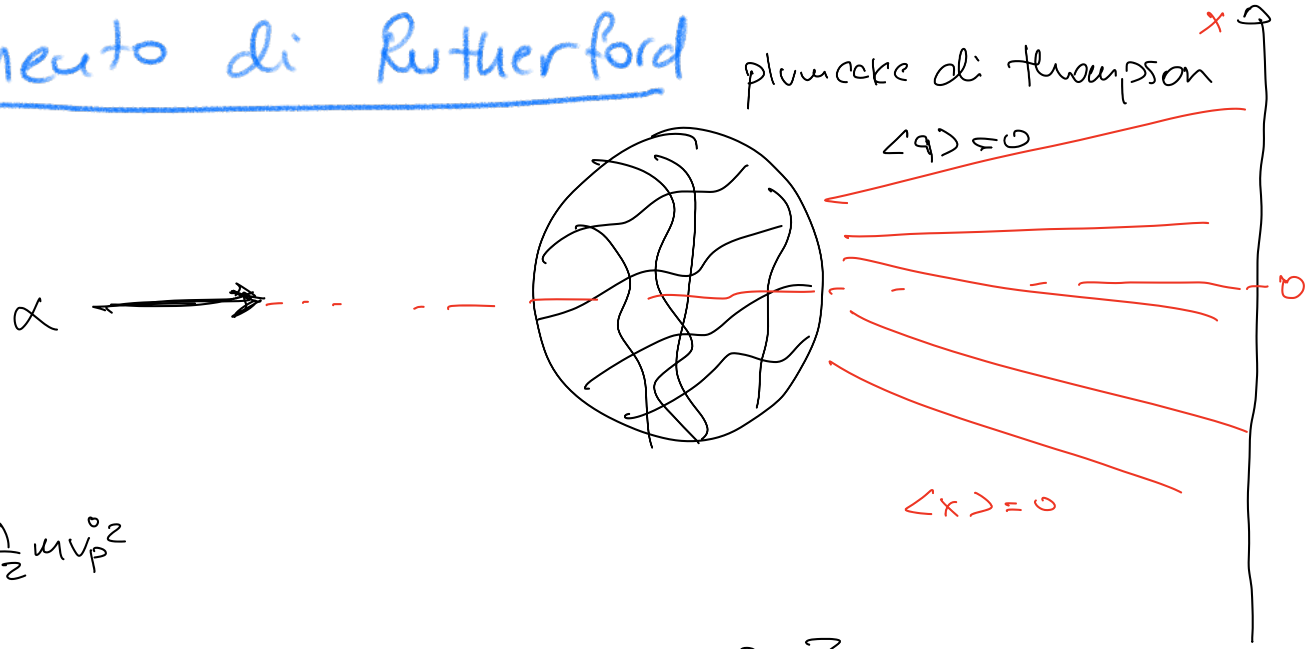
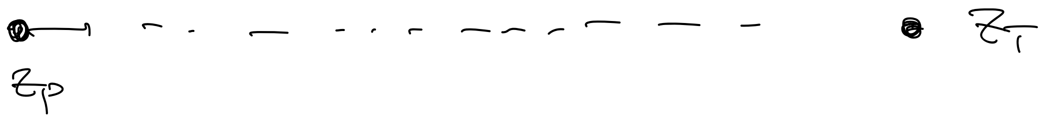
Esperimento di Rutherford



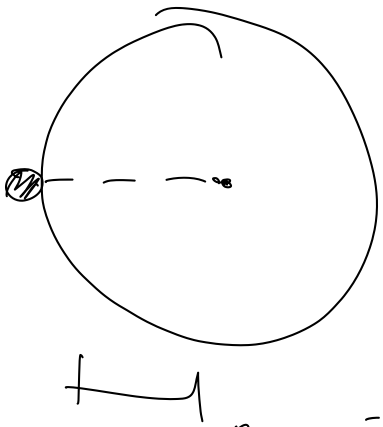
$$E_i = K_p = \frac{1}{2} m v_p^2$$



$$E = K + U = \frac{1}{2} m v^2 - \frac{\alpha Z_p Z_T}{r}$$

$$r_{min} \Rightarrow K + V = 0. \Rightarrow r_{min} = \left(\frac{1}{2} m v^2\right)^{-1} \alpha Z_p Z_T$$

α $\quad Z_p = 2$



Au $\quad Z = 79$

$$r = 1\,\mathring{A} = 10^{-10}\ m$$

$$U = \frac{\alpha Z_p Z_T}{r} = \frac{2 \times 79}{137} \frac{1}{10^{10}\ m} = \frac{160}{137} \frac{1}{10^5\ fm}.$$

$$10^{-10}\ m = 10^5\ 10^{-15}\ m = 10^5\ fm.$$

$$200\ MeV \times fm = 1 = \hbar c \Rightarrow 1\ fm = \frac{1}{200\ MeV}.$$

$$U = \frac{160}{137} 10^{-5} (200\ MeV) \sim \sim 2 \times 10^{-3}\ MeV$$

particelle α da RaBr$_2$: $K = 5-10$ MeV

$K >> U \Rightarrow$ nucleo non può fermare α

Viceversa:

$K = 5$ MeV $\Rightarrow$ Calcolare U inversione

$$5 \text{ MeV} = \frac{\alpha Z_P Z_T}{r_{min}} \Rightarrow r_{min} = \frac{\alpha Z_P Z_T}{5 \text{ MeV}}$$

$$r_{min} = \frac{160}{137} \frac{1}{5 \text{MeV}}$$

200 MeV fm $\simeq 1$.

40 fm $\times$ 5 MeV $\simeq 1$.

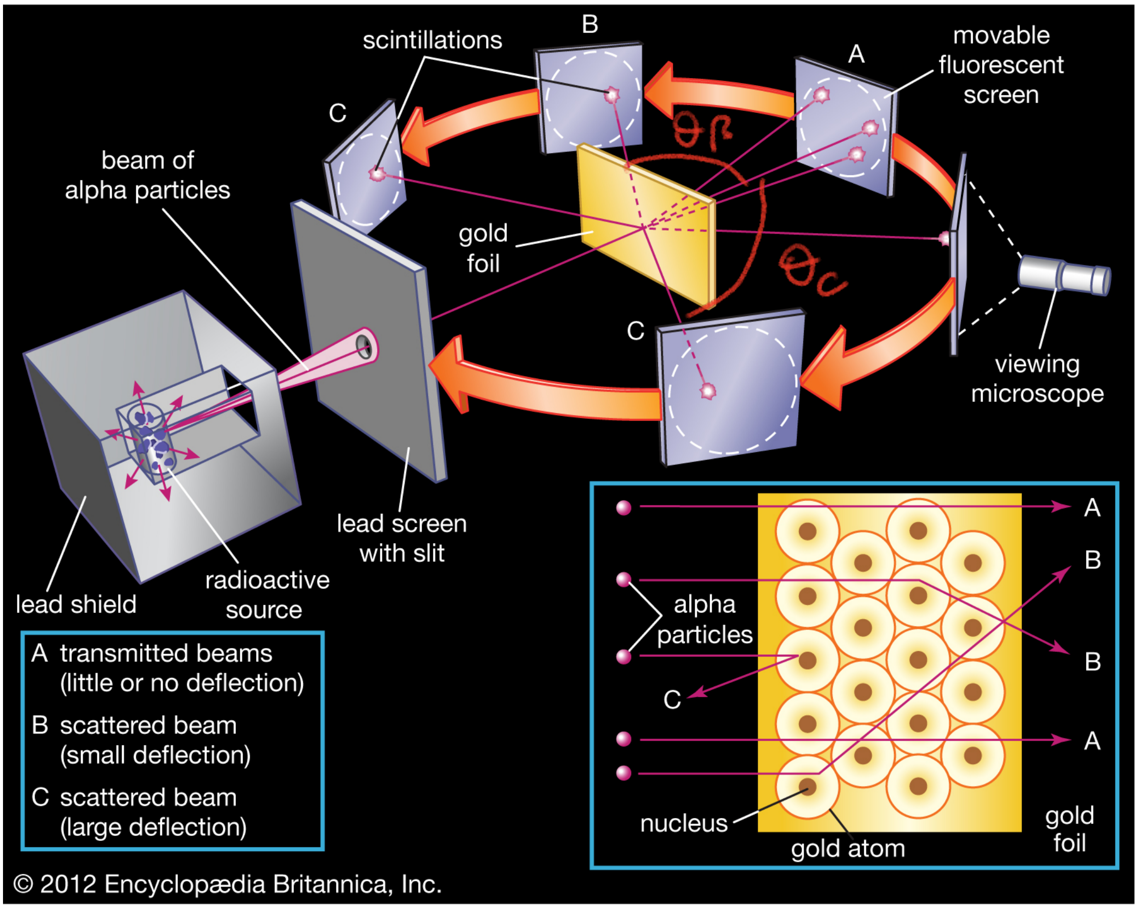
$\Rightarrow 5 \text{ MeV} \simeq (40 \text{ fm})^{-1}$

$\Rightarrow r_{min} = 1.15 \times 40 \text{ fm} \simeq 46 \text{ fm}$

$r_{min} << 1 \text{ Å}$ dimensione atomica

$Z_P = 2$ $\qquad$ $Z_T = 79$

$r_{min} = 46$ fm



contare numero di eventi al variare dell'angolo di deflessione

α_{fin}

α_{in} θ

Ipotesi di urto elastico contro il muro

$M_{Au} >> M_\alpha \Rightarrow$ nell'urto cambia solo direzione di $\vec{P}_\alpha$

$$\alpha + N \longrightarrow \alpha + N$$

α: $M_\alpha = 3.7$ GeV.

$K = 5$ MeV.

$$E = M + K.$$

$$E = \gamma m. \qquad \Rightarrow \gamma m = m + K$$

$$\Rightarrow \quad K = m(\gamma - 1) \Rightarrow \quad \gamma - 1 = \frac{K}{m} \Rightarrow \gamma = 1 + \frac{K}{m}.$$

$$\gamma = 1 + \frac{5\ MeV}{3700\ MeV} = 1 + \frac{5}{37}\times 10^{-2} = 1 + 1.35\times 10^{-3}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \simeq 1 + \frac{1}{2}\beta^2$$

$$\Rightarrow \quad \beta^2 \simeq 2\times 1.35\times 10^{-3} = 0.26\times 10^{-6}.$$

$\Rightarrow \beta \simeq 0.05 \Rightarrow$ non relativistica.

$$K = \frac{p^2}{2m} \Rightarrow \quad p = \sqrt{2mK} = \sqrt{2\times 5\ MeV\times 3.7\times 10^3\ MeV}$$

$$= 190\ MeV$$

sezione d'urto: $\dfrac{dN_r}{dt} = \sigma \dfrac{dN_p}{dt}\, n_b \cdot d$

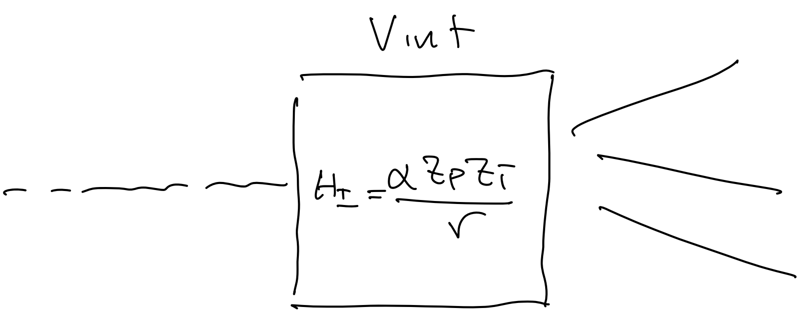
$$\Rightarrow v_p \sigma = \frac{dN_r}{dt}\frac{1}{n_p}\frac{1}{N_B} \Rightarrow \quad \sigma = \frac{1}{v_p}\frac{dN_r}{dt}\frac{V_{int}}{N_p}\frac{1}{N_B}$$

$\Gamma(i\to f)$

prob / unità di tempo

$$\frac{1}{2} m v_p^2 = K \Rightarrow v_p = \sqrt{\frac{2K}{m_\alpha}}$$

$$\Gamma(i\to f) = 2\pi |M_{fi}|^2 \left(\frac{dn}{dE}\right)_{E_f = E_i} \qquad M_{if} = -i\int \psi_f^* H_I \psi_i\, d^3r$$



$|i\rangle = |\alpha + N\rangle$

$|f\rangle = |\alpha' + N\rangle$

impulso modificato in direzione.

Approssimazione di Born: onde piane per particelle libere incidenti e finali:

$$\psi_i(x) = A\, e^{i \vec{P}_{in}\cdot\vec{x}}$$

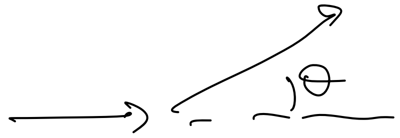
$$\psi_f(x) = A\, e^{i \vec{P}_{fin}\cdot\vec{x}}$$

Normalizzazione: $$\int_{V_{int}} d^3r\, \psi_i^* \psi_i = 1. = A^2 \int_{V_{int}} d^3r\, e^{\cancel{-ip}x}\, e^{\cancel{+i}px}$$

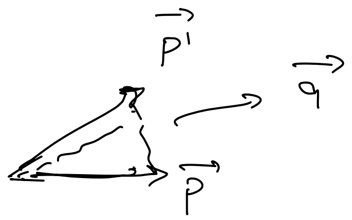
$$\Rightarrow A = (\sqrt{V_{int}})^{-1} \quad \Rightarrow \psi = \frac{1}{\sqrt{V}} e^{i\vec{p}\cdot\vec{x}} \qquad \vec{x} = \vec{r}$$

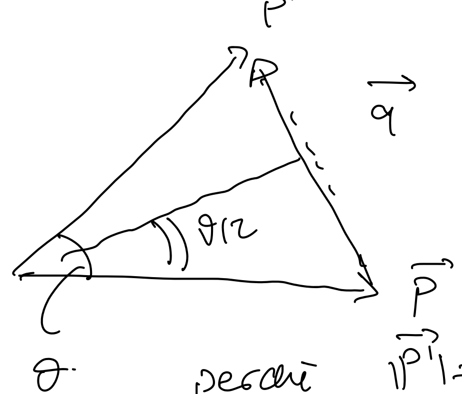
$$\vec{P}_{fin} = \vec{p}' \qquad \vec{q} = \vec{p} - \vec{p}'$$

$\vec{p}' = \vec{P}_{in}$



$$\mathcal{M}_{fi} = -i \int_{V_{int}} d^3r\, \frac{e^{-i\vec{p}'\cdot\vec{x}}}{\sqrt{V}} \frac{\alpha Z_p Z_T}{r} \frac{e^{+i\vec{p}\cdot\vec{x}}}{\sqrt{V}} = \frac{-i\alpha Z_p Z_T}{V_{int}} \int d^3r\, \frac{e^{+i\vec{q}\cdot\vec{x}}}{r}$$





$$|\vec{q}| = 2|\vec{P}_{in}| \sin\frac{\theta}{2}$$

perché $|\vec{p}'| = |\vec{p}|$ urto elastico.

$$\mathcal{M}_{fi} = \frac{-i\alpha Z_p Z_T}{V_{int}} \underbrace{\int_{V_{int}} d^3r\, \frac{e^{i\vec{q}\cdot\vec{r}}}{r}}_{\sim \frac{1}{q^2}}$$

$$d^3r = r^2 dr\, d\varphi\, d\cos\theta.$$

$$e^{i\vec{q}\cdot\vec{r}} = e^{iqr\cos\theta}$$

$$\int d\varphi$$

$$\mathcal{M}_{fi} = -2i\alpha Z_p Z_T \frac{2\pi}{V_{int}} \frac{1}{q^2}$$

$$|\mathcal{M}_{fi}|^2 = \frac{4(\alpha Z_p Z_T)^2}{q^4} \frac{(2\pi)^2}{(V_{int})^2}$$

elemento di matrice

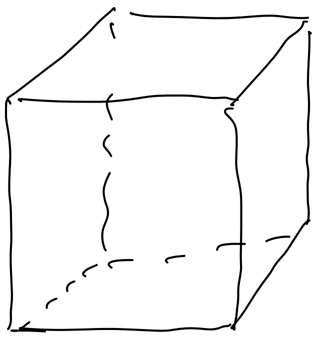
$q = 2p\sin\frac{\theta}{2}$

P: impulso iniziale.

Calcolo dello spazio delle fasi:

$$\rho(E) = \left.\frac{dn}{dE}\right|_{E_f = E_i} = \int dn\, \delta(E_i - E_f)$$

Vint



Buco di potenziale

$$\Rightarrow \quad \psi \sim \sin(P_x L) = 0.$$

$$\sin(P_x \emptyset) = \emptyset.$$

$$\Rightarrow \quad P_x = \frac{\pi}{L}\, n_x$$

$$P_x \cdot L = n_x \pi$$

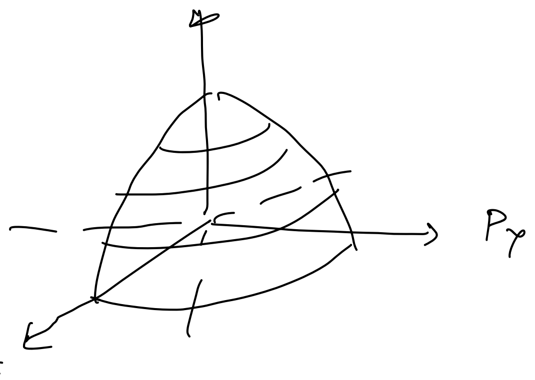
$$dP_x = dn_x \frac{\pi}{L_x}.$$

$$dP_y = dn_y \frac{\pi}{L_y}$$

$$dP_z = dn_z \frac{\pi}{L_z}$$

$$\Rightarrow \quad dn = dn_x dn_y dn_z = \underbrace{\frac{1}{8}}_{P_x, P_y, P_z > 0.} \frac{L_x L_y L_z}{\pi^3}\, dP_x dP_y dP_z$$

$$\Rightarrow \quad dn_z = \frac{L_z}{\pi}\, dP_z$$



$$dn = \frac{V_{int}}{(2\pi)^3}\, d^3 p = \frac{V_{int}}{(2\pi)^3}\, p^2 dp\, d\varphi\, d\cos\theta$$

$$\text{spazio delle fasi} = \int dn\, \delta(E_f - E_i)$$

$$\rho(E) = \frac{V_{int}}{(2\pi)^3} \int p^2 dp\, \delta(E_f - E_i)\, \underbrace{d\cos\theta\, d\varphi}_{d\Omega}$$

$$E = \frac{p^2}{2m} \Rightarrow dE = \frac{\cancel{2} P dP}{\cancel{2} m} \Rightarrow P dP = m dE.$$

$$p^2 dP\, \delta(E_f - E_i) = P m\, dE\, \delta(E_f - E_i)$$

$$P = \sqrt{2mE} \Rightarrow \int p^2 dP\, \delta(E_f - E_i) = \int \sqrt{2mE}\, m\, dE\, \delta(E_f - E_i)$$

$$E_{in} = \frac{1}{2} m V_{in}^2 \qquad = m\sqrt{2m E_{in}}$$

$$\Rightarrow V_{in} = \sqrt{\frac{2E_{in}}{m}}.$$

$$\Gamma = 2\pi |M_{fi}|^2 \rho(E).$$

$$\sigma = \frac{1}{v_p} \Gamma(i \to f)\, V_{int}$$

$$= \int \sqrt{\frac{m}{\cancel{2E_{in}}}}\, \cancel{V_{int}}\, \cancel{(2\pi)} \frac{4(\alpha Z_p Z_T)^2}{q^4} \frac{\cancel{(2\pi)^2}}{\cancel{(V_{int})^2}} \frac{\cancel{V_{int}}}{\cancel{(2\pi)^3}}\, m\sqrt{\cancel{2mE}}\, d\Omega$$

$$\frac{d\sigma}{d\Omega} = \frac{4(\alpha Z_p Z_T)^2}{q^4} m^2 = \frac{4(\alpha Z_p Z_T)^2}{\left(2P \sin\frac{\theta}{2}\right)^4}$$

1) $\frac{d\sigma}{d\Omega} \propto \alpha^2 Z_p^2 Z_T^2$ quadrato della carica.

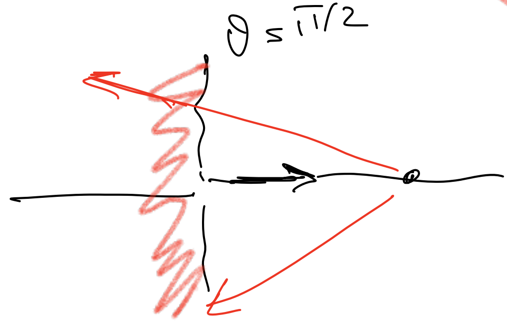
Segno della carica (sia bersaglio che proiettile) sono irrilevanti

2) $\frac{d\sigma}{d\Omega} \propto Z_T^2$ carica del bersaglio.

$\Rightarrow$ variazione quadratica del numero di eventi

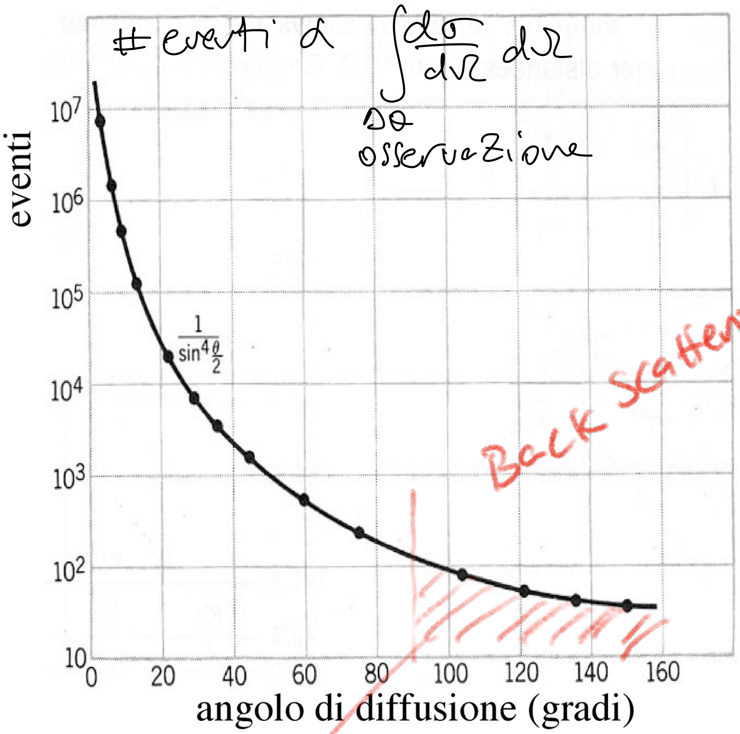
3) $\frac{d\sigma}{d\Omega} \propto \frac{1}{\sin^4 \frac{\theta}{2}}$

$$\int_{\theta > \pi/2} \left(\frac{d\sigma}{d\Omega}\right) d\cos\theta > 0$$



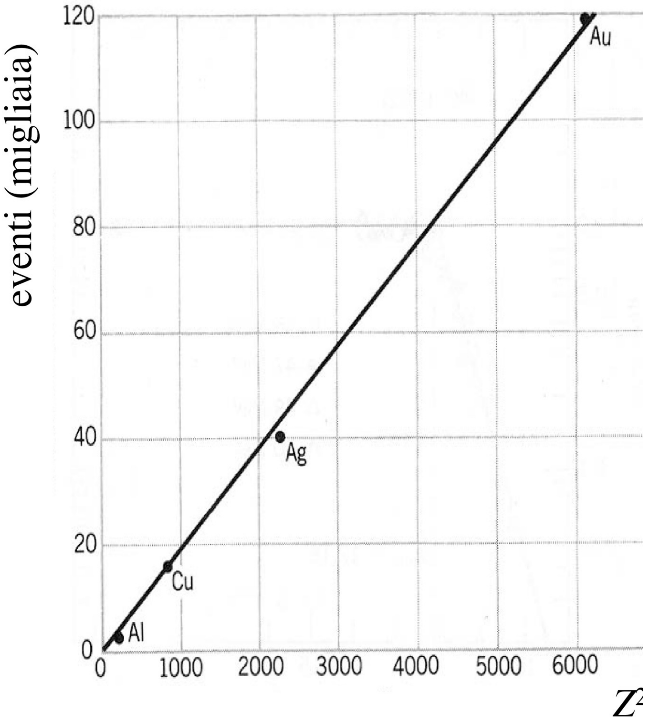
Back-scattering

$\Rightarrow$ mi aspetto eventi con $\theta > \pi/2$.



$\#$ eventi $\propto \int_{\Delta\theta \text{ osservazione}} \frac{d\sigma}{d\Omega} d\Omega$

Back scattering



$\#$ eventi con $\theta > \pi/2$ $= \int_{\theta > \frac{\pi}{2}} \left(\frac{d\sigma}{d\Omega}\right) d\cos\theta$

Conclusione: $r_N < 30$ fm $\ll 1$ Å raggio atomico

$r_N = r_0 A^{1/3}$ $\qquad r_0 = 1.1$ fm

Au: $Z = 79$ $\quad A = 197$ $\quad \Rightarrow \quad r_{Au} = 1.1 \text{ fm} \times \sqrt[3]{197} = 6.4$ fm

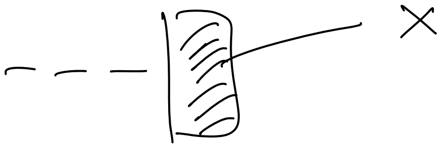
$\Rightarrow$ dimostro plum cake errato.

$\Rightarrow$ carica + dell'atomo concentrata nel nucleo $r \sim$ fm

Nucleo = Z protoni + (A−Z) neutroni

Scoperta del protone (1918)

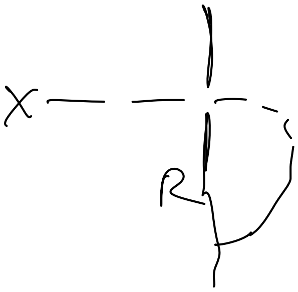
$$\alpha + {}^{14}_{7}N \longrightarrow {}^{17}_{8}O + X$$



$\alpha = {}^{4}_{2}He$

spettrometro di massa

$\Rightarrow$ X carica



$\odot B$

$\Rightarrow$ X ha la stessa massa e carica dello ione H^+

Scoperta del neutrone Chadwick 1931

$$\alpha + {}^{9}_{4}Be \longrightarrow {}^{12}_{6}C + X$$

Li

B

Trasmutazione nucleare