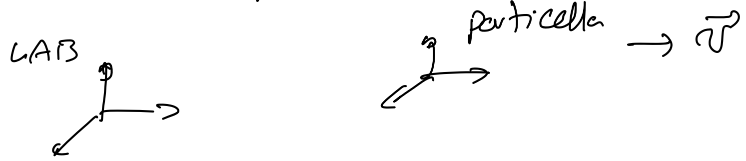
4-impulso: $\underline{P} = (E, \vec{P})$ $\qquad E = \gamma m \qquad \vec{P} = \gamma m \vec{v}$

$$u^\mu = \underline{u} = \frac{dx^\mu}{d\tau} = \left(\frac{dt}{d\tau}, \frac{d\vec{x}}{d\tau}\right) \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \frac{v}{c}$$

τ: tempo nel rif. solidale con la particella.



$$t_{LAB} \equiv t = \gamma \tau \Rightarrow dt = \gamma d\tau.$$

$$\underline{u} = \left(\gamma, \gamma \frac{d\vec{x}}{dt}\right) = (\gamma, \gamma \vec{v}) = (\gamma, \gamma \vec{\beta})$$

$$\vec{\beta} = \left(\frac{v_x}{c}, \frac{v_y}{c}, \frac{v_z}{c}\right)$$

$E = \gamma m \qquad \vec{p} = \gamma m \vec{v}$ (dalla lagrangiana).

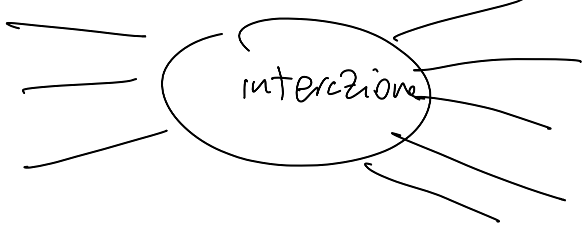
$$\underline{P} = m \underline{u}$$

Legge di conservazione di 4-impulso:

$$|i\rangle \longrightarrow |f\rangle$$

$$\underline{P}_{in} = \sum_{j=1}^{N_{in}} \underline{p}_j^{in} = \sum_{K=1}^{N_{fin}} \underline{p}_K^{fin} = \underline{P}_f$$

stato iniziale $\qquad\qquad$ finale



$$E^2 = p^2 + m^2 \Rightarrow E = \pm\sqrt{p^2+m^2}$$

Eq. schrödinger: $\quad H\psi = E\psi$

$p \to -i\vec{\nabla}$ $\qquad\qquad i\frac{d}{dt}\psi = H\psi$

$E \to i\frac{d}{dt}$

mescola $\frac{d}{dt}$ con ∇^2

$E^2 = p^2 + m^2 \implies \left(i\frac{d}{dt}\right)^2 = (-i\vec{\nabla})^2 + m^2$

$$-\partial_t^2 \psi = (-\nabla^2 + m^2)\psi$$

$$(\partial_t^2 - \nabla^2 + m^2)\psi(x,t) = 0$$ Equazione di Klein-Gordon per particelle senza spin.

$$(\Box + m^2)\psi(x,t) = 0$$

$\Box = \partial_t^2 - \nabla^2$

$$\psi(x,t) = A e^{-i \underline{p} \cdot \underline{r}} = A e^{-iEt} e^{+i\vec{p}\cdot\vec{x}}$$

$$E = \pm\sqrt{p^2+m^2}$$

Equazione di Dirac 1928

$$(i\gamma^\mu \partial_\mu - m)\psi(x,t) = 0 \qquad \psi: \text{spinore}$$

γ^μ: matrici 4x4 di Dirac $\mu = 0, 1, 2, 3,$

∂_μ: $\frac{\partial}{\partial x^\mu}$

$\mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\gamma^0 = \gamma_0 = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix}; \quad \gamma^i = -\gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

σ_i: di Pauli

Rapp. di Pauli-Dirac

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\psi(x^\mu) = u^{(r)} e^{-ip^\mu x_\mu} \quad \text{[positive energy solutions]} \longrightarrow +\sqrt{p^2+m^2}$$

$$\psi(x^\mu) = v^{(r)} e^{ip^\mu x_\mu} \quad \text{[negative energy solutions]} \longrightarrow -\sqrt{p^2+m^2}$$

$$u^{(r)}(\mathbf{p}) = \frac{1}{\sqrt{E_p + m}} \begin{pmatrix} (E_p + m)\chi^{(r)} \\ (\boldsymbol{\sigma} \cdot \mathbf{p})\chi^{(r)} \end{pmatrix}$$

$$v^{(r)}(\mathbf{p}) = \frac{1}{\sqrt{E_p + m}} \begin{pmatrix} (\boldsymbol{\sigma} \cdot \mathbf{p})\chi^{(r)} \\ (E_p + m)\chi^{(r)} \end{pmatrix}$$

$$\chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; ; \quad \chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

e^{-iEt} — $E>0$ particelle

$e^{-i(-E)(-t)}$ — $E<0$ anti-particelle (propagate indietro nel tempo)

e^- — esp. di Thompson 1897

p

e^+ — positrone Anderson.

$\bar{p}$ anti-protone — Segrè-Chamberlain.

Interazioni: EM, forte, debole

particelle elementari:

$$\begin{array}{l} q=-1 \\ q=0 \end{array} \begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

muone

neutrino elettronico; muonico

$s = 1/2$

leptoni: (fermioni) interagiscono solo con EM e debole

p, n: neutrone → barioni fatti da 3 quark

adroni: anche interazione forte. ma non elementari.

$q=+\frac{2}{3}e$ $\begin{pmatrix} up \\ down \end{pmatrix}$ $\begin{pmatrix} Charm \\ strange \end{pmatrix}$ $\begin{pmatrix} top \\ bottom/beauty \end{pmatrix}$ quark non esistono liberi ma solo confinati negli adroni

$q=-\frac{1}{3}$

(Fermioni) $s=1/2$

protone: uud

$q_p = \frac{2}{3}+\frac{2}{3}-\frac{1}{3} = +\frac{3}{3} = 1$

neutrone: udd

$q_n = +\frac{2}{3}-\frac{1}{3}-\frac{1}{3} = 0$

Flavor dei quark: 6

3 colori per ciascun quark
r, g, b

mesoni: $q, \bar{q}$

anti-quark: $\bar{q}$

$q=+\frac{1}{3}$ $\begin{pmatrix} \text{anti-down } \bar{d} \\ \bar{u} \end{pmatrix}$ $\begin{pmatrix} \bar{s} \\ \bar{c} \end{pmatrix}$ $\begin{pmatrix} \bar{b} \\ \bar{t} \end{pmatrix}$

$q=-\frac{2}{3}$

Adroni: interazione forte + EM + debole

Barioni $(q_i q_j q_k)$

$q_i \bar{q}_j$ Mesoni

i, j, k: u, d, c, s, ~~t~~, b

$|\underline{P}|^2 = E^2 - p^2 = m^2$ invariante di Lorentz in tutti i riferimenti

$|\underline{P}|^2 = \underline{P}\cdot\underline{P}$

ϕ: $s\bar{s}$

$\left.\begin{array}{l} \pi^+ : u\bar{d} \\ \pi^- : \bar{u}d \\ \pi^0 = \frac{\bar{u}u + d\bar{d}}{\sqrt{2}} \end{array}\right\}$ pioni

Kaoni:
$s\bar{d}$ K^0
$s\bar{u}$ K^-
$\bar{s}u$ K^+
$\bar{s}d$ $\bar{K}^0$

Bosoni: Fotone γ $s=1$ $m=0$.

debole bosoni: $W^{\pm}, Z^0$

$q_e = 0, \pm 1$ $S = 1$

gluoni: 8 gluoni $S=1$, $q=0$
mediatori di interazione forte q colore $\neq 0$.

Bosone di Higgs: $S = \phi$
Ruolo di H è generare masse per tutte le particelle.

Decadimento delle particelle instabili

e^-, p : stabili
neutrone instabile $n \to p + e^- + \bar{\nu}_e$

$\pi^+ \to \mu^+ \nu_\mu$
$e^+ \nu_e$

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$

Decadimento: $1 \longrightarrow 2 + 3 + \dots + N$

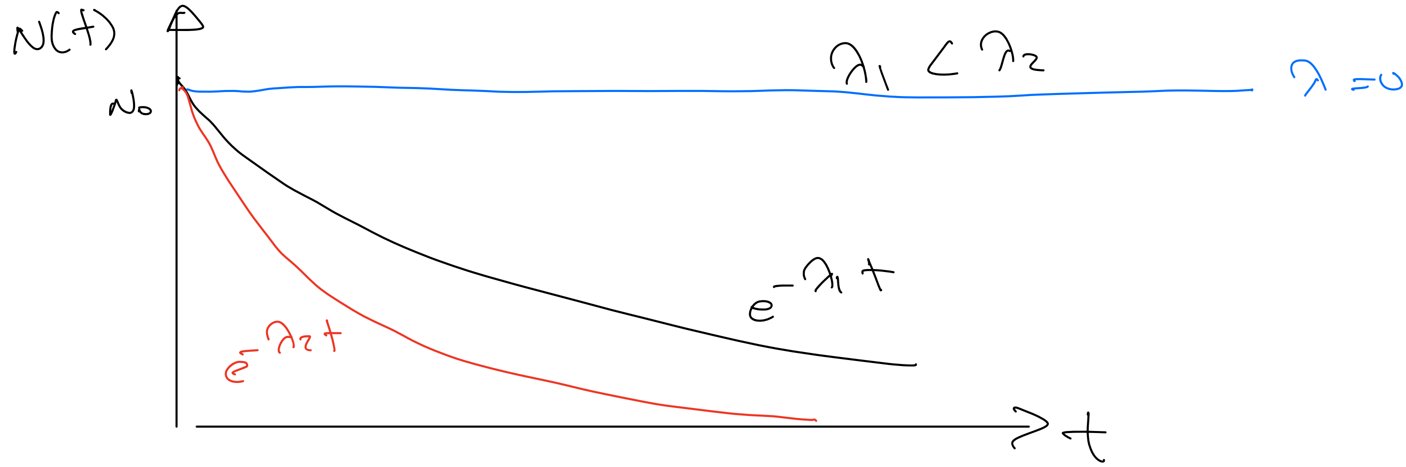
$t = 0$ $N = N_0$ particelle instabili identiche

probabilità di decadimento per unità di tempo λ
- λ non dipende dal numero di particelle
- λ non dipende dal tempo

caratteristica intrinseca delle particelle.

t: $dN = -\underbrace{(\lambda dt)}_{\text{prob. di decad. in tempo } dt} N$

$$\frac{dN}{N} = -\lambda dt \Rightarrow N(t) = N(0)\, e^{-\lambda t}$$



$$\frac{N(t)}{N(0)} = e^{-\lambda t}$$

vita media propria

Definizione $\quad \langle t \rangle =: \dfrac{\int_0^\infty t\, e^{-\lambda t}\, dt}{\int_0^\infty e^{-\lambda t}\, dt} = \dfrac{1}{\lambda}$

$\langle t \rangle = \dfrac{1}{\lambda} \equiv \tau$ caratteristica delle particelle.

$$N(t) = N_0\, e^{-t/\tau}$$

$$N(t=\tau) = N_0\, e^{-\tau/\tau} = \frac{N_0}{e} = 37\%\, N_0$$ sopravvissuti

$$N(t=10\tau) = \frac{N_0}{e^{10}}$$

$$T_{1/2} = N(t') = \frac{N_0}{2} = \cancel{e}\, N_0\, e^{-t'/\tau}$$

$$\Rightarrow \frac{1}{2} = e^{-t'/\tau}$$

$$\ln\frac{1}{2} = \ln\left(e^{-t'/\tau}\right)$$

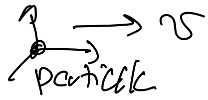
$$-\ln 2 = -t'/\tau$$

$$\boxed{T_{1/2} = \tau \cdot \ln 2}$$

tempo di dimezzamento

τ: tempo medio nel rif. solidale con le particelle.

$N(t) = N_0\, e^{-t^*/\tau}$ rif solidale.

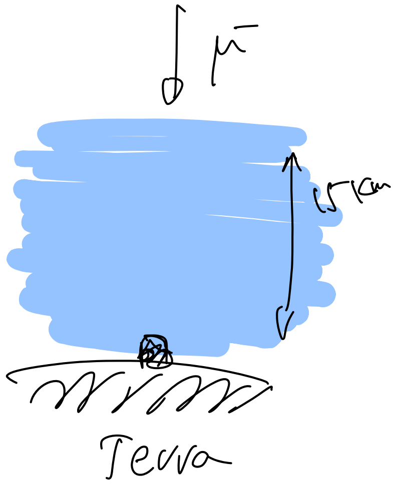


LAB: $N_0\, e^{-\frac{t_{LAB}}{\gamma\tau}}$

muoni μ^-: $\tau = 2.2\,\mu s = 2.2\times 10^{-6}\, s$

$p = 100\ \text{GeV} = 100\times 10^{9}\ \text{eV}$

$m_\mu = 106\ \text{MeV} = 100\times 10^{6}\ \text{eV}$



$$\gamma = \frac{E}{m}$$

$$E^2 = p^2 + m^2 = (10^{11}\ \text{eV})^2 + \cancel{(10^8\ \text{eV})}^2$$

$$t_{LAB} = \frac{x_{LAB}}{v} = \frac{x_{LAB}}{\beta c}$$

$$e^{-\frac{t_{LAB}}{\gamma\tau}} = e^{-\frac{x_{LAB}}{\beta\gamma c\tau}} = e^{-\frac{x_{LAB}}{\langle x\rangle}}$$

$$\beta\gamma = \frac{|\vec{p}|}{m} = \frac{100\ \text{GeV}}{100\ \text{MeV}} = 1000$$

$$\beta\gamma c\tau = 10^3 \times 3\times 10^8\ \frac{m}{\cancel{s}} \times 2.2\times 10^{-6}\ \cancel{s}$$

$$= 6.6\times 10^5\ m = 660\ km$$

$\beta\gamma c\tau = \langle l \rangle$ lunghezza media di sopravv. in laboratorio.

$$N_\mu(15\ km) = N_0\, e^{-15/660}$$

200 MeV fm = 1.

$1\ fm = \frac{1}{200}\ \frac{1}{MeV}$

Spessore dell'atmosfera in MeV

$15\ km = 15\times 10^3\ m.$

$= 15\times 10^3 \times 10^{15}\ fm.$