$E^2 = p^2 + m^2$

$K = E - m$

$p \ll m$ $\quad E = \sqrt{m^2(1+p^2/m^2)}$ $\quad (1+\varepsilon)^\alpha \simeq 1+\alpha\varepsilon$

$\simeq m(1+\frac{1}{2}\frac{p^2}{m^2}) = \underset{\text{massa a riposo}}{m} + \frac{p^2}{2m}$ limite non-relat.

$K = E - m = \frac{p^2}{2m}$

limite di MQ

principio di indeterm. $\quad \Delta p\, \Delta x \simeq \hbar = 1$

$\lambda_{dB} = \frac{h}{p} = \frac{2\pi\hbar}{p} = \frac{2\pi}{p}$ lunghezza d'onda di de Broglie

scala nucleare $\simeq 10^{-15}$ m $= 1$ fm

$\Delta x \simeq 1$ fm $\Rightarrow \Delta p \simeq \frac{1}{\Delta x} = \text{fm}^{-1} = 200 \text{ MeV} = 2\times10^8$ eV

$1 = \hbar c = 200$ MeV fm

$m_e = 9.1\times10^{-31}$ kg $= 0.511$ MeV

$m_p = 1.7\times10^{-27}$ kg $= 938 \simeq 1000$ MeV

$p = 200$ MeV $\quad e^-$: $E = \sqrt{m_e^2+p^2} = 200.00065$ MeV

p: $E = \sqrt{m_p^2+p^2} = 959$ MeV

En. cinetica

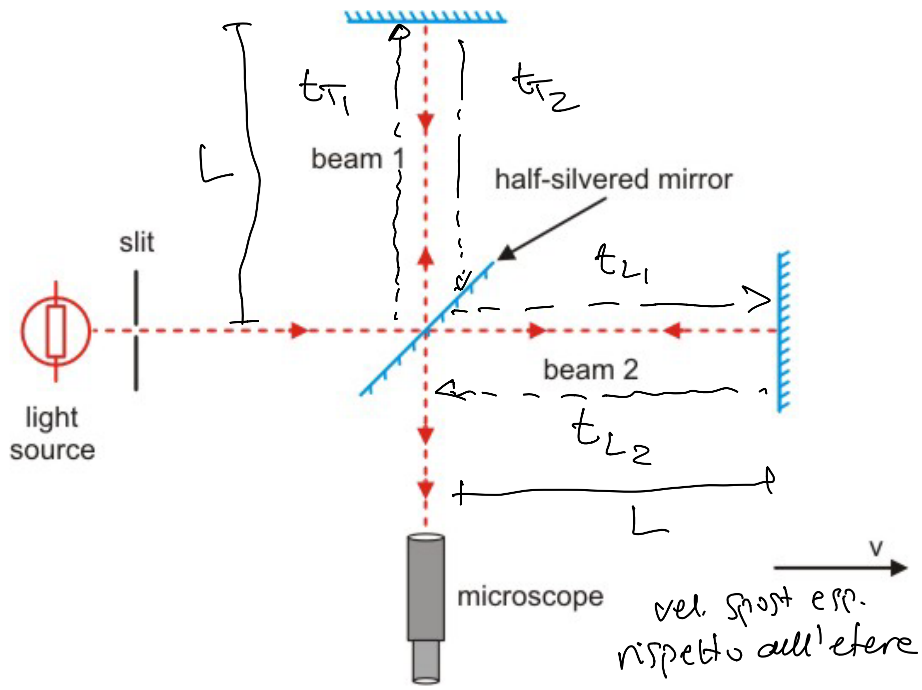
$K_e = E_e - m_e \simeq 199.5$ MeV

$K_p = 959 - 938 = 21$ MeV

sondare $r \simeq 10^{-18}$ m struttura interna dei protoni, quark.

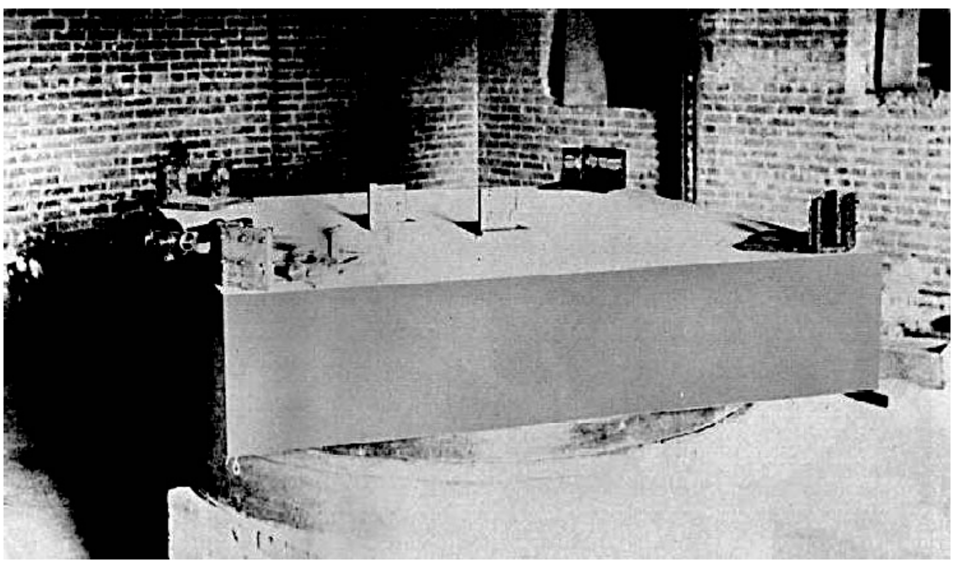
$p \gtrsim 200$ GeV

Esperimento di Michelson-Morley (1887)



$$t_T = t_{T_1} + t_{T_2}$$

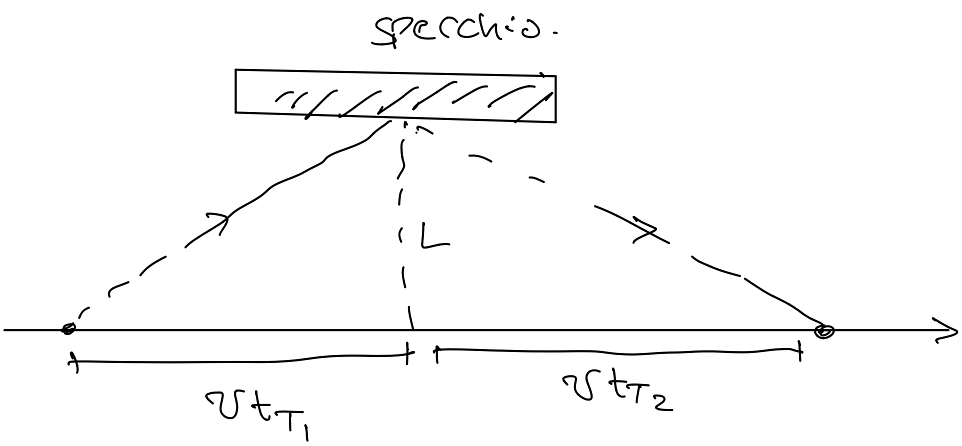
$$t_L = t_{L_1} + t_{L_2}$$



$$t_{L_1} = \frac{L}{c-v}$$

$$t_{L_2} = \frac{L}{c+v}$$

$$t_L = \frac{L}{c-v} + \frac{L}{c+v} = \frac{L(c+\cancel{v}+c-\cancel{v})}{c^2-v^2} = \frac{2L}{c^{\cancel{2}}}\,\frac{\cancel{c}}{1-v^2/c^2}$$



$$(c t_{T_1})^2 = (v t_{T_1})^2 + L^2 \Rightarrow (c^2 - v^2)\, t_{T_1}^2 = L^2$$

$$t_{T_1}^2 = \frac{L^2}{c^2 - v^2} \Rightarrow t_{T_1} = \frac{L}{\sqrt{c^2}} \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{L}{c} \frac{1}{\sqrt{1-v^2/c^2}}$$

$$t_{T_2} = t_{T_1}$$

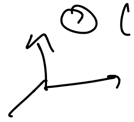
$$t_T = \frac{2L}{c} \frac{1}{\sqrt{1-v^2/c^2}}$$

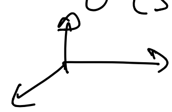
$$t_T = \frac{2L_T}{c} \frac{1}{\sqrt{1-v^2/c^2}} \stackrel{?}{=} \frac{2L_L}{c} \frac{1}{(\sqrt{1-v^2/c^2})^2} = t_L$$

sperimentalmente $t_L = t_T$
nessuno spostamento delle fig di interf.

$$L_T = \frac{L_L}{\sqrt{1-v^2/c^2}} \Rightarrow L_L = \sqrt{1-v^2/c^2}\, L_T$$

$\underline{x} = (t, \vec{x})$


O (LAB)


O' (solidale con part.)
$(t', \vec{x}')$

$$\underline{x} = L(\beta)\, \underline{x}' \qquad L = \begin{pmatrix} \gamma & \beta\gamma & & \\ \beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$|\underline{x}|^2 = t^2 - |\vec{x}|^2$ inv.

$E^2 = p^2 + m^2$ da dove viene?

punto materiale in un sist di rif inerziale.
- spazio isotropo
- spazio omogeneo
- tempo omogeneo

azione $\quad S = \int_{t_1}^{t_2} L\,dt = \int_{t_1}^{t_2} L(\vec{x},\dot{\vec{x}},t)\,dt$

invariante in tutti sist. di rif. inerz.

$$\underline{dx} = (dt, d\vec{x})$$

$$ds^2 = dt^2 - |d\vec{x}|^2 = d\tau^2$$

$$S = A\int d\tau = A\int \frac{dt}{\gamma} = A\int \sqrt{1-v^2/c^2}\,dt$$

$$\boxed{dt = \gamma\, d\tau}$$

$$\lim_{v\to 0} A\sqrt{1-v^2/c^2} = A\left(1-\frac{1}{2}\frac{v^2}{c^2}\right) = A - \frac{A}{2}\frac{v^2}{c^2} = L_{\text{class.}}$$

Lag. part. libera non relat.

cost. arbit

$$-\frac{A}{\cancel{2}}\frac{\cancel{v^2}}{c^2} = \frac{1}{\cancel{2}} m \cancel{v^2} \Rightarrow A = -mc^2$$

$$L = -mc^2\sqrt{1-v^2/c^2} \qquad v = |\dot{\vec{x}}| = \left|\frac{d\vec{x}}{dt}\right|$$

$$v_i = \frac{dx_i}{dt}$$

$$\frac{\partial L}{\partial x_i} = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i}$$

$$L = L(\dot{x}_i^2) = \frac{\partial L}{\partial x_i} = \emptyset \Rightarrow \frac{\partial L}{\partial \dot{x}_i} = P_i = \text{cost}$$

$$\frac{\partial L}{\partial \dot{x}_i} = \cancel{-}m\cancel{c^2}\frac{1}{\cancel{2}\sqrt{1-v^2/c^2}}\left(\cancel{-}\cancel{2}\dot{x}_i\frac{1}{\cancel{c^2}}\right) = \frac{m\dot{x}_i}{\sqrt{1-v^2/c^2}}$$

$$= m\gamma\dot{x}_i \qquad \boxed{\vec{p} = \gamma m\vec{v}}$$

$$E = H = \sum_i \dot{x}_i P_i - L = \gamma m v^2 + \frac{mc^2}{\gamma} = \gamma m c^2\left(\frac{v^2}{c^2}+\frac{1}{\gamma^2}\right)$$

$$\gamma m\vec{v}\cdot\vec{v}$$

$$= \gamma m c^2 \left(\cancel{\beta^2} + 1 - \cancel{\beta^2} \right) = \gamma m c^2$$

$$E = \gamma m c^2 = \frac{m c^2}{\sqrt{1 - v^2/c^2}}$$

$$\underline{P} = \left(\frac{E}{c}, \vec{p} \right) = \left(\gamma m c, \gamma m \vec{v} \right)$$

$$|\underline{P}|^2 = \gamma^2 m^2 c^2 - \gamma^2 m^2 v^2 = \text{inv.} = \cancel{\gamma^2} m^2 c^2 \left(1 - \cancel{\frac{v^2}{c^2}} \right)$$

$$= m^2 c^2$$

$$|\underline{P}|^2 = \frac{E^2}{c^2} - |\vec{p}|^2 = m^2 c^2 .$$

$$\Rightarrow \boxed{E^2 = m^2 c^4 + p^2 c^2}$$

In unità naturali $\quad E^2 = p^2 + m^2$

$$\underline{P} = \left(\gamma m, \gamma m \vec{\beta} \right)$$

$$\gamma = \frac{E}{m} \qquad \beta = \frac{|\vec{p}|}{E} \qquad \beta\gamma = \frac{|\vec{p}|}{m}$$

$$\vec{\beta} = \frac{\vec{p}}{E}$$

$$\underline{u} = \frac{d\underline{x}}{d\tau} = \left(\frac{dt}{d\tau}, \frac{d\vec{x}}{d\tau} \right) \qquad dt = \gamma d\tau . \qquad d\tau = \frac{1}{\gamma} dt$$

$$= \left(\gamma, \gamma \frac{d\vec{x}}{dt} \right) = \left(\gamma, \gamma \vec{v} \right)$$