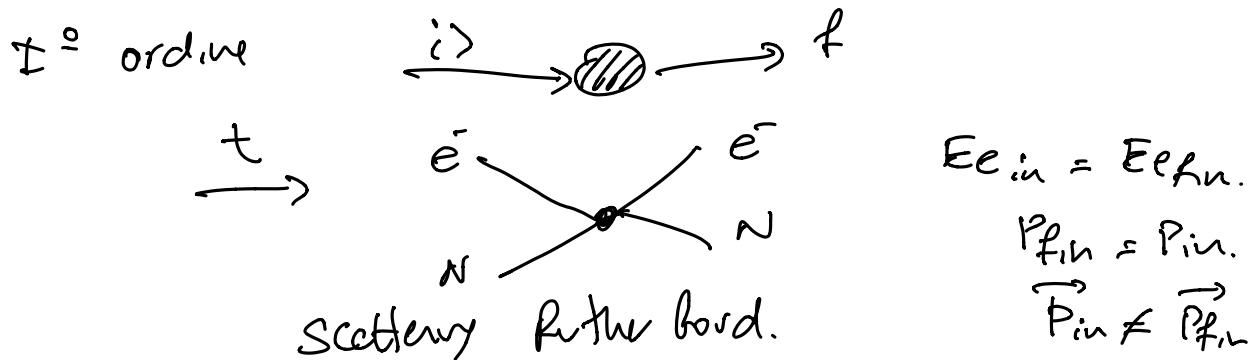


Elementi di Matrice al II ordine

$$T(i \rightarrow f) = 2\pi |M_{fi}^{(2)}|^2 \rho(E) \Big|_{E_f = E_i}$$

$$M_{fi}^{(2)} = -i V_{fi}^{(2)} \quad V_{fi}^{(2)} = V_{fi} + \sum_{n \neq i} \frac{V_{fn} V_{ni}}{E_i - E_n}$$

Nelle regole d'oro $\sum (E_f - E_i) \Rightarrow E_{fin} = E_i$



II° ordine

$$M_{fi} \sim \sum_{n \neq i} \frac{V_{fn} V_{ni}}{E_i - E_n}$$

i) Stato iniziale

f) Stato finale

n) Stati finali accessibili:

per definizione: $E_n \neq E_i \neq E_f$

$$i) \xrightarrow{V_{ni}} n) \xrightarrow{V_{fn}} f)$$

$E_i \neq E_n$: Energia non s. conservata.

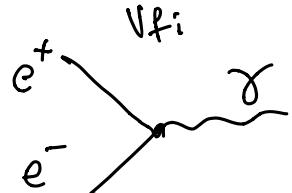
$E_n \neq E_f$: non s. conservata l'energia.

$$E_f = E_i$$

Stati con $E_n \gg E_i$ sono stevoviti
 $E_n \ll E_i$

E sequi.:

\overrightarrow{t}



$$S_{in} = 2m_e c^2$$

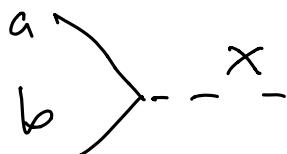
$$S_{fin} = \emptyset$$

$$|i\rangle = |e^+ e^-\rangle$$

$$|f\rangle = |\gamma\rangle$$

$S_{(ff - E_i)}$
non valido.

In generale

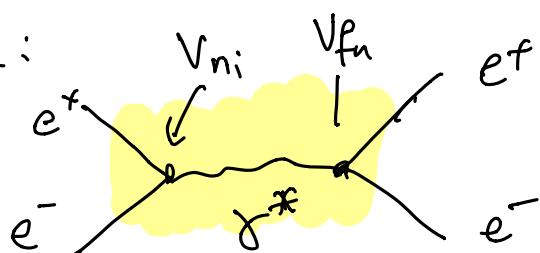


$$S_{in} \geq m_a c^2 + m_b c^2$$

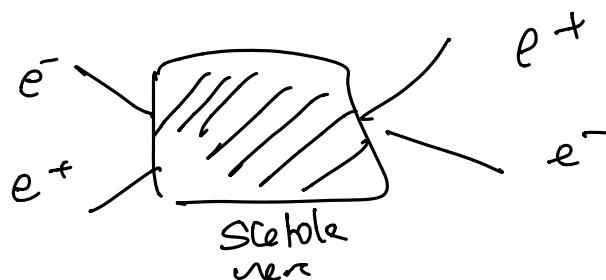
$$S_{fin} = m_X c^2$$

$S_{in} > S_{fin}$. non avviene.

Al II^{-o} ordine:



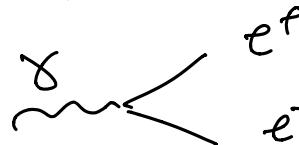
γ^o potere virtuale



S_{in} solo processi al I^{-o} ordine.



$|i\rangle \rightarrow |n\rangle$



$|n\rangle \rightarrow |f\rangle$

al II^{-o} ordine

stato fisico
on shell



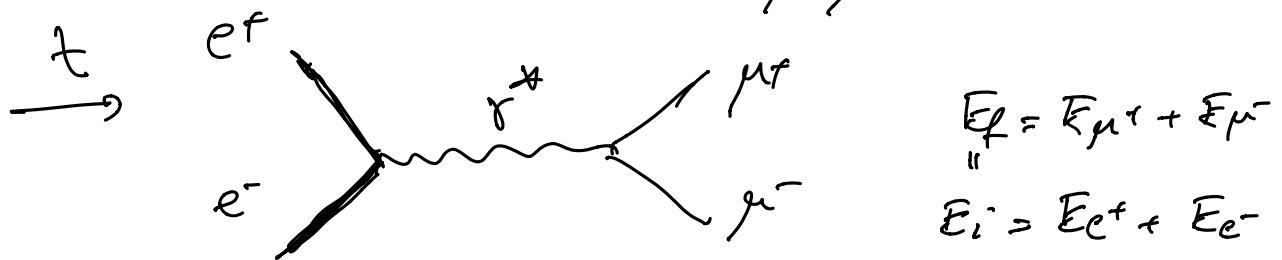
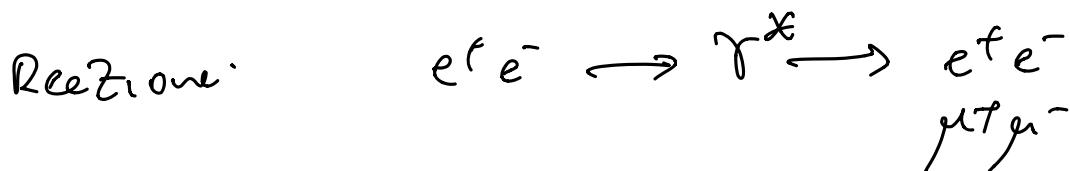
on shell

Abbiamo violato la conservazione dell'energia
per creare γ^* e ricouvertirlo in e^+e^-

Principio di indeterminazione di Heisenberg:

$$\Delta E \cdot \Delta t \approx \hbar$$

$$\Delta t \leq \frac{\hbar}{\Delta E} \quad \text{Violatione di } \Delta E \text{ possibile per } t < \Delta t = \frac{\hbar}{\Delta E}$$



$$E_F = E_{\mu^+} + E_{\mu^-}$$

$$E_i = E_{e^+} + E_{e^-}$$

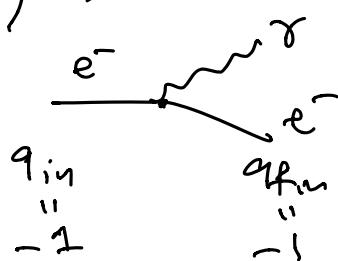
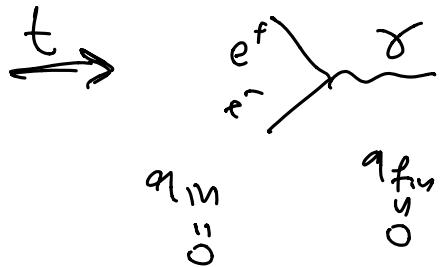
potere come misurazione delle forze EM.

Interazione EM tra part. caricate avviene per lo scambio di un fotone.

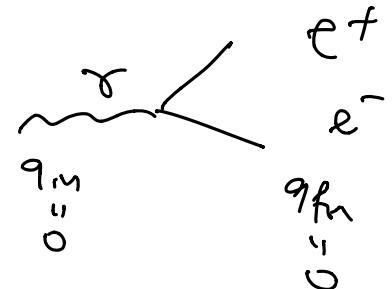
In ciascun vertice: si conserva la carica.

+ tutti i numeri quant. c.

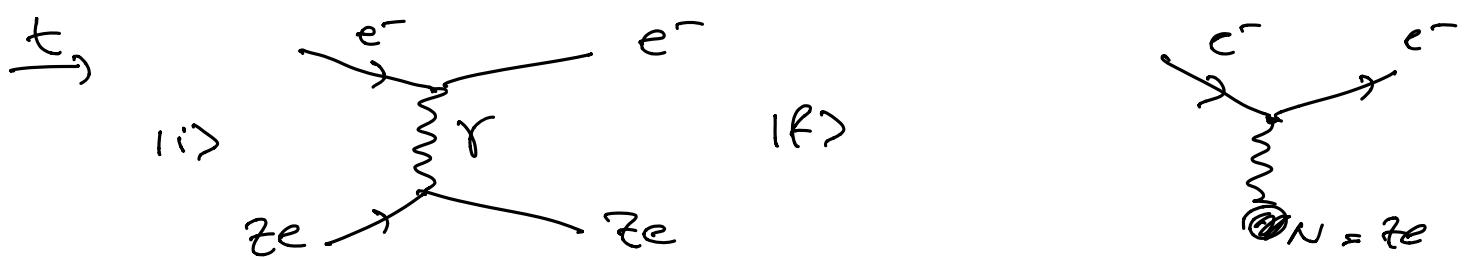
Inter. EM. vertici possibili:



J



Difusione Rutherford $e^- + N \rightarrow e^- + N$



$$Q_{in} = -e + Ze$$

$$Q_{fin} = -e + Ze$$

$$M_{fi} = -i \int d^3r \psi_f^* H_I \psi_i = -i V_{fi}$$

$$H_I = -eV = -\frac{Ze^2}{4\pi\epsilon_0} \frac{1}{r}$$

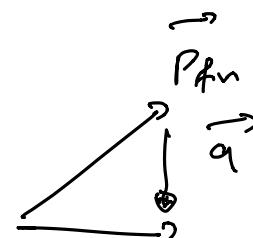
e^-
 $q = -1$
 Ze
 $q = Z$

$$= -\frac{Z\alpha}{r}$$

Nell'approssimazione di Born: stato iniziale e finale approssimati da onde piane.

$$\psi_{in}: \quad \psi_i = \frac{1}{\sqrt{V}} e^{i \vec{P}_{in} \cdot \vec{r}}$$

$$\psi_f = \frac{1}{\sqrt{V}} e^{i \vec{P}_{fin} \cdot \vec{r}}$$



$$V_{fi} = -\frac{Z\alpha}{V} \int d^3r e^{i (\vec{P}_m - \vec{P}_{fin}) \cdot \vec{r}} \frac{1}{r}$$

$$\vec{P}_{in} = \vec{P}_{fin} + \vec{q}$$

\vec{q} : momento trasferito degli e^- al nucleo.

$$= -\frac{Z\alpha}{V} \int_0^\infty r dr \int_0^{2\pi} \int_0^\pi \int_0^\pi \sin\theta d\theta e^{i \vec{q} \cdot \vec{r}} \frac{1}{r}$$

$$\vec{q} \cdot \vec{r} = q \cdot r \cos\theta$$

$$= -\frac{Z\alpha}{V} (2\pi) \int_0^\infty r dr \int_0^\pi \int_0^\pi \sin\theta d\theta e^{iqr \cos\theta}$$

$$\frac{d}{d\theta} \cos \theta = -\sin \theta. \quad \theta \rightarrow \gamma = \cos \theta.$$

$$\rightarrow \frac{1}{iqr} \int_{-\infty}^{\infty} d\omega i\theta e^{iqr\omega i\theta}$$

$$= \frac{1}{iqr} [e^{iqr} - e^{-iqr}]$$

$$V_{fi} = -\frac{Z\alpha}{V} (\bar{z}\bar{u}) \int_0^{\infty} r dr \frac{1}{iqr} [e^{iqr} - e^{-iqr}]$$

$$\int_0^{\infty} e^{iqr} dr = \lim_{\epsilon \rightarrow 0} \int_0^{\infty} e^{iqr} e^{-\epsilon r} dr$$

$$\lim_{\epsilon \rightarrow 0} \int_0^{\infty} dr e^{-\epsilon r} [e^{iqr} - e^{-iqr}] =$$

$$= \lim_{\epsilon \rightarrow 0} \left(\frac{1}{iqr - \epsilon} (0 - 1) - \frac{1}{-iqr - \epsilon} (0 - 1) \right)$$

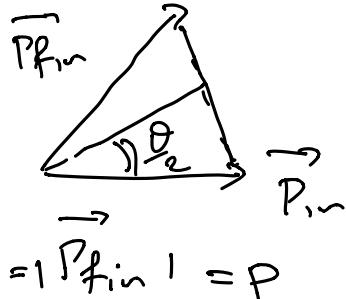
$$= \lim_{\epsilon \rightarrow 0} \left[-\frac{1}{iqr - \epsilon} - \frac{1}{iqr + \epsilon} \right] = \lim_{\epsilon \rightarrow 0} \left[\frac{iq + \epsilon + iq - \epsilon}{-q^2 - \epsilon^2} \right]$$

$$= \frac{\epsilon i q}{q^2}$$

$$V_{fi} = -\frac{Z\alpha}{V} (\bar{z}\bar{u}) \frac{1}{iqr} \frac{\epsilon i q}{q^2} = -\frac{Z\alpha}{V} (\bar{u}\bar{u}) \frac{1}{q^2}$$

$$\sigma = \bar{z}\bar{u} |M_{fi}|^2 \rho(\epsilon)$$

$$\sigma \propto |M_{fi}|^2 \propto Z^2 \alpha^2 \frac{1}{q^4}$$

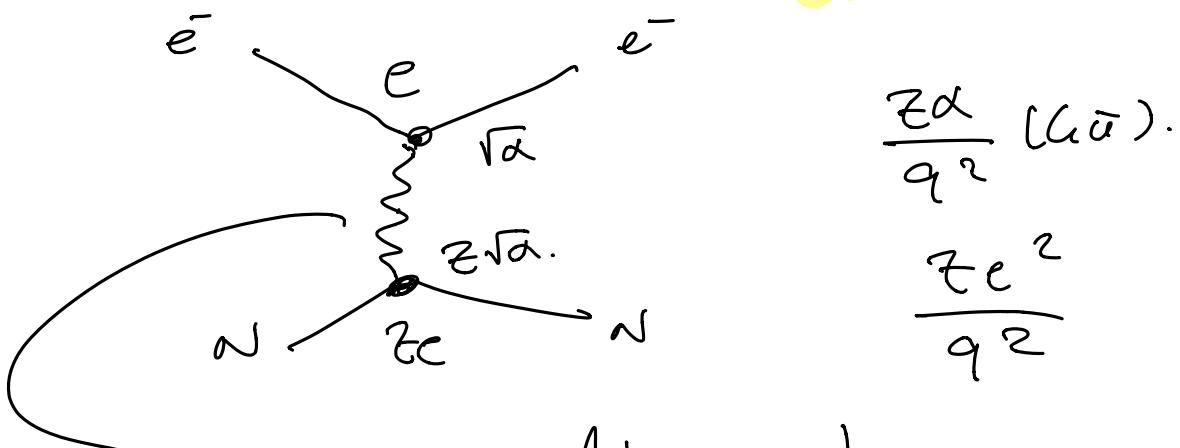


$$q = 2P \sin \frac{\theta}{2}$$

$$|\vec{P}_{in}| = |\vec{P}_{fin}'| = p$$

$$\Im \alpha \frac{z^2 \alpha^2}{16 P^4 \sin^4 \frac{\theta}{2}}$$

$M_{fi} = -i V_{fi} = -i \frac{z\alpha}{q^2} (\text{L} \bar{n})$



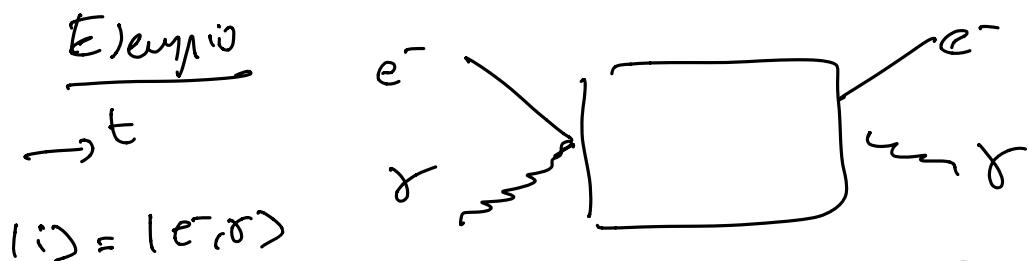
$$\frac{z\alpha}{q^2} (\text{L} \bar{n}).$$

$$\frac{ze^2}{q^2}$$

propagazione del fotone $\sim \frac{1}{q^2}$

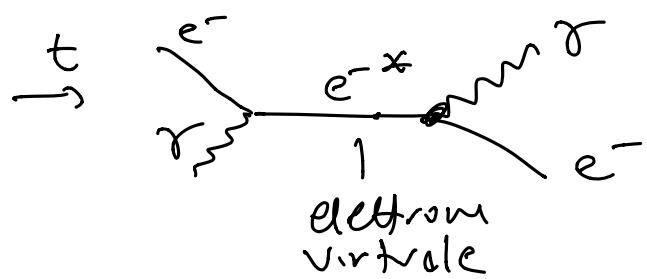
Risultato dei diagrammi di Feynman.

$$M_{fi} = \sqrt{\alpha} \cdot \frac{1}{q^2} z\sqrt{\alpha} = \frac{z\alpha}{q^2}$$



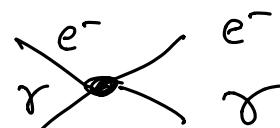
$$(\vec{i}) = (e^-, \vec{\sigma})$$

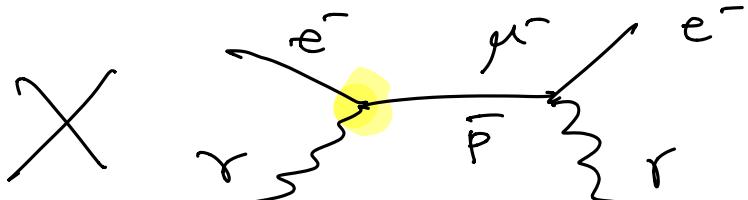
al I° ordine Compton



$$M_{fi} \sim \sqrt{\alpha} \cdot \sqrt{\alpha} \cdot \alpha = \alpha$$

$$\Rightarrow \sigma \propto (M_{fi})^2 \sim \alpha^2$$



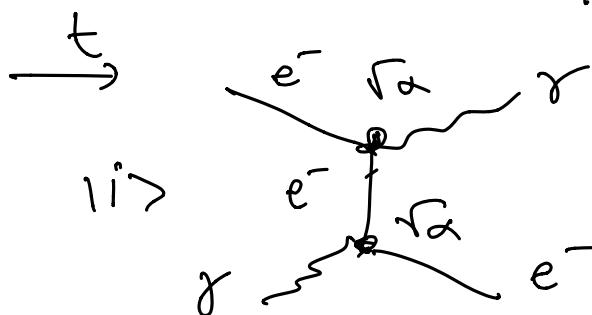


non è possibile

perché viola $\# \text{lept. } e^- (\text{Le})$

Al vertice si conserva
l'efficienza numeri quantici:
numero leptoni co.
numero barioni co.

$\# \text{barioni} (\text{B})$



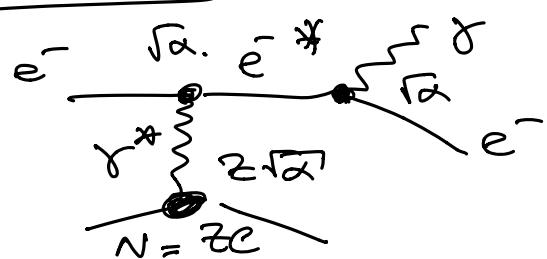
$|f\rangle$

$$M_{fi} \sim \sqrt{\alpha} \cdot \sqrt{\alpha} = \alpha$$

$$\sigma \sim \alpha^2$$

$$M_{fi} \text{ Compton} = \left(\begin{array}{c} e^- \\ \gamma \\ \gamma \end{array} \right) + \left(\begin{array}{c} e^- \\ \gamma \\ e^- \end{array} \right)$$

Bremsstrahlung $|i\rangle = |e^-, N\rangle$ $|f\rangle = |e^-, \gamma, N\rangle$



3 vertici di interazione.

$$M_{fi} \sim Z \sqrt{\alpha} \cdot \sqrt{\alpha} \cdot \sqrt{\alpha} \frac{1}{q^2}$$

$$= \frac{Z \alpha^{3/2}}{q^2}$$

$$\sigma \sim \frac{Z^2 \alpha^3}{q^4}$$

Differenze rispetto a Rutherford

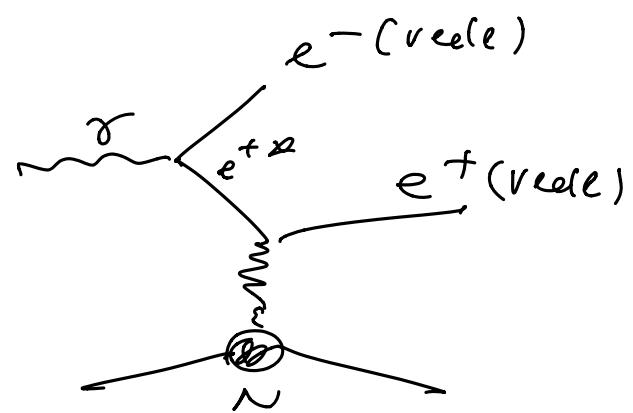
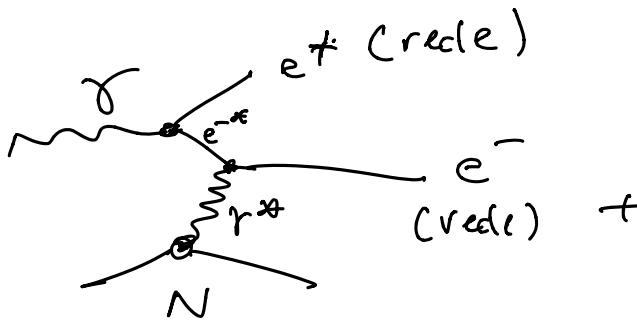
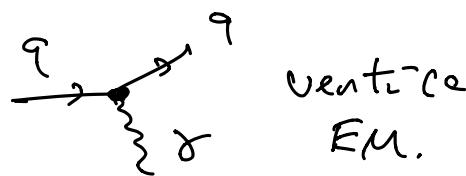
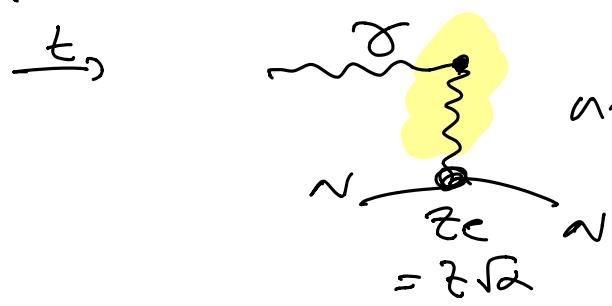
$$\sigma_{\text{Rdm}} \sim \alpha^2$$

$$\sigma_{\text{Br.}} \sim \alpha^3$$

$$\alpha \approx \frac{1}{137}$$

Produzione di Coppia

$$|i\rangle = |\gamma, n\rangle \quad |f\rangle = |e^+ e^- \rangle$$

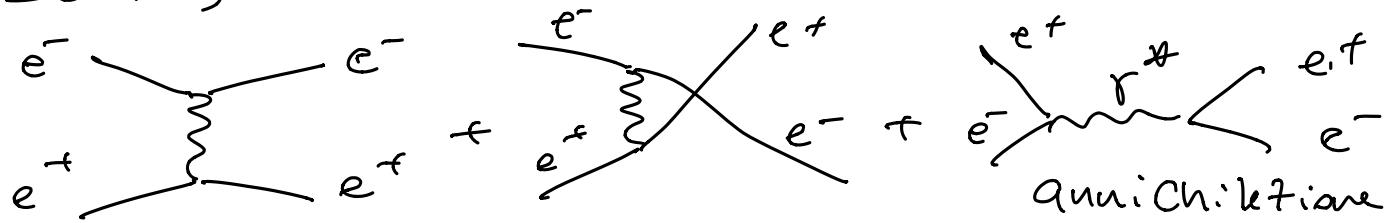


Le linee interne sono sempre particelle virtuali:

$$M_F \sim \sqrt{\alpha} \cdot 2\sqrt{\alpha} \cdot \sqrt{\alpha} \cdot \frac{1}{q^2} \sim \frac{2\alpha^{3/2}}{q^2}$$

off-shell

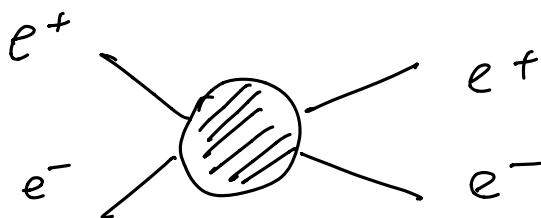
Scattering Bubble: $e^+ e^- \rightarrow e^+ e^-$. $\sigma \sim \alpha^2$



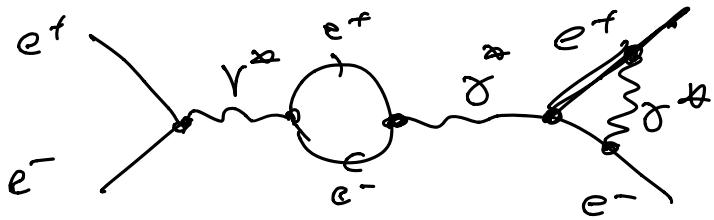
Osservabile: $|i\rangle = |e^+ e^- \rangle$.

$$|f\rangle = |e^+ e^- \rangle.$$

dipende $\mu^+ \mu^-$
dall'ordine $\{ p + \bar{p} \}$
 $\bar{\tau}^+ \tau^-$



\hookrightarrow dipende dall'ordine delle teorie perturbative.



$$M_{fi} \sim \sqrt{\alpha} \cdot \sqrt{\alpha} \cdot \sqrt{\alpha} \cdot \sqrt{\alpha} \cdot \sqrt{\alpha} \cdot \frac{1}{q^6}$$

$$= \alpha^3$$

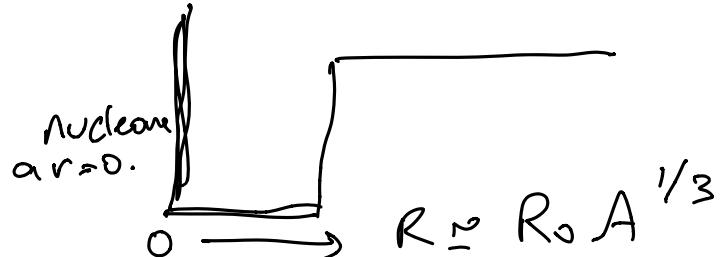
$$\rightarrow \sigma \sim \alpha^6 \sim \left(\frac{1}{137}\right)^6 \text{ Order } \text{II}_{\text{ordine}}.$$

Potenziale di Yukawa per l'interazione nucleare

Yukawa nel 1935 uscire il medietore per spiegare le forze nucleari tra nucleoni.

Fatti sperimentali: forze a corto raggio.
Simmetria tra n, p

Forze abbiano vita



Rappresento campo elettostatico.

$$V(r) = \frac{e}{4\pi} \frac{1}{r}$$

Campo centrale generato dalla carica e all'origine.

$$\nabla^2 V = -\rho(\vec{r}) = -e \delta(\vec{r})$$

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right)$$

Soluzione di Poisson per campo centrale.

$$V(r) = \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3 \vec{r}'$$

Nel Ca/0 dipendente del tempo: eq. di Maxwell.

$$\nabla^2(\vec{E}, \vec{B}) = \emptyset.$$

$$\nabla^2 = \frac{\partial^2}{\partial t^2} - \nabla^2$$

nel Ca/0 stazionario $\nabla^2 \rightarrow -\nabla^2$

$$\vec{E} = -\nabla \phi - \frac{\partial \vec{A}}{\partial t}$$
$$\vec{B} = \vec{D} \times \vec{A}$$

$\vec{A}(\phi, \vec{A})$ potenziale vettore.

Gauge di Lorentz.

$$\frac{\partial^2}{\partial t^2} \phi - \nabla^2 \phi = \rho$$

$$\vec{D} \cdot \vec{A} + \frac{\partial \phi}{\partial t} = \emptyset$$

$$\frac{\partial^2}{\partial t^2} \vec{A} - \nabla^2 \vec{A} = \vec{J}$$

Potenziale elettrico:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 \right) \phi = \rho$$

nel Ca/0 stazionario: $\frac{\partial}{\partial t} \rightarrow 0 \Rightarrow \nabla^2 \phi = -\rho$

Definire pot. nucleare simile a ϕ

$$\nabla^2 = \frac{\partial^2}{\partial t^2} - \nabla^2$$

$$E \rightarrow i \frac{\partial}{\partial t}$$
$$\vec{P} = -i \vec{D}$$

$$\int E^2 - P^2$$

$$\nabla^2 \phi = 0 \Rightarrow (E^2 - P^2) \phi = 0. \quad \forall \phi.$$

$$\boxed{E^2 = P^2}$$

$$E = P$$

per particelle relativistiche con $m = \emptyset$.

Particelle meccaniche relativistiche:

$$E^2 = p^2 + m^2.$$

Particelle mediato da un mediatore massivo scalare

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2 \right) \phi = 0.$$

Eq. di Klein-Gordon.

Particelle scalari relativistici.

Nel caso stazionario

$$EM: \quad \nabla^2 \phi = -\rho = -e \delta(\vec{r}).$$

nucleo. $(\nabla^2 - m^2) \phi = -g \delta(\vec{r})$.
vulcano

ϕ pote.

nucleo scalare.

Carica nucleare.

$g > 0$ Sempre.

Forze nucleare Sempre attrattive.

nucleone con carica g positivo $\vec{r} = \phi$

$g > 0$ H nucleone.

$$\phi(r) = -\frac{g}{4\pi} \frac{e^{-mr}}{r}$$

Pot. Coulomb
Schwarzschild.