} \exp\left[-(r^2+u^2t^2)/2\sigma^2\right] I_0(rut/\sigma^2)$$

where r = dist fm origin
u = speed of target
t = time.

Probability spreads outwards with time so that the target is most likely to be in an expanding ring about O

Target Detection

a) Instantaneous probability of detection

- continuous case.

γ - instantaneous probability density of detection

probability of detection during time t

$$p(t) = 1 - e^{-\gamma t}$$

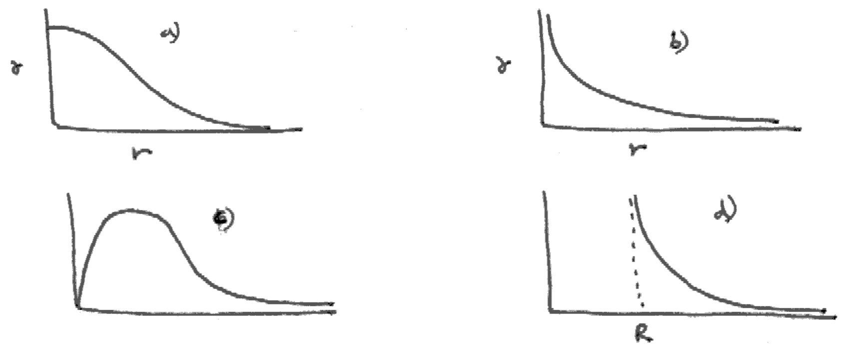
mean or expected time at which detection occurs

$$\bar{t} = \frac{1}{\gamma}$$

γ will be dependant on range $\therefore$ write

$$\gamma = \gamma(r)$$

ie may follow form of



evaluation of γ may be approximated by

$$\gamma = k/r^3 \qquad \{\text{inverse cube law of sighting}\}$$

as the target moves relative to the search so will the range change with time $\therefore$ probability of detection will be given by

$$p = 1 - \exp\left[- \int_{t_0}^{t_1} \gamma(r_t)\, dt \right]$$

the integral is a line integral along the relative path c, if w is relative speed and s = arc length of c from initial position

$$p_c = 1 - \exp\left[- \int_c \gamma(r)\, ds / w \right]$$

let $F(c) = \int_c \gamma(r)\, ds/w$ = sighting potential then

$$p_c = 1 - e^{-F(c)}$$

$F(c)$ has the property of additivity

Using co-ordinates (x, y) where x = lateral range

$$F(c) = \frac{1}{w} \int_{y'}^{y''} \gamma\left(\sqrt{x^2+y^2}\right) dy \qquad \begin{array}{l} y' = wt' \\ y'' = wt'' \end{array}$$

<u>Lateral range distribution</u> –

If obs and target are on straight courses at const sp for a long time before and after closest approach $p(x)$ is then a function of lateral range x. For the inverse cube law

$$p(x) = 1 - e^{-2m/x^2} \qquad \text{where } m = kh/w$$

h = height of obs
w = rel speed tgt

effective search width

$$W = \int_{-\infty}^{+\infty} p(x)\, dx$$

and wW = effective search rate

with N per unit area targets uniformly dist. the av <u>no</u> detected per unit time $N_0 = NwW$

where speed of targets is known but course is not w must be replaced by average $\bar{w}$ uniformly dist in track angle ϕ

$$\bar{w} = (1/2\pi) \int_0^{2\pi} w \, d\phi$$

In the case of the inverse cube law

$$\bar{W} = 2 \sqrt{2\pi k h} \,/\, w$$

$p(x)$ — expected distribution of target.

$b(x, z)$ — conditional cumulative probability of finding target at pt x given it is there.

$c(x, z)$ = time required per unit area to search at point x with intensity z

$$\frac{\partial}{\partial z}\left(p(x)\, b(x,z) - \ell\, c(x,z)\right) = \phi$$

for $z \geq \phi$ $\quad \ell \geq \phi$

References – Search Theory

Koopman B.O. (1956) The Theory of search, Parts I, II, III
Oper Res. 4 pp 324-348 and 508-531,
5 [1957] pp 613-623

Koopman B.O. 1979 An operations critique of detection laws
Oper Res 27 : 115-133

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①

Hoffman 1983 Isopod search behv

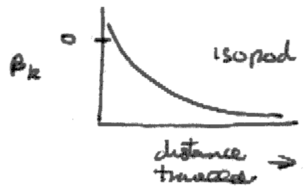
① Random elements in search behav

Recognizable systematic subunits of spatial patter.

1) First stage of search crudely resembles a spiral.

2) Later stages incorporate meanders which alternate with long straight sections.

β_k Integrated turning angle (number of full turns). calculated by adding the signed turning angle between segments of the search path.



Subunits of search arranged so that during a long search distance to starting point always remains small in relation to total search path length. — not due to external cues. Surroundings of the starting point are searched evenly in every direction.

Comparison with spatiotemporal pattern of a brownian search

Search pattern of different animals show "random" variations ie pattern is not strictly regular. within or between animals. To determine extent of randomness compare behav with a simple successful type of random search.
(see definitions of other authors p 86).

defined as

① same search strategy used for all regions entered.

② constant in time

③ Once a direction is chosen it continues for a certain distance. No preference for directions.

→ <u>discrete brownian search without correlations</u>

∴ can compare two characteristic parameters of search behaviour with predictions of a brownian search.

1) the probability distribution of the distance from a preceding point on the track. $R(s)$

2) the respective mean square value (MSD), $\overline{R^2(s)}$ as a function of the intervening search path length

Showed that the form of discrete brownian motion without correlations that best described behav. for search path segments of 1m to 7m had an average step length of 33 cm.
For segments < 1m step length 20mm and strong correlation between directions.

Both describe the tendency of the woodlouse to continue in a chosen direction for some time.

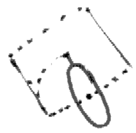
The tendency to sporadically return to the starting point may also be explained by the same search strategy if the directional constancy were not too high. Shown by a modification of Rayleigh's formula of directional statistics. (Mardia 1972) which is also based on the assumption that an animal moves in a brownian way. The probability P that an isopod after a search path of length S is at a distance from the starting point no greater than R is given by

$$P(s,R) = 1 - \exp - \left(R^2 / \overline{R^2(s)} \right) \qquad — 3.1$$

P is always > ϕ but in a long search with high directional constancy becomes v small for small values of R. Can be calculated if the MSD is known as a function of search path length.

If a search strategy is to be regarded as brownian search its MSD curve must become a straight line at longer path lengths s. Can then calculate with (3.1) minimal and maximal distances from the starting point with a P of 99% and compare with actual search paths.

To compare success of search behaviour compared with a brownian search



- need to calculate success of brownian search under conditions that resemble experimental situation used to measure success of search behaviour.

- depends in a complex way on the starting distance, search path length and directional constancy of the searching animal.

If the search path length is large enough ($s \gg r_0$) the success of any kind of brownian search first increases with increasing directional constancy and then falls. The probability of success of a discrete brownian search without correlations is maximal when the searcher moves in an approximately straight line over a distance corresponding roughly to that from which it can detect the target. - if directional constancy too low the animal searches only in the immediate vicinity of the ~~target~~ start point and ∴ does not detect the target. With a search in a completely straight line in a random direction

the target is detected with a probability of
$(\arcsin(a/r_0))/180$

Between these limits is a brownian search with a higher success – results of simulation show that it has the given directional constancy.

∴ to test whether search behaviour is more successful than that kind of brownian search it only has to be shown that it is more successful than the 'optimal' form of brownian search.

Necessary to know <u>detection radius a</u> – the greatest distance at which target can be detected.

② <u>Comparison with systematic search.</u>

During search for a distance S isoped at best can search an area of $2aS$ where a = detection radius
The ratio $2aS/A$ is taken as a measure of search intensity z in this region of A.

Probability of detection is not 1 even when within <u>a</u> but becomes v. high ~~when~~ with time or repeated visits.
∴ CPD – cumulative probability of detection at position x (burrow). depends on the total intensity z with which it is searching there.
This is a conditional cumulative probability as definition is based on assumption that the search is at the right place ie at point x. → <u>detection function</u> $b(x, z)$

$b(x, \emptyset) = \emptyset$ ie cannot detect if search has not occurred at position x (target).

Assume that probability of detecting target increases by $e\Delta z$ when search starts at target with the small intensity Δz, e is then the instantaneous probability density (w.r.t. z) of detection.

Returns of search to target position x assume no retention of information necessary for detection ∴ failure to detect during a revisit to target position will have same probability $(1 - e\Delta z)$. The probability $(1 - b(x, z+\Delta z))$ that the target will not be detected although searching at the right place with intensity $z + \Delta z$ is given by

$$1 - b(x, z+\Delta z) = (1 - b(x,z))(1 - e\Delta z)$$

∴ detection function takes form

$$b(x,z) = 1 - \exp(-ez) \qquad \text{for} \quad \begin{array}{l} e > 0 \\ z \geq 0 \end{array}$$

The probability of detection when searching in the right place is independent of the starting point of search.

<u>Prior information of target position</u>

For the isopod, the animal appears to have information that the burrow entrance is very probably located near the starting point.

∴ the probability density $p(x)$ that the burrow entrance is located at an arbitrary point x – <u>target density</u> has a maximum at $x = 0$ and falls rapidly with increasing distance.

$$p(x) = \frac{1}{2\pi w^2} \exp\left(\frac{-|x|^2}{2w^2}\right)\left[\frac{1}{cm^2}\right]$$

ie a two dimensional normal dist^n.

Comparison with spiral search.

A spiral search in which the surroundings are explored comprehensively but no more than once has a pitch twice as large as the maximal distance a at which the object can be detected.

Simply described as

$$r(t) = \frac{2a}{2\pi}\left(\delta(t) - \delta(0)\right)$$

where $r(t)$, $\delta(t)$ give the polar coordinates relative to the starting point at time t. From the relationship between velocity v, time t, and search path length S and $\delta(t)$, $r(t)$ can be determined.

For long times it is approximated by

$$r(t) = \sqrt{\frac{2avt}{\pi}}$$

probability of burrow detection is ϕ while $r(t) < r_0 - a$

if search intensity = 1 at the target posn ($r(t) = r_0 + a$) the probability of success is given by the detection function with $3 = 1$ ie $1 - \exp(-e.1)$

If a spiral search is executed then there exists a propability of failure to detect $(1 - (1 - \exp(-e.1))$ this probability would increase with the difficulty of executing a precise spiral search — once passed then continuing search will be in vain.

Best soln of a search problem

Only in rare cases is there a unique form of search path of a given overall length that will maximize the effectiveness of the search for any search problem.

But under general assumptions it is possible to calculate an optimal search plan $d^\circ(x,t)$ that gives the intensity $z = d^\circ(x,t)$ with which the animal should search at point x if the total duration of the search must not exceed time t. (~~et~~ Koopman 1946, Stone 1975)

The cumulative (w.r.t z) probability density (w.r.t x) $q(x)$ that the animal will find the target at point x is equal to the product of the conditional cumulative probability $b(x,z)$ of finding the target at point x, given it is there, and the probability density $p(x)$ that the target actually is in point x.

$$q(x) = p(x)\, b(x,z) = p(x)\, b(x, d(x,t))$$

For the probability of success of the search plan d we have

$$P[d] = \int_X q(x)\, dx = \int_X p(x)\, b(x, d(x,t))\, dx$$

where X = entire region within which the target can be located.

The total effort that the animal can expend according to search plan $d(x,t)$ within the head duration is limited by the condition

$$t = C[d] := \int_X c(x, d(x,t))\, dx \qquad — 3.2$$

where $c(x,z)$ is the time required per unit area to search at point x with intensity z.
For a search path $s = vt$ and by definition of $z = \frac{2aS}{A}$

$$c(x,z) = \frac{t}{2aS} z =: kz \left[\frac{s}{cm^2}\right] \quad \text{where } k = \frac{1}{8}\left[\frac{s}{cm^2}\right]$$

$v = 2\,cm/s$

— 3.3

For a successful search the problem can be described as:

For a given target density and detection function one must find the search plan $d^\circ(x,t)$ that maximizes the probability of success during the given search duration t.
Since cost of search and detection probability are independent of search location it is sufficient to optimize the search plan locally at each point.
To determine whether it is advantageous for an animal to increase the intensity of search z in point x by the amount Δz the corresponding change of the effectiveness function

$$p(x)\, b(x,z) - \ell c(x,z)$$

must be checked. It relates the increase in success

$$\Delta z\, \partial / \partial z\, p(x)\, b(x,z)$$

to the weighted increase of the search costs

$$\ell \Delta z\, \partial / \partial z\, c(x,z)$$

the weight ℓ is the marginal rate of return, because for an optimal search it is the ratio of the marginal increase in probability of detection to the increase in cost

$$(\ell = (\partial/\partial z\ p(x)\ b(x,z)) / (\partial/\partial z\ c(x,z))$$

in these points where the animal searches at all (Stone, 1975 pp 84)

To calculate the optimal search plan, according to these considerations one need only solve the fundamental equation for z

$$\frac{\partial}{\partial z}\left(p(x)\ b(x,z) - \ell\, c(x,z)\right) = \phi \qquad \text{— 3.4}$$

for $z \geq 0$ and $\ell \geq \phi$

That is, the effectiveness function is maximized at every point by varying the search intensity z for $z \geq \phi$. Using eqn 3.2 the value of ℓ is then fixed such that the duration of the optimal search so found does not exceed the prescribed value t. The optimal values of search intensity z as a function of search duration establish the optimal search plan $d^{\circ}(x,t)$

Optimal search plan for isopod is obtained:

$$d^{\circ}(x,t) = \begin{cases} \frac{1}{e}\left[\left(\frac{et}{k\pi w^2}\right)^{\frac{1}{2}} - \frac{r^2}{2w^2}\right] & \text{for } r \leq R_M(t) \\ \phi & \text{for } r > R_M(t) \end{cases} \qquad \text{— 3.5}$$

$$\text{with } R^2_M(t) = 2w^2\left[\frac{et}{k\pi w^2}\right]^{\frac{1}{2}} \text{ and } r := |x| \qquad \text{— 3.6}$$

The isopod should thus search only within a small circular region, the radius R_M of which slowly increases in time. Search further away is unprofitable — the increase in probability of success is too small in comparison with the additional time required. (see 3.4)

Within R_M the guard should search with decreasing intensity from the center to the periphery. This is a consequence of the decreasing probability that the entrance is located at increasing distances from the starting point of the search.

Effectiveness of search behavior compared with a systematic search

The cumulative probability CPD that an isopod will find its burrow by time t using a particular form of search depends on the intensity $d(x_B, t)$ with which the animal has so far searched at the position x_B where the burrow is in fact located. The effectiveness of the search thus satisfies the equation $CPD = b(x_B, d(x_B, t))$

The measure of success of search can be the cumulative frequency $CFD(t, r_0)$ with which an isopod displaced from its burrow by distance r_0 manages to find its way back by time t.

If the burrow entrance is initially at the distance r_0 from the isopod the probability of finding the entrance by time t given the most effective type of search is

$$CPD(t, r_0) = \begin{cases} 1 - \exp\left[\dfrac{r_0^2}{2w^2} - \left(\dfrac{et}{k\pi w^2}\right)^{\frac{1}{2}}\right] & \text{for } r_0 \leq R_M(t) \\ \phi & \text{for } r_0 > R_M(t) \end{cases}$$

Found no significant difference between search effectiveness found experimentally and optimal search plan calc above.

Correlation $c(i,j)$ between direction of locomotion in two segments of the search path Δx_i and Δx_j can be defined as follows (Tchen 1952 Random flight with multiple partial correlations. J. Chem Phys 20: 214-217)

$$c(i,j) := \overline{\cos \measuredangle(\Delta x_i, \Delta x_j)}$$

where $\measuredangle(\Delta x_i, \Delta x_j)$ is the angle enclosed by Δx_i and Δx_j

The partial correlation (of track directions) between immediately adjacent path segments is defined as ~~fo~~ follows.

$$Pc(1) := c(i, i+1)$$

The partial correlation between one segment and the next but one is given by

$$Pc(2) := \frac{c(i, i+2) - Pc(1)^2}{1 - Pc(1)^2}$$