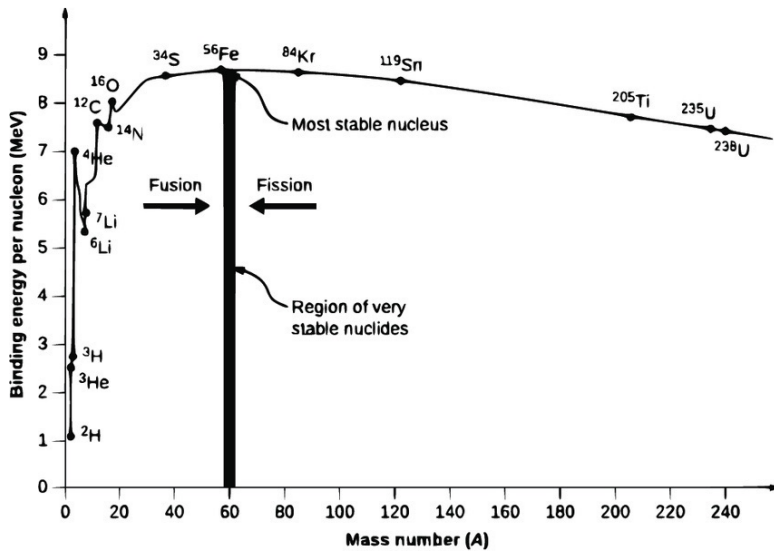# Forza Nucleare

$$m(Z,A) = M_p \cdot Z + m_n(A-Z) - B(Z,A)$$

$B(Z,A) \geq 0$.
energia di legame nucleare

$7.5 \rightarrow 8.5$ MeV/nucleone

Binding energy per nucleon (MeV) vs Mass number (A): ^{34}S, ^{56}Fe, ^{84}Kr, ^{119}Sn, ^{16}O, ^{12}C, ^{14}N, ^4He, ^7Li, ^6Li, ^3H, ^3He, ^2H, ^{205}Ti, ^{235}U, ^{238}U. Fusion → ← Fission. Most stable nucleus. Region of very stable nuclides.

Forze nucleare

$r = 0$
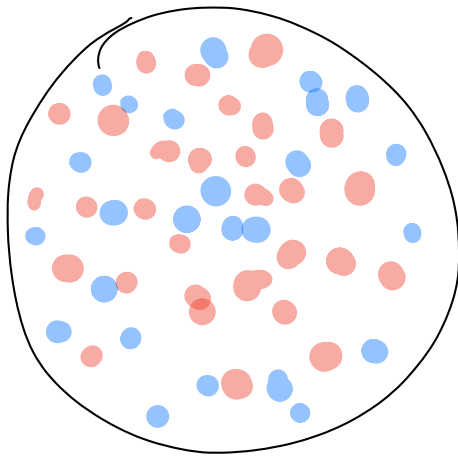
$r > r_N$: forza di coulomb

$r_N = r_0 A^{1/3}$

$r >> r_N$: coulomb / EM

$r < r_N$: Forte/nucleare

nucleone: protone / neutrone
$$S = 1/2$$

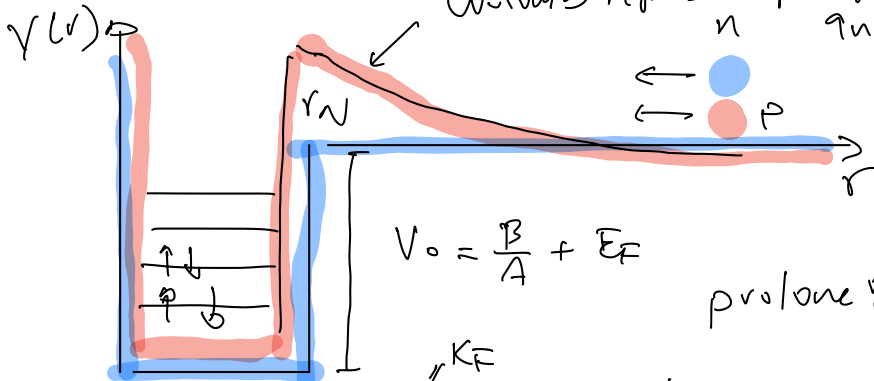$$R \simeq r_0 A^{1/3} \qquad r_0 = 1.1 \text{ fm} = 1.1 \times 10^{-15} \text{ m}$$

All'interno del nucleo $(r < r_N)$: gas di Fermi di nucleoni

Coulomb repulsive per protoni:

$q_n = 0 \Rightarrow$ nessuna coulomb repulsive

$V(r)$

r_N

$$V_0 = \frac{B}{A} + E_F$$

K_F

E_F: en. cinetica massime per i nucleoni

protone: $U(r) = \begin{cases} \dfrac{1 \times Z}{r} \propto & r > r_N \\ -V_0 & r < r_N \end{cases}$
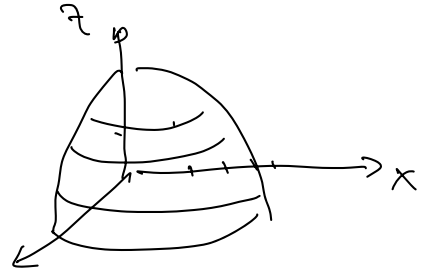
P_F: impulso di Fermi

Gas di Fermi nella buca di potenziale

$$E_F = \sqrt{P_F^2 + m_n^2} \simeq m_n + \frac{P_F^2}{2m_n} \qquad K_F = \frac{P_F^2}{2m_n} \qquad M_n \simeq 1 \, GeV$$

$$\int_0^{P_F} dn_{neut} = 2 \int_0^{P_F} dn = (A - Z) \qquad \text{neutroni}$$

$$\int_0^{P_F} dn_{protoni} = \boxed{2} \int_0^{P_F} dn = Z \qquad \text{protoni}$$

$\uparrow \downarrow \quad$ x2 per spin di nucleone

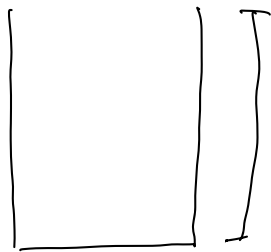$$dn = \frac{V}{(2\pi)^3} d^3P = \frac{V}{(2\pi)^3} 4\pi \, P^2 dP$$

$$P_F = (240 \, MeV) \times \begin{cases} \left(\frac{2Z}{A}\right)^{1/3} & \text{protoni} \\[2mm] \left(\frac{2(A-Z)}{A}\right)^{1/3} & \text{neutroni} \end{cases} \quad \left. \begin{array}{l} \text{mom/impulso} \\ \text{massimo dei} \\ \text{nucleoni} \end{array} \right\}$$

per maggior parte dei nuclidi leggeri $\quad \frac{Z}{A} \simeq \frac{1}{2}$

$$P_F \simeq 240 \, MeV \ll m_p/m_n \implies K_F = \frac{P_F^2}{2m_n}$$

$K_F \simeq 20 \, MeV \ll m_{n/p}$: En. cinetica massima dei nucleoni nella buca di potenziale.

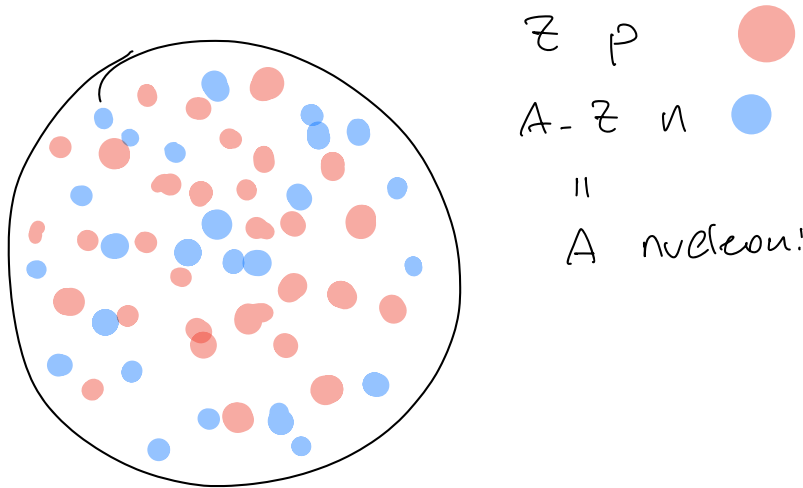$$V_0 \simeq K_F + \frac{B}{A} \simeq 20 + 10 \, MeV = 30 \, MeV$$
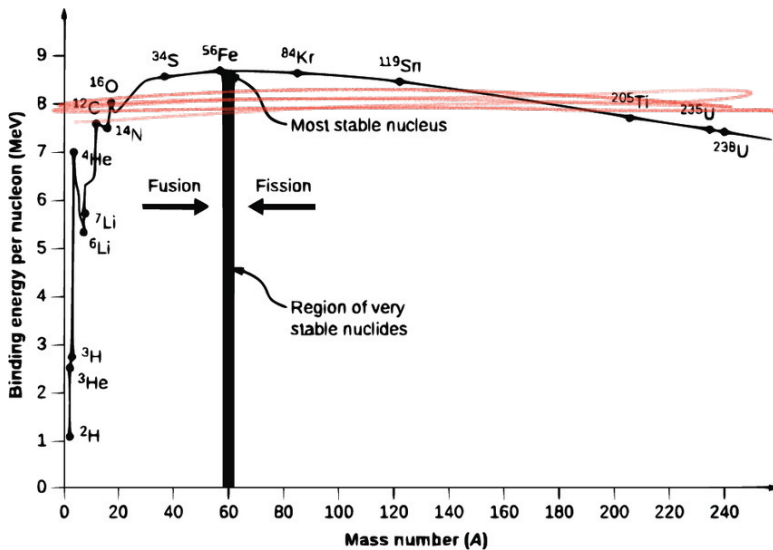$\uparrow$
40 MeV

Ener. Cinetica media

$$\langle E_C \rangle = \frac{\int_0^{P_F} \frac{P^2}{2m} \, dn}{\int_0^{P_F} dn} = (9 \, MeV) \left[\left(\frac{2Z}{A}\right)^{5/3} + \left(\frac{2(A-Z)}{A}\right)^{5/3} \right]$$

Come stimare $B(Z, A)$?

1) tutti nucleoni interagiscono con tutti nucleoni:
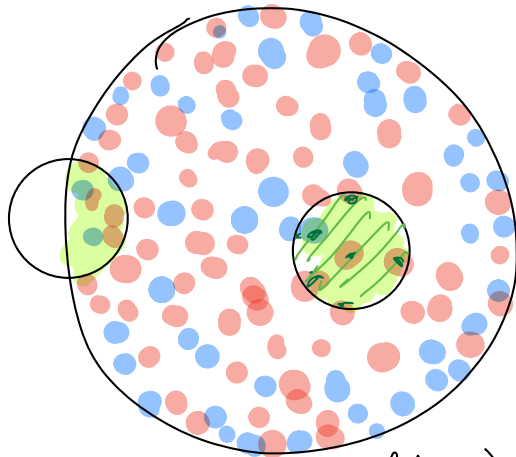

Z p ●(red)

$A-Z$ n ●(blue)

$\|$

A nucleoni

$$B(Z,A) \propto \frac{A(A-1)}{2} \text{ contributi} \Rightarrow \frac{B(Z,A)}{A} \propto A$$

contraddetto dai dati

$$\frac{B}{A} \simeq cost. = 7.-8 \text{ MeV}$$

2) **Modello a goccia**

$r_N \simeq r_0 A^{1/3}$

nucleoni interagiscono con tutti gli altri nucleoni:
con $r < r_{int}$

V_{int}: entro il quale contano contributi a $B(Z,A)$

$$B(Z,A) = \sum_{i \atop nucleoni} \frac{A(A-1)}{2} \underbrace{\frac{V_{int}}{V_{nucleare}}}_{} \langle U \rangle$$

$\langle U \rangle$ en media di legame

< 1 $r_{int} < r_N$

$$V_{nucleone} = \frac{4\pi}{3} r_N^3 = \frac{4\pi}{3} r_0^3 A \qquad r_N = r_0 A^{1/3} : \text{viene dalle misure sperim.}$$

$$B(Z,A) \simeq \frac{A^2}{A} \times (\text{cost}) \simeq A$$

$$\Rightarrow \frac{B(Z,A)}{A} \simeq \text{cost}$$

$$B(Z,A) = a_V A \qquad : \quad \text{termine di Volume}$$
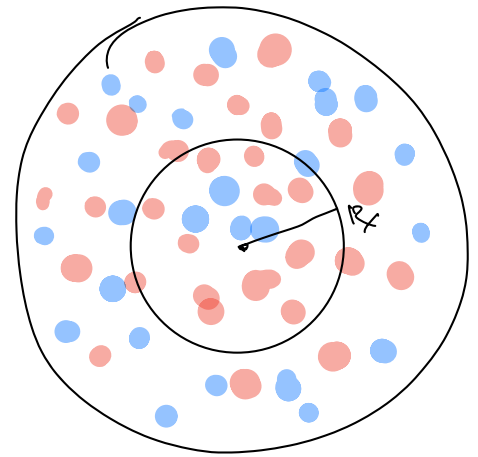
$$a_V : \text{costante}$$

Termine di superficie

$$S_N \simeq 4\pi r_N^2 = 4\pi r_0^2 A^{2/3}$$

$$B(Z,A) = a_V A - a_S A^{2/3}$$

Repulsione di Coulomb tra i protoni:

$$\rho(r) = \frac{Ze}{V_{nuc.}} = \frac{Ze}{\frac{4\pi}{3} r_0^3 A} = \frac{Ze}{\frac{4\pi}{3} R_N^3}$$

Per $R < R_X$ R_X $\curvearrowright$ Pot. di Coulomb

$$E_{coul.} = \int_0^{} d^3r \; \rho(r) \, V(r)$$

$$V(r) = \frac{1}{4\pi} \frac{q(r)}{r} \qquad\qquad \text{Volume di } r$$

$$q(r) = \rho(r) \cdot V(r) = \rho \cdot \frac{4\pi}{3} r^3$$

$$E_{coul.} = \int d^3r \; \rho \; \frac{4\pi}{3} r^3 \; \frac{1}{r} \; \frac{1}{4\pi} \; \rho \qquad \propto \rho^2 \int r^4 dr \propto \rho^2 R_X^5$$

$$\underbrace{\quad}_{4\pi r^2 dr}$$

$$E_{coul} = \int_0^R d^3r \; \rho(r) V(r) \sim \rho^2 R_N^5 \simeq \frac{Z^2}{R_N^6} R_N^5 \sim R_N^{-1} Z^2$$

$$E_{coul} \propto Z^2 A^{-1/3}$$

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3}$$

Termine correzione dovuta a en. cinetica dei nucleoni

$$K_{tot} = A \langle \bar{k} \rangle = (20 \text{ MeV}) \times \left(A + \frac{5}{9} \frac{(A-2Z)^2}{A} \right)$$

$\underbrace{\phantom{\langle \bar{k} \rangle}}$ en. cinetica media fino a impulso di Fermi

$$B(Z,A) = a_V A - a_S A^{+2/3} - a_C Z^2 A^{-1/3} - a_F \frac{(A-2Z)^2}{A}$$

modello a goccia

Fit ai dati sperimentali

$a_V \approx 16$ MeV

$a_S \approx 18$ MeV

$a_C \approx 0.7$ MeV

$a_F \approx 93$ MeV

La Formula di Bethe — Weizsäcker

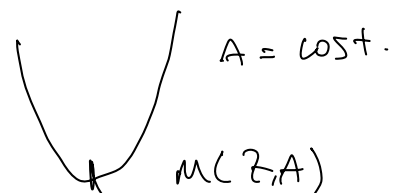$$M(Z,A) = M_p \cdot Z + M_n (A-Z) - B(Z,A)$$

Caratteristiche principali

1) $A = \text{cost}$ (nuclidi isobari) $\frac{Z}{A} \approx \frac{1}{2}$

$$B(Z,A) \approx \alpha + \beta Z + \gamma Z^2 \quad \text{funzione parabolica di } Z$$

$M(Z,A) \approx$ parabola con un minimo
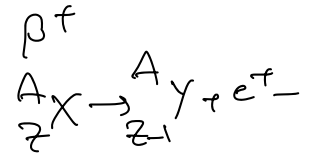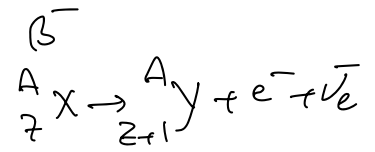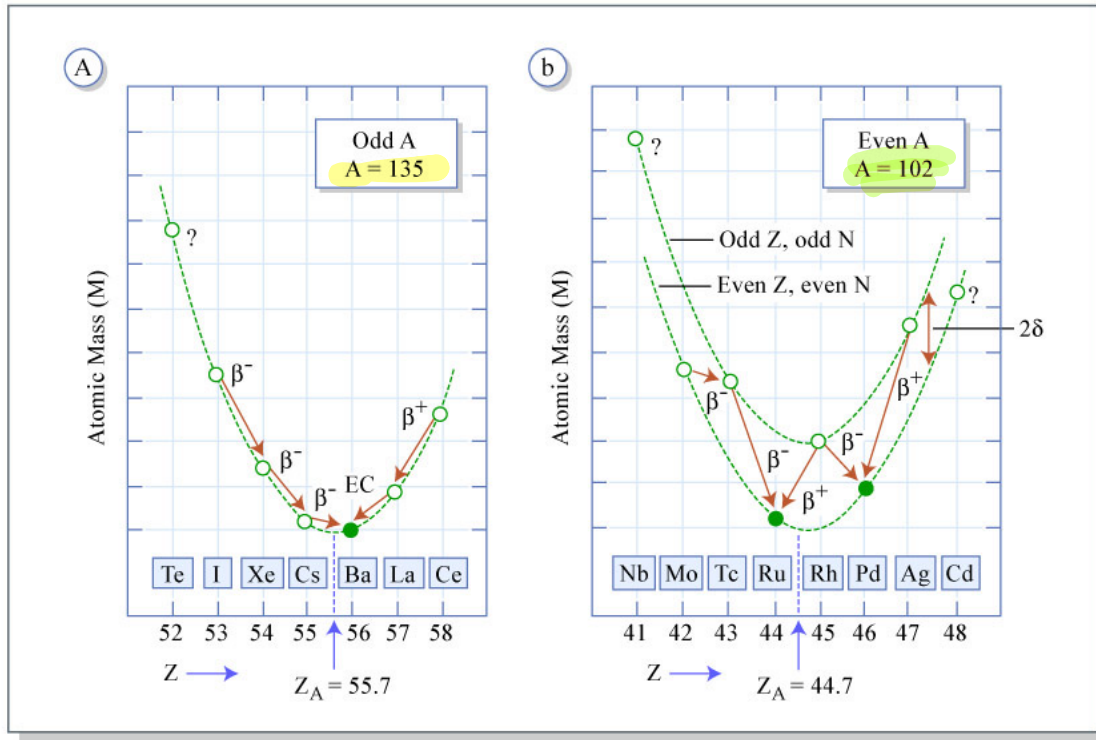
$\Rightarrow$ esiste Z_{min} @ A cost

A = cost.

$M(Z,A)$

2) sperimentalmente differenze per nuclei pari/dispari

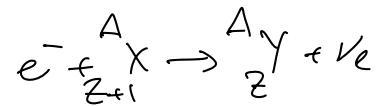| Z	A-Z	A	ΔB
pari	pari	pari	$+\delta$
		dispari	$\emptyset$
	dispari		
dispari	dispari	pari	$-\delta$

$$\delta = a_\delta A^{-1/2}$$

$a_\delta \approx 11$ MeV

$$B(Z,A) = a_V A - a_S A^{+2/3} - a_C Z^2 A^{-1/3} - a_F \frac{(A-2Z)^2}{A} - \begin{cases} +\delta \\ \emptyset \\ -\delta \end{cases}$$

β^-
$$^A_Z X \rightarrow ^A_{Z+1} Y + e^- + \bar{\nu}_e$$

β^+
$$^A_Z X \rightarrow ^A_{Z-1} Y + e^+ -$$

EC: electronic capture
$$e^- + ^A_{Z+1} X \rightarrow ^A_Z Y + \nu_e$$

$$m(Z,A) = Z \cdot m_p + (A-Z) m_n - B(Z,A) + \begin{cases} +\delta \\ \emptyset \\ -\delta \end{cases}$$

$$\left. \frac{\partial m}{\partial Z} \right|_{A=\text{cost}} = \emptyset \quad \Rightarrow \quad Z_{min} = \frac{A}{2} \underbrace{\frac{1}{1 + 0.0076 \, A^{2/3}}}_{\leq 1}$$

$$Z \leq \frac{A}{2}$$