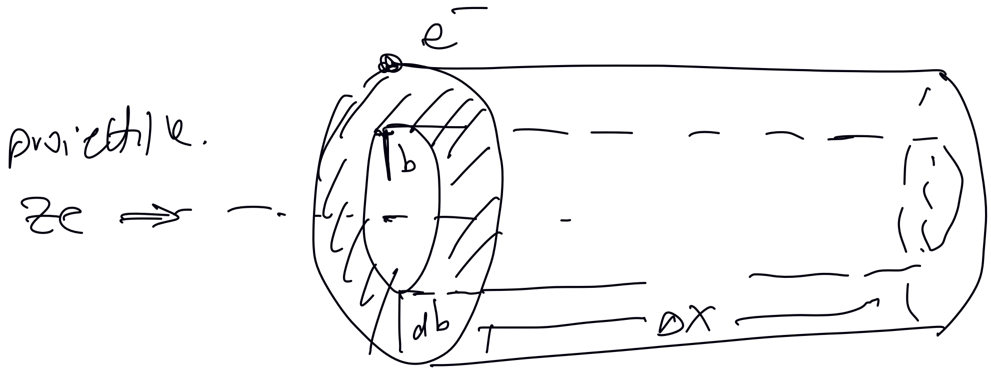
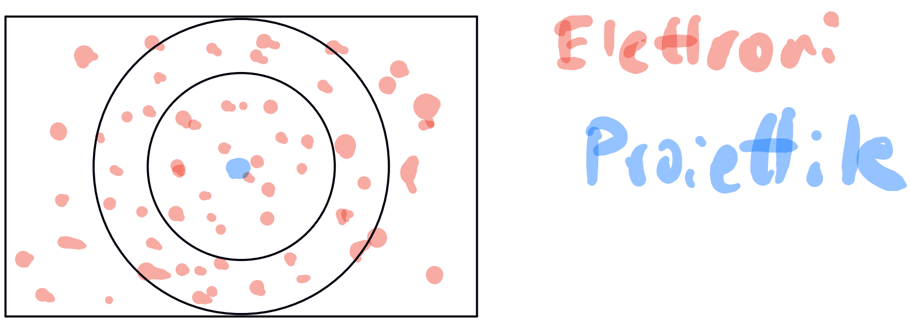
Token 912 989



$$\frac{\rho}{A} N_A \cdot Z_{Bers.}$$

$$V = \Delta x \, dS = 2\pi b db \, \Delta x$$

$$\Delta E = -\underbrace{\Delta E (\underbrace{2\pi \, b db \, \Delta x}_{V}) \times n_e}_{N_{urti}}$$

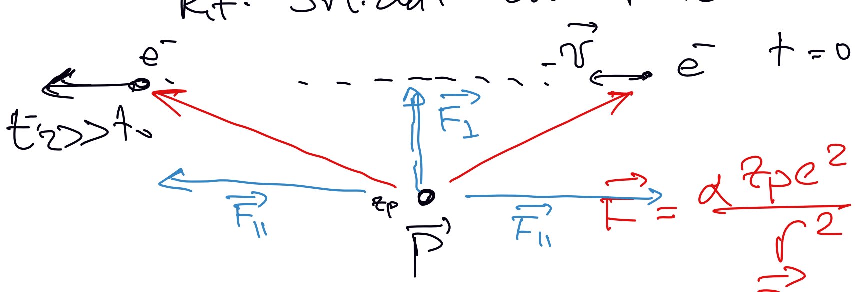


CAB

e^-

$\vec{p} = m\vec{v}$

Rif. solidale con Proiettile



$$\vec{F} = \alpha \frac{z_p e^2}{r^2} = e\vec{E}$$

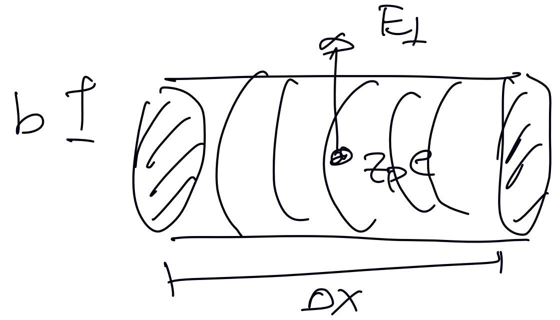
$$\Delta\vec{p} = \int \vec{F} \cdot dt$$

$$dx = v \cdot dt \Rightarrow dt = \frac{dx}{v}$$

$$\Delta\vec{p} = \int_{-\infty}^{+\infty} \vec{F} \frac{dx}{v} = \int_{-\infty}^{+\infty} \vec{F}_\perp \frac{dx}{v} + \int_{-\infty}^{+\infty} F_\parallel \frac{dx}{v} = \varnothing$$

$$|\Delta \vec{p}| = \int_{-\infty}^{+\infty} F_\perp \frac{dx}{v} = \frac{e}{v} \int_{-\infty}^{+\infty} E_\perp \cdot dx \qquad \vec{F} = e\vec{E}$$

↓ generato $Z_p e$



$$\phi(E) = \int_S \vec{E} \cdot dS = \int_{\text{caut.}} E_\perp \cdot dS + \int_{\text{tappi}} E \cdot dS$$

tappi $\simeq 0$

$$= \int_{\text{caut}} E_\perp \cdot dx = (2\pi b \cdot \Delta x) E_\perp$$

$$= \frac{Q}{\varepsilon_0} = \frac{Z_p e}{\varepsilon_0}$$

teorema di Gauss

$$E_\perp = \frac{Q}{\varepsilon_0} \frac{1}{2\pi b \Delta x}$$

$$2\pi b \int E_\perp \cdot dx = \frac{Z_p e}{\varepsilon_0}$$

$$\int E_\perp \cdot dx = \frac{Z_p e}{\varepsilon_0} \frac{1}{2\pi b}$$

$$|\Delta p| = \frac{Z_p \cdot e}{v} \int E_\perp \cdot dx = \frac{(Z_p \cdot e^2)}{\varepsilon_0} \frac{1}{2\pi b} \frac{1}{v}$$

↪ impulso trasferito

Conto classico di Bohr

≃ 1915

↓

Conto quantistico Bethe-Bloch 1930

$$\overline{\Delta E} = \frac{|\Delta p|^2}{2m}$$

$$= \frac{(Z_p e^2)^2}{\varepsilon_0^2} \left(\frac{1}{2\pi b}\right)^2 \frac{1}{2m} \left(\frac{1}{v}\right)^2$$

$$v = \beta c$$

$$T = \overline{\Delta E} \propto \frac{1}{b^2} \frac{1}{\beta^2 c^2}$$

$$T = \overline{\Delta E} = \left(\frac{z_p \cdot e^2}{\varepsilon_0}\right)^2 \frac{1}{(2\pi b)^2} \frac{1}{2mc^2\beta^2} \propto \frac{A}{b^2}$$

$$\Delta E = -\overline{\Delta E} \cdot \underbrace{2\pi b\, db}\ \Delta x$$

$$F = \frac{q^2}{4\pi\varepsilon_0}\frac{1}{r^2}$$

$$T \propto \frac{A}{b^2}.$$

$$b^2 \propto \frac{1}{T} \qquad 2|b\,db| \quad \frac{dT}{T^2}$$

$$2\pi|b\,db| \simeq d\sigma \propto \frac{dT}{T^2}.$$

$\frac{d\sigma}{dT} \propto \frac{1}{T^2} \Rightarrow$ urti con T piccoli avvengono più spesso.

$$\frac{d\sigma}{dT} = \left(\frac{ze^2}{4\pi\varepsilon_0}\right)^2 \frac{2\pi}{mc^2}\frac{1}{\beta^2}\frac{1}{T^2}\left(1-\underbrace{\beta^2\frac{T}{T_{max}}}\right)$$

sezione d'urto classica di Bohr

calcolo quantistico Bethe-Bloch

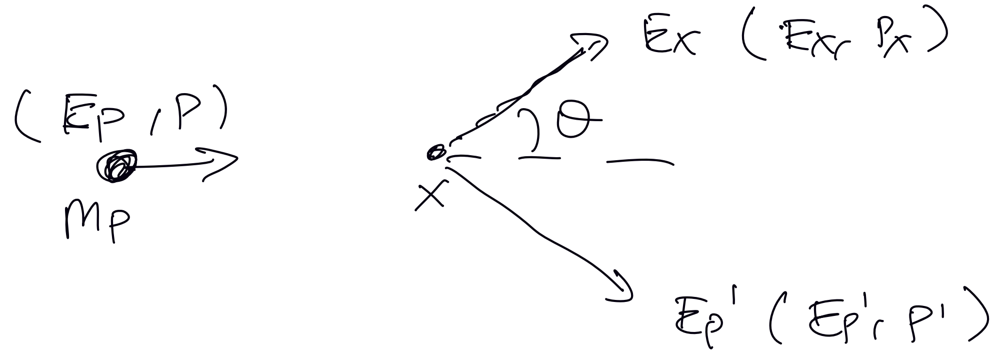
en. media persa per ionizzazione nel tratto Δx

$$\Delta E = -n_e \cancel{T} \cdot \frac{A\pi}{T^{\cancel{2}}}\, dT\ \Delta x$$

$(\overline{\Delta E}$ $2\pi|b\,db|$

$$\Rightarrow \frac{\Delta E}{\Delta x} =: \frac{dE}{dx} = -n_e A\pi \frac{dT}{T}$$

Integrare fra $(T_{min} \to T_{max})$



T_{max}: si ottiene per $\theta = 0$

1) $|\underline{p}|^2$ invariante
2) E si conserva
3) $\vec{p}$ si conserva
4) $E_x = m_x + K_x = m + T$

proiettile

$E_p = \gamma m$

$p = \gamma \beta m c$
$= \gamma m v$

$$T_{max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma (m_e/m_p) + (m_e/m_p)^2}$$

β, γ: del proiettile

$m_x \equiv m_e$

2 casi limite.

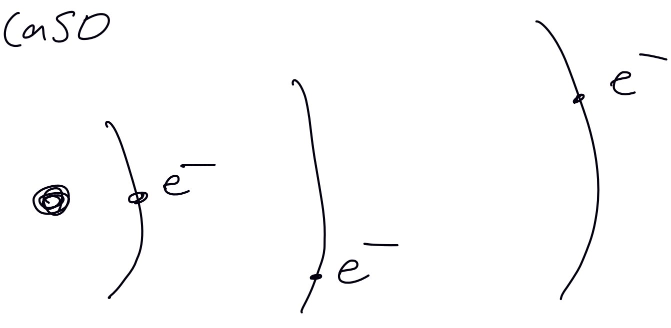
1) $m_p \gg m_e$: proiettile molto massivo

$$T_{max} = 2 m_e c^2 \beta^2 \gamma^2$$

2) $m_p \ll m_e$: $T_{max} = m_p c^2 \beta^2 \gamma$

non il nostro caso

T_{min}:



$I \propto 10 eV \cdot Z$

$$T_{min} \propto \sqrt[n]{\prod I_n} = \sqrt[n]{\prod_{i=1}^{n} I_i \, w_n}$$

$$x_1 = 10 \qquad x_2 = 1 \qquad \bar{x} = \frac{11}{2} = 5.5$$

$$\bar{x}_{geom} = \sqrt{10 \cdot 1} \simeq 3.3$$

$$x_1 \simeq 0.5 \quad x_2 = 2 \quad x_3 = 5 \quad \ldots \quad x_N = 100$$

$$T_{min} = \bar{I}$$
↪ en. media ionizzazione.

$$\frac{\Delta E}{\Delta x} =: \frac{dE}{dx} = -\, n_e \, A \pi \int_{T_{min}}^{T_{max}} \frac{dT}{T} \Rightarrow \left\langle \frac{dE}{dx} \right\rangle \propto \ln \frac{T_{max}}{T_{min}}$$

materiale

$$\frac{dE}{dx} = -\, C \underbrace{\frac{z_p^2}{\beta^2}}_{\Delta E} \underbrace{\rho N_A \frac{Z}{A}}_{n_e} \ln \frac{m_e c^2 \beta^2 \gamma^2}{\bar{I}}$$

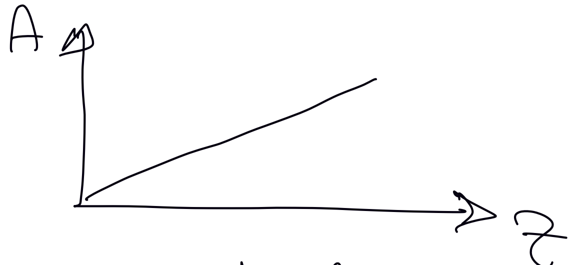
$m_e c^2 \beta^2 \gamma^2 \rightarrow T_{max}$

$\bar{I} \rightarrow T_{min}$

$\frac{Z}{A} \rightarrow$ adimensionali

$$[C]\,[\rho] = \frac{\text{MeV}}{\text{cm}} \qquad C = 0.3 \; \frac{\text{MeV cm}^2}{\text{g}}$$

$$\frac{dE}{dx} \rightarrow \frac{\text{MeV}}{\text{cm}} \qquad \frac{\text{cm}^2}{\text{g}}\,\text{MeV} \; \frac{\text{g}}{\text{cm}^3} = \frac{\text{MeV}}{\text{cm}}$$



Carta di Segrè

$$\frac{Z}{A} \simeq \frac{1}{2}$$

$$-\frac{1}{\rho}\frac{dE}{dx} = C\, N_A \left(\frac{Z}{A}\right)^{\simeq \frac{1}{2}} \frac{z_p^2}{\beta^2} \ln \frac{m_e c^2 \beta^2 \gamma^2}{\bar{I}}$$

$$\bar{I} \propto Z$$

$\ln \frac{1}{Z}$

$\ln 2 = 0.7$
$\ln 60 = 4$

⇒ è in pratica indipendente dal materiale
⇒ solo funzione del proiettile.

Formula di Bethe Bloch

$$-\frac{1}{\rho}\frac{dE}{dx} = C\, N_A \left(\frac{Z}{A}\right) \frac{z_p^2}{\beta^2} \frac{1}{2} \left[\ln \frac{2 m_e c^2 \beta^2 \gamma^2 \overbrace{T_{max}}^{2 m_e c^2 \beta^2 \gamma^2}}{\bar{I}^2} - \beta^2 - \frac{\delta}{2}\right]$$

risalita relativistica.

Correzione dovuta
alla polarizzazione del mezzo.
funzione del mezzo.
⇒ misurata empiricamente negli esp.

$\circledcirc$

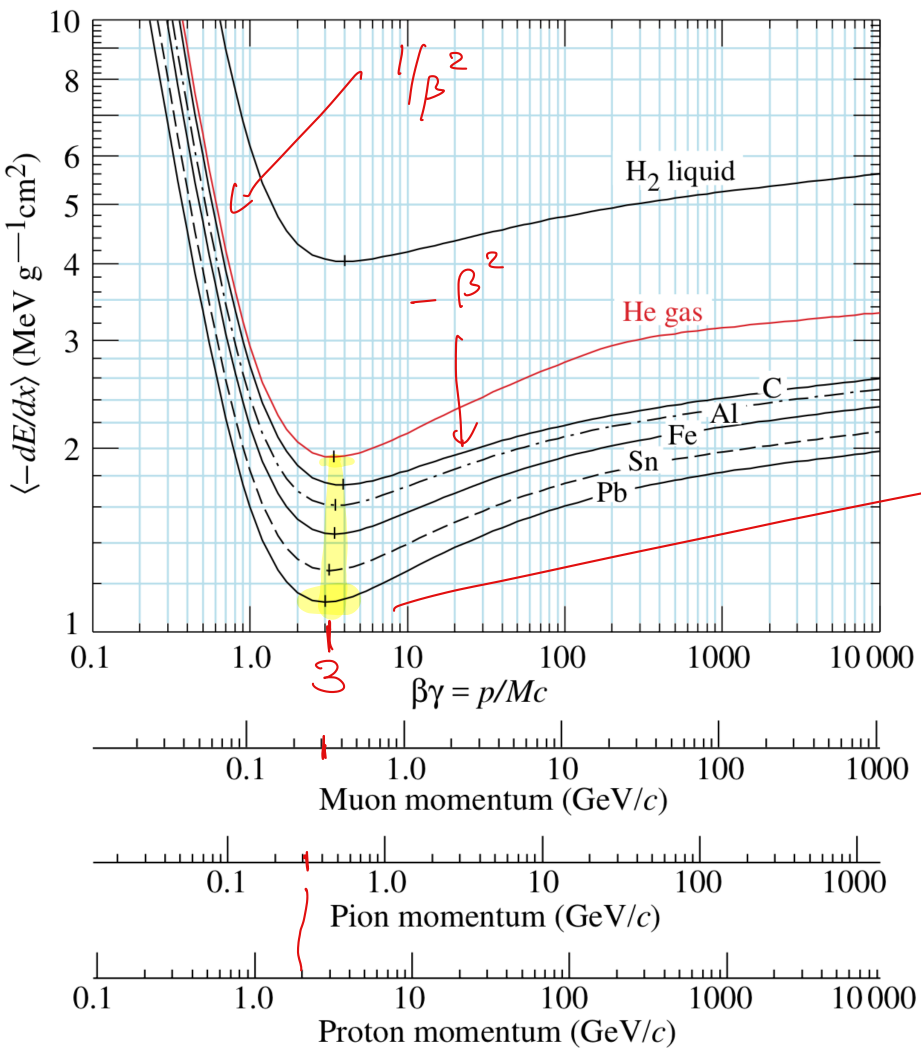
$\delta p > 0$

Porcan importante $\beta\gamma = \frac{p}{m}$

$$\beta = \frac{\beta\gamma}{\sqrt{1+(\beta\gamma)^2}}$$

$-\frac{1}{\rho}\frac{dE}{dx}$

Bethe-Bloch



$\beta\gamma = \frac{p}{m}$

minimo di ionizz.

logaritmica

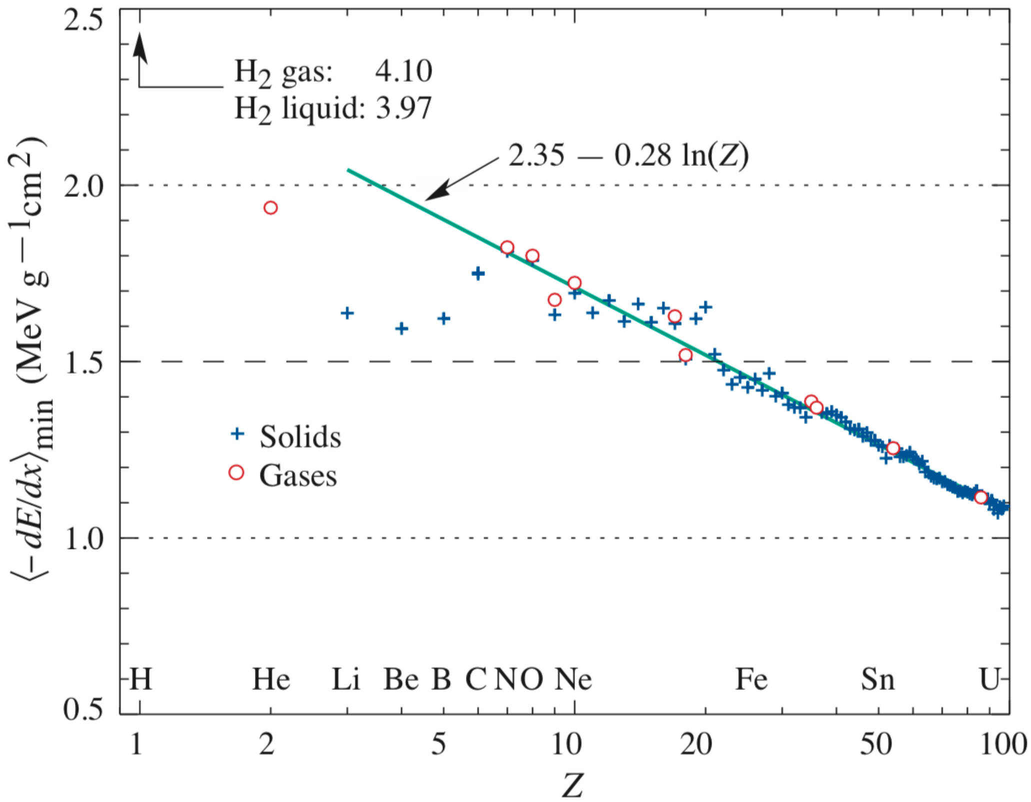
$m_\mu = 106$ MeV

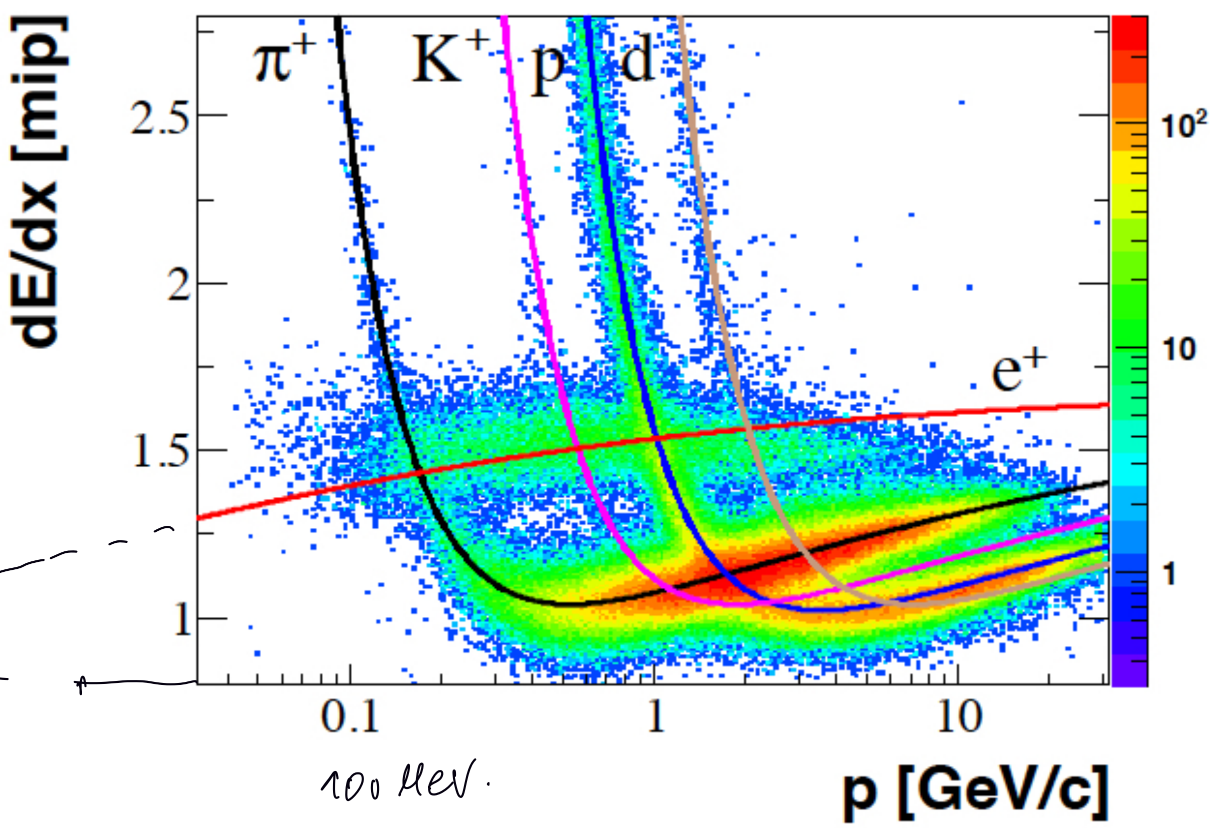
$m_\pi = 140$ MeV

$m_p \simeq 1000$ MeV

$$-\frac{1}{\rho}\frac{dE}{dx}\Big|_{min} \simeq 1-2\ \frac{\text{MeV}}{\text{cm}}\ \frac{1}{\rho}$$

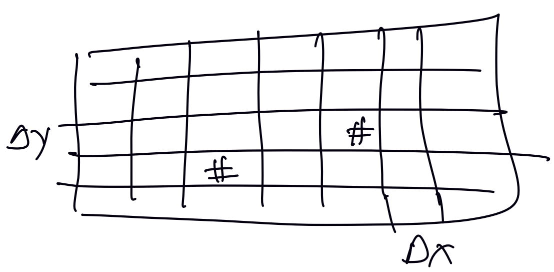
$$\rho_{H_2O} = 1\ g/cm^3 \Rightarrow 1\ \frac{\text{MeV}}{\text{cm}}$$





1.5 MeV

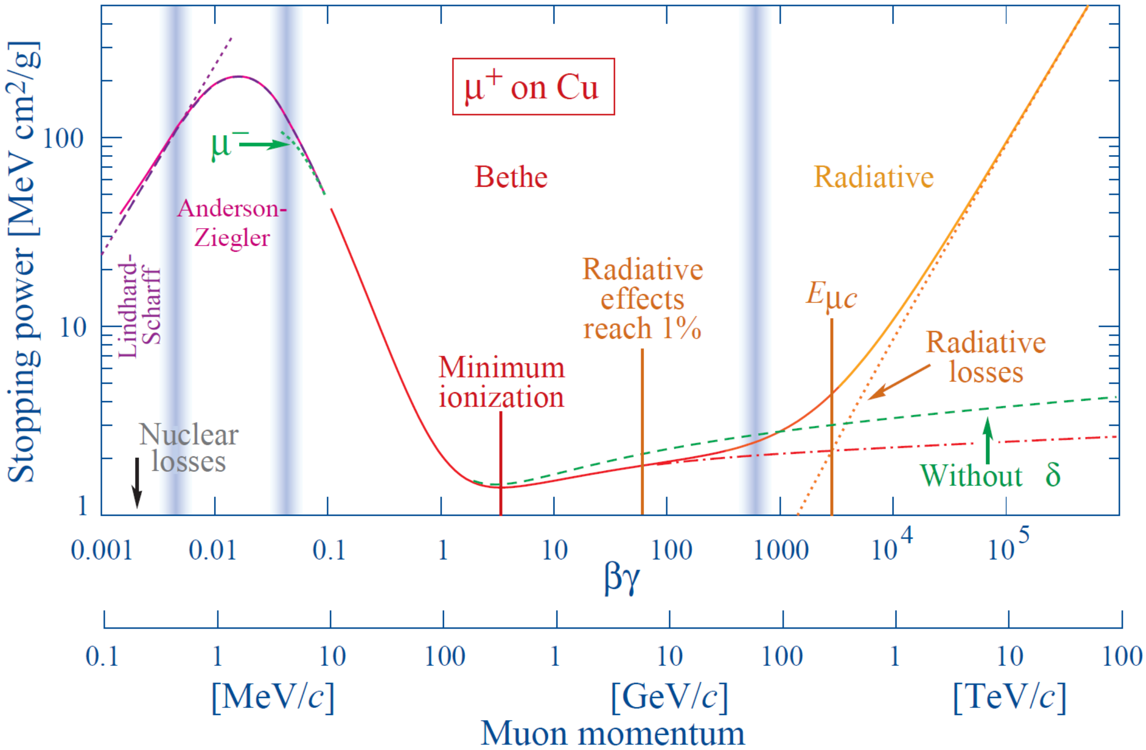
100 MeV.

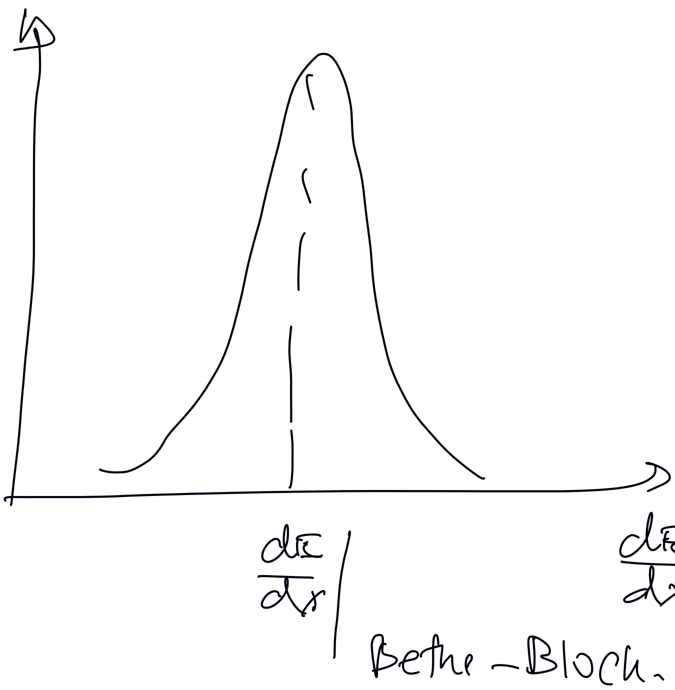


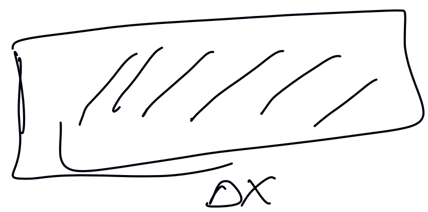
$\beta\gamma = \frac{p}{m}$.

$p = \beta\gamma m$

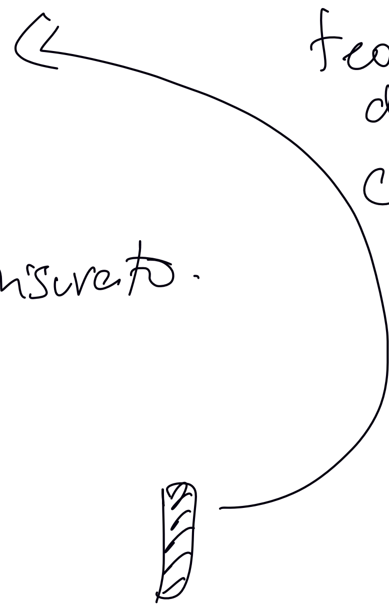
$\beta\gamma|_{min} = 3$ $\quad m_e = 0.5$ MeV $\Rightarrow P^e_{min} = 3 \cdot m_e = 1.5$ MeV







$\frac{dE}{dx}$ misurato.



teorema del limite centrale.

Landau