$P = (E, \vec{p})$

$$P^2 = \underbrace{E^2 - |\vec{p}|^2}_{= M^2} = E^2 - p_x^2 - p_y^2 - p_z^2$$

$$P_{TOT} = \left(\sum_i E_i , \sum_i \vec{p}_i \right)$$

$$|P_{TOT}^2| = \sqrt{s} \quad \underline{\text{inv.}}$$

$$\sqrt{s}_{LAB} = \sqrt{s}_{CdM}$$

$$\begin{cases} E_i = E_f \\ p_{x_i} = p_{xf} \\ p_{y_i} = p_{yf} \\ p_{z_i} = p_{zf} \end{cases}$$

EX PER CASA ($\sqrt{s}$)

(A) $\xrightarrow{e^+} \; \xleftarrow{e^-}$

$p(e^+) = p(e^-) = 1 \text{ GeV}$

$m(e^+) = m(e^-) = 0.511 \text{ MeV}$

$$P_{TOT} = \left(E_+ + E_- \, , \; \vec{p}_+ + \vec{p}_- \right)$$

$$E_+ = E_- = E = \sqrt{\underbrace{m_e^2}_{0.0005^2} + \underbrace{p^2}_{1^2}} \simeq 1 \text{ GeV}$$

$$\Rightarrow \quad P_{tot} = \left(2E \, , \; \vec{0} \right)$$

$$\Rightarrow \quad \sqrt{s} = \sum_i E_i^* = 2 \text{ GeV}$$

(B) $\xrightarrow{e^-} \; \xleftarrow{p}$

$p(e^-) = p(p) = 1 \text{ GeV}$

$m_e = 0.511 \text{ MeV}$

$m_p = 938 \text{ MeV}$

$$\vec{P}_{tot} = \vec{0}$$

$$\sqrt{s} = \underbrace{E_e}_{1 \text{ GeV}} + E_p = 1 + 1.37 = 2.37 \text{ GeV}$$

$$E_p = \sqrt{\underbrace{m_p^2}_{0.938} + \underbrace{p^2}_{1}} = 1.37 \text{ GeV}$$

(c) $e^- \longrightarrow \overset{p}{\bullet}$

$p_e = 2$ GeV

$p_p = 0$

$\vec{p}_{TOT} \neq \vec{0}$

LAB $\neq$ CdM

$$P_{TOT} = \left(E_e + \overset{m_p}{E_p},\ \vec{p}_e + \overset{0}{\vec{p}_p}\right) =$$

$$= \left(E_e + m_p,\ \vec{p}_e\right)$$

$$\sqrt{s} = |P_{TOT}| = \sqrt{(E_e + m_p)^2 - p_e^2} =$$

$$= \sqrt{E_e^2 + m_p^2 + 2m_pE_e - p_e^2}$$

$$E^2 = m^2 + p^2$$

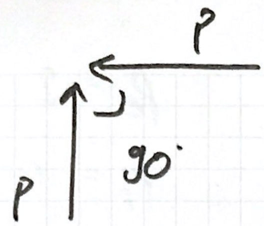
$$\Rightarrow E_e^2 - p_e^2 = m_e^2$$

$$\Rightarrow \sqrt{s} = \sqrt{m_e^2 + m_p^2 + 2m_pE_e}$$

0.511^2 $\quad$ 938^2

$$\sqrt{s} \simeq \sqrt{m_p^2 + 2m_pE_e} = 2.15 \text{ GeV}$$

(D)



$p = 1\ \text{GeV}$

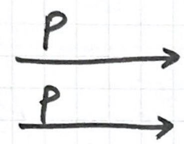
$E = \sqrt{m_p^2 + p^2} = 1.37\ \text{GeV}$

$$P_{TOT} = (E_p + E_p,\ \overset{x}{-p},\ \overset{y}{p},\ \overset{z}{0}) \quad \sqrt{E^2 - P_x^2 - P_y^2 - P_z^2}$$

$$\sqrt{s} = \sqrt{(2E_p)^2 - p^2 - p^2} =$$

$$= \sqrt{4E_p^2 - 2p^2} = 2.35\ \text{GeV}$$

(E)



$p = 100\ \text{GeV}$

$$P_{TOT} = (2E_p,\ 2\vec{p})$$

$$\Rightarrow \sqrt{s} = \sqrt{(2E_p)^2 - |2\vec{p}|^2} =$$

$$= \sqrt{4E_p^2 - 4p^2} =$$

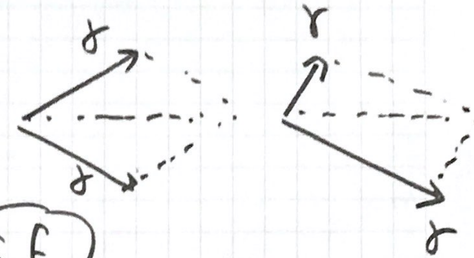
$$= 2\sqrt{\underbrace{E_p^2 - p^2}_{m_p^2}} = 2m_p = 1.88\ \text{GeV}$$

$$\sqrt{s}_{CdM} = \sum_i E_i^*$$

$H \to \gamma\gamma$

$m_H = 125$ GeV $\qquad (m_\gamma = 0)$

$\boxed{\text{LAB}}$ $\qquad H : (E_H, \vec{p}_H)$



s.i. $\longrightarrow$ s.f.

$\boxed{\text{CdM}}$ $\qquad H : (m_H, \vec{0})$

$$\sqrt{s}\Big|_{CdM} = m_H$$

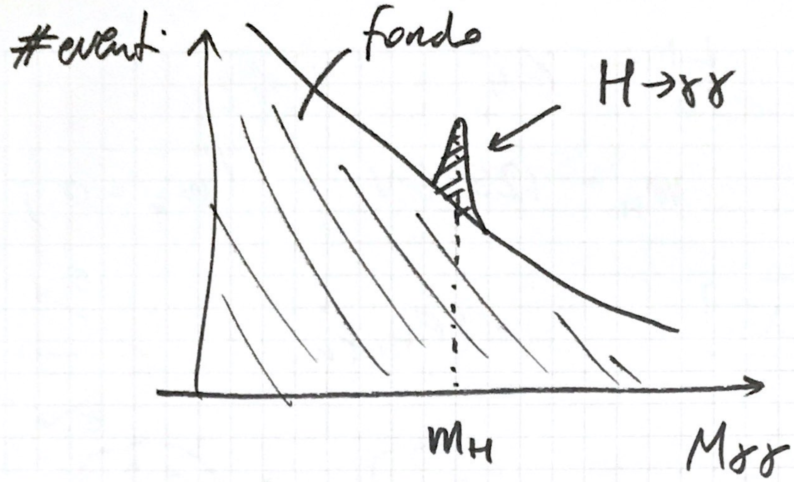
	s.i.	s.f.
CdM	·	·
LAB	·	·

$$P_{TOT} = \left(E_{\gamma_1} + E_{\gamma_2}, \ \vec{p}_{\gamma_1} + \vec{p}_{\gamma_2}\right)$$

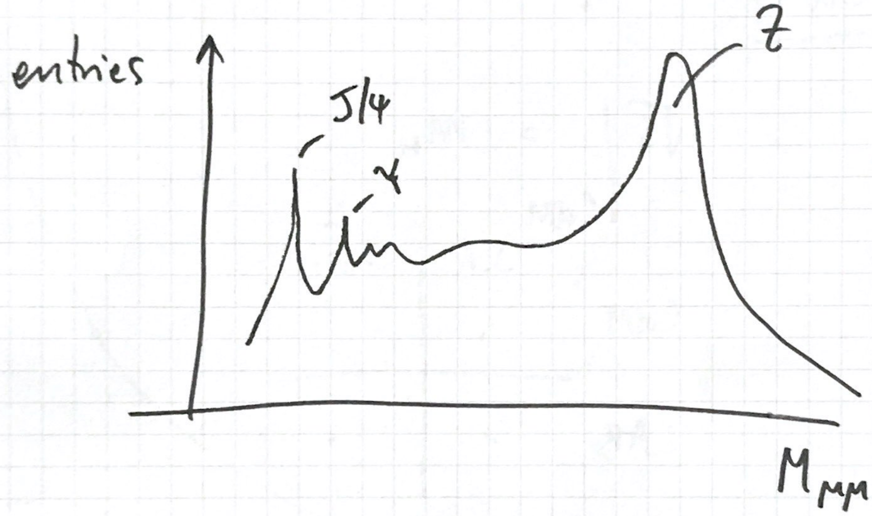
$$\sqrt{s} = \sqrt{(E_{\gamma_1} + E_{\gamma_2})^2 - |\vec{p}_{\gamma_1} + \vec{p}_{\gamma_2}|^2}$$

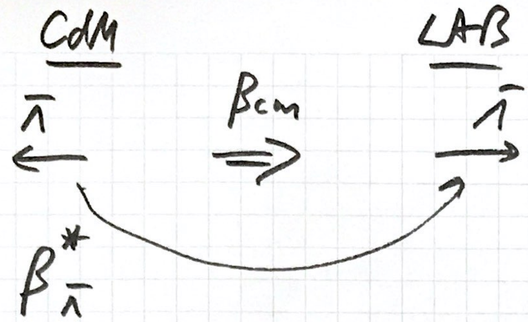
$(= m_H)$

x segnale

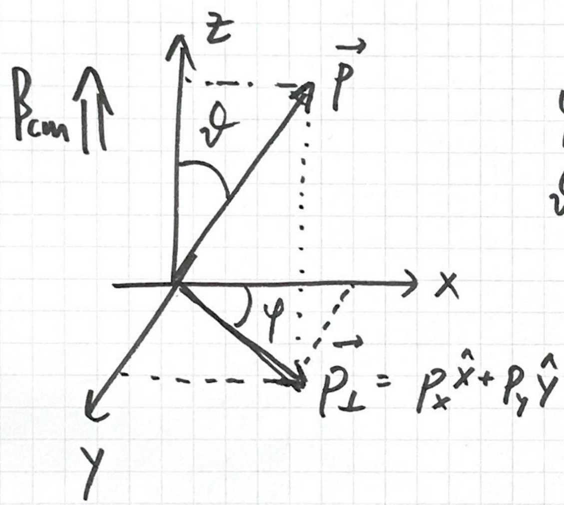


$X \to \underbrace{\mu^+ \mu^-}$





$$\begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} E \\ |\vec{p}|\sin\vartheta\cos\varphi \\ |\vec{p}|\sin\vartheta\sin\varphi \\ |\vec{p}|\cos\vartheta \end{pmatrix}$$



φ: azimuth $\quad \varphi \in [0, 2\pi)$

ϑ: polar $\quad \vartheta \in [0, \pi)$

$|\vec{p}| \equiv p$

$$\begin{pmatrix} E \\ p\sin\vartheta\cos\varphi \\ p\sin\vartheta\sin\varphi \\ p\cos\vartheta \end{pmatrix} = \begin{pmatrix} \gamma_{cm} & 0 & 0 & \beta_{cm}\gamma_{cm} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta_{cm}\gamma_{cm} & 0 & 0 & \gamma_{cm} \end{pmatrix}$$

(*)

$$\begin{pmatrix} E \\ p\sin\vartheta\cos\varphi \\ p\sin\vartheta\sin\varphi \\ p\cos\vartheta \end{pmatrix} = \begin{pmatrix} \gamma_{cm} & 0 & 0 & \beta_{cm}\gamma_{cm} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta_{cm}\gamma_{cm} & 0 & 0 & \gamma_{cm} \end{pmatrix} \begin{pmatrix} E^* \\ p^*\sin\vartheta^*\cos\varphi^* \\ p^*\sin\vartheta^*\sin\varphi^* \\ p^*\cos\vartheta^* \end{pmatrix}$$

$$\beta_{cm} \parallel \hat{z}$$

$$\vec{p} = \vec{p}_{\parallel} + \vec{p}_{\perp}$$

$$\vec{p}_{\parallel} = p_z \hat{z}$$

$$p_{\perp} = \sqrt{p_x^2 + p_y^2} \leftarrow \text{e' } \underline{\text{invariante}}$$

$$p_{\perp}^2 = p_x^2 + p_y^2 = p^2\sin^2\vartheta\cos^2\varphi + p^2\sin^2\vartheta\sin^2\varphi$$

$$= p^2\sin^2\vartheta\,(\underbrace{\cos^2\varphi + \sin^2\varphi}_{=1})$$

$$= p^2\sin^2\vartheta$$

uguale nel CdM : $p_{\perp}^{*2} = p^{*2}\sin^2\vartheta^*$

$p_{\perp} = p_{\perp}^*$ (e' invariante)

$$\Rightarrow p\sin\vartheta = p^*\sin\vartheta^*$$

$P_\perp$ è invariante $\qquad p\sin\vartheta = p^*\sin\vartheta^*$

$P_x \; , P_y$

$$x: \quad p\sin\vartheta\cos\varphi = p^*\sin\vartheta^*\cos\varphi^*$$

$$\Rightarrow \cos\varphi = \cos\varphi^*$$

$$y: \quad p\sin\vartheta\sin\varphi = p^*\sin\vartheta^*\sin\varphi^*$$

$$\Rightarrow \sin\varphi = \sin\varphi^*$$

$$\Rightarrow \varphi = \varphi^*$$

Come si trasforma ϑ?

$$(*) \quad \frac{P_y}{P_z} = \frac{p\sin\vartheta\sin\varphi}{p\cos\vartheta} = \tan\vartheta\sin\varphi$$

$$P_y = P_y^* = p^*\sin\vartheta^*\sin\varphi^*$$

$$P_z = \beta_{cm}\gamma_{cm}E^* + \gamma_{cm}p^*\cos\vartheta^*$$

$$\frac{P_y}{P_z} = \frac{p^*\sin\vartheta^*\sin\varphi^*}{\beta_{cm}\gamma_{cm}E^* + \gamma_{cm}p^*\cos\vartheta^*} = \tan\vartheta\sin\varphi$$

dividere sopra e sotto per p^*

$$\tan\vartheta = \frac{\sin\vartheta^*}{\gamma_{cm}\left(\beta_{cm}\, E^*/p^* + \cos\vartheta^*\right)}$$

$$\beta^* = \frac{p^*}{E^*}$$

$$\Rightarrow \boxed{\tan\vartheta = \frac{\sin\vartheta^*}{\gamma_{cm}\left(\beta_{cm}/\beta^* + \cos\vartheta^*\right)}} \quad (**)$$

β^* = velocità (in unità di c) della p.lla in CdM

β_{cm} = velocità del CdM nel LAB

$\Leftrightarrow$ BOOST fra CdM $\leftrightarrow$ LAB

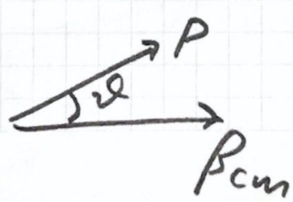
CASO: $\beta_{cm} > \beta^*$

denominatore di (**) > 0

numeratore di (**) > 0 $\quad \vartheta^* \in [0,\pi)$

$\Rightarrow \sin > 0$

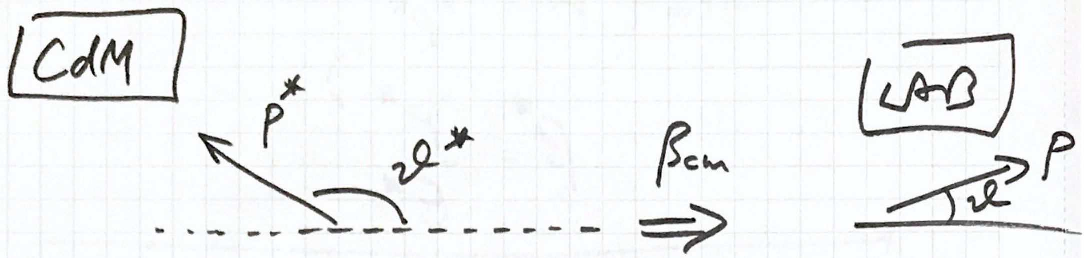
$$\Rightarrow \quad \vartheta \in \left[0, \frac{\pi}{2}\right)$$



sempre in avanti nel LAB

$\vartheta = 0$ sia per $\vartheta^* = 0$

$\vartheta^* = \pi$



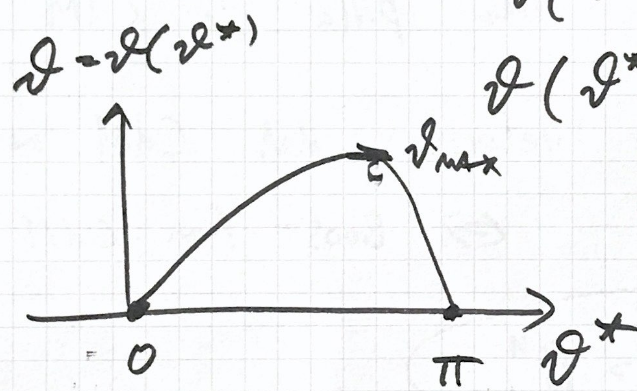
$\xrightarrow{p^*} \; \vartheta^* = 0 \quad \Rightarrow \quad \xrightarrow{p} \; \vartheta = 0$

$\xleftarrow{p^*} \; \vartheta^* = \pi \quad \Rightarrow \quad \xrightarrow{p} \; \vartheta = 0$

$\vartheta^* \in [0, \pi)$ $\qquad \vartheta(\vartheta^*)$

$\vartheta(\vartheta^* = 0) = 0$

$\vartheta(\vartheta^* = \pi) = 0$



$\vartheta_{MAX} = ?$

$$(**) \qquad \frac{d}{d\vartheta^*}(\tan\vartheta) = \frac{1 + \cos\vartheta^* (\beta_{cm}/\beta^*)}{\gamma_{cm}\left(\beta_{cm}/\beta^* + \cos\vartheta^*\right)^2}$$

$$\stackrel{!}{=} 0$$

$$1 + \cos\vartheta^* \left(\frac{\beta_{cm}}{\beta^*} \right) = 0 \quad \left(\begin{matrix} \text{NEL} \\ \text{MAX} \end{matrix} \right)$$

$$\Rightarrow \boxed{\cos\vartheta^* = -\frac{\beta^*}{\beta_{cm}}}$$

in corrispondenza di questo ϑ^* si ha ϑ_{MAX}

$$\Leftrightarrow \sin\vartheta^* = \sqrt{1 - \cos^2\vartheta^*} =$$

$$= \sqrt{1 - \left(\beta^* / \beta_{cm} \right)^2}$$

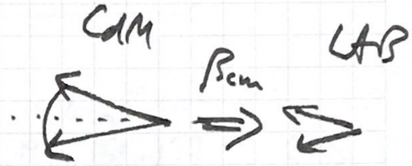
$$\Rightarrow \tan(\vartheta_{MAX}) = \frac{\sqrt{1 - \beta^{*2} / \beta_{cm}^2}}{\gamma_{cm} \left(\beta_{cm} / \beta^* - \frac{\beta^*}{\beta_{cm}} \right)} =$$

$$= \frac{\sqrt{1 - \frac{\beta^{*2}}{\beta_{cm}^2}}}{\gamma_{cm} \left(\frac{\beta_{cm}^2 - \beta^{*2}}{\beta_{cm} \beta^*} \right)} =$$

$$= \frac{\cancel{\frac{1}{\beta_{cm}}} \sqrt{\beta_{cm}^2 - \beta^{*2}}}{\gamma_{cm} \frac{\beta_{cm}^2 - \beta^{*2}}{\cancel{\beta_{cm}} \beta^*}}$$

$$\boxed{\tan(\vartheta_{MAX}) = \frac{\beta^*}{\gamma_{cm}\sqrt{\beta_{cm}^2 - \beta^{*2}}}}$$

$$(\beta_{cm} > \beta^*)$$



CASO: $\beta_{cm} < \beta^*$

$$\exists : \underset{\text{CdM}}{\longleftarrow} \quad \overset{\beta_{cm}}{\Longrightarrow} \quad \underset{\text{LAB}}{\leftarrow}$$

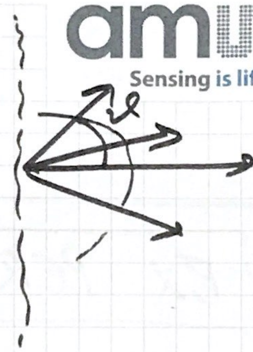
CASO LIMITE $\beta^* = \beta_{cm}$

Velocità della particella annulla esattamente il boost

$$\overset{\beta^*}{\longleftarrow} \quad \overset{\beta_{cm}}{\longrightarrow} \qquad \underline{\text{LAB}}$$

$$\cdot \; \vec{p} = 0$$

ϑ^* CdM



~~$\vartheta^* = 0$ CdM~~

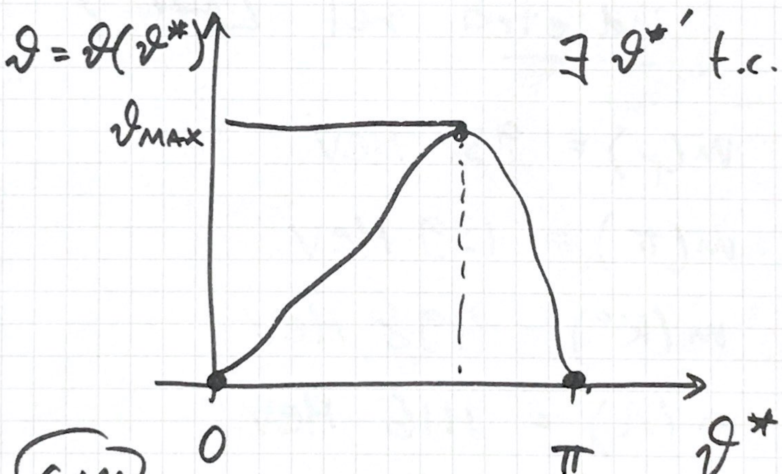
CdM		LAB

$\vartheta^* = 0 \quad \xrightarrow{p^*} \quad \overset{\beta_{cm}}{\Longrightarrow} \quad \xrightarrow{p} \quad \vartheta = 0$

$\vartheta^* = \pi \quad \xleftarrow[\vartheta^* = \pi]{p^*} \quad \overset{\beta_{cm}}{\Longrightarrow} \quad \xrightarrow{p} \quad \vartheta = 0$

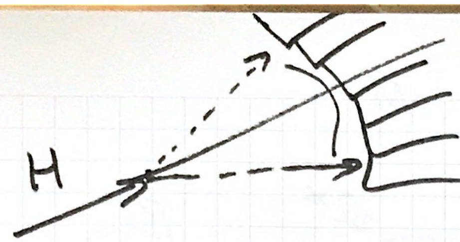
$$\vartheta^* \longrightarrow \vartheta$$

$$\vartheta = \vartheta(\vartheta^*)$$



$\exists\, \vartheta^{*\prime}$ t.c. ϑ è MAX

ϑ_{MAX}



$\boxed{\text{EX PER CASA}}$

$$\pi^- \, p \longrightarrow K^0 \, \Lambda$$

$$\sqrt{s} = 3 \text{ GeV}$$

1. Qual e' l'impulso nel CdM di π^- e Λ ?

2. Supponendo il protone fermo nel LAB può il K^0 essere emesso <u>indietro</u> nel LAB ?

$m(p) = 938$ MeV

$m(\pi) = 139$ MeV

$m(K^0) = 498$ MeV

$m(\Lambda) = 1116$ MeV

PDG Particle Data Group