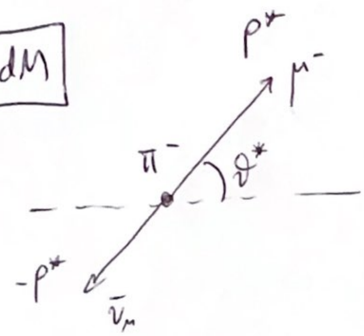
# EX PER CASA

$\pi^- \to \mu^- \bar{\nu}_\mu$

Per che impulso del $\pi$ i muoni sono sempre emessi in avanti nel LAB?

$$m_\pi = 140 \text{ MeV} \qquad m_\mu = 106 \text{ MeV} \qquad m_\nu = 0$$

$\boxed{\text{CdM}}$




stato iniziale : $\begin{pmatrix} m_\pi \\ \vec{0} \end{pmatrix}$

stato finale : $\begin{pmatrix} E_\mu^* \\ p^* \end{pmatrix} + \begin{pmatrix} E_\nu^* \\ -p^* \end{pmatrix}$

$m_\nu = 0 \Rightarrow E_\nu^* = p^*$

$$\Leftrightarrow \quad P_i = P_f$$

$$\Rightarrow \begin{pmatrix} m_\pi \\ \vec{0} \end{pmatrix} = \begin{pmatrix} E_\mu^* + p^* \\ \vec{0} \end{pmatrix}$$

$$\Leftrightarrow \quad m_\pi = E_\mu^* + p^*$$

$m_\pi - p^* = E_\mu^*$ e quadro

$$m_\pi^2 + \cancel{p^{*2}} - 2 m_\pi p^* = E_\mu^{*2} = m_\mu^2 + \cancel{p^{*2}}$$

$$\Rightarrow p^* = \frac{m_\pi^2 - m_\mu^2}{2 m_\pi} = \frac{140^2 - 106^2}{2 \cdot 140} = 30 \text{ MeV}$$

$\Rightarrow \quad E_\mu^* = \sqrt{p^{*2} + m_\mu^2} = \sqrt{30^2 + 106^2} = 110 \text{ MeV}$

(MONOCROMATICO!)

$\Rightarrow \quad \beta_\mu^* = \dfrac{p^*}{E_\mu^*} = \dfrac{30}{110} = 0.27$

$\mu$ sempre in avanti se $\beta_{cm} > \beta^*$

$\beta_{cm} = \beta_\pi = \dfrac{p_\pi}{E_\pi} \leftarrow$ nel LAB

$\Rightarrow$ serve $\beta_\pi > 0.27$

$\Leftrightarrow \quad \gamma_\pi > \dfrac{1}{\sqrt{1-\beta_\pi^2}} = 1.04 = \dfrac{E_\pi}{m_\pi}$

$\Rightarrow \quad E_\pi > 1.04\, m_\pi = 1.04 \cdot 140 \text{ MeV} = 145 \text{ MeV}$

$\Leftrightarrow \quad p_\pi = \sqrt{E_\pi^2 - m_\pi^2} = \sqrt{145^2 - 140^2} = 38 \text{ MeV}$

**EX** Un $\bar{p}$ con $p_{\bar{p}} = 2.2$ GeV urta contro un bersaglio dando luogo a

$$\bar{p} + p \rightarrow \Lambda + \bar{\Lambda}$$

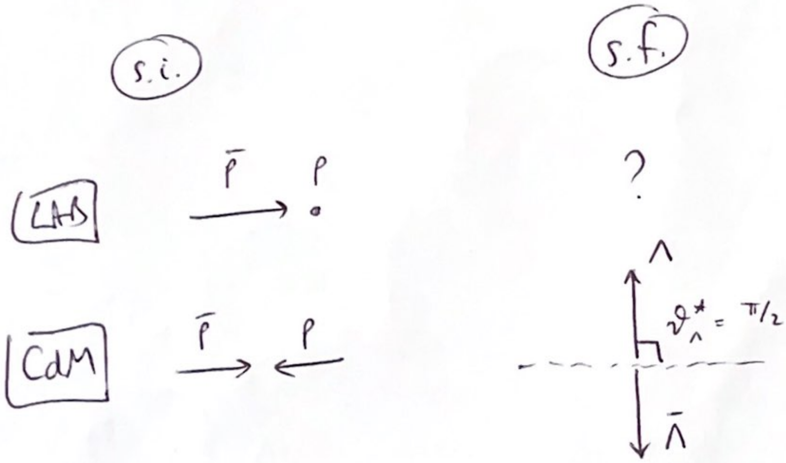
$m_p = 938$ MeV

$m_\Lambda = 1116$ MeV

Supponiamo che $\Lambda$ nel CdM è prodotta con

$$\vartheta^*_\Lambda = 90°$$

(a) $p^*_\Lambda = ?$




Comincio con calcolare $\sqrt{s}$ che è uguale ovunque

$\Rightarrow$ per comodità lo calcolo nel LAB nello s.i.

$\bar{p} \longrightarrow p$ con $p_{\bar{p}} = 2.2$ GeV

$$\begin{pmatrix} E_{\bar{p}} \\ p_{\bar{p}} \end{pmatrix} \qquad \begin{pmatrix} m_p \\ \vec{0} \end{pmatrix}$$

$$\Rightarrow E_{\bar{p}} = \sqrt{m_p^2 + p_{\bar{p}}^2} = 2.39 \text{ GeV}$$

$$p_{TOT} = \begin{pmatrix} E_{\bar{p}} + m_p \\ \vec{p}_{\bar{p}} \end{pmatrix}$$

$$\Rightarrow \sqrt{s} = |p_{TOT}| = \sqrt{(E_{\bar{p}} + m_p)^2 - p_{\bar{p}}^2} = \sqrt{E_{\bar{p}}^2 + m_p^2 + 2E_{\bar{p}} m_p - p_{\bar{p}}^2}$$

$$= \sqrt{m_p^2 + m_p^2 + 2E_{\bar{p}} m_p} = \sqrt{2m_p^2 + 2E_{\bar{p}} m_p}$$

$$= 2.50 \text{ GeV}$$

Poi sappiamo che

$$\sqrt{s} = \sum_i^{\text{stato}} E_i^* \quad \leftarrow \text{in (s.i.) ma anche (s.f.)}$$

$$\Rightarrow \sqrt{s} \overset{\text{s.f.}}{=} E_\Lambda^* + E_{\bar{\Lambda}}^* = 2E_\Lambda^*$$

$|p_\Lambda^*| = |p_{\bar{\Lambda}}^*|$ e $m_\Lambda = m_{\bar{\Lambda}}$

$$\Rightarrow E_\Lambda^* = \frac{\sqrt{s}}{2} = 1.25 \text{ GeV}$$

$$p_\Lambda^* = \sqrt{E_\Lambda^{*2} - m_\Lambda^2} = 0.56 \text{ GeV}$$

**(b)** $E_\Lambda, p_\Lambda = ?$ nel LAB

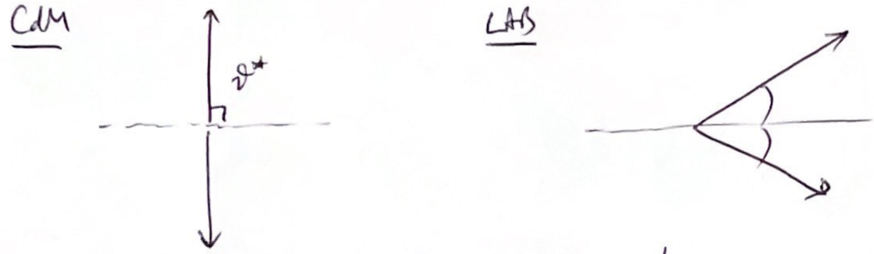
nel LAB:

$$\overset{\text{(s.i.)}}{\begin{pmatrix} E_{\bar{p}} + m_p \\ \vec{p}_{\bar{p}} \end{pmatrix}} = \overset{\text{(s.f.)}}{\begin{pmatrix} E_\Lambda + E_{\bar{\Lambda}} \\ \vec{p}_\Lambda + \vec{p}_{\bar{\Lambda}} \end{pmatrix}}$$

↳ dalla prima $E_{\bar{p}} + m_p = E_\Lambda + E_{\bar{\Lambda}} \equiv 2E_\Lambda$

infatti ⟶

se $\vartheta^* = \frac{\pi}{2}$




deve rimanere simmetrico

$\Rightarrow E_\Lambda = \frac{E_{\bar{p}} + m_p}{2} = 1.66 \text{ GeV}$ $\quad p_\Lambda = \sqrt{E_\Lambda^2 - m_\Lambda^2} = 1.23 \text{ GeV}$

(c) $\vartheta_\Lambda = ?$ $\quad p_\perp = p_\perp^*$ (è in generale) e qui $p_\perp^* = p^* = 0.56 \text{ GeV}$

$\Rightarrow (p_\Lambda)_\perp = 0.56 \text{ GeV}$

e poi $(p_\Lambda)_\parallel = \sqrt{(p_\Lambda)^2 - (p_\Lambda)_\perp^2} = 1.1 \text{ GeV}$

$\Rightarrow \vartheta = \tan^{-1}\left(\frac{p_\perp}{p_\parallel}\right) = 0.47 \simeq 27°$

(d) se $\tau_\Lambda = 2.63 \cdot 10^{-10} \text{ s}$ calcolare cammino libero medio di $\Lambda$ nel LAB

$\gamma_\Lambda = \frac{E_\Lambda}{m_\Lambda} = 1.49$ $\quad \beta_\Lambda = \frac{p_\Lambda}{E_\Lambda} = 0.74$

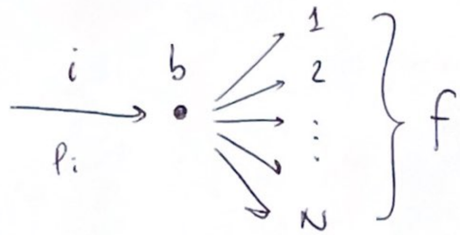
$\tau_\Lambda \rightarrow \gamma_\Lambda \tau_\Lambda$ (nel LAB)

$\Rightarrow \lambda_\Lambda = \beta_\Lambda \gamma_\Lambda c \tau_\Lambda = 0.087 \text{ m} = 8.7 \text{ cm}$

# ENERGIA DI SOGLIA

nel LAB:

s.i. $\begin{pmatrix} E_i \\ p_i \end{pmatrix} + \begin{pmatrix} m_b \\ \vec{0} \end{pmatrix}$




s.f. $\sum_f \begin{pmatrix} E_f \\ p_f \end{pmatrix}$

$K \equiv E - m$ energia cinetica

$$\sqrt{s}\Big|_{\substack{s.f.\\ CdM}} = \underbrace{\sum_f E_f^*}_{} = \sum_f \left( m_f + K_f^* \right)$$

visto altre volte

$$\sqrt{s}\Big|_{\substack{s.i.\\ LAB}} = \left| P_{TOT} \right|_{s.i.} = \sqrt{E_{TOT}^2 - P_{TOT}^2}$$

$$\text{s.i. nel LAB} = \begin{pmatrix} E_i \\ p_i \end{pmatrix} + \begin{pmatrix} m_b \\ \vec{0} \end{pmatrix}$$

$$\Rightarrow \sqrt{s}\Big|_{\substack{s.i.\\ LAB}} = \sqrt{(E_i + m_b)^2 - p_i^2} = \sqrt{\underbrace{E_i^2 + m_b^2 + 2E_i m_b - p_i^2}_{m_i^2}}$$

$$= \sqrt{m_i^2 + m_b^2 + 2E_i m_b}$$

ora $\sqrt{s}\Big|_{\substack{s.f.\\ CdM}} = \sqrt{s}\Big|_{\substack{s.i.\\ LAB}}$

$$\Rightarrow \sqrt{m_i^2 + m_b^2 + 2E_i m_b} = \sum_f \left( m_f + K_f \right) \geq \sum_f m_f$$

quadro

$$\Leftrightarrow \quad m_i^2 + m_b^2 + 2E_i m_b \geq \left(\sum_f m_f\right)^2$$

$$\Rightarrow \quad E_i \geq \frac{\left(\sum_f m_f\right)^2 - m_i^2 - m_b^2}{2m_b} \equiv \text{energia di soglia}$$

$$\Leftrightarrow \quad K_{soglia} = E_{soglia} - m_i = \frac{\left(\sum_f m_f\right)^2 - (m_i + m_b)^2}{2m_b}$$

$\boxed{EX}$ Nell'esercizio di prima $\bar{p} + p \to \Lambda + \bar{\Lambda}$

$m_p = 938$ MeV
$m_\Lambda = 1116$ MeV

$\xrightarrow{\bar{p}} \overset{p}{\bullet}$

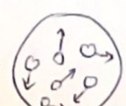
$$E_{soglia}(\bar{p}) = \frac{(2m_\Lambda)^2 - m_p^2 - m_p^2}{2m_p} = \frac{4m_\Lambda^2 - 2m_p^2}{2m_p} = 1.72 \text{ GeV}$$

$$\Leftrightarrow \quad K_{soglia} = E_{soglia} - m_p = (1.72 - 0.938) = 780 \text{ MeV}$$

$\boxed{\text{EX PER CASA}}$ Calcolare energia di soglia di

$$p + p \to p + p + p + \bar{p}$$

come cambia soglia se invece consideriamo ~~atomi~~ nuclei del bersaglio come gas di Fermi

$\xrightarrow{p} \overset{p}{\bullet} \quad \Rightarrow \quad \xrightarrow{p}$ 

$\langle p \rangle \sim 240$ MeV $\equiv p_F$
direzione casuale