भारतीय प्रबंध संस्थान बेंगलूर
INDIAN INSTITUTE OF MANAGEMENT BANGALORE

Certificate Program in Business Analytics and Intelligence (BATCH 9)

Module 4: Prescriptive Analytics

Instructions

1. This is a **take-home** assignment. You are free to discuss the assignment questions with your classmates. However, you are not allowed to copy the answers from other students.
2. Answer all questions; there are 10 questions in the assignment.
3. **Show all work and give adequate explanations to get credit. The mathematical equations of the solutions should be clearly mentioned and Outputs must be interpreted and clearly explained. Software outputs without mathematical explanation will NOT be given credit.**
4. Codes run in tools need not be shared. Excel/other software outputs alone shall not be treated as complete answers
5. **Encircle or underline your final answer for each part.**
6. Use significance of 0.5 (α = 0.05) where ever required.
7. **Completed assignment should be submitted in Moodle by 16th December 2018 23.55 Hours. Assignments will not be accepted after 16th December 2018, the grades of such students will be marked Incomplete (I) in the grade sheet.**

Question 1 (20 points)

Appukuttan Menon is the co-founder and CEO of Appukuttan Halva (AH) with headquarters in Kuttanad, Kerala. AH produces 4 different types of Halva: 1. Fruit and Nut (FN), 2. Death by Halva (DH), 3. Kuttanad Halva (KH) and 4. Kerala Black Halva (KBH). 5 different main ingredients are used for making these 4 varieties of Halva. Quantity of Ingredients required per 1 kg of Halva is given in table 1.

Table 1. Ingredients (in Kg) required for different Halva (Per Kg)

Halva variety	Ingredient (in Kg)				
	Maida	Sugar	Fruits and Nuts	Coconut Milk	Ghee
Fruit and Nut (FN)	0.3	0.4	0.4	0.2	0.5
Death by Halva (DH)	0.2	0.5	0.3	0.3	0.5
Kuttanad Halva (KH)	0.4	0.8	0.1	0.6	0.4
Kerala Black Halva (KBH)	0.4	0.8	0.1	0.5	0.8

The profit from FN, DH, KH and KBH per kilogram are INR 45, 50, 60 and 10 respectively. The maximum daily demand for FN, DH, KBH and KRH are 250 Kg, 120 kg, 90 Kg and 550 Kg respectively. Appukuttan Menon is a big fan of the Japanese lean management concept and followed Just in Time (JIT) procurement. All the raw material necessary for the daily production are delivered on the day of production at 6.00 am and AH maintained no safety stock. The suppliers of the ingredients are located in Coimbatore, which is about 260 Km from Kuttanad.

In the evening of 3rd November 2018, all the political parties of Kerala called for a state wide bandh on 4th November 2018 starting from midnight 12 to 12 Noon against World Sports committee's (WSC) refusal to exclude "Kilikiti" at the Worlds Sports Meet (WSM). Kilikiti is a national sport of Tuvalu, a small island nation in Pacific Ocean. The bandh was called to express solidarity with the people of Tuvalu at the time of this crisis according to Kurian Vadake Tharavad, the leader of Democratically United Left-Right Front (DULRF). The supplier called Appukuttan Menon to express his inability to deliver the ingredients due to bandh on 4th November as per their contract. Since the bandh was closer to Diwali, one of the most famous festivals of India, Appukuttan wanted to ensure that the production is not stopped.

The ingredients used by Appukutan Halva are of a specific brand and there was only one supplier in Kuttanad. The supplier in Kuttanad told Mr Menon that he has 500, 450, 75, 300 and 200 (all in Kgs) of Maida, Sugar, Fruits and Nuts, Coconut Milk and Ghee and was ready to supply at the same price as his regular supplier. Appukauutan remembered his prescriptive analytics class at IIM Bangalore (IIMB) and formulated a LP problem to decide on the optimal production plan. Since AH supplied the products after 12 Noon every day, he did not see any change in the demand for the products.

Question 1.1 (5 Points)

Formulate a linear programming problem to maximize the profit generated for Appukuttan Halva. Clearly define the decision variables and constraints. Solve the formulation using Excel Solver (LINDO or LINGO).

Question 1.2 (1 point)

Write the quantity of KBH to be produced on 4th November with reasons.

Question 1.3 (2 points)

Appukuttan Menon's friend has offered to supply him 50 Kg of Maida of the same brand. Should he use this additional Maida? What will be the change in the objective function value? Clearly state your reasons.

Question 1.4 (2 points)

One of the Halva shops in Kuttanad has placed order for 20 Kg of Kuttanad Halva. Should AH accept this order of KH? If yes, what should be the minimum profit on KH?

Question 1.5 (2 points)

AH plans to give a discount on DH that will result in reduction in profit by INR 10. Will this change the current optimal production plan? Explain

Question 1.6 (3 points)

Mr Menon would like to increase the profit on all his products by 20% simultaneously. State whether this will change the current optimal solution? What will be the new profit value?

Question 1.7 (2 points)

Which ingredient among the 5 ingredients has the highest impact on the current optimal solution (that is change in the availability of this ingredient will change the optimal solution and profit)?

Question 1.8 (3 points)

If the profit on DH is reduced to 42 (from 50) and profit on KH is reduced to 36 (from 60) simultaneously, will the current optimal solution change? Clearly state your arguments.

Question 2 (20 Points)

An oil tanker owned by ABS Company has five compartments that can hold up to 2,700, 2,800, 1,100, 1,800, and 3,400 litres of petrol, respectively. The company must deliver three types of fuel (super, regular, and unleaded) to a distribution outlet. In case the company the outlet's demand is not met ABC Company has to pay a penalty for the shortage. The demands, penalty per litre short, and the maximum allowed shortages for the three types of petrol are given in Table 1 below. Each compartment of the tanker can carry only one type of petrol. However, more than one compartments can be used to carry the same kind of petrol. ABS would like to plan delivery such that the shortage costs are minimized and is interested in using an appropriate mathematical programming model to make the decision. It would like to keep the model generic enough for it to work for different combinations of demands, shortages allowed and their penalties.

TABLE 1

Type of Petrol	Demand (litres)	Penalty per Litre Short (Rs.)	Maximum allowed Shortage (litres)
Super	2,900	10	500
Regular	4,000	8	500
Unleaded	4,900	6	500

a) Define the variables needed for the mathematical programming model that can help ABS in its decision. (3 points)

b) List the constraints and label them clearly. (9 points)

c) State the objective. (2 points)

d) Suppose the outlet increases the penalty by 10% for shortages above 250 litres. State the modifications required in the formulation (parts a-c) above to incorporate the new penalty structure. (3 points)

e) Now suppose the increased penalty of 10% is applicable for all units short whenever the shortage is over 250 litres. State the modifications required in the formulation (parts a-c) above to incorporate this penalty structure. (3 points)

Question 3 (20 Points)

The Sidon company produces four types of alloys which we label 1, 2, 3 and 4. Each type of alloys (per gram) requires 3 different types of metals as shown below:

	Metal 1 (in grams)	Metal 2 (in grams)	Metal 3 (in grams)	Selling Price
Alloy 1	0.2	0.4	0.4	200.8
Alloy 2	0.2	0.6	0.2	125.4
Alloy 3	0.3	0.3	0.4	300.3
Alloy 4	0.5	0.5	0	215.5

During the coming month, Sidon can acquire up to 2000 grams of metal 1, 3000 grams of metal 2 and 500 grams of metal 3. The unit costs are INR 5 per gram for metal 1, 5 per gram for metal 2 and 7 per gram for metal 3. The demand for alloys 1, 2, 3 and 4 are 1000, 2000, 500 and 1000 respectively. The company wants to maximise its monthly profit.

Let x1, x2, x3 and x4 alloys (in grams) 1, 2, 3 and 4 manufactured. The corresponding formulation is given by:

Maximize 186x1 + 111x2 + 281x3 + 188x4

Subject to constraints

0.2x1 + 0.2x2 + 0.3x3 + 0.5x4 <= 2000 (metal 1 constraint)

0.4x1 + 0.6x2 + 0.3x3 + 0.5x4 <= 3000 (metal 2 constraint)

0.4x1 + 0.2x2 + 0.4x3 <= 500 (metal 3 constraint)

x1 <= 1000 (alloy 1 demand)

x2 <= 2000 (alloy 2 demand)

x3 <= 500 (alloy 3 demand)

x4 <= 1000 (alloy 4 demand)

x1, x2, x3 and x4 >= 0

The Excel Solver output is provided below:

Microsoft Excel 14.0 Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
A2	x1	0		186		1E+30
B2	x2		0	111	29.5	18
C2	x3		0	281	1E+30	59
D2	x4		0	188	1E+30	188

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
A5	Metal 1	950		2000	1E+30	1050
A6	Metal 2	1550		3000	1E+30	1450
A7	Metal 3	500		500	100	300
A8	Alloy 1	0		1000	1E+30	1000
A9	Alloy 2	1500		2000	1E+30	500
A10	Alloy 3	500		500	750	250

A11	Alloy 4	1000		1000	2100	1000

ANSWER Questions 3.1 to 3.8 based on the Excel Output (NOT BY SOLVING THE FORMULATION)

Question 3.1 (3 points)

Write the optimal production plan (in terms of number of grams of alloys 1, 2, 3 and 4 manufactured). What is the objective function value?

Question 3.2 (3 points)

200 additional grams of metal 3 can be imported and it would cost Sadon INR 20000. Should Sadon import this additional metal 3. State your answers clearly.

Question 3.3 (2 points)

What is the impact on the objective function value if the demand for metal 3 is increased by 200 units?

Question 3.4 (3 points)

One of the customers of Sedon has placed order for alloy type 1. To maintain a long term relationship Sedon would like to accept this order. How much should be profit on alloy 1 so that Sedon can accept the order.

Question 3.5 (2 points)

One of the main competitors of Sedon goes bankrupt and using this opportunity, Sedon plans to increase the profits earned from different alloys to 279, 166.5, 421.5 and 282 respectively. State whether these changes to the profit will the current optimal production plan.

Question 3.6 (2 points)

Government imposes a restriction that Sedon can sell only 1000 grams of alloy 2. What will be the impact of this restriction on the optimal solution and the optimal profit?

Question 3.7 (2 points)

In addition to the restriction imposed in question 3.6, the demand for alloy 3 increases to 1000 (from 500 grams). Do you expect the current basis to change due to these changes?

Question 3.8 (3 points)

An alloy 5 can be manufactured using 0.5 units of metal 1, 0.4 units of metal 2 and 0.1 unit of metal 3 per gram of alloy 5. The profit earned by alloy 5 is 220 per gram and the demand is 1500 grams per month. Should Sedon manufacture this new alloy?

Question 4 (15 Points)

The best and brightest from top colleges and universities in the UK vie for the lucrative and demanding position of analyst at the Lester and Myers bank's London office. After graduating with a Bachelor's degree, analysts would join the bank to work in any number of areas providing analysis, research, and support skills to the MBA professional staff. The requirement for analysts in the London office varies from month to month. The following table provides the projected requirement of analysts for next six months in the office.

Month	Analyst Requirement
September	110
October	105
November	90
December	65
January	80
February	90

Shortfall

If the London office falls short of the required analysts, then analysts from other European offices are transferred temporarily to take care of the shortfall for that month. Due to cultural and language differences, a transferred analyst is expected to work at 80% capacity of local analysts. As a result, the total number of transferred workers over the six-month period should be no more than 20% of the total number of London-based analysts over the period.

Cost

The cost-to-company of an analyst in each office is 6000 pounds per month. However, if analysts are transferred from other European offices, the cost-to-company increases to 8000 pounds per month. The increase in cost is attributed to the hotel expenses and allowances that the bank pays to the visiting analysts.

Recruitment

The bank recruits on college campuses in UK anywhere between zero and two times during a year. The recruitment, if it takes place, is scheduled in April and/or December. For the analysts recruited in April, the joining date is decided by the bank and could be either 1st September or 1st October, as per the requirement. On the other hand, for analysts recruited in December, the joining date is either 1st January or 1st February. Every recruitment effort, if undertaken, costs 20,000 pounds regardless of the number of analysts recruited.

All fresh recruits are on probation for two months. After the completion of probation, every fresh recruit's performance is assessed to decide if he/she be given a permanent position or let go. In the past, 5% of all fresh recruits have been asked to leave at the end of their probation period. This trend is likely to continue.

As of March 1, the company has 60 permanent analysts and 20 analysts on probation, whose review is due at the end of March. Write a formulation that can be used to determine the optimal recruitment plan. Define all variables precisely and label all constraints clearly.

Question 5 (10 points)

In the year 1240 B.C, Egyptian Pharaoh Ramses II commissioned a granite causeway (elevated road) to be built from the gates of the temple at Karnak down to the east bank of the Nile River. The causeway will commemorate the Pharaoh's triumphs at the battle of Kadesh, so he insists it be completed before the scheduled visit of several kings from that region. This means that Imhotep, the Chief architect of Public Works of Ramses II, has exactly six months to finish the project. The project is fairly simple: Imhotep needs to transport a total of 7,000 carefully carved blocks from the granite quarries of Lower Egypt, downstream to the current site at Karnak. The stonemasons at Karnak do not need all 7,000 blocks at once; rather, the monthly requirements of the granite blocks are shown in table 1:

Table 1: Monthly requirements of granite blocks

Month	1	2	3	4	5	6
Blocks Required	700	700	1,000	1,200	2000	1400

The requirements of the stonemasons must be met. Imhotep has a choice of two granite quarries from which to purchase his blocks: the quarry at Deir El Medinah, which can produce a maximum of 800 blocks a month for the next six months, or the quarry near Fayum, which can produce a maximum of 1400 blocks a month during months 1, 2, 3 and 6. Unfortunately, the annual flooding of the Nile River, which will happen during months 4 and 5, makes the Fayum

quarry inaccessible during that time. Hence production during those months is zero (at Fayum). In addition, while the stonemasons at Karnak do not mind having more blocks than they need in a given month, they cannot have more than 1,200 blocks "left over" at the end of a month, due to space considerations.

Imhotep needs to decide how many blocks to have delivered to Karnak each month. He also needs to decide on which quarry to order those blocks from - he is free to order blocks from both quarries in the same month, if he wishes. His objective is to minimise the cost of the entire project. The labour costs involved in moving a single block of granite can vary, according to the month. In months 1 through 3, Imhotep must rely primarily on the Pharaoh's military to do the transporting: they will charge the modern day equivalent of Rs 15000 per block moved from either quarry to Karnak. In months 4, 5 and 6, Imhotep does not have to contract with the military; the flooding of the Nile will leave all local farmers idle, they will move as many blocks as he wants for the equivalent of Rs 7500 per block. In addition to the transportation costs, Imhotep must consider the actual purchase price of the blocks: the quarry at Deir El Medinah charges Rs 200,000 per block it produces, while the Fayum quarry charges Rs 225,000 per block it produces. Finally, it costs Rs 1000 per block "left over" at the end of the month in inventory carrying costs.

Assuming that blocks produced during a particular month can be transported and used during the same month, formulate Imhotep's problem as a linear programming model. Clearly write all the decision variables and constraints and label them.

Question 6 (10 points)

Clara International Marketing (CIM) assists companies to advertise their products in television channels.

The advertisement cost depends on the average television rating points (TRP). The programs, average

TRP, cost of advertisement per minute are shown in Table 15.23.

Table 15.23 TRP and cost of advertisement per minute

	Program				
	Cricket Matches	Other sporting event	Hindi Serials	Hindi Movies	English News

					Channels
Average TRP	4.2	3.5	2.8	2.5	0.2
Cost of advertisement per minute in (INR)	120,000	85,000	70,000	60,000	25,000

A customer of CIM has sent a proposal to promote their new product and has set the following goals:

1. The gross rating points (GRP) should be at least 100, where GRP = TRP × Number of Spots of 1 minute advertisement.

2. The GRP through sports events should be at least 20.

3. The GRP for English news channels should not exceed 5.

The total budget for advertisement is INR 2 million. Develop a goal programming model that can be used for solving the problem; treat all three goals as equally important. Solve the problem using Excel Solver. The duration of the advertisement should be in integer multiple of minutes.