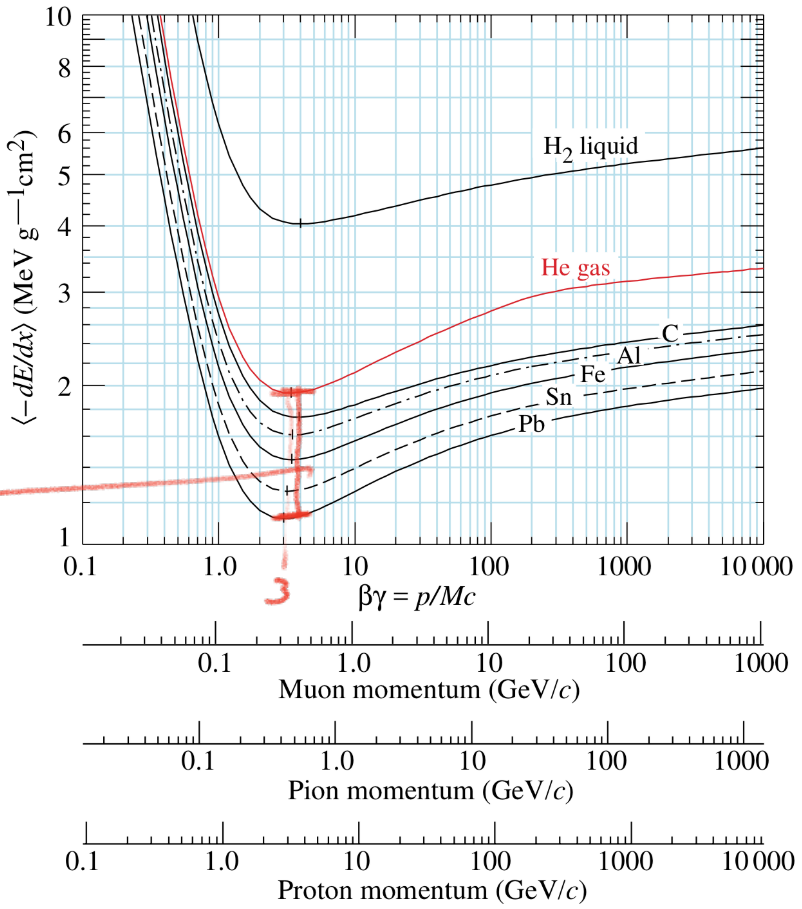
Formula di Bethe-Bloch

$$-\frac{1}{\rho}\frac{dE}{dx} = C\,\frac{Z}{A}\,\frac{z_p^2}{\beta^2}\left(\ln\frac{\beta\gamma^2}{I^2} - \beta^2 - \frac{\delta}{2}\right)$$

$$0.3\ \left[\frac{\text{MeV}}{\text{cm}}\ \frac{\text{cm}^3}{\text{g}}\right]$$



minimo $\sim 1.5-2$

3

del proiettile

$\beta\gamma = \frac{p}{m}$ $\qquad \beta = \frac{p}{E}$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$(\beta\gamma)^2 = \frac{\beta^2}{(\sqrt{1-\beta^2})^2} \qquad \text{invertite.}$$

$$\Rightarrow \quad \beta = \frac{(\beta\gamma)^2}{\sqrt{1+(\beta\gamma)^2}}$$

$$(\beta\gamma) = \frac{p}{m}$$

Misure di $p \Longrightarrow \beta\gamma = \frac{p}{m}$

Bethe-Bloch fornisce il valore medio $\left\langle \frac{dE}{dx} \right\rangle$

ed. perse medie ΔE in un tratto dx

Al minimo $\beta\gamma = 3$ $\quad \left(\frac{1}{\rho}\frac{dE}{dx}\right)\Big|_{\beta\gamma=3} = 2\ \text{MeV/cm}\ \frac{\text{cm}^3}{\text{g}}$

10 cm di H_2O con $\rho = 1\ \text{g/cm}^3$

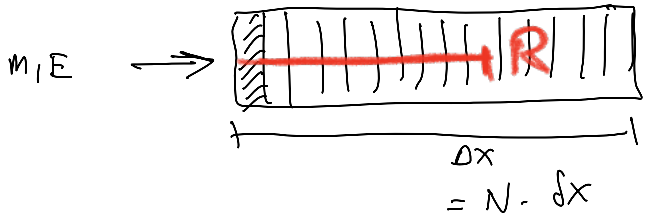
$$\overline{\Delta E} = 2\ \frac{\text{MeV}}{\text{cm}}\ \frac{\cancel{\text{cm}^3}}{\cancel{\text{g}}} \times 1\ \frac{\cancel{\text{g}}}{\cancel{\text{cm}^3}} \times 10\ \text{cm}$$

$$= 20\ \text{MeV}$$

per Δx fissato

Ripetendo le misure
Distribuzione sperimentale
è di Landau

Range



$$\left(\frac{dE}{dx}\right)_{BB} \cdot \delta x \qquad \text{per } \delta x \text{ sottili}$$

Range: distanza percorsa prima che la particella incidente si fermi

$E = m + T$ $\qquad E_{in} = E \longrightarrow T = E - m$

$E_{fin} = m \qquad T = 0$

$$R(E) = \int_E^m dx = \int_{T=E-m}^{T=0} dx = \int_{T=E-m}^{T=0} -\frac{1}{\underbrace{\left(-\frac{dE}{dx}\right)}_{\text{Bethe-Bloch}}}\, dE$$

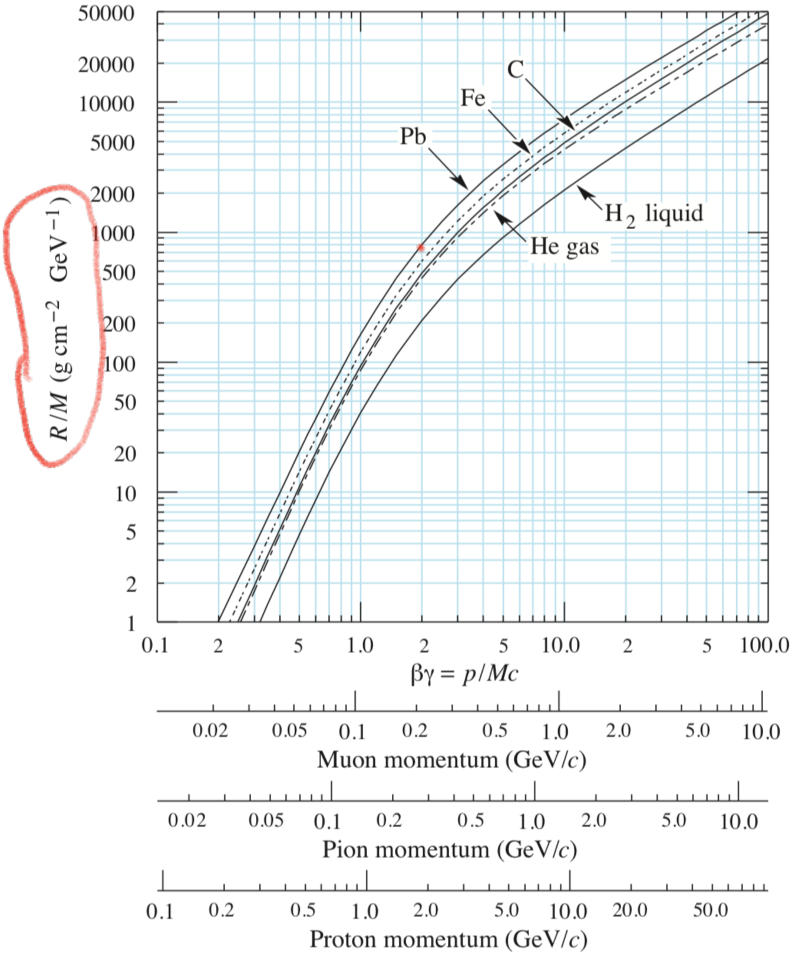
$$= \int_m^E \frac{d}{\left(-\frac{dE}{dx}\right)}\, dE$$

$$\frac{R}{M} = \frac{R(E)}{\text{massa particella incidente}}$$

$$\left[\frac{R}{M}\right] = g\,cm^{-2}\,GeV^{-1}$$

massa in GeV

$$R = \int \frac{1}{\left(-\frac{1}{\rho}\frac{dE}{dx}\right)}\, dE$$



protone $M = 1$ GeV $\qquad p = 2$ GeV

$$\beta\gamma = \frac{p}{M} = 2$$

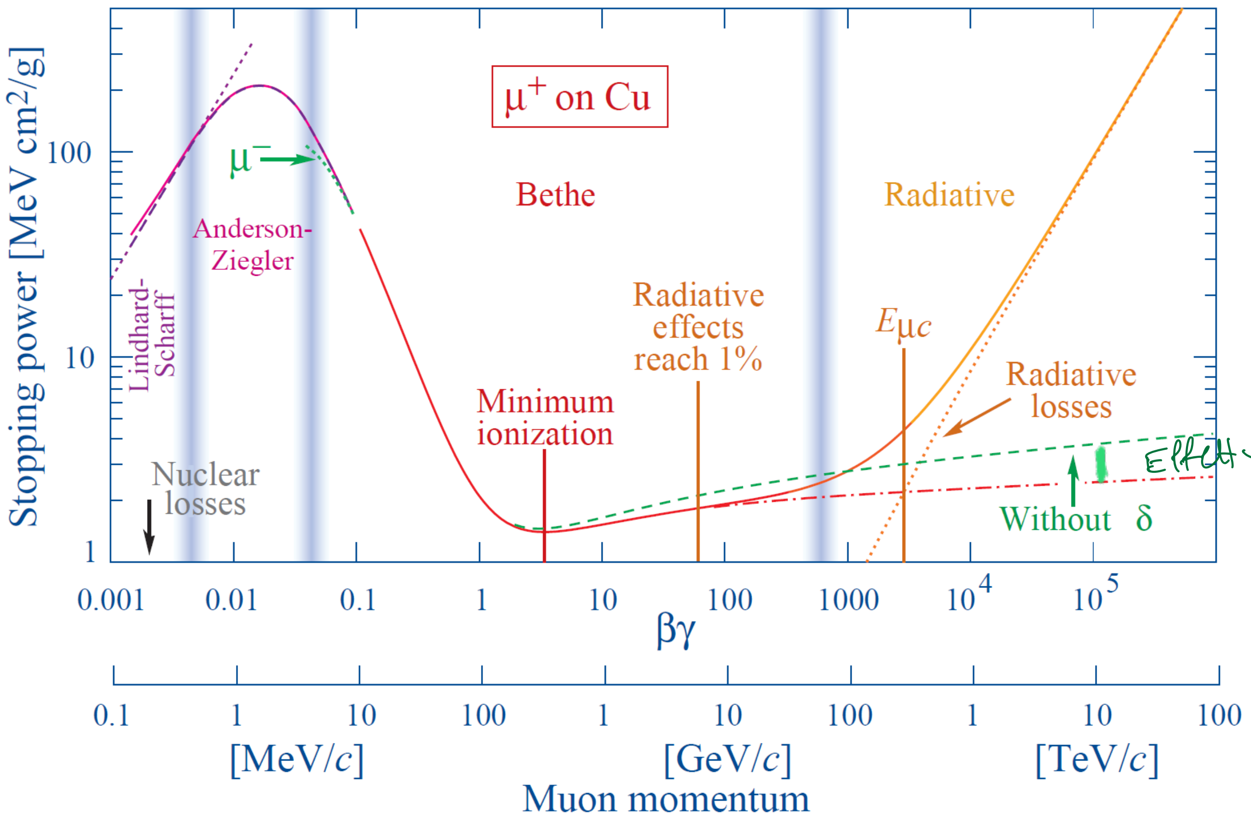
Pb: $\rho_{Pb} \simeq 11\ g/cm^3$ $\qquad \beta\gamma = 2 \Rightarrow \frac{R}{M} = 800$

$$R = (800) \times M = 800\ \frac{g}{cm^2}$$

$$R[cm] = \frac{800}{11 (\text{dens. Pb})}\ cm \simeq 72\ cm$$

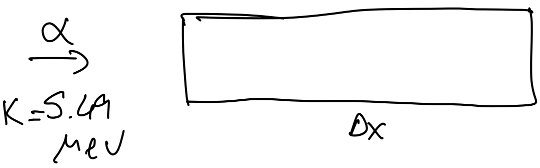
Range: spessore percorso prima di perdere tutta l'energia cinetica

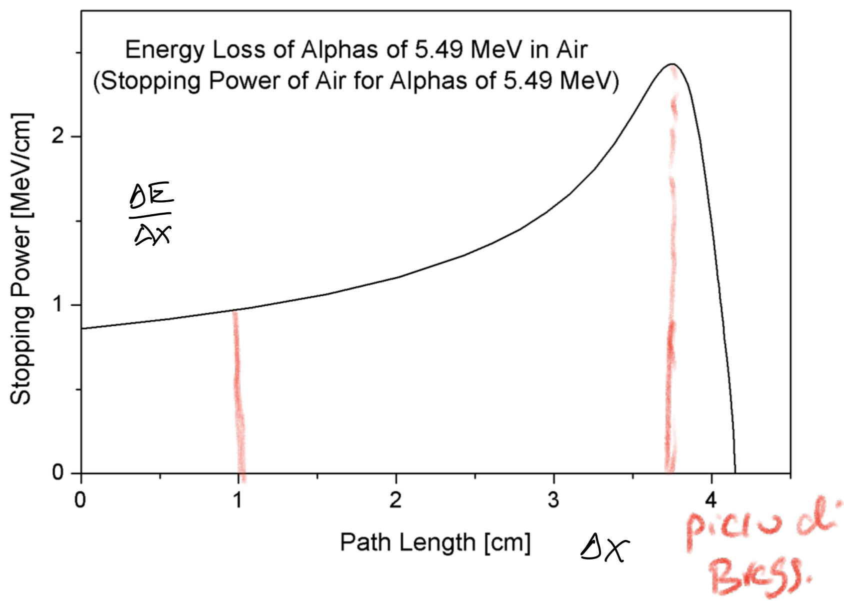
$\frac{1}{\rho}\frac{dE}{dx}$



Effetto densità del mezzo

Picco di Bragg





$M_\alpha = 3.7$ GeV

$\beta\gamma = \frac{p}{m}$

$K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}$

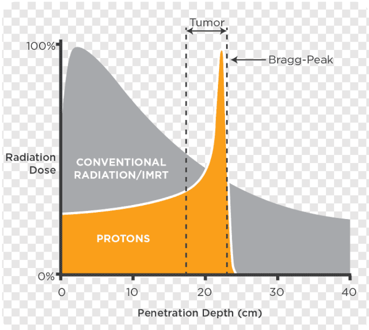
$= \sqrt{10 \text{ MeV} \times 3.7 \text{ GeV}}$

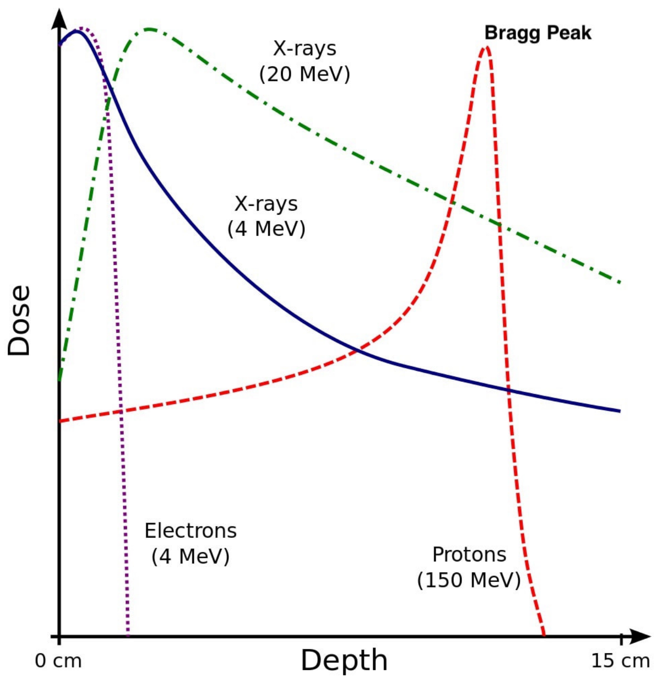
$= 6\sqrt{\text{MeV} \times \text{GeV}} = 6\sqrt{(\text{MeV})^2 \times 1000} = 6\sqrt{1000}$ MeV

Spessore con max di perdita di energia

Dose: energia depositata nel mezzo

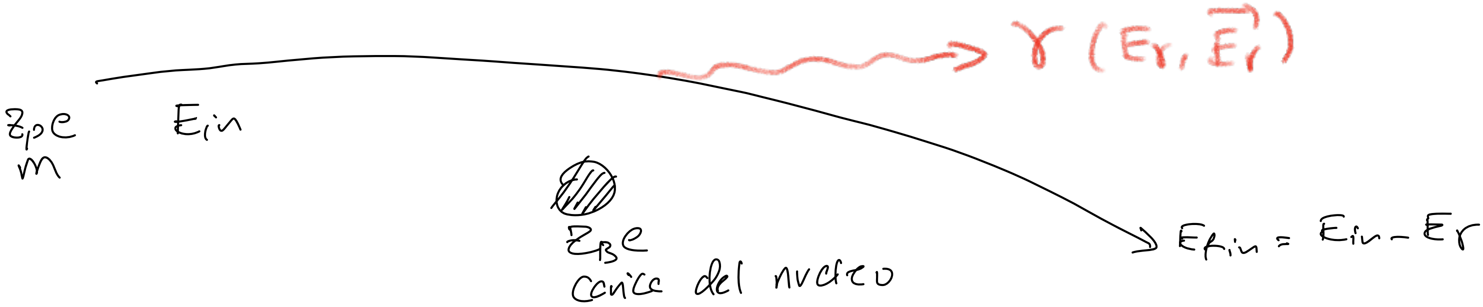
Applicazione medica del picco di Bragg per ottimizzare deposito di energia nella vicinanza del tumore.





Elettroni nel mezzo

Elettroni ionizzano come tutte le particelle cariche.



carica Z_p accelerata in un campo elettrico esterno emette radiazione

Effetto Larmor

$$\cancel{m} a = \frac{1}{4\pi\varepsilon_0} \frac{Z_p Z_B e^2}{r^2} \frac{1}{m}$$

Effetto Larmor: emissione di fotone da particelle carice accelerate in un campo elettrico.

potenza di irraggiam.

$$P = \frac{2}{3} \frac{e^2}{m^2 c^3} |\dot{v}|^2$$

$\dot{v} = a$

$a \propto \frac{1}{m}$

$\Rightarrow P$ minore per $m \nearrow$

Effetto Larmor Relativistico: $P = -\frac{2}{3} \frac{e^2}{m^2 c^3} \frac{dp_\mu}{d\tau} \frac{dp^\mu}{d\tau}$

τ: tempo nel rif. solidale con le particella. $t_{LAB} = \gamma \tau$

$P_\mu = (E, \vec{p}) = (\gamma m, \gamma m \vec{v}) = (\gamma m, \gamma m \vec{\beta})$ $\quad \beta = v/c$

$$P = \frac{2}{3}\frac{e^2}{c}\gamma^6\left[(\dot{\vec{\beta}})^2 - (\vec{\beta}\times\dot{\vec{\beta}})^2\right] \qquad c\dot{\vec{\beta}} = \vec{a}$$

$\dot{\vec{\beta}} \parallel \vec{\beta}$ oppure $\dot{\vec{\beta}} \nparallel \vec{\beta}$ componenti diverse.

Accelerazione lineare:

$\dot{\vec{\beta}} \parallel \vec{\beta} \Rightarrow \vec{\beta}\times\dot{\vec{\beta}} = 0 \Rightarrow P = \frac{2}{3}\frac{e^2}{c}\gamma^6\frac{a^2}{c^2}$ Lineare

$c\dot{\vec{\beta}} = \vec{a} \Rightarrow \dot{\beta} = \frac{a}{c}$

successione di accel. circolare a raggi diversi

Accelerazione Curvilinea

$\vec{\beta} \perp \dot{\vec{\beta}} \qquad \vec{a} \perp \vec{v}$ (
(moto circolare)

$\frac{a}{c} = \dot{\beta}$

$$\dot{\vec{\beta}}^2 - (\vec{\beta}\times\dot{\vec{\beta}})^2 = \dot{\beta}^2 - \dot{\beta}^2\beta^2 = \dot{\beta}^2(1-\beta^2) = \dot{\beta}^2\frac{1}{\gamma^2}$$

potenza

$$P = \frac{2}{3}\frac{e^2}{c^3}\gamma^6\frac{a^2}{\gamma^2} = \frac{2}{3}\frac{e^2}{c^3}a^2\gamma^4$$ Curvilineo

$$\frac{P_{curv.}}{P_{line}} \simeq \frac{1}{\gamma^2} \qquad \gamma = \frac{E}{m}$$ particella accelerata

a parità di energia E con acceleratore circolare

$$\frac{P_e}{P_{Prot}} = \gamma_e^4\frac{1}{\gamma_p^4} \sim \frac{(m_p)^4}{(m_e)^4} = \left(\frac{1000\ MeV}{(0.5)\ MeV}\right)^4 \simeq 2^4 \times 10^{12}$$

LHC: $P_p = 6.5$ TeV $\quad$ Ragsio di Curvatura $\sim$ 5 Km.

$\Rightarrow$ ad alta energia perdite di energia per radiazione dominante per elettroni: $\boxed{\text{Bremsstrahlung}}$

Effetto Bremsstrahlung domina ad Alta Energia

$$\left.\frac{dE}{dx}\right|_{e^-} = \left.\frac{dE}{dx}\right|_{ion} + \left.\frac{dE}{dx}\right|_{Brem.}$$

$$\left.\frac{dE}{dx}\right|_{Brem} \simeq \frac{E}{X_0}$$

$E \longrightarrow$ En. e^-

$X_0 \longrightarrow$ Lunghezza di radiazione del mezzo

caratteristica de materiale attraversato

$$E(x) = E_0 \, e^{-x/X_0}$$

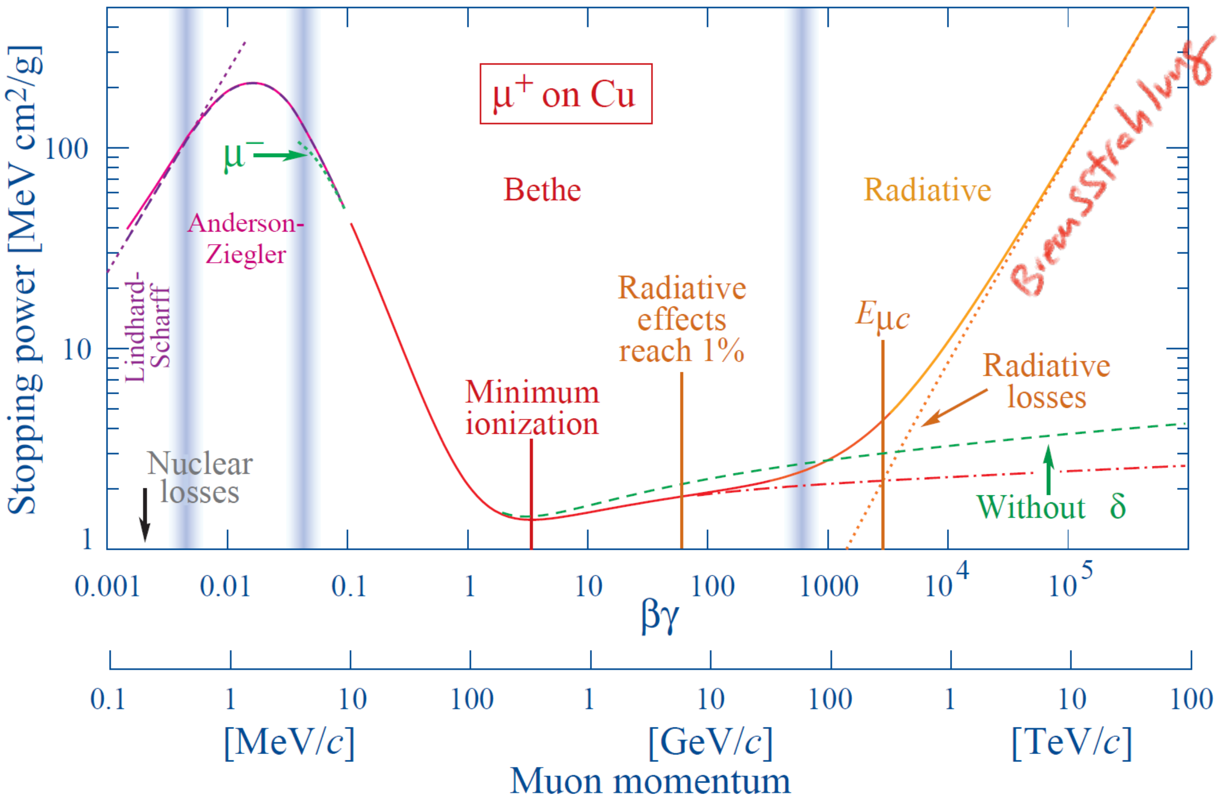
$$\frac{1}{X_0} \sim \rho \, \frac{N_A}{A} \, Z_B^2 \, \ln\left(183 \, Z_B^{-1/3}\right)$$

Formula Empirica

prop. del mezzo

$$\rho X_0 \sim 170 \, \frac{A}{(Z_B)^2} \quad \frac{g}{cm^2}$$

Dopo $x = X_0$ $\quad E = E_0 e^{-1} \simeq 30\% \, E_0$



Esempio: confronto di $\left.\frac{dE}{dx}\right|_{ion}$ vs. $\left.\frac{dE}{dx}\right|_{Brem}$

per e^- con $E_0 = 100$ MeV e $X_0 \sim 1$ cm.