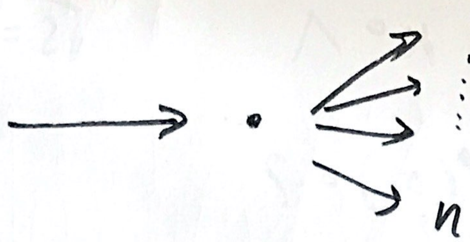


E_{min} del proiettile

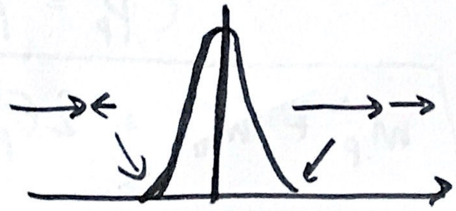
$$K_{min} = K_{soglia} = \frac{\left(\sum_f m_f\right)^2 - (m_i + m_b)^2}{2 m_b}$$

S.i. $\longrightarrow$ S.f.

$$\sqrt{s_i} \geq \sqrt{s}_{MIN,\, s.f.} = \sum_f m_f$$

$$\sqrt{s}\left[\longrightarrow \; \leftarrow\right] > \sqrt{s}\left[\longrightarrow \; \rightarrow\right]$$

fissata $E_{proiettile}$



$E_{proiettile}$ t.c. si ottiene s.f.

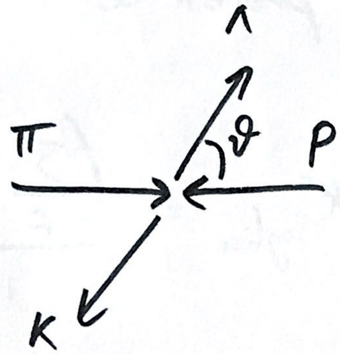
$\boxed{EX}$

$$\pi^- p \to K^0 \Lambda \qquad \sqrt{s} = 3\ \text{GeV}$$

Ⓐ nel CdM $p^*(\pi) = ?$

$p^*(\Lambda) = ?$

CdM



$$\begin{cases} p^*(\pi) = p^*(p) \\ p^*(\Lambda) = p^*(K) \end{cases}$$

$$\sqrt{s}\Big|_{s.i.} = \sqrt{(E_p^* + E_\pi^*)^2 - |\underbrace{\vec{P}_p^* + \vec{P}_\pi^*}_{=\vec{0}}|^2}$$

$$\sqrt{s} = E_p^* + E_\pi^*$$

$$\sqrt{s} = \sqrt{E_p^{*2} + E_\pi^{*2} + 2E_p E_\pi}$$

$$= \sqrt{m_p^2 + p_p^{*2} + m_\pi^2 + p_\pi^{*2} + 2E_p^* E_\pi^*}$$

$$(p_p^* = p_\pi^*)$$

$$= \sqrt{m_p^2 + m_\pi^2 + 2E_p^* E_\pi^* + 2p_\pi^{*2}}$$

$$s = m_p^2 + m_\pi^2 + 2E_p^* E_\pi^* + 2p_\pi^{*2}$$

$$p_\pi^{*2} = \frac{s - m_p^2 - m_\pi^2 - 2E_p^* E_\pi^*}{2}$$

$$\sqrt{s} = E_p^* + E_\pi^* \Rightarrow E_p^* = \sqrt{s} - E_\pi^*$$

$$\Rightarrow p_\pi^{*2} = \frac{s - m_p^2 - m_\pi^2 - 2E_\pi^*(\sqrt{s} - E_\pi^*)}{2}$$

$$s - m_p^2 - m_\pi^2 - 2E_\pi^*\sqrt{s} + 2E_\pi^{*2} - 2p_\pi^{*2} = 0$$

$$p_\pi^{*2} = E_\pi^{*2} - m_\pi^2$$

$$s - m_p^2 - m_\pi^2 - 2E_\pi^*\sqrt{s} + \cancel{2E_\pi^{*2}} - \cancel{2E_\pi^{*2}} + 2m_\pi^2 = 0$$

$$s - m_p^2 + m_\pi^2 - 2E_\pi^*\sqrt{s} = 0$$

$$\Rightarrow E_\pi^* = \frac{s - m_p^2 + m_\pi^2}{2\sqrt{s}}$$

$$= \frac{9 - 0.938^2 + 0.140^2}{2 \cdot 3} =$$

$$= 1.356 \text{ GeV}$$

$$E_\pi^* = 1.356 \text{ GeV}$$

$$\Rightarrow \quad P_\pi^* = \sqrt{E_\pi^{*2} - m_\pi^2} = 1.349 \text{ GeV}$$

$$E_\Lambda^* = \frac{s - m_K^2 + m_\Lambda^2}{2\sqrt{s}} = 1.67 \text{ GeV}$$

$$\Rightarrow \quad P_\Lambda^* = \sqrt{E_\Lambda^{*2} - m_\Lambda^2} = 1.24 \text{ GeV}$$

(B) $\vec{P}_p = \vec{0}$

$$\sqrt{s}\,\Big|_{s.i.,\,LAB} = \sqrt{(E_\pi + m_p)^2 - P_\pi^2} =$$

$$= \sqrt{E_\pi^2 + m_p^2 + 2E_\pi m_p - P_\pi^2}$$

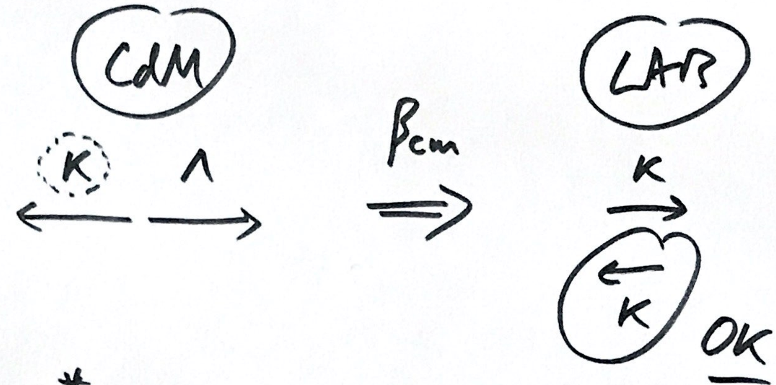
$$= m_\pi^2$$

$$= \sqrt{m_\pi^2 + m_p^2 + 2E_\pi m_p}$$

$$E_\pi = \frac{s - m_\pi^2 - m_p^2}{2m_p} = 4.31 \text{ GeV}$$

$$P_\pi = \sqrt{E_\pi^2 - m_\pi^2} \cong 4.31 \text{ GeV}$$

$$\beta_{cm} = \frac{|\vec{P}_{TOT}|}{E_{TOT}} = \frac{P_\pi}{E_\pi + m_p} = 0.82$$



$$\beta_K^* < \beta_{cm}$$

$$p_K^* = 1.24 \text{ GeV} \quad \text{(da prima)}$$

$$E_K^* = \sqrt{p_K^{*2} + m_K^2} = 1.34 \text{ GeV}$$

$$\beta_K^* = \frac{p_K^*}{E_K^*} = 0.93 > \beta_{cm}$$

il K e' emesso (anche) all'indietro nel LAB

$\boxed{EX}$

$$\gamma p \to \pi^0 p$$

$$\xrightarrow{\gamma} \overset{p}{\cdot}$$

$m_\gamma = 0$

(A) E_γ^{MIN}

$$K_{\gamma,soglia} = \frac{(m_{\pi^0} + m_p)^2 - m_p^2}{2 m_p}$$

$m_\gamma = 0 \Leftrightarrow E = K$

$$= 0.145 \text{ GeV}$$
$$= 145 \text{ MeV}$$

Alla soglia calcolare:

(B) $$\beta_{cm} = \frac{|\vec{P}_{TOT}|}{E_{TOT}} = \frac{\overset{= E_\gamma}{p_\gamma}}{E_\gamma + m_p} = \frac{E_\gamma}{E_\gamma + m_p} = 0.134$$

$$\gamma_{cm} = \frac{1}{\sqrt{1 - \beta_{cm}^2}} = 1.009$$

(C) $E_\gamma^* = ?$

$$E_\gamma = \gamma_{cm}\left(E_\gamma^* + \beta_{cm} \overset{p_\gamma^*}{\overset{\parallel}{E_\gamma^*}}\right) =$$
$$= \gamma_{cm} E_\gamma^* (1 + \beta_{cm})$$

$$\Rightarrow E_\gamma^* = \frac{E_\gamma}{\gamma_{cm}(1 + \beta_{cm})} = 0.127 \text{ GeV}$$
$$= 127 \text{ MeV}$$

$$\sqrt{s}\Big|_{s.i.,\,LAB} = \sqrt{(E_\gamma + m_p)^2 - p_\gamma^2} =$$

$$= \sqrt{\cancel{E_\gamma^2} + m_p^2 + 2E_\gamma m_p - \cancel{p_\gamma^2}}$$

$$= \sqrt{m_p^2 + 2E_\gamma m_p} = 1.073 \text{ GeV}$$

Nel <u>CdM</u>

$$p_\gamma^* = p_p^* = \sqrt{E_p^{*2} - m_p^2} = \sqrt{(\sqrt{s} - E_\gamma^*)^2 - m_p^2}$$

$$= E_\gamma^* \qquad\qquad \sqrt{s} = E_p^* + E_\gamma^*$$

$$\Leftrightarrow \quad E_\gamma^* = \sqrt{(\sqrt{s} - E_\gamma^*)^2 - m_p^2}$$

$$\Leftrightarrow \quad E_\gamma^{*2} = (\sqrt{s} - E_\gamma^*)^2 - m_p^2$$

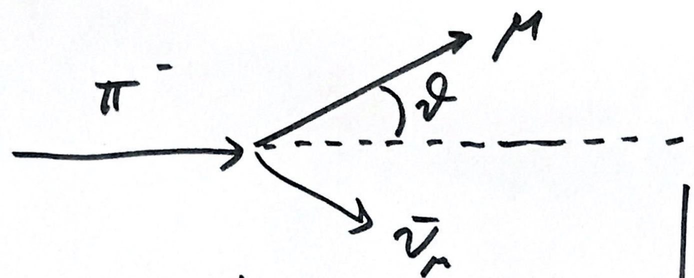
$$\Leftrightarrow \quad \cancel{E_\gamma^{*2}} = (\sqrt{s})^2 + \cancel{E_\gamma^{*2}} - 2E_\gamma^* \sqrt{s} - m_p^2$$

$$\Rightarrow \quad E_\gamma^* = \frac{s - m_p^2}{2\sqrt{s}} = 127 \text{ MeV}$$

$\boxed{\text{EX}}$ $\quad \pi^-$ $\qquad p_\pi = 2\ \text{GeV}$

$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$$

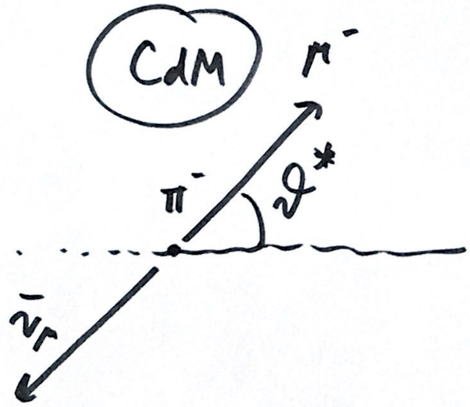
(A) ϑ_{MAX} del μ nel LAB

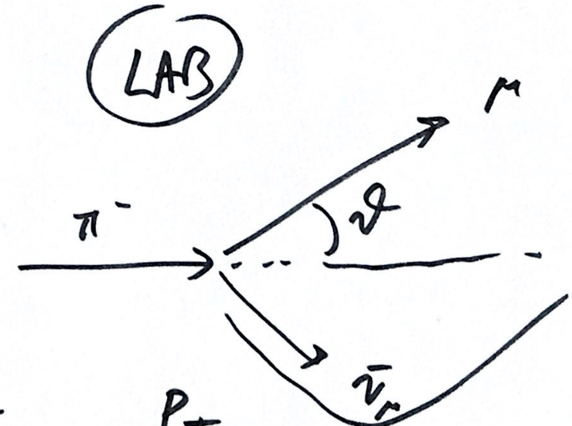


$$\tan \vartheta_{MAX} = \frac{\beta^*_\mu}{\gamma_{cm}\sqrt{\beta_{cm}^2 - \beta^{*2}_\mu}}$$

IN CORRISP. DI

$$\cos\vartheta^* = -\beta^*/\beta_{cm}$$





$$\beta_{cm} = \beta_\pi = \frac{p_\pi}{E_\pi} = \frac{p_\pi}{\sqrt{p_\pi^2 + m_\pi^2}} = 0.9975$$

$$\gamma_{cm} = \frac{1}{\sqrt{1-\beta_{cm}^2}} = 14.32$$

$\pi^- \to \mu^- \bar{\nu}_\mu$

($m_\nu = 0$)

MONO ENERGETICO NEL CdM!

$$p_\mu^* = p_\nu^* = \frac{m_\pi^2 - m_\mu^2}{2 m_\pi} = 31 \text{ MeV}$$

$$\Rightarrow E_\mu^* = \sqrt{p_\mu^{*2} + \underbrace{m_\mu^2}_{105 \text{ MeV}}} = 109 \text{ MeV}$$

$$\beta_\mu^* = \frac{p_\mu^*}{E_\mu^*} = 0.28 < \beta_{cm} = 0.9975$$

$$\tan \vartheta_{MAX} = \frac{\beta_\mu^*}{\gamma_{cm} \sqrt{\beta_{cm}^2 - \beta_\mu^{*2}}}$$

$$\Rightarrow \vartheta_{MAX} = \tan^{-1}\left[\;\cdots\; \right] = 1.2^\circ$$

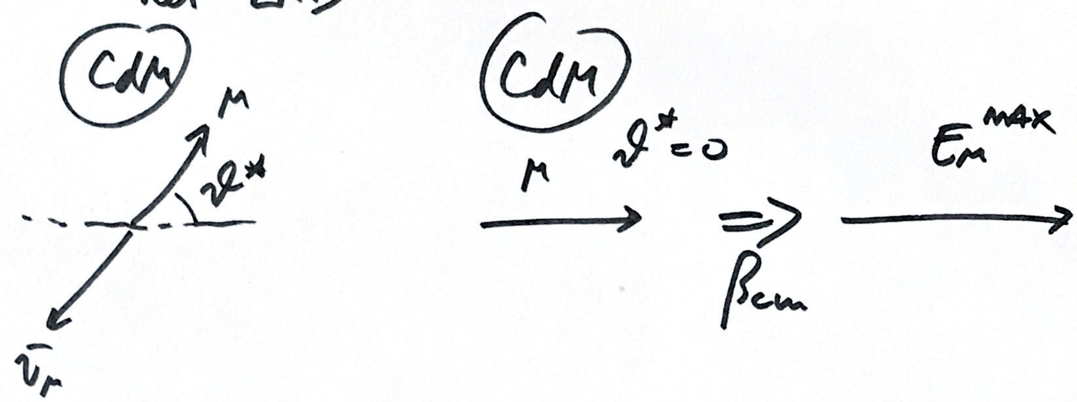
$$E_\mu(\vartheta_{MAX}) = \gamma_{cm}\left(E_\mu^* + \beta_{cm}\, p_\mu^* \cos\vartheta_\mu^* \right)$$

in ϑ_{MAX}: $\cos\vartheta_\mu^* = -\dfrac{\beta_\mu^*}{\beta_{cm}}$

$$\beta_{cm} \cos\vartheta_\mu^* = -\beta_\mu^*$$

$$E_\mu(\vartheta_{MAX}) = \underset{14.32}{\gamma_{cm}}\left(\underset{109\ MeV}{E_\mu^*} - \underset{0.28}{\beta_\mu^*}\ \underset{31\ MeV}{p_\mu^*}\right) = 1440\ MeV$$

(B) E_μ^{MAX} nel LAB



E^{MAX} si ottiene quando $\vartheta_\mu^* = 0$

$\vartheta_\mu = 0$

$$E_\mu = \gamma_{cm}\left(E_\mu^* + \beta_{cm}\, p_\mu^* \underbrace{\cos\vartheta_\mu^*}_{=1}\right)$$

$$= \underset{14.32}{\gamma_{cm}}\left(E_\mu^* + \underset{0.9975}{\beta_{cm}}\ p_\mu^*\right) = 2004\ MeV$$

$$\overset{\nu}{\longleftarrow} \quad \overset{\nu}{\longrightarrow}$$

$$\underset{=1}{\beta_\nu^*} > \beta_{cm}$$

(c) E_μ^{MAX} percorso medio del μ prima di decad.

$$\tau_0 = 2.2 \cdot 10^{-6}\ s$$

$$L = \beta\gamma c\tau_0$$

$$E_{MAX} = 2.004\ GeV$$

$$\Rightarrow P_{MAX} = \sqrt{E_{MAX}^2 - m_\mu^2} = 2001\ MeV$$

$$\beta = \frac{P_{MAX}}{E_{MAX}} = 0.9985$$

$$\gamma = \frac{E_{MAX}}{m_\mu} = 19.1$$

$$L = \beta\gamma c\tau = 0.9985 \cdot 19.1 \cdot 3 \cdot 10^8 \cdot 2.2 \cdot 10^{-6} =$$

$$= 12587\ m = 12.6\ Km$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\beta = \frac{P}{E} \Rightarrow \gamma = \frac{E}{m}$$

$$E^2 = p^2 + m^2$$

$\boxed{\text{EX PER CASA}}$

$\gamma \to e^+ e^-$ $\qquad m_{e^+} = m_{e^-} = m_e = 0.511$ MeV

se e sotto che condizioni e' possibile?

$\pi^0 \to \gamma\gamma$ $\qquad m_{\pi^0} = 135$ MeV

~~sotto~~ per quali P_{π^0} e' possibile avere che i due fotoni sono emessi entrambi a $\vartheta = 0$