$\alpha + N \to \alpha + N.$

$|i\rangle = |\alpha N\rangle \qquad |f\rangle = |\alpha N\rangle$

assumiamo N fermo; urto elastico contro il muro.

$\Rightarrow$ solo $\vec{p}_\alpha$ può variare

$$\Gamma(i \to f) = 2\pi |M_{fi}|^2 \rho(E).$$

$$M_{fi} = -i \int_V d^3V \, \psi_f^* H_I \psi_I$$

Approssimazione di Born.

$\psi_i = \dfrac{e^{i\vec{p}\cdot\vec{r}}}{\sqrt{V}}$ onde libere

$\int_V |\psi|^2 d^3r = 1$

$\Rightarrow \psi \propto \dfrac{1}{\sqrt{V}}$

V: volume di rif per normalizz. ψ.

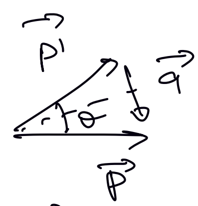
$$\psi_i = \frac{e^{i\vec{p}\cdot\vec{r}}}{\sqrt{2\pi}} \qquad \psi_f = \frac{e^{i\vec{p}'\cdot\vec{r}}}{\sqrt{2\pi}}$$

$$H_I = \frac{e^2}{4\pi\varepsilon_0} \frac{Z_\alpha \cdot Z_N}{r} = \frac{\alpha Z_\alpha Z_N}{r} = \frac{A}{r}$$

$$M_{fi} = -i \int d^3r \, \frac{e^{-i\vec{p}'\cdot\vec{r}}}{\sqrt{V}} \frac{A}{r} \frac{e^{+i\vec{p}\cdot\vec{r}}}{\sqrt{V}} = -i\frac{A}{V} \int d^3r \, \frac{e^{-i(\vec{p}'-\vec{p})\cdot\vec{r}}}{r}$$

$q = 2p \sin\frac{\theta}{2}$

$|\vec{p}| = |\vec{p}'|$



$-\vec{p}' + \vec{p} = \vec{q} \Rightarrow \vec{p} = \vec{p}' + \vec{q}$

$$M_{fi} = -i\frac{A}{V} \int d^3r \, \frac{e^{i\vec{q}\cdot\vec{r}}}{r} \qquad d^3r = \sin\theta \, d\theta \, d\varphi \, r^2 dr$$

$$\vec{q}\cdot\vec{r} = qr\cos\theta \qquad \int_0^\pi \sin\theta \, d\theta = -\int_1^{-1} d\cos\theta = \int_{-1}^{1} d\cos\theta.$$

$$\int_{-1}^{1} d\cos\theta \, e^{iqr\cos\theta} = \frac{1}{iqr}\left[e^{iqr} - e^{-iqr}\right].$$

$$M_{fi} = -i\frac{A}{V}\int_0^{2\pi} d\varphi \int_0^\infty r^2 dr \; \frac{1}{r}\; \frac{1}{iqr}\left[e^{iqr} - e^{-iqr}\right]$$

$$= -i\,\frac{A}{V}\,(2\pi)\,\frac{1}{iq}\int_0^\infty \left[e^{iqr} - e^{-iqr}\right] dr$$

anzichè usare $V(r) = \frac{A}{r}$ usiamo $V(r) = A\,\frac{e^{-\lambda r}}{r}$

e poi alla fine $\lim_{\lambda\to 0}$

$$M_{fi} = -\frac{iA}{V}\,\frac{2\pi}{iq}\,\lim_{\lambda\to 0}\int_0^\infty e^{-\lambda r}\left(e^{iqr} - e^{-iqr}\right) dr$$

$\alpha = \lambda - iq$

$$\int_0^\infty e^{-\lambda r}\, e^{iqr}\, dr = \frac{1}{\lambda - iq}\int_0^\infty \alpha\, e^{-\alpha r}\, dr$$

$$= \frac{1}{\lambda - iq}\int_0^\infty e^{-z} dz = \frac{1}{\lambda - iq}\left[0 - 1\right]$$

analogamente

$$\int_0^\infty e^{-\lambda r}\, e^{-iqr}\, dr = \frac{1}{\lambda + iq}\left[0-1\right].$$

$$\int_0^\infty e^{-\lambda r}\left(e^{iqr} - e^{-iqr}\right) dr = \frac{1}{\lambda - iq} - \frac{1}{\lambda + iq} = \frac{\lambda + iq - \lambda + iq}{\lambda^2 - (-q^2)}$$

$$= \frac{2iq}{\lambda^2 + q^2}$$

$$M_{fi} = \lim_{\lambda\to 0}\left[-i\,\frac{2\pi}{V}\,\frac{A}{i\cancel{q}}\;\frac{2i\cancel{q}}{\lambda^2+q^2}\right] = -2iA\,\frac{1}{q^2}\,\frac{2\pi}{V}$$

$$\Rightarrow |M_{fi}|^2 = \frac{4A^2}{q^4}\,\frac{(2\pi)^2}{V^2}$$

$$\rho(E) = \int dn\, \delta(E_f - E_i)$$

$$dn = \frac{V}{(2\pi)^3} 4\pi \, p^2 dp$$ p: impulso delle particelle α

particelle α non relativistiche.

$$E = \frac{p^2}{2m} \Rightarrow \quad p^2 = 2mE \Rightarrow \cancel{2} p\,dp = \cancel{2} m\,dE$$

$$p\,dp = m\,dE \qquad\qquad p^2 dp = \quad p\,m\,dE$$
$$p = \sqrt{2mE} \quad \Rightarrow$$

$$\rho(E) = \int dn\, \delta(E_f - E_i) = \frac{V}{(2\pi)^3} 4\pi \int p^2 dp\, \delta(E_f - E_i)$$

$$= \frac{V}{(2\pi)^3} 4\pi \int p\,m\, \delta(E_f - E_i)\, dE = \frac{V}{(2\pi)^3} (4\pi)\, m \underbrace{\sqrt{2mE_i}}_{\substack{\shortparallel \\ p_i}}$$

p del proiettile.

Ricordiamo di nuovo che

$$\Gamma(i \to f) = \frac{dN_r}{dt} \frac{1}{N_B} \frac{1}{N_p} = \sigma \cdot \frac{v_p}{V}$$

velocità proiettile

$$= 2\pi \, |M_{fi}|^2 \, \rho(E).$$

$$\Rightarrow (2\pi) \frac{4A^2}{q^4} \frac{(2\pi)^2}{\cancel{V^2}} \frac{\cancel{V}(4\pi)}{(2\pi)^3} m \cancel{\sqrt{2mE_i}} = \sigma \frac{v_p}{\cancel{V}} = \sigma \sqrt{\frac{\cancel{2}}{m}} \cancel{\sqrt{E_i}}$$

$$E = \frac{p^2}{2m} = \frac{1}{2} m v_p^2 \Rightarrow v_p = \sqrt{\frac{2E_i}{m}}$$

$$\Rightarrow \sigma = \frac{4A^2}{q^4} \times 2 \times m^2 = \frac{8A^2}{q^4} m^2$$

$$q = 2p \sin\frac{\theta}{2} \Rightarrow q^4 = 16\, p^4 \sin^4\frac{\theta}{2}$$

$$E = \frac{p^2}{2m} \Rightarrow p^4 = 4m^2E^2$$

$$\Rightarrow \quad \Gamma = \frac{\cancel{8}^{\,2} A^2}{16 \times \cancel{4} \times \cancel{m}^2 E^2 \sin^4\frac{\theta}{2}} \qquad \cancel{m^2}$$

$$= \underbrace{2\times} \left(\frac{A}{4E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}} \qquad A = \alpha Z_\alpha Z_N$$

controllare