$A \qquad [A,H]=0 \qquad$ simmetria.

$A|\psi\rangle \qquad \langle\psi|A|\psi\rangle = \langle A\rangle$

$$\frac{d}{dt}\langle A\rangle = \frac{d}{dt}\langle\psi|A|\psi\rangle = \left(\frac{d}{dt}\langle\psi|\right)A|\psi\rangle + \langle\psi|\frac{\partial A}{\partial t}|\psi\rangle + \langle\psi|A\frac{d}{dt}|\psi\rangle$$

$$i\frac{d}{dt}|\psi\rangle = H|\psi\rangle \qquad -i\langle\psi|\frac{d}{dt} = \langle\psi|H$$

$$= \langle\psi|[A,H]|\psi\rangle + \left\langle\frac{dA}{dt}\right\rangle$$

se A non dipende dal tempo

$$\frac{d\langle A\rangle}{dt} = 0 \quad \longleftrightarrow \quad [A,H]=0$$

Traslazione	$\vec{r}$	$\vec{P}$
traslazione temporale	t	E
rotazione in $\vec{r}$		$\vec{J} = \vec{r}\times\vec{p}$

Simmetria invariante del sistema sotto una trasf. $\longleftrightarrow$ quantità conservate

Teorema di Nöther 1917

$P|\psi\rangle \quad , \quad \vec{r} \to -\vec{r}$

parità discrete $\longleftrightarrow$ $\lambda = \pm 1 \quad e^{i\alpha}$

$T \qquad t \to -t$

C : coniugazione di carica.

$\pi^+ \longrightarrow \pi^-$

trasf $\quad TT = T^2 = \mathbb{1}$

CP $\longrightarrow$ cariche opposte

$\vec{r} \to -\vec{r}$

particelle $\longrightarrow$ antiparticelle.

$K^0 \to \pi^+\pi^-$

$\pi^+\pi^-\pi^0$ $\quad$ 1964

Interazioni deboli $\cancel{P}$ $\cancel{CP}$

CPT

$e^+ + e^- \longrightarrow 2\gamma$

Isospin : Heisenberg

rotazione nello spazio di isospin

nucleoni: protoni, neutroni

interazioni forti non distinguono n da p.

m_p $\quad \Delta m = 1.6$ MeV

m_n $\quad m \simeq 1000$ MeV.

$$\text{nucleone} = \begin{pmatrix} p \\ n \end{pmatrix} \qquad I = \frac{1}{2}$$

I_3

$$|p\rangle = |\tfrac{1}{2}, \tfrac{1}{2}\rangle \qquad |n\rangle = |\tfrac{1}{2}, -1/2\rangle$$

Per interazioni forti conta solo I

H dipende solo da I non da I_3 $\Rightarrow$ non distingue n da p.

$\pi^{\pm} \simeq 140$ MeV

$\pi^{0} \simeq 135$ MeV

$$|\pi\rangle = \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} = \begin{pmatrix} I_3 = +1 \\ 0 \\ -1 \end{pmatrix} \quad I=1$$

Isospin : nuovo numero quantico.

spin: num. quantico diverso

deuterio

$d = pn$

$|p\rangle = |1/2\rangle |1/2\rangle \qquad |n\rangle = |1/2, -1/2\rangle$

(pn) tripletto

$|1,1\rangle = |1/2, 1/2\rangle \, |1/2, 1/2\rangle \longrightarrow$ (pp)

$|1,0\rangle = \frac{1}{\sqrt{2}}\left(|1/2, 1/2\rangle + |1/2, -1/2\rangle\right)$ (pn)

$|1,-1\rangle = |1/2, -1/2\rangle \, |1/2 -1/2\rangle$ (nn)

(pn) singoletto

$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|1/2, 1/2\rangle - |1/2, -1/2\rangle\right)$ (pn)

Per natura solo pn come deuterio.

(i) $|1,1\rangle$

$p + p \longrightarrow d + \pi^+$ (a) $|1,1\rangle$

$\frac{1}{2} \quad \frac{1}{2} \qquad 0 \quad +1 = I$

$+\frac{1}{2} \; +\frac{1}{2} \qquad 0 \quad +1 = I_3$

$p + n \longrightarrow d + \pi^0$ (b) $|1,0\rangle$

$\frac{1}{\sqrt{2}}\left(|1,0\rangle + |0,0\rangle\right)$

$0 \quad 1 = I$

$+\frac{1}{2} \; -\frac{1}{2} \qquad 0 \quad 0 = I_3$

$$n + n \rightarrow d + \pi^-$$

$I_3 = -\frac{1}{2} \; -\frac{1}{2} \qquad 0 \quad -1$

(c) $|1,-1\rangle$

$0 \quad 1$

$I = 1 \; |1,-1\rangle$

$$\sigma(a+b \rightarrow c+d) \propto |M_{fi}|^2$$

$\langle f | H_I | i \rangle$

(a) $\langle 1,1 | H_I | 1,1 \rangle = M_a$

(b) $\langle 1,0 | H_I \left(\frac{1}{\sqrt{2}} |1,0\rangle + |0,0\rangle \right) = M_b$

Se Int. forte conserva isospin.

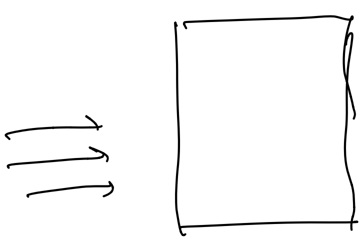
$$\frac{1}{\sqrt{2}} \langle 1,0 | H_I | 1,0 \rangle + \frac{1}{\sqrt{2}} \underbrace{\langle 1,0 | H_I | 0,0 \rangle}_{=0}$$

Misuro

$$\frac{\sigma_a}{\sigma_b} \simeq \frac{|M_a|^2}{|M_b|^2} \simeq \frac{1}{|\frac{1}{\sqrt{2}}|^2} \simeq 2$$

$$\frac{dN_r}{dt} = \sigma \cdot \frac{dN_p}{dt} \cdot n_b \cdot d.$$

p



mi aspetto.
$N(a) \simeq 2N(b)$

(c)

verifica sperimentale:

nucleoni contro pioni:

$I = \frac{1}{2}$ $\qquad$ $I = 1$

$\pi^+ + p \longrightarrow \pi^+ + p$ $\qquad |3/2, 3/2\rangle$

$|\frac{1}{2}, \frac{1}{2}\rangle |1, 1\rangle$

$|3/2, +3/2\rangle$

conservano

$(\pi^+, \pi^0, \pi^-) + (p, n) \longrightarrow$ q, I, N barionico.

$\pi^0 + p \longrightarrow \pi^0 + p$

$\pi^- + p \longrightarrow \pi^- + p$

$|1, -1\rangle |\frac{1}{2}, \frac{1}{2}\rangle$ $\qquad I_3 = -1/2$

$(|3/2, -1/2\rangle + |1/2, -1/2\rangle)$

$\pi^- + p \longrightarrow \pi^0 + n$

pione + nucleone.

$|I = 1, a\rangle + |I = 1/2, b\rangle =$

$= \alpha |I = 3/2, a+b\rangle + \beta |I = 1/2, a+b\rangle$

$|i\rangle = \alpha |I = 3/2\rangle + \beta |I = 1/2\rangle$

$|f\rangle = \gamma |I = 3/2\rangle + \delta |I = 1/2\rangle$

Se H_I forte conserva isospin.

$M_{fi} = \langle f | H_I | i\rangle = a \langle I = 3/2 | H_3 | I = 3/2\rangle +$

$b \langle I = 1/2 | H_1 | I = 1/2\rangle$

$\sigma \propto |\mathcal{M}_{fi}|^2 = a^2 |\mathcal{M}_3|^2 + b^2 |\mathcal{M}_1|^2$

+ termini di interferenza.

$|\mathcal{M}_{fi}|^2 = \mathcal{M}_{fi} \cdot \mathcal{M}_{fi}^* = (a\mathcal{M}_3 + b\mathcal{M}_1)(a^* \mathcal{M}_3^* + b^* \mathcal{M}_1^*)$

Table 3.3. *Clebsch–Gordan coefficients in pion–nucleon scattering*

		$I = \frac{3}{2}$				$I = \frac{1}{2}$	
Pion	Nucleon	$I_3 = \frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
π^+	p	1					
π^+	n		$\sqrt{\frac{1}{3}}$			$\sqrt{\frac{2}{3}}$	
π^0	p		$\sqrt{\frac{2}{3}}$			$-\sqrt{\frac{1}{3}}$	
π^0	n			$\sqrt{\frac{2}{3}}$			$\sqrt{\frac{1}{3}}$
π^-	p			$\sqrt{\frac{1}{3}}$			$-\sqrt{\frac{2}{3}}$
π^-	n				1		

a) $\pi^+ + p \to \pi^+ + p$ $\qquad \sigma_a \propto |\mathcal{M}_3|^2$

c) $\pi^- + p \to \pi^- + p$ $\qquad \sigma_c \propto \frac{1}{9} |\mathcal{M}_3 + 2\mathcal{M}_1|^2$

j) $\pi^- + n \to \pi^- + n$ $\qquad \sigma_j \propto \frac{2}{9} |\mathcal{M}_3 - \mathcal{M}_1|^2$

① $\mathcal{M}_3 >> \mathcal{M}_1$ $\qquad \sigma_a : \sigma_b : \sigma_c = 1 : \frac{1}{9} : \frac{2}{9}$

② $\mathcal{M}_1 >> \mathcal{M}_3$ $\qquad \sigma_a : \sigma_b : \sigma_c = 0 : \frac{4}{9} |\mathcal{M}_1|^2 : \frac{2}{9} |\mathcal{M}_1|^2$

$\pi^+ + p$ avviene $\Rightarrow$ $\cancel{|\mathcal{M}_1| >>} \mathcal{M}_3$

$\sigma_a : (\sigma_c + \sigma_j)$ $\qquad \dfrac{\pi^+ + p \to \pi^+ + p}{(\pi^- + (p,n) \to \pi^- + (p,n))}$

$\Rightarrow \sigma(\pi^+ + p) \simeq 3\, \sigma(\pi^- + \ldots)$

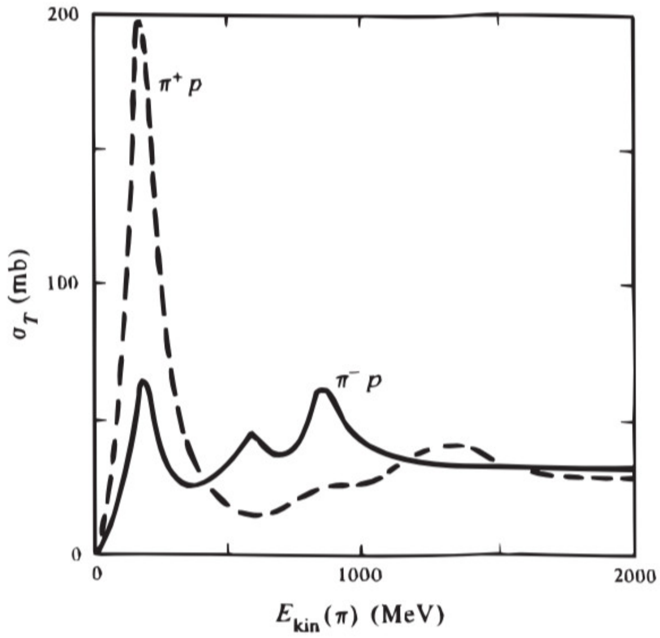


Figure 5.35: Total cross section as a function of pion kinetic energy for the scattering of positive and negative pions from protons. (1 mb = 1 millibarn = 10^{-27} cm^2.)