EX PER CASA

$$\pi^+ \to \mu^+ \nu_\mu$$

$m_\pi = 140$ MeV

$m_\mu = 106$ MeV

$m_\nu = 0$

$p(\pi^+) = 500$ MeV

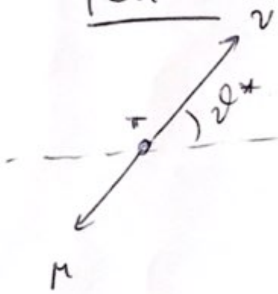
Determinare ϑ e ϑ^* in configurazioni:

① E_ν^{MAX} ② E_ν^{MIN} ③ $E_\nu = \frac{E_\nu^{MAX}}{2}$ ④ ϑ^{MAX}

$$E_\pi = \sqrt{m_\pi^2 + p_\pi^2} = 519 \text{ MeV}$$

$$\Rightarrow \beta_\pi = \frac{p_\pi}{E_\pi} = 0.963 \qquad \gamma_\pi = \frac{E_\pi}{m_\pi} = 3.70$$

$\boxed{\text{CdM}}$



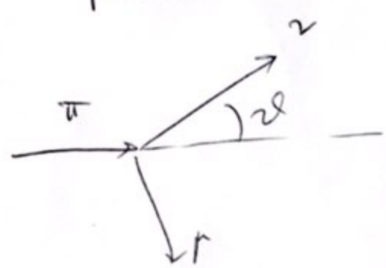
$E_\nu = p_\nu$

$p_\nu = p^*$

frutto delle masse

→ conta solo angolo

$\boxed{\text{LAB}}$



visto che $p_\perp = p_\perp^*$ $\Rightarrow$ conf. di E_{MAX} (nel LAB) è quando $\vartheta^* = 0$ $\Rightarrow$ INFATTI il neutrino si "prende" tutto il boost

$\vartheta^* = 0 \qquad \xrightarrow{p_2^*} \quad \Longrightarrow \quad \xrightarrow{\quad p_2 \quad}$

$\vartheta^* \neq 0$

$p_\parallel^* < p^*$ ← boost agisce solo su $p_\parallel^*$

$\Rightarrow$ (1): $\vartheta^* = 0$
$\vartheta = 0$

per (2) stesso ragionamento: E_2^{MIN} deve succedere per $\vartheta^* = \pi$

$\vartheta = 0?$ o $\pi?$

bisogna vedere se boost riesce a "flippare" momento

$\Rightarrow \quad \beta_{cm} \overset{?}{\gtrless} \beta_2^*$

$\hookrightarrow$ ma $\beta_2^* = 1$ $(= \beta_2)$ visto che $m_2 = 0$

$\Rightarrow$ non può essere flippato

$\Rightarrow$ (2): $\vartheta^* = \pi$
$\vartheta = \pi$

visto che $\vartheta = \pi$ è possibile $\Rightarrow \vartheta_{max} = \pi$!

$\Rightarrow$ (4): $\vartheta^* = \pi$
$\vartheta = \pi$

infine ③ $E_\nu = \dfrac{E_\nu^{max}}{2}$

in generale $E_\nu = \gamma_\pi \left(E_\nu^* + \beta_\pi p_\nu^* \cos\vartheta^* \right)$

$$= \gamma_\pi E_\nu^* \left(1 + \beta_\pi \cos\vartheta^* \right)$$

$$\uparrow$$

$$p_\nu^* = E_\nu^*$$

$$E_\nu^{max} = \gamma_\pi E_\nu^* \left(1 + \beta_\pi \right)$$

invece $E_\nu = \frac{1}{2} E_\nu^{max}$

$$\Leftrightarrow \quad \gamma_\pi E_\nu^* \left(1 + \beta_\pi \cos\vartheta^* \right) = \frac{1}{2} \gamma_\pi E_\nu^* \left(1 + \beta_\pi \right)$$

$$\Leftrightarrow \quad 1 + \beta_\pi \cos\vartheta^* = \frac{1}{2} \left(1 + \beta_\pi \right)$$

$$\rightarrow \quad \cos\vartheta^* = \frac{\beta_\pi - 1}{2\beta_\pi} = -0.017$$

$$\Leftrightarrow \quad \vartheta^* = \cos^{-1}(-0.017) = 1.59$$

$$\rightarrow \quad \vartheta = \tan^{-1}\left[\frac{\sin\vartheta^*}{\gamma_\pi \left(\frac{\beta_\pi}{\beta^*} + \cos\vartheta^* \right)} \right] = 0.25$$

EX Un $\bar{p}$ con $p_{\bar{p}} = 2.2$ GeV urta contro un bersaglio dando luogo a

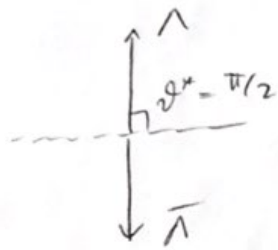
$$\bar{p} + p \rightarrow \Lambda + \bar{\Lambda}$$

$m_p = 938$ MeV
$m_\Lambda = 1116$ MeV

s.i. s.f.

LAB $\quad \overset{\bar{p}}{\longrightarrow} \overset{p}{\cdot}$?

CdM $\quad \overset{\bar{p}}{\longrightarrow} \overset{p}{\longleftarrow}$



Cominciamo con calcolare $\sqrt{s}$ che è uguale <u>OVUNQUE</u> per comodità me lo calcolo nel <u>LAB</u> nello <u>s.i.</u>

$\overset{\bar{p}}{\longrightarrow} \overset{p}{\cdot}$

$\bar{p} : \begin{pmatrix} E_{\bar{p}} \\ \vec{p}_{\bar{p}} \end{pmatrix} \quad p : \begin{pmatrix} m_p \\ \vec{0} \end{pmatrix}$

$$p_{\bar{p}} = 2.2 \text{ GeV}$$

$$\Rightarrow E_{\bar{p}} = \sqrt{m_{\bar{p}}^2 + p_{\bar{p}}^2} = 2.39 \text{ GeV}$$

$$\Rightarrow P_{tot} = \begin{pmatrix} E_{\bar{p}} + m_p \\ \vec{p}_{\bar{p}} \end{pmatrix}$$

$$\sqrt{s} = |P_{tot}| = \sqrt{(E_{\bar{p}} + m_p)^2 - p_{\bar{p}}^2} =$$

$$= \sqrt{\underbrace{E_{\bar{p}}^2 + m_p^2 + 2E_{\bar{p}} m_p - p_{\bar{p}}^2}_{= m_p^2}} = \sqrt{2m_p^2 + 2E_{\bar{p}} m_p}$$

$$= 2.50 \text{ GeV}$$

Sappiamo anche che

$$\sqrt{s} = \sum_i E_i^* \longleftarrow \text{in s.i. ma anche in s.f.}$$

$\Rightarrow$ in s.f. $\sqrt{s} = E_\Lambda^* + E_{\bar\Lambda}^* = 2E_\Lambda^*$

$|p_\Lambda^*| = |p_{\bar\Lambda}^*|$ $\quad (m_\Lambda = m_{\bar\Lambda})$

$\Rightarrow E_\Lambda^* = E_{\bar\Lambda}^*$

$\Rightarrow E_\Lambda^* = E_{\bar\Lambda}^* = \frac{\sqrt{s}}{2} = 1.25$ GeV

$p_\Lambda^* = p_{\bar\Lambda}^* = p^* = \sqrt{E_\Lambda^{*} - m_\Lambda^2} = 0.56$ GeV

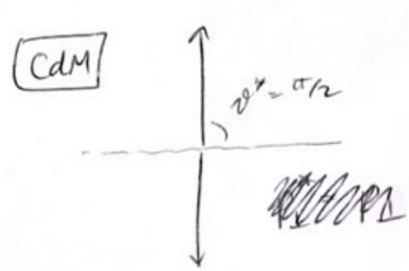
(2) $E_\Lambda, p_\Lambda = ?$ (nel LAB)

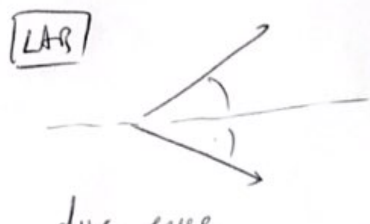
$$\begin{array}{ccc} \underline{\text{s.i.}} & & \underline{\text{s.f.}} \end{array}$$

nel LAB: $\begin{pmatrix} E_{\bar p} + m_p \\ \vec{p}_{\bar p} \end{pmatrix} = \begin{pmatrix} E_\Lambda + E_{\bar\Lambda} \\ \vec{p}_\Lambda + \vec{p}_{\bar\Lambda} \end{pmatrix}$

dalla prima $\quad E_{\bar p} + m_p = E_\Lambda + E_{\bar\Lambda} = 2E_\Lambda$

infatti se $\vartheta^* = \frac{\pi}{2}$





due curve simmetriche

$\Rightarrow E_\Lambda = \dfrac{E_{\bar{p}} + m_p}{2} = 1.66 \text{ GeV}$

$\rightarrow P_\Lambda = \sqrt{E_\Lambda^2 - m_\Lambda^2} = 1.23 \text{ GeV}$

(3) $\vartheta_\Lambda = ?$

ora $p_\perp = p_\perp^*$ e $p_\perp^* = p^* = 0.56 \text{ GeV}$

$\Rightarrow (P_\Lambda)_\perp = 0.56 \text{ GeV}$

$\Rightarrow (P_\Lambda)_\parallel = \sqrt{P_\Lambda^2 - (P_\Lambda)_\perp^2} = 1.1 \text{ GeV}$

$\Rightarrow \vartheta = \tan^{-1}\left(\dfrac{P_\perp}{P_\parallel}\right) = 0.47 \sim 27^\circ$

(4) se $\tau_\Lambda = 2.63 \cdot 10^{-10} s$ calcolare cammino libero medio di Λ nel LAB

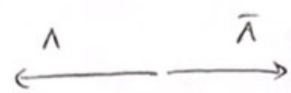
$\gamma_\Lambda = \dfrac{E_\Lambda}{m_\Lambda} = 1.49 \qquad \beta_\Lambda = \dfrac{P_\Lambda}{E_\Lambda} = 0.74$

$\tau_\Lambda \longrightarrow \gamma_\Lambda \tau_\Lambda$

$\Rightarrow \lambda_\Lambda = \beta_\Lambda \gamma_\Lambda c \tau_\Lambda = 0.087 m = 8.7 \text{ cm}$

(B) se $\vartheta^* = 0°$ e $180°$ $\rightarrow$ $\vartheta = ?$

$\boxed{\text{CdM}}$ $\boxed{\text{LAB}}$



se $\bar{\Lambda}$ ha $\vartheta^*_{\bar{\Lambda}} = 0 \Rightarrow \vartheta_{\bar{\Lambda}} = 0$

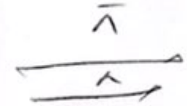
se Λ ha $\vartheta^*_{\Lambda} = \pi \Rightarrow \vartheta_{\Lambda} = 0$ oppure π

dobbiamo confrontare β^*_Λ con β_{cm}

$$\beta^*_\Lambda = \frac{p^*_\Lambda}{E^*_\Lambda} = \frac{0.56}{1.25} = 0.45$$

$$\beta_{cm} = \frac{|\vec{p}_{TOT}|}{E_{TOT}} = \frac{p_{\bar{p}}}{E_{\bar{p}} + m_p} = 0.66$$

$\beta_{cm} > \beta^*_\Lambda \Rightarrow$ flippa $\Rightarrow \vartheta_\Lambda = 0$

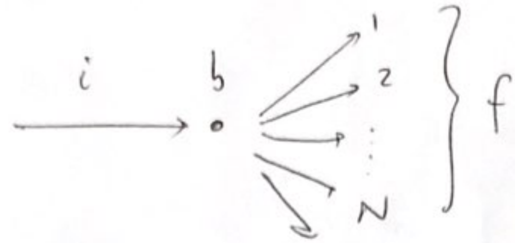


~~(C) Si può avere $\vartheta = 90°$ nel LAB?~~

(C) $\vartheta_{MAX} = ?$

$$\vartheta_{max} = \tan^{-1}\left(\frac{\beta^*_\Lambda}{\gamma_{cm}\sqrt{\beta_{cm}^2 - \beta^{*2}_\Lambda}}\right) = 0.61 \sim 35°$$

ENERGIA DI SOGLIA



$$\sqrt{s}\Big|_{\substack{s.f.\\ CdM}} = \sum_f E_f^* = \sum_f \left(m_f + K_f^* \right)$$

$$\sqrt{s}\Big|_{\substack{s.i.\\ LAB}} = \left| P_{TOT} \right| = \sqrt{E_{TOT}^2 - p_{TOT}^2}$$

nello s.i. nel LAB: $\quad i : \begin{pmatrix} E_i \\ p_i \end{pmatrix} \quad b : \begin{pmatrix} m_b \\ \vec{0} \end{pmatrix}$

$$\Rightarrow \sqrt{s}\Big|_{\substack{s.i.\\ LAB}} = \sqrt{(E_i + m_b)^2 - p_i^2} = \sqrt{\underbrace{E_i^2} + m_b^2 + 2E_i m_b \underbrace{- p_i^2}}$$

$$= \sqrt{m_i^2 + m_b^2 + 2E_i m_b}$$

ora $\quad \sqrt{s}\Big|_{\substack{s.f.\\ CdM}} = \sqrt{s}\Big|_{\substack{s.i.\\ LAB}}$

!

$$\Rightarrow \sqrt{m_i^2 + m_b^2 + 2E_i m_b} = \sum_f \left(m_f + K_f^* \right) \geq \sum_f m_f$$

(quadro)

$$\Leftrightarrow \quad m_i^2 + m_b^2 + 2E_i m_b \geq \left(\sum_f m_f \right)^2$$

$$\Rightarrow \quad E_i \geq \frac{\left(\sum_f m_f\right)^2 - m_i^2 - m_b^2}{2m_b} \equiv E_{soglia}$$

$$\Leftrightarrow \quad K_{soglia} = E_{soglia} - m_i = \frac{\left(\sum_f m_f\right)^2 - (m_i + m_b)^2}{2m_b}$$

$\boxed{\text{EX}}$ esercizio di prima

$$\bar{p} + p \longrightarrow \Lambda + \bar{\Lambda}$$

$m_p = 938$ MeV

$m_\Lambda = 1116$ MeV

$$\xrightarrow{\ \bar{p}\ } \overset{p}{\bullet}$$

$$E_{soglia}(\bar{p}) = \frac{(2m_\Lambda)^2 - m_p^2 - m_p^2}{2m_p} = \frac{4m_\Lambda^2 - 2m_p^2}{2m_p} =$$

$$= 1.72 \text{ GeV}$$

$$\Leftrightarrow \quad K_{soglia} = E_{soglia} - m_p = (\underbrace{1.72 - 0.938}_{\text{GeV}}) = 780 \text{ MeV}$$

$\boxed{\text{EX}}$

Calcolare energia di soglia di

$$p + p \longrightarrow \bar{p} + p + p + p$$

NON È NECESSARIO RICORDARSI LA FORMULA A MEMORIA!

s.i. $\quad p \longrightarrow p$ $\qquad$ s.f. $\quad p\; p\; p\; \bar{p}$

$$\underbrace{\begin{pmatrix} E_p \\ \vec{p}_p \end{pmatrix} + \begin{pmatrix} m_p \\ 0 \end{pmatrix}}_{\text{nel LAB}} \Rightarrow \sqrt{s}\Big|_{\substack{s.i.\\ LAB}} = \sqrt{(E_p+m_p)^2 - p_p^2} =$$

$$= \sqrt{E_p^2 + m_p^2 + 2E_p m_p - p_p^2}$$

$$= \sqrt{2m_p^2 + 2E_p m_p}$$

poi mi calcolo $\sqrt{s}\Big|_{\substack{s.f.\\ CdM}}$ ALLA SOGLIA $\Leftrightarrow$ tutte le particelle sono ferme NEL CENTRO DI MASSA

$$\sqrt{s}\Big|_{\substack{s.f.\\ CdM\\ soglia}} = \sum_f m_f = 4m_p$$

$$\Rightarrow \sqrt{s}\Big|_{\substack{s.i.\\ LAB}} = \sqrt{s}\Big|_{\substack{s.f.\\ CdM\\ soglia}}$$

$$\Leftrightarrow \sqrt{2m_p^2 + 2E_p m_p} = 4m_p$$

(quadro) $\quad 2m_p^2 + 2E_p m_p = 16\, m_p^2$

$$\Leftrightarrow \quad E_{p_{soglia}} = \frac{16m_p^2 - 2m_p^2}{2m_p} = \frac{14m_p^2}{2m_p} = 7m_p \sim 6.9 \text{ GeV}$$

$$\Rightarrow \quad K_{soglia} = E_{soglia} - m_p = 5.9 \text{ GeV}$$

DOMANDA: come cambia se ~~gas~~ atomi bersaglio = gas di Fermi

$\xrightarrow{p} \overset{p}{\bullet} \quad \Longrightarrow \quad \xrightarrow{p}$

$\langle p \rangle \sim 240$ MeV $= p_F$
direzione casuale
$\Rightarrow E_F = \sqrt{m_p^2 + p_F^2} = 1.01$ GeV

$$\begin{pmatrix} E \\ \vec{p} \end{pmatrix} + \begin{pmatrix} m_p \\ \vec{0} \end{pmatrix} \Rightarrow \begin{pmatrix} E \\ \vec{p} \end{pmatrix} + \begin{pmatrix} E_F \\ \vec{p}_F \end{pmatrix}$$

$$\Rightarrow \sqrt{s}\Big|_{s.r.\,LAB} = \sqrt{(E+E_F)^2 - (\vec{p}+\vec{p}_F)^2} =$$

$$= \sqrt{E^2 + E_F^2 + 2EE_F - p^2 - p_F^2 - 2(\vec{p}\cdot\vec{p}_F)} =$$

$$= \sqrt{2m_p^2 + 2EE_F - 2\,p\,p_F \cos\alpha}$$

$\uparrow$ angolo di p di bersaglio

$\Leftrightarrow$

$\sqrt{s} = \sqrt{s}(\alpha)$

$\sqrt{s}$ è MIN quando $\alpha = 0$ $\quad \xrightarrow{p} \overset{p_F}{\rightarrow}$

$\sqrt{s}$ è MAX quando $\alpha = 180°$ $\quad \xrightarrow{p} \overset{p_F}{\leftarrow}$

LO STATO INIZIALE È MIX DI TUTTI GLI α

Quello che ci interessa è caso MAX

$\to$ sono con maggior energia a disposizione

infatti $\quad \sqrt{s}\Big|_{MAX} = \sqrt{2m_p^2 + 2EE_F + 2pp_F}$

la soglia non cambia $\sqrt{s}\Big|_{soglia} = 4m_p$

per ottenere s.f. deve essere che $\sqrt{s} \geq \sqrt{s}\Big|_{soglia}$

$$\Leftrightarrow \quad \sqrt{2m_p^2 + 2EE_F + 2pp_F} \geq 4\, m_p$$

(quadro) $\quad 2m_p^2 + 2EE_F + 2pp_F \geq 16\, m_p^2$

$$\Leftrightarrow \quad 2m_p^2 + 2EE_F - 16\, m_p^2 \geq 2p_F \sqrt{E^2 - m_p^2}$$

e quadro e ottengo

$$AE^2 + BE + C = 0$$

con
$$A = 4m_p^2 = 3.86 \text{ GeV}^2$$
$$B = -56\, m_p^2 E_F = -54.6 \text{ GeV}^3$$
$$C = (14\, m_p^2)^2 + 4p_F^2 m_p^2 = 184 \text{ GeV}^4$$

$$\Rightarrow \quad E_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \begin{cases} 8.6 \text{ GeV} \\ 5.5 \text{ GeV} \end{cases}$$

MINORE DI 6.9 GeV

ottenuta con $\vec{p} = \vec{0}$

$\Rightarrow \exists$ configurazione $(\alpha = 180°)$

con $E_{soglia} < E_{soglia}\,(p = 0)$

è possibile creare stati finali anche con

fascio con energia minore