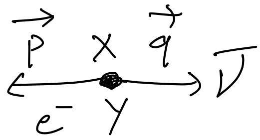
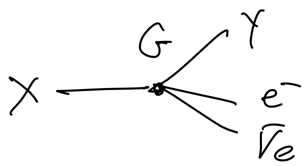
Token: 566 803

$Q = m_X - m_Y - m_e \simeq O(\text{1 MeV})$

$${}^A_Z X \longrightarrow {}^{A}_{Z+1} Y + e^- + \bar\nu_e \qquad \Gamma(X \to Y + e^- + \bar\nu_e) = 2\pi |M_{fi}|^2 \rho \Big|_{E_f = E_i}$$

$$M_{fi} = -i \int d^3r\, \psi_Y^* \psi_X \underbrace{\psi_e^* \psi_\nu^*}_{\hookrightarrow\ \frac{e^{-i(\vec p + \vec q)\cdot \vec r}}{\sqrt{V}\sqrt{V}}} H_I \qquad \simeq 1 + O(\dots)$$





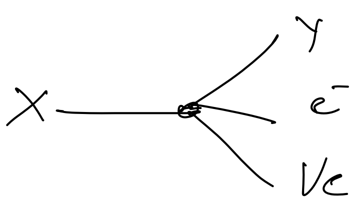
$H_I = G \qquad \psi \propto \frac{1}{\sqrt V}$

Ipotesi: $\quad |\vec p + \vec q \cdot|\, |\vec r| \simeq \frac{1}{200}$

$$N = \int \underset{V}{d^3r}\ \underset{\frac{1}{\sqrt V}}{\psi_Y^*}\ \underset{\frac{1}{\sqrt V}}{\psi_X} \qquad |N| \simeq 1$$

$$M_{fi} = -i \frac{G}{V} N \Rightarrow |M_{fi}|^2 = \frac{G^2}{V^2} |N|^2$$

$$\rho(E)\Big|_{E_f = E_i} = \int dn\, \delta(E_f - E_i)$$



$\vec P_Y + \vec P_{e^-} + \vec P_\nu = \emptyset$

$\vec P_Y = -(\vec P_{e^-} + \vec P_\nu)$

scelgo a piacere $\vec P_e \equiv \vec p$, $\vec P_\nu \equiv \vec q \Rightarrow \vec P_Y$ fissato.

$$dn = \frac{V}{(2\pi)^3} \underbrace{4\pi\, p^2 dp}_{\text{sp. fasi } e^-} \quad \frac{V}{(2\pi)^3} \underbrace{(4\pi)\, q^2 dq}_{\nu_e}$$

$E_f = E_i$: $\quad M_X = M_Y + E_e + E_\nu \Rightarrow \underbrace{M_X - M_Y}_{E_T} = E_e + E_\nu.$

Massima en. trasferibile a e^-, ν_e

$$\rho(E) = \int \delta(E_e + E_\nu - E_T) \frac{V^2}{(2\pi)^6} (4\pi)^2 \, p^2 dp \; q^2 dq$$

$m_\nu \simeq \emptyset \Rightarrow q = E_\nu \qquad q^2 dq = E_\nu^2 dE_\nu$

$m_e \neq 0 \qquad p^2 = E_e^2 - m_e^2 \quad \Rightarrow \cancel{2} p\,dp = \cancel{2} E_e dE_e$

$$p^2 dp = p\, E_e dE_e$$

$$\Gamma = 2\pi \frac{G^2}{V^2} |M|^2 \frac{V^2}{(2\pi)^6} (4\pi)^2 \int \underbrace{\delta(E_e + E_\nu - E_T)} p\, E_e dE_e \underbrace{E_\nu^2 dE_\nu}$$

$$\Rightarrow E_\nu = E_T - E_e$$

$$\Gamma \propto G^2 \int p\, E_e (E_T - E_e)^2 \, dE_e$$

$$\propto \int_{m_e}^{E_T} E_e \sqrt{E_e^2 - m_e^2} \, (E_T - E_e)^2 dE_e \quad \propto$$

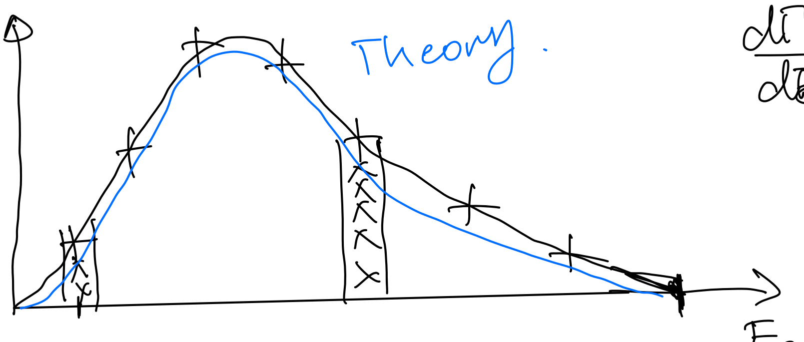
$n \to p\, e^- \bar\nu_e \qquad E_T \gtrsim m_e \qquad$ simili

per A, Z grandi $\quad E_T \gg m_e \quad \longrightarrow \int_0^{E_T}$

\# decadimenti

$\frac{d\Gamma}{dE}$

larghezza differenziale



$\frac{d\Gamma}{dE}$ = # decadimenti per intervallo di energia

$$\frac{d\Gamma}{dE} \propto G^2 \; E_e \sqrt{E_e^2 - m_e^2} \, \underbrace{(E_T - E_e)^2}$$

endpoint

$$\Gamma = \frac{G^2}{V^2} \rho(E) \qquad \delta(E) \qquad E^{-1} \qquad E$$

$$\int \delta(E)\, dE = 1$$

E

$$\underbrace{\delta(E_f - E_i)}_{\frac{1}{E}} \quad \frac{V}{(\ )} p^2 dp \quad \frac{V}{(\ )} q^2 dq.$$

Γ

Dim. $$E = [G]^2 \frac{1}{\cancel{V^2}} \frac{1}{E} \cancel{V^2} (E)^6.$$

$$E^2 = [G]^2 \quad E^6$$

$$[V] = [L]^3 = [E]^{-3}$$

$$V^2 = [E]^{-6}$$

$$[G] = [E]^{-2}$$ costante di Fermi

$$\Gamma \propto G^2 \int P E_e (E_T - E_e)^2 \, dE_e$$

$$\propto \int_{m_e}^{E_T} E_e \sqrt{E_e^2 - m_e^2} \, (E_T - E_e)^2 \, dE_e \quad \propto$$

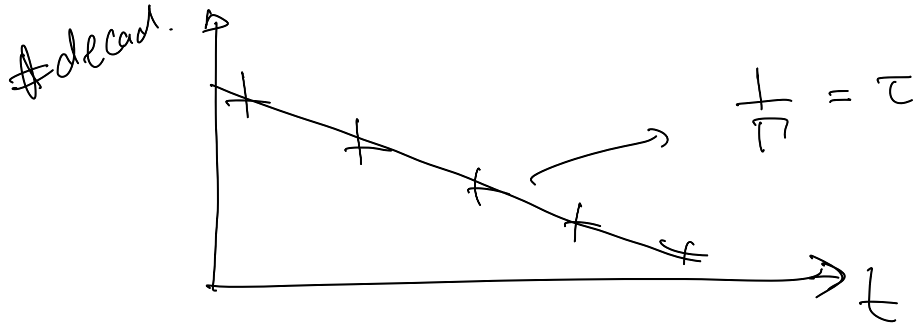
$$E_T \gg m_e \implies E_e \gg m_e$$

$$\Gamma \propto \int E_e^2 (E_T - E_e)^2 \, dE_e \propto \int_0^{E_T} E_e^4 \, dE_e$$

$$\implies \Gamma \propto G^2 E_T^5 \qquad E_T = M_X - M_Y$$

$$\frac{1}{\tau} = \Gamma = (\sim) G^2 E_T^5 |N|^2$$

legge di Sargent



$$\frac{1}{\Gamma} = \tau$$

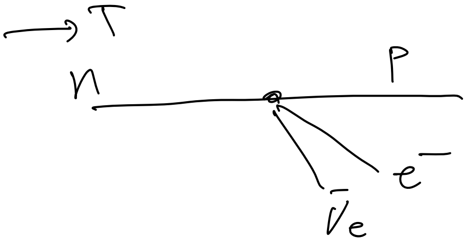
$$G = 1.1664 \times 10^{-5}\ \text{GeV}^{-2}$$

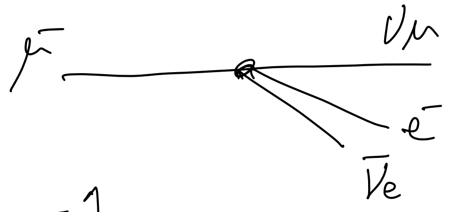
$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \qquad \tau = 10^{-6}\ \text{s} \cdot \times 2.2$$

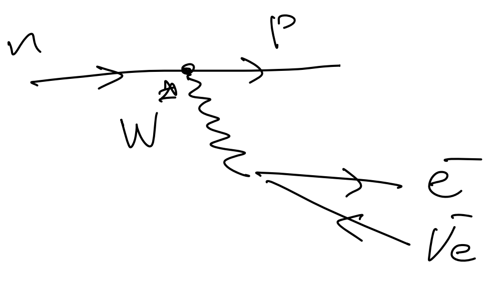
$$\frac{1}{\tau} = \Gamma(\mu \rightarrow e \nu \bar{\nu}) = \frac{G^2}{192\pi^3}\, m_\mu^5$$

$\rightarrow T$

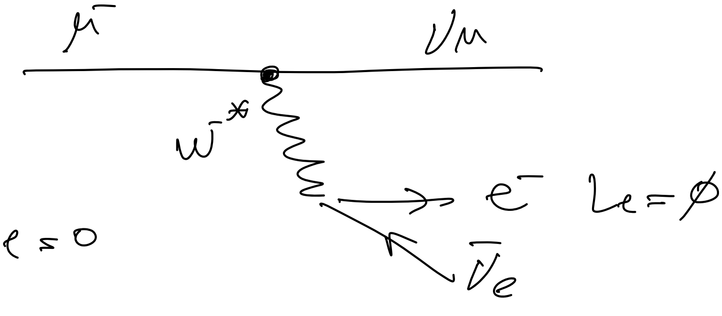




$L_\mu = 1$ $\qquad$ 1



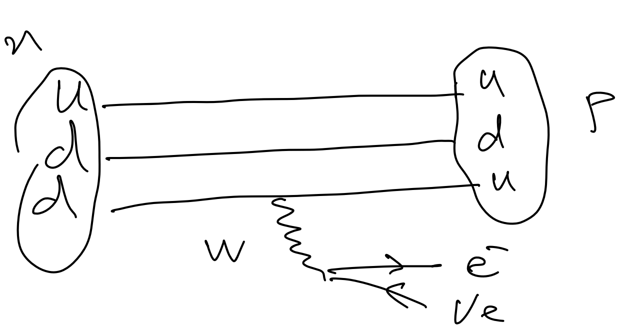
$L_e = 0$



$$E^2 = p^2 + m^2$$

$$\psi \propto e^{-iEt}$$

$$n = \begin{pmatrix} u \\ d \\ d \end{pmatrix} \qquad p = \begin{pmatrix} u \\ u \\ d \end{pmatrix}$$



$n \rightarrow p + e^- + \bar{\nu}_e$

$\Rightarrow \quad \bar{\nu}_e + p \longrightarrow n + e^+$ possibile?

$Q = -1.8\ MeV$

$E_\nu \geq 1.8\ MeV$

1 reazione fissione

$Q \simeq 200\ MeV$

$\langle \nu \rangle = 6$

$P = 1000\ MW = 1\ GW$

$1\ eV \approx 1.6 \times 10^{-19}\ J$

$\Rightarrow \quad 1\ W = \frac{1\ J}{1\ s.} = \frac{1}{1.6} \times 10^{19}\ \frac{eV}{s}$

$$\frac{P_{reattore}}{200\ MeV} \times 6 = \frac{\#\nu}{s} = 2 \times 10^{20}\ Hz$$

$\bar{\nu}_e + p \longrightarrow n + e^+$

$$\frac{dN_r}{dt} = \sigma \cdot \frac{dN_p}{dt} n_b \cdot d = \sigma \frac{dN_p}{dt\, S} \underbrace{(n_b \cdot d \cdot S)}_{N_B}$$

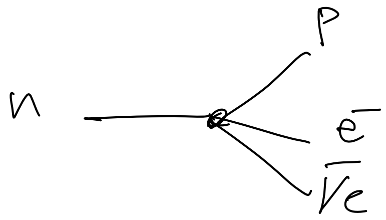
$$= \sigma \cdot \phi_p \cdot N_B = \sigma \cdot n_p \cdot v_p\, N_B$$

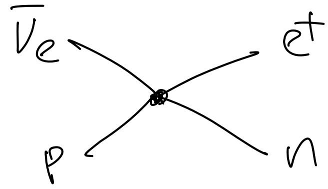
$$= \sigma \frac{N_p}{V} v_p \cdot N_B$$

$$\frac{1}{N_B} \frac{1}{N_p} \frac{dN_r}{dt} = \sigma \cdot \frac{v_p}{V}$$

$$\Gamma(\bar{\nu}_e + p \longrightarrow n + e^+) = 2\pi |M_{fi}|^2 \rho(E)$$

$$\sigma = \frac{V}{v_p} 2\pi |M_{fi}|^2 \rho(E)$$



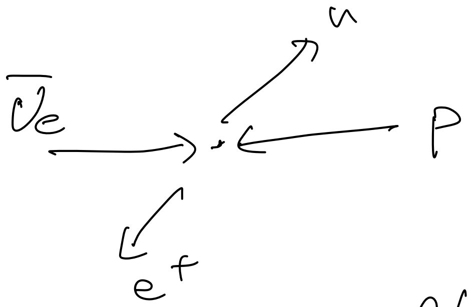


β^-

$$M_{fi} = -i\int d^3r\, \psi_F^* H_I \psi_I$$

$$H_I = G$$

$$= -i\,\frac{G}{V}\,|N|$$



$\vec{P}_n + \vec{P}_{e^+} = 0$ nel centro di massa.

$$\rho(E) = \int \delta(E_f - E_i)\,\frac{V}{(2\pi)^3}\,(4\pi)\,p^2 dP.$$

nel. c.d.m.

$E_f = E_n + E_{e^+}$ $\quad E_i = E_{\bar\nu} + E_p$

$$\sigma = \frac{V}{v_p}\,2\pi\,|M_{fi}|^2\,\rho(E) =$$

$$= \frac{V}{v_p}\,2\pi\,\frac{G^2}{V^2}\,|N|^2 \int \frac{V}{(2\pi)^3}\,(4\pi)\,p^2 dp\,\delta(E_f - E_i)$$

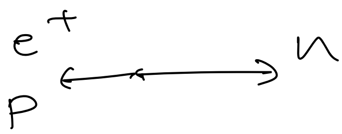
$\underbrace{v_p}_{\text{proiettile } \bar\nu}$

$$p^2 = E^2 - m^2.$$

$$2PdP = 2EdE.$$

$$\Rightarrow\ p^2 dp = PEdE$$

$$\int p^2 dp\,\delta(E_f - E_i) = \int p\,E\,\delta(E_f - E_i)\,dE$$



$$\sigma \propto \frac{1}{v_p}\,G^2\,p\,E$$

$E_i = E_\nu + E_p$ $\quad E_f = E_{e^+} + E_n.$

$$v_p = \beta = \frac{p}{E} \qquad PE = p^2 \frac{E}{p} = \frac{p^2}{\beta_\nu}$$

$$\sigma \propto \frac{1}{\beta_\nu} G^2 p^2 \frac{1}{\beta_{fin.}}$$

$$\sigma \propto G^2 p^2 \qquad \beta \simeq 1$$

$$G^2 = \left[10^{-5}\ \text{GeV}^{-2}\right]^2 = 10^{-1}\ \text{GeV}^{-4}.$$

$$200\ \text{MeV} = 1\ \text{fm}^{-1}$$

$$\sigma = 10^{-43}\ \text{cm}^2 \left(\frac{E}{\text{MeV}}\right)^2$$

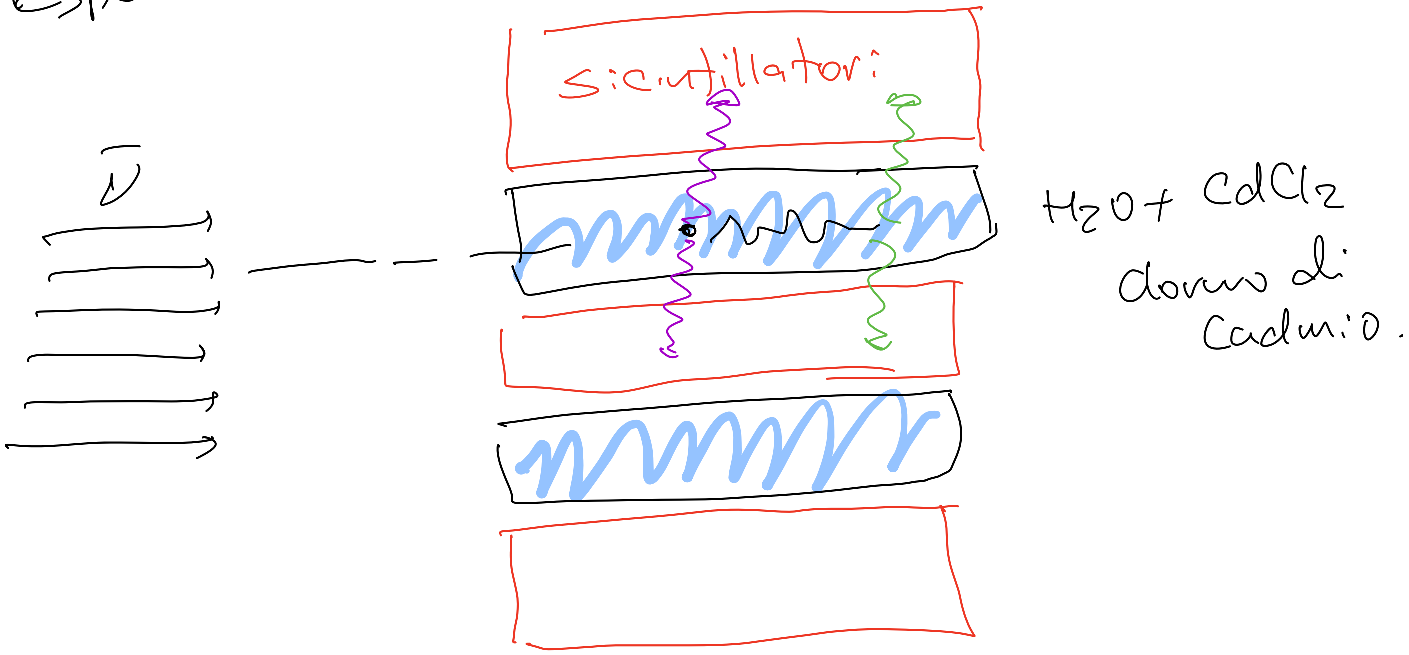
$$\frac{dN_r}{dt} = \underset{10^{-43}\ \text{cm}^2}{\sigma}\ \underset{10^{20}\ \text{Hz}}{\frac{dN_p}{dt}}\ \underbrace{n_b \cdot d.}$$

$$\lambda = \frac{1}{\sigma \cdot n_b} \quad \text{Cammino libero medio.}$$

$$\rho = 1\ g/\text{cm}^3$$

$$\lambda_\nu \simeq 10^{16}\ \text{Km.}$$

Esperimento di Reines-Cowan 1956



$\bar{\nu}_e + p \to n + e^+$

$\hookrightarrow e^+ + e^- \to$ positronio $\to 2\gamma$ $\quad \tau = 10^{-10}$ s.

n: $\hookrightarrow$ urti elastici in H_2O
si termalizza

$n + Cd \to Cd^* \to Cd + \gamma + \gamma$
decad. gamma.
$E_\gamma \simeq 6$ MeV

$T \simeq 10^{-5}$ sec.