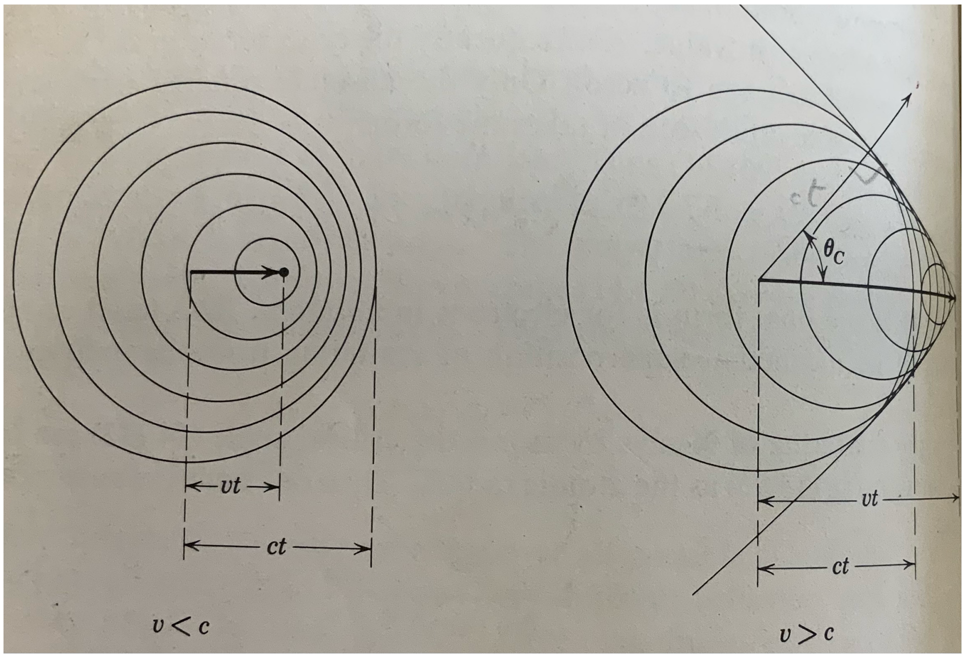
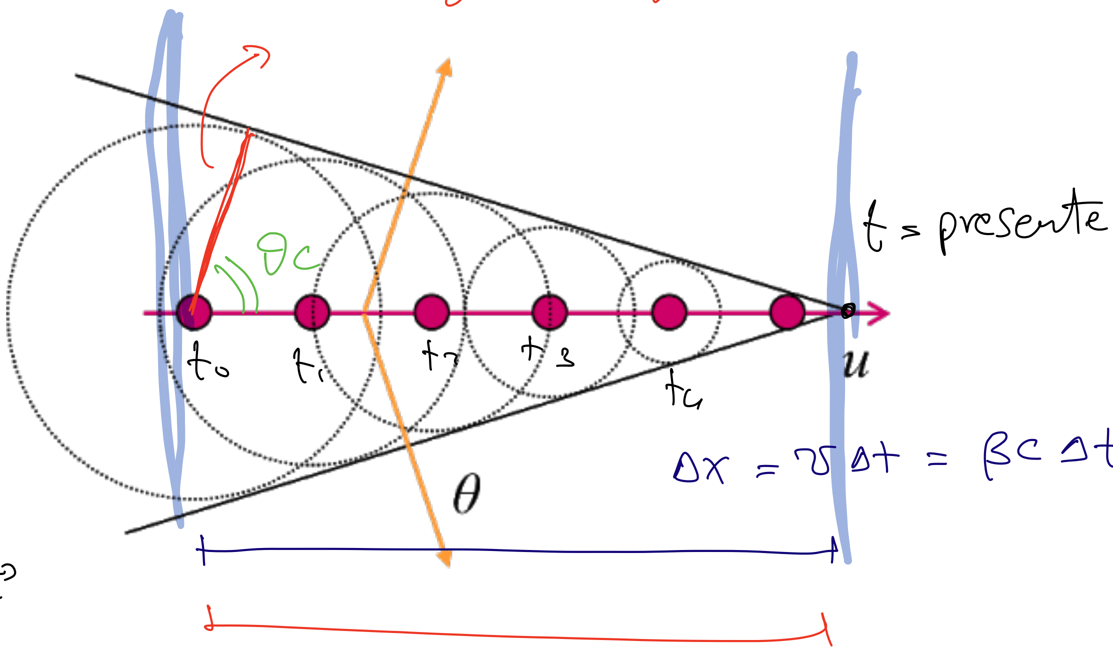
Token: 128 718

Effetto Cerenkov 1937 — Jackson 14.9



$L = v_g \Delta t = \frac{c}{n} \Delta t$



$\Delta x = v \Delta t = \beta c \Delta t$

m $\vec{v}$

$\Delta t = t_{presente} - t_0$

$L = \Delta x \cdot \cos\vartheta_c \Rightarrow \frac{c}{n} \Delta t = \cos\vartheta_c \, \beta c \Delta t$

$\cos\vartheta_c = \frac{1}{\beta n} \leq 1 \Rightarrow \beta \geq \frac{1}{n}$ — Effetto a soglia

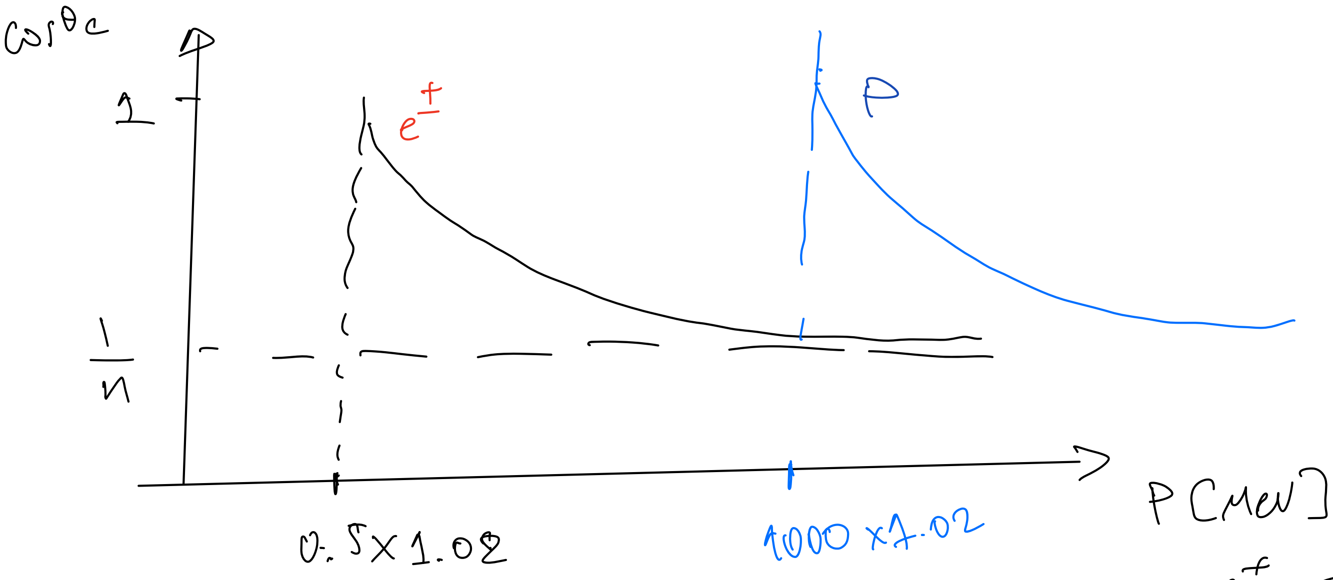
$$\beta \geq \frac{1}{n} = \beta_{th} = \frac{p_{th}}{E_{th}} = \frac{p_{th}}{\sqrt{p_{th}^2 + m^2}} = \frac{\cancel{p_{th}}}{\cancel{p_{th}}\sqrt{1+(m/p_{th})^2}}$$

$$n^2 = 1 + \left(\frac{m}{p_{th}}\right)^2$$

$$p_{th} = \frac{m}{\sqrt{n^2-1}}$$ massa proiettile

$n = 1.4 \Rightarrow \quad p_{th} \simeq m \times 1.02$

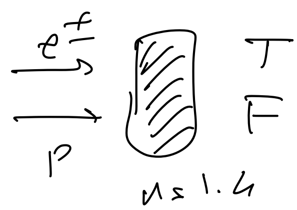
$$\cos\theta_c = \frac{1}{n}\sqrt{1+\left(\frac{m}{p}\right)^2} \qquad p \geq p_{th}$$

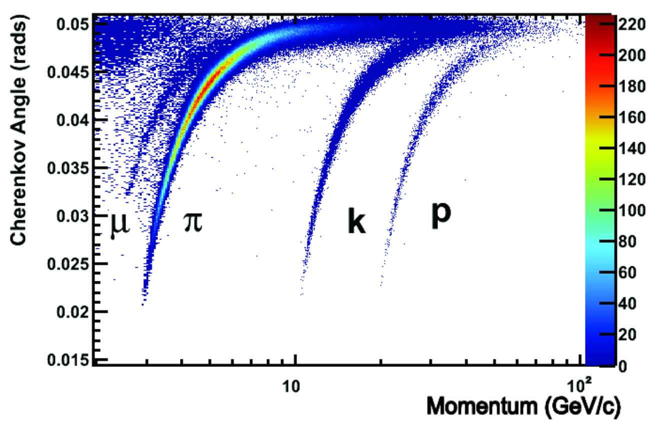


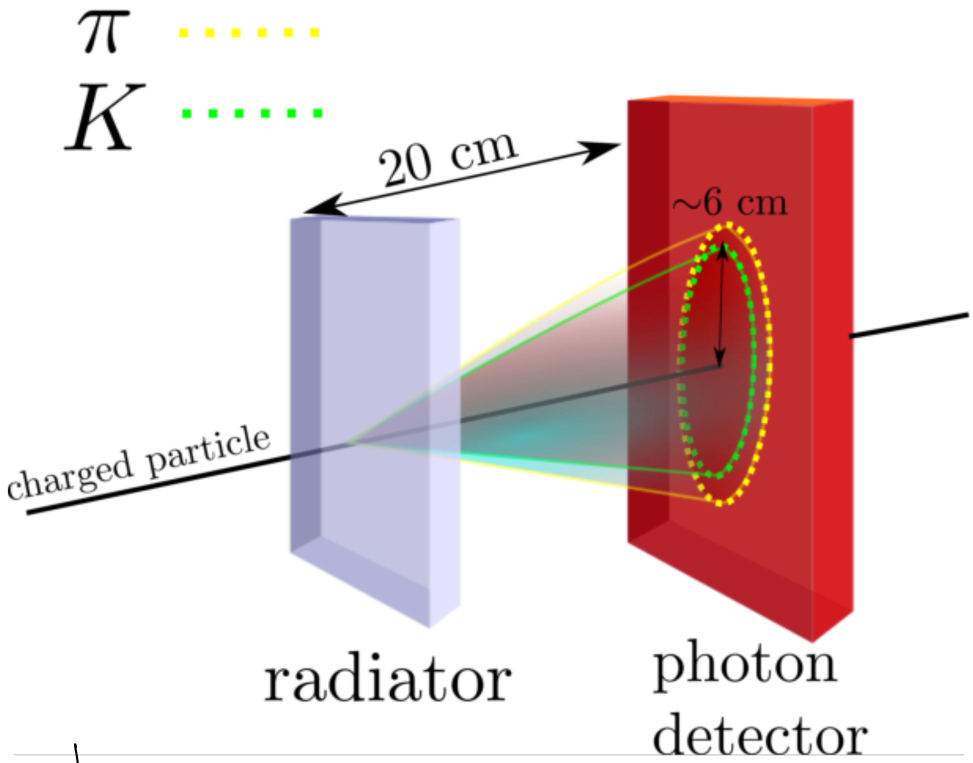
e^-

p

p = 10 MeV







$$\cos\vartheta_c = \frac{1}{n}\,\frac{1}{\beta}$$

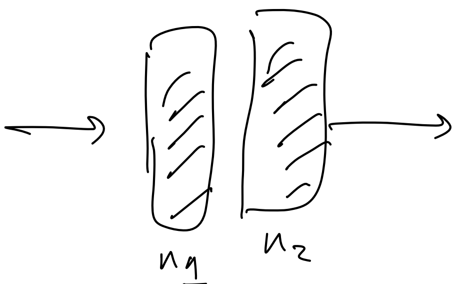
$m_\pi = 140$ MeV

$m_K = 500$ MeV

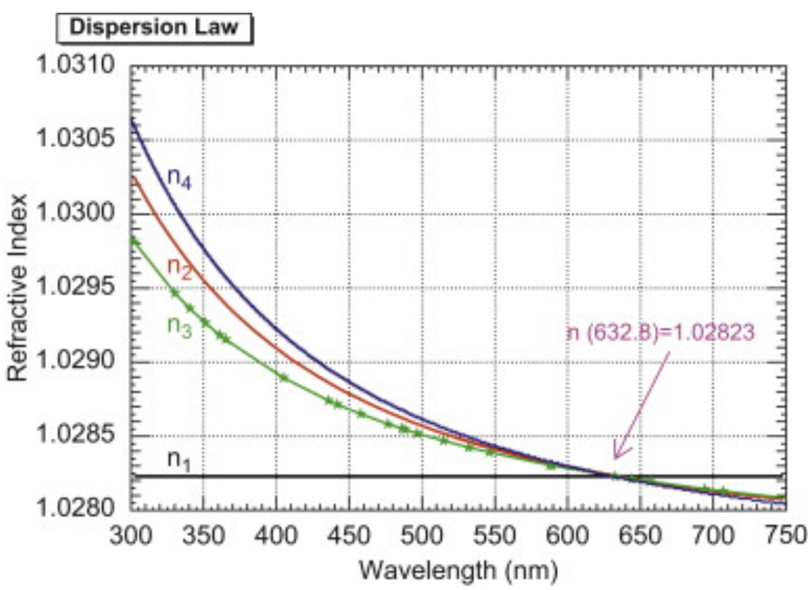
$p = 4$ GeV

$\beta_K \neq \beta_\pi$.

$\Rightarrow \quad \theta_\pi \neq \theta_K$



$$\frac{1}{n_1} < \beta < \frac{1}{n_2}$$



aerogel

$n = n(\lambda)$

$n = \sqrt{\varepsilon_r(\lambda)}$

$\beta \geqslant \frac{1}{n}$

Numero e Energia di photoni Cerenkov

$$\frac{d^2N}{dx\,dE} = \frac{\alpha Z^2}{\hbar c}\left(1 - \frac{1}{\beta^2 n^2(E)}\right) = \frac{\alpha Z^2}{\hbar c} \sin^2\theta_c$$

per $\beta \geqslant \frac{1}{n}$

$$E = h\nu = \frac{2\pi\hbar}{T} = 2\pi \frac{\hbar c}{\lambda}$$

$$\frac{d^2N}{dx\,dE} \Longrightarrow \frac{d^2N}{dx\,d\lambda} = \frac{2\pi\alpha Z^2}{\lambda^2} \sin^2\theta_c$$

$\lambda \in [\lambda_1, \lambda_2]$ $\quad \beta \geqslant \frac{1}{n(\lambda)}$

$$\frac{dN}{dx} = \int_{\lambda = 400nm}^{\lambda = 700nm} \frac{d^2N}{dx\,d\lambda}\, d\lambda = 2\pi\alpha Z^2 \sin^2\theta_c \left(\frac{1}{400nm} - \frac{1}{700nm}\right)$$

$$\simeq 6 \times \frac{1}{10^2}\left(Z^2 \sin^2\theta_c\right)\left[\;\right]$$

$$\underbrace{0.1 \times 10^9 \; m^{-1} \times 10^{-2}}$$

$$= 6 \times \frac{10^6}{10^2} Z^2 \sin^2\theta_c = 6 \times 10^4 \; Z^2 \sin^2\theta_c \; m^{-1}$$

fotoni/m $\Rightarrow \simeq 10^2$ /cm

$E_{vis} \simeq 1.8 \to 3.1$ eV

$$\Rightarrow \frac{dE}{dx} \simeq \frac{100\ eV}{cm} \qquad \left.\frac{dE}{dx}\right|_{BB} \simeq 1.5 \frac{MeV}{cm}$$

H_2O

$$\left.\frac{dE}{dx}\right|_{\substack{cerenkov \\ H_2O}} \simeq \frac{1\ keV}{cm}$$

Fotoni nelle materia γ

Dipende da E_γ $\longrightarrow$ $\sigma_{\gamma \to \text{in materia}} = \sigma(E_\gamma, Z)$

del mezzo

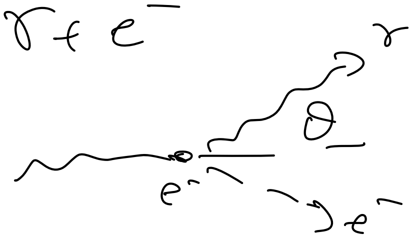
- basse energie: Effetto foto elettrico.

$$\gamma + A \longrightarrow e^- + A^+$$

$I_B \simeq 10\ eV\ Z$ esterni

Effetto a soglia $E_\gamma \geq I_B$

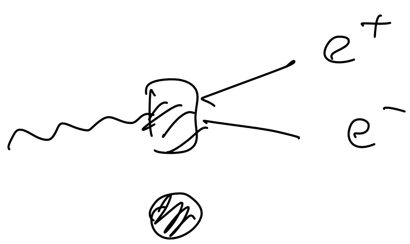
- Effetto Compton: $\gamma + e^- \longrightarrow \gamma + e^-$

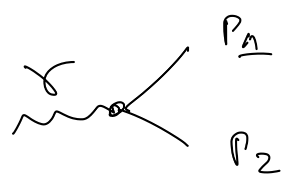


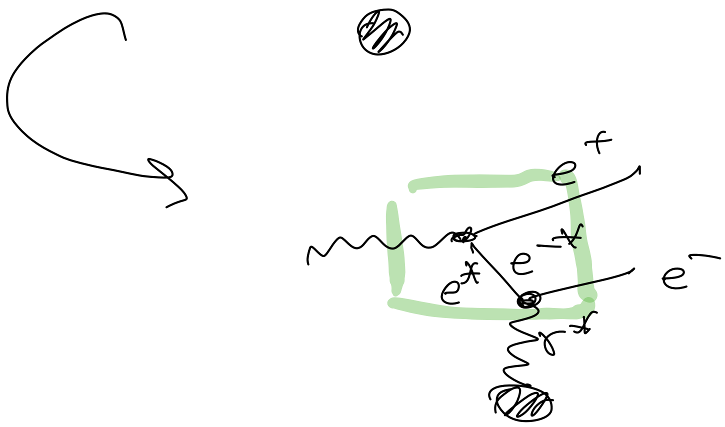
$$E_\gamma' = \frac{E_\gamma}{1 + (\ \ )\ \cos\theta}$$

- produzione di coppia.

$$\gamma + N \longrightarrow e^+ e^- + N$$







$$\gamma \longrightarrow e^+ e^-$$

$$\sqrt{s_{in}} = \sqrt{|P|^2} = \emptyset$$

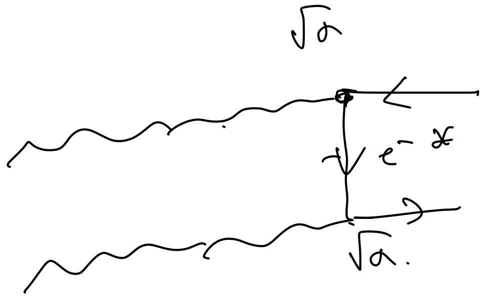
$$(\sqrt{s_{fin}})^2 = |P_{e^-}|^2 + |P_{e^+}|^2 + 2\, P_{e^-} \cdot P_{e^+}$$

$$= m_e^2 + m_e^2 + 2(\sim)$$

$$= 2m_e^2 + (\sim)$$

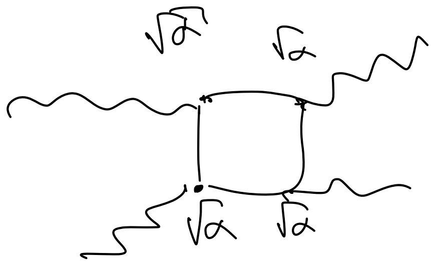
$\gamma \to e^+e^-$ non è possibile

$e^+e^- \to \gamma + \gamma$



$A \propto \alpha$

$\sigma \propto \alpha^2$



$A \propto \alpha^2 \Rightarrow \sigma \propto \alpha^4$

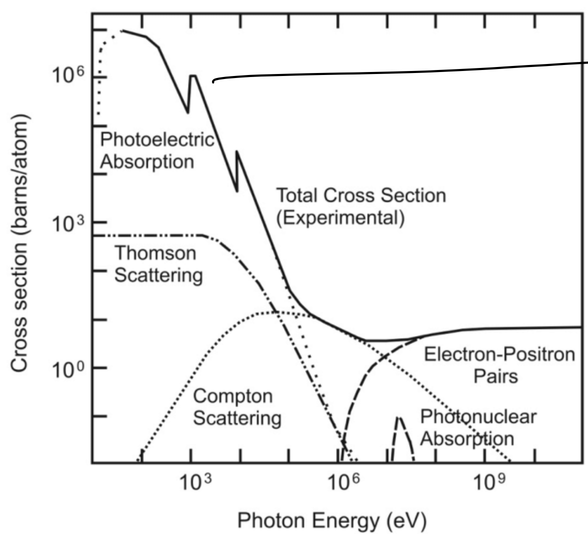
$\gamma + \gamma \to \gamma + \gamma$

$\gamma + N \to e^+e^- + N$ produzione a soglia.

$E_\gamma \simeq 2m_e = 0.511 \times 2 = 1.022$ MeV

- photoelettrico $E_\gamma \simeq$ eV — 100 keV dominante
- Compton $E_\gamma \sim 100$ keV $\to$ 10 MeV.
- produzione di coppia $E_\gamma \geq 1.022$ MeV

σ



$\to \dfrac{Z^5}{E_\gamma^3}$ $E_\gamma \leq m_e$

$\dfrac{Z}{E_\gamma}$ $E_\gamma > m_e$

$\sigma \simeq$ cost.

$E_\gamma \geq 10$ MeV

$$\frac{dW_\gamma}{dt} = (\sigma_{comp} + \sigma_{ph}) \text{ --- --- }$$

$$I(x) = I_0 e^{-\mu x} = I_0 e^{-(\sigma \cdot n) x}$$

$$[\sigma \cdot n] \quad L^2 L^{-3} = L^{-1}$$

$$\mu = \sigma \cdot n = \frac{1}{X_\gamma}$$ lunghezza di attenuazione del fotone.

$$\frac{1}{X_\gamma} \simeq \frac{7}{9} \frac{1}{X_0}$$

↪ lunghezza di radiazione di $e^{\pm}$

$E_\gamma \geq 2m_e$ $\qquad$ $\gamma \to e^+ e^-$

$E_\gamma = 100$ MeV.

$E_e \simeq 50$ MeV $\qquad$ $e^{\pm} \longrightarrow e^{\pm} + \gamma$

25 MeV $\quad$ 25 MeV