भारतीय प्रबंध संस्थान बेंगलूर
INDIAN INSTITUTE OF MANAGEMENT
BANGALORE

Certificate Program in Business Analytics and Intelligence
BATCH 9

Module 5: Stochastic Models
(Marketing and Retail Analytics)

Instructions:

1. This is a **take-home** assignment. You are free to discuss the assignment questions with your classmates. However, you are **not allowed to copy** the answers from other students.
2. Answer all questions; there are 10 questions in the assignment. You have to download the required case studies from the Moodle
3. **Show all work and give adequate explanations to get credit. The mathematical equations of the solutions should be clearly mentioned.**
4. Encircle or underline your final answer for each part.
5. Use significance of 0.5 ($\alpha = 0.05$) where ever required.
6. **Completed assignment must be submitted in Moodle by 31 January 2019: 23.00 Hours. Assignments will not be accepted after 31st January 2019, the grades of such students will be marked Incomplete (I) in the grade sheet.**

Question 1 (20 Points)

Read the case "Using Markov Chain to Forecast Sales Booking", and answer to the following questions:

1. Refer to the monthly transition matrices in the given data set. Calculate the one-step transition matrix for 3 months and aggregate transition probability matrix for WSES.

2. Check whether the aggregate data follows a Markov Chain?

3. Based on the aggregate data provided in the data set can you calculate the time it will take for an opportunity in stage B to get converted into 'contract signing'?

4. Jack was quite excited by seeing the results. He asked Ben what is the pipeline for the current month by different stages? Ben mentioned the pipeline by stages as below. Can you calculate the forecast which Jack can give to his stakeholders?

   Stage A: $ 1 Bn
   Stage B: $ 1.8 Bn
   Stage C: $ 1.6 Bn

Question 2 (5 points)

An automobile insurance company charges vehicle insurance premium depending on whether the customer had accidents during the previous two years. Each year insurance holders have an accident with a probability of 0.10. Accidents in different years are independent events. The next year's premium is determined according to the following rules:

1. If an insurance holder had accidents in each of the past two years, they pay additional INR 1000.
2. If an insurance holder had accident in the current year, but not the previous year, they pay an additional INR 600.
3. If an insurance holder had an accident last year but not the current year, they pay an extra INR 300.
4. If an insurance holder had no accidents in the past two years, they are given a discount of INR 200.

2.1 Formulate the above problem using Markov chain. Clearly identify the states and the state transition probability matrix (TPM) (5 points).

2.2 A customer in the current year has not committed any accident in the past two years. What will be the expected additional insurance premium for this customer in year t+3? (5 points)

Question 3 (10 points)

Bouygues Telecom (BT) is a mobile phone service provider based in Nepal that provides several value added services such as mobile data, video mail etc. The market is highly competitive due to the presence

of several mobile companies from China. The customers of BT are categorized in to different states as listed below.

State 1 Customer churn that generated no revenue/profit

State 2 Customer churn that generated Nepali Rupee (NR) 200 profit per month on average (customer uses the service only for incoming calls)

State 3 Customer state that generated NR 300 profit per month on average

State 4 Customer state that generated NR 400 profit per month on average

State 5 Customer state that generated NR 600 profit per month on average

State 6 Customer state that generated NR 800 profit per month on average

The transitions between different states are shown in Table 4:

Table 4. Transition probability matrix (based on monthly data)

	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0.05	0.05	0.90	0	0	0
4	0.10	0.05	0	0.80	0.05	0
5	0.20	0.10	0	0.05	0.60	0.05
6	0.10	0.20	0	0	0	0.70

3.1 Customer in which state has the longest expected life (that is longest life before churn to either of the churn states). (3 points)

3.2 If a customer is in state 6, calculate the probability of eventual absorption in state 2? (2 points)

3.3 Round off the expected duration obtained in question 2.1 to the nearest integer and calculate the expected value of the customer life time value of the customers in state 6 using all the information that you have from questions 2.1 and 2.2 Use a discount factor of 0.99. (5 points)

Question 4 (25 Points)

Monthly revenue generated by a software company depends on its NPS Score. The state space and the corresponding revenue generated are shown in tables 5 and 6.

Table 5. State Space

State	Description
1	NPS of less than 10%
2	NPS score between 10 and 20%
3	NPS score between 20 and 30%
4	NPS Score more than 30%

Table 6. Revenue generated in millions of rupees per month

State	1	2	3	4
Revenue	200	250	300	500

To improve the NPS Score, the company uses the following strategy:

Strategy 1 = Free maintenance support for one year, leading to 10% reduction in revenue in that state.

Strategy 2 = Free Maintenance Support for two years, leading to 20% reduction in revenue in that state.

Strategy 3 = Do nothing (no change in the revenue)

State Transitions under different strategies are shown in Tables 7, 8 and 9.

Table 7. State transition under strategy 1 (one year free maintenance):

	1	2	3	4
1	0.5	0.25	0.15	0.1
2	0	0.75	0.15	0.1
3	0	0.1	0.8	0.1
4	0	0	0.2	0.8

Table 8 State transition under strategy 2 (2 years free maintenance):

	1	2	3	4
1	0.4	0.30	0.15	0.15
2	0	0.75	0.10	0.15
3	0	0.05	0.8	0.15
4	0	0	0.10	0.90

Table 9. State transition under strategy 3 (Do nothing):

	1	2	3	4
1	0.6	0.2	0.1	0.1
2	0.1	0.7	0.1	0.1
3	0	0.2	0.7	0.1
4	0	0	0.3	0.7

4.1 Calculate the long term revenue generate for the policy (2, 2, 1, 3) using policy iteration algorithm. (5 points)

4.2 Is policy (2, 2, 1, 2) is better than policy (2, 2, 1, 3)? (5 points)
4.3 Use linear programming formulation to find the optimal policy (5 points)
4.4 Calculate the optimal policy for 4 months planning horizon using value iteration algorithm. (10 points)

Question 5 (25 Points)

Read the case, "Customer analytics at Flipkart.com", and answer the following questions:

1. Discuss whether the churn problem can be modelled as a Markov chain? What are the assumptions made while modelling customer churn as a Markov chain?
2. How can churn be defined for e-commerce companies such as Flipkart? In **Exhibit 6**, the recency state 13 is identified as the churn state. Comment on the use of state 13 as churn state.

3. Using **Exhibit 7** (Markov chain with recency states), Ravi and his team wanted to find out on average how many months customers in each non-absorbing state (states 1 to 12) take to reach the churn state (state 13).
4. Given a hypothetical case of 1,000 customers in state 1, 1,000 customers in state 2, and 1,000 customers in state 3, predict the distribution of the customers after a period of 4 months from now? (Use **Exhibit 4**.)
5. Ravi wants to evaluate Flipkart's relationship with a customer by calculating the expected life time value (CLV) for infinite horizon. Assuming Flipkart is risk neutral, and willing to make decisions based on expected net present value, calculate the CLV of a customer using **Exhibit 7** and information given below:
   Discount rate d = 0.2 and the reward in recency state (state 1) is 1000 and states 2 to 12 is -200 (interpreted as cost of promotion) and state 13 is 0.
6. Karan and Arun are two Flipkart customers who made their first purchase in April 2013. Karan purchased products on Flipkart every month, except in August 2013, whereas Arun made his next purchase only in September 2013. From the months to churn (lifetime) calculated in Q2, calculate the estimated remaining lifetime for both Karan and Arun at the end of September 2013. (Round all decimals to the higher integer.)
7. The Analytics team built a Recency–Monetary Discrete Time Markov Chain model to predict revenue from existing customers using the information in **Exhibit 8**. The state space of the DTMC using recency monetary information is described in **Exhibit 9**. The Transition Probability Matrix (TPM) of the Recency–Monetary DTMC is uploaded on Moodle as a separate spreadsheet. Using the distribution of existing customers as of October 2013 (provided in the spreadsheet) for a particular segment and information in **Exhibit 10** predict the number of customers in November 2013 and calculate the estimated revenue from this customer segment.
8. Using **Exhibit 14**, identify states where an intervention could be undertaken to reduce churn.

Question 6 (10 points)

A new product company's business is described as in one of the two states, 1. Successful product launch and 2. Unsuccessful product launch. The company has the following actions for each of the two states (Table 10):

Table 10 States and corresponding actions

State i	Action k	Description of action
1 (successful product launch)	1 (k = 1)	Do not promote
	2 (k = 2)	Promote
2 (unsuccessful product launch)	1 (k = 1)	Do not research
	2 (k = 2)	Research

The transition probabilities for each action and the corresponding rewards are shown in the following table (Table 11):

Table 11.

State (i)	Action (k)	Transition Matrix $p_{ij}^{(k)}$		Reward
		1	2	
1 (i = 1)	1	0.6	0.4	20
	2	0.8	0.2	30
2 (i = 2)	1	0.3	0.7	- 2
	2	0.8	0.2	-12

6.1 Use policy iteration algorithm to calculate the value function for states 1 and 2 for the policy (1, 1) using a discount rate of 0.9. Check whether the policy (1, 2) is better than policy (1,1) in the long run. Clearly state all the steps. (8 points)

6.2 Find optimal policy using linear programming formulation

Question 7: (5 points)

A sample data of 20 failures (time between failures) of a laptop batteries (measured in months) are shown below:

Table 12. Time between failures

9	11	8	12	6	4	14	11	6	10
16	8	6	7	4	6	7	13	8	16

Assume that the data follows a Poisson process (that is, the time between failures follow an exponential distribution).

Question 7.1 (2 points)
If the manufacturer provides 12 months warranty, what is the expected number of failures during the warranty period?

Question 7.2 (3 points)
The manufacture would like to ensure they can meet the demand for batteries 90% of the times from the stock. How many batteries they should stock to meet this condition for 2 year period?

END OF ASSIGNMENT