Videolezione-2020-03-13

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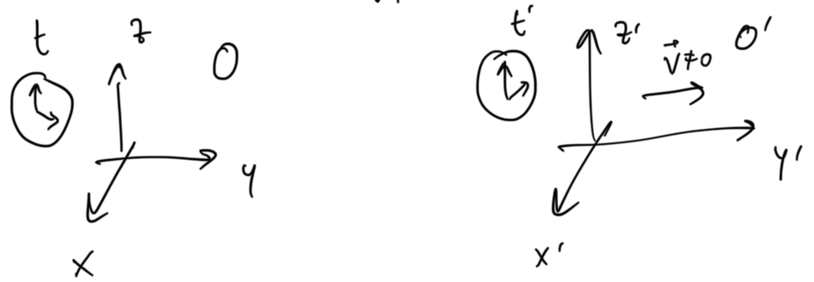
$\boxed{\text{EX 0}}$

$v = ?$ $\qquad \Delta t = 2\Delta t'$

Δt: $v \neq 0$ $\qquad$ $\Delta t'$: $v = 0$



$O \rightarrow O'$

$$v \Rightarrow \beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\rightarrow \boxed{\Delta t = \gamma\, \Delta t'} \quad \text{dilat. dei tempi}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \overset{!}{=} 2$$

$$\frac{1}{1-\frac{v^2}{c^2}} = 4 \quad \Leftrightarrow \quad \frac{1}{4} = 1 - \frac{v^2}{c^2}$$

$$\Rightarrow \frac{v^2}{c^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \frac{v}{c} = \frac{\sqrt{3}}{2} = \underline{0.866}$$

$\boxed{\text{EX 1}}$ Un fascio di 10^{12} π^+ al secondo

$$p = 2 \ \mathrm{GeV}/c$$

Qual è l'intensità del fascio (in Ampère) dopo $\Delta x = 120$ m nel vuoto

$$m_\pi = 139.6 \ \mathrm{MeV}/c^2 \qquad \tau_0(\pi) = 2.6 \cdot 10^{-8} \ \mathrm{s}$$

$$\boxed{\beta = \frac{p}{E} \qquad \gamma = \frac{E}{m} \qquad E = \sqrt{m^2 + p^2}}$$

$E = m + K$

E cinetica

$E \geq m$

$p = 0$ in quiete

$0 \leq \beta < 1$

β: 0 ($v = 0$) → 1 ($v = c$)

$1 \leq \gamma < +\infty$

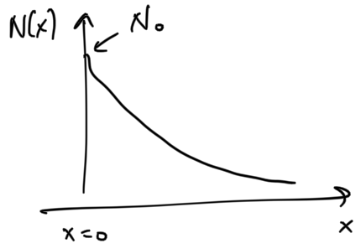
γ: 1 (fermo) ---- $+\infty$ (ultrarelativ.)

$$p_\pi = 2 \ \mathrm{GeV}$$

$$E_\pi = \sqrt{m_\pi^2 + p_\pi^2} = \sqrt{0.1396^2 + 2^2} = 2.005 \ \mathrm{GeV}$$

$$\beta_\pi = \frac{p_\pi}{E_\pi} = \frac{2}{2.005} = 0.9976$$

$$\gamma_\pi = \frac{E_\pi}{m_\pi} = \frac{2.005}{0.1396} = 14.4$$



$$N(x) = N_0 \, e^{-x/\lambda}$$

$$\lambda = v \cdot \tau$$

$v = \beta c \qquad \tau = \gamma \tau_0$

$$N(x) = N_0 \, e^{-x/\beta\gamma c\tau_0}$$

$$\dot{N}(x) = \dot{N}_0 \, e^{-x/\beta\gamma c\tau_0}$$

$$I = q\dot{N} \qquad q(\pi) = e = 1.6 \cdot 10^{-19}\,C$$

$$I(x) = q\,\dot{N}_0\, e^{-x/\beta\gamma c\tau_0}$$

$q = 1.6 \cdot 10^{-19}\,C$ $\qquad \gamma = 14.4$

$\dot{N}_0 = 10^{12}\,s^{-1}$ $\qquad c = 3 \cdot 10^{8}\,m/s$

$x = 120\,m$ $\qquad \tau_0 = 2.6 \cdot 10^{-8}\,s$

$\beta = 0.9976$

$$I(x = 120m) = (1.6 \cdot 10^{-19}) \cdot 10^{12}\, e^{-120/0.9976 \cdot 14.4 \cdot 3 \cdot 1\cancel{0^{8}} \cdot 2.6 \cdot 1\cancel{0^{-8}}}$$

$$= 1.6 \cdot 10^{-7}\, e^{-1.07} = 0.55 \cdot 10^{-7}\,A$$

$$= 55 \cdot \underbrace{10^{-9}}_{nA}\,A = \underline{55\,nA}$$

$\boxed{EX2}$

$\bar{p}$ con $p = 2.2$ GeV

Contro un protone fermo nel LAB

$$p + \bar{p} \rightarrow \Lambda + \bar{\Lambda}$$

$m_p = m_{\bar{p}} = 938$ MeV

$m_\Lambda = 1116$ MeV

<u>LAB</u> $\xleftrightarrow[\beta_{cm},\,\gamma_{cm}]{}$ <u>CdM</u>

LAB: $\bar{p} \longrightarrow \quad p\ \bullet$

CdM: $\bar{p} \longrightarrow \ \longleftarrow p$

nel CdM

$$\sum \vec{p}^{\,*} = \vec{0}$$

$p(\bar{p}) = 2.2$ GeV $\qquad \vec{p}^{\,*}(\bar{p}) = -\vec{p}^{\,*}(p)$

$$\bar{p}: \ (E_{\bar{p}}, \vec{p}_{\bar{p}}) = \left(\sqrt{m_p^2 + |\vec{p}_{\bar{p}}|^2},\ \vec{p}_{\bar{p}}\right) = \left(\sqrt{m_p^2 + p^2},\ \vec{p}\right)$$

$$p: \ (E_p, \vec{p}_p) = (m_p, \vec{0})$$

$$E = \sqrt{m^2 + p^2} \overset{p=0}{=} m$$

4-vettore del sistema

$$(E_{TOT}, \vec{P}_{TOT}) = \left(\sum_i E_i,\ \sum_i \vec{p}_i\right)$$

$$\beta_{cm} = \frac{|(\sum \vec{p})_{LAB}|}{\sum E_{LAB}} = \frac{p}{E_{\bar{p}} + m_p}$$

$$\hookrightarrow E_{\bar p} = \sqrt{m_p^2 + p^2} = \sqrt{0.938^2 + 2.2^2} =$$
$$= 2.39 \text{ GeV}$$

$$\Rightarrow \beta_{cm} = \frac{2.2}{2.39 + 0.938} = 0.66$$

$$\gamma_{cm} = \frac{1}{\sqrt{1-\beta_{cm}^2}} = 1.33$$

<u>TRASF. DI LORENTZ</u>

$$p^*_{\bar p} = \gamma_{cm}\big(\overset{2.2}{p_{\bar p}} - \beta_{cm}\overset{2.39}{E_{\bar p}}\big) = 0.83 \text{ GeV}$$
$$p^*_{p} = \gamma_{cm}\big(\underbrace{p_p}_{=0} - \beta_{cm}\underbrace{E_p}_{=m_p}\big) = -0.83 \text{ GeV}$$

$\Big\}$ uguali e opposti

$$E^*_{\bar p} = \sqrt{m_p^2 + (p^*_{\bar p})^2} = \sqrt{0.938^2 + 0.83^2} = 1.25 \text{ GeV}$$
$$= E^*_p$$

<u>LAB</u> $\quad \overset{\beta_{cm},\,\gamma_{cm}}{\Longleftrightarrow} \quad$ <u>CdM</u>

LAB: $\bar p \longrightarrow \; p\,\bullet$ $\qquad$ CdM: $\bar p \longrightarrow \;\longleftarrow p$

$\bar p : (E, \vec p)$ $\qquad$ $\bar p : (E^*, \vec p^{\,*})$

$p : (m_p, \vec 0)$ $\qquad$ $p : (E^*, -\vec p^{\,*})$

$$\sqrt{s} \overset{def}{=} \sqrt{\Big(\sum_i E_i\Big)^2 - \Big|\sum_i \vec p_i\Big|^2} \qquad (E_{TOT}, \vec p_{TOT})$$

nel CdM: $\quad \sqrt{s}_{CdM} \equiv \sqrt{s} = \sqrt{\Big(\sum_i E_i^*\Big)^2 - \underbrace{\Big|\sum_i \vec p_i^{\,*}\Big|^2}_{=0\,!}} = \sum_i E_i^*$

nel LAB: $\quad \sqrt{s}_{LAB} \equiv \sqrt{s} = \sqrt{\Big(\sum_i E_i\Big)^2 - \Big|\sum_i \vec p_i\Big|^2} =$

$$= \sqrt{(E_{\bar p} + m_p)^2 - p_{\bar p}^2} =$$

$$= \sqrt{E_{\bar p}^2 + m_p^2 + 2E_{\bar p} m_p - p_{\bar p}^2}$$

$$E_{\bar p}^2 = m_{\bar p}^2 + p_{\bar p}^2 \qquad E_{\bar p}^2 - p_{\bar p}^2 = m_p^2$$

$$= \sqrt{m_p^2 + m_{\bar{p}}^2 + 2E_{\bar{p}} m_p}$$

$$= \sqrt{2m_p^2 + 2E_{\bar{p}} m_p} = 2.50 \text{ GeV}$$

$\rightarrow$ nel CdM $\quad \sqrt{s}_{CdM} = \sum_i E_i^* = E_p^* + E_{\bar{p}}^* = 2E^*$

$(\sqrt{s}_{CdM} = \sqrt{s}_{LAB})$

$$\left.\begin{array}{l} p_p^* = -p_{\bar{p}}^* \\ m_p = m_{\bar{p}} \end{array}\right\} \Rightarrow E_p^* = E_{\bar{p}}^* \equiv E$$

$$2E^* = 2.50 \text{ GeV}$$

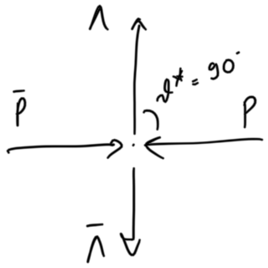
$$\Rightarrow E^* = 1.25 \text{ GeV}$$

$$\Leftrightarrow p^* = \sqrt{E^{*2} - m_p^2} = 0.83 \text{ GeV}$$

$$\bar{p} + p \longrightarrow \Lambda + \bar{\Lambda}$$

la produzione di Λ è simmetrica nel CdM con $\vartheta^* = 90°$

$\boxed{\text{CdM}}$



① p^*, E^* di Λ e $\bar{\Lambda}$ nel CdM = ?

	s.i.	s.f.
LAB	$\bar{p}: (E_{\bar{p}}, \vec{p})$	$\Lambda: (E_\Lambda, \vec{p}_\Lambda)$
	$p: (m_p, \vec{0})$	$\bar{\Lambda}: (E_{\bar{\Lambda}}, \vec{p}_{\bar{\Lambda}})$
CdM	$\bar{p}: (E_p^*, \vec{p}_p^*)$	$\Lambda: (E_\Lambda^*, \vec{p}_\Lambda^*)$
	$p: (E_p^*, -\vec{p}_p^*)$	$\bar{\Lambda}: (E_{\bar{\Lambda}}^*, \vec{p}_{\bar{\Lambda}}^*)$

$\sqrt{s}$ non dip. da SdR

$\Leftrightarrow \sqrt{s}_{LAB} = \sqrt{s}_{CdM}$

$\sqrt{s}$ si conserva da s.i. a

$\sqrt{s}_i = \sqrt{s}_f$

$$P_{TOT} = (\underbrace{E_{TOT}}, \underbrace{\vec{P}_{TOT}})$$

$E_{TOT,i} = E_{TOT,f}$

$\vec{P}_{TOT,i} = \vec{P}_{TOT,f}$

$\sqrt{s} = 2.50$ GeV

$$\sqrt{s}\,(\text{CdM}, sf) = \sum_i E_i^* = E_\Lambda^* + E_{\bar{\Lambda}}^*$$

$$\vec{p}_\Lambda^* = -\vec{p}_{\bar{\Lambda}}^*$$

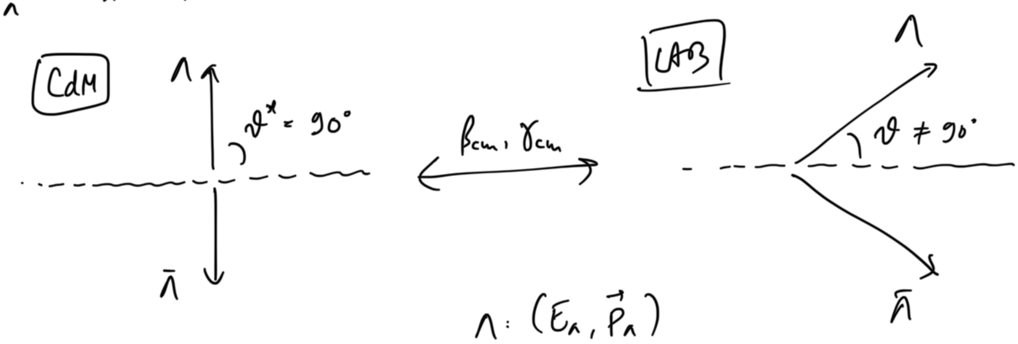
$$\text{e} \quad m_\Lambda = m_{\bar{\Lambda}} \Rightarrow E_\Lambda^* = E_{\bar{\Lambda}}^*$$

$$\Rightarrow \sqrt{s} = 2E_\Lambda^* \quad \Rightarrow E_\Lambda^* = \frac{\sqrt{s}}{2} = 1.25 \text{ GeV}$$

$$\Rightarrow p_\Lambda^* = \sqrt{E_\Lambda^{*2} - m_\Lambda^2} = 0.56 \text{ GeV}$$

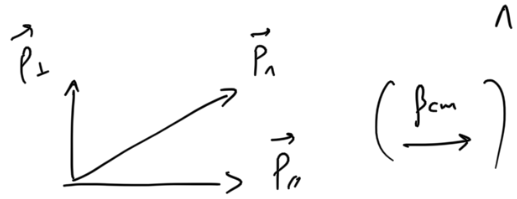
1.116 GeV

② p_Λ e E_Λ nel LAB = ?



$\Lambda : (E_\Lambda, \vec{p}_\Lambda)$

$\bar\Lambda : (E_{\bar\Lambda}, \vec{p}_{\bar\Lambda})$ $\quad E_\Lambda = E_{\bar\Lambda} \quad |\vec{p}_{\bar\Lambda}| = |\vec{p}_\Lambda$



$\vec{p}_\perp^{\,\Lambda} = -\vec{p}_\perp^{\,\bar\Lambda}$

$$\begin{cases} \bullet\ E_i^{TOT} = E_f^{TOT} \Rightarrow E_{\bar p} + m_p = E_\Lambda + E_{\bar\Lambda} = 2E_\Lambda \\ \bullet\ (p_{/\!/}^{TOT})_i = (p_{/\!/}^{TOT})_f \Rightarrow p_{\bar p} = (p_{/\!/})_\Lambda + (p_{/\!/})_{\bar\Lambda} = 2p_{/\!/} \\ \bullet\ (p_\perp^{TOT})_i = (p_\perp^{TOT})_f \Rightarrow 0 = (p_\perp)_\Lambda + (p_\perp)_{\bar\Lambda} \end{cases}$$

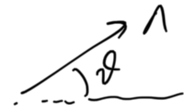
$$E_\Lambda = \frac{1}{2}(\underset{2.39}{E_{\bar p}} + \underset{0.938}{m_p}) = 1.66 \text{ GeV}$$

$$\Rightarrow p_\Lambda = \sqrt{E_\Lambda^2 - m_\Lambda^2} = \sqrt{1.66^2 - 1.116^2} = 1.23 \text{ GeV}$$

$$\left.\begin{aligned} p_\Lambda^2 &= \sqrt{p_\perp^2 + p_{/\!/}^2} \\ p_{/\!/} &= \frac{1}{2}p_{\bar p} = 1.1 \text{ GeV} \end{aligned}\right\} \Rightarrow p_\perp = \sqrt{p_\Lambda^2 - p_{/\!/}^2} = \underline{\underline{0.56}} \text{ G}$$

$$(= p_\perp^*)$$

③ angolo ϑ nel LAB fra Λ e $\vec{p}_{\bar p}$



$$\tan\vartheta = \frac{p_\perp}{p_{/\!/}} = \frac{0.56}{1.1} = 0.51$$

$$\Rightarrow \vartheta = \tan^{-1}(0.51) = 0.47 \text{ rad} \simeq 27°$$

④ sapendo che $\tau_0(\Lambda) = 2.63 \cdot 10^{-10}$ s

calcolare il cammino libero medio di decadim. Λ (nel LAB)

$$L_\Lambda = v \cdot \tau = (\beta_\Lambda c)(\gamma_\Lambda \tau_0)$$

1.66

$$\beta_\Lambda = \frac{p_\Lambda}{E_\Lambda} = \frac{1.23}{1.66} = 0.74 \quad \Big\} \quad \gamma_\Lambda = \frac{E_\Lambda}{m_\Lambda} = \frac{\;}{1.116} = 1.49$$

$$\Rightarrow L_\Lambda = 0.74 \cdot 3 \cdot 10^{8} \cdot 1.49 \cdot 2.63 \cdot 10^{-10} = 0.087 \text{ m}$$
$$= 8.7 \text{ cm}$$

(5) nel $\boxed{\text{CdM}}$: a $\vartheta^* = 0°$ e $180°$

$$\overset{\bar\Lambda}{\longleftarrow}\cdot\overset{\Lambda}{\longrightarrow} \quad \overset{?}{\Rightarrow} \quad \boxed{\text{LAB}}$$

(A): $\overset{\bar\Lambda}{\leftarrow} \overset{\Lambda}{\longrightarrow}$

$$p_\perp^* = 0 \quad \Rightarrow \quad p_\perp = 0$$

(B): $\overset{\bar\Lambda}{\longrightarrow}$ $\overset{\Lambda}{\longrightarrow}$

$\beta_{cm} \leftarrow$ quanto è "forte" il boost

$\beta^*_{\bar\Lambda}$ se $\beta_{cm} > \beta^*_{\bar\Lambda} \rightarrow$ flippa

$\beta_{cm} < \beta^*_{\bar\Lambda} \rightarrow$ non flippa

$$\beta_{cm} = 0.66 \qquad \beta^*_{\bar\Lambda} = \frac{p^*_{\bar\Lambda}}{E^*_{\bar\Lambda}} = \frac{0.56}{1.25} = 0.45$$

$\Rightarrow$ flippa $\Rightarrow$ (B)

(6) è possibile avere $\vartheta = 90°$ nel LAB?

$\boxed{\text{EX PER CASA}}$

Calcolare $\sqrt{s}$ per:

(A) $\overset{e^-}{\longrightarrow}\overset{e^+}{\longleftarrow}$ $\quad p(e^+) = p(e^-) = 1$ GeV

(B) $\overset{e^-}{\longrightarrow}\overset{p}{\longleftarrow}$ $\quad p(e^-) = p(p) = 1$ GeV

(C) $\overset{e^-}{\longrightarrow}\overset{p}{\cdot}$ $\quad p(e^-) = 2$ GeV $\quad p(p) = 0$

(D) $p \uparrow$ $\overset{p}{\longleftarrow}$ $\vartheta = 90°$ $\quad p = 1$ GeV

(E) $\overset{p}{\longrightarrow}$ $\overset{p}{\longrightarrow}$ $\quad p = 100$ GeV

$$m_{e^-} = m_{e^+} = 0.511 \text{ MeV}$$
$$m_p = 938 \text{ MeV}$$