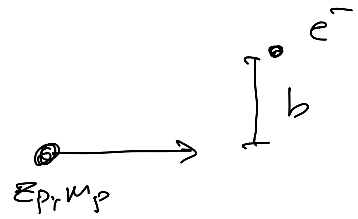
$$T \equiv \Delta E_p = \frac{\Delta p^2}{2m} = \left(\frac{z_p e^2}{2\pi\varepsilon_0}\right)^2 \frac{1}{\beta^2 c^2}\,\frac{1}{b^2}\,\frac{1}{2m}.$$

$$= \left(\frac{z_p e^2}{4\pi\varepsilon_0}\right)^2 \frac{2}{m\beta^2 c^2}\,\frac{1}{b^2}$$

$q \longrightarrow \quad \cdot\, e^-$ $\qquad P_e^i = 0$

$$\vec p_p^{\,i} = \vec P_e^f + \vec P_p^f$$

$$\Delta p = \vec P_p^{\,i} - \vec P_p^f = \vec P_e^f$$

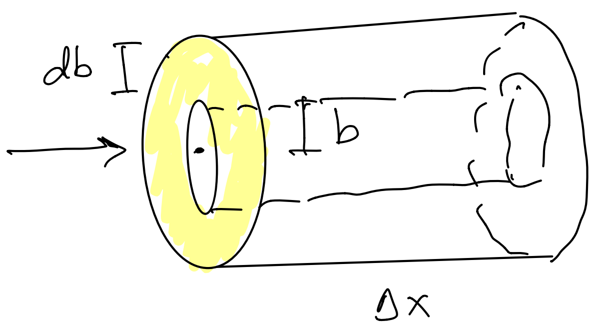


$\Delta E = T_{\text{trasf}} = \dfrac{P_e^{f\,2}}{2m}$ var. en. cinetica del proiettile.

$$T \propto \frac{z_p^2}{\beta^2}\,\frac{1}{b^2}$$

energia persa $\quad dE = -\overline{\Delta E}\, N_{urti} = -\overline{\Delta E}\cdot 2\pi b\, db\, dx\, n_e$

$d\sigma = 2\pi\,|b\,db|$ $\qquad dE \propto \dfrac{z_p^2}{\beta^2}$



$$b^2 \propto \frac{1}{T} \quad \Rightarrow \quad 2|b\,db| = \frac{1}{T^2}\,dT$$

$$d\sigma = 2\pi\,|b\,db| = \frac{\pi}{T^2}\,dT$$

$\dfrac{d\sigma}{dT}$: σ per $T' \in [T, T+dT]$

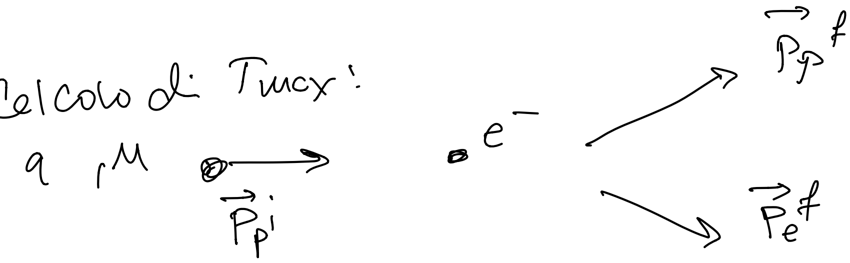
$$dE = -T\,\frac{\pi}{T^2}\,dT\,dx\,n_e\,\frac{z_p^2}{\beta^2} = T\,\frac{d\sigma}{dT}\,dT\,dx\,n_e\cdot\frac{z_p^2}{\beta^2}$$

$$\frac{dE}{dx} = -n_e T \frac{d\sigma}{dT} dT \Rightarrow \frac{dE}{dx} = -n_e \pi \int_{T_{min}}^{T_{max}} \frac{dT}{T} \frac{z_p^2}{\beta^2}$$

$$\Rightarrow \quad \frac{dE}{dx} = -n_e \pi \ln \frac{T_{max}}{T_{min}} \frac{z_p^2}{\beta^2}$$

$$T_{min} \simeq I = \hbar \langle \omega \rangle \simeq 10 \text{ eV } Z$$

Calcolo di T_{max}:



$E_e^f = m_e + T_e^f$

M: massa del proiettile

$$T_{max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \frac{m_e}{M} + \left(\frac{m_e}{M}\right)^2}$$

β, γ: del proiettile.

$$\beta = \frac{P_p}{E_p}$$

$$\gamma = \frac{E_p}{M}$$

1/ $2\gamma m_e \ll M$

proiettile = protone $\Rightarrow M \simeq 1$ GeV.

$2\gamma \cdot 0.5 \text{ MeV} \simeq 1$ GeV.

$\Rightarrow 2\gamma \simeq 2 \times 10^3$

$\Rightarrow \gamma \simeq 1000$

$$T_{max} = 2 m_e c^2 \beta^2 \gamma^2$$

per proiettile pione $m_\pi = 140$ MeV @ $p = 100$ GeV

errore $\frac{\delta T}{T} \simeq 6\%$

2/ $2\gamma m_e \gg M \Rightarrow T_{max} = \frac{\cancel{2 m_e} c^2 \beta^2 \gamma^{\cancel{2}}}{\cancel{2\gamma} \frac{\cancel{m_e}}{M}}$

$$= M c^2 \beta^2 \gamma$$

$$\frac{dE}{dx} \propto -n_e \frac{z_p^2}{\beta^2} \ln \frac{T_{max}}{T_{min}} = -Z \frac{\rho}{A} N_A \frac{z_p^2}{\beta^2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2}{10 eV\, Z}$$

Conto quantistico di Bethe-Bloch.

$$\frac{d\sigma}{dT} \propto \frac{z_p^2}{\beta^2} \frac{1}{T^2} \left(1 - \beta^2 \frac{T}{T_{max}}\right)$$

correzione dovute allo spin dei fermioni

$$\frac{dE}{dx} \propto \int T \frac{d\sigma}{dT} \, dT$$

$$\int T \frac{1}{T^2} \left(1 - \beta^2 \frac{T}{T_{max}}\right) dT$$

Bohr.

$$\frac{dE}{dx} = -Z \frac{\rho}{A} N_A \frac{z_p^2}{\beta^2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2}{10 eV\, Z}$$

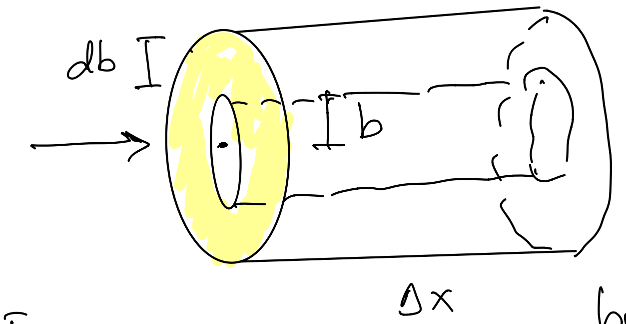
$$\simeq -\frac{1}{2} \rho \frac{z_p^2}{\beta^2} \ln \frac{(\ )\, \beta^2 \gamma^2}{(\ )\, Z}$$

$$\frac{1}{\rho} \frac{dE}{dx} \propto \frac{z_p^2}{\beta^2} \ln \frac{\beta^2 \gamma^2}{Z}$$

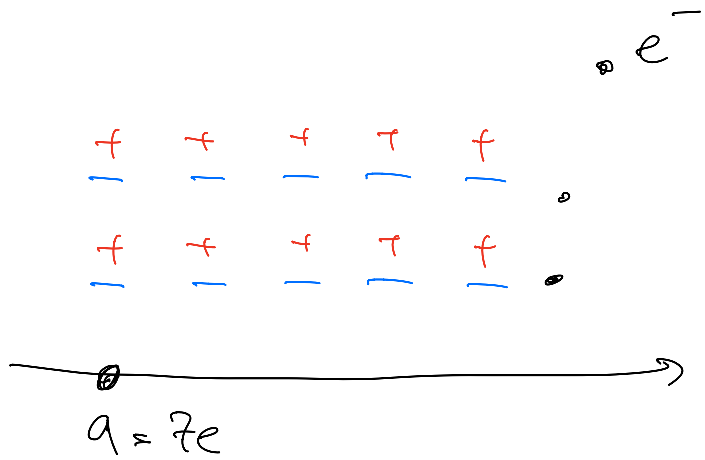
$\ln(2) \simeq 0.7 \qquad \ln(60) \simeq 4$

Perdite di energia normalizzate a spessore densità.

non dipende dal mezzo ma solo dal proiettile.



$$\frac{dE}{dx} \propto \int_{T_{min}}^{T_{max}} T \frac{d\sigma}{dT}\, dT \simeq \int_{b_{min}}^{b_{max}} T\, |b\, db|$$



correzione per effetto densità polarizzazione

$$\frac{\text{MeV}}{\text{cm}}$$

$$-\frac{dE}{dx} = C \rho \frac{Z}{A} \frac{z_p^2}{\beta^2} \frac{1}{2} \left[\ln\left(\frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} \right) - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

$T_{max} \simeq 2 m_e c^2 \beta^2 \gamma^2$ con buona approssimazione.

Simile al calcolo classico.

Correzione quantistica/relativistica.
Risalita relativistica

$$C \simeq 0.3 \ \frac{\text{MeV cm}^2}{\text{g}}$$

$\frac{1}{\rho}\frac{dE}{dx}$ funzione universale per tutti i mezzi.

Caratteristiche.

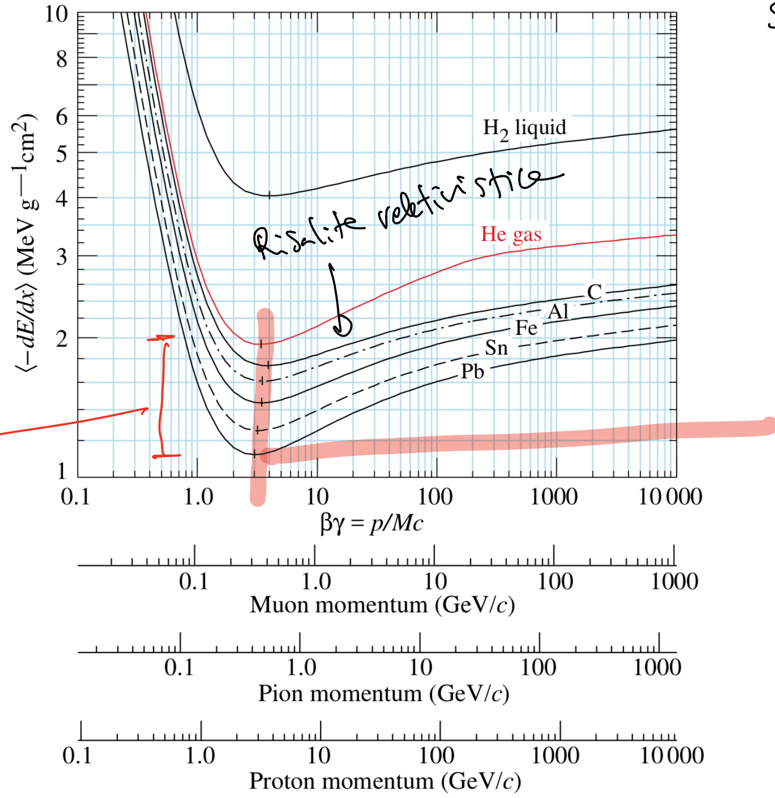
1/ Non appare m del proiettile.

2/ $\beta \to 0 \implies \frac{1}{\rho}\frac{dE}{dx} \propto \frac{1}{\beta^2}$

$\left[\frac{1}{\rho}\frac{dE}{dx}\right]$

$\frac{MeV}{\cancel{cm}} \quad \frac{cm^{\cancel{3}\,2}}{g}$

Scala logaritmica.



$\left.\frac{dE}{dx}\right|_{min} \simeq 1-2 \ \frac{MeV\,cm^2}{g}$

minimo per $\beta\gamma \simeq 3$ ascissa.

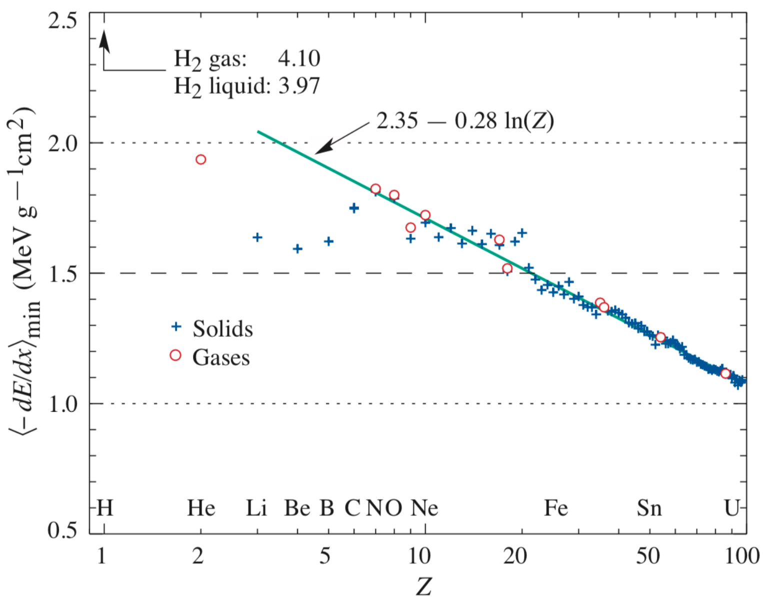
punto di minimo ionizzazione.

$H_2O \quad \rho = 1\ g/cm^3$ al minimo. $\frac{dE}{dx} \simeq 1.5\ \frac{MeV}{cm}$.

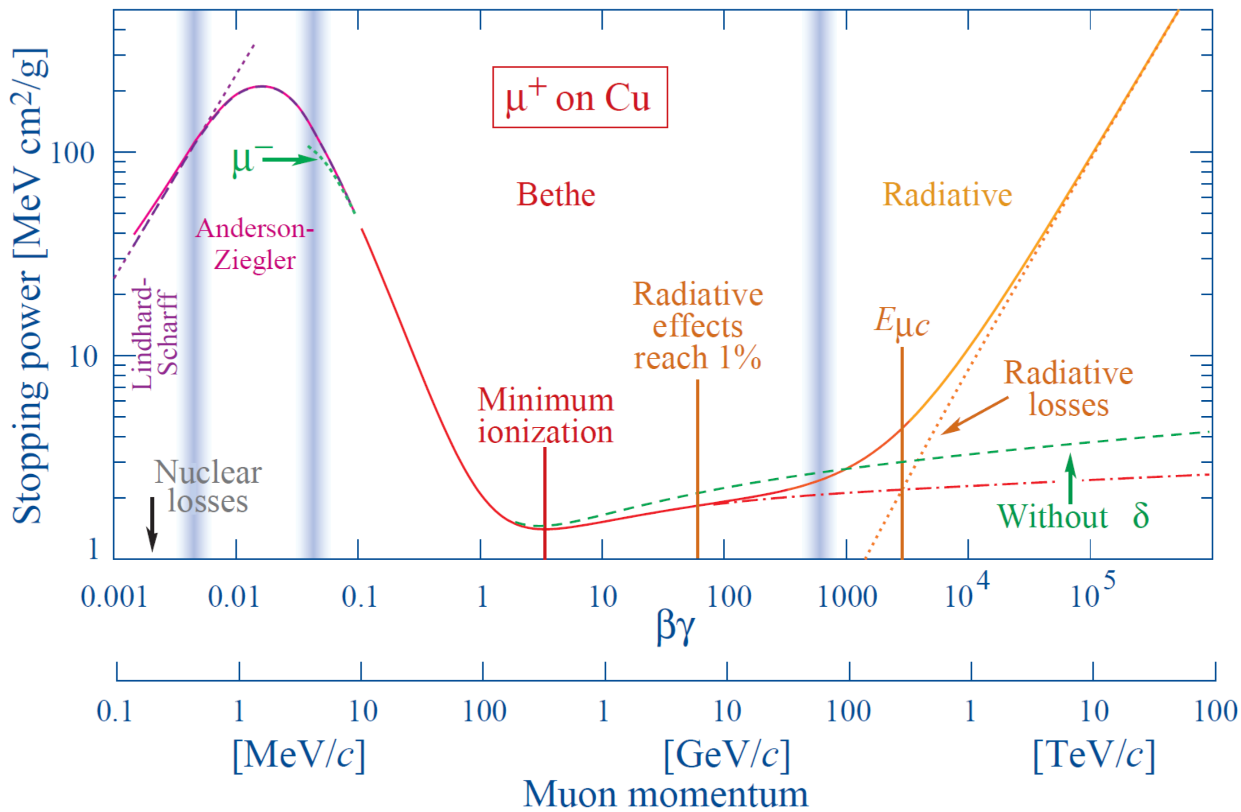
Se mi trovo con $\beta\gamma \geq 3$ suff considerare.

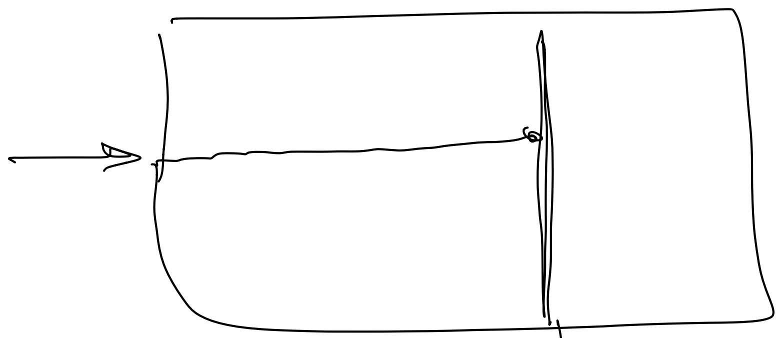
$\left\langle \frac{1}{\rho}\frac{dE}{dx} \right\rangle$ al minimo.

posizione del minimo di Bethe-Bloch.



$-\frac{1}{\rho}\frac{dE}{dx}$





X_{max} : percorso residuo.

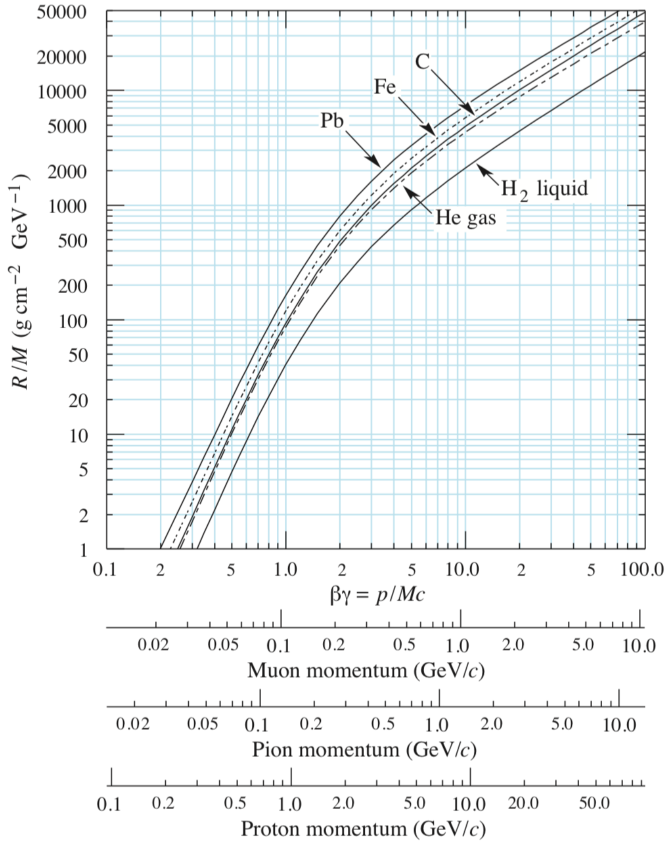
Range

distanza di penetrazione nel mezzo prima di perdere tutta l'energia

$$dE = -\left(\frac{dE}{dx}\right)\cdot dx.$$

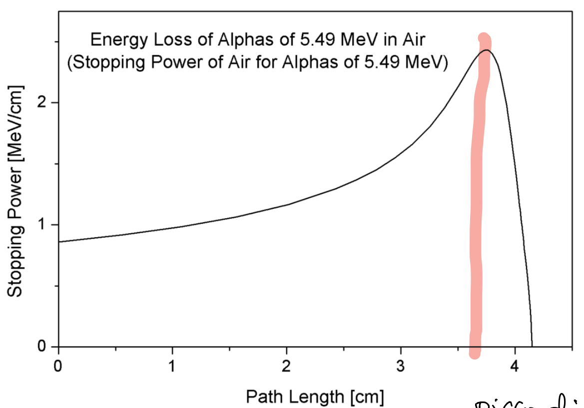
$$\Rightarrow \qquad dx = \frac{dE}{-\left(\frac{dE}{dx}\right)}$$

$$\int_0^{X_{max}} dx = X_{max} = \int_{E_0}^{0} \frac{dE}{-\left(\frac{dE}{dx}\right)} \qquad \text{Range} \quad R(E_0)$$



$\rho = \rho_{aria}.$

$\frac{dE}{dx}$



picco di Bragg

energia depositata