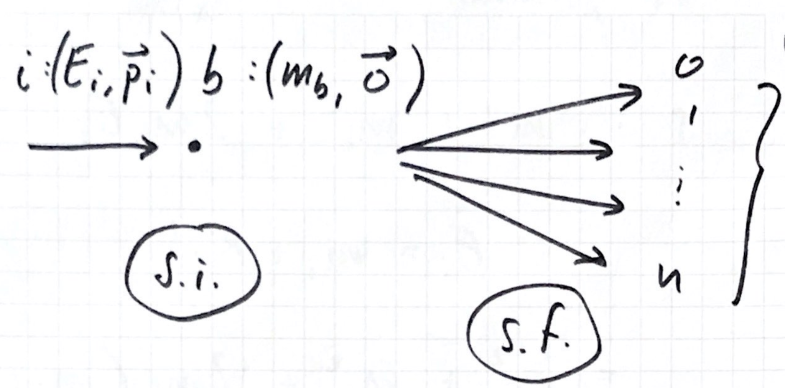


∃ una energia di soglia t.c. avviene la reazione

$$\sqrt{s}\Big|_{CdM,\, s.f.} = \sum_f E_f^* = \sum_f \left(m_f + K_f^*\right)$$

$$(E = m + K)$$

$$\sqrt{s}_{MIN} = \sum_f m_f$$

anche nello s.i.

$$\sqrt{s}\Big|_{LAB,\, S.i.} = \sqrt{(E_i + m_b)^2 - |\vec{p}_i|^2}$$

$$= \sqrt{E_i^2 + m_b^2 + 2E_i m_b - p_i^2}$$

$$= \sqrt{m_i^2 + m_b^2 + 2E_i m_b}$$

$$E^2 = m^2 + p^2$$

al quadrato

$$s = m_i^2 + m_b^2 + 2 m_b E_i$$

$$E_i = m_i + K_i$$

$$= m_i^2 + m_b^2 + 2 m_b (m_i + K_i)$$

$$= (m_b + m_i)^2 + 2 m_b K_i$$

$$\Rightarrow (m_b + m_i)^2 + 2 m_b K_i = \left(\sum_f m_f + K_f^* \right)^2$$

$$\Rightarrow K_i = \frac{\left[\sum_f m_f + K_f^* \right]^2 - (m_b + m_i)^2}{2 m_b}$$

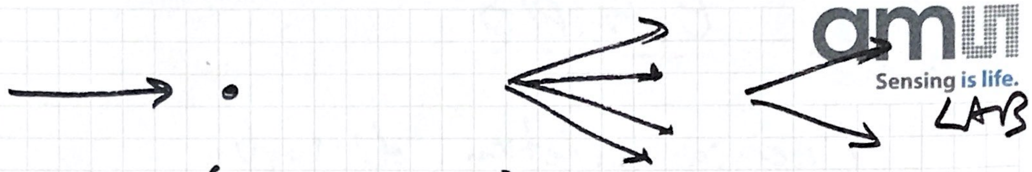
$$K_i \geq \underbrace{\frac{\left(\sum_f m_f \right)^2 - (m_b + m_i)^2}{2 m_b}}_{= K_{soglia}}$$

<u>DUE CASI</u>

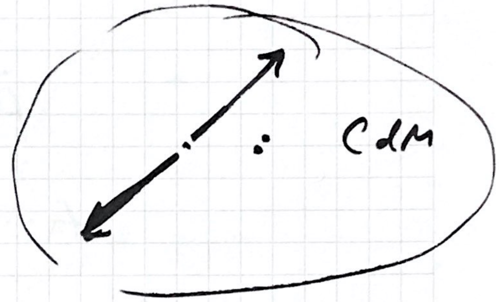
$K_{soglia} \leq 0$ sempre permessa

$K_{soglia} > 0$ $\exists$ soglia

$$K_{soglia} \longrightarrow E_{soglia} = K_{soglia} + m_i$$

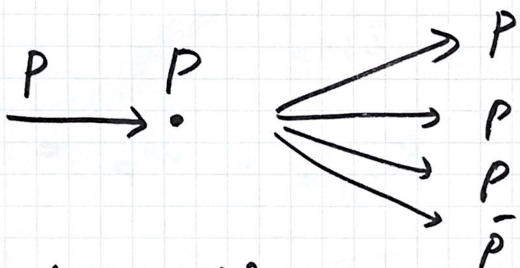


$$P_{TOT} = \left(E_{TOT},\ \vec{P}_{TOT}\right)$$



Fascio di protoni
contro un bersaglio di protoni

$$p\,p \to p\,p\,p\,\bar{p}$$



$m_p = 938$ MeV
$(= m_{\bar{p}})$

$$K_{soglia} = \frac{\left(\sum_f m_f\right)^2 - (m_i + m_b)^2}{2 m_b} =$$

$$= \frac{(4m_p)^2 - (2m_p)^2}{2m_p} = \frac{16\,m_p^2 - 4m_p^2}{2m_p}$$

$$= 6\,m_p = 5.6 \text{ GeV}$$

$$\Rightarrow E_{soglia} = K_{soglia} + m_p \simeq 6.6 \text{ GeV}$$

ESEMPIO

Energia cinetica di soglia

$$\bar{p} + p \longrightarrow \chi \bar{\chi}$$

$\uparrow$ fascio $\qquad$ $\uparrow$ bersaglio

$m(\chi) = 1$ TeV

$m(p) = 938$ MeV

$$K_{soglia} = \frac{\left(\sum_f m_f\right)^2 - (m_b + m_i)^2}{2m_b} =$$

$$= \frac{(2m_\chi)^2 - (2m_p)^2}{2m_p} \simeq 2.1 \text{TeV}$$

$$E_{soglia} = K_{soglia} + m_p = 2.1 \text{ TeV}$$

proiettile + bersaglio $\qquad\qquad$ collider

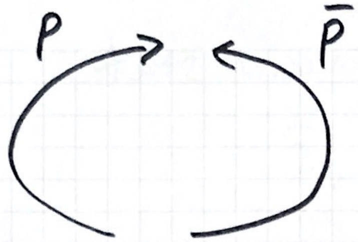
$\longrightarrow \cdot \qquad \longrightarrow \qquad \longrightarrow\ \longleftarrow$

SIMMETRICO $\qquad$ $m_1 = m_2$

$E_1 = E_2$

$\vec{p}_1 = -\vec{p}_2$

1. LAB = CdM
2. facile



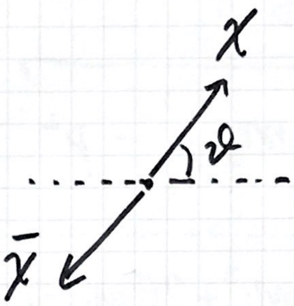
B

$q(p) = -q(\bar{p})$

E, p

$\overset{p}{\longrightarrow} \; \overset{\bar{p}}{\longleftarrow}$

s.i.



χ : $\bar{\chi}$

$\boxed{\text{LAB} = \text{CdM}}$

$$P_{TOT}\Big|_{s.f.} = \Big(E_\chi + E_{\bar{\chi}},\ \underbrace{\vec{p}_\chi + \vec{p}_{\bar{\chi}}}_{=\vec{0}}\Big)$$

$$\sqrt{s} = 2E_\chi \qquad E_\chi = \sqrt{m_\chi^2 + p_\chi^2}$$

$$\sqrt{s}_{MIN} = 2m_\chi$$

$$\sqrt{s}\Big|_{s.i.} = 2E_p \Rightarrow 2E_p \geq 2m_\chi$$

$$\Leftrightarrow E_p \geq m_\chi$$

$E_{soglia} = 1$ TeV

(bersaglio 2.1 TeV)

Voglio produrre uno stato

con $\sqrt{s}$

BERSAGLIO

$p : (E, \vec{p}) \quad (m_b, \vec{0})$

$$\sqrt{s} = \sqrt{(E + m_b)^2 - p^2}$$

$$= \sqrt{E^2 + m_b^2 + 2 E m_b - p^2}$$

$$= \sqrt{m_i^2 + m_b^2 + 2 E m_b}$$

$$\sqrt{s} \propto \sqrt{E}$$

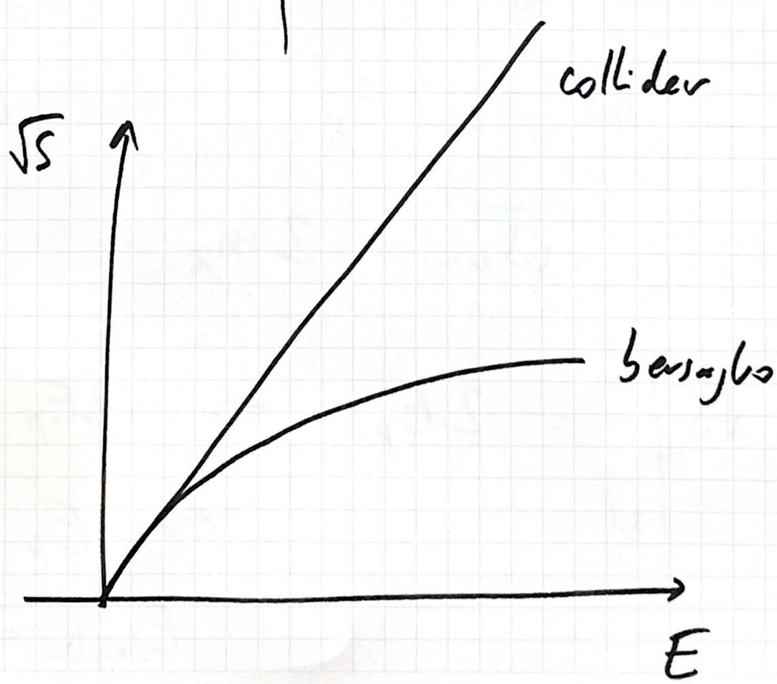
COLLIDER

$(E, \vec{p}) \quad (E, -\vec{p})$

LAB = CdM

$$\sqrt{s} = 2E$$

$$\Leftrightarrow \sqrt{s} \propto E$$



$\boxed{\text{ESERCIZIO}}$ fascio di protoni su bersaglio di RAME

$$p + p \rightarrow p + p + \pi^+ + \pi^-$$

$m(p) = 938$ MeV

$m(\pi) = 139$ MeV

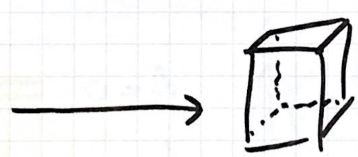
① $E_{soglia} = ?$

$$K_{soglia} = \frac{\left(\sum_f m_f\right)^2 - (m_i + m_b)^2}{2m_b} =$$

$$= \frac{(2m_p + 2m_\pi)^2 - (2m_p)^2}{2m_p} = 0.602\,\text{GeV}$$

$$E_{soglia} = K_{soglia} + m_p = 1.540\ \text{GeV}$$

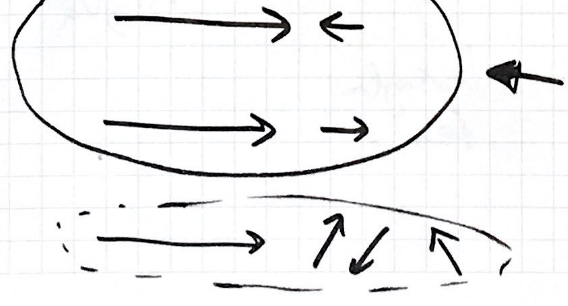
②



nucleo

GAS DI FERMI

$p_F \simeq 240$ MeV



$$\overset{p_1}{\longrightarrow} \; \overset{p_2}{\longleftarrow} \qquad\qquad \overset{p_1}{\longrightarrow} \; \overset{p_2}{\rightarrow}$$

$$p_1 = (E_1 , \vec{p}_1) \qquad p_2 = (E_2 , \vec{p}_2)$$

$$\sqrt{s} = \sqrt{(E_1 + E_2)^2 - |\vec{p}_1 + \vec{p}_2|^2} =$$

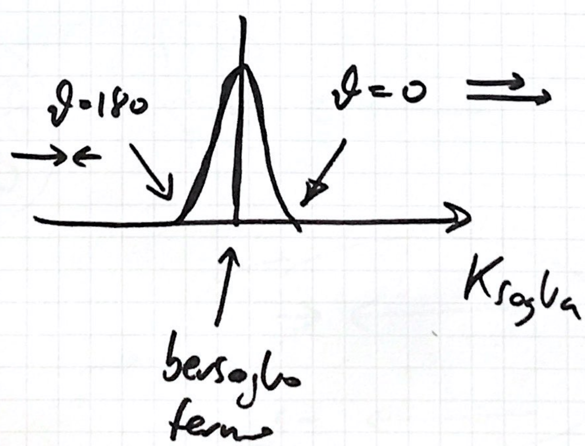
$$= \sqrt{E_1^2 + E_2^2 + 2E_1E_2 - p_1^2 - p_2^2 - 2\vec{p}_1 \cdot \vec{p}_2}$$

$$= \sqrt{m_1^2 + m_2^2 + 2E_1E_2 - 2p_1p_2 \cos\vartheta}$$

$$\sqrt{s}(\vartheta = 180) = \sqrt{m_1^2 + m_2^2 + 2E_1E_2 + 2p_1p_2}$$

$$\rightarrow\leftarrow \qquad\qquad > \sqrt{s}(\vartheta = 0)$$

$$\longrightarrow\leftarrow$$



$$p_i \longrightarrow \longleftarrow p_b$$

$$p_i = (E_i , \vec{p}_i) \qquad p_b = (E_b , \vec{p}_F)$$

$$p_F = 240 \text{ MeV}$$

$$\sqrt{s} = \sqrt{2m_p^2 + 2E_iE_b + 2\,p_ip_F}$$

ALLA SOGLIA le particelle dello s.f. sono FERME NEL CdM

$$\sqrt{s}_{MIN}\Big|_{CdM} = \sum_f m_f = 2m_p + 2m_\pi$$

$$\sqrt{2m_p^2 + 2E_iE_b + 2p_ip_F} = 2m_p + 2m_\pi$$

$$\Leftrightarrow \quad 2m_p^2 + 2E_iE_b + 2p_ip_F = (2m_p + 2m_\pi)^2$$

$$= 4m_p^2 + 4m_\pi^2 + 8m_pm_\pi$$

$$\Leftrightarrow \quad m_p^2 + E_iE_b + p_ip_F = 2m_p^2 + 2m_\pi^2 + 4m_pm_\pi$$

$$\cancel{A = 4.44 \text{ GeV}}$$

$$E_iE_b + \underbrace{\sqrt{E_i^2 - m_i^2}}_{p_i}\, p_F = \underbrace{m_p^2 + 2m_\pi^2 + 4m_pm_\pi}_{A = 1.44 \text{ GeV}}$$

$$E_i E_b - A = \sqrt{E_i^2 - m_p^2}\, p_F$$

al quadrato!

$$E_i^2 E_b^2 + A^2 - 2E_i E_b A = (E_i^2 - m_p^2)\, p_F^2$$

$p_F = 240$ MeV

$$\Rightarrow E_b = \sqrt{m_p^2 + p_F^2} = 968 \text{ MeV}$$

equ. di II grado in E_i

$$\Rightarrow E_i = \frac{AE_b}{m_p^2} \pm \sqrt{\left(\frac{AE_b}{m_p^2}\right)^2 - \frac{A^2}{m_p^2} - p_F^2}$$

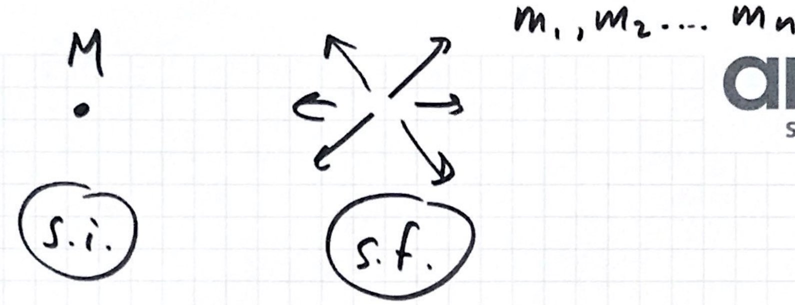
$$= (1.589 \pm 0.312) \text{ GeV}$$

ALLA SOGLIA

$$E_{MIN} = (1.589 - 0.312) \text{ GeV}$$

$\vartheta = 180°$

$\rightarrow \leftarrow$

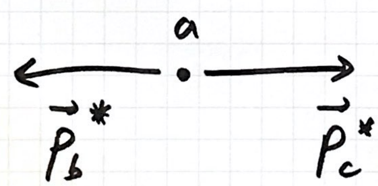


$$\sqrt{s}\,\Big|_{s.i.} = M \qquad (LAB = CdM)$$

$$\sqrt{s}\,\Big|_{s.f.} = \sum_f m_f + K_f^* \geqslant \sum_f m_f$$

$$\Rightarrow \quad M \geqslant \sum_f m_f$$

$$a \rightarrow b + c \qquad \boxed{E^2 = p^2 + m^2}$$



$$\vec{p}_b^{\,*} = -\vec{p}_c^{\,*} = p^*$$

$$\sqrt{s} = M_a = E_b^* + E_c^*$$

$$= \sqrt{(p^*)^2 + m_b^2} + \sqrt{(p^*)^2 + m_c^2}$$

$$M_a - \sqrt{(p^*)^2 + m_b^2} = \sqrt{(p^*)^2 + m_c^2}$$

$$M_a^2 + \cancel{(p^*)^2} + m_b^2 +$$

$$- 2 M_a \sqrt{(p^*)^2 + m_b^2} = \cancel{(p^*)^2} + m_c^2$$

$$M_a^2 + (m_b^2 - m_c^2) = 2M_a \sqrt{(p^*)^2 + m_b^2}$$

al quadrato!

$$\Leftrightarrow M_a^4 + (m_b^2 - m_c^2)^2 + 2M_a^2 (m_b^2 - m_c^2) = 4M_a^2 \left((p^*)^2 + m_b^2\right)$$

$$\Leftrightarrow M_a^4 + (m_b^2 - m_c^2)^2 + 2M_a^2 (m_b^2 - m_c^2) = 4M_a^2 (p^*)^2 + 4M_a^2 m_b^2$$

$$\Leftrightarrow 4M_a^2 (p^*)^2 = M_a^4 + (m_b^2 - m_c^2)^2 - 2M_a^2 (m_b^2 + m_c^2)$$

$$(p^*)^2 = \frac{M_a^4 + (m_b^2 - m_c^2)^2 - 2M_a^2 (m_b^2 + m_c^2)}{4 M_a^2}$$

$$p^* = \sqrt{\frac{M_a^4 + (m_b^2 - m_c^2)^2 - 2M_a^2 (m_b^2 + m_c^2)}{4M_a^2}}$$

$$E_b^* = \sqrt{(p^*)^2 + m_b^2}$$

$$= \sqrt{\frac{M_a^4 + (m_b^2 - m_c^2)^2 - 2M_a^2 (m_b^2 + m_c^2)}{4M_a^2} + m_b^2}$$

$$= \sqrt{\frac{M_a^4 + (m_b^2 - m_c^2)^2 - 2M_a^2(m_b^2 + m_c^2) + 4M_a^2 m_b^2}{4M_a^2}}$$

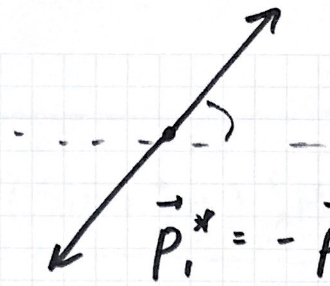
$$= \sqrt{\frac{M_a^4 + (m_b^2 - m_c^2)^2 + 2M_a^2(m_b^2 - m_c^2)}{4M_a^2}}$$

$$= \sqrt{\frac{\left(M_a^2 + (m_b^2 - m_c^2)\right)^2}{4M_a^2}}$$

$$\Rightarrow \quad E_b^* = \frac{M_a^2 + (m_b^2 - m_c^2)}{2M_a}$$

$$E_c^* = \frac{M_a^2 + (m_c^2 - m_b^2)}{2M_a}$$

$$(m_b = m_c) \qquad E_b^* = E_c^* = \frac{M_a}{2}$$



$\vec{p}_1^{\,*} = -\vec{p}_2^{\,*}$

decad. in 3 corpi $\quad a \to b+c+d$

$n \to p\; e^-\; \nu_e$

$n \to p\, e^-$