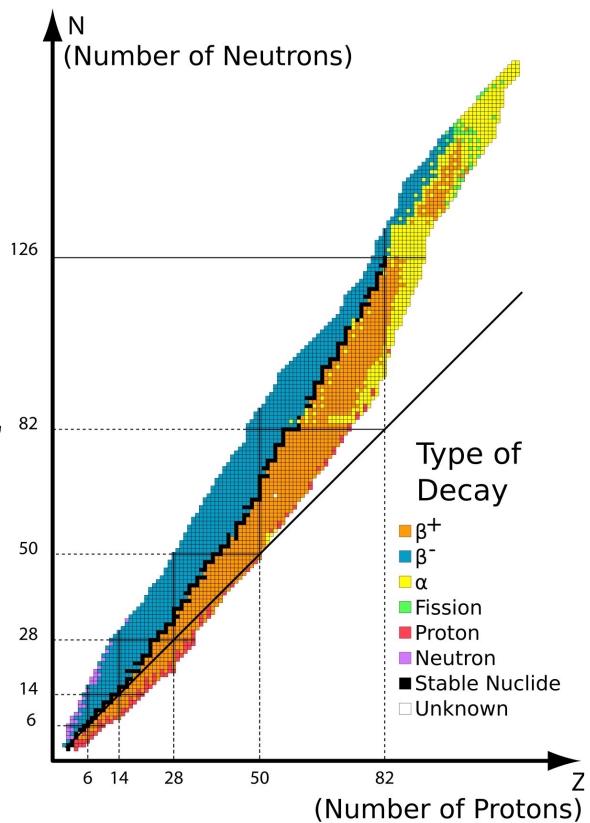
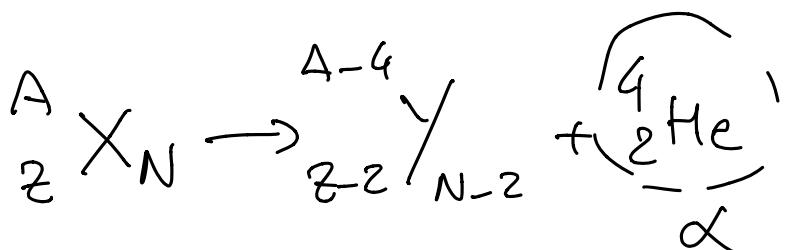


Decadimento  $\alpha$   
appare  $N > 82$   
 $Z \geq 50$

$A \geq 160$

tipici per nuclidi con  $A > 200$



### Energia di legame

$$B(F, A) = Z \mu(^1H) + (A - Z) \mu_N - \mu(Z, A)$$

Cause dell'energ. di legame:

- interazione di Coulomb (repulsiva)
- interazione forte (attrattiva)

Fattori di correzione:

- energia cinetica dei nucleoni nel nucleo.

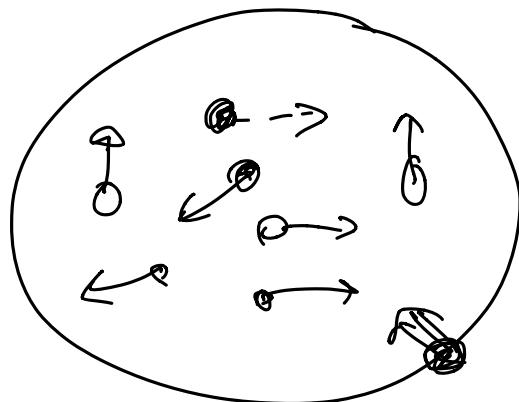
Nucleoni non sono fermi  $\square$

Principio di esclusione di Pauli:

## Modello a gas di Fermi

stione di  
energie cinetiche.

Modello statistico di particelle indipendenti



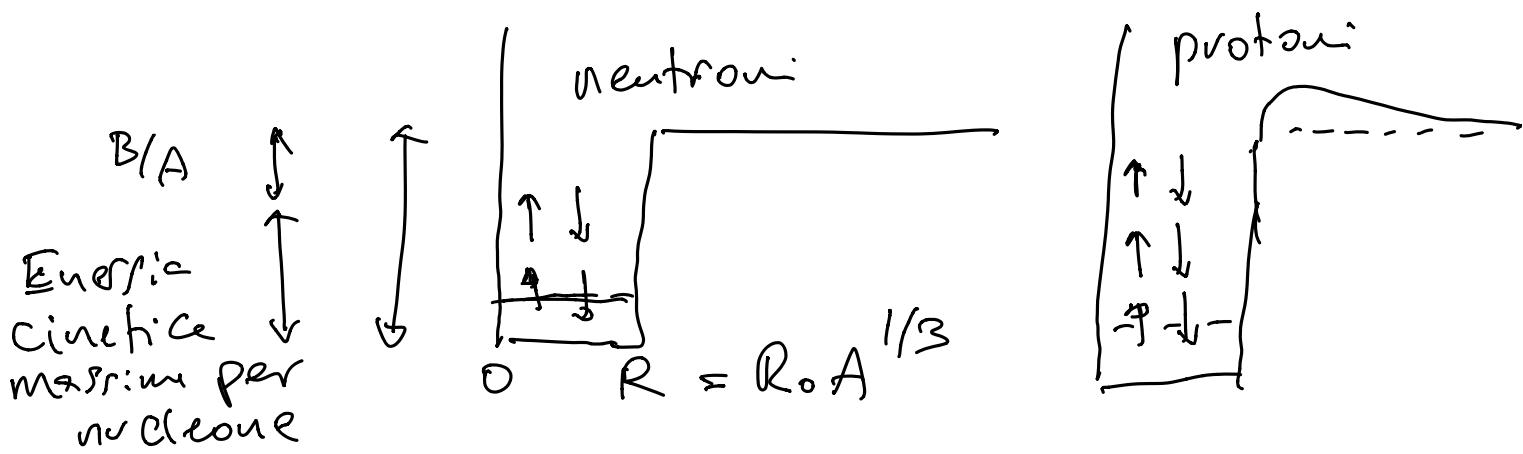
nucleo.

si muovono libere-  
mente nel nucleo.

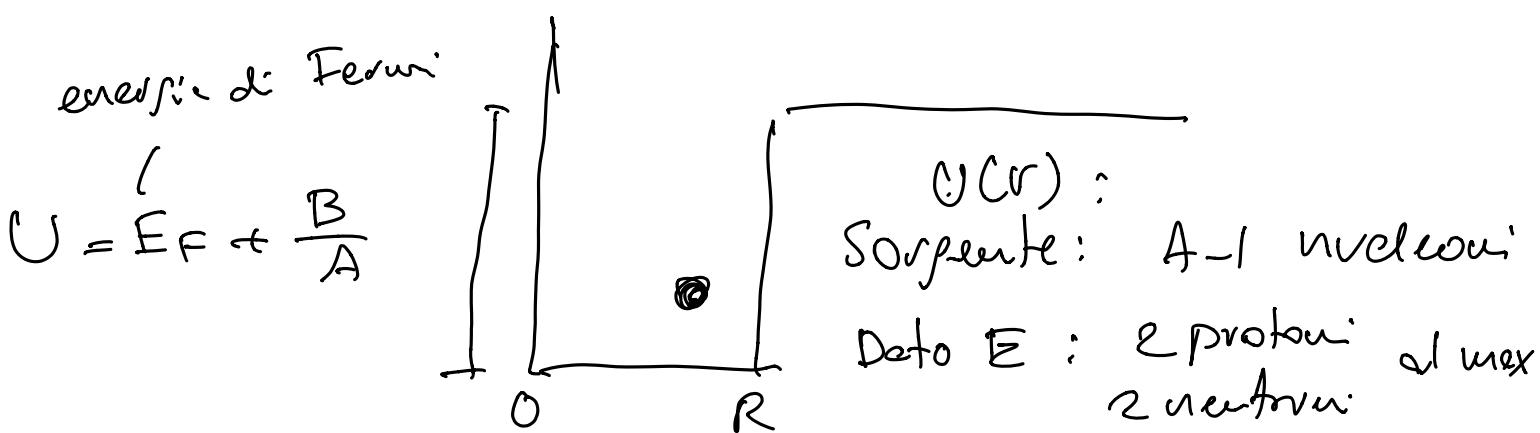
forte attrazione dei  
nucleoni se si  
avvicina sul bordo.

- azione di A-1 nucleoni RPP.

da un buco di potere totale a Sfera-sfera

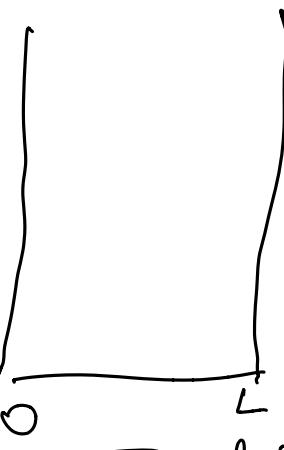


-  $E_K \gg kT$  oppure  $T \approx 0$



$$\psi(x) = A \sin(Kx)$$

$$K = \frac{P}{\hbar} \quad \hbar \approx 1 \quad \Rightarrow$$



$$\psi(x) = A \sin(Px)$$

$$\psi(0) = \psi(L) = 0$$

$$PL = n\hbar \quad \Rightarrow$$

$$P = n \frac{\pi}{L}$$

large  $\pi$   
base

In 3D:

$$P_x = n_x \frac{\pi}{L}$$

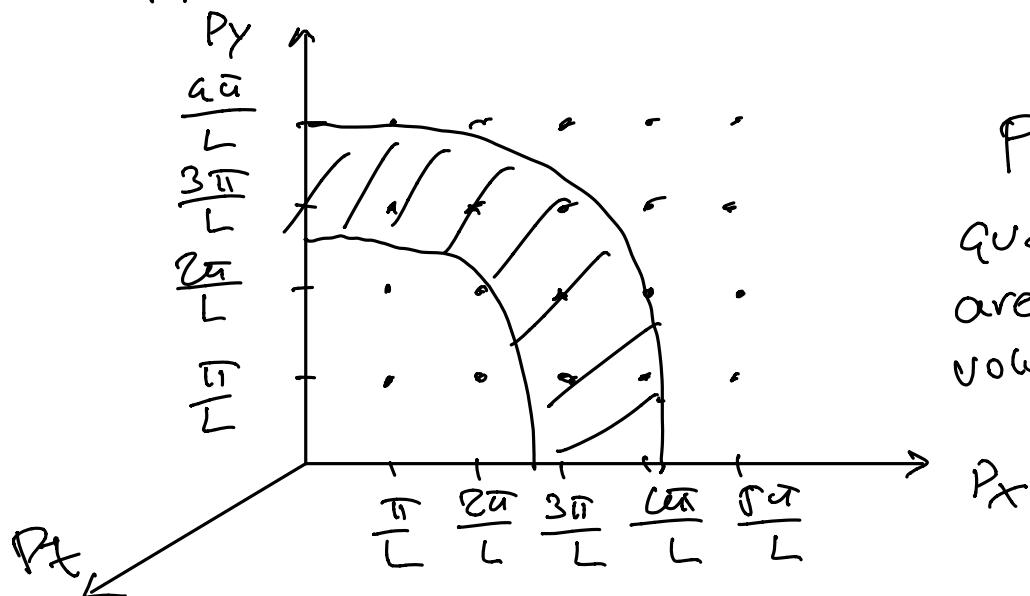
$$P_y = n_y \frac{\pi}{L}$$

$$P_z = n_z \frac{\pi}{L}$$

$$E = \frac{P^2}{2m} = \frac{\pi^2}{2mL^2} \left( n_x^2 + n_y^2 + n_z^2 \right) = \frac{\pi^2}{2m} n^2$$



Quanti stati possibili dato impulso  $P$ , oppure energia  $E$



$$P \rightarrow P_f dP$$

Quanti stati  
area corona  
volume guscio

$dN$  con  $P \in [P_1, P_2 dP]$

$$dN = \frac{1}{Z} \frac{4\pi P^2 dP}{\left(\frac{\pi}{L}\right)^3} \quad P_x, P_y, P_z > 0 \\ n_i > 0.$$

↳  $V_F$  def volum del guscio

$$dN = \frac{4\pi a^3}{(2\pi)^3} P^2 dP = \frac{V}{(2\pi)^3} 4\pi P^2 dP$$

$$dN = \frac{V}{(2\pi)^3} 4\pi P^2 dP$$

$$\frac{dN}{dP} \propto P^2$$

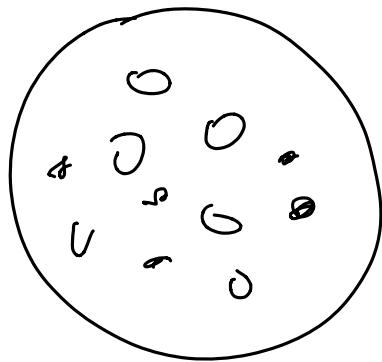
$$\frac{dN}{dE} = \rho(E) \quad E = \frac{P^2}{2m}$$

$$P = \sqrt{2mE} \quad dP \approx dE = \frac{EPdP}{2m}$$

$$\frac{dN}{dE} \propto E^{1/2}$$

$$E_F(P) \quad E_F(F)$$
$$\int_0^P dN_P = \int_0^F Z dN = Z \quad \text{protoni}$$

$$E_F(n) \quad E_F(n)$$
$$\int_0^P dN_n = \int_0^F N dN = A - Z \quad \text{neutroni}$$



A nuclear

$A - Z$  neutron  
 $Z$  proton

Da  $Z, A - Z \Rightarrow$  Stilme  $E_F(p)$   
 $E_F(a)$

$$dn = \frac{V}{(2\pi)^3} 4\pi p^2 dp$$

$$Z = Z \int_0^{E_F} \frac{V}{(2\pi)^3} 4\pi p^2 dp$$

$$= \frac{2V}{(2\pi)^3} \frac{1}{3} p_F^3 \quad E_F = \frac{p_F^2}{2m}$$

$$V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} R_0^3 A \quad \begin{matrix} \text{volume} \\ \text{nucleo} \end{matrix}$$

$$Z = \frac{g}{g\pi} R_0^3 A p_F^3$$

$$p_F(p) = \left(\frac{g\pi}{4}\right)^{1/3} \frac{1}{R_0} \left(\frac{Z}{A}\right)^{1/3}$$

$$R_0 = 1.25 \text{ fm.}$$

$$= (240 \text{ Mev}) \left(\frac{Z}{A}\right)^{1/3}$$

$$A - \varepsilon = \varepsilon \int_0^{E_F(n)} dn = \dots = (260 \text{ MeV}) \left( \frac{\varepsilon(A-\varepsilon)}{A} \right)^{1/3}$$

Nuclei lesser  $A \approx 27$   $A - \varepsilon \approx 2$

$$P_F(n) \approx P_F(p) \approx 260 \text{ MeV}$$

$$E_F = \frac{P^2}{2m} \approx 80 \text{ MeV}$$

Energic Chetic mesime per i nucleon  
nel nucleo

$\Rightarrow$  possono usare MQ non rel.

per calcoli nucleon

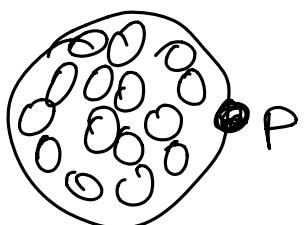
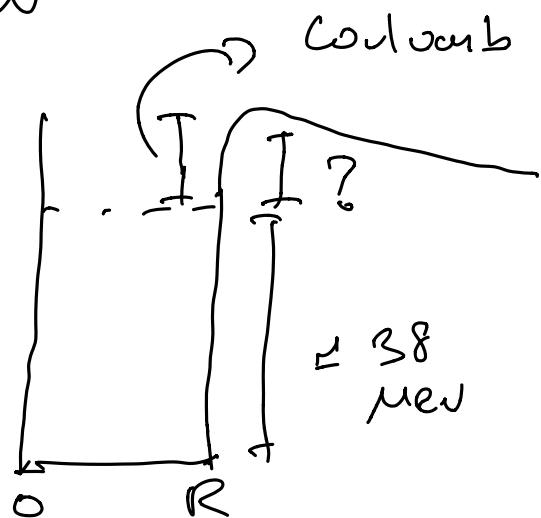
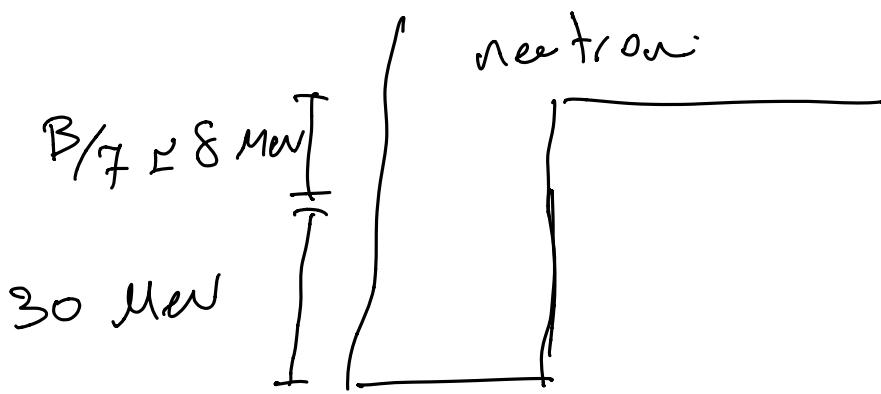
$A$  grande

238

92 U

$$E_F^n = 82 \text{ MeV}$$

$$E_F^p = 28 \text{ MeV}$$



$$U(R) \approx \frac{Ze^2}{4\pi} \frac{1}{R} = Z \frac{e^2}{4\pi} \frac{1}{R_0} A^{-1/3}$$

$$= \alpha Z \frac{1}{R_0} A^{-1/3}$$

$$\hookrightarrow \frac{1}{137} \frac{1}{R_0} = \frac{1}{1.25 \text{ fm}} = 158 \text{ MeV}$$

$$U(R) \approx (1.2 \text{ MeV}) Z A^{-1/3}$$

$$^{238}_{92} \text{U} : U(R) = 1.2 \text{ MeV} \times 92 \times (238)^{-1/3} \approx 18 \text{ MeV.}$$

$\frac{B}{A}$  ci serve  $\langle E_c \rangle$  energia critica media

$$\langle E_c \rangle = \frac{\int \frac{P^2}{2m} dn}{\int dn}$$

$$\int dn = 2 \int du_p + 2 \int du_n = Z + (A - Z) \approx A$$

$$dn = 2 \frac{V}{(2\pi)^3} 4\pi P^2 dP$$

già calcolato.

$$\int \frac{P^2}{2m} dn \approx C \int_0^P P^4 dP$$

$$\langle E_C \rangle = \frac{4}{3\pi} R_0^3 \left[ \frac{P_F(P)^5}{10^{mp}} + \frac{P_F(n)^5}{10^{mn}} \right]$$

$P_F(P)$ ,  $P_F(n)$  e sostitute.   
 massimo  
 1 GeV

$$= \underbrace{\frac{4}{3\pi} \frac{R_0^3}{10^{mn}} \left( \frac{9\pi}{8} \right)^{5/3}}_{C = 9 \text{ MeV}} \frac{1}{R_0^5} \left[ \left( \frac{2Z}{A} \right)^{5/3} + \left( \frac{2(A-Z)}{A} \right)^{5/3} \right]$$

$$\langle E_C \rangle = (9 \text{ MeV}) [ ]$$

Mass: or parte dei nuclei:  $A \geq 2Z$   
nuclei presenti:  $A \geq 2Z$

$$\frac{2Z}{A} = 1 - x \quad x = \frac{A - 2Z}{A}$$

$$\frac{2(A-Z)}{A} = 1 + x$$

$$(1-x)^{5/3} + (1+x)^{5/3} \approx 1 - \cancel{\frac{5x}{3}} + \frac{5}{9}x^2 + \cancel{\frac{5x}{3}} + \frac{5}{9}x^2 + O(x^3)$$

$$\propto x^2$$

$$\langle E_C \rangle \approx (9 \text{ MeV}) 2 \left[ 1 + \frac{5}{9} \left( \frac{A - 2Z}{A} \right)^2 \right]$$

$$\approx 18 \text{ MeV} [ ]$$

Per maggior perute dei nuclei

$\langle E_c \rangle$  per nucleone  $\leq 20$  MeV

$$\langle E_c \rangle_{A=2n} \neq \langle E_c \rangle_{A=2n+1}$$

$$K(Z, A) = A \langle E_c \rangle = (20 \text{ MeV}) \left( A + \frac{5}{a} \frac{(A - 2Z)^2}{A} \right)$$

Correzione nel calcolo di  $B(Z, A)$

Calcolo di  $B(Z, A) \rightarrow$  modello a goccia  
del nucleo.

analogie con le gocce di un liquido.

energ. di legame si calcola dall'inter.  
di doppio-

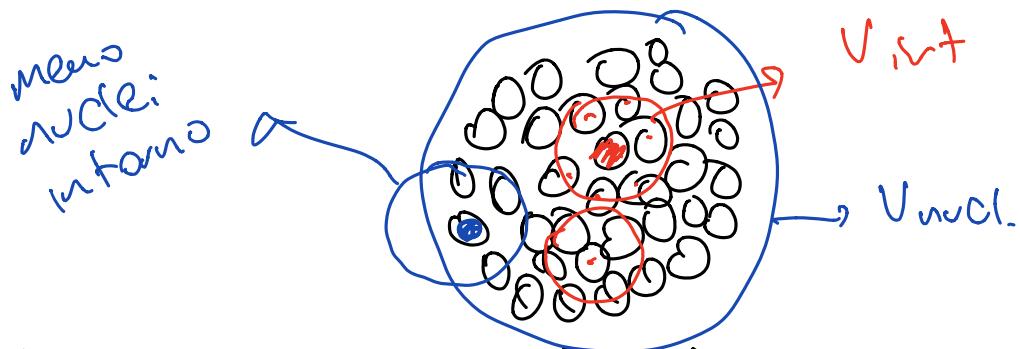
-  $V_{int}$  indipendente da  $A$ , dal tipo

- int. attrattiva per  $r \leq R_{int}$

- delle misure  $\frac{B}{A} \cong 7.5 \text{ MeV} = \text{cost.}$

→ i)  $B(Z, A) \propto A$  parametro.

Termine di volume:  $B_V = \overrightarrow{a}_V A$ .



Icat. Coulomb:  $\frac{A(A-1)}{2}$  app. e.

$$B \propto A^2. \Rightarrow \frac{B}{A} \propto A$$

Invece.  $\frac{B}{A}$  così  $\Rightarrow$  Raggio di azione  
fisico

Terme di volume  $B_1 = a_V A$

Correzione di superficie

$$S = 4\pi R^2 = 4\pi R_0^2 A^{2/3}.$$

$$B_2 = -\frac{\alpha_S}{R} A^{2/3}$$

termine di superficie.  
riduce l'energia di legame.

Correzione di Coulomb

Sfera uniforme di carica  $z$  e raggio  $R$

$$\rho = \frac{ze}{V} \quad \rho \text{ costante}$$

$$E_{Coul.} = \int_0^R d^3r \rho V(r)$$



$$V(r) = \frac{1}{4\pi} \frac{q(r)}{r}$$

$$q(r) = \frac{4\pi}{3} r^3 \rho$$

$$V(r) = \frac{1}{4\pi} \frac{4\pi}{3} \rho r^3$$

$$E_{\text{Coul.}} = (-) \int_0^R r^2 dr \frac{r^3}{r} \propto R^5$$

$$= \frac{e^2 e^2}{15} \frac{1}{\left(\frac{4\pi}{3}\right)^2 R_0} \frac{1}{A^{1/3}}$$

$$E_{\text{Coul}} \propto e^2 A^{-1/3}$$

$$B_3 = -\alpha_C e^2 A^{-1/3}$$

Correzione energetica Coulomb

$$B_5 = -\alpha_F \frac{(A-2Z)^2}{A}$$

dal modello gas di Fermi

$$B(Z/A) = \alpha_V A - \alpha_S A^{2/3} - \alpha_C e^2 A^{-1/3} - \alpha_F \frac{(A-2Z)^2}{A}$$

Formule di Bethe-Weizsäcker

$$\frac{B(Z, A)}{A} = \alpha_V - \alpha_S A^{-1/3} - \alpha_C Z^2 A^{4/3} - \alpha_F \left(\frac{A-2Z}{A}\right)^2$$

Fit ai dati:

$$\alpha_V \approx 16 \text{ MeV}$$

$$\alpha_C \approx 0.7 \text{ MeV}$$

$$\alpha_S \approx 18 \text{ MeV}$$

$$\alpha_F \approx 93 \text{ MeV}$$

Caratteristiche principali:

1/  $A = \text{cost}$  (isobarici)

$$B(Z, A = \text{cost}) = f(Z, Z^2) \text{ parabolica}$$

$$M(Z, A) = Z M(^1H) + (A-Z) M_n - B(Z, Z^2)$$

→ forma parabolica dipende da  $Z$   
fissato  $A = \text{cost}$

2/ sperimentalmente diff. sistematica

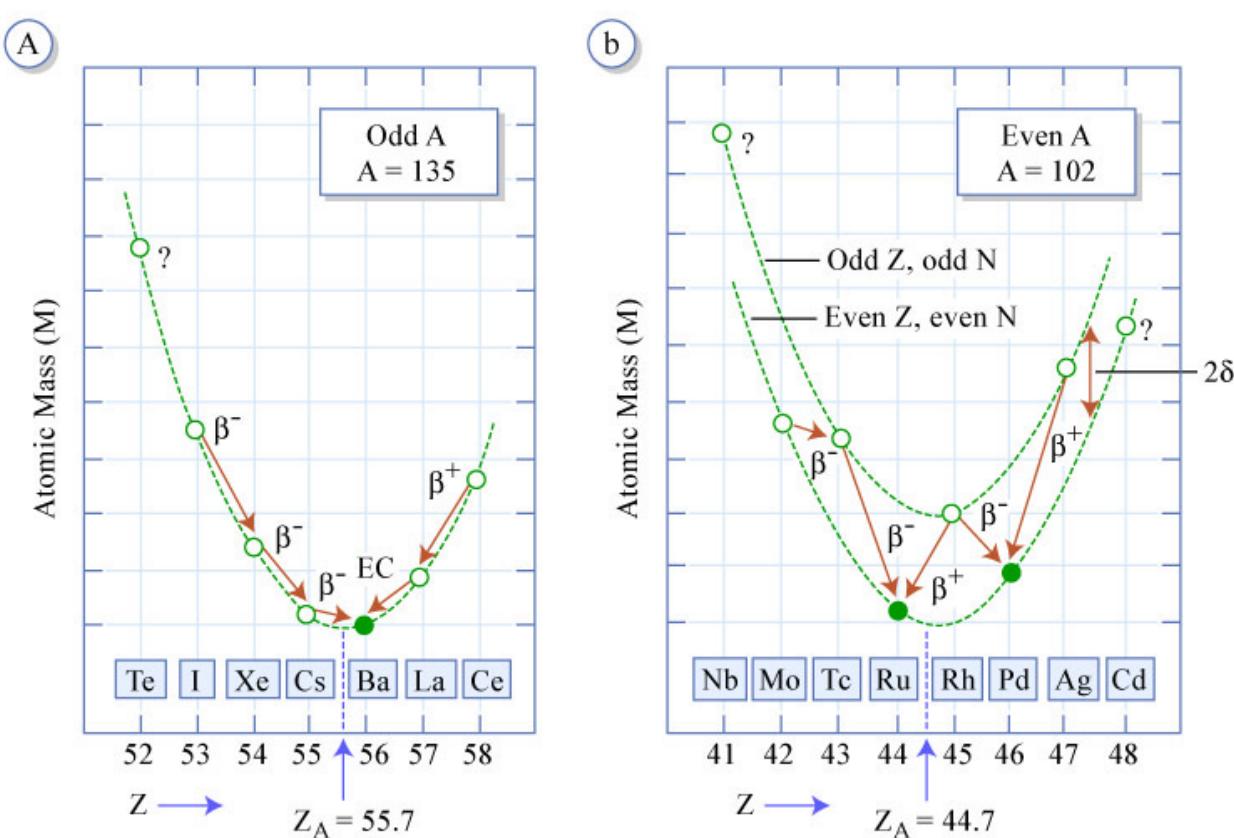
fra dati e parabola

$Z$	$A-Z$	$A$	$\Delta B$
poni	poni	poni	$+ \delta$
disponi	disponi	poni	$- \delta$

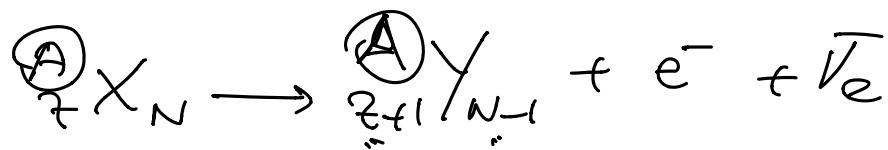
$$\delta \propto A^{-1/2}$$

$$\left. \frac{\partial M}{\partial Z} \right|_{A \gg \omega r t} = 0$$

$$Z_{\text{turn}} \approx \frac{A}{2} \cdot \frac{1}{1 + 0.0076 A^{2/3}}$$



$Z < Z_{\text{min}}$ : decadimento  $\beta^-$



A constante

$Z > Z_{\text{turn}}$ :

