proiettile.

$Ze \Rightarrow$

$\dfrac{\rho}{A} N_A \cdot Ri\,Bers.$

$$V = \Delta x \, dS = 2\pi \, b\,db \, \Delta x$$

$$\Delta E = -\Delta \overline{E} \left(2\pi \, b\,db \, \Delta x \right) \times n_e$$

$\underbrace{}_{V}$

$\underbrace{\phantom{\Delta \overline{E} (2\pi \, b\,db \, \Delta x) \times n_e}}_{N \text{ urti}}$

Elettroni
Proiettile

CAB

$\cdot e^-$

Rif. solidale con Proiettile

e^-

$t_2 \gg t_0$

$\vec{v} \quad e^- \quad t=0$

$\vec{F_\perp}$

$\vec{F_\parallel} \quad \varepsilon_p \quad \vec{F_\parallel} \quad \vec{F} = \alpha \dfrac{Z_p e^2}{r^2}$
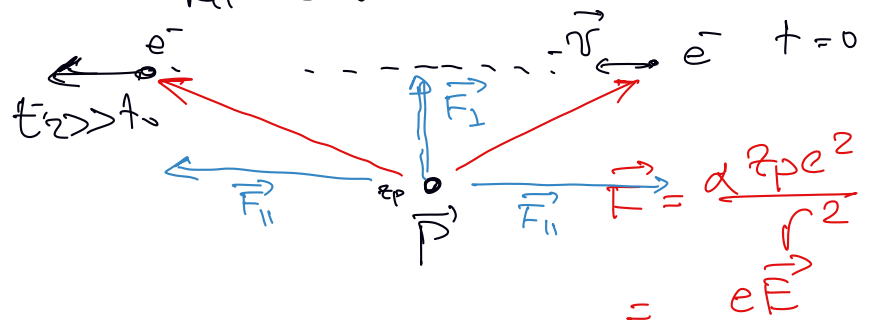
$\vec{P}$

$= e\vec{E}$

$$\vec{P} = m\vec{v}$$

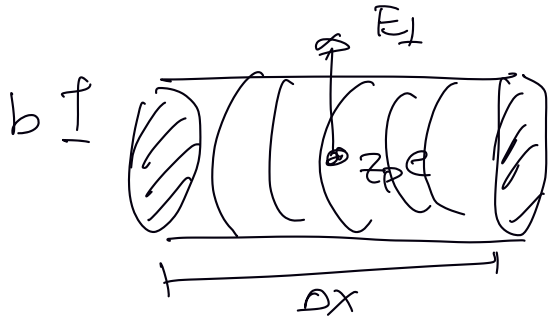$$\Delta \vec{P} = \int \vec{F} \cdot dt$$

$$dx = v \cdot dt \Rightarrow dt = \dfrac{dx}{v}$$

$$\Delta \vec{P} = \int_{-\infty}^{+\infty} \vec{F} \, \dfrac{dx}{v} = \int_{-\infty}^{+\infty} \vec{F_\perp} \dfrac{dx}{v} + \int_{-\infty}^{+\infty} \vec{F_\parallel} \dfrac{dx}{v} = \emptyset$$

$$|\Delta \vec{P}| = \int_{-\infty}^{+\infty} F_\perp \frac{dx}{\upsilon} = \frac{e}{\upsilon} \int_{-\infty}^{+\infty} E_\perp \cdot dx \qquad \vec{F} = e\vec{E}$$

$$\underset{\text{genericO } Zpe}{\downarrow}$$

$$\phi(E) = \int_S \vec{E} \cdot dS = \int_{\text{lat.}} E_\perp \cdot dS + \int_{\text{teppi}} E \cdot dS$$

$$\underset{0}{\overset{\eta}{\downarrow}}$$

$$= \int_{\text{lat}} E_\perp \cdot dx = (2\pi b \cdot \Delta x) E_\perp$$

$$= \frac{Q}{\varepsilon_0} = \frac{Zpe}{\varepsilon_0} \qquad \begin{array}{l}\text{teorema}\\ \text{di Gauss}\end{array}$$

$$E_\perp = \frac{Q}{\varepsilon_0} \; \frac{1}{2\pi b \, \Delta x}$$

$$2\pi b \int E_\perp \cdot dx = \frac{Zpe}{\varepsilon_0}$$

$$\int E_\perp \cdot dx = \frac{Zpe}{\varepsilon_0} \; \frac{1}{2\pi b}$$

$$|\Delta P| = \frac{Zp \cdot e}{\upsilon} \int E_\perp \cdot dx = \frac{(Zp \cdot e^2)}{\varepsilon_0} \; \frac{1}{2\pi b} \; \frac{1}{\upsilon}$$

$$\hookrightarrow \text{ impulso trasferito}$$

Conto classico
di Bohr

$$\overline{\Delta E} = \frac{|\Delta P|^2}{2m}$$

$$\downarrow \quad \asymp 1915$$

$$= \frac{(Zpe^2)^2}{\varepsilon_0^2} \left(\frac{1}{2\pi b}\right)^2 \frac{1}{2m}\left(\frac{1}{\upsilon}\right)^2$$

Conto quantistico

Bethe-Bloch

$$\upsilon = \beta c$$

$$1930$$

$$T = \overline{\Delta E} \; \alpha \; \frac{1}{b^2} \; \frac{1}{\beta^2 c^2}$$

$$T = \overline{\Delta E} = \boxed{\left(\frac{z_p \cdot e^2}{\varepsilon_o} \right)^2 \frac{1}{(2\pi b)^2} \frac{1}{2mc^2 \beta^2}} = \frac{A}{b^2}$$

$$\Delta E = -\overline{\Delta E} \cdot 2\pi \underline{bdb} \, \Delta x$$

$$F = \frac{q^2}{4\pi\varepsilon_o} \frac{1}{r^2}$$

$$T = \frac{A}{b^2} \cdot$$

$$b^2 \, \alpha \, \frac{1}{T} \qquad\qquad 2|bdb| = A \frac{dT}{T^2}$$

$$2\pi |bdb| = d\sigma = A\pi \frac{dT}{T^2} \cdot$$

$$\frac{d\sigma}{dT} \, \alpha \, \frac{1}{T^2} \quad \Rightarrow \quad \text{urti con } T \text{ piccoli}$$
$$\text{avvensovo più spesso.}$$

$$\frac{d\sigma}{dT} = \left(\frac{z e^2}{4\pi\varepsilon_o} \right) \frac{2\pi}{mc^2} \frac{1}{\beta^2} \frac{1}{T^2} \left(1 - \beta^2 \frac{T}{T_{max}} \right)$$

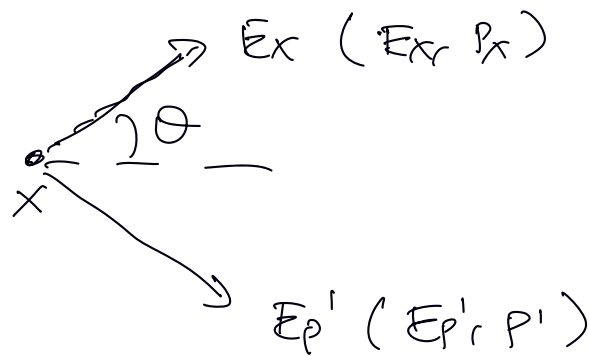sezione d'uto Classica
di Bour

Calcolo
quantistico
Beth - Bloch

$\rightarrow$ en. media Dusc Deriouizza zione nel fretto Δx

$$\Delta E = -ne \, \int \cdot \underbrace{\frac{A\pi}{T^{x}} dT}_{\overline{\Delta E}} \, \Delta x$$
$$\Bigg(\underbrace{\phantom{\frac{A\pi}{T^x}dT}}_{2\pi |bdb|}$$

$$\Rightarrow \quad \frac{\Delta E}{\Delta x} =: \frac{dE}{dx} = -ne \, A\pi \, \frac{dT}{T}$$

Integrare fra $(T_{min} \to T_{max})$

(E_p, P)
M_p

$E_x (E_x, P_x)$

2θ

$E_p' (E_p', P')$

T_{max}: si ottiene per $\theta = 0$

1) $|\underline{P}|^2$ invariante

2) E si conserva

3) $\vec{P}$ si conserva

4) $E_x = M_x + K_x = M + T$

$$T_{max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma (m_e / m_p) + (m_e / m_p)^2}$$

proiettile

$E_p = \gamma m$

$P = \gamma \beta mc$
$\quad = \gamma m v$

β, γ: del proiettile

$M_x \equiv m_e$

2 casi limite.

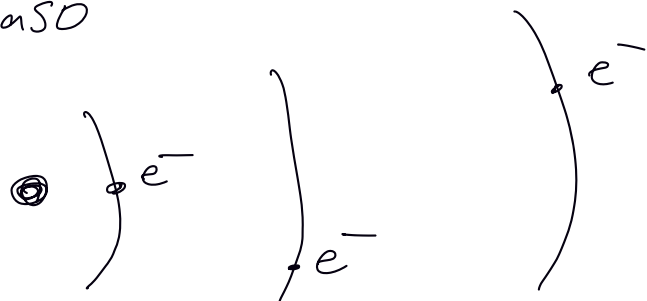1) $m_p \gg m_e$: proiettile molto massivo

$$T_{max} = 2 m_e c^2 \beta^2 \gamma^2$$

2) $m_p \ll m_e$: $T_{max} = m_p c^2 \beta^2 \gamma$

non il nostro caso

e^-
e^-
e^-

T_{min}:

$\bar{I} \propto 10 eV \cdot Z$

$$T_{min} \propto \sqrt[n]{\prod F_n} = \sqrt[n]{\prod_{i=1}^{n} t_i c w_n}$$

$$X_1 = 10 \qquad X_2 = 1 \qquad \bar{X} = \frac{11}{2} = 5.5$$

$$\bar{X}_{geom} = \sqrt{10 \cdot 1} \simeq 3.3$$

$$X_1 \simeq 0.5 \quad X_2 = 2 \qquad X_3 = 5 \quad -- \quad X_N = 100$$

$$T_{min} \simeq \bar{I} \quad \rightsquigarrow \text{en. media ionizzazione.}$$

$$\frac{\Delta E}{\Delta x} =: \frac{dE}{dx} = - n_e \, A\pi \int_{T_{min}}^{T_{max}} \frac{dT}{T} \implies \left\langle \frac{dE}{dx} \right\rangle$$

$$\propto \ln \frac{T_{max}}{T_{min}}$$

$$\frac{dE}{dx} = -\,\mathbb{C}\, \frac{z_p^2}{\beta^2} \underbrace{\rho \, N_A \left(\frac{Z}{A}\right)}_{n_e} \overbrace{\ln \frac{m_e c^2 \beta^2 \gamma^2}{\bar{I}}}$$

materiale

$$\longrightarrow T_{max}$$
$$\longrightarrow T_{min}.$$

$$\underbrace{\qquad}_{\overline{\Delta E}}$$

adimensionali

$$[\mathbb{C}] \, [\rho] = \frac{MeV}{cm}. \qquad \mathbb{C} = 0.3$$

$$\frac{MeV}{cm} \qquad \frac{cm^2}{g} MeV \qquad \frac{g}{cm^3} = \frac{MeV}{cm} \qquad \frac{MeV \, cm^2}{g}$$

A

carta di Segrè
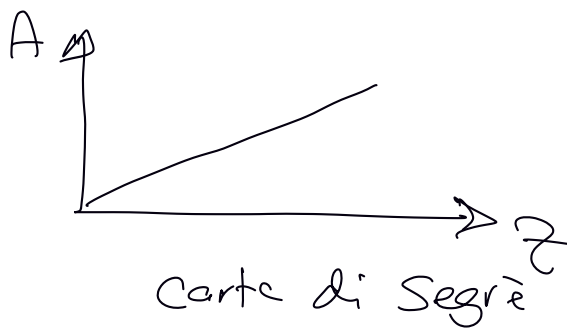
$$\frac{Z}{A} \simeq \frac{1}{2}$$

$$-\frac{1}{\rho} \frac{dE}{dx} = \mathbb{C} \, N_A \left(\frac{Z}{A}\right)^{\frac{1}{2}} \frac{z_p^2}{\beta^2} \ln \frac{m_e c^2 \beta^2 \gamma^2}{I}$$

$$I \propto Z$$

$$\ln \frac{1}{2}$$

$$\ln 2 = 0.7$$
$$\ln 60 = 4$$

$\not{I}$ in pratica indipendente dal materiale
$\Rightarrow$ solo funzione del proiettile.

Formula di Bethe Bloch

$$-\frac{1}{\rho} \frac{dE}{dx} = \mathbb{C} \, N_A \left(\frac{Z}{A}\right) \frac{z_p^2}{\beta^2} \frac{1}{2} \left[\ln \frac{2 m_e c^2 \beta^2 \gamma^2 \, T_{max}}{I^2} - \beta^2 - \frac{\delta}{2} \right]$$

$$\overset{2 m_e c^2 \beta^2 \gamma^2}{\diagdown}$$

risolte relativistica.

correzione dovuta
alla polarizzazione del mezzo.
funzione del mezzo.
$\Rightarrow$ misurato empiricamente negli esp.

$$\frac{\boxed{+}}{\boxed{-}} \quad \frac{\boxed{+}}{\boxed{-}} \quad \frac{\boxed{+}}{\boxed{-}}$$

⊙

$\varepsilon_p > 0$

$$\boxed{\frac{-}{+}} \quad \boxed{\frac{-}{+}} \quad \boxed{\frac{-}{+}}$$
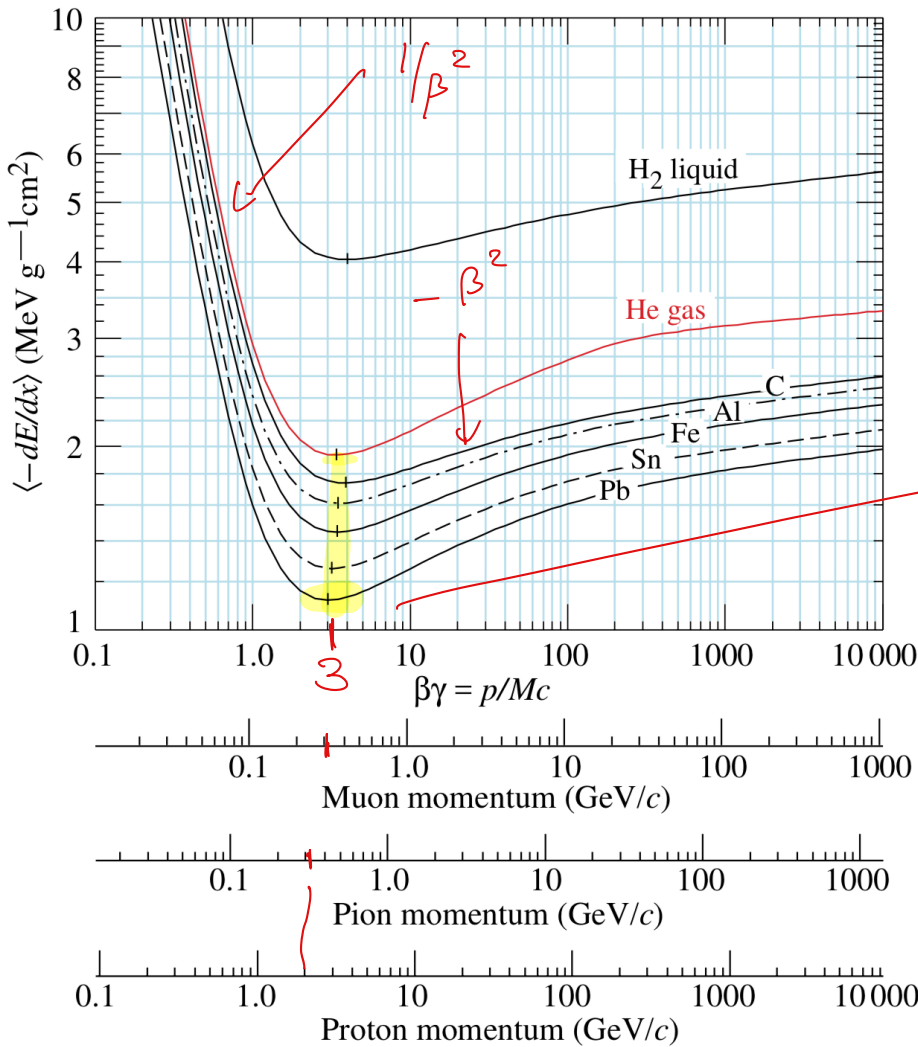
Parcan import cufe $\quad \beta\gamma = \dfrac{P}{m}$

$$\beta = \frac{\beta\gamma}{\sqrt{1 + (\beta\gamma)^2}}$$

$-\dfrac{1}{\rho}\dfrac{dE}{dx}$

$\beta\gamma = \dfrac{P}{m}$

Bethe–Bloch

$1/\beta^2$

$-\beta^2$

minimo di ioniZZ.

logaritmica

$m_\mu = 106 \text{ MeV}$

$m_\pi = 140 \text{ MeV}$

$m_p \simeq 1000 \text{ MeV}$

$$-\frac{1}{\rho}\frac{dE}{dx}\bigg|_{min} \simeq 1{-}2 \ \frac{MeV}{cm} \cdot \frac{1}{\rho}$$

$$\rho_{H2O} = 1 \ g/cm^3 \implies 1 \ \frac{MeV}{cm}$$

Upper figure:

$\langle -dE/dx \rangle_{min}$ (MeV g^{-1}cm^2) vs Z

H$_2$ gas: 4.10
H$_2$ liquid: 3.97

$2.35 - 0.28 \ln(Z)$

+ Solids
○ Gases

H   He   Li   Be   B   C   N   O   Ne   Fe   Sn   U

Lower figure:

dE/dx [mip] vs p [GeV/c]

π^+ K^+ p   d e^+

10^2
10
1

Handwritten annotations:

1.5 MeV

100 MeV.

$\beta\gamma = \frac{p}{m}$.

$p = \beta\gamma m$

Δy #   # Δx

$\beta\gamma\big|_{min} = 3$

$M_e = 0.5 \ MeV \Rightarrow P_{min}^e = 3 \cdot M_e = 1.5 \ MeV$

μ⁺ on Cu

$\frac{dE}{dx}\Big|$ Bethe-Bloch.

$\frac{dE}{dx}$ misurato.
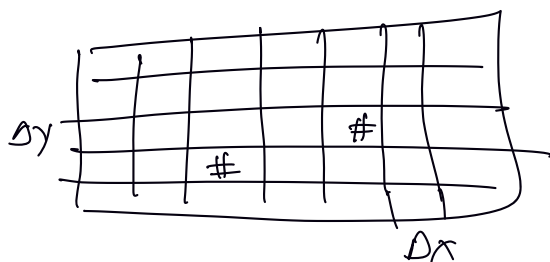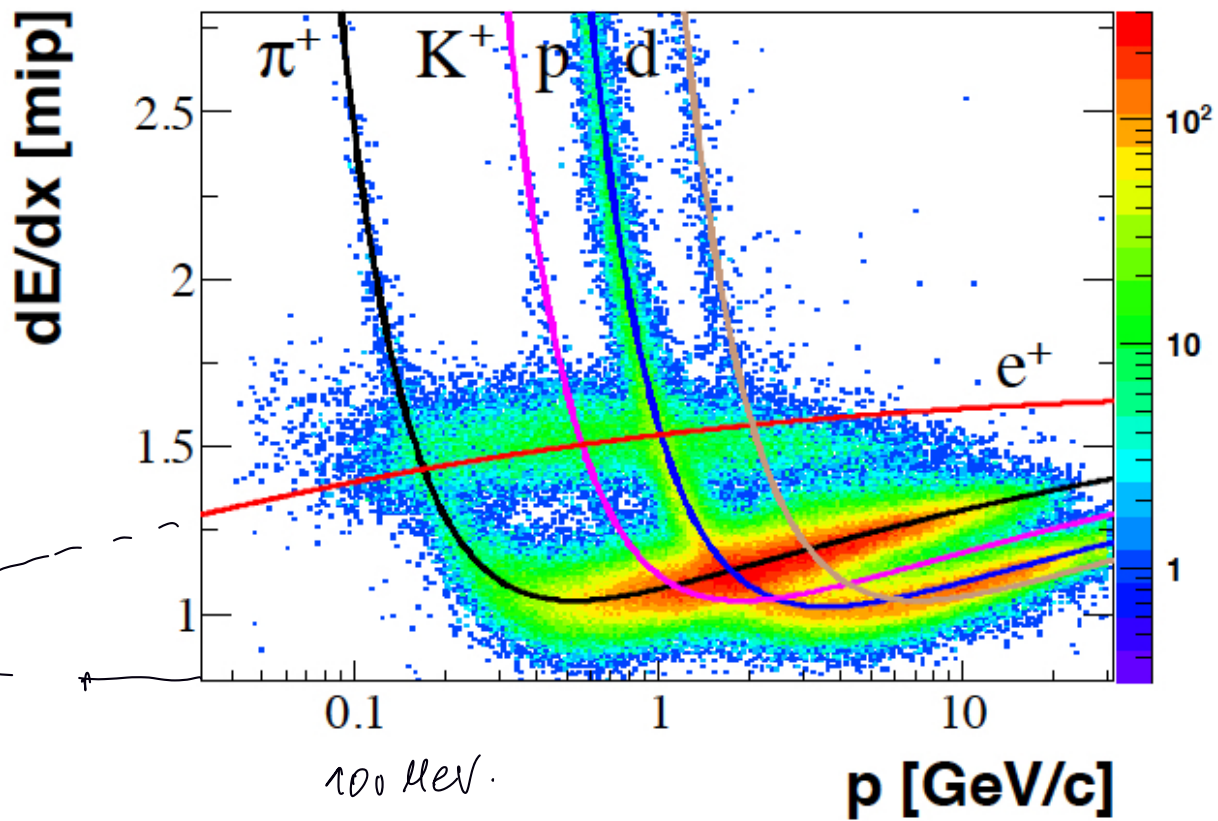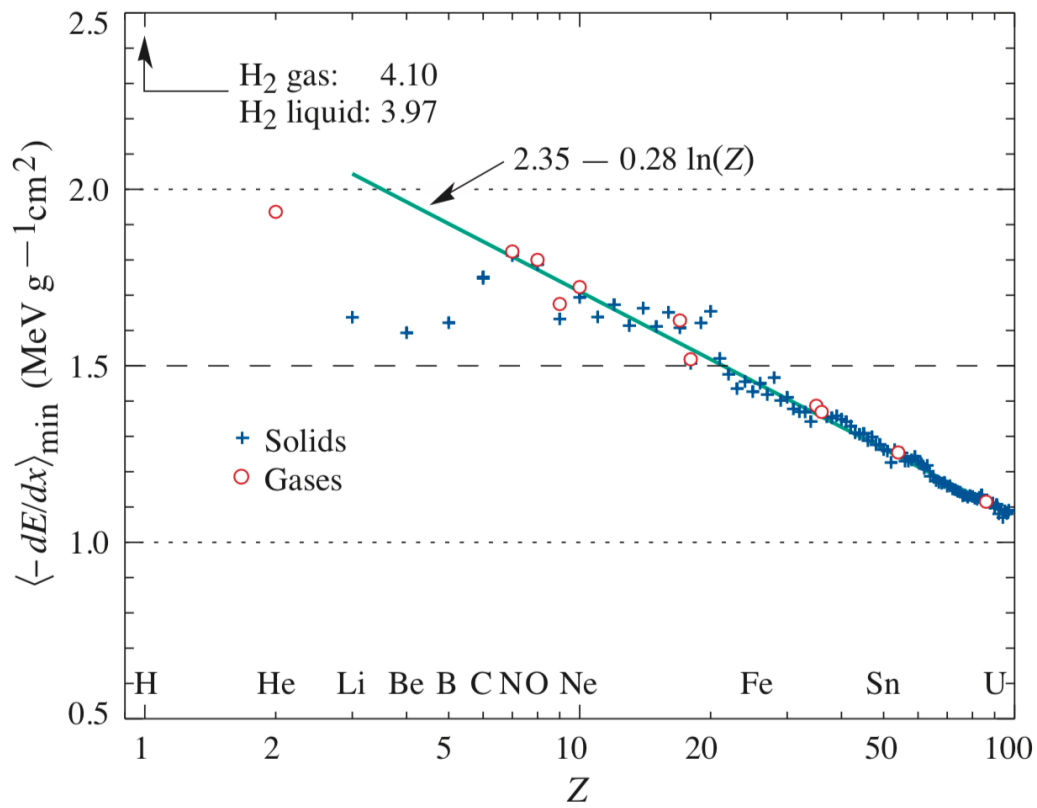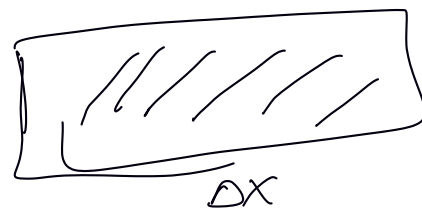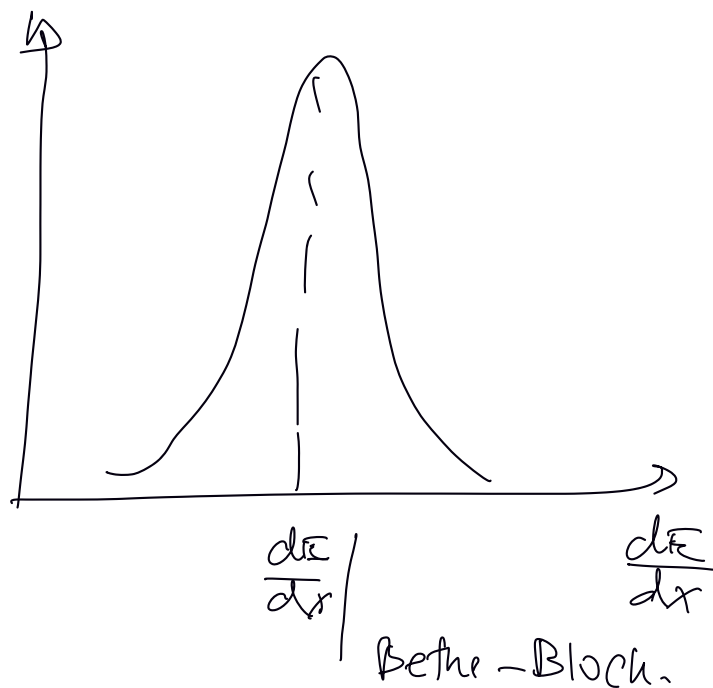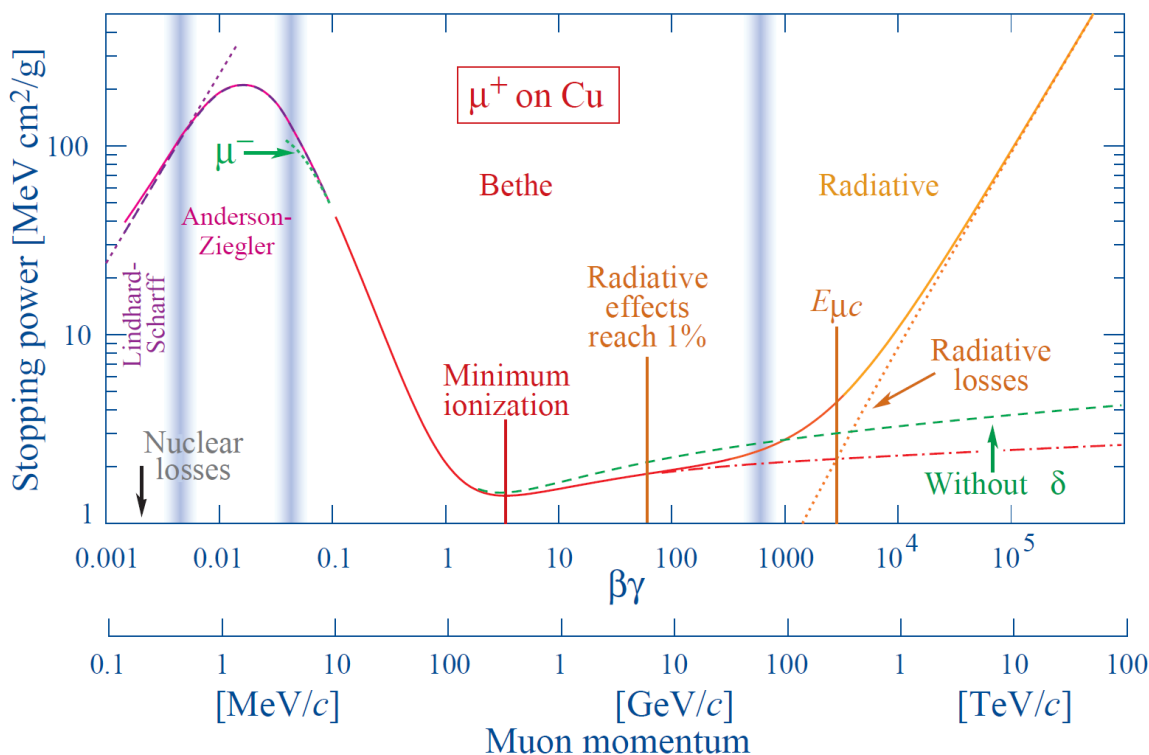
teorema del limite centrale.

Landau

$\langle \frac{dE}{dx} \rangle$