EX PER CASA

Fascio di pioni $10^{12}\ \pi/s$

$p = 2\ GeV$

Intensità del fascio dopo 120 m nel moto ?

$m(\pi^+) = 140\ MeV \qquad \tau_0(\pi^+) = 2.6 \cdot 10^{-8}\ s$

$$I_\pi(t=0) = e\,\dot{N}_\pi = 1.6 \cdot 10^{-19}\ C \cdot 10^{12}\ s^{-1} = 0.16\ \mu A$$

$$N(t) = N_0\, e^{-t/\tau} \xrightarrow[\text{tempi}]{\text{dilatazione}} N_0\, e^{-t/\gamma\tau}$$

ma io voglio $\underline{\underline{N(x)}}$

$$N(x) = N_0\, e^{-x/\bar{x}}$$

con $\bar{x} = v \cdot \bar{\tau} = v \cdot \gamma\tau_0 = \beta c \cdot \gamma\tau_0$

$$\Rightarrow N(x) = N_0\, e^{-x/\beta\gamma c\tau_0}$$

$$\Leftrightarrow \frac{N(x)}{N_0} = e^{-x/\beta\gamma c\tau_0} \xrightarrow{x=120m} e^{-\frac{120m}{\beta\gamma c\tau_0}}$$

$$\beta = \frac{p}{E}$$

$$E = \sqrt{m_\pi^2 + p^2} = \sqrt{0.140^2 + 2^2} = 2.005 \text{ GeV}$$

$$\Rightarrow \beta = \frac{2}{2.005} = 0.9976$$

$$\Rightarrow \gamma = \frac{1}{\sqrt{1-\beta^2}} = 14.4$$

$$\Rightarrow e^{-x/\beta\gamma c\tau_0} = e^{-120/0.9976 \cdot 14.4 \cdot 3 \cdot 10^8 \cdot 2.6 \cdot 10^{-8}} = 0.33$$

$$\equiv \frac{N(x = 120\,m)}{N_0}$$

ALTRO ESERCIZIO

10^{10} μ^+ con $p = 200$ GeV in anello di accumulatore $R = 100$ m

$\tau_0(\mu) = 2.2 \cdot 10^{-6}$ s $\quad m_\mu = 106$ MeV

Quante rivoluzioni prima che corrente si riduca di un fattore 10^6?

VITA MEDIA NEL LAB:

$\boxed{2}$

$t = \gamma \tau_0$

200 GeV

106 MeV

$$\gamma = \frac{E}{m} = \frac{\sqrt{p^2+m^2}}{m} = \frac{\sqrt{200000^2 + 106^2}}{m} \simeq \frac{200000\ \text{MeV}}{m}$$

$$\gamma = 1887$$

$$(\beta \simeq 1)$$

$$\tau = \gamma \tau_0 = 1887 \cdot 2.2 \cdot 10^{-6}\,s \simeq 4.2 \cdot 10^{-3}\,s$$

Assumendo che $v = c$ $(\beta = 1)$

$$\frac{N(x)}{N_0} = e^{-x/\beta\gamma c\tau_0}$$

$$\parallel$$

$$10^{-6} = e^{-x/\beta\gamma c\tau_0}$$

$$\ln(10^{-6}) = -\frac{x}{\beta\gamma c\tau_0}$$

$$x = -\beta\gamma c\tau_0 \ln(10^{-6})$$

$$= -1 \cdot 1887 \cdot 3 \cdot 10^{8} \cdot 2.2 \cdot 10^{-6} \cdot (-13.8)$$

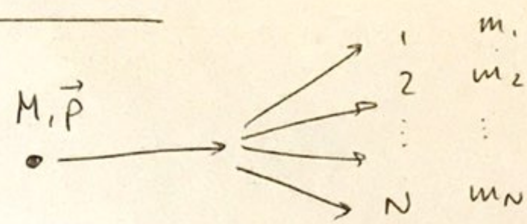
$$= 17.2 \cdot 10^{6}\ m$$

1 giro è $2\pi R = 2 \cdot 3.14 \cdot 100\,m = 628\ m$

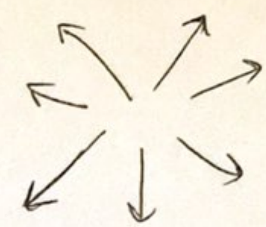
$$\Rightarrow N_{riv} = \frac{x}{2\pi R} = \frac{17.2 \cdot 10^6}{628} = 27.4 \cdot 10^3$$

DECADIMENTO IN DUE CORPI

IN GENERALE:



Nel SdR del CdM della particella:



Conservazione del 4 impulso

stato iniziale

$$\begin{pmatrix} M \\ \vec{0} \end{pmatrix} = \sum_i \begin{pmatrix} E_i^* \\ \vec{p}_i^* \end{pmatrix}$$

la particella è a riposo ($\vec{p} = \vec{0}$)

$$\Rightarrow \quad E = \left(M^2 + |\vec{p}|^2 \right)^{1/2} = \underline{\underline{M}}$$

$$\Rightarrow \text{ho} \quad \begin{cases} M = \sum_i E_i^* \\ \vec{0} = \vec{p}_i^* \end{cases}$$

$$M = \sum_i E_i^* = \sum_i \sqrt{m_i^2 + |\vec{p}_i|^2} \geq \sum_i m_i$$

$$\Leftrightarrow \quad \sum_i m_i \leq M$$

La somma delle masse delle particelle figlie ≤ massa madre!

consideremos $\quad a \to b + c$

$$M_a \quad m_b \quad m_c$$

$$(E_b, \vec{p}_b^{\,*}) \xleftarrow{\quad} \overset{(M_a, \vec{0})}{\underset{\bullet}{a}} \xrightarrow{\quad} (E_c, \vec{p}_c^{\,*})$$

$$\underset{\boxed{s.i.}}{\begin{pmatrix} M_a \\ \vec{0} \end{pmatrix}} \underset{\uparrow}{=} \underset{\boxed{s.f.}}{\begin{pmatrix} E_b^* + E_c^* \\ \vec{p}_b^{\,*} + \vec{p}_c^{\,*} \end{pmatrix}}$$

$$\Rightarrow \quad M_a = E_b^* + E_c^*$$

$$\vec{0} = \vec{p}_b^{\,*} + \vec{p}_c^{\,*} \iff \vec{p}_b^{\,*} = -\vec{p}_c^{\,*} \equiv \vec{p}^{\,*}$$

back-to-back

$$M_a = \sqrt{(p^*)^2 + m_b^2} + \sqrt{(p^*)^2 + m_c^2}$$

$$\iff M_a - \sqrt{(p^*)^2 + m_b^2} = \sqrt{(p^*)^2 + m_c^2}$$

quadrado

$$M_a^2 + \left(\cancel{(p^*)^2} + m_b^2\right) - 2M_a\sqrt{(p^*)^2 + m_b^2} = \cancel{(p^*)^2} + m_c^2$$

$$M_a^2 + (m_b^2 - m_c^2) = 2M_a\sqrt{(p^*)^2 + m_b^2}$$

quadrado

$$M_a^4 + (m_b^2 - m_c^2)^2 + 2M_a^2(m_b^2 - m_c^2) = 4M_a^2\left((p^*)^2 + m_b^2\right)$$

$$M_a^4 + (m_b^2 - m_c^2)^2 + 2M_a^2(m_b^2 - m_c^2) = 4M_a^2(p^*)^2 + 4M_a^2 m_b^2$$

$$M_a^4 + (m_b^2 - m_c^2)^2 - 2M_a^2(m_b^2 - m_c^2) = 4M_a^2(p^*)^2$$

$$\Rightarrow \quad p^* = \sqrt{\frac{M_a^4 + (m_b^2 - m_c^2)^2 - 2M_a^2(m_b^2 - m_c^2)}{4M_a^2}}$$

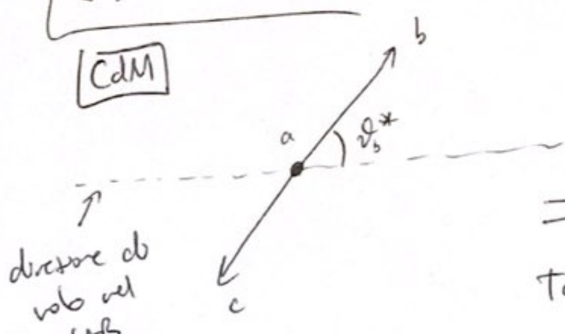
MONOCROMATICO!

EX: ricavare:

$$\begin{cases} E_b^* = \sqrt{(p^*)^2 + m_b^2} = \dfrac{M_a^2 + (m_b^2 - m_c^2)}{2M_a} \\ E_c^* = \sqrt{(p^*)^2 + m_c^2} = \dfrac{M_a^2 + (m_c^2 - m_b^2)}{2M_a} \end{cases}$$

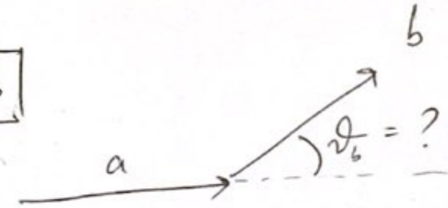
$\boxed{\text{CASO GENERICO}}$

$\boxed{\text{CdM}}$



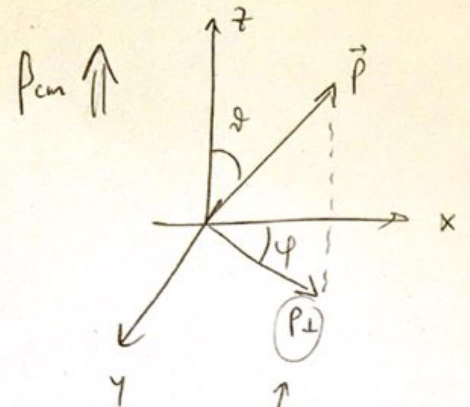
$\Longrightarrow$ TdL con $\beta_{cm} = \beta_a$, $\gamma_{cm} = \gamma_a$

$\boxed{\text{LAB}}$



LAB

$$\Rightarrow \quad (*) \quad \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} E \\ |\vec{p}| \sin\vartheta \cos\varphi \\ |\vec{p}| \sin\vartheta \sin\varphi \\ |\vec{p}| \cos\vartheta \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} E^* \\ |\vec{p}|^* \sin\vartheta^* \cos\varphi^* \\ |\vec{p}|^* \sin\vartheta^* \sin\varphi^* \\ |\vec{p}|^* \cos\vartheta^* \end{pmatrix}$$



$0 \le \vartheta \le \pi$

$0 \le \varphi \le 2\pi$

ora come sappiamo $p_\perp$ e' invariante per TdL

$$\Rightarrow \quad p_\perp = p_\perp^*$$

$$\Leftrightarrow \quad \sqrt{p_x^2 + p_y^2} = \sqrt{(p_x^*)^2 + (p_y^*)^2}$$

$$\Leftrightarrow \quad \sqrt{p^2 \sin^2\vartheta \cos^2\varphi + p^2 \sin^2\vartheta \sin^2\varphi} = \sqrt{p^{*2} \sin^2\vartheta^* \cos^2\varphi^* + p^{*2} \sin^2\vartheta \sin^2\varphi}$$

$$\Leftrightarrow \quad \sqrt{p^2 \sin^2\vartheta \underbrace{(\cos^2\varphi + \sin^2\varphi)}_{=1}} = \sqrt{p^{*2} \sin^2\vartheta^*}$$

$$\Leftrightarrow \quad p^* \sin\vartheta = p^* \sin\vartheta^*$$

sostituendo in $(*)$

$$\Rightarrow \quad \begin{cases} \cos\varphi = \cos\varphi^* \\ \sin\varphi = \sin\varphi^* \end{cases} \quad \forall\varphi \ \dots \Rightarrow \varphi = \varphi^*$$

invece per ϑ divido (y) per (z)

$$\Rightarrow \quad \frac{p_y}{p_z} = \frac{\cancel{|\vec{p}|} \sin\vartheta \cancel{\sin\varphi}}{\cancel{|\vec{p}|} \cos\vartheta} = \frac{|\vec{p}^*| \sin\vartheta^* \cancel{\sin\varphi^*}}{\beta_{cm}\gamma_{cm} E^* + \gamma_{cm} |\vec{p}^*| \cos\vartheta^*}$$

$$\Leftrightarrow \quad \tan\vartheta = \frac{\sin\vartheta^*}{\gamma_{cm}\left(\beta_{cm} \, E^*/p^* + \cos\vartheta^*\right)}$$

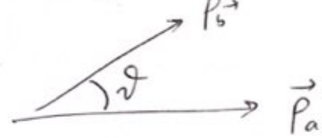
$\frac{p^*}{E^*} = \beta^*$ velocità di (b) in $\underline{\underline{\text{CdM}}}$

$$\Rightarrow \quad \tan\vartheta = \frac{\sin\vartheta^*}{\gamma_{cm}\left(\beta_{cm}/\beta^* + \cos\vartheta^*\right)} \qquad (1)$$

$\boxed{\text{CASI}}$

$\beta_{cm} > \beta^* \Rightarrow$ denom è sempre > 0 $\quad (\forall \vartheta^*)$

$\Rightarrow 0 \leq \vartheta \leq \frac{\pi}{2}$



(b) è emessa in avanti nel LAB

<u>sempre</u> $(\forall \underline{\vartheta^*})$

ora noto che $\quad \vartheta^* = 0 \longrightarrow \vartheta = 0$

$\qquad\qquad\qquad\quad \vartheta^* = \frac{\pi}{2} \longrightarrow \vartheta = 0$

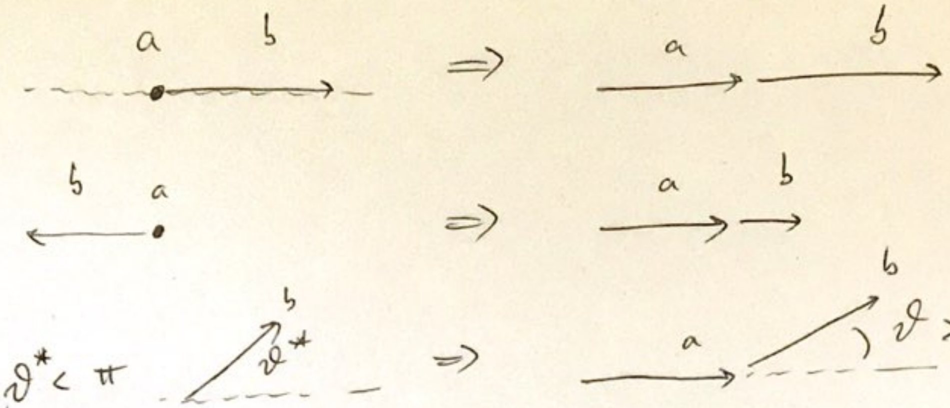
$\rightarrow \exists\, \vartheta^*$ t.c. ϑ è <u>MAX</u>

$\boxed{\text{CdM}}$ $\boxed{\text{LAB}}$

$\vartheta^* = 0$ ⟹ $\vartheta = 0$

$\vartheta^* = \pi$ ⟹ $\vartheta = 0$

$0 < \vartheta^* < \pi$ ⟹ $\vartheta > 0$



vogliamo trovare l'angolo di apertura MASSIMO

per trovare MAX trovo il punto dove si annulla la derivata di (1)

EX

$$\Rightarrow \quad 0 = \frac{d}{d\vartheta^*}(tg\,\vartheta) = \frac{1 + \cos\vartheta^* (\beta_{cm}/\beta^*)}{\gamma_{cm}\left(\beta_{cm}/\beta^* + \cos\vartheta^*\right)}$$

si annulla in corrispondenza di

$$\cos\vartheta^* = -\beta^*/\beta_{cm}$$

$$\Rightarrow \sin\vartheta^* = \sqrt{1-\cos^2\vartheta^*} = \sqrt{1 - \beta^{*2}/\beta_{cm}^2} = \frac{1}{\beta_{cm}}\sqrt{\beta_{cm}^2 - \beta^{*2}}$$

e il corrispondente angolo ϑ_{MAX} è

$$tg(\vartheta_{MAX}) = \frac{\frac{1}{\beta_{cm}}\sqrt{\beta_{cm}^2 - \beta^{*2}}}{\gamma_{cm}\left(\frac{\beta_{cm}}{\beta^*} - \frac{\beta^*}{\beta_{cm}}\right)} = \frac{\sqrt{\beta_{cm}^2 - \beta^{*2}}}{\beta_{cm}\gamma_{cm}\left(\frac{\beta_{cm}^2 - \beta^{*2}}{\beta^*\beta_{cm}}\right)}$$

$$\Rightarrow \quad \operatorname{tg}(\vartheta_{MAX}) = \frac{\beta^*}{\gamma_{cm}\left(\beta_{cm}^2 - \beta^{*2}\right)^{1/2}}$$

<u>OK</u>

$-\beta^*/\beta_{cm}$

in corrispondenza di questo angolo abbiamo

$$E(\vartheta_{MAX}) = \gamma_{cm}\left(E^* + \beta_{cm}\, p^* \cos\vartheta^*\right) =$$

$$= \gamma_{cm}\left(E^* + \beta_{cm}\, p^*\left(-\frac{\beta^*}{\beta_{cm}}\right)\right) =$$

$$= \gamma_{cm}\left(E^* - \beta^* p^*\right) \qquad \beta^* = \frac{p^*}{E^*}$$

$$= \gamma_{cm}\left(E^* - \frac{p^{*2}}{E^*}\right) = \qquad E^* = m^2 + p^{*2}$$

$$= \gamma_{cm}\left(\frac{E^{*2} - p^{*2}}{E^*}\right) = \gamma_{cm}\frac{m^2}{E^*} =$$

$$= \gamma_{cm}\frac{m}{\gamma^*} = m\,\gamma^*/\gamma_{cm}$$

$$\gamma^* = \frac{E^*}{m}$$

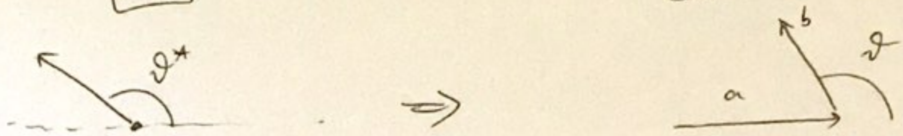
(che <u>NON</u> è E_{MAX})

ok quello era caso $\beta_{cm} > \beta^*$

CASO : $\beta_{cm} < \beta^*$

$\exists\ \vartheta^*$ t.c. $\vartheta_{LAB} > \pi/2$

CdM $\qquad\qquad\qquad$ LAB



$\vartheta = \frac{\pi}{2}$ quando $\cos\vartheta^* = -\beta_{cm}/\beta^*$

NON esiste ϑ_{MAX} (ovvero $\vartheta_{MAX} = \pi$)

↑

denominatore non si annulla mai

CASO : $\beta_{cm} = \beta^* \longrightarrow \cos\vartheta^* = -1 \rightarrow \vartheta^* = \pi$

il boost annulla esattamente β^*

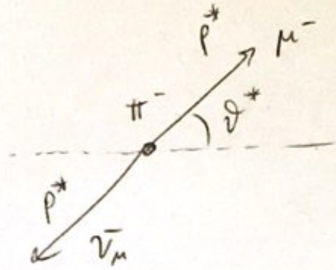
CdM $\qquad\qquad\qquad$ LAB

EX $\qquad \pi^- \to \mu^- \bar{\nu}_\mu$

Per che impulso del π^- i muoni sono emessi sempre in avanti nel LAB?

$m(\pi^-) = 140$ MeV $\qquad m(\mu^-) = 106$ MeV $\qquad m(\nu) = 0$

CdM



$m_\nu = 0$

$\Rightarrow E_2^* = p^*$

s.i. $\begin{pmatrix} m_\pi \\ \vec{0} \end{pmatrix}$ $\qquad$ s.f. $\begin{pmatrix} E_\mu^* \\ p^* \end{pmatrix} + \begin{pmatrix} E_2^* \\ p^* \end{pmatrix}$

$$\Rightarrow m_\pi = E_\mu^* + p^*$$

$$\Rightarrow m_\pi - p^* = E_\mu^*$$

quadro

$$m_\pi^2 + \cancel{p^{*2}} - 2m_\pi p^* = E_\mu^{*2} = m_\mu^2 + \cancel{p^{*2}}$$

$$p^* = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = \frac{140^2 - 106^2}{2 \cdot 140} = \cdot 30 \text{ MeV}$$

$$\Rightarrow E_\mu^* = \sqrt{p^{*2} + m_\mu^2} = \sqrt{30^2 + 106^2} = 110 \text{ MeV}$$

$$\Rightarrow \beta_\mu^* = \frac{p^*}{E_\mu^*} = \frac{30}{110} = 0.27$$

μ sempre in avanti se $\beta_{cm} > \beta^*$

$$\beta_{cm} = \beta_\pi = \frac{p_\pi}{E_\pi} \longleftarrow \text{nel } \underline{LAB}$$

$$\Rightarrow \text{ se } \beta_\pi > 0.27$$

$$\Leftrightarrow \gamma_\pi = \frac{1}{\sqrt{1-\beta_\pi^2}} = 1.04 = \frac{E_\pi}{m_\pi}$$

$$\Rightarrow E_\pi \gtrsim 1.04 \cdot m_\pi = 1.04 \cdot 140 \text{ MeV} = 145 \text{ MeV}$$

$$\Leftrightarrow p_\pi \gtrsim \sqrt{E_\pi^2 - m_\pi^2} = \sqrt{145^2 - 140^2} = 38 \text{ MeV}$$