Decadimento α

$${}^{A}_{Z}N \longrightarrow {}^{A-4}_{Z-2}X + \alpha \qquad \alpha = {}^{4}_{2}He \qquad K_\alpha \simeq 5-8 \text{ MeV}$$

$$X \longleftarrow \overset{N}{\circ} \longrightarrow \alpha \qquad \vec{P}_X^* + \vec{P}_\alpha^* = \emptyset \Rightarrow |\vec{P}_X^*| = |\vec{P}_\alpha^*| = p^*$$

$$M_N = E_X + E_\alpha = M_X + K_X + M_\alpha + K_\alpha$$

$m_\alpha = 3.6 \text{ GeV} \ll m_X$

$m_N \sim A \cdot m_p \qquad A \gg 4$

$A \geq 60$

$$Q = M_N - M_X - M_\alpha = K_X + K_\alpha$$

$$Q = -B(A,Z) + B(A-4, Z-2) + B(A=4, Z=2) \simeq O(\text{MeV})$$

$Q \ll M_N, m_X \Rightarrow$ moto non relativistico

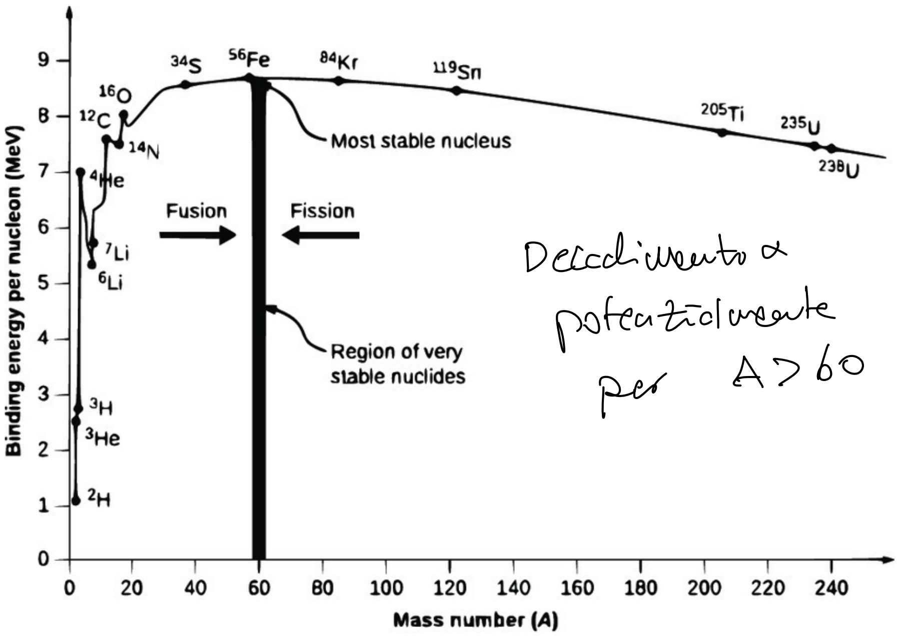
$$Q = K_X + K_\alpha \simeq \frac{p^{*2}}{2m_X} + \frac{p^{*2}}{2m_\alpha} = \frac{p^{*2}}{2m_\alpha}\left(1 + \frac{m_\alpha}{m_X}\right)$$

$$\frac{p^{*2}}{2m_\alpha} = \frac{Q}{\left(1 + \frac{m_\alpha}{m_X}\right)} \simeq Q\left(1 - \frac{m_\alpha}{m_X}\right)$$

$$\frac{m_\alpha}{m_X} \simeq \frac{4}{A-4} \ll 1 \quad \text{per} \quad A > 60 \qquad \frac{\partial B(Z,A)}{\partial A} < 0$$

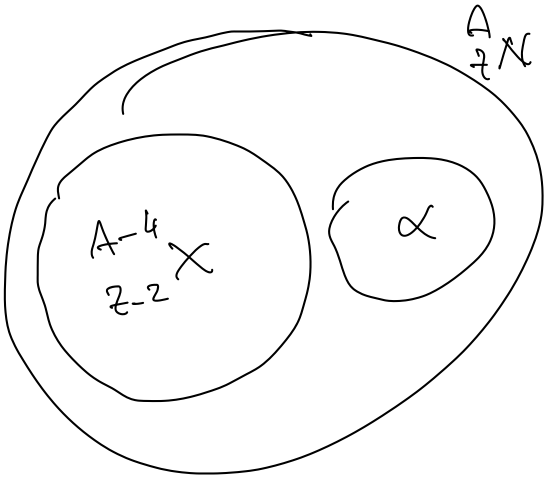
$$\Rightarrow K_\alpha = \frac{p^{*2}}{2m_\alpha} \simeq Q \qquad p^* = \frac{1}{2m_N}\left(m_N^2 - m_X^2 - m_\alpha^2\right)$$

K_α non varia tanto al variare di A

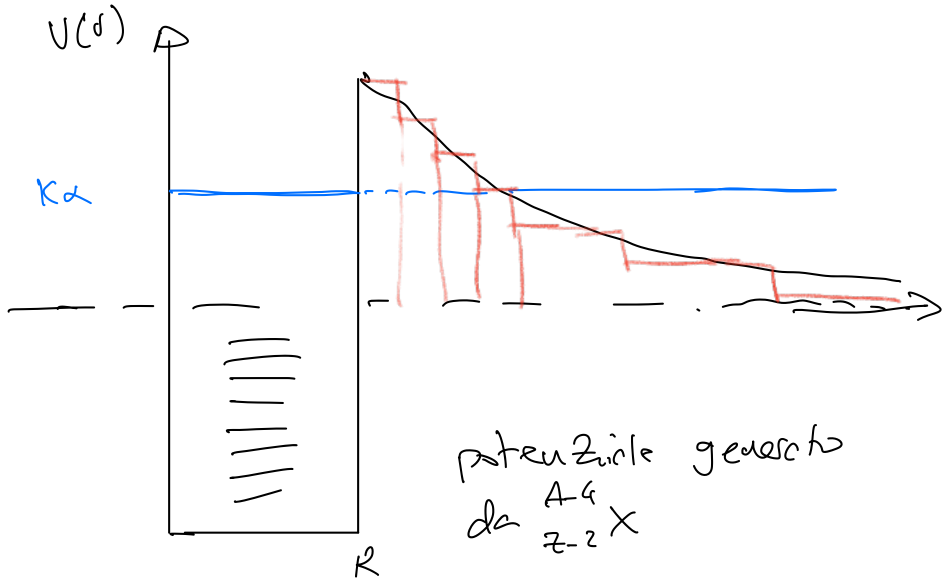


Decadimento α potenzialmente per $A > 60$

Modello di Gamow per decadimento α



α: intrappolata nel potenziale generato da $^{A-4}_{Z-2}X$

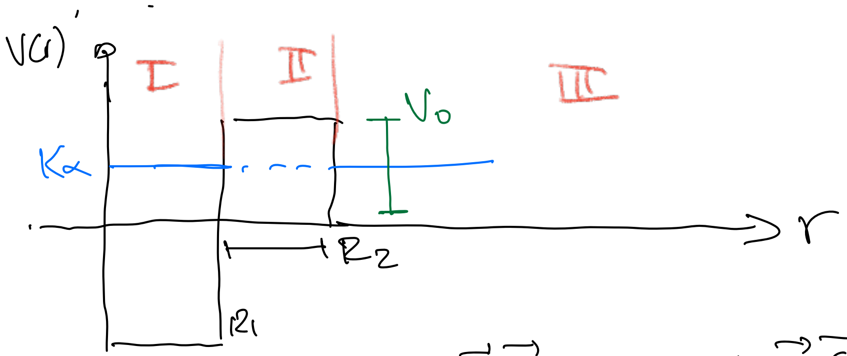


potenziale generato da $^{A-4}_{Z-2}X$

$$V(r) = \begin{cases} -V_0 & r < R \\ \dfrac{z(Z-2)\alpha}{r} & r > R \end{cases}$$

$\to z_\alpha$ $\quad \to Z_X$

particella α supera la barriera per l'effetto tunnel quantistico



$$\psi(r) = \begin{cases} \psi_I(r) = A\, e^{i \vec{p}_1 \cdot \vec{r}} + B\, e^{-i \vec{p}_1 \cdot \vec{r}} & \text{I} \quad r < R_1 \\ \psi_{II}(r) = C\, e^{-\vec{p}_2 \cdot \vec{r}} + D\, e^{+\vec{p}_2 \cdot \vec{r}} & \text{II} \quad R_1 < r < R_2 \\ \psi_{III}(r) = E\, e^{+i \vec{p}_3 \cdot \vec{r}} & \text{III} \quad r > R_2 \end{cases}$$

sperimentalmente $K_\alpha \ll m_\alpha$

$$E_1 = \frac{p_1^2}{2m_\alpha} \quad \text{(regime non relativistico)}$$

$$E_2 = \frac{p_3^2}{2m_\alpha} = \frac{p_1^2}{2m_\alpha} \Rightarrow \qquad p_1 = \sqrt{2m_\alpha E_1}$$

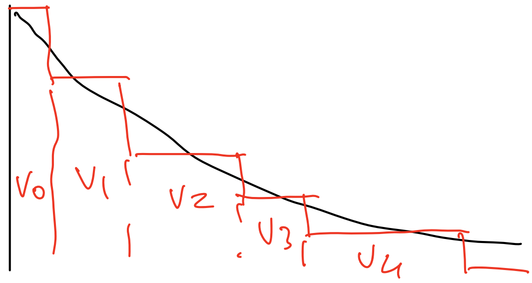
$$E_3 = V_0 - \frac{p_2^2}{2m_\alpha} \qquad p_2 = \sqrt{2m(V_0 - E_2)}$$

prob. di tunnelling $= \dfrac{|E|^2}{|A|^2} = T$

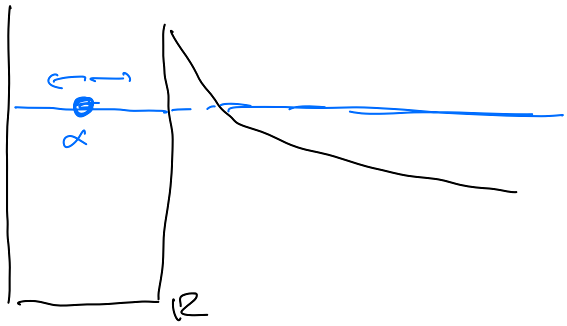
$$T \sim e^{-2\sqrt{2m(V_0 - E_\alpha)}\,(R_2 - R_1)} = e^{-2G}$$

Fattore di Gamow.

Se potenziale continuo



$$G = \int_{R_1}^{R_2} dr \sqrt{2m(V(r) - E_\alpha)}$$



f : frequenza di urto contro barriera di potenziale.

$$f = \frac{1}{\Delta T} = \frac{v}{2R}$$

$$\frac{p^2}{2m} = E \Rightarrow p = \sqrt{2mE} = mv \Rightarrow v = \sqrt{\frac{2E}{m}}$$

$$f = \frac{1}{2R}\sqrt{\frac{2E}{m}} = \sqrt{\frac{E}{2m}}\,\frac{1}{R}$$

$$\lambda_{tunn.} = f \cdot T = \sqrt{\frac{E}{2m}}\,\frac{1}{R}\,e^{-2G}$$ (prob di tunneling / unità di tempo)

Legge di decadimento $P(t) = e^{-\lambda \cdot t}$

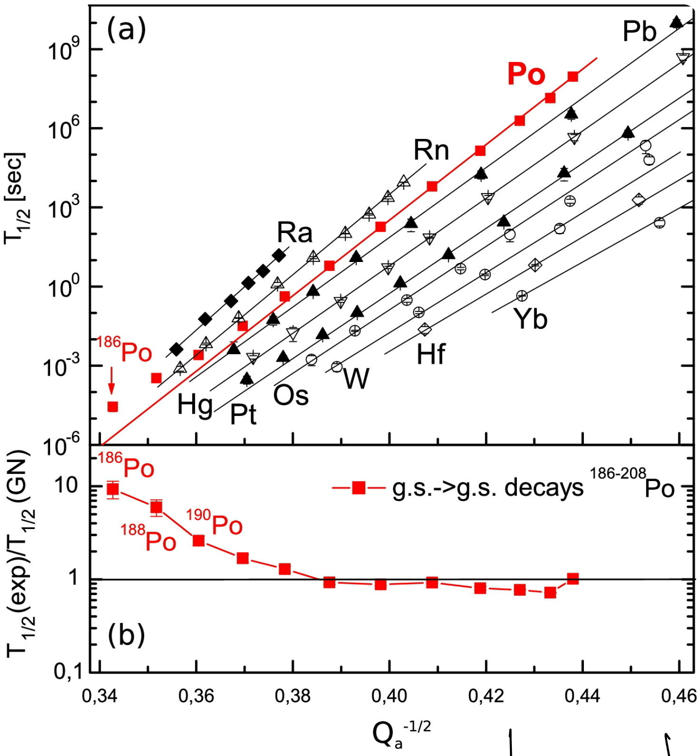
$$\tau = \frac{1}{\lambda} \Rightarrow \tau \propto E_\alpha^{-1/2}\, e^{2G}$$ vita media propria del nucleo instabile

$$\tau = \mathbb{C}\, E_\alpha^{-1/2}\, e^{2G}$$ $\mathbb{C}, \mathbb{C}', \mathbb{B}$ costanti

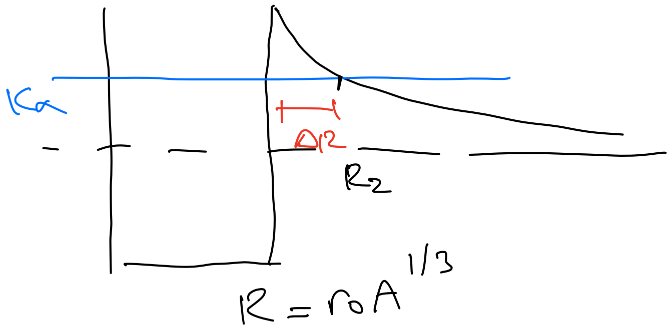
$$\ln \tau = \mathbb{C}' - \frac{1}{2}\mathbb{B} \ln E_\alpha = \mathbb{C}' - \mathbb{B} \ln \frac{1}{\sqrt{E_\alpha}}$$

Sperimentalmente: 1) misurare E_α : en. cinetica di α

2) misurare τ : tempo di decadimento.



legge di Geiger–Nuttal



$$R = r_0 A^{1/3}$$

${}^{A-4}_{Z-2}X$: $R \simeq r_0 (A-4)^{1/3}$

$$Q_\alpha = m_N - m_X - m_\alpha = -B(A,Z) + B(A-4,Z-2) + B(4,2)$$

$T_2(Exp)$: misurato

$T_2(GN)$: calcolato

$$\frac{1}{E_\alpha^{1/2}} = \frac{1}{\sqrt{E_\alpha}}$$

$$U(r) = \frac{2(Z-2)\alpha}{r} \qquad r > R \Rightarrow U(R_2) = \frac{2(Z-2)\alpha}{R_2} = K_\alpha$$

$$K_\alpha = 4-8 \text{ MeV} \Rightarrow R_2 = \frac{2(Z-2)\alpha}{K_\alpha}$$

$$1 = 200 \text{ MeV}\cdot\text{fm} \Rightarrow 40 \text{ fm} \simeq \frac{1}{5 \text{ MeV}}$$

Per $Z = 100 \Rightarrow$ $R_2 = \frac{2\times(Z-2)\alpha}{K_\alpha} \simeq \frac{200}{137} \times 40 \text{ fm} \simeq 60 \text{ fm}$.

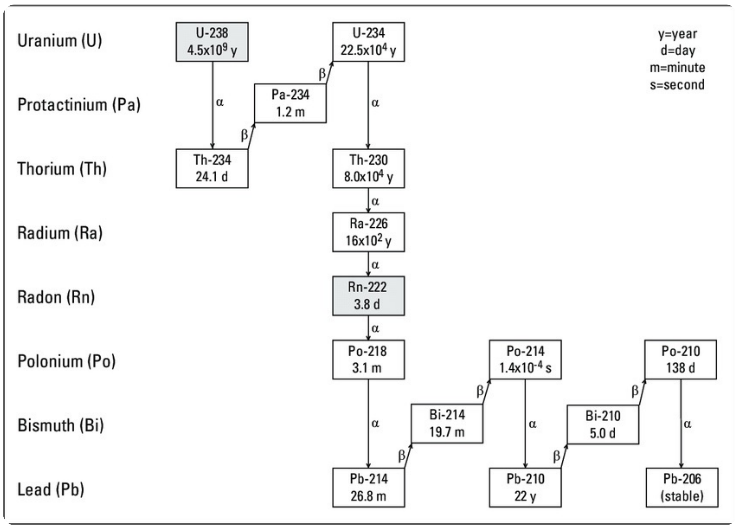
$K_\alpha = 5$ MeV

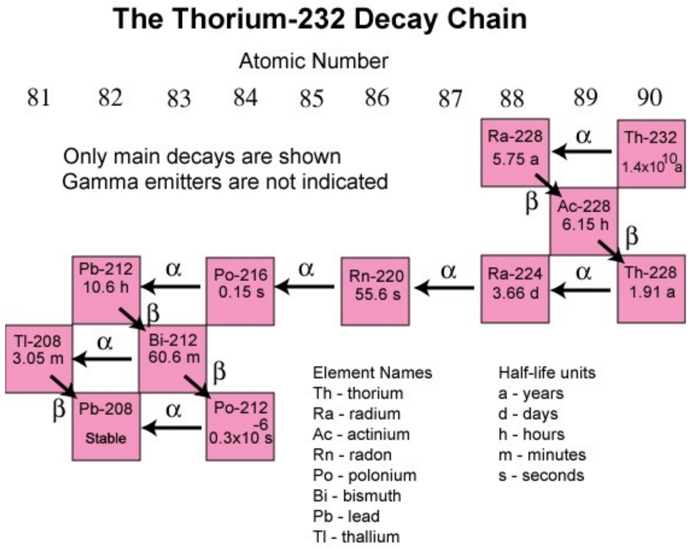
$$B_\alpha = B(4,2) = 28 \text{ MeV}$$

$$Q_\alpha = -B(A,Z) + B(A-4,Z-2) + 28 \text{ MeV} \geq 0$$

$$B(A-4,Z-2) \geq B(A,Z) - 28 \text{ MeV} \qquad \frac{\partial B}{\partial A} < 0 \quad (A > 60)$$

Decadimento α possibile per $A \geq 60$





Decadimenti α, β, γ: elementi con $\tau \gtrsim \tau_{SS} = 5 \times 10^9\ yr$ sopravvissuti

Reazioni Nucleari

$$a + b \longrightarrow c + d + \underbrace{Q}_{\text{energia}}$$

$$Q = m_a + m_b - m_c - m_d$$

$Q > 0$: reazione esotermica (massa si trasforma in Ene)

$Q < 0$: reazione endotermica (K_a, K_b necessarie per far avvenire la reazione)

Scoperta del protone:

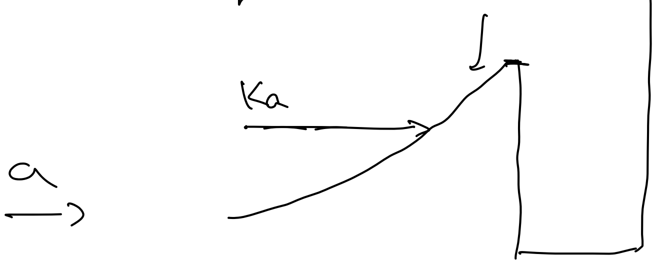
$$\alpha + {}^{14}_{7}N \longrightarrow {}^{17}_{8}O + p \qquad Q = -1.19\ MeV$$

$K_\alpha \simeq 4-8\ MeV > |Q| \Rightarrow$ reazione possibile

Barriera di Coulomb

reazione $a + {}^{A}_{Z}X$

$a = \alpha$, altri nuclei: $Z_a = 2$



$R = r_0 A^{1/3}$.

${}^{A}_{Z}X$: $Z_X > 2$

$$K_a = \frac{Z_a Z_X \alpha}{r_0 A^{1/3}} \geq U(R)$$

per reazioni nucleari con protoni

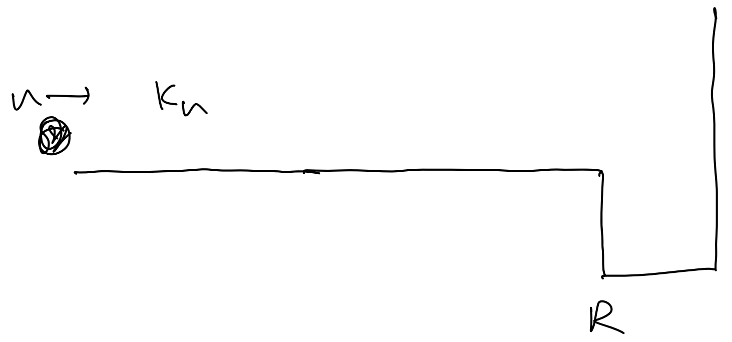
$Z_\alpha = 2$ alpha: $K_\alpha = \frac{2 \times 100}{137} \frac{1}{r_0} \frac{1}{A^{1/3}}$ $r_0 = 1$ fm.

$Z_X = 100$

$$\simeq 1.3 \times \frac{1}{A^{1/3}} \; 200 \text{ MeV}.$$

$$\Rightarrow K_\alpha \simeq 40 \text{ MeV}$$

Neutroni: nessuna barriera di Coulomb.



$$n + X \longrightarrow Y + Z + Q.$$

Fornire energia cinetica ai neutroni

$p + {}^{7}_{3}Li \longrightarrow \alpha + \alpha$ $\quad Q = 17.35$ MeV

$\alpha + \underbrace{{}^{9}_{4}Be}_{\text{bersaglio fermo}} \longrightarrow \underbrace{{}^{12}_{6}C}_{\text{fermo}} + n$ $\quad Q = 5.7$ MeV

al crescere di $K_\alpha \Rightarrow$ cresce K_n

$n + {}^{A}_{Z}X \longrightarrow \begin{cases} n + {}^{A}_{Z}X & \text{urto elastico} \\ \text{cattura neutronica: } {}^{A+1}_{Z}X^* & \text{inizio di cascata di decadim.} \\ \text{fissione indotta} \end{cases}$

1938 Hahn-Strassman

$n + {}^{238}_{92}U \longrightarrow {}^{137}_{56}Ba + {}^{84}_{36}Kr + 17n.$

Bario Kripton

Fissione indotta

Fissione spontanea: $\quad {}^{A}_{Z}N \longrightarrow {}^{A-a}_{Z-b}X + {}^{a}_{b}Y + Q$

$Q \geq 0$ per essere spontanea.

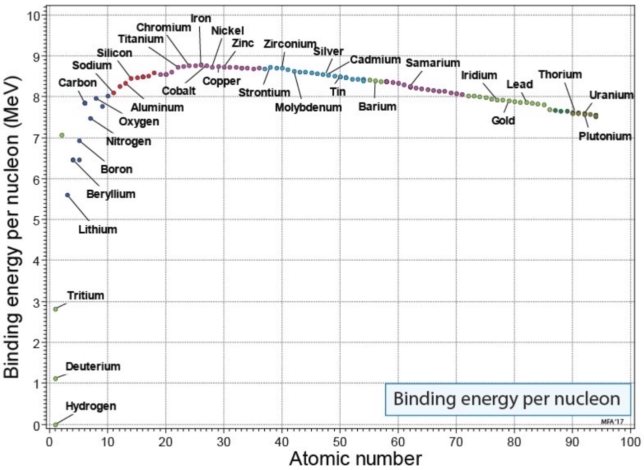
$$Q = m_N - m_X - m_Y$$

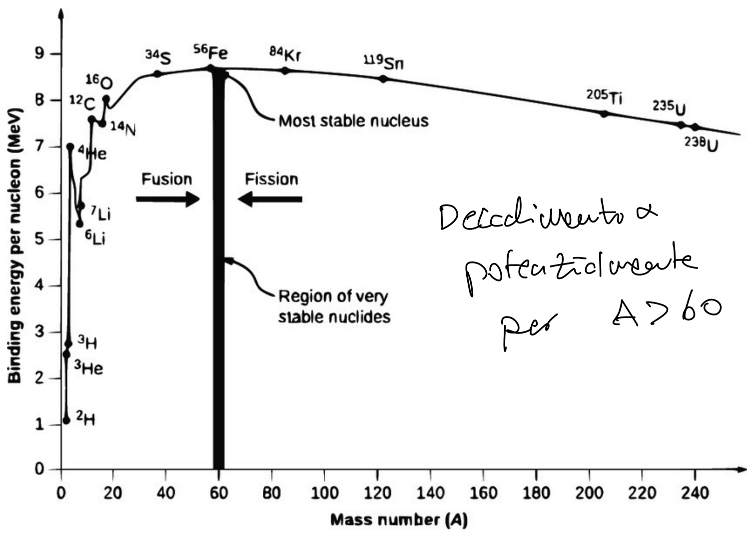
$$= -B(A,Z) + B(A-a, Z-b) + B(a,b) \geq 0.$$

$$B(A,Z) \leq B(A-a, \cancel{Z}-b) + B(a,b).$$

$A \searrow \qquad B \nearrow \qquad \frac{\partial B}{\partial A} < 0$

succede solo per $A \geq 60$





Decadimento α potenzialmente per $A > 60$

1939 Meitner - Frisch fissione spontanea

$${}^{238}_{92}U \longrightarrow {}^{137}_{56}Ba + {}^{84}_{36}Kr + 16n.$$

$\bar{B} = \frac{B}{A}$ $\qquad \bar{B}(A=240) = 7.6$ MeV

$\bar{B}(A=120) = 8.5$ MeV

$\bar{B}(A=84) = 8.7$ MeV

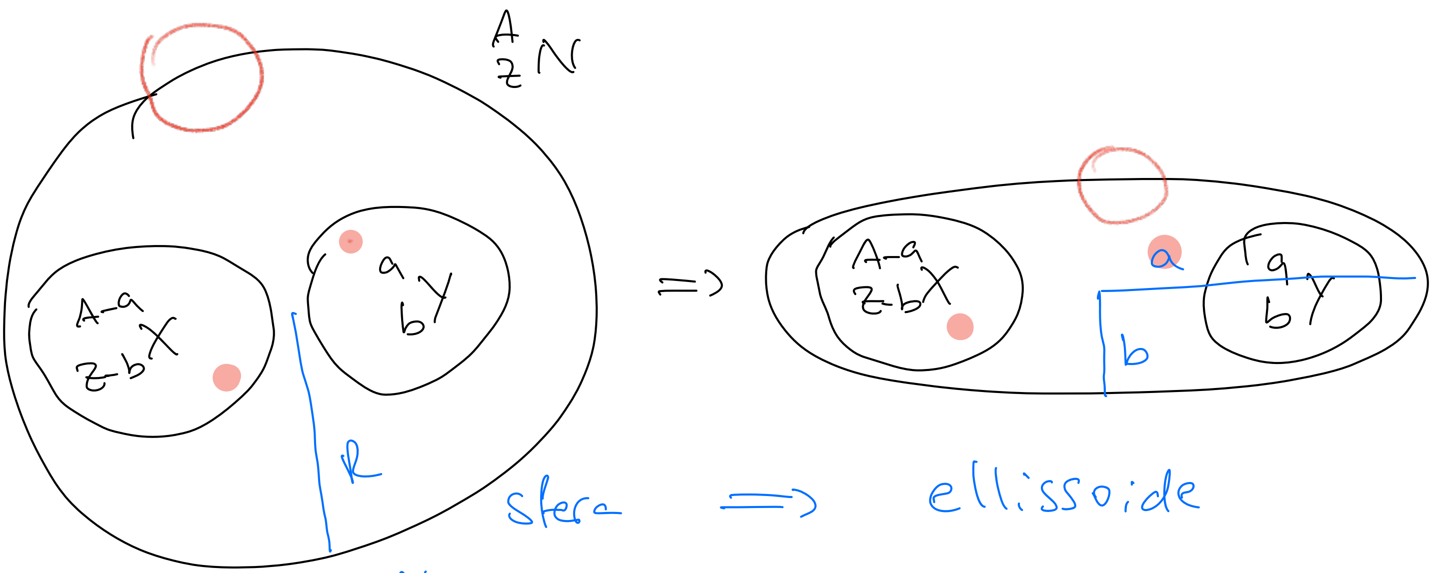
$$Q = -B(U) + B(Ba) + B(Kr) =$$

$$= 238 \text{ MeV} - 137 \times \bar{B}(Ba) - 84 \times \bar{B}(84) =$$

$$= 216 \text{ MeV}$$

$Q = m_U - m_{Ba} - m_{Kr} = -B(U) + B(Ba) + B(Kr)$

Fissione Nucleare spontanea



sfera $\Longrightarrow$ ellissoide

$R \simeq r_0 A^{1/3}$

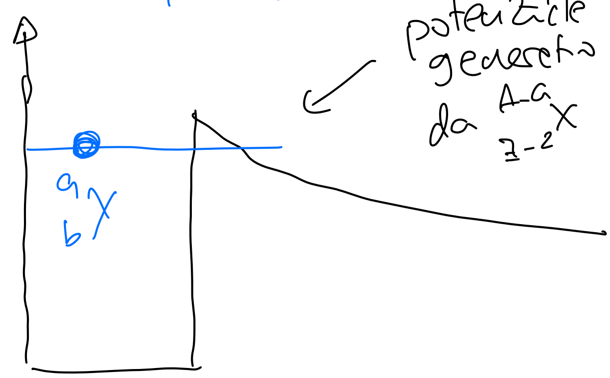
raggio del nucleo iniziale.

$R \longrightarrow (a,b)$

Deformazione del volume nucleare

$\Downarrow$

modifica l'energia di legame nucleare totale di $^{A}_{Z}N$



potenziale generato da $^{A-a}_{Z-b}X$

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3} + a_F(\ ) \pm \delta$$

Volume:

sfera $V = \frac{4\pi}{3} R^3$

$= \frac{4\pi r_0^3}{3} A$

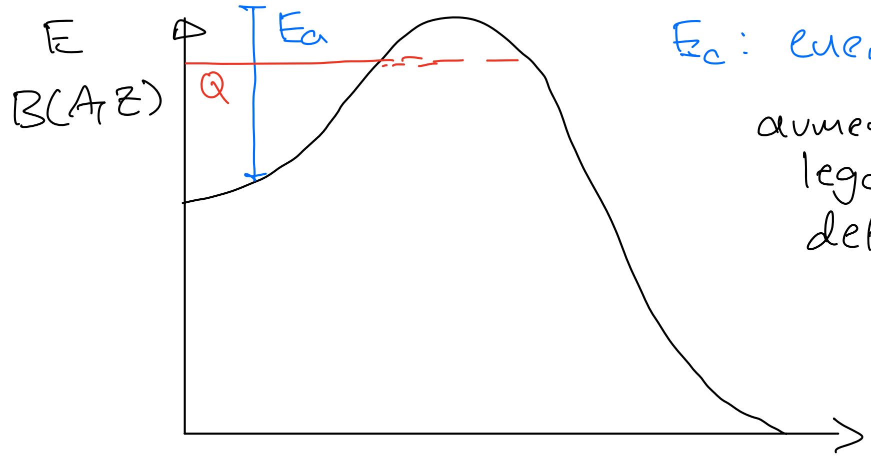
ellissoide:

$V = \frac{4\pi}{3} a b^2$

$\Delta B_{volume} \simeq$ trascurabile.

$\Delta B_{sup.} \neq 0$

$\Delta B_{coul} \neq 0$



E_a: energia di attivazione.

aumento di energia di legame a causa di deformazione

ε: deformazione del nucleo.

$$\Delta B \simeq -a_s A^{2/3}\left(\frac{2}{5}\varepsilon^2\right) + \\ - a_c Z^2 A^{-1/3}\left(-\frac{1}{5}\varepsilon^2\right)$$

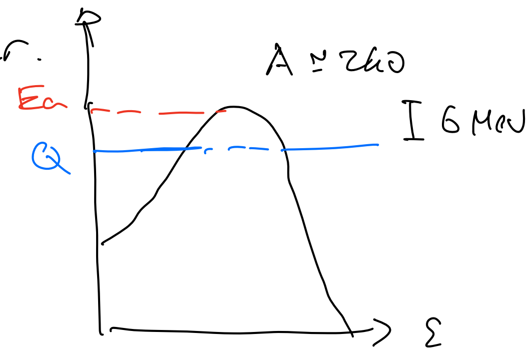
Fissione spontanea: $Q \geq E_a$

Fissione indotta: neutroni forniscono l'energia mancante

Fissione favorita per $\Delta B < 0$. nessuna variazione con deformazione

$\Delta B = 0 \Rightarrow A \simeq 200$

Dalla Formula di Bethe-Weizsäcker.



$A = 300 \qquad Q \geq E_a$

$A \simeq 240 \qquad E_a - Q \simeq 6\text{ MeV}$

Fissione spontanea possibile per effetto tunnel

$A \simeq 100 \qquad E_a - Q \simeq 60\text{ MeV}$ fiss. spontanea non avviene.