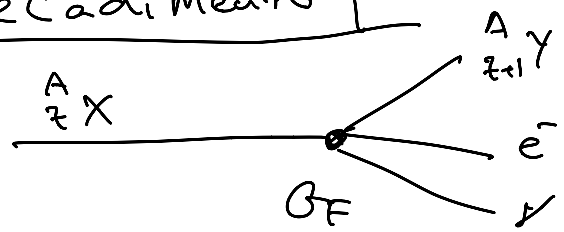
Decadimento β



$$\Gamma = 2\pi |M_{fi}|^2 \rho(E)$$

$$M_{fi} = -i \int d^3r\, \psi_f^* H_I \psi_i = \frac{G}{V} N$$

$$N = \int d^3r\, \psi_p^* \psi_n$$

Elemento nucleare $\simeq 1$ · matrice

$$|M_{fi}|^2 = \frac{G_F^2}{V^2} |N|^2$$

$$\Gamma = 2\pi \frac{G_F^2}{V^2} |N|^2 \rho(E)$$

oggi calcoliamo $\rho(E)$

spazio delle fasi:

$$\vec{P}_X + \vec{P}_e + \vec{P}_\nu = \emptyset \quad \text{per la conservazione}$$

$$\rho(E) = \left.\frac{dn}{dE_f}\right|_{E_f = E_i} = \int \delta(E_f - E_i) \cdot dn$$

$$\vec{p} = \vec{P}_e \qquad \vec{q} = \vec{P}_\nu$$

$$dn = \underbrace{\frac{V}{(2\pi)^3} d^3p}_{e^-} \; \underbrace{\frac{V}{(2\pi)^3} d^3q}_{\nu}$$

$E_i = E_X = M_X$ ci siamo messi nel c.d.m di X

$E_f = E_Y + E_e + E_\nu \simeq M_Y + K_Y + E_e + E_\nu$

$M_X, M_Y \gg m_e \Rightarrow K_Y \simeq \emptyset.$ (trascuro).

$E_f \equiv E_i \qquad M_X \simeq M_Y + E_e + E_\nu$

energia disponibile $E_T = M_X - M_Y = E_e + E_\nu$

$$Q = M_X - M_Y - \underset{\swarrow}{m_e} - \underbrace{m_\nu}_{\simeq 0} \simeq E_T$$

Caso limite $X \equiv p$, $Y \equiv n$ $\longrightarrow$ $E_T \simeq 1$ MeV

$$\delta(E_f - E_i) = \delta(E_T - E_e - E_\nu)$$

$m_\nu = 0 \qquad E_\nu = q \qquad q^2 dq = E_\nu^2 dE_\nu$

$m_e \neq 0 \qquad p = \sqrt{E_e^2 - m^2} \qquad dp = \frac{E_e}{p} dE_e$

$$p^2 dp = p E_e dE_e$$

$$\iint \delta(E_f - E_i) p^2 dp\, q^2 dq = \iint \delta(E_T - E_e - E_\nu) E_\nu^2 dE_\nu E_e p dE_e$$

$$= \int_0^{E_T} p E_e (E_T - E_e)^2 dE_e$$

$E_e^{max} = E_T = M_X - M_Y$

$E_e \in [$

$m_\nu = 0 \qquad E_\nu \to 0$

$E_e \to E_T$

$$\Gamma = 2\pi \frac{G_F^2}{V^2} |N|^2 \frac{V^2}{(2\pi)^6} (4\pi)^2 \int_{m_e}^{E_T} E_e \sqrt{E_e^2 - m_e^2} (E_T - E_e)^2 dE_e$$

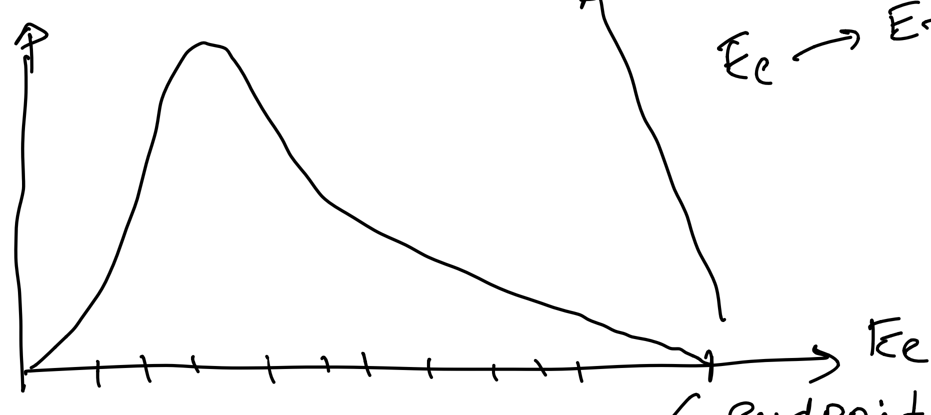
$$= \frac{G_F^2}{2\pi^3} |N|^2 \int (- \; -) dE_e$$

$\frac{d\Gamma}{dE_e}$ ossia $\frac{N(n \to p e \nu)}{\Delta E_e}$

$$\frac{d\Gamma}{dE_e} \propto G_F^2 E_e \sqrt{E_e^2 - m_e^2} (E_T - E_e)^2$$

$E_e \to E_T$

$\frac{\# decad.}{\Delta E}$



1 MeV

$n \to p e \nu \qquad E_T \simeq 1 \text{ MeV}$

$M_X > M_Y$ (alcuni MeV) $\qquad E_T \simeq$ alcuni MeV

$E_e \gg m_e$

Nel limite $E_e \gg m_e \rightarrow m_e \simeq 0$. (trascuro).

$$\int_0^{E_T} f(E_T^3, E_T^4, E^2) \ldots dE_e.$$

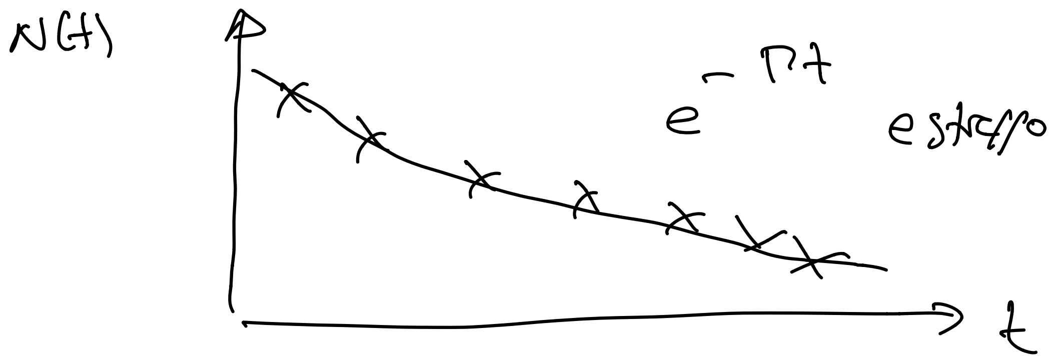
$$\Gamma = \frac{G_F^2}{2\pi} |N|^2 \frac{E_T^5}{30}$$ Regola di Sargent

$M_X - M_Y$ cresce $\Rightarrow$ E_T cresce

$\Rightarrow$ cresce spazio delle fasi

$\Gamma \propto G_F^2 \; E_T^5$ $\quad [G_F][E]^5 = [E]^{-4}[E]^5 = [E$

$\Gamma = \frac{1}{\tau} \Rightarrow \tau \propto G_F^{-2} \; E_T^{-5}$



$e^{-\Gamma t}$ estraggo $\Gamma = 1/\tau$

Γ misurato $\Rightarrow$ $G_F^2 |N|^2$

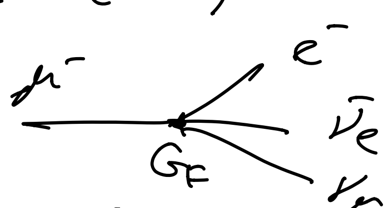
se mi calcolo N dalla teoria.

$\Rightarrow$ determinare G_F.

N: elem. matrice nucleare.

$\mu^- \longrightarrow e^- \bar{\nu}_e \nu_\mu$ $\quad \tau = 2.2 \; \mu s$

$$\Gamma(\mu \to e\nu\nu) = \frac{G^2}{192\pi^3} m_\mu^5$$



$\Rightarrow G_F = 1.166 \times 10^{-5} \; \text{GeV}^{-2}$

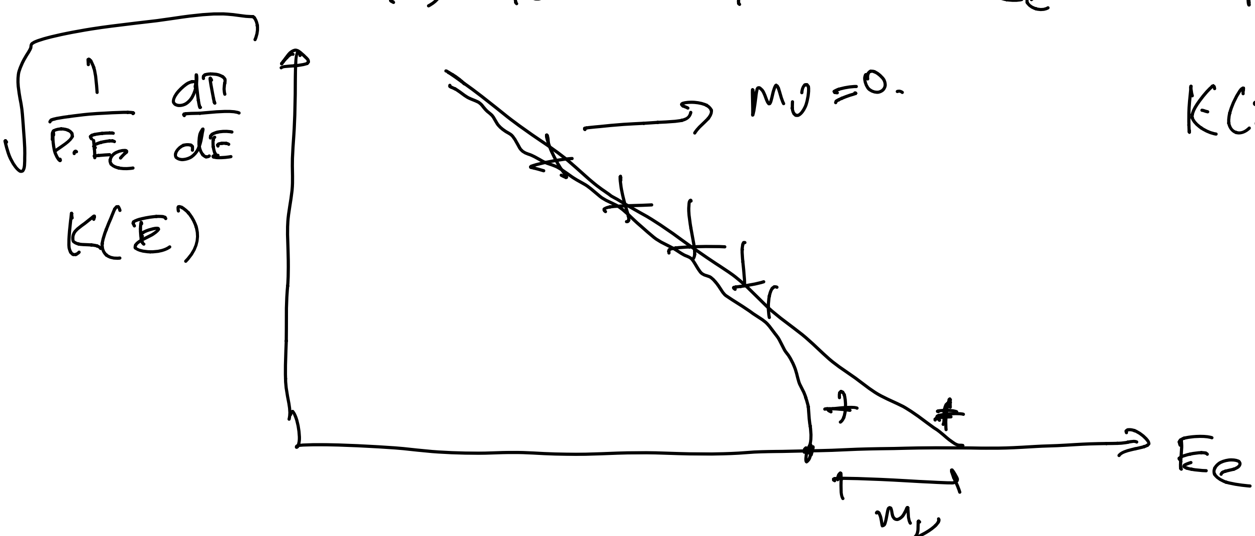
Misurare o stimare m_ν

Ipotesi finora: $m_\nu = 0 \Rightarrow \frac{d\Gamma}{dE_e} \propto P \cdot E_e (E_T - E_e)^2$

$$E_T = M_X - M_Y = E_e + E_\nu$$

$$m_\nu \neq 0 \Rightarrow E_T = M_X - M_Y - m_\nu$$

$\Rightarrow$ riduce E_T ossia E_e^{max} disponibile.

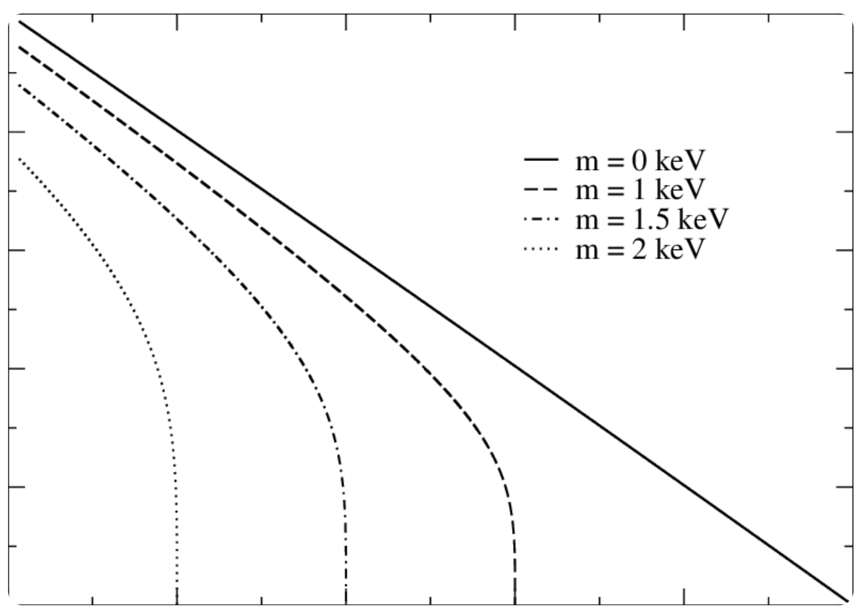


$K(E) \propto (E_T - E_e)$

Kurie

Grafico di Kurie

ci serve risoluzione in energia per risolvere E_T, $E_T - m_\nu$



Teoria di Fermi spiega decadimento β

$\Rightarrow$ esiste il neutrino

Finora $n \to p\, e^- \bar{\nu}$

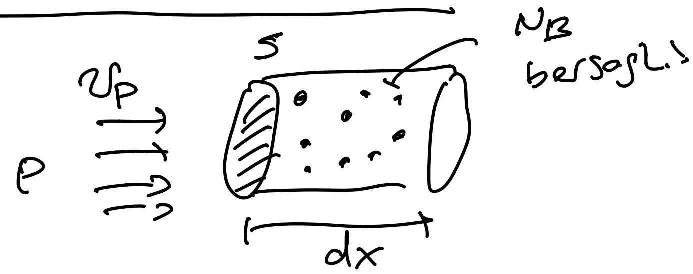
doveva essere possibile

$$\bar{\nu} + p \to n + e^+$$

- generare neutrini
- bersaglio ricco di p
- osservare n, e^+
- contare # eventi
- misurare sezione d'urto σ

Teoria di Fermi può predire σ per questo processo?

<u>Calcolo di σ con la regola d'oro di Fermi</u>



$$\frac{dN_r}{dt} = \sigma \frac{dN_p}{dt} n_b \cdot dx$$

$$= \sigma \frac{dN_p}{dt} \frac{1}{S} \underbrace{n_b \cdot dx \cdot S}_{N_B}$$

$$\frac{1}{N_B} \frac{dN_r}{dt} = \sigma \cdot \Phi_p = \sigma \cdot v_p \cdot n_p = \sigma \cdot v_p \cdot \frac{N_p}{V}$$

v_p: vel. relativa tra proiettile e bersaglio

$$v_p = \frac{p_p}{E_p}$$

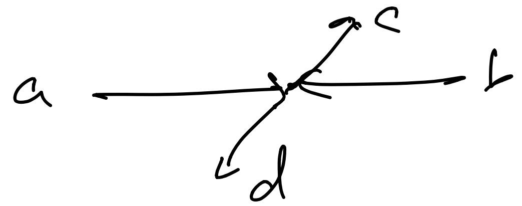
$$\underbrace{\frac{1}{N_p} \frac{1}{N_B} \frac{dN_r}{dt}}_{?} = \sigma \cdot \frac{v_p}{V}$$

reazioni in unità di tempo per singolo proiettile sul singolo bersaglio.

$$a + b \to c + d.$$

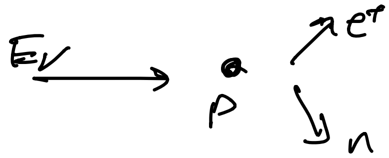
$$\frac{1}{N_p N_B}\frac{dN_r}{dt} = \Gamma(a+b \to c+d) = 2\pi\,|M_{fi}|^2 \rho(E_f)\Big|_{E_f = E_i}$$

$|i\rangle$: $a+b$. $|f\rangle$: $c+d$.

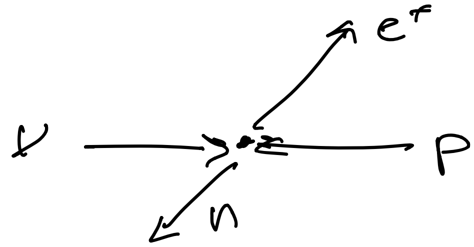


$$\sigma(a+b \to c+d) = 2\pi\,|M_{fi}|^2 \rho(E_f)\cdot\frac{V}{v_p}$$

$\bar{\nu} + p \to n + e^+$



mi pongo nel c.d.m.



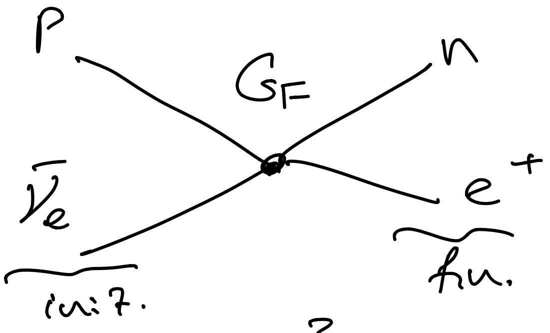
$$(\beta\gamma)_{CM} = \frac{P_{LAB}}{\sqrt{s}} = \frac{E_\nu}{\sqrt{m_p^2 + 2m_p E_\nu}}$$

$m_\nu = 0 \Rightarrow E_\nu = P_\nu$

$\vec{P}^*_{e^+} + \vec{P}^*_n = \emptyset$

$P = |P^*_e| = |P^*_n|$

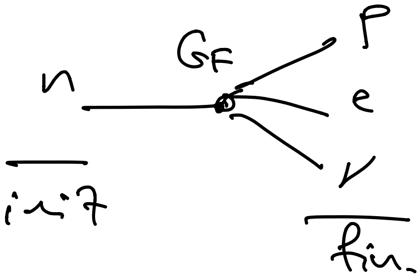
$E^* = \sqrt{m_p^2 + 2m_p E_\nu}$



$$|M_{fi}|^2 = \frac{G_F^2}{V^2}|N|^2$$

$$\sigma = 2\pi\,\frac{G_F^2}{V^2}|N|^2\rho(E_f)\times\frac{V}{v_i}$$

↳ velocità relative ν e protoni



decadim β.

$$v_i = \frac{P^*_{\nu\,in.}}{E^*_\nu} + \frac{P^*_{p\,in.}}{E^*_p}$$

Teoria di Fermi: $H_I = G_F$

Conserv. del mom. $\vec{P}_n^* + \vec{P}_{e^+}^* = \emptyset$. $P = P_n^* = P_{e^+}^*$

$$\rho(E_f) = \frac{dn}{dE_f} = \frac{V}{(2\pi)^3}(4\pi)\, P^2 \frac{dP}{dE_f}$$

$$E_f = \sqrt{p^2 + m_n^2} + \sqrt{p^2 + m_e^2}$$

$$\frac{dE_f}{dP} = \frac{P(E_n^* + E_e^*)}{E_n^* E_{e^+}^*} = \frac{P E_f}{E_n^* E_e^*} \Rightarrow \frac{dP}{dE_f} = \frac{E_n^* E_e^*}{P E_f}$$

$$\sigma = 2\pi \frac{G_F^2}{\cancel{V^2}} |N|^2 \frac{\cancel{V}}{v_i} \frac{\cancel{V}}{(2\pi)^3} 4\pi\, p^2 \frac{E_n^* E_e^*}{P E_f}$$

$$v_f = \frac{P}{E_e^*} + \frac{P}{E_n^*} = \frac{P E_f}{E_n^* E_e^*}$$ velocità relativa tra e^+ e n

$$\sigma = \frac{G_F^2}{\pi} |N|^2 \frac{1}{v_i} \frac{1}{v_f}\, p^2 \qquad [\sigma] = [E]^{-2} = [L]^2$$

p: impulso di e^+ (n) finale nel c.d.m.

$G_F = 1.17 \times 10^{-5}$ GeV^{-2}

$m_e \simeq 0.5$ $m_\nu \simeq 0$. $v_i \simeq v_f \simeq 1$

$G_F^2 \propto 10^{-10}$ GeV^{-4}.

200 MeV x fm = 1. 0.2 GeV x fm = 1.

GeV^{-1} = 0.2 fm.

$$\sigma = \frac{1.4\times10^{-10}}{\pi} \times \underbrace{4\times10^{-28}}\ \frac{cm^2}{GeV^{-2}} \left(\frac{P}{GeV}\right)^2$$

$\hookrightarrow G_F^{\,2}$

$$= 10^{-37}\ cm^2 \left(\frac{P}{GeV}\right)^2$$

$\nu_e + p \to n + e^+.$

$Q = m_p - m_n - m_{e^+} = -1.3 - 0.5 \simeq 1.1$ MeV.

processo a soglia $E_\nu > 1.8$ MeV

$$= 10^{-43}\ cm^2 \left(\frac{P}{MeV}\right)^2$$

$\sigma \cdot n_b. = \frac{1}{\lambda}$ Cammino libero medio

$\rho_{H_2O} = 1\ g/cm^3$ H_2O: $A = 16+2 = 18\ \frac{g}{mol.}$

$n_b. = \frac{1}{A} N_A\ cm^{-3} = 3\times10^{22}\ cm^{-3}$

$$\lambda^{-1} = 10^{-43}\ cm^2 \times 3\times10^{22}\ cm^{-3} \left(\frac{E}{MeV}\right)$$

$$= 3\times10^{-22}\ cm^{-1} \left(\frac{E}{MeV}\right)$$

$E \simeq$ (MeV) ordine.

$\lambda^{-1} \simeq 10^{-22}\ cm^{-1} \Rightarrow \lambda \simeq 10^{19}$ m.

$R_{terra} \simeq 6000$ km

Dist. terra-Sole $\simeq 150\times10^6$ km.

Per fare esperimenti:

- aumento E_ν in ingresso $\Rightarrow$ ρ aumenti.
- n_b grandi
- fonti forti neutrini:

$$\bar{\nu} + p \longrightarrow n + e^+$$

Fissione di Uranio $\quad Q \simeq 200$ MeV

$$\langle N_\nu \rangle \simeq 6$$

Reines-Cowan 1956 a usare un reattore

$$P_{termica} = 1000\ MW \equiv 1\ GW = 10^9\ J/s$$

$$1\ eV = 1.6 \times 10^{-19}\ J$$

$$P_{termico} = \frac{1\ GW}{e} = 0.6 \times 10^{28}\ eV/s$$

$$\#\text{reazioni} = \frac{P_{reattore}}{Q} = 3 \times 10^{19}\ Hz$$

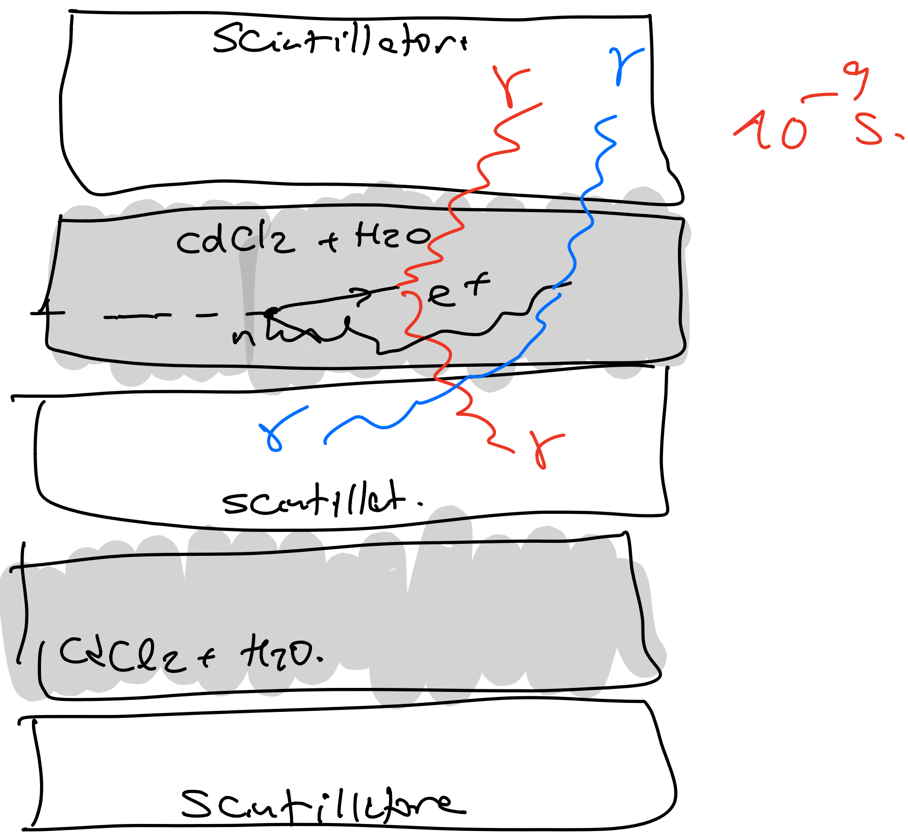
$$\#\nu = 20 \times 10^{19}\ Hz = 10^{20}\ Hz \qquad \frac{dN_\nu}{dt}$$

$$E_{\bar{\nu}} \simeq 3\ MeV$$

a poca distanza dal core del reattore

$$\phi = \frac{dN_\nu}{dt\,dS} = 10^{+13}\ cm^{-2}\,s^{-1}$$

angolo solido del rivelatore



$\nu + p \to n + e^+$

$e^+e^- \to \gamma + \gamma$

tempi $\sim 10^{-9}$ s

H_2O rallenta neutrini

neutroni termalizzano negli urti elastici H_2O.

$$n + {}^{A}_{Z}X \longrightarrow {}^{A+1}_{Z}Y^{*} \longrightarrow {}^{A+1}_{Z}Y + \gamma + \gamma$$

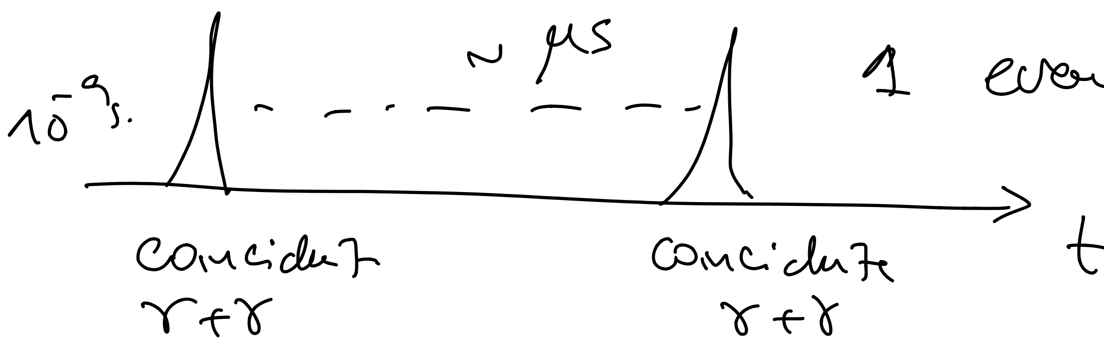
Decad. γ

$E_\gamma \simeq 6$ MeV.

dopo circa 10^{-5} s.

$e^+e^- \to \gamma\gamma$

$n + Cd \to Cd + \gamma\gamma$



1 evento.

Presi dati con reattore acceso. $N_{tot} = 567 \} \Delta T$

eventi. spento: $N_B = 209 \} \Delta T$

γγ coincid ⟷ γγ coincid.

lo stesso tempo.

$$N_S^{\text{stim.}} = N_{tot} - N_B = 358$$ eventi di eccesso quando acceso.

$$\text{Significanza} = \frac{N_{tot} - N_B}{\sqrt{N_B}} \simeq 25\,\sigma$$ deviazioni standard.

Signif: $$\frac{N_{tot}^{obs} - N_B^{stim}}{\delta N_B}$$ fondo: B

poisson: $$\delta N_B = \sqrt{N_B}$$

Evento: $$\bar{\nu} + p \longrightarrow n + e^+$$

$$e^+ \hookrightarrow e^+ e^- \to \gamma\gamma$$

$$n \hookrightarrow n + Cd \to Cd^* \to Cd + \gamma\gamma.$$

$$\Delta T \sim 1\,\mu s$$

Acceso: 250.
spento 209.
} ΔT uguale

$$\frac{N_{tot} - N_B}{\sqrt{N_B}} = \frac{40}{\sqrt{200}}.$$ Comp. con $\emptyset$.