Videolezione-2020-03-11

Decadimento $A \longrightarrow a+b$

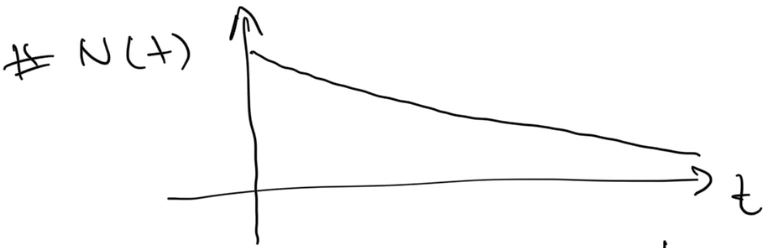
$dt \quad : \quad P = \lambda\, dt$
prob. decad.

$$dN = -PN = -\lambda\, dt\, N$$

$$\Rightarrow \quad N(t) = N_0\, e^{-\lambda t}$$

$$\langle t \rangle := \tau = \frac{1}{\lambda} \quad \text{vita media propria}$$

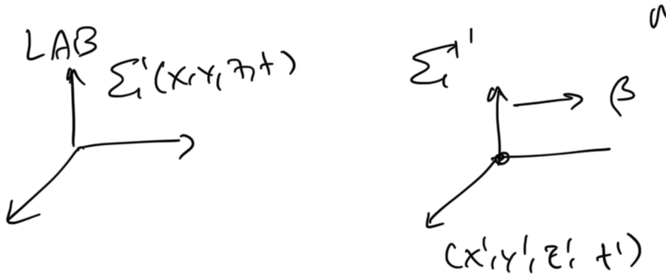
$N_0 = N(t=0)$ particelle instabili



$$N(t) = N_0\, e^{-\frac{t'}{\tau_0}}$$

t': misurato in rif. c.d.m.

τ_0: vita media propria



c.d.m.
Rif solidale con la particella

$$\begin{pmatrix} ct \\ x \\ y \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma & 0 \\ \beta\gamma & \gamma & \end{pmatrix} \begin{pmatrix} ct' \\ x' \\ y' \end{pmatrix}$$

$$\begin{pmatrix} \\ z \end{pmatrix} \quad \begin{pmatrix} 0 & & \\ & & 1 \end{pmatrix} \begin{pmatrix} y' \\ z' \end{pmatrix}$$

Nel c.d.m: $x' = 0 = y' = z'$

LAB: $$ct = \gamma c t' + \underbrace{\cancel{\beta\gamma} x'}_{=0}$$

vita media $\tau_{LAB} = \gamma \tau_0$

LAB: $$N(t) = N(t=0)\, e^{-\frac{t}{\gamma \tau_0}}$$

τ_0: prop propria della particella

muoni attraversano la terra

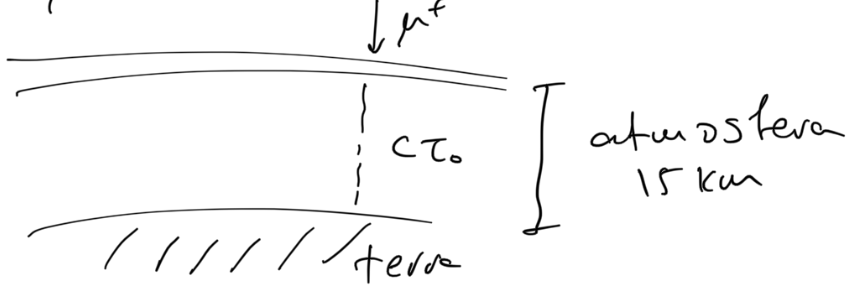
$\mu^{\pm}$: $m_\mu = 106$ MeV

$e^{\pm}$: $m_e = 0.511$ MeV

$$\mu^+ \longrightarrow e^+ \nu_e \bar{\nu}_\mu$$

$$\mu^- \longrightarrow e^- \bar{\nu}_e \nu_\mu$$

$$\tau_\mu = 2.2\ \mu s = 2.2 \times 10^{-6}\ s$$



$$c\tau_0 = 3 \times 10^8 \frac{m}{\cancel{s}} \times 2.2 \times 10^{-6}\ \cancel{s}$$

$$= 6.6 \times 10^2\ m = 660\ m.$$

τ_μ in LAB $\neq \tau_0$

τ_μ in LAB = ? τ_0

muoni $|\vec{P}| = 10$ GeV

$$\beta\gamma = \frac{P}{m} = \frac{10\ \text{GeV}}{0.1\ \text{GeV}} = 100$$

$$\beta\gamma = \frac{\beta}{\sqrt{1-\beta^2}} \Rightarrow \beta = \frac{\beta\gamma}{\sqrt{1+(\beta\gamma)^2}}$$

$$\beta = \frac{(P/m)}{\sqrt{1+(P/m)^2}} = \frac{100}{\sqrt{1+100^2}}$$

$$= 0.99995$$

$$\gamma = \frac{100}{0.99995} = 100$$

$$\tau_{LAB} = 100\,\tau_0 = 100 \times 2.2 \times 10^{-6}\ \text{s}$$

$$= 2.2 \times 10^{-4}\ \text{s} = \underline{0.2\ \text{ms}}$$

$$\Delta x_{LAB} = v \cdot \tau_{LAB} = \beta c \gamma \tau_0$$

$$= \beta\gamma c \tau_0$$

$$= \underline{100} \times 3 \times 10^8\ \frac{\text{m}}{\text{s}} \times 2.2 \times 10^{-6}\ \text{s}$$

$$= 6.6 \times 10^4\ \text{m} = \boxed{66\ \text{km}}$$

in LAB

$/$ τ

$$\boxed{T = \gamma \tau_0}$$

Significato di τ e λ in MQ

$$A \to a + b$$

$$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$$

interazione debole

$$H = H_0 + H_w$$

$\hookrightarrow$ int debole

Solo 2 stati fisici

$$\left| \begin{array}{l} |\mu^+\rangle \\ |e^+ \nu_e \bar{\nu}_\mu\rangle \end{array} \right.$$

$$|\Psi(t)\rangle = a(t)\,|\mu^+\rangle + c(t)\,|e^+ \nu_e \bar{\nu}_\mu\rangle$$

$t = 0$ No muoni

$a(0) = 1$

$c(0) = 0$

t: tempo proprio del μ^+

$a(t) = ?$

$c(t)$ non mi interessa

Weisskopf-Wigner 1930

$$|\mu^+, t\rangle = ?$$

$$i \frac{d}{dt} |\mu^+, t\rangle = H_{eff}\, |\mu^+, t\rangle$$

Se $H_{eff} = H_{eff}^{\dagger} \Rightarrow E_i$ reali

$$|\mu^+, t\rangle = a(t)|\mu^+\rangle$$
$$= e^{-iE_i t}|\mu^+\rangle$$

$E_i = m_\mu$

$$|\mu^+, t\rangle = e^{-imt}|\mu^+\rangle$$

$$|\langle\mu^+|\mu^+, t\rangle|^2 \equiv 1$$

$\Rightarrow \mu^+$ non decade

Invece μ^+ decade

usiamo H_{eff} non hermit.

$$H_{eff} = M - i\frac{\Gamma}{2}$$

$M = M^\dagger \qquad \Gamma = \Gamma^\dagger$

legame tra M, Γ e H_W ?

$\hookrightarrow \mu^+$ debole?

teoria delle perturb. al II° ordine

M, Γ si possono calcolare da H_W

$$|\mu^+, t\rangle = e^{-imt} e^{-\frac{\Gamma}{2}t} |\mu^+\rangle$$

$$a(t) = \underbrace{e^{-imt}}_{} e^{-\frac{\Gamma}{2}t}$$

$2 \qquad -\Gamma t$

$$|\langle \mu^+ | \mu^+, t\rangle|^{-} = e$$

μ^+ @ $t=0$ potrebbe non essere μ^+ a $t>0$

$$\underset{a(t)}{|\mu^+\rangle}, \quad \underset{c(t)}{|e^+\nu_e\bar{\nu}_\mu\rangle}$$

$$|a(t)|^2 + |c(t)|^2 = 1$$

$$P(\mu^+ \to e^+\nu_e\bar{\nu}_\mu) = |c(t)|^2 = $$
$$= 1 - e^{-\Gamma t}$$

$$N_\mu(t) = N_0\, e^{-\Gamma t}$$

$t=0$ nessun decad.

$t \to \infty$ tutti muoni decadono

$$\underbrace{\lambda \equiv \Gamma}_{\text{larghezza totale di decadimento}} = \frac{1}{\underbrace{\tau}_{\text{vita media propria}}}$$

Γ: si può calcolare da H_W usando la regola d'oro di Fermi

prob. di decad. in dt

$$dP = \Gamma dt = \lambda dt$$

Γ: prob. di decadim. in unità di tempo.

$$\Gamma \equiv \frac{dP}{dt}$$

misura in Δt

misuro $\tau \Rightarrow$ stimo Γ_{mis}

Calcolo Γ_{th} da Hw + perturb.

confronto Γ_{mis} con Γ_{th}

Γ_{mis}: misurato sperim. $\Gamma_{mis} \pm \delta\Gamma$

Γ_{th}: stimato dalla teoria calcolato

Decadim. in generale

$$A \to a+b \quad c_1$$

$$A \to c+d \quad c_2$$

$$A \to e+f \quad c_3$$

$$A \to g+h \quad c_4$$

$$dN = -dP\,N$$

$$dP = ?$$

$$\Gamma_1 : \ P(A \to c_1)$$

$$\vdots$$

$$\Gamma_4 \quad P(A \to c_4)$$

$$dP = (\Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \dots + \Gamma_N)\,dt$$

$$dN = -(\Gamma_1 + \Gamma_2 + \dots + \Gamma_N)\,dt\,N.$$

$$N(t) = N_0\, e^{-\Gamma_1 t}\, e^{-\Gamma_2 t} \dots e^{-\Gamma_N t}$$

$$= N_0\, e^{-\Gamma_{tot} t}$$

$$\Gamma_{tot} := \sum_i \Gamma_i$$

$\Gamma_i \hookrightarrow$ larghezze di decad. parziale.

$\Gamma_{tot} \downarrow$ larghezza di decad. totale

$$\langle t \rangle = \tau = \frac{1}{\Gamma_{tot}}$$

N larghezze parziali

Γ_i : Prob$(A \longrightarrow C_i)$

1 larghezza totale $\Gamma_{tot} = \sum_i \Gamma_i$

1 vita media: $\tau = \dfrac{1}{\Gamma_{tot}} = \dfrac{1}{\sum_i \Gamma_i}$

$$BF(A \longrightarrow C_i) = B_i = \frac{\Gamma_i}{\sum_i \Gamma_i}$$

Branching fraction

Branching ratio

$$\sum_i BF(A \to C_i) = 1$$

$$
A \to \begin{cases} C_1 & 10\% \\ C_2 & 20\% \\ C_3 & 70\% \quad \dfrac{\Gamma_3}{\Gamma_{tot}} = 0.7 \end{cases}
$$

Conosco tutti i decadimenti.

Sperimentalmente non sempre conoscete tutti i canali.

$$
B \to \begin{cases} C_1 & 20\% \\ C_2 & 10\% \\ C_3 & 20\% \\ C_4 & 20\% \end{cases}
$$

$$
\overline{70\%}
$$

$\Rightarrow$ Esiste almeno un altro canale.

$$
BF(A \to X) = 1 - \sum_i^{BF_{note}} BF_i
$$

$$
= 0.3
$$

esiste canale $\left.\begin{matrix} C5 \\ C6 \\ C7 \\ \vdots \end{matrix}\right\} 30\%$

Bosone $Z^0 \rightarrow e^+e^-$ 3.3%

$\mu^+\mu^-$ 3.3%

$\tau^+\tau^-$ 3.3%

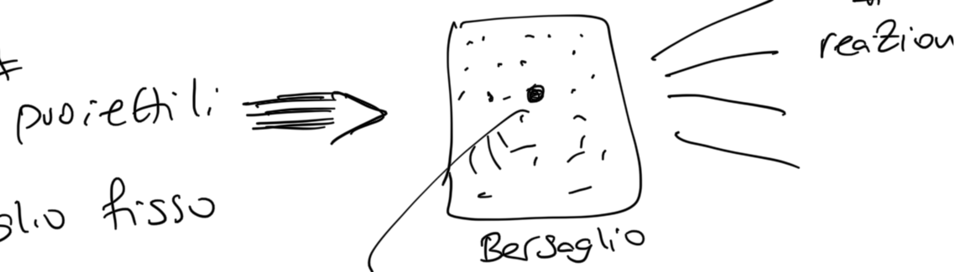
$N_{Z_0} = 10^9$ $\quad N(Z \to e^+e^-) = 3 \times 10^{-2} \times 10^9$

$= 3.3 \times 10^7$

$= 33$ millioni

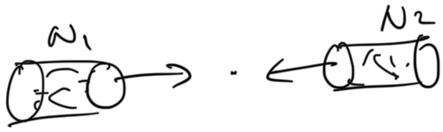
Investigazione subnucleare, nucleare

- decadimenti
- esperim. di diffusione



Bersaglio fisso

Fascio contro fascio:



I esperim. di Rutherford fine '800
scoperta del nucleo

Definizione operativa di sezione d'urto

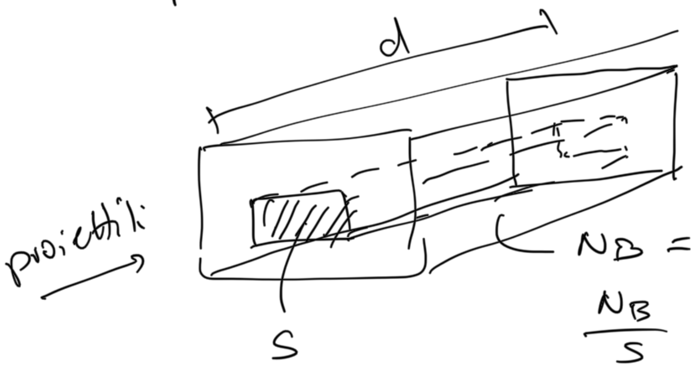
Ipotesi: - 1 proiettile interagisce al più con 1 bersaglio

— prob. di interazione con B non dipende dalla vicin. di altri Bersagli

$$\underset{\sim}{N_r} \propto \underset{\sim}{N_p}\, N_B\, \underset{\sim}{T}$$

$$\frac{\#\text{reazioni}}{T} : \frac{dN_r}{dt}$$

$$\frac{\#\text{proiettili}}{T} : \frac{dN_p}{dt}$$



$$N_B = n_b \cdot S \cdot d$$

$$\frac{N_B}{S} = n_b \cdot d$$

$$\frac{dN_r}{dt} \propto \frac{dN_p}{dt} \cdot n_b \cdot d$$

$[\ ] = T^{-1}$ $\qquad T^{-1}$ $\qquad [n_b \cdot d] = L^{-3} L = L^{-2}$

$$\frac{dN_r}{dt} = \sigma_r \frac{dN_p}{dt}\, n_b \cdot d$$

d — spess

n_b → densità num. bersagli

$\frac{dN_r}{dt}$ → $\frac{\#\text{reazioni}}{\text{tempo}}$

σ_r → sezione d'urto

$\frac{dN_p}{dt}$ → $\frac{\#\text{proi}}{\text{tempo}}$

Esperienza in tempo finito T

exp.

Ideale $T \to \infty$

Misurare $\sigma_r = \dfrac{\left(\frac{dN_r}{dt}\right)}{\left(\frac{dN_p}{dt}\right)} \dfrac{1}{n_b \cdot d}$

Sezione d'urto totale di reazione

- natura del proiettile
- natura del bersaglio
- tipo di interazione tra loro

Dalla teoria se conosco H_int
usando la regola d'oro di Fermi
calcolo/stimo σ per processo.

σ_{th} vs. σ_{exp}

σ dipende da energia
num. quantici
del proiettile
bersaglio

In tempo T mando N_p proiettili
contro bersaglio.

$$\frac{dN_p}{dt} = \frac{N_p}{T}$$

$$\frac{dN_r}{dt} = \frac{N_r}{T}$$

$$\sigma_r = \frac{N_r}{N_p} \frac{1}{n_b \cdot d} \quad \text{in tempo } T$$

costanti

$$N_p = \frac{dN_p}{dt} \cdot T$$