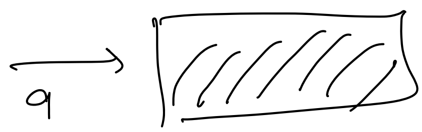
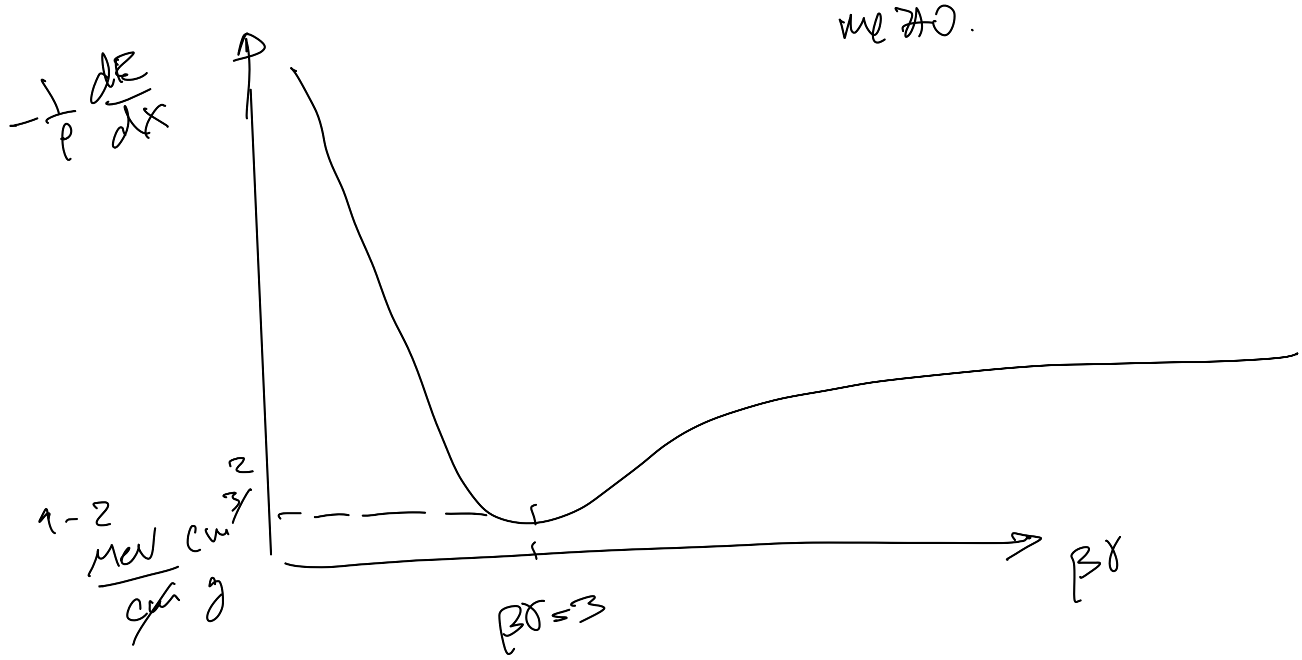
Token 551 799

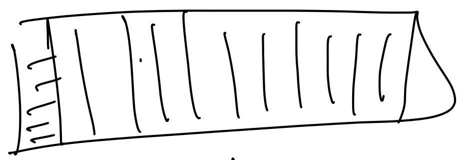


ionizzazione $-\frac{1}{\rho}\frac{dE}{dx} = C \left(\frac{Z}{A}\right) \frac{z_p^2}{\beta^2}\left[\ln \frac{\beta\gamma^2}{2} - \beta^2\right]$

mezzo.



$\beta\gamma_{in} > 3$



d = 10 cm.

$\Delta E = \rho \underbrace{\Delta x}_{10\ cm} \cdot \left(\frac{dE}{dx}\right)\Big|_{\beta\gamma}$ X NO.

Se $\beta\gamma \gtrsim 3$ per spessore sottile $\Rightarrow$ $\Delta E = \frac{dE}{dx}\Big|_{\beta\gamma} \cdot \rho \cdot \Delta x$

spesso come stime al 10–20%

$\frac{dE}{dx}\Big|_{\beta\gamma=3} = 1.5 \frac{MeV}{cm} \cdot \rho$

quando $\beta\gamma \lesssim 3 \Rightarrow$ integrare

$$E = m + K$$

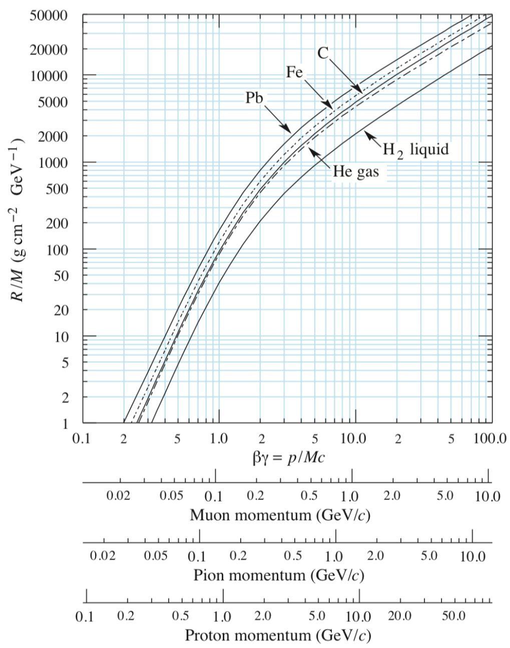
Range: percorso residuo nel mezzo.

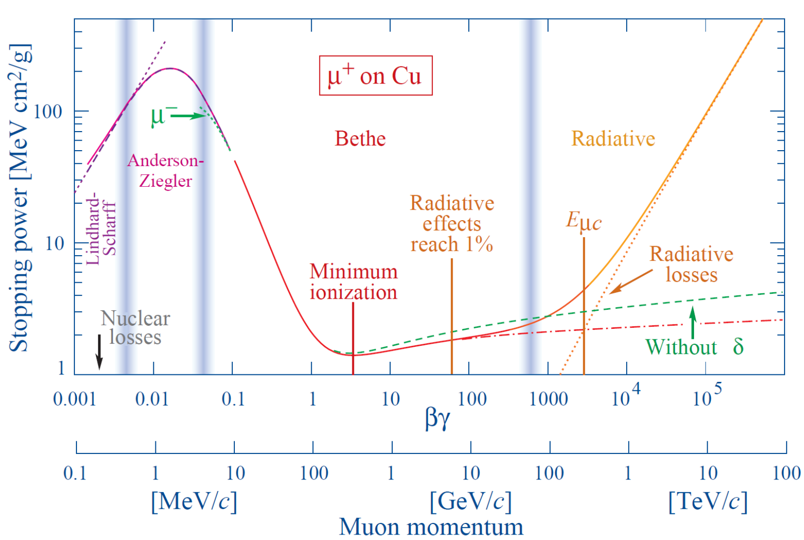
$$R(E) = \int_E^0 dx \qquad -\frac{dE}{dx}$$

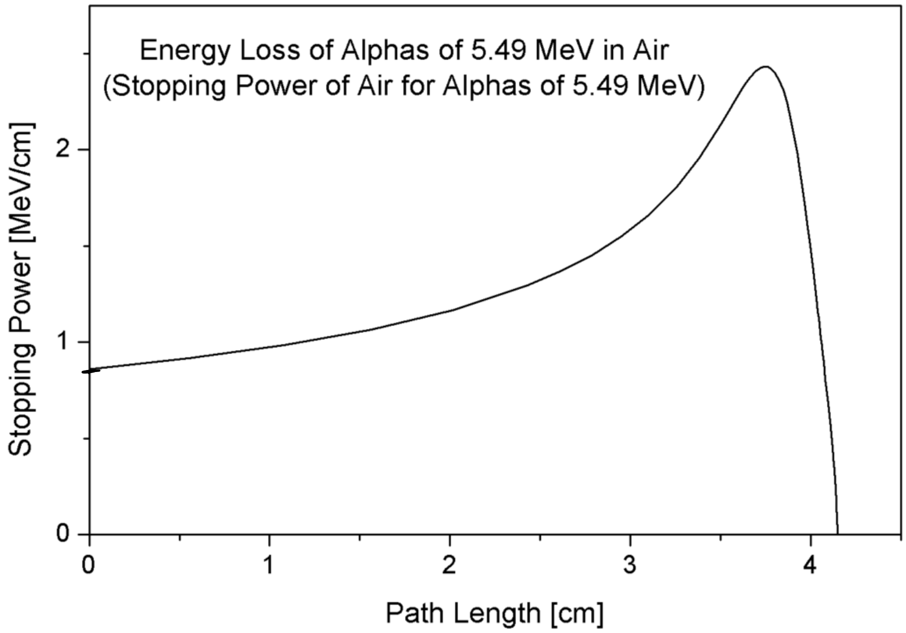
$$= \int_E^0 -\frac{1}{-\frac{dE}{dx}}\, dE = \int_m^E \frac{1}{\left(-\frac{dE}{dx}\right)}\, dE$$

↳ Bethe-Bloch

$$\frac{R(E)}{M}$$





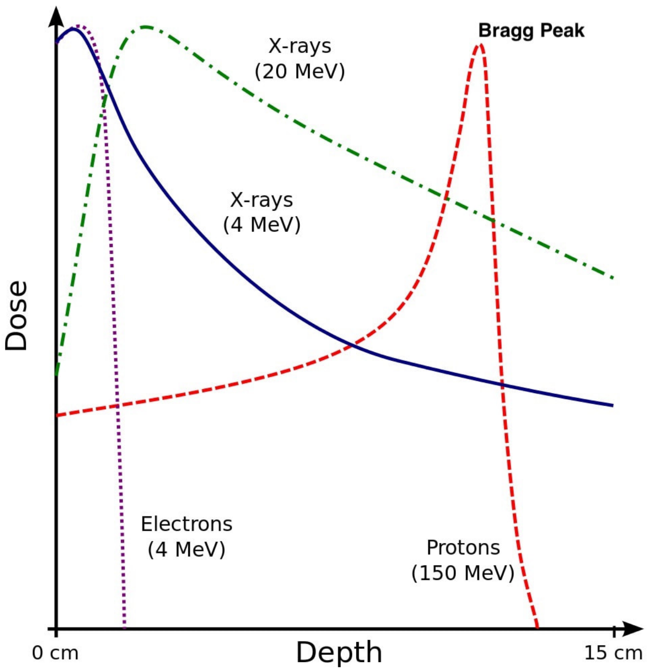


Picco di Bragg

α $K_\alpha = 5.49$ MeV.

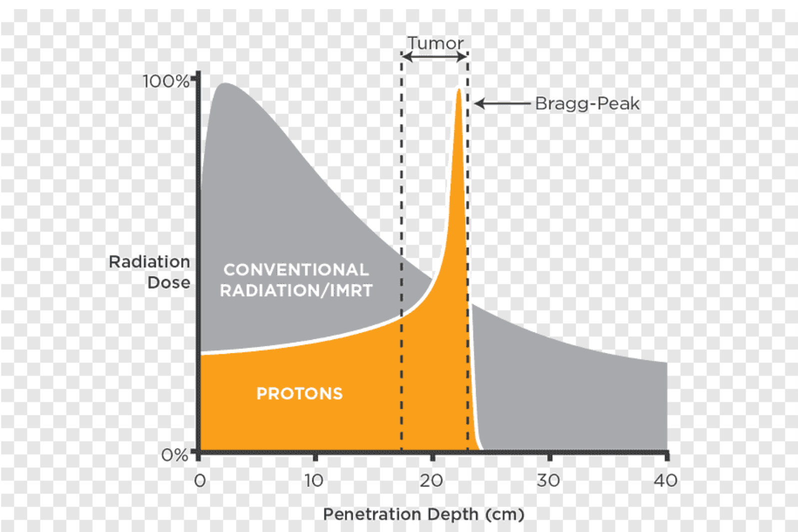
nell'aria.

Dose
energia
rilasciata
nel tessuto

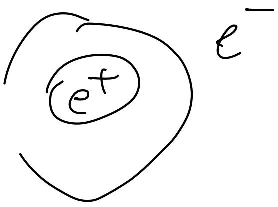


adroterapia

RC

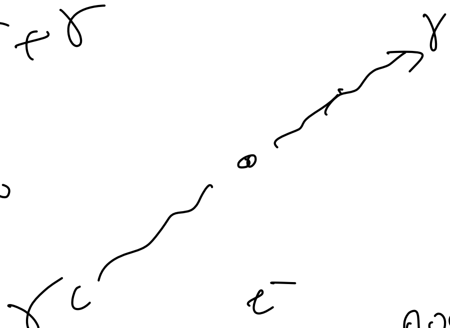


e^+ - - e^+ si ferma



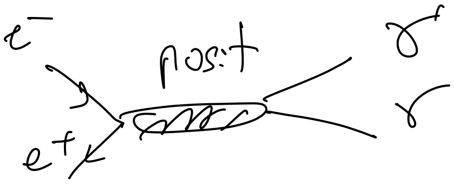
positronio stato instabile

$(e^+e^-) \longrightarrow \gamma+\gamma$

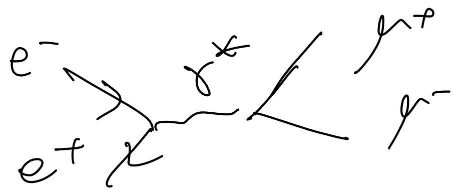


positronio praticamente fermo

$e^+ + e^- \longrightarrow$ positronio $\longrightarrow$ decade



$e^+ + e^- \longrightarrow \mu^+ + \mu^-$



Perdite di energia per e^+e^-

Radiazione emessa nel mezzo.

$$\cancel{m}a = \frac{1}{4\pi\varepsilon_0}\frac{Z_p Z_B e^2}{r^2} = \frac{\alpha Z_p Z_B}{r^2}\frac{1}{m}$$

Effetto Larmor: $P = \frac{2}{3}\frac{e^2}{m^2c^3}|\dot{v}|^2 \qquad \dot{v} = a$

potenza di irraggiamento

Larmor Relativistico:
$$P = -\frac{2}{3}\frac{e^2}{m^2c^3}\frac{dP_\mu}{d\tau}\frac{dP^\mu}{d\tau}$$

$$P_\mu = (E, \vec{p})$$

$$E = \gamma m c \qquad \vec{p} = \gamma m \vec{v} = \gamma m \vec{\beta} c$$

$$P = \frac{2}{3}\frac{e^2}{c}\gamma^6\left[(\dot{\vec{\beta}})^2 - (\vec{\beta}\times\dot{\vec{\beta}})^2\right]$$

$$c\dot{\vec{\beta}} = \vec{a}$$

Accelerazione lineare.

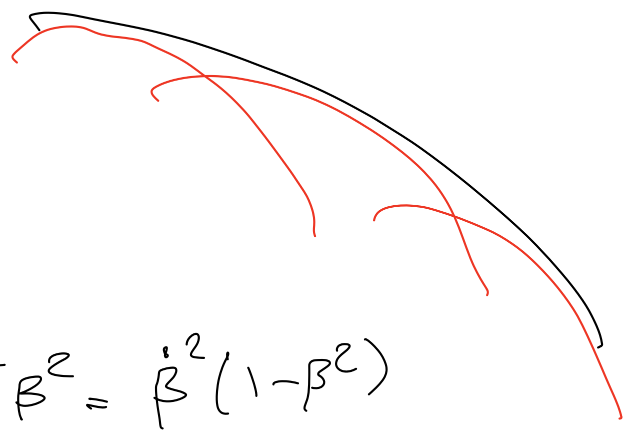
$$\dot{\vec{\beta}} \parallel \vec{\beta} \Rightarrow \dot{\vec{\beta}} \times \vec{\beta} = 0$$

$$c\dot{\beta} = a \qquad \dot{\beta}^2 = \frac{a^2}{c^2}$$

$$P = \frac{2}{3}\frac{e^2}{c}\gamma^6\frac{a^2}{c^2}$$

$$= \frac{2}{3}\frac{e^2}{c^3}\gamma^6 a^2$$

accelerazione curvilinea.



$$\vec{\beta} \perp \dot{\vec{\beta}} \qquad \vec{a} \perp \vec{v}$$

moto circolare

$$\dot{\vec{\beta}}^2 - (\vec{\beta}\times\dot{\vec{\beta}})^2 = \dot{\beta}^2 - \dot{\beta}^2\beta^2 = \dot{\beta}^2(1-\beta^2)$$

$$= \dot{\beta}^2\frac{1}{\gamma^2}$$

$$P = \frac{2}{3}\frac{e^2}{c^3}\gamma^6\frac{a^2}{\gamma^2} = \frac{2}{3}\frac{e^2}{c^3}a^2\gamma^4$$

$$\gamma = \frac{E}{m} \Rightarrow P \propto \left(\frac{E}{m}\right)^4$$

E uguale per e^-, p

Zpe

$Z_B e$

$$\frac{P_e}{P_p} = \frac{1}{(m_e)^4} (m_p)^4$$

$$\simeq (2\times 10^3)^4 \simeq 10^{13}$$

LHC: $E, p = 6.5$ TeV

$\Rightarrow$ Radiazione nel mezzo dominante per e^-, e^+

$$\left.\frac{dE}{dx}\right|_{e^\pm} = \left.\frac{dE}{dx}\right|_{ion} + \underbrace{\left.\frac{dE}{dx}\right|_{rad.}}_{\text{Bremsstrahlung}}$$

$$\left.\frac{dE}{dx}\right|_{rad.} \simeq \frac{E}{X_0}$$

X_0 lunghezza di radiazione
prop. del mezzo.

$$\frac{1}{X_0} \propto \rho \frac{N_A}{A} Z_B^2 \ln\left(183 Z_B^{-1/3}\right)$$

$$\rho X_0 \sim 170 \frac{A_B}{Z_B^2} \frac{g}{cm^2}$$

Formula empirica
prop. del mezzo.

$$E(x) = E_0 e^{-x/X_0}$$

$$x = X_0 \quad E(X_0) = E_0 e^{-1} \simeq 30\% \, E_0$$

Confrontare rad. con ioniz.

en. cin. di $e^\pm$

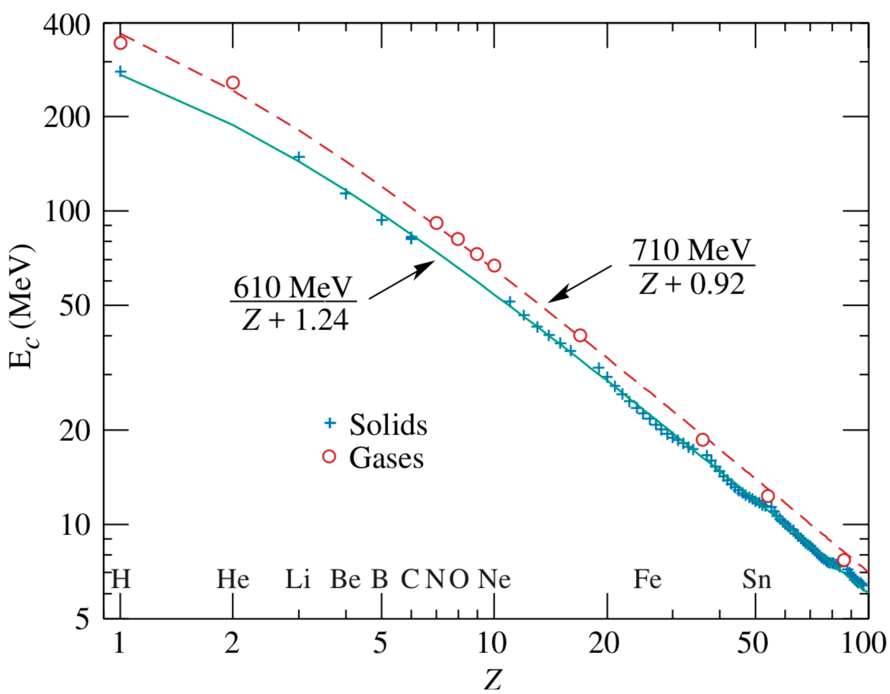
$$\frac{\left.\frac{dE}{dx}\right|_{Brem}}{\left.\frac{dE}{dx}\right|_{ion}} \simeq \frac{K_e Z_B}{1200\, m_e} \qquad E_e = K_e + m_e$$

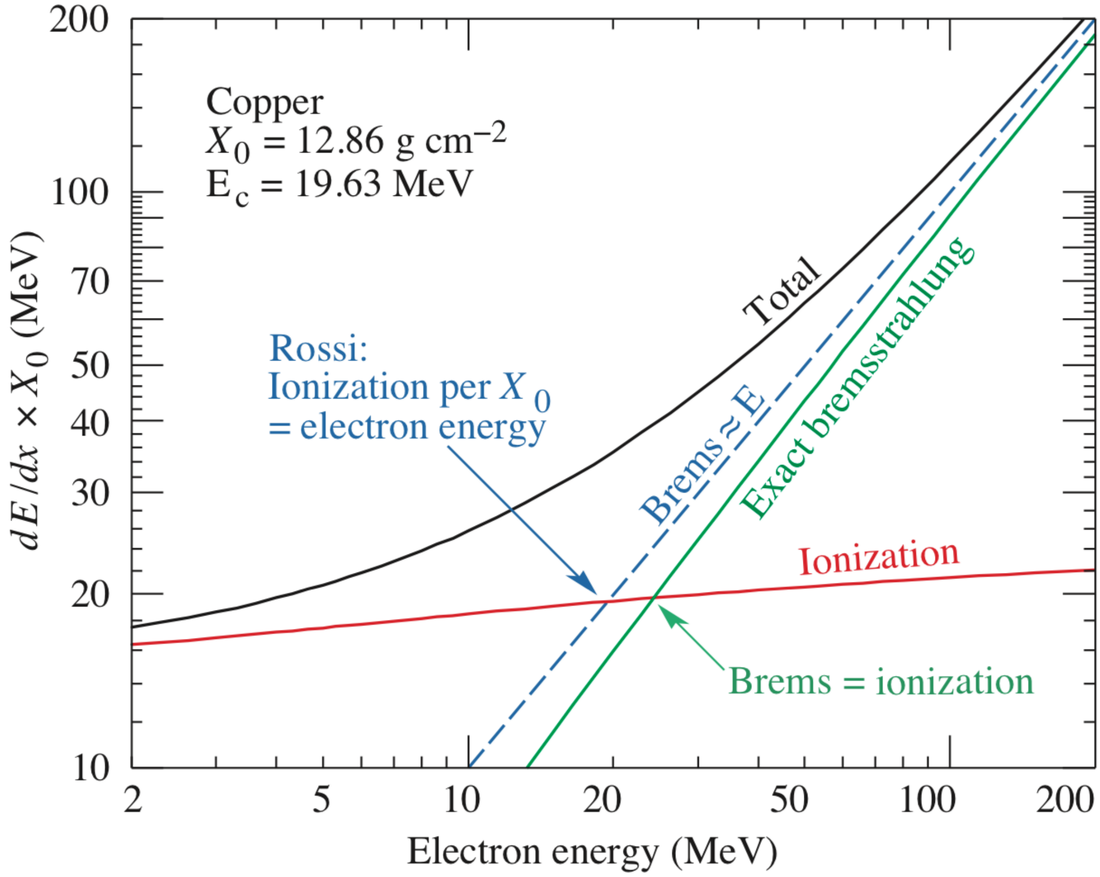
$$= 1 \Rightarrow E_c = \frac{1200\, m_e}{Z_B} \simeq \frac{600\ \text{MeV}}{Z_B}$$

Energia critica per $e^\pm$
prop. del mezzo.

per $E_e > E_c \Rightarrow$ Bremss. prevalgono.

$E_e < E_c \Rightarrow$ ion.





$$\beta\gamma = \frac{p}{m}.$$

$$E_e \simeq p_e$$

$$\frac{E_c}{\rho X_0} = \frac{600\,MeV}{\cancel{Z_B}} \; \frac{1}{170} \underbrace{\frac{\cancel{Z_B}}{A}}_{\simeq \frac{1}{2}} \simeq \frac{600}{340} = 1.7\,\frac{MeV}{cm}\left[\frac{1}{\rho}\right]$$