$[\rho] = \frac{[M]}{[V]}$ $\quad g/cm^3$

$\frac{\rho}{A}$ $\quad \frac{\#mol}{cm^3} \Rightarrow [A] = g/mole$

$\frac{\rho}{A} : \frac{\cancel{g}}{cm^3} \frac{mole}{\cancel{g}}$

N_A : # atomi / mole

$\frac{\rho}{A} N_A \Rightarrow \frac{\#atomi}{cm^3}$

Decadimento β : $\quad {}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e$

	$n \to$	p	e^-	$\bar{\nu}_e$
Q	0	+1	−1	$\emptyset$

$n \to p\, e^-$ $\quad$ sperimentalmente vedo e^-

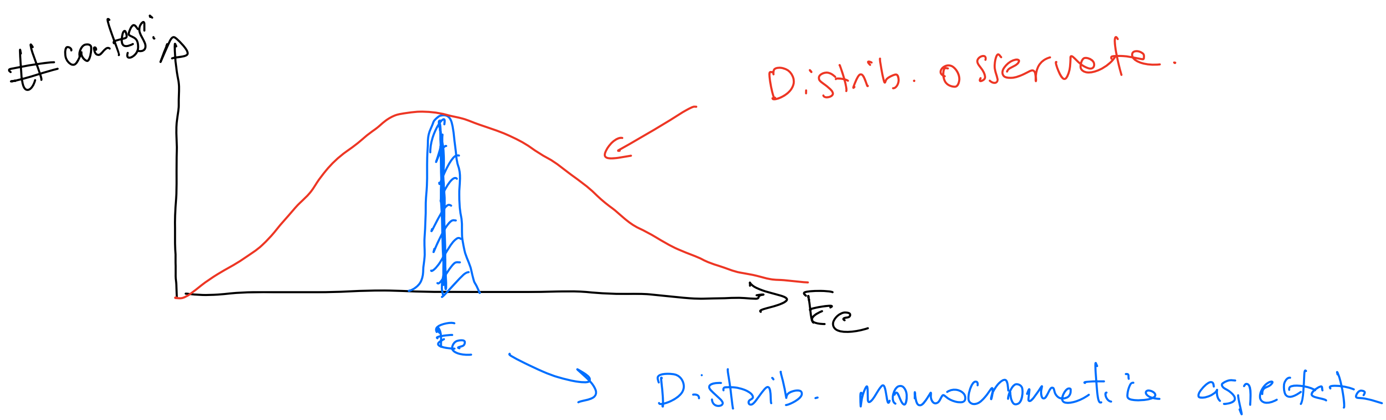
Decadim a due corpi: $\quad p \longleftarrow \overset{n}{\circ} \longrightarrow e^-$

$\vec{\emptyset} = \vec{P}_p + \vec{P}_{e^-} \Rightarrow \vec{P}_p = -\vec{P}_e = \vec{P}^*$

$m_n = E_p + E_{e^-}$ $\quad$ p^* fissato da m_n, m_p, m_e

$= \sqrt{m_p^2 + p^{*2}} + \sqrt{m_e^2 + p^{*2}}$

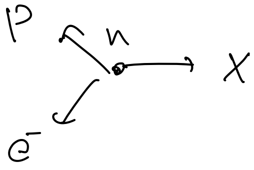
$\Rightarrow E_e = \sqrt{m_e^2 + p^{*2}}$ sempre lo stesso valore.

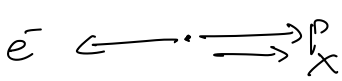


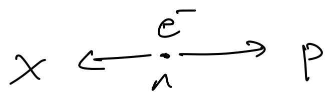
Pauli @ 1930 ipotesi di una nuova particella.

$\Rightarrow \quad n \to p + e^- + X$

decadim a tre corpi $\Rightarrow E_e$ non monocromatica







Carica elettrica $Q_X = 0$!

$E_e = m_e + (T_e = 0)$
elettrone fermo.

$$Q \doteq m_n - m_p - m_e - m_X = 939.6 - 938.3 - 0.5 - m_X \text{ MeV}$$

$$= 1.3 - 0.5 - m_X \text{ MeV} = 0.8 - m_X \text{ MeV}.$$

$$Q \geq 0 \Rightarrow m_X \leq 0.8 \text{ MeV}$$

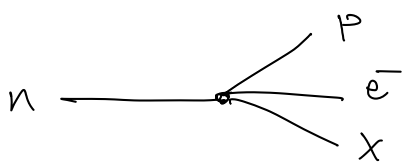
$$n \to p + e^- + \gamma$$

$$J \quad \tfrac{1}{2} \qquad \tfrac{1}{2} \quad \tfrac{1}{2} \quad 1 \qquad J_{in} \neq J_{fin}$$

$$J_X = \tfrac{1}{2}$$

$\Rightarrow$ X: fermione, $S = 1/2$, $Q_e = 0$, $m < 0.8$ MeV.

Fermi: neutrino



$\Gamma(n \to p e^- X)$ = prob. di transizione / unità di tempo.

$$[\Gamma] = [T]^{-1} = [E].$$

$\Gamma = \frac{1}{\tau}$ τ: vita media del neutrone

Regola d'oro di Fermi

$$\Gamma(n \to p e^- X) = 2\pi |M_{fi}|^2 \rho(\text{spazio delle fasi}) \Big|_{E_f = E_i}$$

$$E_f = E_e + E_p + E_X = E_i$$

nel rif. solidale con n: $E_i = m_n = E_p + E_e + E_X$

$$= m_p + T_p + E_e + E_X.$$

$Q = 0.8$ MeV $(\ll m_p, m_n) \Rightarrow T_p \simeq \emptyset$ trascurabile

$$E_T = m_n - m_p = E_e + E_\nu$$

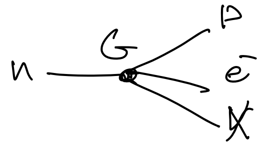
$$X \to Y + e^- + X$$

$$E_T = m_X - m_Y$$

in generale.

$$\mathcal{M}_{fi} = -i \int d^3r \, \psi_f^* H_I \psi_i$$

Fermi: $H_I = G$ costante.

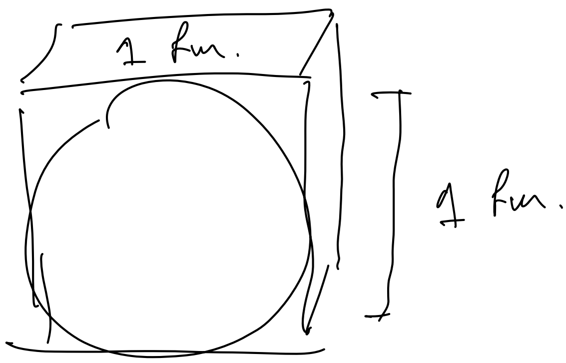


onda piana per particelle libere normalizzata al volume

$$\psi(\vec r) = \frac{1}{\sqrt{V}} e^{+i\vec p \cdot \vec r}$$

particelle libere con impulso $\vec p$

$$\int d^3r \, \psi^* \psi = 1$$



volume nucleo.

$$\mathcal{M}_{fi} = -i \int d^3r \, \psi_p^* \psi_{e^-}^* \psi_X^* G \psi_n$$

$$n \to p + e^- + X$$

$$\vec P_p + \vec P_e + \vec P_X = \vec 0$$ nel rif. solidale con n.

$$|\vec P_p| \simeq 0 \qquad \vec P_e \equiv \vec p \qquad \vec P_\nu \equiv \vec q$$

$$\vec P_p = -(\vec p + \vec q)$$

$$\psi_e^* = \frac{e^{-i\vec p \cdot \vec r}}{\sqrt{V}} \qquad \psi_X^* = \frac{1}{\sqrt{V}} e^{-i\vec q \cdot \vec r}$$

$$\mathcal{M}_{fi} = -i G \int d^3r \, \underbrace{\psi_p^* \psi_n}_{\text{Funzione d'onda adroni}} \frac{1}{\sqrt{V}} \frac{1}{\sqrt{V}} e^{-i(\vec p + \vec q)\cdot \vec r}$$

$$|\vec p + \vec q| \simeq Q = 0.8 \text{ MeV} \simeq 1 \text{ MeV}$$

$$|\vec r| \simeq 1 \text{ fm} = \frac{1}{200 \text{ MeV}}$$

$$(\vec p + \vec q)\cdot \vec r \simeq \frac{1 \text{ MeV}}{200 \text{ MeV}} = 5 \times 10^{-3}$$

$$e^{-i(\vec{p}+\vec{q})\cdot\vec{r}} \simeq 1 + O(\ )$$

$$\mathcal{M}_{fi} = -iG\frac{1}{V}\int d^3r\, \psi_p^* \psi_n = -i\frac{G}{V} N$$

↳ Fattore di corr. nucleare

In generale $X \to Y + e^- + \bar{\nu}$

$$N = \int d^3r\, \psi_X \psi_Y^*$$

$$|N| \simeq 1$$

$$|\mathcal{M}|^2 = \frac{G^2}{V^2}|N|^2$$

Spazio delle fasi: $\rho(E)\big|_{E_i=E_f} = \int dn\, \delta(E_f - E_i)$

particella con mom $\vec{p}$ $\quad dn = \frac{V}{(2\pi)^3}d^3p = \frac{V}{(2\pi)^3}p^2 dp\, d\Omega.$

3 particelle: p, e^-, X

$$\vec{P}_p + \vec{P}_e + \vec{P}_X = \vec{0} \Rightarrow \vec{P}_p = -(\vec{P}_p + \vec{P}_X) = -(\vec{p}+\vec{q}).$$

$\Rightarrow$ $\vec{P}_p$ fissato dalla conservazione del momento.

$$dn = \underbrace{\frac{V}{(2\pi)^3}(4\pi)p^2dp}_{e^-} \quad \underbrace{\frac{V}{(2\pi)^3}(4\pi)q^2dq}_{X}$$

$$\int dn\, \delta(E_p + E_e + E_X - E_n)$$

$E_X = |\vec{q}|$ $\quad$ massa trascurabile $m_X \simeq \emptyset$

$E_e = \sqrt{m_e^2 + p^2}$

$$E_p + E_e + E_X - E_n = m_p + T_p + E_e + E_X - m_n.$$

$T_p \simeq \emptyset$. $\quad E_n = m_n$ rif. solidale con n

$m_p - m_n + E_e + E_X = 0.$ $\quad E_T = m_n - m_p.$

$$\delta(E_f - E_i) = \delta(E_T - E_e - E_X) \quad m_n - m_p = E_T = E_e + E_X$$

$$E_e = \sqrt{m_e^2 + p^2} \qquad p^2 = E_e^2 - m_e^2$$

$$p^2 dp \longrightarrow \qquad \cancel{2} p\,dp = \cancel{2} E_e dE$$

$$p^2 dp = p E_e dE_e$$

$$E_x = \sqrt{m_x^2 + q^2} \simeq q \qquad m_x \simeq \emptyset \qquad \text{trascuriamo massa di } x$$

$$q^2 dq = E_x^2 dE_x$$

$$\Gamma = 2\pi \frac{G^2}{\cancel{V^2}} |N|^2 \frac{\cancel{V^2}}{(2\pi)^6} (4\pi)^2 \int \delta(E_T - \cancel{E_e - E_x}) \, p E_e dE_e \, E_x^2 \cancel{dE_x}$$

$$E_x = E_T - E_e$$

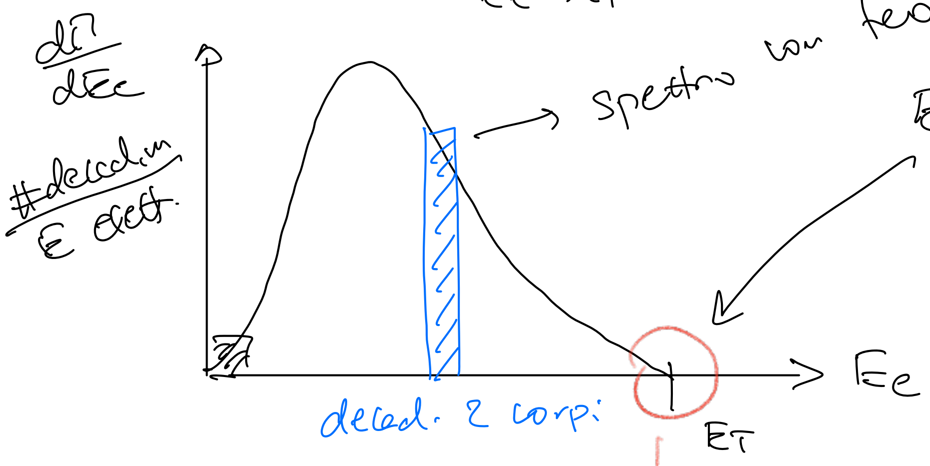
$$\Gamma = 2\pi \frac{G^2}{V^2} |N|^2 \frac{(4\pi)^2}{(2\pi)^6} \int_{m_e}^{E_T} p \, E_e (E_T - E_e)^2 \, dE_e$$

$\frac{d\Gamma}{dE_e}$: # decadimenti / Energie di E_e. Spettro energetico di E_e.

$$\frac{d\Gamma}{dE_e} \propto G^2 \sqrt{E_e^2 - m_e^2}\, E_e (E_T - E_e)^2$$

E_e piccoli $\lim\limits_{E_e \to m_e} \frac{d\Gamma}{dE_e} = \emptyset$

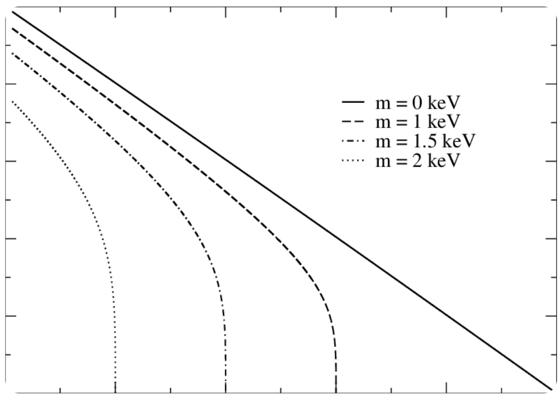
$$\lim_{E_e \to E_T} \frac{d\Gamma}{dE_e} = \emptyset$$



→ spettro con teoria di Fermi con neutrino.

$$E_T = m_n - m_p - m_x$$

$$Q = m_n - m_p - m_e - m_x = E_T - m_e$$



$$X \rightarrow Y + e^- + \bar{\nu}_e$$

$$E_T = m_X - m_Y - m_\nu \simeq m_X - m_Y$$

${}^A_Z X$ con A, Z grandi $\Rightarrow$ $E_T \gg m_e$

$$\Gamma \propto G^2 \int_{m_e \to 0}^{E_T} E_e \sqrt{E_e^2 - m_e^2}\,(E_T - E_e)^2\, dE_e$$

$E_T \gg m_e$.

$$\frac{1}{\tau} = \Gamma \propto G^2 E_T^5 |M|^2$$

Vita media τ del nucleo ${}^A_Z X$ $\propto$ $1 / G^2 (m_X - m_Y)^5$

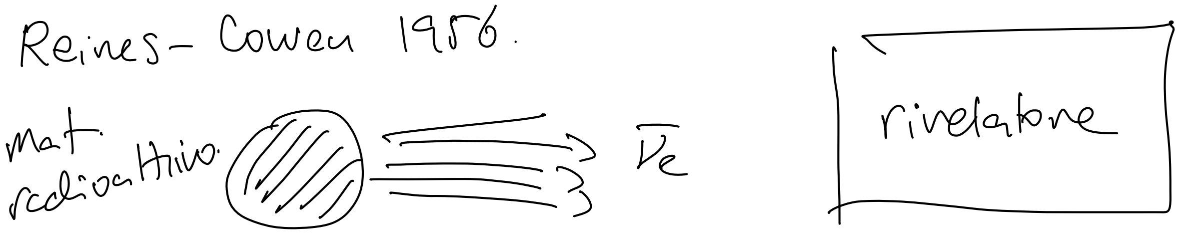
legge Sargent

spettro $\frac{d\Gamma}{dE_e}$ di Fermi in accordo con dati $\Rightarrow$ prove indirette di esistenza di neutrino.

prove di interazione debole.

$$n \rightarrow p\, e^- \bar{\nu}_e \quad \Rightarrow \quad \bar{\nu}_e + p \rightarrow n + e^+$$

Reines - Cowen 1956.



$\bar{\nu}_e + p \longrightarrow n + e^+$

$Q = m_\nu + m_p - m_n - m_{e^+} = -1.8$ MeV.

$\Rightarrow$ Fascio di neutrini con $E_\nu \geq 1.8$ MeV Soglia di reazione.

Sorgente: nucleo di un reattore da 1 GW di potenza.

$1\ eV = 1.6 \times 10^{-19}\ J$

$$1\ W = \frac{1\ J}{1\ s} = \frac{1}{1.6} \times 10^{19}\ \frac{eV}{s}$$

Reattore nucleare a fissione:

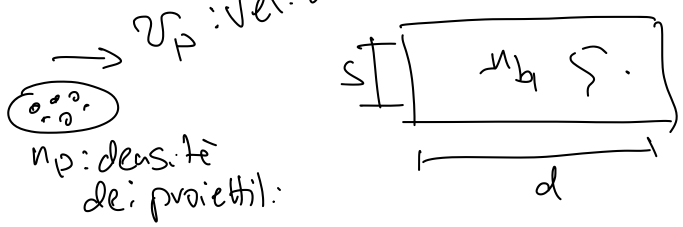
per ogni reazione $\langle n_\nu \rangle = 6$

$Q \simeq 200$ MeV. En. rilasciate per reazione

$$\frac{\#\ \text{neutrini}}{\text{sec}} = \frac{\text{Potenza}}{200\ \text{MeV}} \times 6 \quad \#\ \text{neut.}/\text{secondo}$$

$$= 2 \times 10^{20}\ \text{Hz}$$

$$\frac{dN_r}{dt} = \sigma \underbrace{\frac{dN_p}{dt}}_{\#\nu/\text{sec.}} n_b \cdot d$$



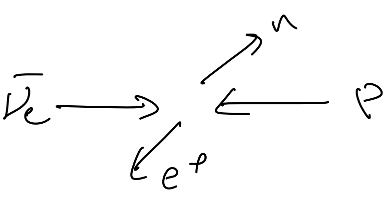
$$\frac{dN_r}{dt} = \sigma \underbrace{\frac{dN_p}{dt} \frac{1}{S}}_{\Phi} \underbrace{n_b \cdot d \cdot S}_{N_B} = \sigma \Phi_p N_B = \sigma n_p \cdot v_p \cdot N_B$$

$$= \sigma \frac{N_p}{V} v_p \cdot N_B.$$

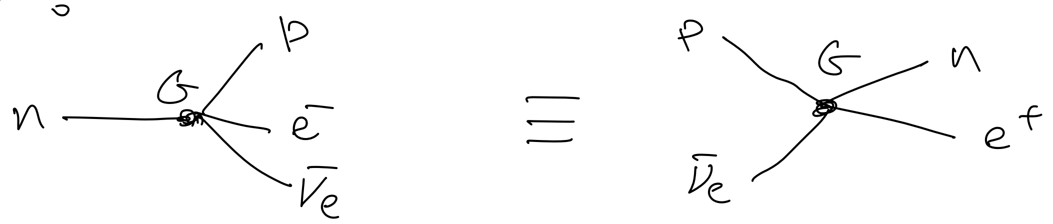
$$\frac{1}{N_B} \frac{1}{N_p} \frac{dN_r}{dt} = \sigma \cdot \frac{v_p}{V}$$

$\hookrightarrow \Gamma(\bar{\nu}_e + p \longrightarrow n + e^+) = 2\pi |M_{fi}|^2 \rho(E)$

Nel rif. Centro di masse



$\mathcal{M}_{fi} = ?$



$\vec{P}_{\nu_e} + \vec{P}_p = \emptyset$ centro di massa (c.d.m.) $\vec{P}_p = -\vec{P}_\nu \equiv \vec{P}$

$$\rho(E) = \int \delta(E_f - E_i) \frac{V}{(2\pi)^3} (4\pi) P^2 dP$$

$E_f = E_n + E_{e^+}$ $\quad E_i = E_\nu + E_p$ nel c.d.m.

$$\sigma = \frac{V}{v_p} \Gamma = \frac{V}{v_p} 2\pi |\mathcal{M}_{fi}|^2 \rho(E)$$

$$= \frac{V}{v_p} 2\pi \frac{G^2}{V^2} |M|^2 \int \frac{V}{(2\pi)^3} (4\pi) P^2 dP \, \delta(E_f - E_i)$$

lo stesso $\mathcal{M}_{fi}$ del decad. β.

$$P^2 dP = P E dE$$

$$\int P^2 dP \, \delta(E_f - E_i) = \int P E \, \delta(E_f - E_i) dE \propto PE$$

$$\sigma \propto \frac{1}{v_p} G^2 P E \qquad v_p = \beta_p = \frac{P}{E}$$

$\beta \simeq 1$ particelle relativistiche.

$G_F = 10^{-5} \text{ GeV}^{-2}$ misurata nei decadim. β

$$\sigma \propto G^2 P E \sim [10^{-5}]^2 \text{ GeV}^{-4} \times PE$$