

Videolezione-2020-04-01

Esperimento Thompson: misura $\frac{q}{m}$ di e^-

Esperimento di Milliken \rightarrow misura di q

Diagramma dell'esperimento di Milliken: una goccia di olio nebulizzato (oil nebulized) è sospesa tra due piastre metalliche. Una piastra è collegata a un potenziale V_0 . La gravitazione g agisce verso il basso. L'attrito viscoso (viscous drag) è indicato.

Formula per la massa della goccia:

$$m = \frac{4\pi}{3} r^3 \rho$$

densità

Equazione della forza:

$$F = mg - 6\pi \eta r \underbrace{v_0}_{\text{Si misura}} = 0$$

$$\frac{4\pi}{3} r^3 \rho = mg = 6\pi \eta r v_0$$

$$r = 3 \sqrt{\frac{\eta v_0}{2\rho g}}$$

misura del raggio

$$F = mg - 6\pi \eta r v_i - qE = 0$$

$$qE = \frac{4\pi}{3} r^3 \rho g - 6\pi \eta r v_i$$

$$= 2\pi r \left[\frac{2}{3} r^2 \rho g - 3\eta v_i \right]$$

$$= 6\pi r \eta \left[\underbrace{v_0}_{\text{vel. reg. senza } E}} - \underbrace{v_i}_{\text{vel. reg. con } E \neq 0} \right]$$

vel. reg.
senza E

vel. reg.
con $E \neq 0$

$$\Rightarrow q = \frac{6\pi \eta r (v_0 - v_i)}{E}$$

Estrazione q

he misurato $q = 1.59 \times 10^{-19} \text{ C}$
entro 1% del valore attuale

$$M = \left(\frac{M}{q} \right) \cdot q = \frac{1.59 \times 10^{-19} \text{ C}}{1.76 \times 10^{11} \text{ C kg}^{-1}} \quad \text{Milliken}$$

Thompson

$$= 0.911 \times 10^{-30} \text{ kg}$$

in U.N. $\Rightarrow \underline{m_e = 0.511 \text{ MeV}}$

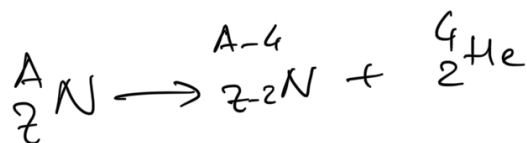
valore di q_m

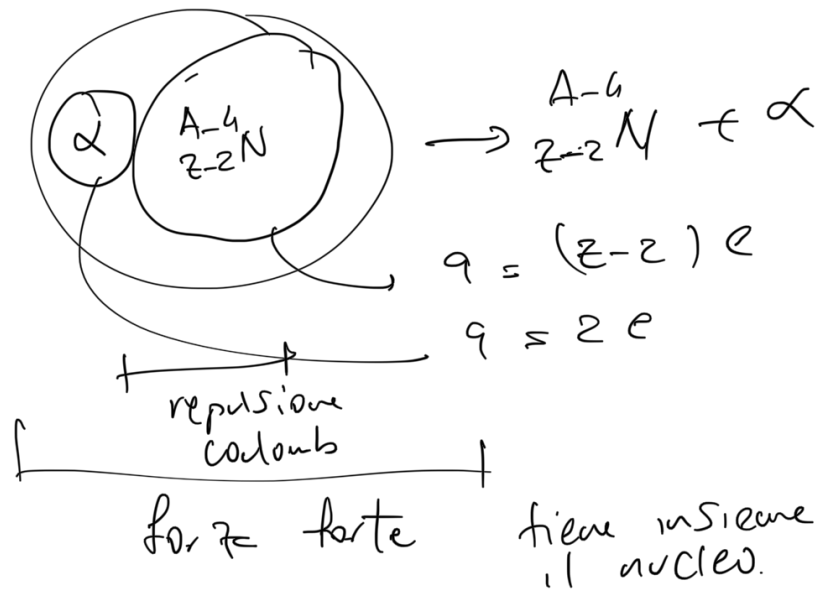
$\Rightarrow \alpha$ e β erano particelle diverse

Esperimenti di Rutherford-Geiger-Marsden
Scoperta del nucleo.

osservazione:
per A grandi: ${}_Z^A N \longrightarrow {}_{Z'}^{A'} N' + \alpha$

$$\alpha = {}_2^4 \text{He} \quad \begin{matrix} m_e \approx 3.7 \text{ GeV} \\ q = 2 \\ A = 4 \end{matrix}$$





particule α : $K \equiv E_{cm} \approx 3 - 7 \text{ MeV}$

$$K = 5 \text{ MeV.}$$

$$K = E - m = (r-1)m = 5 \text{ MeV.}$$

$$r-1 = \frac{5 \text{ MeV}}{3.7 \text{ GeV.}}$$

$$r = 1 + \frac{5}{3.7} \times 10^{-3}$$

$$r = \frac{1}{\sqrt{1-\beta^2}} \approx 1 + \frac{1}{2} \beta^2$$

$$\beta^2 \approx \frac{10^{-2}}{3.7}$$

$$\Rightarrow \beta \approx 0.05 \quad \text{non relativistic.}$$

Uranio $Z = 92$

U_{238} 99%
in nature

U_{235} 1%

$T_{1/2}$

4.5×10^9 anni

700×10^6 anni

Radio	$Z = 88$	$\tau_{1/2}$
	$A = 223$	11.4 gg
	$A = 226$	3.6 gg

Retroscel per fascio di α .

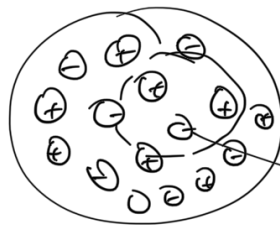
Bromuro di Radio $RaBr_2$

Obiettivo: verificare modello nucleare

e^- esiste con $q < 0$.

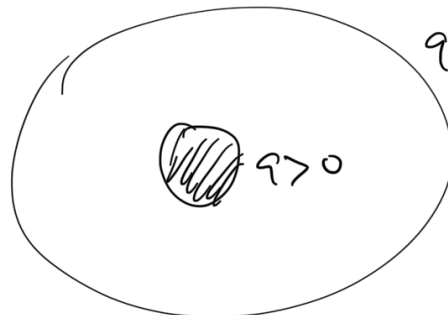
atomo neutro

1) Plum Cake Thompson

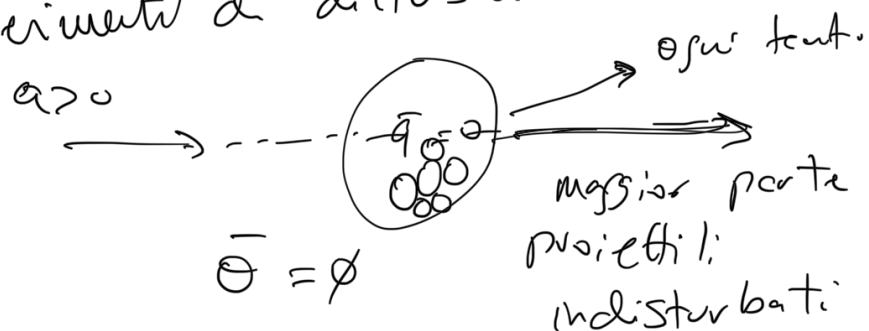


Distrib. continua
con $\bar{q} = 0$
 $\rightarrow \bar{q} \approx 0$

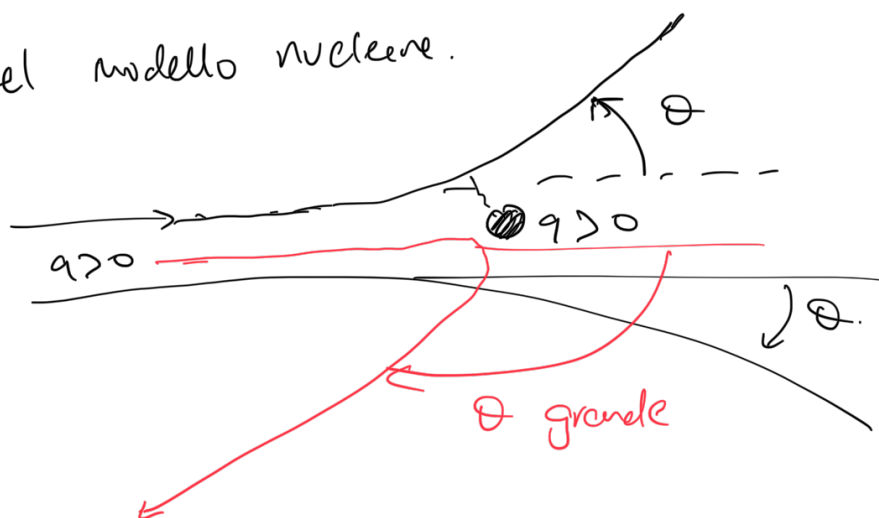
2) Nucleo-



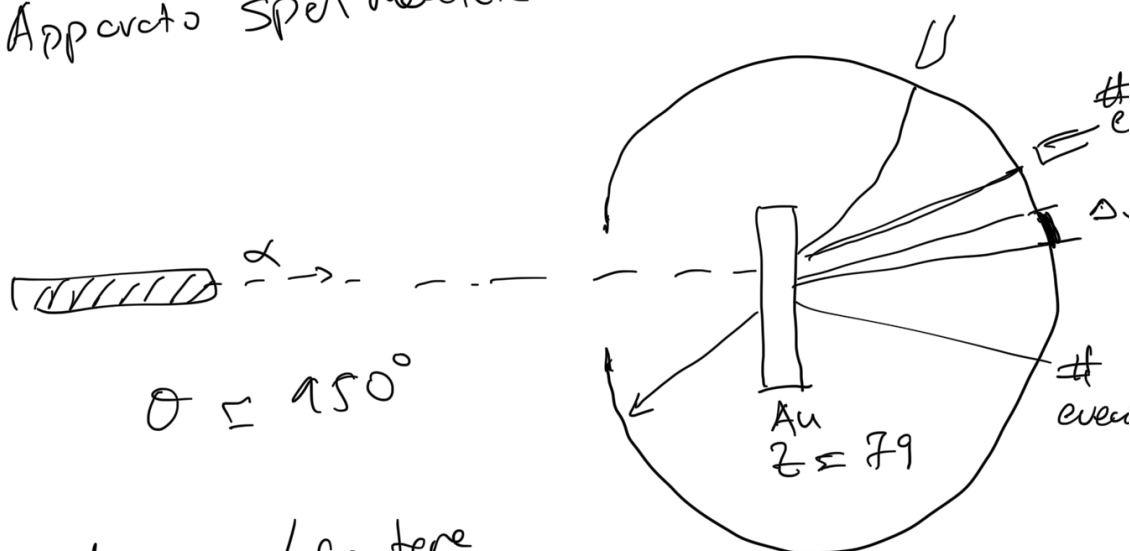
Esperimenti di diffusione



nel modello nucleare.



Apparato sperimentale



Misure/contare

eventi in funzione dell'angolo di deviazione

eventi

Misura sperimentale



Misura della sezione d'urto differenziale

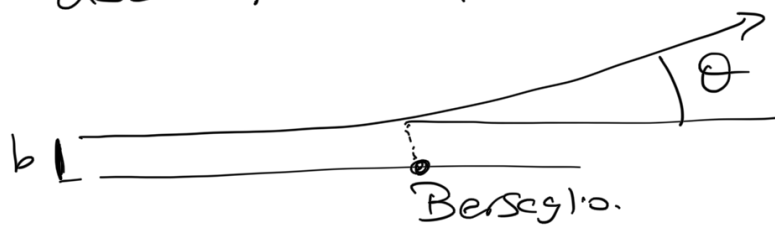
Calcolo Sezione d'urto

Interazioni Coulomb tra α e nucleo.

Campo Centrale

$$\frac{d\sigma}{d\Omega} = \left| \frac{b}{\sin\theta} \right| \left| \frac{db}{d\theta} \right|$$

per. d'impet.



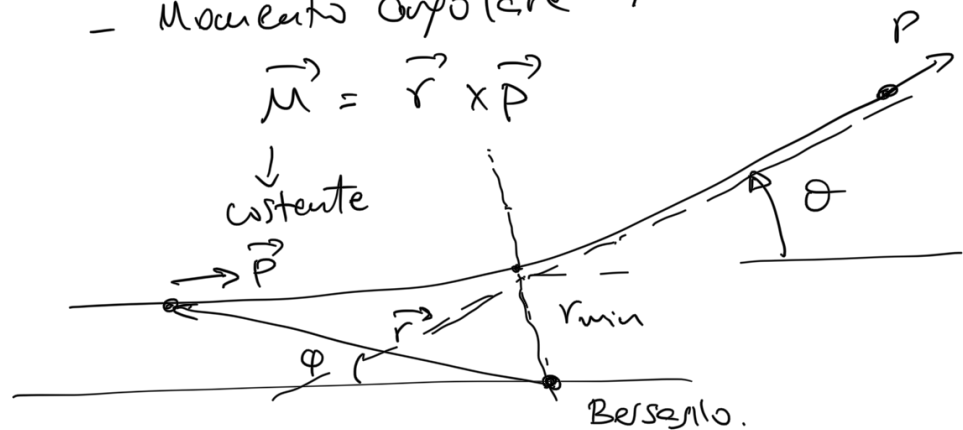
$$b \leftrightarrow \theta$$

Moto nel Campo Centrale (urto elastico)

- Energia si conserva
- Momento angolare \vec{M}

$$\vec{M} = \vec{r} \times \vec{p}$$

costante



$$L = \frac{m}{2} (\dot{r}^2 + r^2 \dot{\phi}^2) - U(r)$$

$$\frac{\partial L}{\partial \phi} = 0 \Rightarrow \frac{d}{dt} \frac{\partial L}{\partial \dot{\phi}} = 0 \Rightarrow \frac{\partial L}{\partial \dot{\phi}} \equiv P_{\phi} = \text{costante}$$

$$P_{\phi} = \frac{\partial L}{\partial \dot{\phi}} = \frac{m}{2} 2 r^2 \dot{\phi} = \frac{m r^2 \dot{\phi}}{\text{costante}} \equiv M$$

$$\vec{L} = \vec{r} \times \vec{p} \quad |\vec{p}| = m r \dot{\varphi}$$

$$\vec{L} \perp \vec{r} / \vec{p}$$

const
||

$$E = \frac{m}{2} (\dot{r}^2 + r^2 \dot{\varphi}^2) + U(r)$$

$$= \frac{m}{2} \dot{r}^2 + \frac{m r^2 \dot{\varphi}^2}{2} + U(r)$$

$$\rightarrow L = m r^2 \dot{\varphi}$$

$$= \frac{m}{2} \dot{r}^2 + \frac{L^2}{2 m r^2} + U(r)$$

pot. eff.

$$\frac{2}{m} \frac{m}{2} \dot{r}^2 = \left(E - U(r) - \frac{L^2}{2 m r^2} \right) \frac{2}{m}$$

$$\left| \frac{dr}{dt} = \sqrt{\frac{2}{m} \left(E - U(r) - \frac{L^2}{2 m r^2} \right)} \right.$$

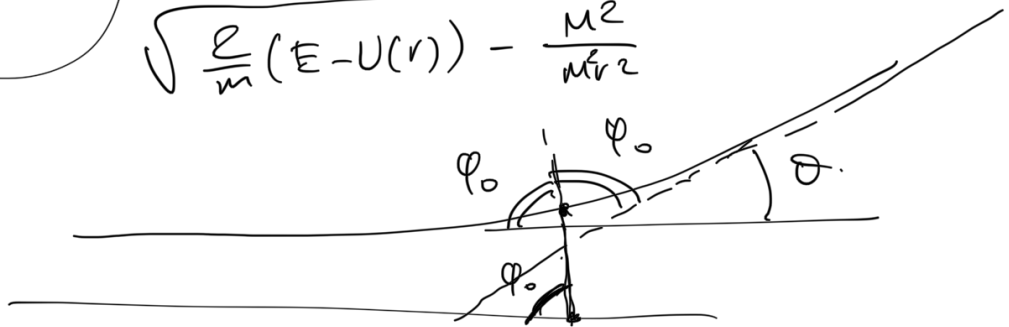
conserv.
dellen.

$$\text{const} = L = m r^2 \dot{\varphi} = m r^2 \frac{d\varphi}{dt}$$

$$\left| d\varphi = \frac{L}{m r^2} dt \right.$$

$$d\varphi =$$

$$\frac{L}{m r^2} \frac{dr}{\sqrt{\frac{2}{m} \left(E - U(r) - \frac{L^2}{2 m r^2} \right)}}$$

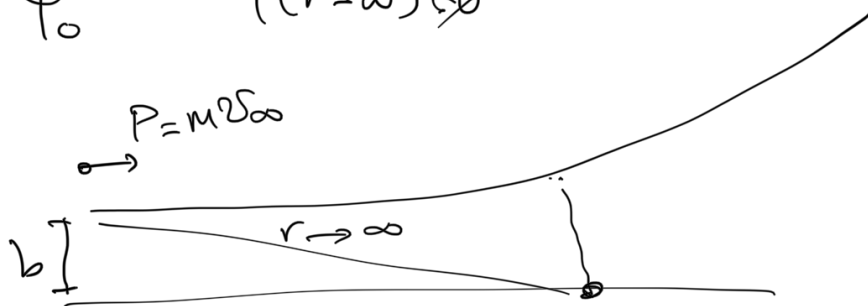


Calcolo di Θ da φ_0

$$\Theta = \pi - 2\varphi_0$$

$$\varphi(r_{\min}) - \varphi(r_{\infty}) = \int_{r_{\min}}^{\infty} \frac{\frac{M}{mr^2}}{\sqrt{\dots}} dr$$

φ_0 $\varphi(r=\infty) = 0$



$$E_0 = \frac{1}{2} m v_{\infty}^2$$

$$M = |\vec{r}_{\infty} \times \vec{p}_{\infty}| = b \cdot m v_{\infty}$$

v_{∞} : velocità iniziale del proiettile

$$\varphi_0 = \int_{r_{\min}}^{\infty} \frac{\frac{b m v_{\infty}}{r^2}}{\sqrt{2m\left(\frac{1}{2} m v_{\infty}^2 - U\right) - \frac{b^2 m^2 v_{\infty}^2}{r^2}}} dr$$

$$m^2 v_{\infty}^2 \left(1 - \frac{b^2}{r^2} - \frac{U}{E_0}\right)$$

$$\varphi_0 = \int_{r_{\min}}^{\infty} \frac{\frac{b}{r^2}}{\sqrt{1 - \frac{b^2}{r^2} - \frac{U}{E_0}}} dr$$

$$U(r) = \frac{A}{r} = \frac{1}{4\pi\epsilon_0} \frac{9.92}{r}$$

$$\alpha = \frac{1}{137} \leftarrow \frac{e^2}{4\pi\epsilon_0} \frac{Z_p \cdot Z_N}{r}$$

$$= \frac{\alpha Z_p \cdot Z_N}{r} \equiv \frac{A}{r}$$

$$\varphi_0 = \int_{r_{\min}}^{\infty} \frac{b/r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{A}{\epsilon_0} \frac{1}{r}}} dr$$

$$= \arccos \frac{A/2\epsilon_0 b}{\sqrt{1 + \left(\frac{A}{2\epsilon_0 b}\right)^2}}$$

$$B = \frac{A}{2\epsilon_0 b}$$

$$\varphi_0 = \arccos \frac{B}{\sqrt{1+B^2}}$$

$$\cos \varphi_0 = \frac{B}{\sqrt{1+B^2}} \Rightarrow \cos^2 \varphi_0 = \frac{B^2}{1+B^2}$$

$$\sin^2 \varphi_0 = 1 - \cos^2 \varphi_0 = \frac{1}{1+B^2}$$

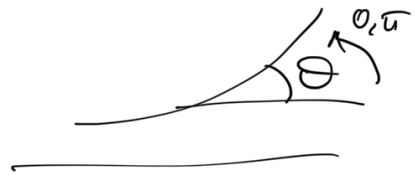
$$\frac{\cos^2 \varphi_0}{\sin^2 \varphi_0} = \cot^2 \varphi_0 = B^2 = \frac{A^2}{(2\epsilon_0 b)^2}$$

$$b^2 = \frac{A^2}{4\epsilon_0^2} \tan^2 \varphi_0$$

$$\theta = |\pi - 2\varphi_0|$$

$$\varphi_0 = \frac{\pi}{2} - \frac{\theta}{2}$$

$$\operatorname{ctg} \varphi_0 = \operatorname{ctg} \frac{\theta}{2}$$



$$b = \frac{A}{2\epsilon_0} \operatorname{ctg} \frac{\theta}{2} \quad b = b(\theta)$$

$$\frac{d\sigma}{d\Omega} = \left| \frac{b}{\sin\theta} \right| \left| \frac{db}{d\theta} \right|$$

$$\frac{d}{dx} \operatorname{ctg} x = -\frac{1}{\sin^2 x}$$

$$\left| \frac{db}{d\theta} \right| = \frac{A}{2\epsilon_0} \frac{1}{2} \frac{1}{\sin^2 \frac{\theta}{2}}$$

$$\sin\theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = \underbrace{\left(\frac{A}{2\epsilon_0} \frac{\cos \frac{\theta}{2}}{\sin \frac{\theta}{2}} \right)}_b \frac{1}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \frac{A}{2\epsilon_0} \frac{1}{2} \frac{1}{\sin^2 \frac{\theta}{2}}$$

$$= \frac{A^2}{16\epsilon_0^2} \frac{1}{\sin^4 \frac{\theta}{2}} = \left(\frac{A}{4\epsilon_0} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}}$$

$$\Delta = \alpha Z_p Z_N$$

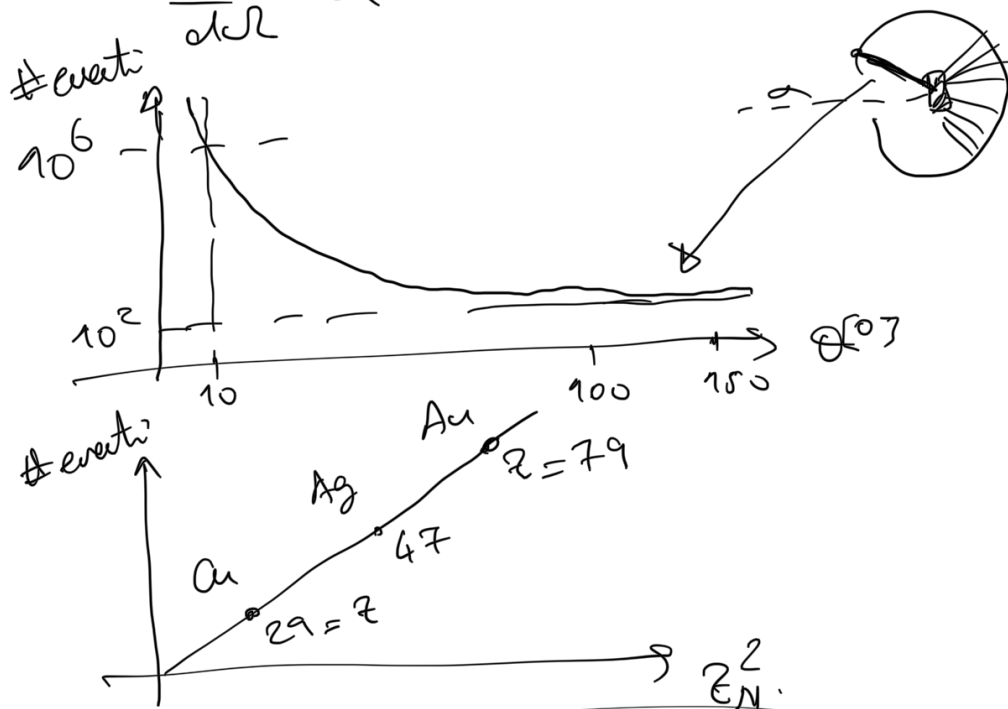
$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha Z_p Z_N}{4\epsilon_0} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}}$$

Formule di Rutherford

E_0 : En. incidente $\approx 5 \text{ MeV}$

$E_0 \nearrow \quad \sigma \searrow$

$$\frac{d\sigma}{d\Omega} \propto Z_N^2$$



\Rightarrow esiste il nucleo. ✓
modello a plum cake scartato ✗

r_{\min} :

$$K = \frac{1}{2} m v_{\infty}^2 = 5 \text{ MeV.}$$

$$U(r_{\min}) = \frac{\alpha Z_p \cdot Z_N}{r_{\min}} = K = \frac{1}{2} m v_{\infty}^2 = 5 \text{ MeV.}$$

$$r_{\min} \approx \frac{\alpha Z_p \cdot Z_N}{5 \text{ MeV}} = \frac{79 \times 2}{137} \frac{\text{MeV}^{-1}}{5} = 0.23 \text{ MeV}^{-1}$$

$$200 \text{ MeV} \times \text{fm} = 1 \Rightarrow \text{MeV}^{-1} = 200 \text{ fm}$$

$$r_{\text{min}} = 0.23 \times 200 \text{ fm} = 46 \text{ fm.}$$

Conclusione di Rutherford.

$$\text{nucleo} \mid r < 30 \text{ fm}$$

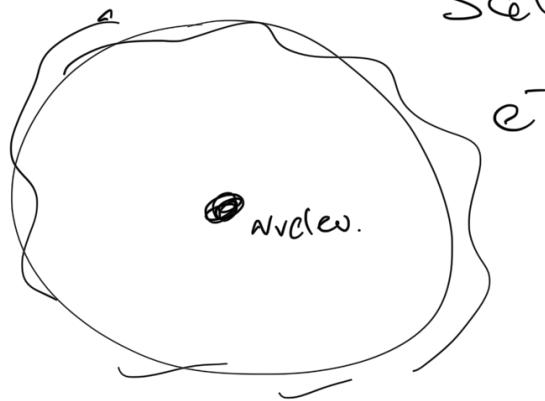
per i nuclei:

$$R = R_0 A^{1/3} \quad R_0 = 1.1 \text{ fm}$$

$$\text{Au} \quad R = 5 \text{ fm.}$$

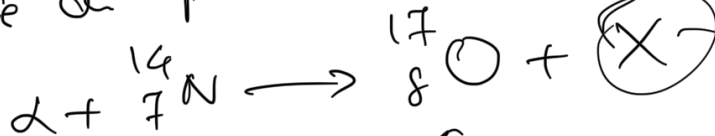
$$r < 30 \text{ fm.}$$

Scale del nucleo fm
Scale dell'atomo 10^{-10} m



Scoperta del protone Rutherford 1918

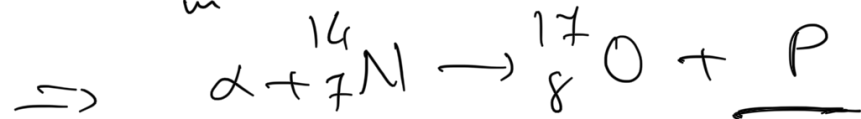
grazie ai particelle α



trasmutazione nucleare
artificiale

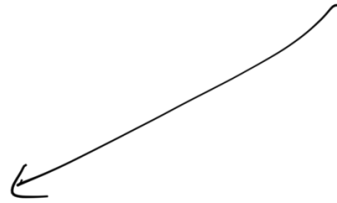
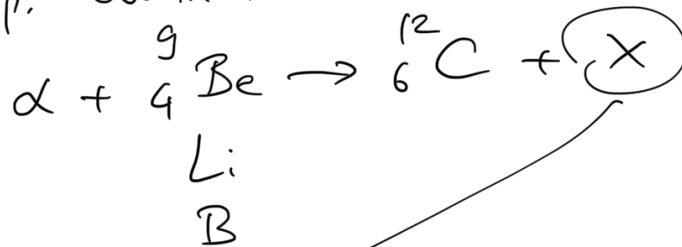
→ spettroscopie e misur di q/m

$\frac{q}{m}$ di X compatibile con H^+



Neutrone Chadwick (1931)

Esp. Cominciati da Curie et al.



Osservazioni

- neutra
- non ionizzante
- molto penetrante

Ipotesi:

- fotoni
- nuova particella presente