

Bremsstrahlung

$$\left. \frac{dE}{dx} \right|_{\text{rad}} \approx \frac{E}{x_0}$$

(e^\pm nelle materie)

$$E(x) = E_0 e^{-x/x_0}$$

x_0 : lunghezza di radiazione

$$\rho x_0 \sim 170 \frac{A}{Z^2} \frac{g}{cm^2} \quad \text{formula empirica}$$

x_0 è prop delle materie (A, Z) → mettere attraversato

$$\left. \frac{dE}{dx} \right|_{e^\pm} = \underbrace{\left. \frac{dE}{dx} \right|_{\text{ion: } Z=1} + \left. \frac{dE}{dx} \right|_{\text{rad}}}^{B-B} + \underbrace{\left. \frac{dE}{dx} \right|_{\text{Brems.}}}_{\text{dipende da } E \text{ delle particelle}}$$

dipende da $\beta \gamma \approx \frac{P}{m}$

$$\text{Energia critica} \quad \left. \frac{dE}{dx} \right|_{\text{ion.}} \approx \left. \frac{dE}{dx} \right|_{\text{Brems.}} ? \quad \text{Energia Critica } E_C$$

$$\frac{\left. \frac{dE}{dx} \right|_{\text{Brems.}}}{\left. \frac{dE}{dx} \right|_{\text{ion.}}} \approx 1 \quad @ E = E_C$$

sperimentalmente

$$\frac{\left. \frac{dE}{dx} \right|_{\text{Brems.}}}{\left. \frac{dE}{dx} \right|_{\text{ion.}}} \approx \frac{K_e Z}{1200 m_e}$$

en. critiche e^\pm

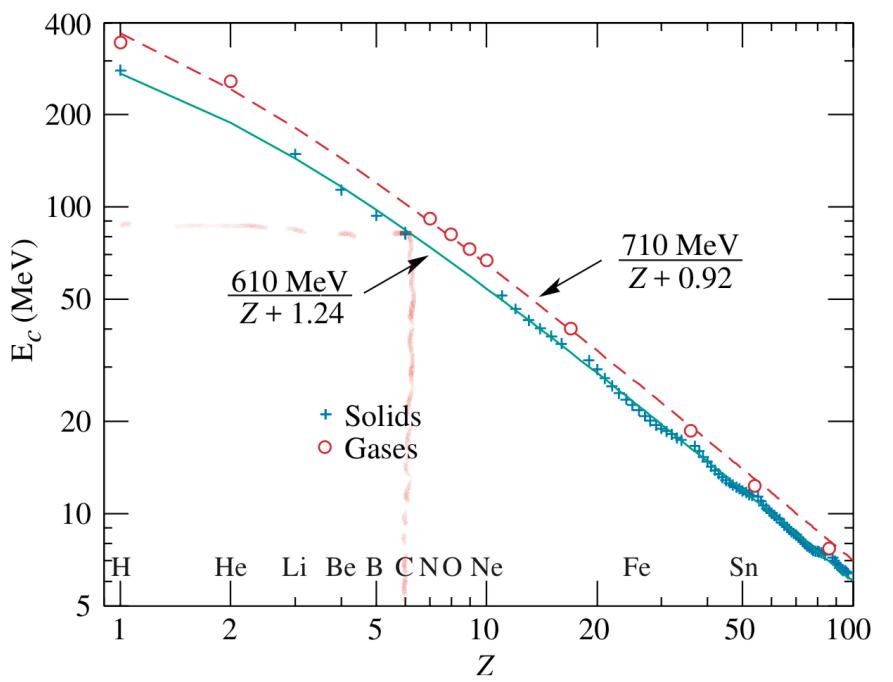
$$K_e := E_e - m_e$$

$$m_e = 0.511 \text{ MeV}$$

$$\Rightarrow E_C \approx \frac{1200 m_e}{Z} \approx \frac{600 \text{ MeV}}{Z}$$

$$E_C: Z=6 \Rightarrow E_C^{\text{carb}} \approx \frac{600}{6} \text{ MeV} = 100 \text{ MeV.}$$

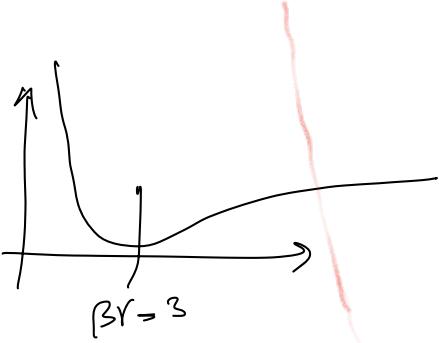
Nel carbonio per $E_e > 100 \text{ MeV}$ prevale Bremsstrahlung



lineare in
E

$$\frac{dE}{dx} \propto \frac{1}{\sqrt{Z}}$$

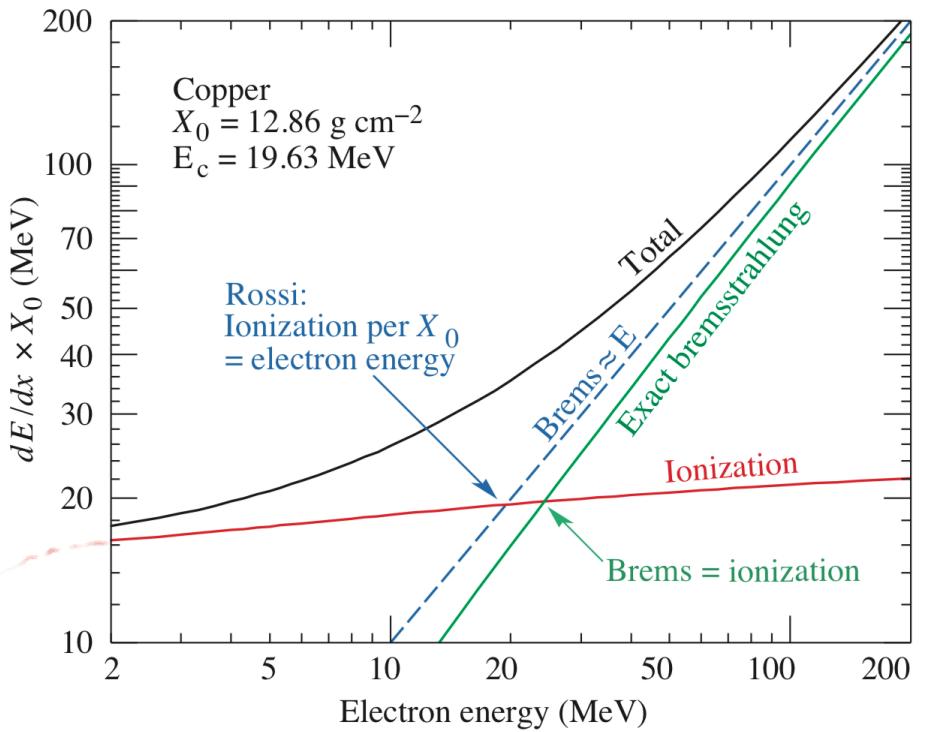
$$\frac{dE}{dx} \propto \frac{1}{\sqrt{Z}}$$



$$\beta\gamma = \frac{P}{m}$$

$$\beta\gamma \approx 3 \Rightarrow P = 3m_e c = 1.5 \text{ MeV}$$

$$E = \sqrt{P^2 + m^2} = \sqrt{1.5^2 + 0.5^2} \approx 1.5$$



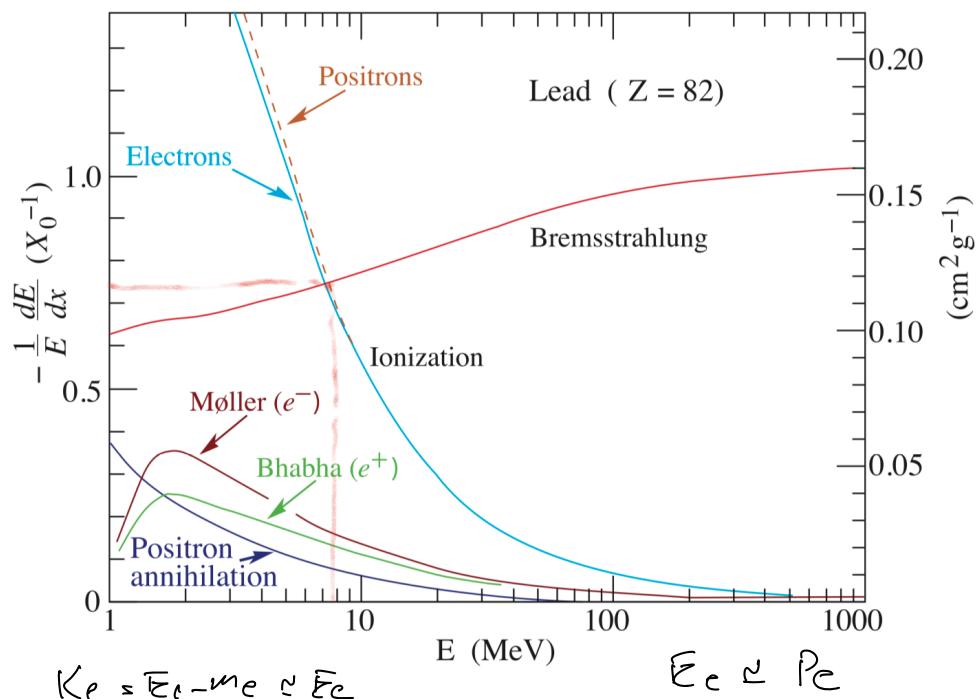
$$E = 20 \text{ MeV} \Rightarrow P \approx 20 \text{ MeV}$$

$$\beta\gamma = \frac{20}{0.5} \approx 40$$

$$P_b : Z = 82$$

$$E_c = \frac{600}{82} \text{ MeV}$$

$$\approx 7.5 \text{ MeV}$$



% en. pers.

$E > E_c$ Bremsr. $\gtrsim 75\%$ di perdite di energia.

$$\frac{E_c}{\rho X_0} = \frac{600 \text{ MeV}}{170} \cdot \frac{1}{170} \cdot \frac{Z}{A} = \frac{600}{170} \cdot \frac{Z}{A}$$

$$= 1.7 \frac{\text{MeV}}{\text{cm}} \frac{\text{cm}^3}{\text{g}}$$

$\approx \frac{1}{2}$ per quasi tutti met. stabili

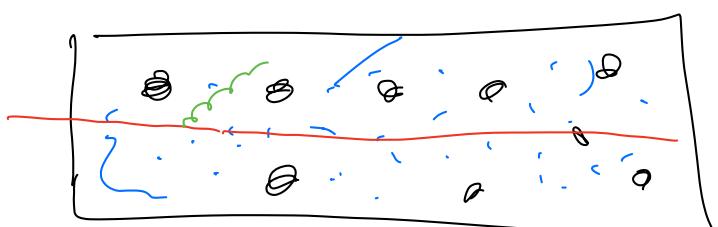
Minimo di Bethe-Bloch. Θ Br ≈ 3

$$\frac{1}{e} \frac{E_c}{X_0} \quad \frac{1}{e} \frac{dE}{dx} \quad \left. \right\} \text{Bethe-Bloch.}$$

particelle caricate nel mezzo perdono energia

1) ionizzazione: trasferiscono energ. e glie⁻ del mezzo

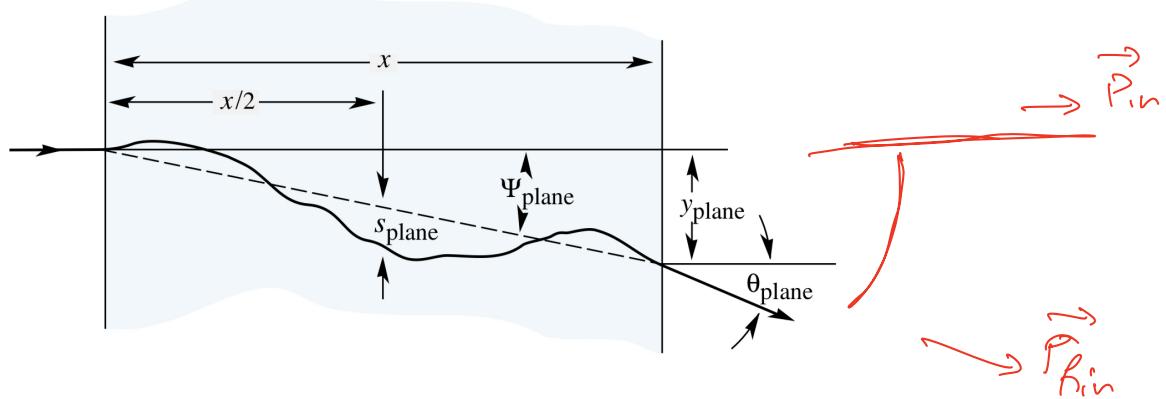
2) radiazione se $E > E_c = \frac{600 \text{ MeV}}{Z}$
vde per e^\pm



e^- del mezzo

\bullet : nuclei nel mezzo

ne nuclei \Rightarrow diffusione colloidale delle particelle cariche



Attraversare met \Rightarrow deflessione rispetto alla direzione iniz.
Calcolo del $\langle \theta^n \rangle$ θ : angolo di deflessione

$$\langle \theta^n \rangle = \frac{\int \theta^n \left(\frac{d\phi}{dr} \right) dr}{\int \left(\frac{d\phi}{dr} \right) dr}$$

$$\frac{d\phi}{dr} \approx \frac{1}{\sin^4 \frac{\theta}{2}}$$

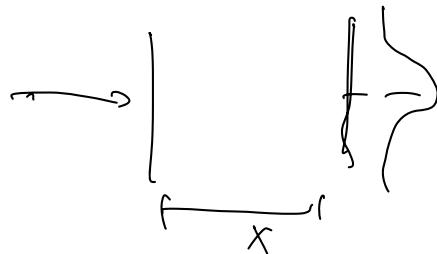
$$\sin \frac{\theta}{2} \approx \frac{\theta}{2}$$

Piccoli angoli $\sin \theta \approx \theta$.

$$\langle \theta \rangle = \int \theta \frac{1}{\theta^n} d\theta = 0$$

$$\langle \theta^2 \rangle \neq 0 = \varepsilon \text{ MeV} \frac{z_p}{BCP} \sqrt{\frac{x}{x_0}}$$

Varianza



z_p : z del proiettile

x_0 : lunghezza di rad.

x : spessore attiv.

$$\theta(x) \approx \frac{1}{\sqrt{2n} \sqrt{\langle \theta^2 \rangle}} e^{-\frac{\theta^2}{2\theta_0^2}}$$

$$\theta_0 = \sqrt{\langle \theta^2 \rangle}$$

Dif. colloidale non fa perdere energia.
solo modifica direzione di \vec{p}

Effetto Cherenkov

osservato 1934

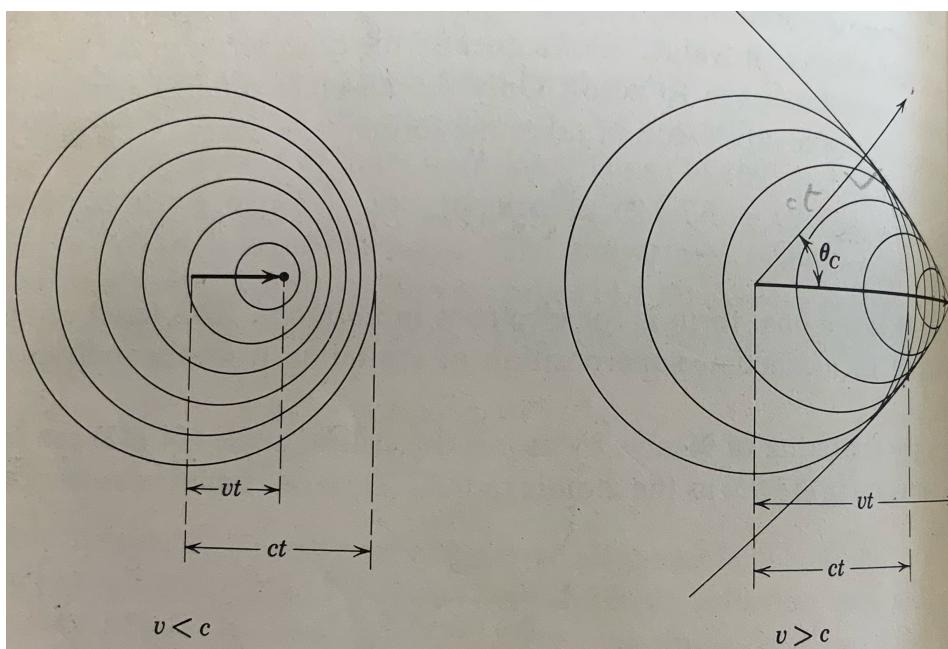
passaggio di particelle caricate nel mezzo. $v > \frac{c}{n}$

v : vel. part. carica.

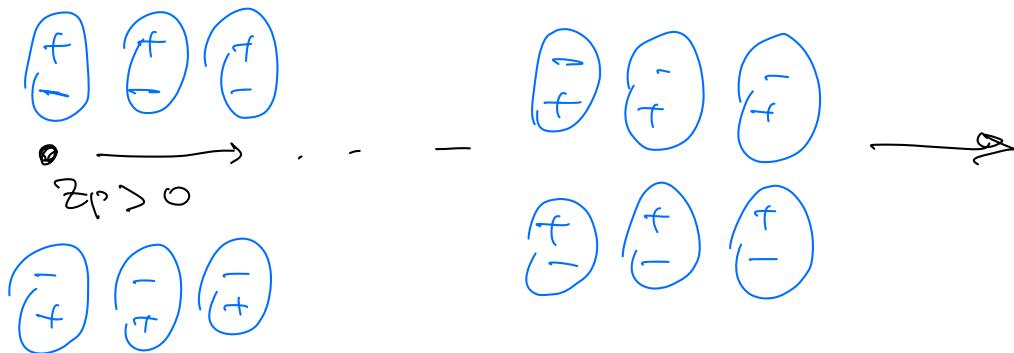
c : vel. luce.

n : l'indice di rifrazione

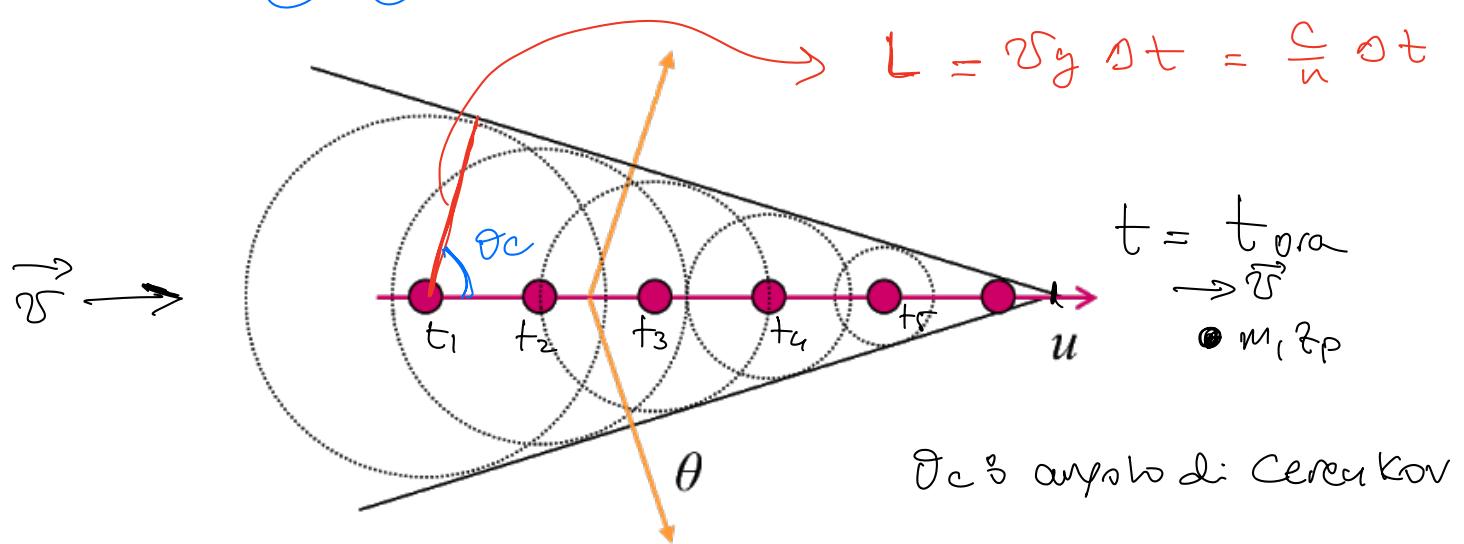
$$n = \sqrt{\epsilon_r}$$



Radiazione emessa durante il passaggio delle particelle cariche nel mezzo.



$$L = v_g \Delta t = \frac{c}{n} \Delta t$$



θ_C è angolo di Cherenkov

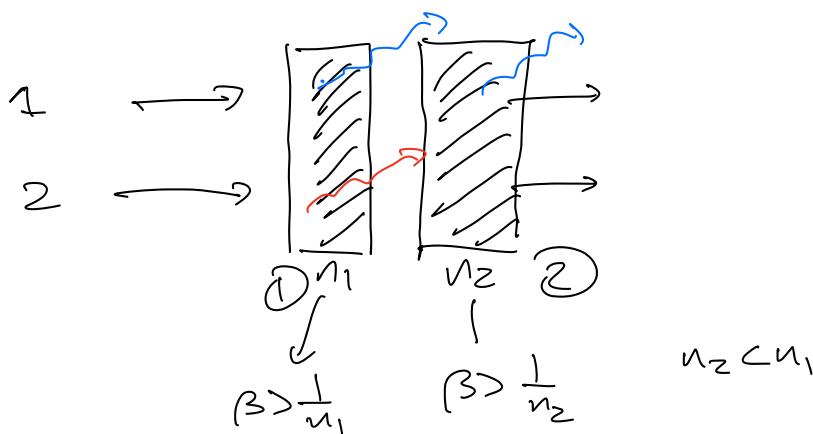
$$L = \Delta x \cdot \cos\theta_C = \underbrace{\beta c \Delta t}_{\text{v}} \cos\theta_C = \frac{c}{n} \Delta t$$

$$\cos\theta_C = \frac{1}{\beta n} \leq 1 \Rightarrow \boxed{\beta \geq \frac{1}{n}}$$

fornisce a soglia
si verifica se particelle più veloci
della luce nel mezzo

$$\beta = \frac{E}{\gamma}$$

$$n > 1 \text{ per i materiali: } \beta_{th} = \frac{1}{n} \text{ veloce minima per } \beta$$



se entrambi in ① + ② $\Rightarrow \beta > \frac{1}{n_2}, \beta > \frac{1}{n_1}$.

entra in ① non in ② $\beta > \frac{1}{n_1} \in \beta < \frac{1}{n_2}$

Rivelazione Cerenkov di diverse forme.

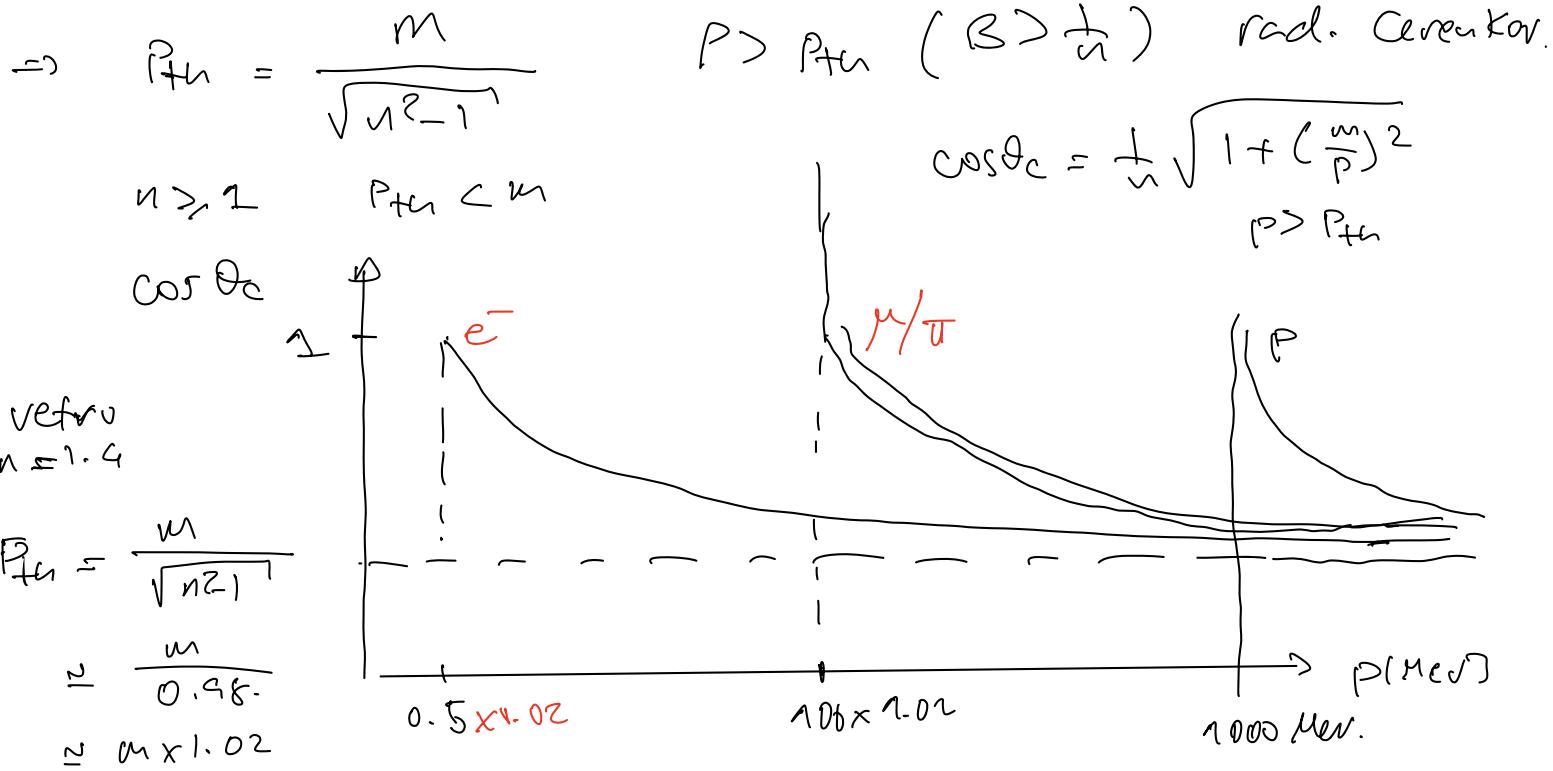
$$\text{misura intervallo } \frac{1}{n_1} < \beta < \frac{1}{n_2}$$

combinazione di Cerenkov + misure di impulso con \vec{B}

\Rightarrow fornisce stime delle masse del proiettile

$$\cos\theta_C = \frac{1}{\beta n} = \frac{1}{n} \frac{E}{P} = \frac{1}{n} \sqrt{\frac{P^2 + m^2}{P^2}} = \frac{1}{n} \sqrt{1 + \left(\frac{m}{P}\right)^2}$$

se conosco P \Rightarrow $\cos\theta_C \Rightarrow$ fornisce massa

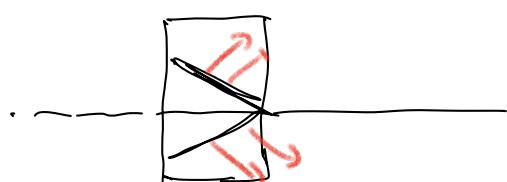
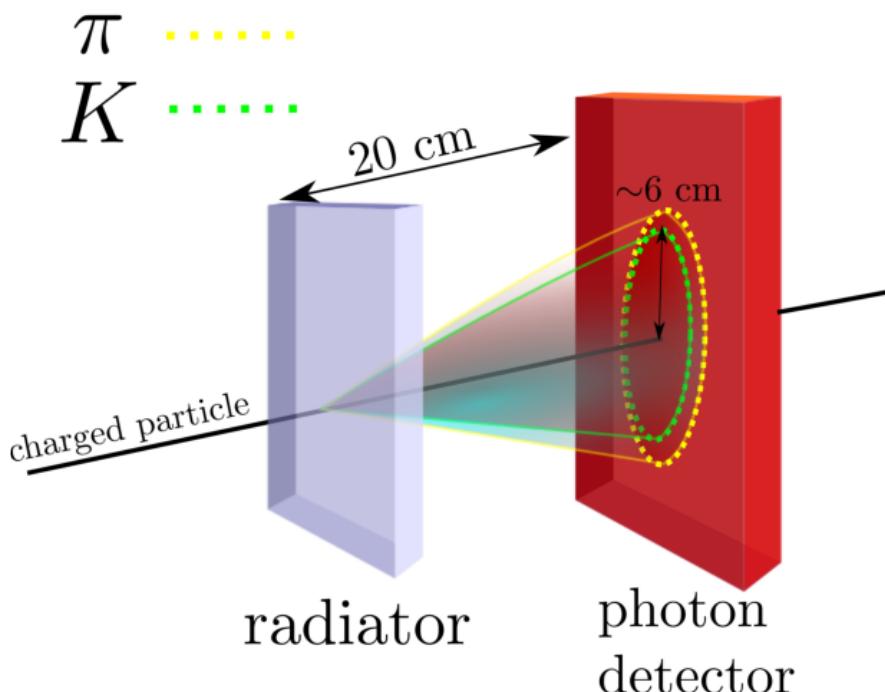


Muone θ $m = 106$ MeV.

$$P_{\text{thr}} = 106 \times 1.02 \approx 108 \text{ MeV}.$$

prose: $m = 140$ MeV

per $p \rightarrow \infty \quad \cos \theta_c \rightarrow \frac{1}{n}$ per tutte le particelle.



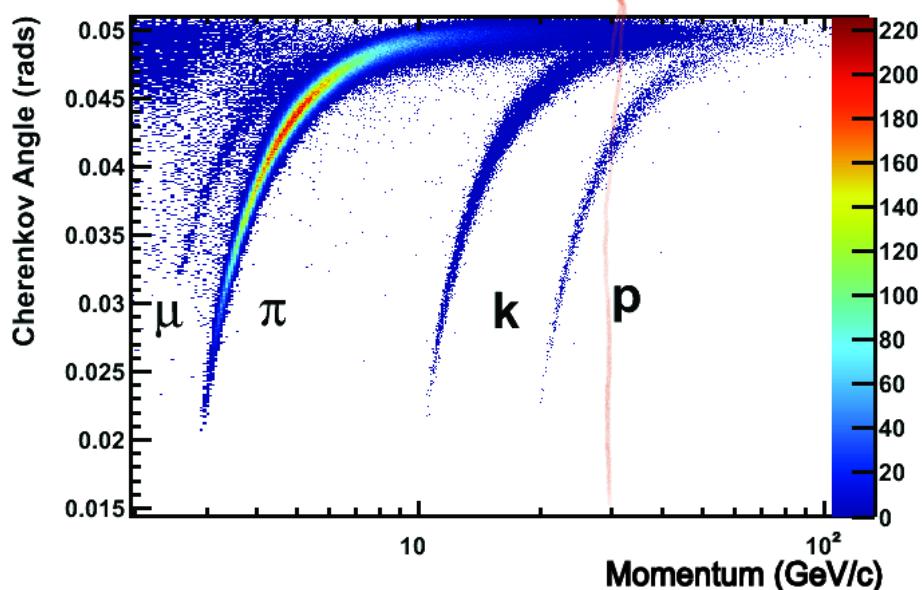
impulsi < 30 GeV

separare particelle con masse diverse.

Effetto Cherenkov

+

misure di impulso

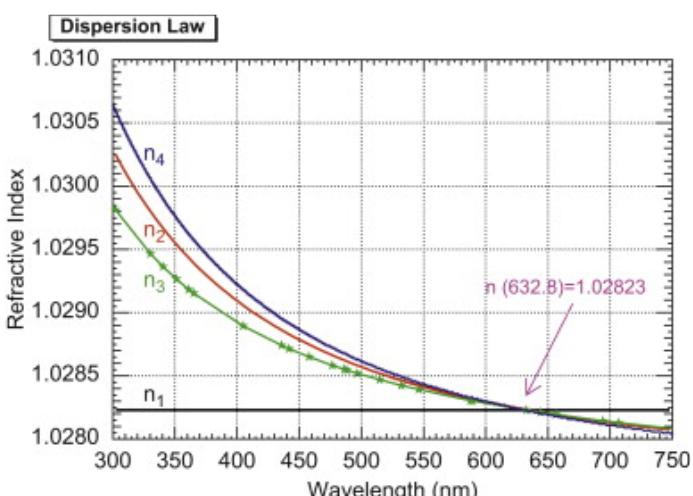


Identificazione

delle particelle cariche.

Aerogel (a base di silicio)

Variando densità δ del
metallo \Rightarrow ottenere n diversi.



Numeri e energie di fotoni Cherenkov

Numeri di fotoni emessi per unità di lunghezza e per unità di energia

$$\frac{d^2N}{dx dE} = \frac{\alpha z^2}{\pi c} \left(1 - \frac{1}{\beta^2 n(E)^2} \right) = \frac{d^2z^2}{\pi c} \sin^2 \theta_C$$

$$E = h\nu = \frac{h}{\lambda} = 2\pi \frac{\pi c}{\lambda}$$

$$dE = 2\pi \frac{\pi c}{\lambda^2} d\lambda$$

$$\frac{d^2N}{dx dE} = \frac{d^2N}{dx d\lambda} \frac{d\lambda}{dE} = \frac{2\pi \alpha z^2}{\lambda^2} \sin^2 \theta_C$$

$n(E)$ è indice di rifrazione
del metallo per fotone
di energia E .

$$n = \sqrt{\epsilon_r}$$

fotoni emessi:
$$\frac{(spessore dx) \times \text{lunghezza d'onda del fotone emesso}}{}$$

$$\frac{dN}{dx} = \int_{\lambda_{min}}^{\lambda_{max}} \frac{d^2N}{dx dE} d\lambda =$$

Nel visibile $600 - 700 \text{ nm} = [4-7] \times 10^{-2} \times 10^9 \text{ m.}$

$$\int \frac{d^2N}{dx dE} d\lambda \sim \int \frac{1}{\lambda^2} d\lambda \approx \left(\frac{1}{\lambda_{min}} - \frac{1}{\lambda_{max}} \right)$$

$$\frac{dN}{dx} \approx 450 \cdot Z^2 \sin^2 \theta_c \quad \# \text{ fotoni emessi/cm.}$$

$$E_{\text{fotoni visibili}} \approx 2 \text{ eV} \Rightarrow \frac{dE}{dx} \approx \frac{dN}{dx} \times 2 \text{ eV} = 2 \text{ eV} \times 450 \cdot Z^2 \approx 900 Z^2 \text{ keV/cm.}$$

$$\Rightarrow \left. \frac{dE}{dx} \right|_{\text{cerenkov}} \approx 1 \text{ keV/cm.} \quad \begin{aligned} &\text{Energia persa dalle particelle} \\ &\text{per emissione luce cerenkov} \end{aligned}$$

$$\left. \frac{dE}{dx} \right|_{\text{ion}} \approx 2 \frac{\text{MeV}}{\text{cm.}} \times \rho (= 1 \text{ g/cm}^3 \text{ per H}_2\text{O}).$$

$$\text{H}_2\text{O } \rho = 1. \text{ g/cm}^3$$

$$\left. \frac{dE}{dx} \right|_{\text{cerenkov}} \approx 10^{-3} \left. \frac{dE}{dx} \right|_{\text{ion}}.$$

\Rightarrow trascurabile