

A

$$[A, H] = 0$$

simmetria.

$$A|\psi\rangle$$

$$\langle\psi|A|\psi\rangle = \langle A \rangle$$

$$\frac{d}{dt} \langle A \rangle = \frac{d}{dt} \langle\psi|A|\psi\rangle = \left(\frac{d}{dt} \langle\psi|\right) A |\psi\rangle + \langle\psi| \frac{\partial A}{\partial t} |\psi\rangle + \langle\psi| A \frac{d}{dt} |\psi\rangle$$

$$i \frac{d}{dt} |\psi\rangle = H |\psi\rangle$$

$$-i \langle\psi| \frac{d}{dt} = \langle\psi| H$$

$$\rangle = \langle\psi| [A, H] |\psi\rangle + \left\langle \frac{\partial A}{\partial t} \right\rangle$$

Se A non dipende dal tempo

$$\frac{d\langle A \rangle}{dt} = 0 \quad \longleftrightarrow \quad [A, H] = 0$$

traslazione \vec{r}

traslazione temporale t

rotazione in \vec{r}

\vec{p}

E

$$\vec{J} = \vec{r} \times \vec{p}$$

Simmetria

invariante del sistema

sotto una transf.

\longleftrightarrow

quantità
conservate

Teorema di Noether

1917

$$P|\psi\rangle, \quad \vec{r} \rightarrow -\vec{r}$$

partic. discrete.

$$\longleftrightarrow \quad \lambda = \pm 1 e^{i\alpha}$$

$$T \quad t \rightarrow -t$$

C : coniugazione di Carice.

$$\pi^+ \longrightarrow \pi^-$$

trasf

$$T T = T^2 = 11$$

CP \longrightarrow Carice opposte
 $\vec{r} \longrightarrow -\vec{r}$

particelle \longrightarrow antiparticelle.

$$K^0 \longrightarrow \begin{matrix} \bar{u}^+ \pi^- \\ \pi^+ u^- \pi^0 \end{matrix} \quad 1964$$

Interazioni deboli ~~/~~ ~~CP~~

CPT

$$e^+ e^- \longrightarrow e^+ e^-$$

ISOSPIN : Heisenberg

rotazione nello spazio di isospin

nucleoni: protoni, neutroni

interazioni forti non distinguono n da p.

m_p

$$\Delta m = 1.6 \text{ MeV}$$

m_n

$$m \approx 1000 \text{ MeV.}$$

$$\text{nucleone} = \begin{pmatrix} p \\ n \end{pmatrix} \quad I = \frac{1}{2}$$

I_3

$$|p\rangle = |\frac{1}{2}, \frac{1}{2}\rangle$$

$$|n\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle$$

Per interazioni forti conta solo I

fl dipende solo da I non da $I_3 \Rightarrow$ non distingue n da p .

$$\pi^\pm \approx 140 \text{ MeV}$$

$$\pi^0 \approx 135 \text{ MeV}$$

$$|u\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} I_3 = +1 \\ 0 \\ -1 \end{pmatrix} I = 1$$

I spin : nuovo numero quantico.

spin : num. quantico diverso

deuterio

$$d = pn$$

$$|p\rangle = |1/2, 1/2\rangle \quad |n\rangle = |1/2, -1/2\rangle$$

$$(pn) \quad |4, 1\rangle = |1/2, 1/2\rangle |1/2, 1/2\rangle \longrightarrow (pp)$$

$$\text{tripletto} \quad |4, 0\rangle = \frac{1}{\sqrt{2}} (|1/2, 1/2\rangle + |1/2, -1/2\rangle) \quad (pn)$$

$$|4, -1\rangle = |1/2, -1/2\rangle |1/2, -1/2\rangle \quad (nn)$$

$$(pn) \quad |0, 0\rangle = \frac{1}{\sqrt{2}} (|1/2, 1/2\rangle - |1/2, -1/2\rangle) \quad (pn)$$

singoleto

Per natura solo pn come deuterio.

$$(i) \quad p + p \longrightarrow d + \pi^+ \quad (a) \quad |1, 1\rangle$$

$$\begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & 0 \quad +1 = I \\ +\frac{1}{2} & +\frac{1}{2} & 0 \quad +1 = I_3 \end{array}$$

$$p + n \longrightarrow d + \pi^0 \quad (b) \quad |1, 0\rangle$$

$$\begin{array}{ccc} 0 & 1 = I \\ +\frac{1}{2} & -\frac{1}{2} & 0 \quad 0 = I_3 \end{array}$$

$$\frac{1}{\sqrt{2}} (|1, 0\rangle + |0, 0\rangle)$$

$$\begin{array}{ccc}
 n + u & \rightarrow & d + \bar{u} \\
 I_3 = -\frac{1}{2} - \frac{1}{2} & & 0 - 1 \\
 I = 1 \quad |1, -1\rangle & & 0 \quad 1
 \end{array}
 \quad (c) \quad |1, -1\rangle$$

$$\sigma(a+b \rightarrow c+d) \propto |M_f|^2$$

$$\langle f | H_I | i \rangle$$

$$(a) \quad \langle 1, 1 | H_I | 1, 1 \rangle = M_a$$

$$(b) \quad \langle 1, 0 | H_I \left(\frac{1}{\sqrt{2}} |1, 0\rangle + |0, 0\rangle \right) = M_b$$

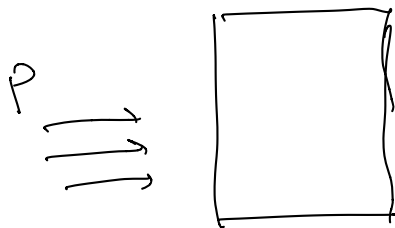
Se Int. forte conserva isospin.

$$\frac{1}{\sqrt{2}} \langle 1, 0 | H_I | 1, 0 \rangle + \underbrace{\frac{1}{\sqrt{2}} \langle 1, 0 | H_I | 0, 0 \rangle}_{=0}$$

Misura

$$\frac{\sigma_a}{\sigma_b} \approx \frac{|M_a|^2}{|M_b|^2} \approx \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} = 2$$

$$\frac{dN_r}{dt} = \sigma \cdot \frac{dN_p}{dt} \cdot n_b \cdot d.$$



di aspetto.

$$N(a) \approx 2 N(b)$$

(c)

verifica sperimentale

Nucleoni contro pioni:

$$I = \frac{1}{2}$$

$$I = 1$$

$$\pi^+ + p \rightarrow \pi^+ + p \quad |3/2, 3/2\rangle$$

$$|\frac{1}{2}, \frac{1}{2}\rangle |1, 1\rangle$$

$$|3/2, 3/2\rangle$$

$$(\pi^+, \pi^0, \pi^-) + (p, n) \rightarrow \text{conservano } q, I, N_{\text{barionico.}}$$

$$\pi^0 + p \rightarrow \pi^0 + p$$

$$\pi^- + p \rightarrow \pi^- + p$$

$$|1, -1\rangle |\frac{1}{2}, \frac{1}{2}\rangle \quad I_3 = -1/2$$

$$\left(|3/2, -1/2\rangle + |1/2, -1/2\rangle \right)$$

$$\pi^- + p \rightarrow \pi^0 + n$$

pione + nucleone.

$$|I=1, a\rangle + |I=1/2, b\rangle =$$

$$= \alpha |I=3/2, a+b\rangle + \beta |I=1/2, a+b\rangle$$

$$|i\rangle = \alpha |I=3/2\rangle + \beta |I=1/2\rangle$$

$$|f\rangle = \gamma |I=3/2\rangle + \delta |I=1/2\rangle$$

Se H_I forte conserva isospin.

$$\mu_{fi} = \langle f | H_I | i \rangle = a \langle I=3/2 | H_3 | I=3/2 \rangle + b \langle I=1/2 | H_1 | I=1/2 \rangle$$

$$\sigma \propto |\mathcal{M}_f|^2 = a^2 |\mathcal{M}_3|^2 + b^2 |\mathcal{M}_1|^2$$

+ termini di interazione.

$$|\mathcal{M}_f|^2 = \mathcal{M}_f \cdot \mathcal{M}_f^* = (a \mathcal{M}_3 + b \mathcal{M}_1) (a^* \mathcal{M}_3^* + b^* \mathcal{M}_1^*)$$

Table 3.3. Clebsch-Gordan coefficients in pion-nucleon scattering

Pion	Nucleon	$I = \frac{3}{2}$				$I = \frac{1}{2}$	
		$I_3 = \frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
π^+	p	1					
π^+	n		$\sqrt{\frac{1}{3}}$			$\sqrt{\frac{2}{3}}$	
π^0	p		$\sqrt{\frac{2}{3}}$			$-\sqrt{\frac{1}{3}}$	
π^0	n			$\sqrt{\frac{2}{3}}$			$\sqrt{\frac{1}{3}}$
π^-	p			$\sqrt{\frac{1}{3}}$			$-\sqrt{\frac{2}{3}}$
π^-	n				1		

a) $\pi^+ + p \rightarrow \pi^+ + p$

$$\sigma_a \propto |\mathcal{M}_3|^2$$

c) $\pi^- + p \rightarrow \pi^- + p$

$$\sigma_c \propto \frac{1}{9} |\mathcal{M}_3 + 2\mathcal{M}_1|^2$$

j) $\pi^- + p \rightarrow \pi^0 + n$

$$\sigma_j \propto \frac{2}{9} |\mathcal{M}_3 - \mathcal{M}_1|^2$$

① $M_3 \gg M_1$

$$\sigma_a : \sigma_b : \sigma_c = 1 : \frac{1}{9} : \frac{2}{9}$$

② $M_1 \gg M_3$

$$\sigma_a : \sigma_b : \sigma_c = 0 : \frac{4}{9} (M_1^2 : \frac{2}{9} M_1^2)$$

$\pi^+ + p$ annihilation \Rightarrow ~~$M_1 \gg M_3$~~

$$\sigma_a : (\sigma_c + \sigma_j) = \frac{\sigma(\pi^+ + p \rightarrow \pi^+ + p)}{\sigma(\pi^- + p \rightarrow \pi^- + p) + \sigma(\pi^- + p \rightarrow \pi^0 + n)}$$

$$\Rightarrow \sigma(\pi^+ + p) \simeq 3 \sigma(\pi^- + p)$$

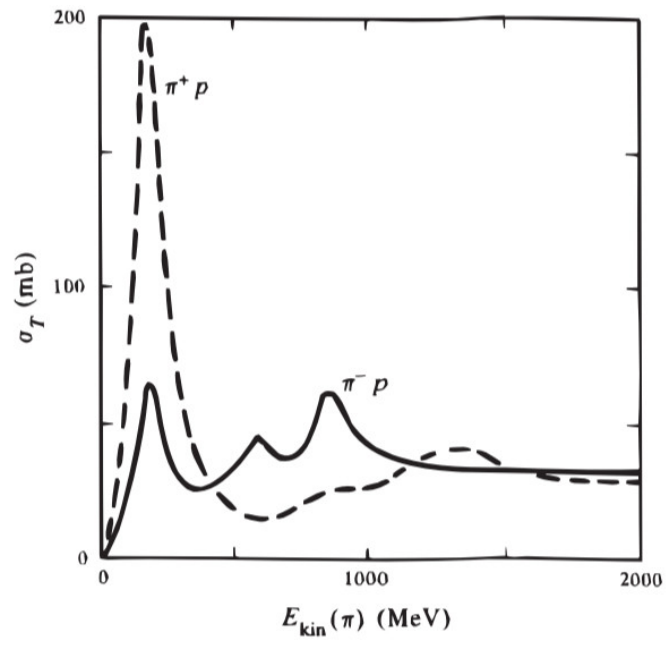


Figure 5.35: Total cross section as a function of pion kinetic energy for the scattering of positive and negative pions from protons. (1 mb = 1 millibarn = 10^{-27} cm².)