$\alpha + N \to \alpha + N.$

$|i\rangle = |\alpha N\rangle \qquad |f\rangle = |\alpha N\rangle$

assumiamo N fermo; urto elastico contro il muro.

$\Rightarrow$ solo $\vec{p}_\alpha$ può variare

$$\Gamma(i \to f) = 2\pi\, |M_{fi}|^2\, \rho(E).$$

$$M_{fi} = -i \int_V d^3V\, \psi_f^* H_I \psi_I$$

Approssimazione di Born.

$$\psi_i = \frac{e^{i\vec{p}\cdot\vec{r}}}{\sqrt{V}}$$ onde libere

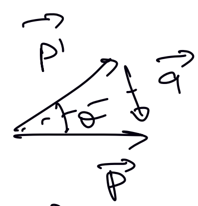
$$\int_V |\psi|^2 d^3r = 1 \Rightarrow \psi \propto \frac{1}{\sqrt{V}}$$

V: volume di rif per normalizz. ψ.

$$\psi_i = \frac{e^{i\vec{p}\cdot\vec{r}}}{\sqrt{2\pi}} \qquad \psi_f = \frac{e^{i\vec{p}'\cdot\vec{r}}}{\sqrt{2\pi}}$$

$$H_I = \frac{e^2}{4\pi\varepsilon_0}\frac{Z_\alpha \cdot Z_N}{r} = \frac{\alpha Z_\alpha Z_N}{r} = \frac{A}{r}$$

$$M_{fi} = -i \int d^3r\, \frac{e^{-i\vec{p}'\cdot\vec{r}}}{\sqrt{V}} \frac{A}{V}\frac{e^{+i\vec{p}\cdot\vec{r}}}{\sqrt{V}} = -i\frac{A}{V}\int d^3r\, \frac{e^{-i(\vec{p}'-\vec{p})\cdot\vec{r}}}{r}$$



$q = 2p\sin\frac{\theta}{2}$

$|\vec{p}| = |\vec{p}'|$

$-\vec{p}' + \vec{p} = \vec{q} \Rightarrow \vec{p} = \vec{p}' + \vec{q}$

$$M_{fi} = -i\frac{A}{V}\int d^3r\, \frac{e^{i\vec{q}\cdot\vec{r}}}{r}$$

$d^3r = \sin\theta\, d\theta\, d\varphi\, r^2 dr$

$\vec{q}\cdot\vec{r} = qr\cos\theta$

$$\int_0^\pi \sin\theta\, d\theta = -\int_1^{-1} d\cos\theta = \int_{-1}^{1} d\cos\theta.$$

$$\int_{-1}^{1} d\cos\theta\, e^{iqr\cos\theta} = \frac{1}{iqr}\left[e^{iqr} - e^{-iqr}\right].$$

$$M_{fi} = -i\frac{A}{V}\int_0^{2\pi} d\varphi \int_0^\infty r^2 dr \ \frac{1}{r} \ \frac{1}{iqr}\left[e^{iqr} - e^{-iqr}\right]$$

$$= -i\frac{A}{V}(2\pi)\frac{1}{iq}\int_0^\infty \left[e^{iqr} - e^{-iqr}\right] dr$$

anzichè usare $V(r) = \frac{A}{r}$ usiamo $V(r) = A\frac{e^{-\lambda r}}{r}$

e poi alla fine $\lim_{\lambda\to 0}$

$$M_{fi} = -\frac{iA}{V}\frac{2\pi}{iq}\lim_{\lambda\to 0}\int_0^\infty e^{-\lambda r}\left(e^{iqr} - e^{-iqr}\right) dr$$

$\alpha = \lambda - iq$

$$\int_0^\infty e^{-\lambda r} e^{iqr} dr = \frac{1}{\lambda - iq}\int_0^\infty \alpha \ e^{-\alpha r} dr$$

$$= \frac{1}{\lambda - iq}\int_0^\infty e^{-z} dz = \frac{1}{\lambda - iq}\left[0 - 1\right]$$

analogamente

$$\int_0^\infty e^{-\lambda r} e^{-iqr} dr = \frac{1}{\lambda + iq}\left[0 - 1\right].$$

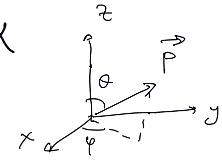
$$\int_0^\infty e^{-\lambda r}\left(e^{iqr} - e^{-iqr}\right) dr = \frac{1}{\lambda - iq} - \frac{1}{\lambda + iq} = \frac{\lambda + iq - \lambda + iq}{\lambda^2 - (-q^2)}$$

$$= \frac{2iq}{\lambda^2 + q^2}$$

$$M_{fi} = \lim_{\lambda\to 0}\left[-i\frac{2\pi}{V}\frac{A}{i\cancel{q}}\frac{2i\cancel{q}}{\lambda^2 + q^2}\right] = -2iA\frac{1}{q^2}\frac{2\pi}{V}$$

$$\Rightarrow |M_{fi}|^2 = \frac{4A^2}{q^4}\frac{(2\pi)^2}{V^2}$$

$$\rho(E) = \int dn \ \delta(E_f - E_i)$$



$$dn = \frac{V}{(2\pi)^3}\, p^2 dp\, d\Omega$$

p: impulso delle particelle α

particelle α non relativistiche.

$$E = \frac{p^2}{2m} \Rightarrow p^2 = 2mE \Rightarrow \cancel{2}p\,dp = \cancel{2}m\,dE$$

$$p\,dp = m\,dE$$
$$p = \sqrt{2mE} \Rightarrow p^2 dp = p\, m\, dE$$

$$\rho(E) = \int dn\, \delta(E_f - E_i) = \frac{V}{(2\pi)^3}\int p^2 dp\, \delta(E_f - E_i)\, d\Omega$$

$$= \frac{V}{(2\pi)^3}\, d\Omega \int p\, m\, \delta(E_f - E_i)\, dE = \frac{V}{(2\pi)^3}\, d\Omega\, m \underbrace{\sqrt{2mE_i}}_{= p_i}$$

p del proiettile.

Ricordiamo di nuovo che

$$\Gamma(i \to f) = \frac{dN_r}{dt}\frac{1}{N_B}\frac{1}{N_P} = \sigma \cdot \frac{v_P}{V}$$

velocità proiettile

$$= 2\pi\, |M_{fi}|^2\, \rho(E)$$

$$\Rightarrow (\cancel{2\pi})\, \frac{4A^2}{q^4}\, \frac{\cancel{(2\pi)^2}}{\cancel{V^2}}\, \frac{\cancel{V}}{\cancel{(2\pi)^3}}\, m \sqrt{\cancel{2}m\cancel{E_i}}\, d\Omega = d\sigma\, \frac{v_P}{\cancel{V}} = d\sigma \sqrt{\frac{\cancel{2}}{m}}\, \cancel{\sqrt{E_i}}$$

$$E = \frac{p^2}{2m} = \frac{1}{2} m v_P^2 \Rightarrow v_P = \sqrt{\frac{2E_i}{m}}$$

$$\Rightarrow d\sigma = \frac{4A^2}{q^4}\, m^2\, d\Omega$$

$$q = 2p\sin\frac{\theta}{2} \Rightarrow q^4 = 16\, p^4 \sin^4\frac{\theta}{2}$$

$$E = \frac{p^2}{2m} \Rightarrow p^4 = 4m^2E^2$$

$$\Rightarrow \quad d\sigma = \frac{\cancel{4} A^2}{16 \times \cancel{4} \times \cancel{m^2} E^2 \sin^4\frac{\theta}{2}} d\Omega \, \cancel{m^2}$$

$$= \left(\frac{A}{4E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}} d\Omega \qquad A = \alpha z_\alpha z_N$$

$$\Rightarrow \quad \frac{d\sigma}{d\Omega} = \left(\frac{A}{4E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$$

Notiamo che $dn = \frac{V}{(2\pi)^3} p^2 dp \, d\Omega$.

$d\Omega$: angolo solido dell'impulso del proiettile.

non possiamo usare $dn = \frac{V}{(2\pi)^3} p^2 dp \, (4\pi)$ ossia integrare su $\int d\Omega = 4\pi$ perchè il termine $\frac{1}{q^4}$ ha proprio la dipendenza da θ.