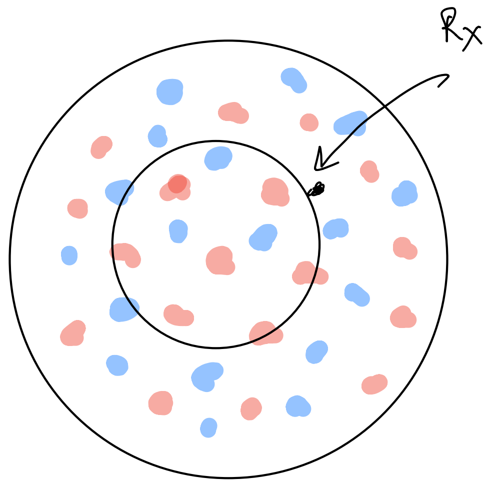
Energia legame per nucleoni

termine di volume $\propto A$.

termine di superficie $\propto A^{2/3}$

$$B(Z,A) = a_V A - a_S A^{2/3}$$



Repulsione di Colomb.

$$\rho(r) = \frac{Ze}{V} = \frac{Ze}{\frac{4\pi}{3} R^3}$$

Per $R < R_x$

$$E_{coul.} = \int_0^{R_x} d^3r \, \rho(r) \, V(r)$$ potenziale di Coulomb.

$$V(r) = \frac{1}{4\pi} \frac{q(r)}{r}$$

$$q(r) = \rho(r) \cdot Vol(r) = \rho \cdot \frac{4\pi}{3} r^3$$

$$E_{coul} = \int_0^{R_x} d^3r \, \rho \frac{1}{\cancel{4\pi}} \rho \frac{\cancel{4\pi}}{3} r^3 \frac{1}{r} = \frac{\rho^2}{3} \int_0^{R_x} d^3r \, r^2$$

$$d^3r = \sin\theta d\theta d\varphi \, r^2 dr$$

$$= \frac{\rho^2}{3} 4\pi \int_0^{R_x} dr \, r^2 r^2 \propto \rho^2 R_x^5$$

Su tutto il nucleo.

$$E_{coul} \propto \rho^2 R^5 \simeq \frac{Z^2}{R^6} R^5 = \frac{Z^2}{R}$$

$$R_N = r_0 A^{1/3}$$ $r_0 = 1.1$ fm costante estratta dai dati

$$E_{coul} \propto \frac{Z^2}{A^{1/3}} = Z^2 A^{-1/3}$$

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3}$$

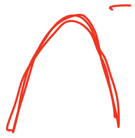
Correzione di energia cinetica

$$K_{tot} = A\langle \bar{K} \rangle = 20\ \text{MeV}\left(A + \frac{5}{9}\frac{(A-2Z)^2}{A}\right)$$

en. cinetica media da impulso di Fermi

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3} - a_F \frac{(A-2Z)^2}{A}$$

$-aZ^2 + -$



Estratti dal fit ai dati sperimentali:

$a_V \simeq 16$ MeV

$a_S \simeq 18$ MeV

$a_C \simeq 0.7$ MeV

$a_F \simeq 23$ MeV

La Formula di Bethe-Weizsäcker

Caratteristiche principali

1/ $A = \text{cost}$ isobari nuclidi con circa le stesse masse. ma #protoni diversi.

$B(Z, A=\text{cost}) =$ parabola in funzione di Z

$M(Z,A)$

massa nuclide

$$M(Z,A) = Z\,M({}^1_1H) + (A-Z)\,m_N - B(Z,A).$$

$\Rightarrow$ esiste un Z_{min}.

2/ sperimentalmente differenze sistematiche rispetto a questa espressione.

Z	$A-Z$	A	ΔB
pari	pari	pari	$+\delta$
		dispari	$\emptyset$
dispari	dispari	pari	$-\delta$

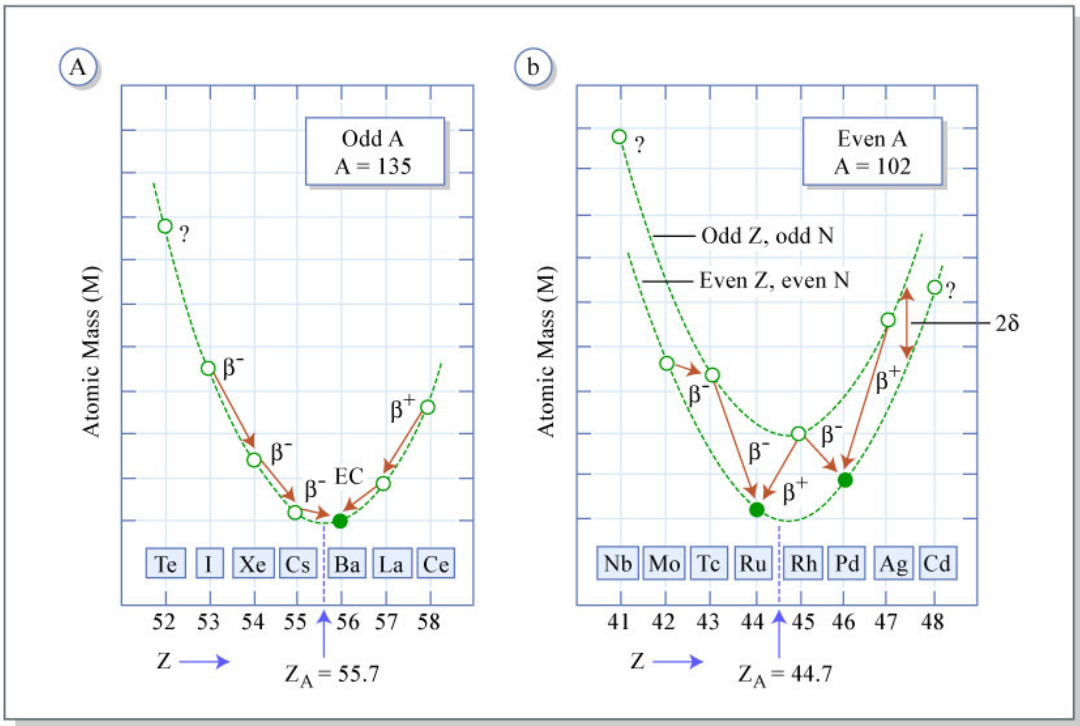
$$\delta = a_\delta A^{-1/2} \qquad a_\delta \simeq 11 \text{ MeV}$$

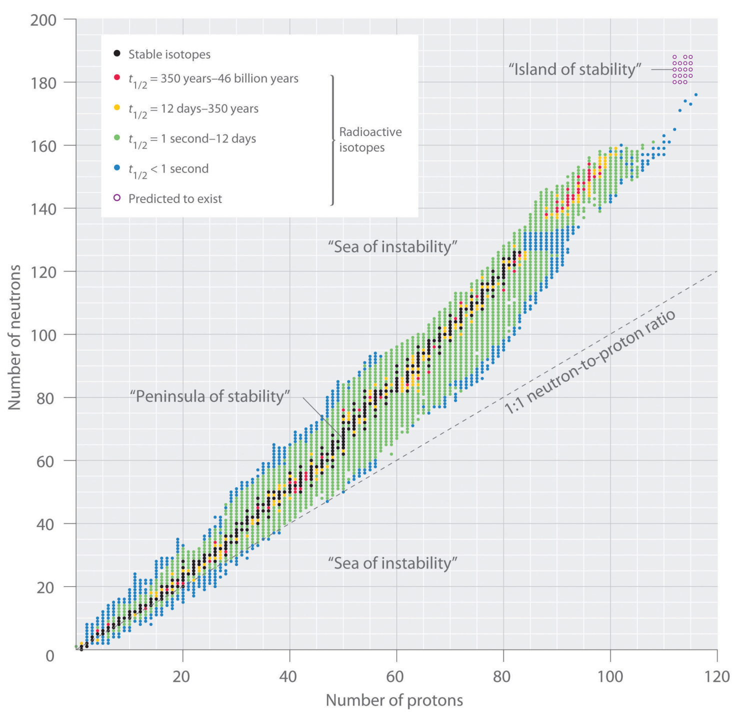
$$M(Z,A) = Z M({}^1H) + (A-Z) M_N - B(Z,A) + \left\{ \begin{array}{c} +\delta \\ \emptyset \\ -\delta \end{array} \right.$$

A fissato

$$\left.\frac{\partial M}{\partial Z}\right|_{A=cost} = 0 \implies Z_{min} \simeq \frac{A}{2}\,\frac{1}{1+0.0076\,A^{2/3}}$$

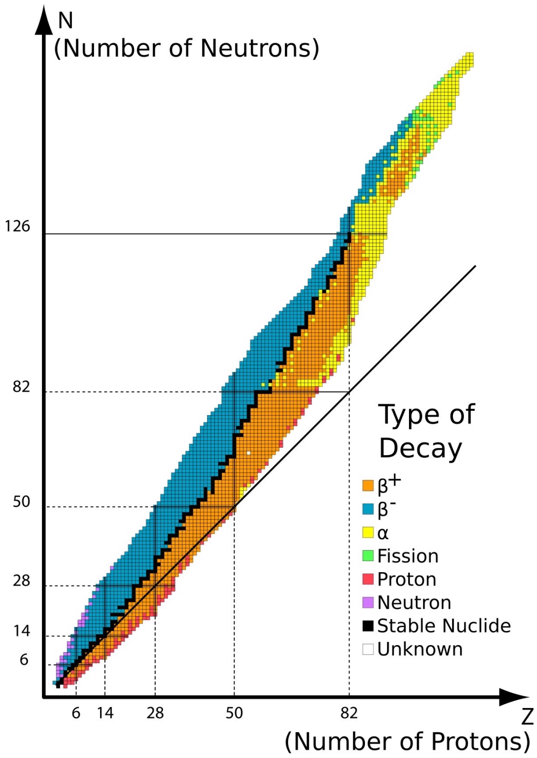
$$Z \leq \frac{A}{2}$$





$$B(Z,A) = a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3} - a_F \frac{(A-2Z)^2}{A}$$

$$\frac{B(Z,A)}{A} = a_V - a_S A^{-1/3} - a_C Z^2 A^{-4/3} - a_F \left(\frac{A-2Z}{A}\right)^2$$



Decadimento α

$$^{A}_{Z}N \to {}^{A-4}_{Z-2}X + \alpha$$

$K_\alpha \simeq 4-8$ MeV. sperimentalmente misurate.

Nel rif. solidale con N.

$$\alpha \longleftarrow N \longrightarrow X \qquad \vec{P}_\alpha = -\vec{P}_X$$

$$M_N = E_\alpha + E_X = M_\alpha + M_X + K_\alpha + K_X$$

$$Q = M_N - M_X - M_\alpha = K_\alpha + K_X$$

$M_\alpha = 3.7$ GeV.

$K_\alpha = 4-8$ MeV Non relativistica

$$Q = K_\alpha + K_X = \frac{P_\alpha^2}{2m_\alpha} + \frac{P_X^2}{2m_X} = \frac{p^2}{2m_\alpha} + \frac{p^2}{2m_X}$$

$$= \frac{p^2}{2m_\alpha}\left(1 + \frac{m_\alpha}{m_X}\right)$$

$m_\alpha \ll m_X$ X: $A-4$ nucleon

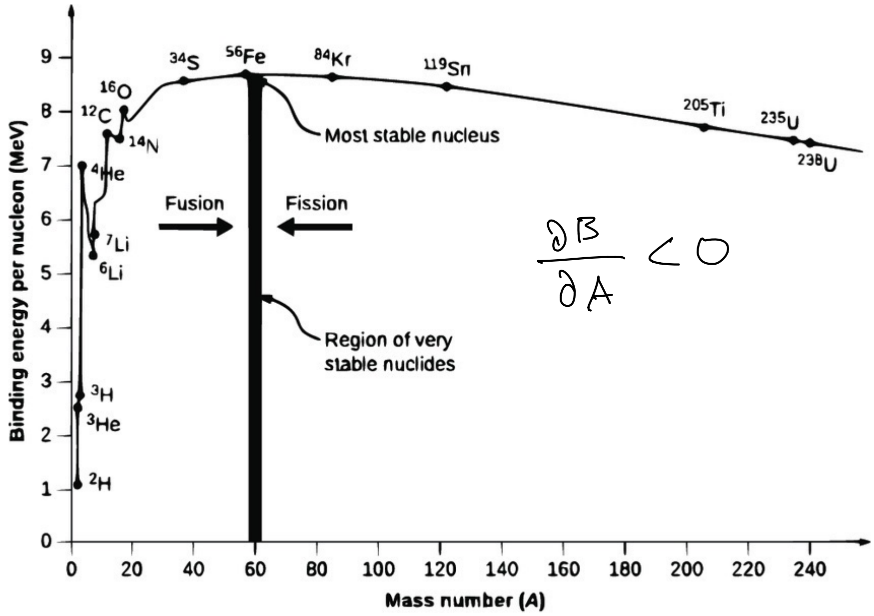
α: 4 $A \geq 160$ - -

$$\frac{p^2}{2m_\alpha} = \frac{Q}{1+\frac{m_\alpha}{m_X}} \simeq Q\left(1 - \frac{m_\alpha}{m_X}\right)$$

Q va quasi tutta in particella α. $\Rightarrow$ X praticam. fermo.

$$Q = M_N - M_X - M_\alpha = -B_N(A,Z) + B_X(A-4, Z-2) + \underbrace{B(4,2)}_{\simeq\, 28\ \text{MeV}}$$

processo avviene se $Q > 0$.



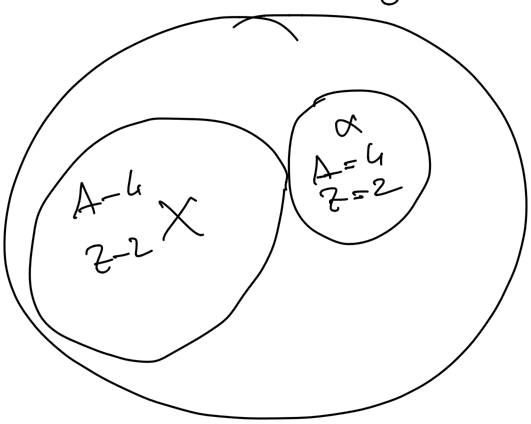
$$\frac{\partial B}{\partial A} < 0$$

α: $B \simeq 28$ MeV. nuclide molto legato.

$$\frac{B}{A} \simeq 7.1\ \text{MeV}$$

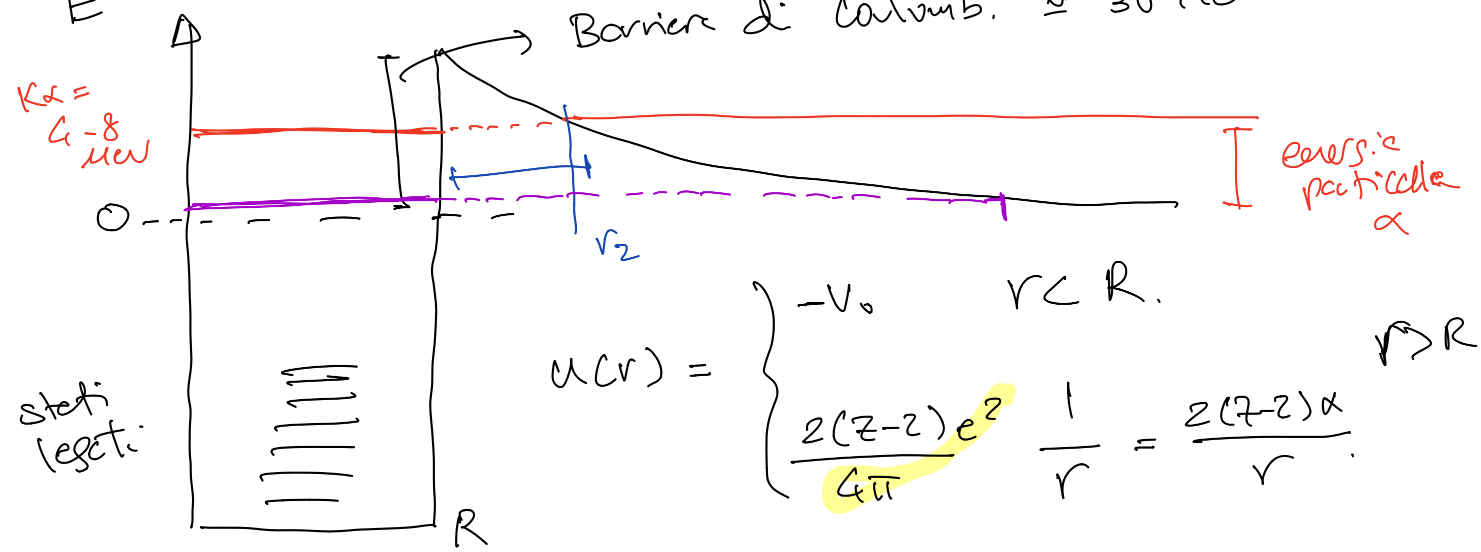
Modello di Gamow

$^{A}_{Z}N$



$$^{A}_{Z}N \longrightarrow {}^{A-4}_{Z-2}X + \alpha$$

α intrappolata nella buca di potenziale generata da $^{A-4}_{Z-2}X$



$$u(r) = \begin{cases} -V_0 & r < R \\ \dfrac{2(Z-2)e^2}{4\pi}\,\dfrac{1}{r} = \dfrac{2(Z-2)\alpha}{r} & r > R \end{cases}$$

$R = r_0 A^{1/3}$

$$U(R) = \frac{2(Z-2)\,\alpha}{r_0 A^{1/3}}$$

$\hbar = 200$ MeV. fm.

$A = 200 \Rightarrow R \simeq 7$ fm.

$$U(R) = \frac{2(Z-2)\,\alpha}{7 \text{ fm}} \simeq 30 \text{ MeV Coulomb.}$$

$$U(r_2) = \frac{2(Z-2)\,\alpha}{r_2} = K_\alpha \Rightarrow r_2 = \frac{2(Z-2)\,\alpha}{K_\alpha}$$

$r_2 \simeq 20-50$ fm. per $K_\alpha = 8-4$ MeV.

Q value piccole per $Z \gtrsim 60$ appena dopo il picco di $\frac{B(Z,A)}{A}$.

$Q \simeq 0 \Rightarrow K_\alpha \simeq 0.$

$\Rightarrow$ effetto tunnel difficile.

$\Rightarrow$ tempo di transizione $\longrightarrow \infty$.

Effetto tunnel con barriere finite