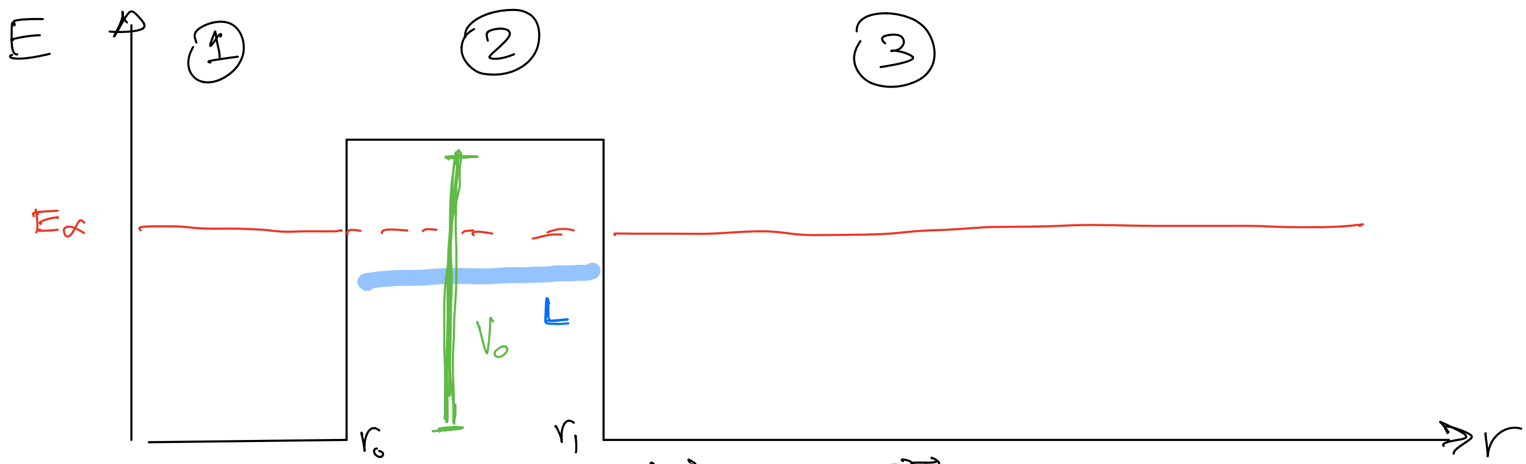
Decadimento α, Modello di Gamow

Effetto tunnel applic. a buca/barriera di potenziale



$$\psi(r) = \begin{cases} \psi_1(r) = A e^{+i\vec{p}_1\cdot\vec{r}} + B e^{-i\vec{p}_1\cdot\vec{r}} \\ \psi_2(r) = C e^{+\vec{p}_2\cdot\vec{r}} + D e^{-\vec{p}_2\cdot\vec{r}} \\ \psi_3(r) = E e^{+i\vec{p}_3\cdot\vec{r}} \end{cases}$$

nelle unità naturali $\hbar = 1$

condizioni al contorno $\begin{cases} \psi_1(r_0) = \psi_2(r_0) & \psi_1'(r_0) = \psi_2'(r_0) \\ \psi_2(r_1) = \psi_3(r_1) & \psi_2'(r_1) = \psi_3'(r_1) \end{cases}$

$$E_1 = \frac{p_1^2}{2m}$$

nel regime non relativistico.

$$E_3 = E_1 = \frac{p_3^2}{2m}$$

$p_1 \equiv p_3$

$$E_2 = V_0 - \frac{p_2^2}{2m}$$

prob. tunnelling: $T = \dfrac{|E|^2}{|A|^2}$

$$p_1 \equiv p_3 = \sqrt{2m E_\alpha}$$

$E_\alpha \equiv K_\alpha$

$$p_2 = \sqrt{2m(V_0 - E_\alpha)}$$

$$T = \frac{1}{1 + \frac{V_0^2}{V_0^2 - (2E - V_0)^2} \sinh^2(P_2 L)} \qquad \sim e^{2P_2L} \qquad x \equiv P_2 L$$

$$\sinh x = \frac{1}{2}(e^x - e^{-x}) \qquad \sinh^2 x = \frac{1}{4}\{e^{2x} + e^{-2x}\} - \frac{1}{2}$$

$$L \simeq 50 - 7 \text{ fm} \simeq 40 \text{ fm} \qquad E_\alpha \simeq 5 \text{ MeV}$$

$$P_2 = \sqrt{2m_\alpha(V_0 - E_\alpha)} = \sqrt{2 \times 3.7 \times 10^3 \text{ MeV } (30-5) \text{ MeV}}$$

$$= 430 \text{ MeV}$$

$$x = P_2 L = 430 \text{ MeV} \times 40 \text{ fm} \qquad 1 = 200 \text{ MeV fm}$$

$$\simeq 80$$

$$\sinh^2 x \simeq \frac{1}{4} e^{2x} \gg 1$$

$$T \simeq 4 \frac{V_0^2 - (2E - V_0)^2}{V_0^2} e^{-2P_2L}$$

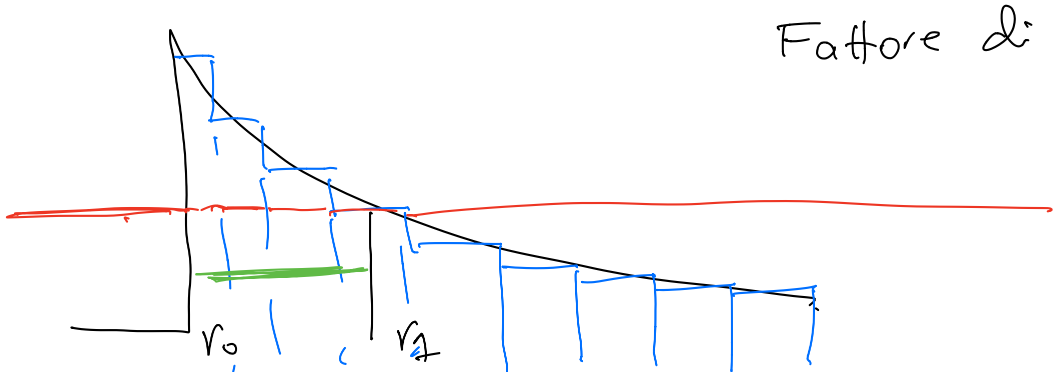
$$P_1^2 = 2m\,E \qquad P_2^2 = 2m(V_0 - E)$$

$$P_1^2 + P_2^2 = \text{—} \quad \text{—}$$

$$T = 16 \frac{P_1^2 P_2^2}{(P_1^2 + P_2^2)^2} e^{-2P_2L} = 4\, e^{-2\sqrt{2m(V_0-E)}\,L}$$

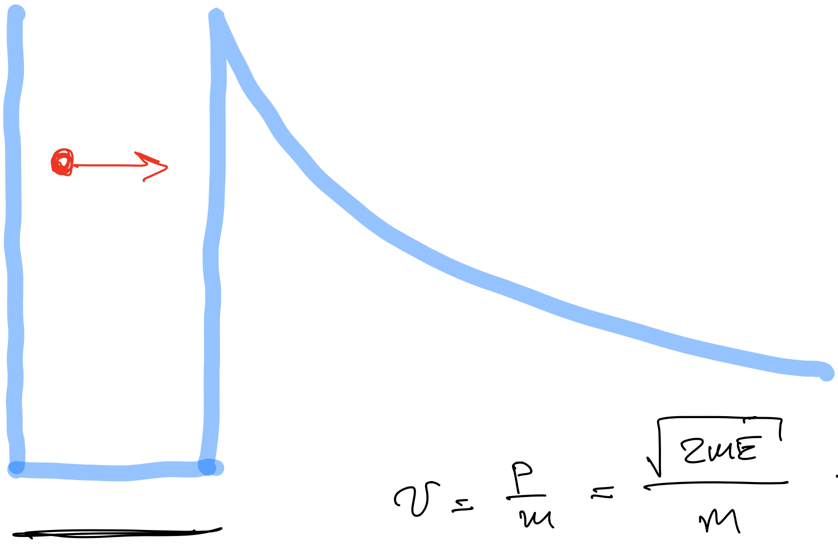
$$\simeq 4 \qquad = 4\, e^{-2G}$$

Fattore di Gamow



$$G = \int_{r_0}^{r_1} dr \sqrt{2m(V(r) - E)} = \int_{r_0}^{r_1} p(r)\, dr$$

$$T = e^{-2G}$$



r_0

$$v = \frac{p}{m} = \frac{\sqrt{2mE}}{m} = \sqrt{\frac{2E}{m}}$$

$$f = \frac{1}{\Delta T} = \frac{v}{2 r_0} = \frac{1}{2}\sqrt{\frac{2E}{m}}\frac{1}{r_0} = \sqrt{\frac{E}{2m}}\frac{1}{r_0}$$

ΔT tempo tra urti successivi contro barriera

$$\lambda = f \cdot T = \sqrt{\frac{E}{2m}}\frac{1}{r_0} e^{-2G}$$ prob. di decad. per unità di tempo.

$\tau = \dfrac{1}{\lambda}$ vita media $\qquad \tau \propto E^{-1/2} e^{2G}$

$$\tau = C\, E^{-1/2} e^{2G}$$

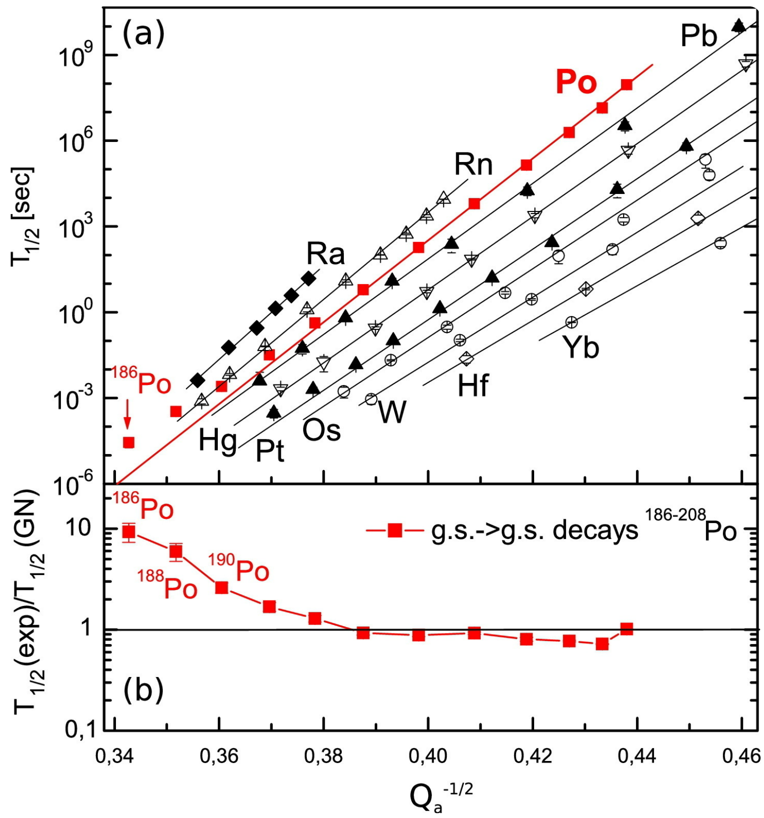
$$\ln \tau = C' - \frac{1}{2}\ln E + 2G$$

$$G = \int dr \sqrt{2m(V_1 - E_\alpha)}$$

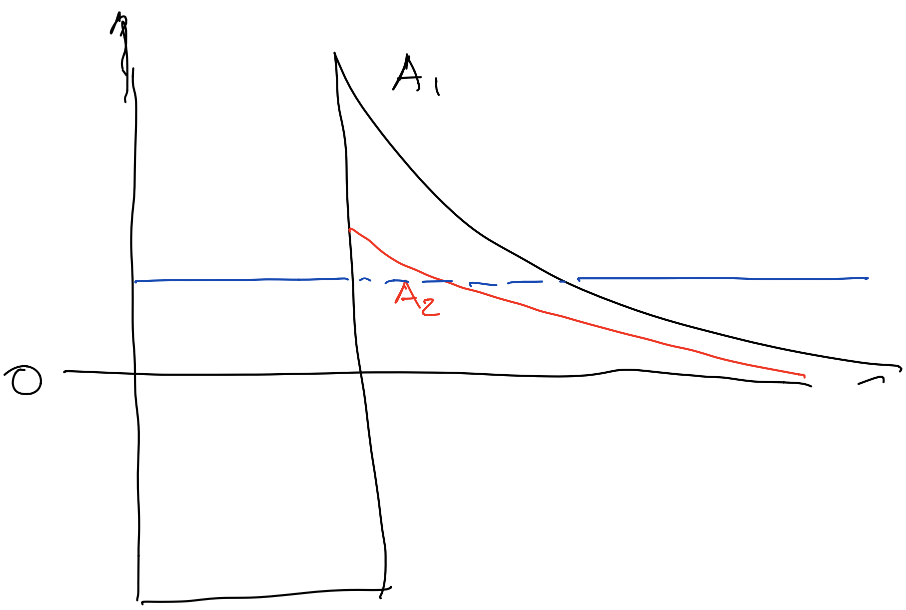
variazione lenta con E_α

$\ln \tau = C - B \ln E_\alpha$ Legge di Geiger-Nuttal

τ



$Q_\alpha \simeq E_\alpha$



$$U(r) = \frac{2(Z-2)\alpha}{r}$$

$$= \frac{2(Z-2)\alpha}{r_0} \frac{1}{A^{1/3}}$$

$$U(r) \propto \frac{2(Z-2)}{A^{1/3}} \simeq$$

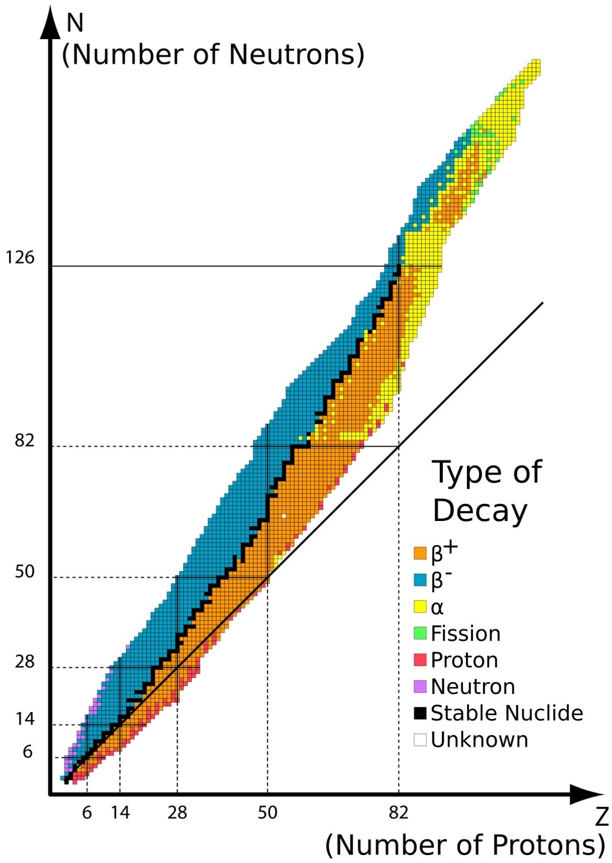
$$\simeq \frac{Z}{A^{1/3}}$$

$$Z \simeq \frac{A}{2}$$

$$U(r) \simeq \frac{A}{A^{1/3}} \simeq A^{2/3}$$

Per E_α fissata.

τ cresce con $A^{2/3}$



$\tau \propto E_\alpha$

$\tau_{SS} \simeq 5 \times 10^9$ yr

Elem. instabili

con $\tau_{1/2} < \tau_{SS}$

già decaduti

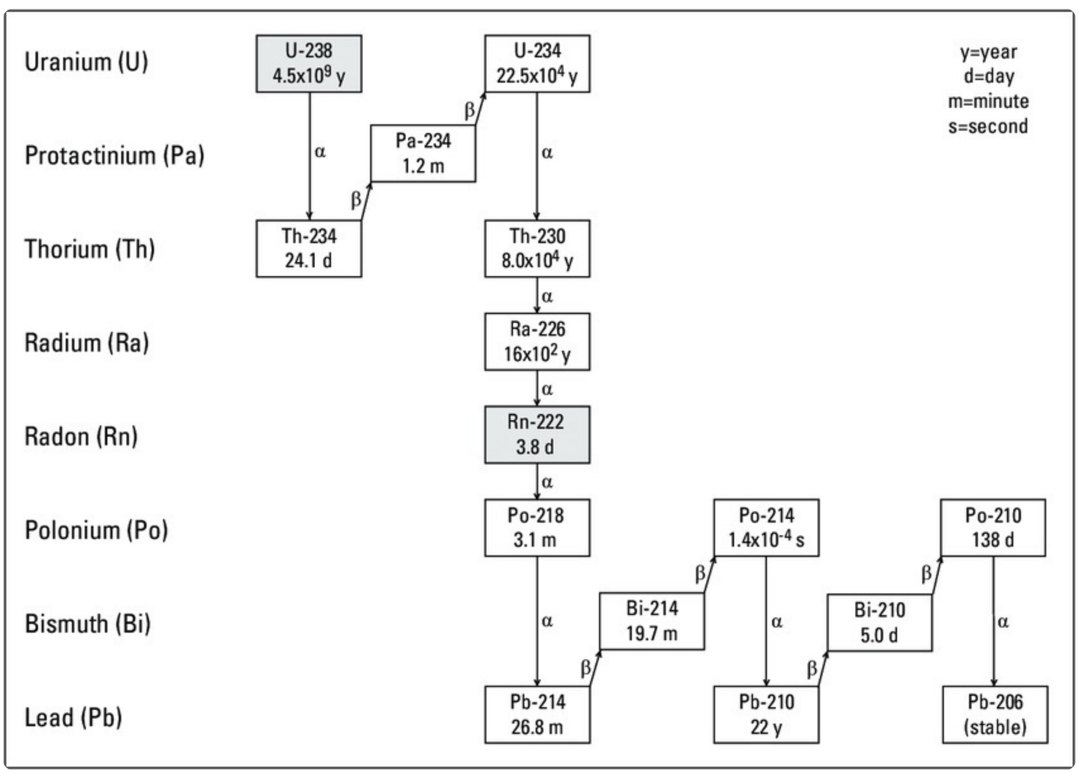
tutti gli elementi con $A > 209$

$^{238}_{92}U$ — 4.5×10^9 yr

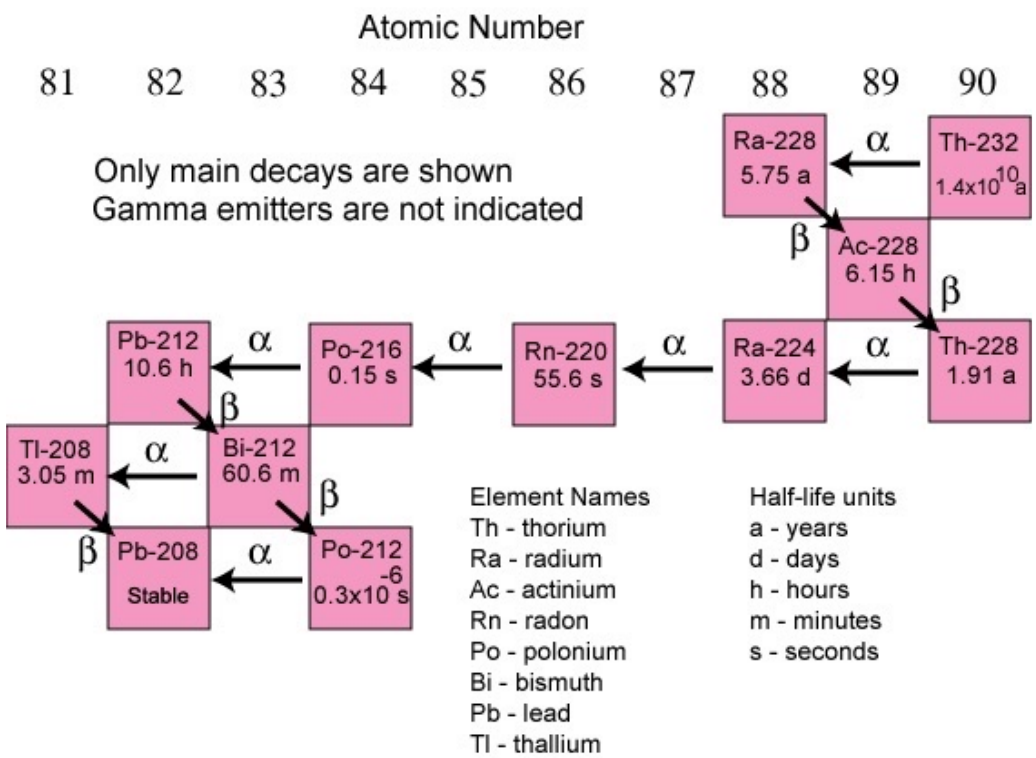
$^{235}_{92}U$ — 7.1×10^8 yr

$^{232}_{90}Th$ — 1.6×10^{10} yr

Questi elementi sono presenti perché fanno parte di alcune catene di decadimento.



The Thorium-232 Decay Chain



Reazione nucleare

$$a + b \longrightarrow c + d + Q$$

$$Q = m_a + m_b - m_c - m_d$$

$Q > 0$: reazione esotermica

$Q < 0$ reazione endotermica

$Q > 0$: masse si trasformano in energia

$Q < 0$: K_a, K_b energia cinetica iniziale necessaria per far avvenire la reazione.

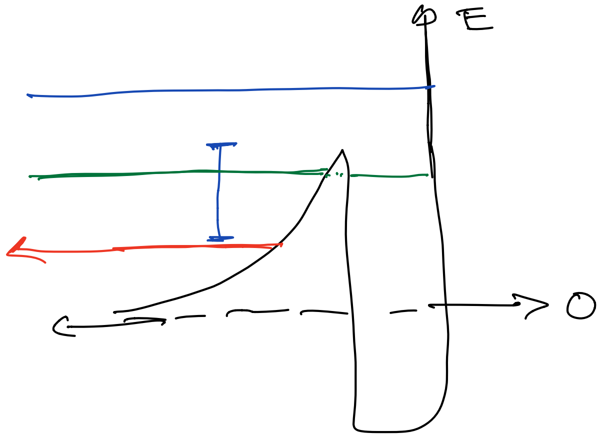
Scoperta del protone. (Rutherford)

$$\alpha + {}^{14}_{7}N \longrightarrow {}^{17}_{8}O + p$$

$$Q = -1.19 \text{ MeV}$$

$K_\alpha \simeq 4-8$ MeV $> |Q| \Rightarrow$ avviene reazione

$q \neq 0$



particelle cariche: servono acceleratori

neutrone

$$n + X \longrightarrow Y + Z + Q.$$

$$Q < 0$$

per compensare $Q < 0$ $\Longrightarrow$ fornire K ai neutroni

$p + {}^{7}_{3}Li \longrightarrow \alpha + \alpha$ 1931

accelero p $K_p \longrightarrow$ si trasferisce in K_α.

Chadwick

$$\alpha + \underbrace{{}^{9}_{4}Be}_{\text{fermo}} \longrightarrow \underbrace{{}^{12}_{6}C}_{\text{fermo}} + n \qquad Q = 5.7 \text{ MeV}$$

trasformazione di K_α in K_n

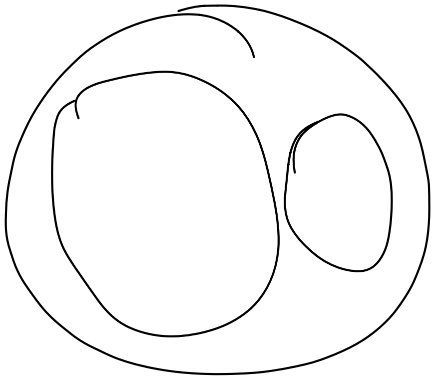
$$n + {}^{A}_{Z}X \longrightarrow \begin{cases} n + {}^{A}_{Z}X \\ \text{cattura neutronica: inizio di cascate} \\ \text{fissione indotta.} \end{cases}$$

1938 Hahn-Strassman.

$$n + {}^{238}_{92}U \longrightarrow {}^{137}_{56}Ba + {}^{84}_{36}Kr + 17n$$

Bario Kripton.

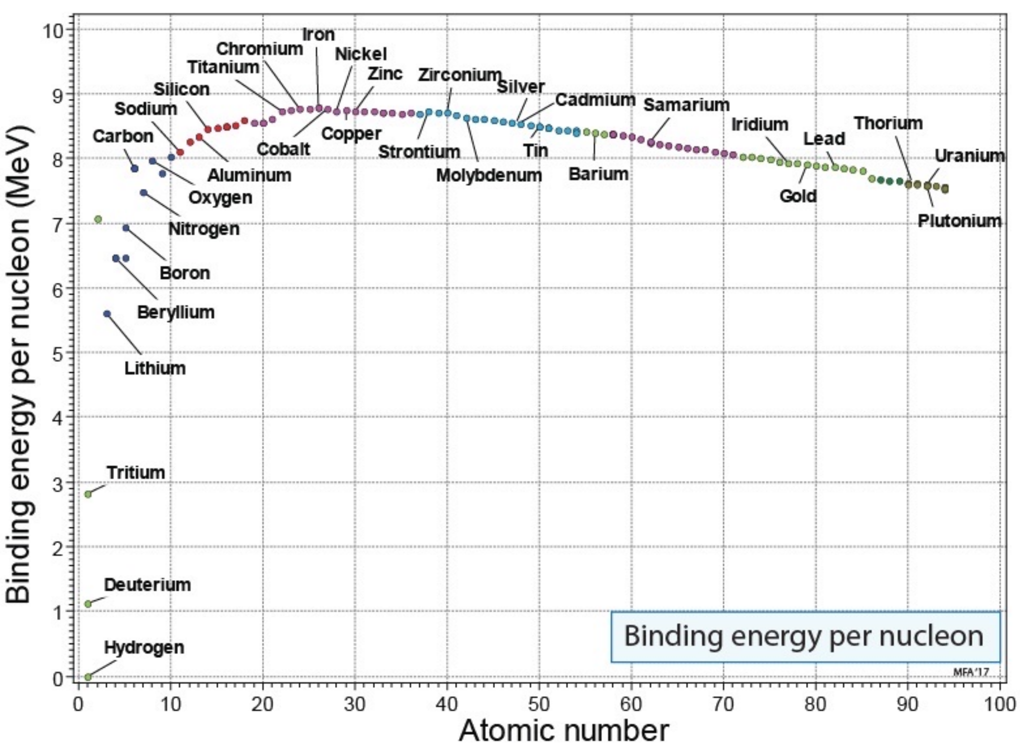
Fissione indotta.



1939 Meitner-Frisch fissione nucleare

$${}^{A}_{Z}N \longrightarrow {}^{A-a}_{Z-b}X + {}^{a}_{b}Y + Q$$

possibile?



sulla Carte fissione possibile per $A > 60$ perchè $\frac{\partial B}{\partial A} < 0$

$$^{238}_{92}U \longrightarrow {}^{137}_{56}Ba + {}^{84}_{36}Kr + 16n + Q$$

$$\bar{B}(A=240) = 7.6\ MeV \qquad \bar{B} = \frac{B}{A}$$

$$\bar{B}(A=120) = 8.5\ MeV$$

$$\bar{B}(A=84) = 8.7\ MeV$$

$$B(^{238}U) - B(Ba) - B(Kr) = 23\cancel{8}\,\bar{B}(U) - 137\times\bar{B}(Ba) - 84\,\bar{B}(Kr)$$

$$\simeq -216\ MeV$$

la fissione di ^{238}U $\quad Q = 210\ MeV$