Isospin

1) n, p hanno masse simili: $\frac{m_p - m_n}{m_n} \simeq \frac{1}{1000} \simeq 1‰$

2) $\sigma_{nucl}(p+X) \simeq \sigma_{nucl}(n+X)$ sezione d'urto adronica

nucleone $= \begin{pmatrix} p \\ n \end{pmatrix}$ doppietto di isospin $I = 1/2$

$|p\rangle = |I = 1/2, I_3 = 1/2\rangle$

$|n\rangle = |I = 1/2, I_3 = -1/2\rangle$

pioni: $m_{\pi^\pm} \simeq 135$ MeV $\quad m_{\pi^0} \simeq 140$ MeV

$\frac{m_{\pi^0} - m_{\pi^\pm}}{m_{\pi^0}} \simeq \frac{5}{135} \simeq 4\%$

pione $= \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$ $I = 1$

$|\pi^+\rangle = |I=1, I_3 = +1\rangle$

$|\pi^0\rangle = |I=1, I_3 = 0\rangle$

$|\pi^-\rangle = |I=1, I_3 = -1\rangle$

Deuterio: ${}^2_1H$ (pn)

$|p\rangle|n\rangle = |\tfrac{1}{2}, +\tfrac{1}{2}\rangle|\tfrac{1}{2}, -1/2\rangle$

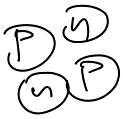
$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

$I_3(\text{deut}) = +\frac{1}{2} - \frac{1}{2} = 0$ $\quad I_3^{tot} = 0 \; \begin{cases} I = 0 \\ I = 1 \end{cases}$?

$$\mathbb{1} = \begin{cases} |+\rangle|+\rangle & |p\rangle|p\rangle \\ (|+\rangle|-\rangle + |-\rangle|+\rangle)\frac{1}{\sqrt{2}} & (|p\rangle|n\rangle + |n\rangle|p\rangle) \quad |I^{tot}=1, I_3^{tot}=0\rangle \\ |-\rangle|-\rangle & |n\rangle|n\rangle \end{cases}$$

$$0 = (|p\rangle|n\rangle - |n\rangle|p\rangle)\frac{1}{\sqrt{2}} \qquad |I^{tot}=0, I_3^{tot}=0\rangle$$

$|d\rangle \in 0$ oppure $\mathbb{1}$?

^4_2He 

$pp = ?$ $\qquad \cancel{^2_2N}$ $\qquad$ stato legato

nn $\qquad \cancel{^{A=2}_{Z=0}N}$ $\qquad$ nè stato legato.

Non esistono

$|d\rangle = pn$ stato legato $\equiv$ deuterio ✓

Ipotesi: $|d\rangle \equiv$ singoletto di isospin $\equiv |I^{tot}=0, I_3=0\rangle$

^1_1H idrogen. $\qquad$ ^2_1H deuterio $\qquad$ ^3_1H trizio

Esperimenti di diffusione

(a)

	p	$+\ p$	$\longrightarrow d$	$+\ X^+$
Q	1	1	1	1
B	1	1	2	0
I_3	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+1$

Se I si conserva nelle int. forti

$$X^+ \equiv \pi^+$$

$|i\rangle = |p\rangle|p\rangle = |I=1, I_3=+1\rangle$

$|f\rangle = |d\rangle + |\pi^+\rangle$

ipotesi: $|d\rangle = |I=0, I_3=0\rangle$

$|\pi^+\rangle = |I=1, I_3=+1\rangle$

$0 + 1 = 1$

f: $I_3 = 0 + 1 = +1 \Rightarrow I^{tot}_f = 1$

$|f\rangle = |d\rangle + |\pi^+\rangle = |I=1, I_3=+1\rangle$

$\sigma \propto |M|^2 \rho$(spazio delle fasi)

$M = \langle f | H_I | i \rangle = \langle I=1, I_3=+1 | H_I | I=1, I_3=+1\rangle$

$\sigma(p+p \to d+\pi^+) \propto |M|^2 \underbrace{\rho(p+p \to d+\pi^+)}_{\text{spazio delle fasi}}$

$H_I = H_{forte}$ $\quad H_I \, |I=1, I_3=+1\rangle = (\text{cost}) \, |I=1, I_3=+1\rangle$

H_I conserva isospin $\equiv [H, I] = 0$

b)

	p	$+\ n$	$\longrightarrow\ d$	$+\ X^0$
Q	$+1$	0	$+1$	0
B	$+1$	$+1$	$+2$	0
I_3	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0

$$X^0 \equiv \pi^0 \hookrightarrow \gamma\gamma$$

$|i\rangle = |p\rangle + |n\rangle$

$I_3 = +\frac{1}{2} - \frac{1}{2} = 0 \quad < \begin{array}{l} I=0 \\ I=1 \end{array}$

$$|i\rangle = \frac{1}{\sqrt{2}} \left(|1,0\rangle + |0,0\rangle \right)$$

$|f\rangle = |d\rangle + |\pi^0\rangle \quad = |I^{tot}=1, I_3^{tot}=0\rangle$

$I_3 = 0. \qquad |d\rangle = |I=0, I_3=0\rangle$

$I = 1 \qquad |\pi^0\rangle = |I=1, I_3=0\rangle$

$$M_{fi}^b = \langle f| H_I |i\rangle = \langle I=1, I_3=0| H_I | \left(\frac{1}{\sqrt{2}} \left(|1,0\rangle + |0,0\rangle \right) \right)$$

$$= \frac{1}{\sqrt{2}} \langle I=1, I_3=0| H_I | I=1, I_3=0\rangle +$$

$$\frac{1}{\sqrt{2}} \underline{\langle I=1, I_3=0| H_I | I=0, I_3=0\rangle}$$

$\propto \langle I=1 | I=0\rangle$

Se H_I conserva isospin.

$$M_b = \frac{1}{\sqrt{2}} \langle I=1, I_3=0| H_I | I=1, I_3=0\rangle$$

$$\frac{\sigma(p+p\to d+\pi^+)}{\sigma(p+n\to d+\pi^0)} = \frac{\overbrace{|\mathcal{M}(pp\to d\pi^+)|^2}^{\mathcal{M}^a}}{\underbrace{|\mathcal{M}(pn\to d\pi^0)|^2}_{\mathcal{M}^b}}\;\frac{\rho(pp\to d\pi^+)}{\rho(pn\to d\pi^0)}$$

$$\mathcal{M}_a(pp\to d\pi^+) \simeq \langle I=1, I_3=1 |\, H_I \,| I=1, I_3=1\rangle$$

$$\mathcal{M}_b(pn\to d\pi^0) \simeq \langle I=1, I_3=0 |\, H_I \,| I=1, I_3=0\rangle \frac{1}{\sqrt{2}}$$

p, n : $\Delta m \simeq 7‰$

$\pi^\pm, \pi^0$: $\Delta m_\pi \simeq 4\%$

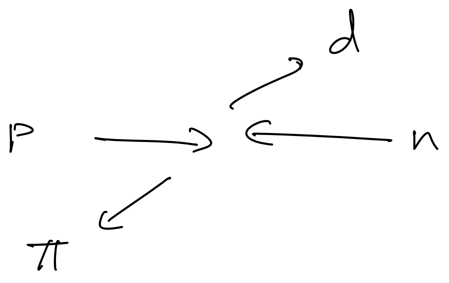
$$p + \begin{pmatrix} p \\ n \end{pmatrix} \longrightarrow \begin{pmatrix} d+\pi^+ \\ d+\pi^0 \end{pmatrix}$$

En. del fascio fissata

$$\frac{\rho(pp\to d\pi^+)}{\rho(pn\to d\pi^0)} \simeq 1 \quad \text{entro } 4\%$$

$$\frac{\#(p+\text{bersaglio}\to d+\pi^+)}{\#(p+\text{bersaglio}\to d+\pi^0)}_{iso} = \frac{\sigma(pp\to d\pi^+)}{\sigma(pn\to d\pi^0)} \simeq \frac{|\mathcal{M}_a(\ldots)|^2}{|\mathcal{M}_b(\ldots)|^2}$$

$$= \frac{|\langle I=1, I_3=1 | H_I | I=1, I_3=1\rangle|^2}{\left|\frac{1}{\sqrt{2}}\right|^2 |\langle I=1, I_3=0 | H_I | I=1, I_3=0\rangle|^2}$$



$$E^* = \frac{\sqrt{s}}{2}$$

$$p^* = \sqrt{E^{*2} - m_\pi^2}$$

$= 2$ Confermato dalle misure sperimentali

Diffusione

$\pi^+ + p \longrightarrow$

$\pi^- + p \longrightarrow$

$\pi^- + n \longrightarrow$

$\pi^- + p \longrightarrow$

$|\pi^+\rangle + |p\rangle$ $\qquad |I = 3/2, I_3 = 3/2\rangle$

$I_3 = +1 + \frac{1}{2} = 3/2$

$\frac{1}{2} \otimes 1 = \frac{1}{2} \oplus \frac{3}{2}$

$\pi^+ + n$:

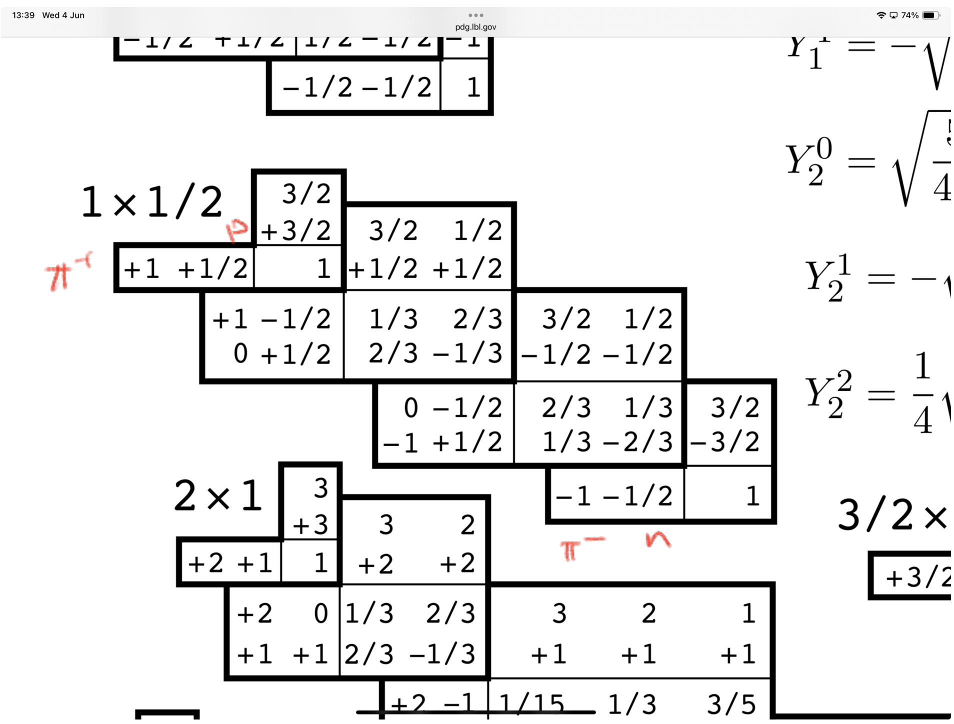
$I_3 = +1 - 1/2 = 1/2$ $\qquad I = 1/2$, $\quad I_3 = 3/2$

$\pi^- + p$:

$I_3 = -1 + 1/2 = -1/2$ $\qquad I = 1/2$, $\quad I = 3/2$

$|\pi^-\rangle + |n\rangle$ $\qquad |I = 3/2, I_3 = -3/2\rangle$

$I_3 = -1 - 1/2 = -3/2$.



Nucleo $\;{}^{A}_{Z}N$ $\qquad Z$: protoni $\qquad A-Z$: neutroni..

$$I_3^{tot} = Z\left(\frac{1}{2}\right) + (A-Z)\left(-\frac{1}{2}\right) = \frac{Z}{2} + \frac{Z}{2} - \frac{A}{2}$$

protone $\qquad\qquad$ neutrone.

$$Z = \frac{A}{2} + I_3$$

Per adroni:

$$Q = \frac{\overbrace{\#\text{Barioni}}^{B}}{2} + I_3$$

Formula di Gell-Mann Nishijima

$B = +1$ barioni $\qquad B = -1$ anti barioni

$= 0$ per mesoni (π, K)

$$\pi^+: \quad Q = \frac{B=0}{2} + I_3 = +1 = 1$$

Kaoni: particelle strane,

$$Q = I_3 + \frac{B + S}{2}$$

$S \rightarrow$ stranezza

$\begin{pmatrix} u \\ d \end{pmatrix}$ up down. $\quad$ sono doppietto di isospin $I = 1/2$.

$c, s, t, b \qquad I = 0 \quad$ singoletto di isospin.

numero quantico
strangeness
Charm
beauty
top

$$Q = I_3 + \frac{B + S + C + b + t}{2}$$

num. quantico di sapore

$$B_q = \frac{1}{3}$$

quark strano $\bar{s}$: $B = 1/3$. $S = 1$. $I_3 = 0$.

$$Q = 0 + \frac{1}{2}\left(\frac{1}{3} + 1\right) = \frac{4}{3}\frac{1}{2} = \frac{4}{6} = \frac{2}{3}$$

quark up: $I_3 = +1/2$. $S = C = b = t = 0$ $B = 1/3$

$$Q = \frac{1}{2} + \frac{1}{2}\frac{1}{3} = \frac{1}{2}\left(1 + \frac{1}{3}\right) = \frac{1}{2}\frac{4}{3} = \frac{2}{3}$$

$\begin{pmatrix} u \\ d \end{pmatrix}$ $\begin{pmatrix} c \\ s \end{pmatrix}$ $\begin{pmatrix} t \\ b \end{pmatrix}$ $\leftarrow$ doppietto di isospin debole.

$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}$ $\begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}$ $\begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$

L_e L_μ L_τ num di sapore leptonico

numero leptonico.

$p + p \longrightarrow p + p$

Q

B 1 1

$p + p \longrightarrow \cancel{\pi^+ + \pi^-}$

	p	$+p$	$\longrightarrow$	p	$+p$	$+\pi^0$
Q	+1	+1		+1	+1	0
B	+1	+1		+1	+1	0

$p + p + \pi^+ \ \pi^- + \pi^0 + \pi^+ + \pi^-$

	p	$+p$	$+K^+$	$+K^-$
S	0	0	+1	-1

$E > E_{soglia}$

$$
\begin{array}{lccccccccc}
 & p & + & p & \longrightarrow & p + p + K^{+} + \pi^{-} & & & \\
Q & + & & +1 & & +1 & +1 & +1 & -1 \\
B & 1 & & 1 & & 1 & 1 & 0 & 0 \\
S & 0 & & 0 & & 0 & 0 & 2 & 0
\end{array}
$$

$$
\begin{array}{lcccccccc}
 & p + p & \longrightarrow & p + \bar{p} & + & p + p \\
B & 1 \quad 1 & & 1 \quad -1 & &
\end{array}
$$