$\gamma \rightarrow e^+ e^-$ IMPOSSIBILE (NEL VUOTO)

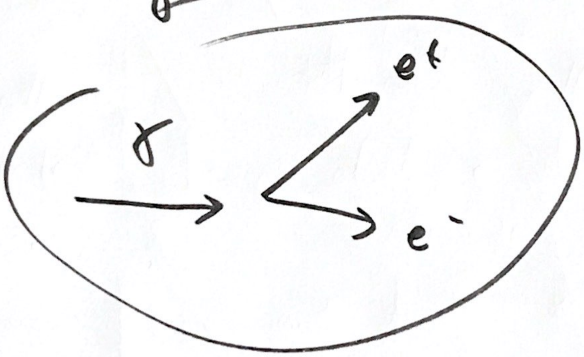
$$\sqrt{s}\Big|_{s.i.} = \sqrt{E_\gamma^2 - p_\gamma^2} = m_\gamma = 0$$

$$\sqrt{s}\Big|_{s.f.,\,CdM} = E_+^* + E_-^* \geq m_e + m_e = 2m_e$$

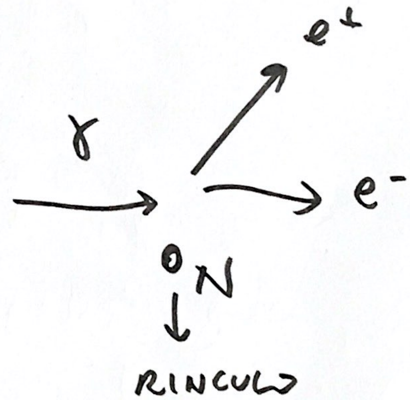
$m_e = 0.511$ MeV

$$\sqrt{s}_{s.i.} < \sqrt{s}_{s.f.}$$

$\cancel{\gamma + N}$



NO



$$\gamma + N \rightarrow e^+ e^- + N'$$

$\pi^0 \to \gamma\gamma \qquad \underbrace{\Delta\vartheta(\gamma,\gamma)}_{\alpha} \neq 0 \quad \Rightarrow \exists\, \alpha_{MIN} > 0$

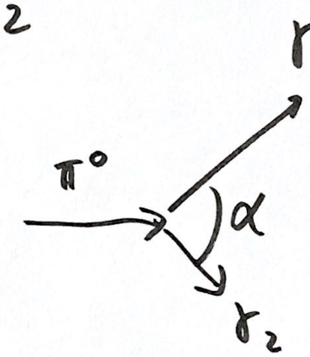
$$\sqrt{s} = m_{\pi^0} = \sqrt{2E_1E_2(1-\cos\alpha)}$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\Leftrightarrow \quad 1 - \cos 2x = 2\sin^2 x$$

$$\Rightarrow \quad m_{\pi^0} = \sqrt{4E_1E_2 \sin^2\frac{\alpha}{2}}$$

$$= 2\sin\frac{\alpha}{2}\sqrt{E_1E_2}$$



$$\Rightarrow \boxed{\sin\frac{\alpha}{2} = \frac{m_{\pi^0}}{2\sqrt{E_1E_2}}}$$

MIN STA NEL MAX di $\sqrt{E_1E_2}$

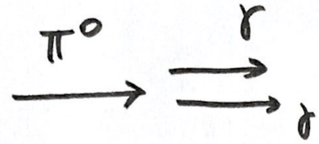
MAX di E_1E_2

$\pi^0: (E_\pi, \vec{p}_\pi)$

$\gamma_1: (E_1, \vec{p}_1) \qquad E_\pi = E_1 + E_2$

$\gamma_2: (E_2, \vec{p}_2) \qquad \Rightarrow E_2 = E_\pi - E_1$

$\pi^0 \to \gamma\gamma$ $\qquad m_{\pi^0} = 135$ MeV



$\Delta\vartheta(\gamma,\gamma) = 0$

$\vartheta_1 = \vartheta_2 = 0$

IMPOSSIBILE

$$\sqrt{s}\Big|_{s.i.} = \sqrt{E_\pi^2 - p_\pi^2} = m_{\pi^0}$$

$$\sqrt{s}\Big|_{s.f.} = \sqrt{(E_1+E_2)^2 - |\vec{p}_1+\vec{p}_2|^2} =$$

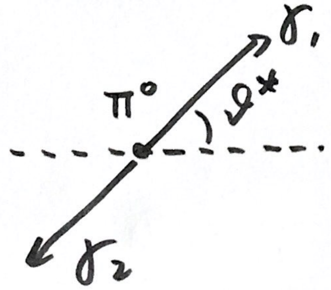
$$= \sqrt{E_1^2 + E_2^2 + 2E_1E_2 - p_1^2 - p_2^2 - 2\vec{p}_1\cdot\vec{p}_2}$$

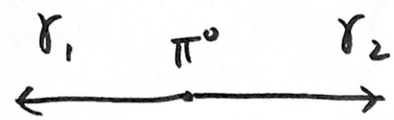
$$= \sqrt{2E_1E_2 - 2p_1p_2\cos\vartheta}$$

$m=0 \Rightarrow E = |\vec{p}|$

$$= \sqrt{2E_1E_2(1-\cos\vartheta)} \xrightarrow[\vartheta=0]{} = 0$$

CdM





$(\vartheta^* = 0)$

$\beta_{cm} \leftrightarrow \beta^* = 1$

FLIPPA SE $\beta_{cm} > \beta^*$

$$\frac{d}{dE_1}(E_1E_2) = \frac{d}{dE_1}\left[E_1(E_\pi - E_1)\right] =$$

$$= E_\pi - 2E_1 \overset{!}{=} 0 \qquad \text{MAX}$$

$$\Rightarrow E_1 = \frac{E_\pi}{2}$$

$$E_2 = E_\pi - E_1 = \frac{E_\pi}{2}$$

ANGOLO MIN $\Rightarrow$ equipartizione energetica

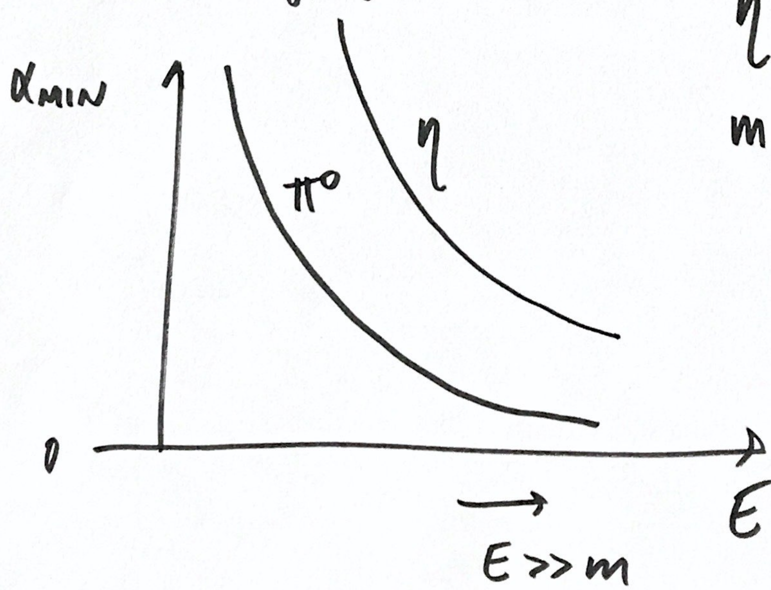
$$\sin\frac{\alpha_{MIN}}{2} = \frac{m_{\pi^0}}{2\sqrt{\frac{E_\pi^2}{4}}} = \frac{m_{\pi^0}}{E_\pi} \quad \left(=\frac{1}{\gamma_\pi}\right)$$

per $\alpha \ll 1$ $\quad \sin\alpha \sim \alpha$

$\eta \to \gamma\gamma$

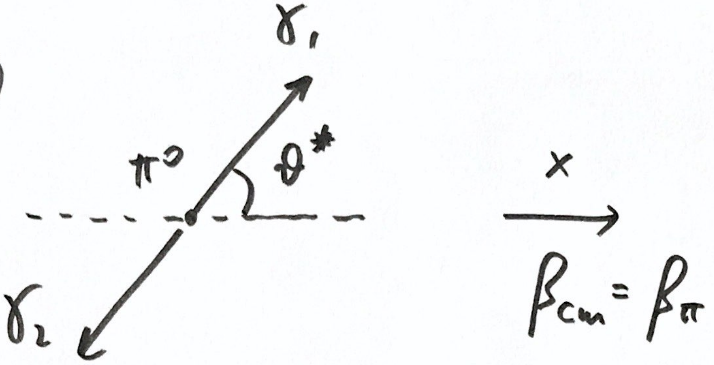
$m(\eta) = 548$ MeV

$> m_{\pi^0}$



DISTRIB. ENERGETICA DEI FOTONI NEL LAB

CM



$\pi^0: (m_{\pi^0}, \vec{0})$

$\gamma_1: (E_1^*, \vec{p}_1^*)$

$\gamma_2: (E_2^*, \vec{p}_2^*)$

$m_\gamma = 0$

$\Rightarrow E = |\vec{p}|$

$$|\vec{p}_1^*| = |\vec{p}_2^*|$$

$$m_{\pi^0} = E_1^* + E_2^* = 2E^*$$

$$E^* = \frac{m_{\pi^0}}{2} = p^*$$

$$E_1 = \gamma_\pi (E_1^* + \beta_\pi p_{1x}^*) = \gamma_\pi (E^* + \beta_\pi p^* \cos\vartheta^*)$$

$$= \gamma_\pi E^* (1 + \beta_\pi \cos\vartheta^*)$$

$$= \gamma_\pi \frac{m_{\pi^0}}{2} (1 + \beta_\pi \cos\vartheta^*)$$

$$\beta_\pi = \frac{p_\pi}{E_\pi} \qquad \gamma_\pi = \frac{E_\pi}{m_{\pi^0}}$$

$$\Rightarrow E_1 = \frac{E_\pi}{\cancel{m_{\pi^0}}} \frac{\cancel{m_{\pi^0}}}{2} \left(1 + \frac{p_\pi}{E_\pi} \cos\vartheta^* \right)$$

$$\Rightarrow \boxed{E_1 = \frac{1}{2} \left(E_\pi + p_\pi \cos\vartheta^* \right)}$$

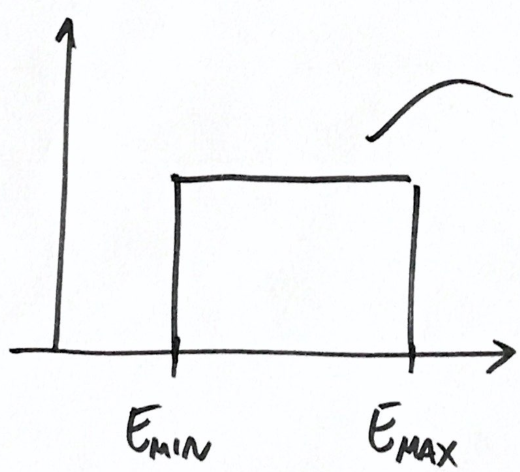
per γ_2 $\quad \vartheta^* \longrightarrow \pi + \vartheta^*$

$$\boxed{E_2 = \frac{1}{2} \left(E_\pi - p_\pi \cos\vartheta^* \right)}$$

$$E_{MIN} = \frac{1}{2} \left(E_\pi - p_\pi \right)$$

$$E_{MAX} = \frac{1}{2} \left(E_\pi + p_\pi \right)$$

PDF
dens. di
prob.



tutti i valori fra MIN e MAX sono equiprobabili

π^0 ha spin = 0

$\Rightarrow \nexists$ direzione preferenziale

$\Rightarrow$ il decadimento e' isotropo

$$\frac{df}{d\Omega^*} = \text{cost.} = \frac{1}{4\pi}$$

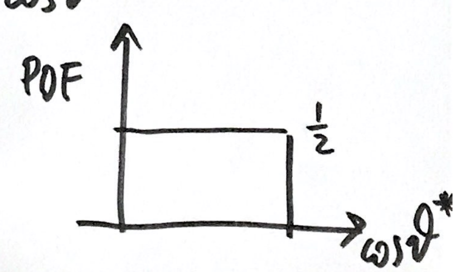
$$d\Omega^* = \sin\vartheta^* \, d\vartheta^* \, d\varphi^*$$

$$\Rightarrow \frac{df}{\sin\vartheta^* \, d\vartheta^* \, d\varphi^*} = \frac{1}{4\pi}$$

integro in $\varphi^* \in [0, 2\pi]$

$$\Rightarrow \frac{df}{2\pi \sin\vartheta^* \, d\vartheta^*} = \frac{1}{4\pi}$$

$$\Leftrightarrow \frac{df}{\underbrace{\sin\vartheta^* \, d\vartheta^*}_{d(\cos\vartheta^*)}} = \frac{1}{2} = \frac{df}{d\cos\vartheta^*}$$



IL DECAD. IN DUE CORPI DI UNA P.LLA

DI SPIN = 0 NEL CdM

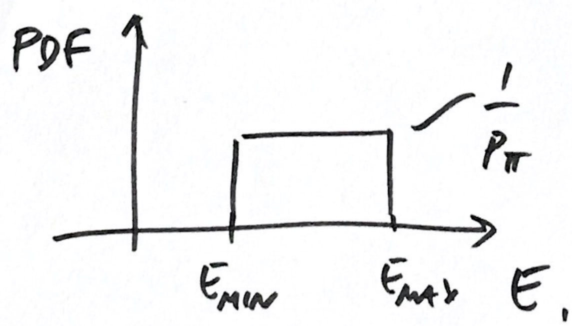
e' equiprobabile in $\cos\vartheta^*$

$$\frac{df}{dE_1} = \underbrace{\frac{df}{d\cos\vartheta^*}}_{=\frac{1}{2}} \frac{d\cos\vartheta^*}{dE_1} = \frac{1}{2} \frac{d\cos\vartheta^*}{dE_1}$$

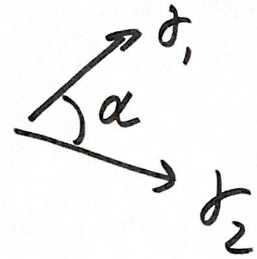
$$\frac{dE_1}{d\cos\vartheta^*} = \frac{d}{d\cos\vartheta^*}\left[\frac{1}{2}\left(E_\pi + p_\pi \cos\vartheta^* \ldots\right)\right]$$

$$= \frac{1}{2} p_\pi$$

$$\frac{df}{dE_1} = \cancel{\frac{1}{2}\frac{p_\pi}{2} = \frac{p_\pi}{4}}\ \frac{1}{2}\frac{2}{p_\pi} = \frac{1}{p_\pi} = \text{cost.}$$



$$\frac{df}{d\alpha} = ?$$



$$\sin\frac{\alpha}{2} = \frac{m_{\pi^0}}{2\sqrt{E_1 E_2}} = \frac{m_{\pi_0}}{2\sqrt{E_1(E_\pi - E_1)}}$$

quadro!

$$\sin^2\frac{\alpha}{2} = \frac{m_{\pi^0}^2}{4E_1(E_\pi - E_1)}$$

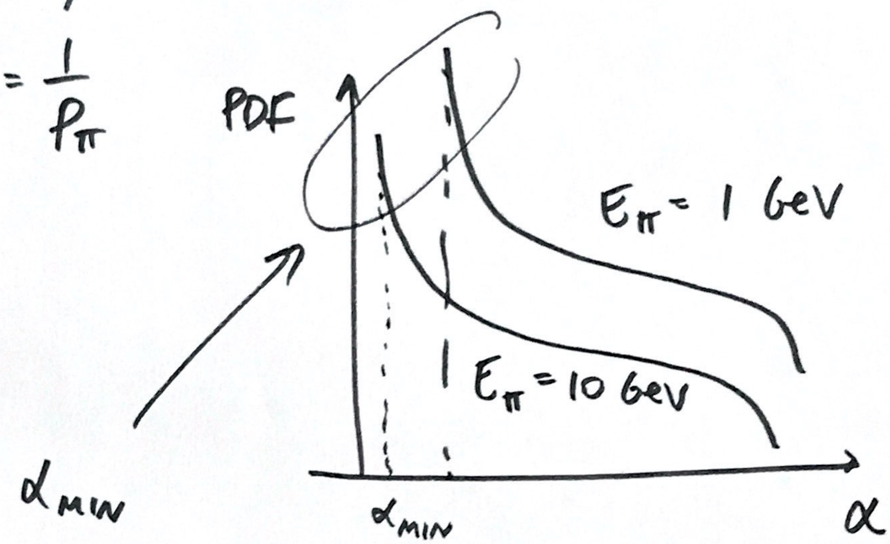
$$4E_1^2 - 4E_\pi E_1 + \frac{m_{\pi^0}^2}{\sin^2\left(\frac{\alpha}{2}\right)} = 0$$

$$\Rightarrow E_1 = \frac{E_\pi \pm \sqrt{E_\pi^2 - m_{\pi^0}^2 / \sin^2\left(\frac{\alpha}{2}\right)}}{2}$$

$$\frac{dE_1}{d\alpha} = \frac{m_{\pi^0}^2 \cos\left(\frac{\alpha}{2}\right)}{4\sin^2\left(\frac{\alpha}{2}\right)\cdot\sqrt{E_\pi^2 \sin^2\left(\frac{\alpha}{2}\right) - m_{\pi^0}^2}}$$

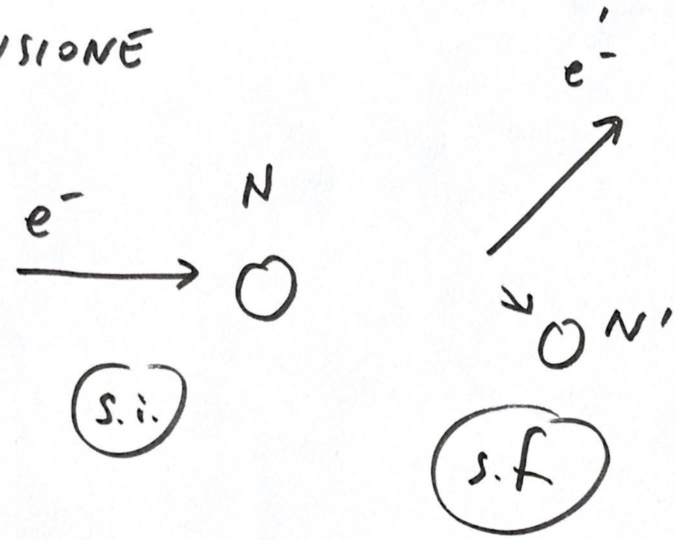
$$\frac{df}{d\alpha} = \frac{df}{dE_1}\frac{dE_1}{d\alpha} = \frac{m_{\pi^0}^2 \cos\left(\frac{\alpha}{2}\right)}{4 p_\pi E_\pi \sin^2(\alpha/2) \sqrt{\sin^2\left(\frac{\alpha}{2}\right) - \frac{m_{\pi^0}^2}{E_\pi^2}}}$$

$$\frac{df}{dE_1} = \frac{1}{p_\pi}$$



SCATTERING ELASTICO

DIFFUSIONE



e: $p = (E, \vec{p})$ $\qquad p' = (E', \vec{p}\,')$

N: $P = (M, \vec{0})$ $\qquad P' = (E_P', \vec{P})$

conserv. 4-impulso : $p + P = p' + P'$

$$p^2 = m_e^2 = p'^2$$

$$P^2 = M^2 = P'^2$$

$$(p+P)^2 = (p'+P')^2$$

$$p^2 + P^2 + 2p\cdot P = p'^2 + P'^2 + 2p'\cdot P'$$

$$\cancel{m_e^2} + \cancel{M^2} + 2p\cdot P = \cancel{m_e^2} + \cancel{M^2} + 2p'\cdot P'$$

$$\Rightarrow p\cdot P = p'\cdot P'$$

$$p+P = p'+P' \Rightarrow P' = p+P-p'$$

$$\Rightarrow \quad p\cdot P = p'\cdot(p+P-p')$$

$$\Leftrightarrow \quad p\cdot P = p'\cdot p + p'\cdot P \underbrace{- p'^2}_{= m_e^2}$$

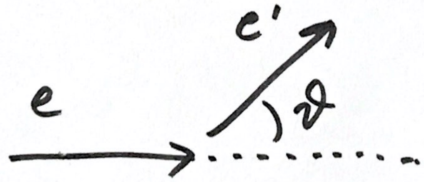
$$p = (E, \vec{p}), \quad p' = (E', \vec{p}\,') \Rightarrow p'\cdot p = E\cdot E' - \vec{p}\cdot\vec{p}\,'$$

$$P = (M, \vec{0}) \Rightarrow p'\cdot P = E'\cdot M - 0$$

$$p\cdot P = E\cdot M - 0$$

$$\Rightarrow \quad EM = E'M + E'E - \vec{p}\cdot\vec{p}\,' - m_e^2$$

$$\rightarrow \quad EM = E'M + E'E - pp'\cos\vartheta - m_e^2$$



$$E, E' \gg m_e = 0.511 \text{ MeV}$$

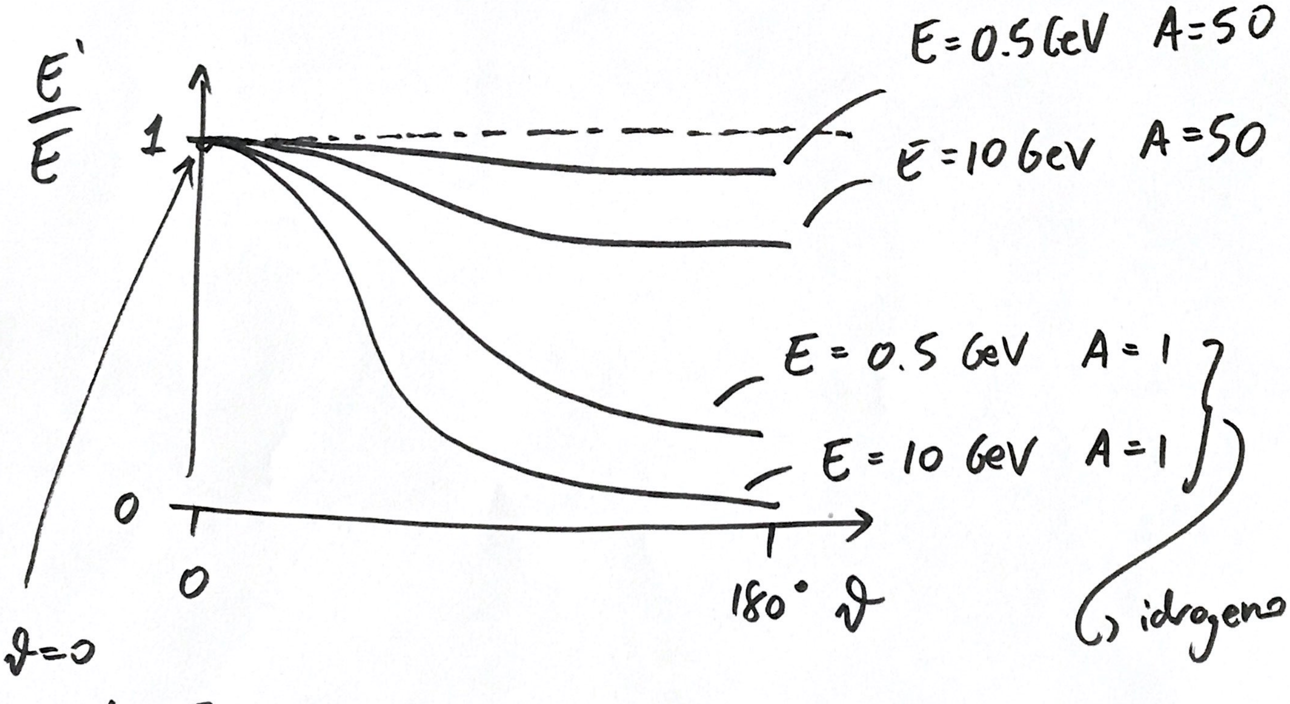
$$\Rightarrow \begin{cases} E \sim p \\ E' \sim p' \end{cases}$$

$$EM \simeq E'M + E'E - E'E\cos\vartheta \qquad \cancel{(-m_e^2)}$$

$$E'\left[E(1-\cos\vartheta) + M\right] = EM$$

$$\frac{E'}{E} = \frac{1}{1 + \frac{E}{M}(1-\cos\vartheta)}$$

$$E' = \frac{E}{1 + \frac{E}{M}(1-\cos\vartheta)}$$



$E' = E$

2 ASSUNZIONI

(1) una p.lla incidente e una **ferma**

(2) energie >> m p.lla incidente

$$\xrightarrow{\ \gamma\ } e^-$$

$m_\gamma = 0$

(2) sempre vera

$E \gg m_\gamma$

(1) elettrone in orbita $\vec{p}_e \ll \vec{p}_\gamma$

$\vec{p}_e \sim \vec{0}$

$$\frac{E'_\gamma}{E_\gamma} = \frac{1}{1 + \frac{E_\gamma}{m_e}(1 - \cos\vartheta)}$$

SCATTERING COMPTON