

Perdite di energia per ionizzazione

Formule di Bohr con un calcolo classico

Jackson
Cap. 13.1

$$dE = DE_{tot} = n_e e \bar{a}_b \cdot \bar{DE} \frac{db}{dx}$$

$$\bar{DE} \equiv T$$

$$\frac{d^2 E}{db dx} = 2\pi n_e b \bar{DE}$$

$$= 4\pi m_e c^2 r_c^2 n_e \underbrace{\frac{e^2}{\beta^2}}_A \frac{1}{b}$$

b_{max} A

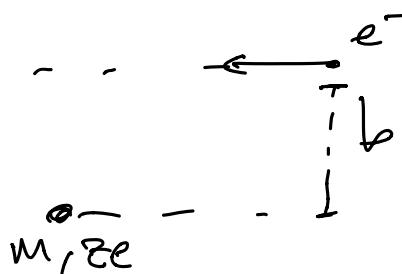
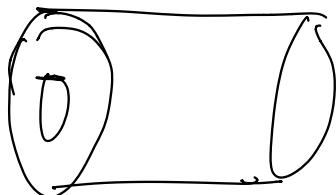
energia cinetica
trasferita dal
proiettile al
singolo elettrone

$$\frac{dE}{dx} = \int_{b_{min}}^{b_{max}} \frac{d^2 E}{db dx} = A \ln \frac{b_{max}}{b_{min}}$$

Stimare

b_{max}

b_{min}

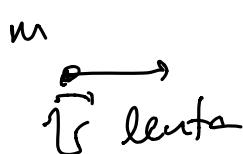


$$\frac{dE}{dx} \propto \frac{e^2}{\beta^2} \ln \frac{b_{max}}{b_{min}}$$

b_{max} : tempo d'urto $\frac{b}{v} \propto$

tempo di rivoltazione
dell'elettrone nell'atomo

mentre e^- come libero



BR del proiettile
lo stesso per e^- ,
ma $\vec{v}_e = -\vec{v}_p$



Sembra nullo
di impulso.

Per avere scambio d. impulso.

$$\Delta p \propto F_{Coul} \left(\frac{1}{b} \right)$$

$$\frac{b}{v} \approx \gamma t_{triv} = \gamma \frac{1}{\langle \omega \rangle} = \gamma \frac{1}{I/b}$$

I : energia d. ioniz. media

$$b_{\text{max}} \approx \gamma \frac{\sigma}{\bar{I}/t_h} = \frac{\gamma \beta c}{\bar{I}/t_h}$$

barr: principio di indeterminazione
e' implicito ρ_e .

$m \equiv m_e$

$$\Delta P \cdot \Delta x \leq t_h \Rightarrow \Delta x \leq \frac{t_h}{\rho_e}$$

$$b_{\text{min}} \leq \Delta x = \frac{t_h}{\rho_e} = \frac{t_h}{\gamma m_e c} = \frac{t_h}{\gamma m \beta c}$$

$$\frac{b_{\text{max}}}{b_{\text{min}}} = \frac{\gamma \beta c}{\bar{I}/t_h} \frac{\gamma m \beta c}{t_h} = \frac{\gamma^2 \beta^2 m_e c^2}{\bar{I}}$$

- $\frac{dE_{\text{part}}}{dx} = 4\pi m_e c^2 r_c^2 \frac{e^2}{\beta^2}$ ne der $\frac{\gamma^2 \beta^2 m_e c^2}{\bar{I}}$

$$n_c = \frac{\rho}{A} N_A \cdot Z \quad \text{per molti materiali:} \\ \bar{I} \approx 10^2 \text{ eV}$$

- $\frac{dE_{\text{projettile}}}{dx} = \underbrace{4\pi m_e c^2}_{[E]} \underbrace{r_c^2}_{[L]^{-1}} \underbrace{N_A}_{P} \sum_i \frac{Z_i^2}{\beta^2}$ ne der $\frac{\gamma^2 \beta^2 m_e c^2}{\bar{I}}$

$\frac{dE}{dx}$: variazione energ. dell'elettrone.
energ. trasp. all'elettrone
del proiettile.

$$\frac{dE_{\text{proiettile}}}{dx} = - \frac{dE}{dx}$$

$\frac{dE}{dx}$: perdite d'energ. del proiettile

$$C = 4\pi m_e c^2 r_c^2 N_A = 0.3 \frac{\text{MeV}}{\text{g cm}^{-2}}$$

- $\frac{dE}{dx} = \underbrace{(0.3 \text{ MeV})}_C \rho \frac{Z}{A} \frac{Z^2}{\beta^2}$ ne der $\frac{\beta^2 \gamma^2 m_e c^2}{\bar{I}}$

$$\frac{Z}{A} \approx \frac{1}{2} \quad \ln \frac{\bar{I}}{I} \approx \ln \frac{z}{z'} \\ z' \rightarrow 100 \\ \text{per Cambio Nucleo.}$$

$$-\frac{1}{\rho} \frac{dE}{dx} = \frac{(0.3 \text{ MeV})}{Z} \frac{\gamma^2}{\beta^2} \ln \frac{\beta^2 \gamma^2 w_{max}^2}{\bar{I}}$$

Formule di Bohr (1915)

- funzione universale per tutti i materiali
- $\frac{dE}{dx}$ non dipende dalle misse del proiettile
vale per $m > m_e$ abb. pesanti.
Verificate con P, d

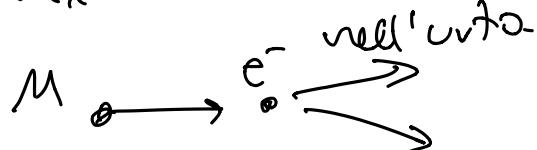
Per tener conto di

- $M \oplus$
- relatività

Bethe-Bloch have calcolato con QED

$$-\frac{1}{\rho} \frac{dE}{dx} = (0.3 \text{ MeV}) \frac{Z}{A} \frac{\gamma^2}{\beta^2} \left[\ln \frac{\beta^2 \gamma^2 w_{max}^2}{\bar{I}^2} - \beta^2 - \frac{\delta}{\epsilon} \right]$$

w_{max} = en. cin. max trasf. dell'elettrone



$$w_{max} = T_e^{max} = 2mc^2 \beta \gamma \epsilon$$

- β^2 : correzione relativistica.

$\frac{\delta(\beta \gamma)}{2}$: correzione dovuta a effetti
di densità e polarizzazione
del mezzo.

$\begin{array}{c} +\gamma c \rightarrow \pi^+ \pi^- \\ \text{--- --- --- ---} \\ \text{--- --- --- ---} \end{array}$
 $\rightarrow \pi^+, \beta \rightarrow 1$

Corruzione dovuta
a effetti di
Scheratura.

$$-\frac{1}{\rho} \frac{dE}{dx} = (0.3 \text{ MeV}) \frac{Z}{A} \frac{e^2}{\beta^2} \left[\ln \frac{\beta^2 e^2 w_{\max}}{\pi^2} - \beta^2 - \frac{\delta}{Z} \right]$$

- β piccoli $\propto \frac{1}{\beta^2}$

- c'è un minimo.
- risolte relativistica

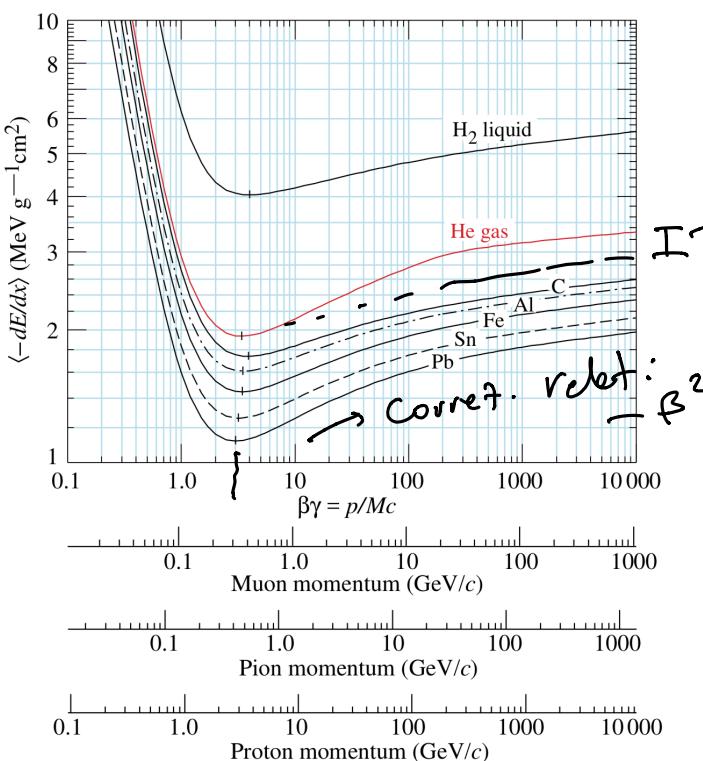
$$\begin{aligned} \text{minimo} &\approx \beta \gamma \approx 3 \\ Z = 7 & \quad \beta \gamma_{\min} \approx 5. \\ Z = 100 & \quad \beta \gamma_{\min} = 3.5 \end{aligned}$$

funzione universale per tutti i materiali

- scelto log
per $\frac{1}{\rho} \frac{dE}{dx}$
per $\beta \gamma$

$$\text{min } \beta \gamma \approx 3$$

$$\beta \gamma = \frac{p}{m}$$



$$\text{Bethe-Bloch} \quad -\frac{\delta(\beta \gamma)}{2}$$

I →
polo dip. del
matereiale.

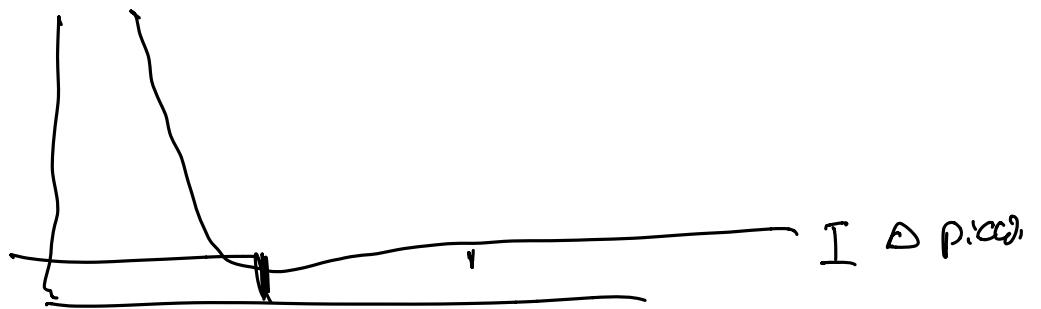
minimo
a p diversi:
per messe
diverse.

cresce relativ dopo il minimo piccolo.
 $\alpha(\max) \approx 50\%$

$$-\frac{1}{\rho} \frac{dE}{dx} \Big|_{\min} \approx 1-2 \text{ MeV/g cm}^{-2}$$

$$\text{H}_2\text{O} \quad \rho = 1 \text{ g/cm}^3$$

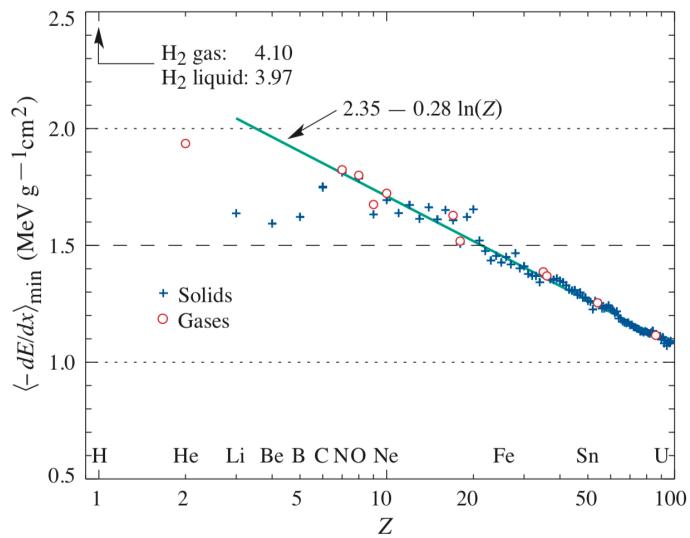
$$-\frac{dE}{dx}\Big|_{\min} \approx 1-2 \text{ MeV/cm}$$



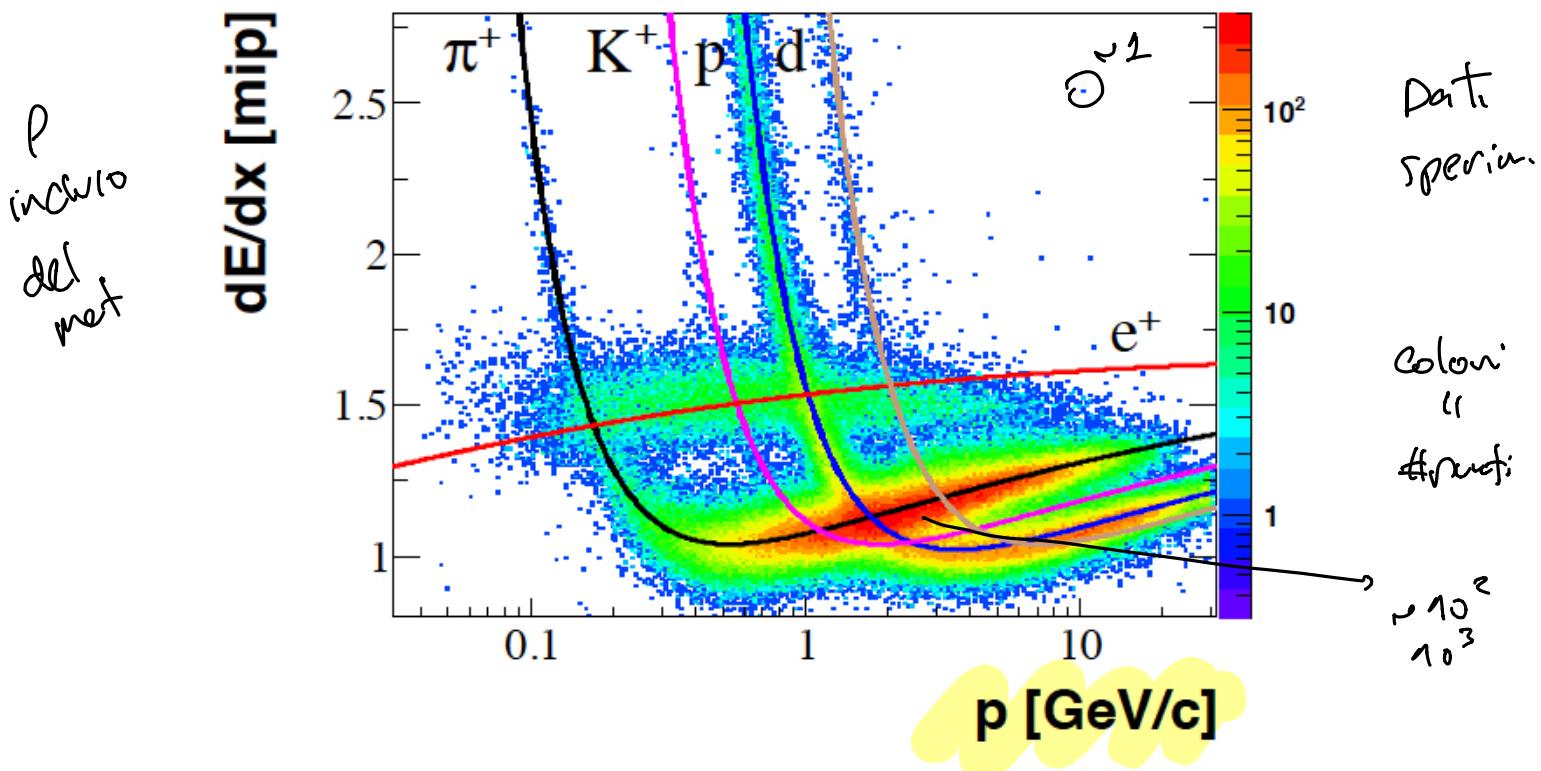
particelle al minimo di ionizzazione. $\beta r = \frac{P}{m} = 3$
 MIP: minimum ionizing particle.

Protone $\beta r = 5 \Rightarrow P = S \cdot m \approx 5 \text{ GeV}$.
 perde $\approx 1 \text{ MeV/cm}$ in H_2O

$$-\frac{1}{\rho} \frac{dE}{dx}\Big|_{\min} \longrightarrow$$



assunzione di $1.5 \text{ MeV/g cm}^{-2}$



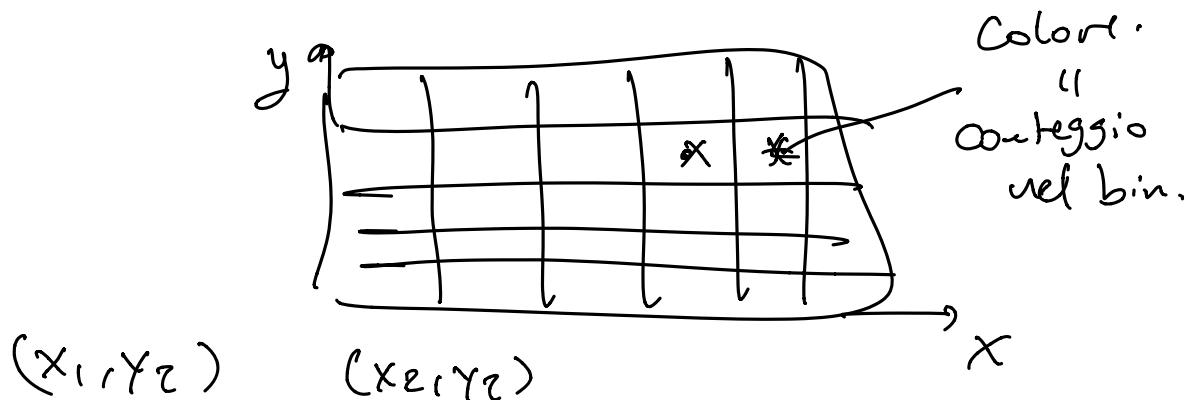
$M_e = 0.5 \text{ MeV}$
 $m_n = 135 \text{ MeV}$

impulso proiettile
 ~ $10^3 \text{ GeV}/c$

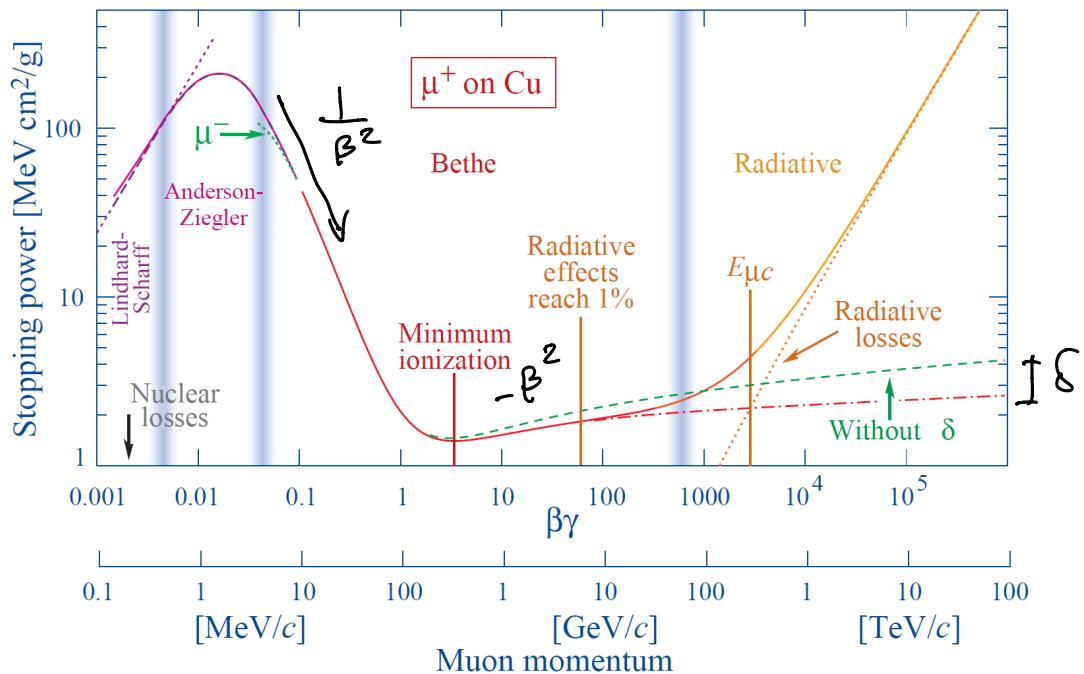
$$e^- \text{ con } P \approx 1 \text{ GeV.} \quad \beta \gamma \approx \frac{P}{m} = \frac{1000 \text{ MeV}}{0.5 \text{ MeV}}$$

Minimo di $\frac{dE}{dx}$ per P molto piccolo. $= 2000$

misur. indip. di $(\frac{dE}{dx}, P)$ \Rightarrow Sono le misure del proiettile.
 \Rightarrow identificare il proiettile.



Validité de Bethe-Bloch $\beta\gamma: 0.1 \rightarrow 1000$



Curve universale per mettuto:
e dipende da $\beta\gamma$

$$\beta\gamma = 2 \quad \beta\gamma = \frac{\beta}{\sqrt{1-\beta^2}}$$

$$(\beta\gamma)^2(1-\beta^2) = \beta^2$$

$$\beta^2(1 + (\beta\gamma)^2) = (\beta\gamma)^2$$

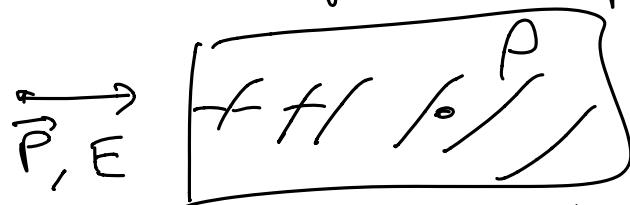
$$\beta = \frac{(\beta\gamma)}{\sqrt{1 + (\beta\gamma)^2}}.$$

$\beta\gamma \approx 2 \Rightarrow$ finito cycle β

P, m

Percorso residuo (Rcye)

quanto cammina un proiettile prima di fermarsi?



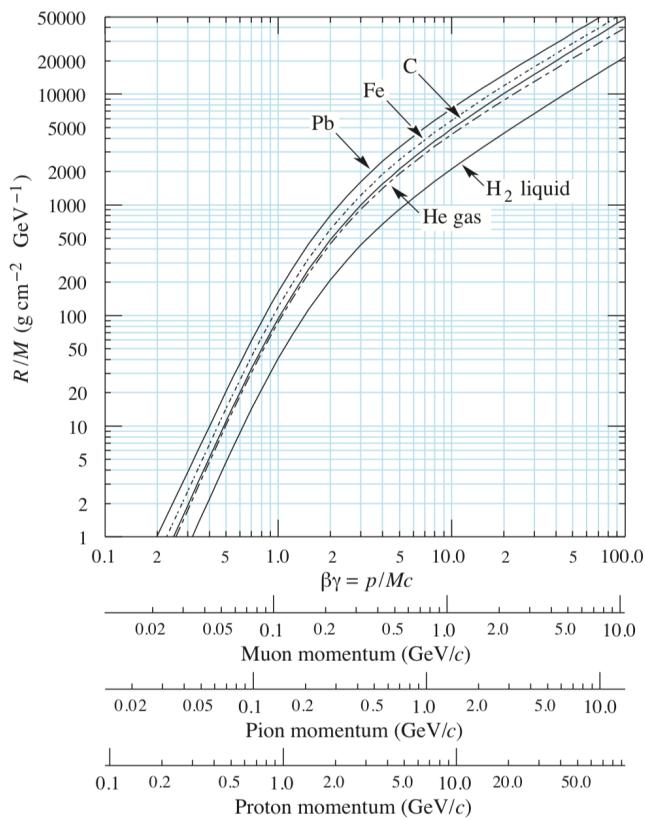
Rapporto: Cammino in cui proiettile perde tutta

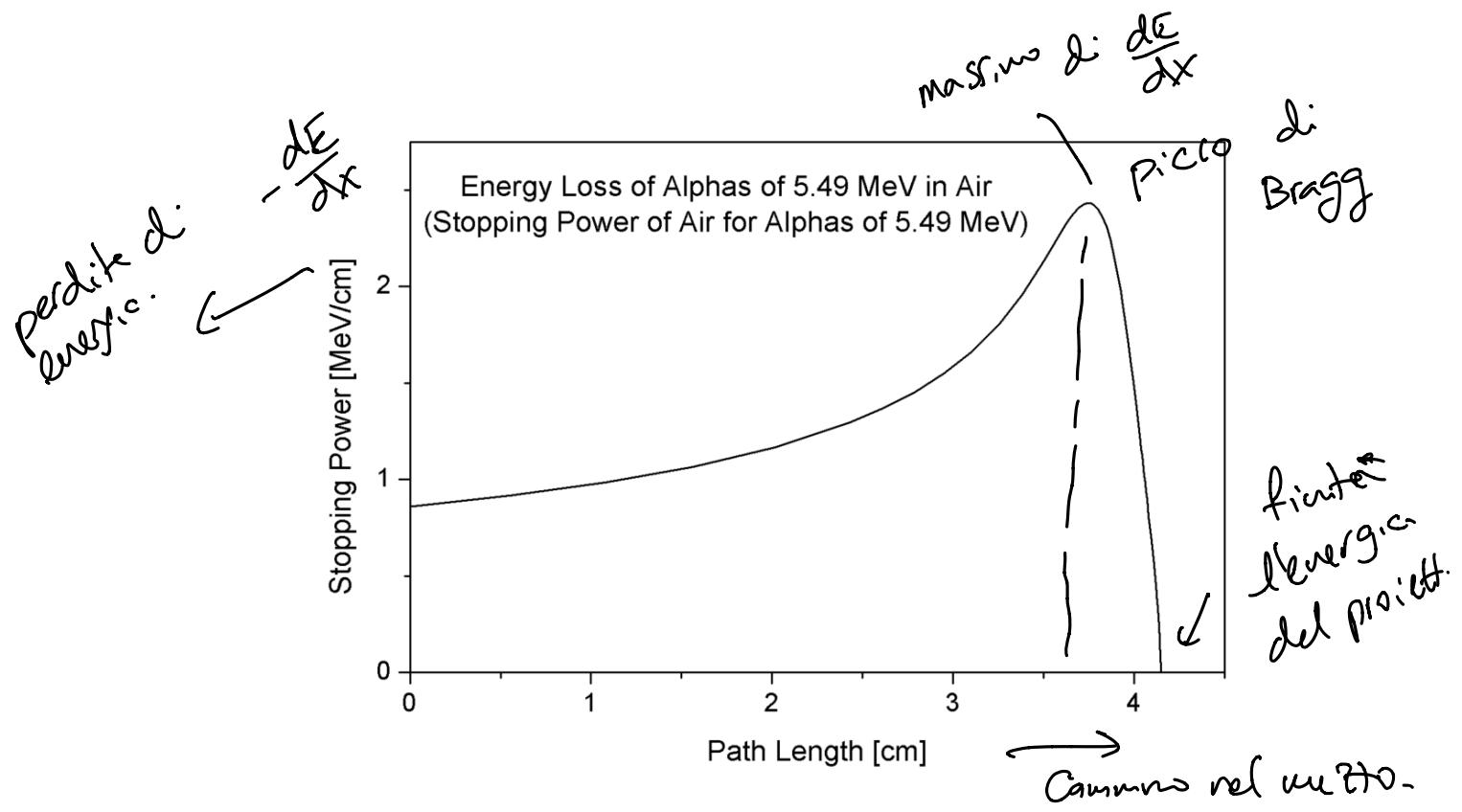
$$R(E) = \int_E^\infty dx = \int_0^E \frac{dE}{\left(-\frac{dE}{dx} \right)} = \int_0^E \frac{dE}{\frac{(dE/dx)}{\text{Bethe-Bloch}}}$$

ϕ l'energia cinetica.

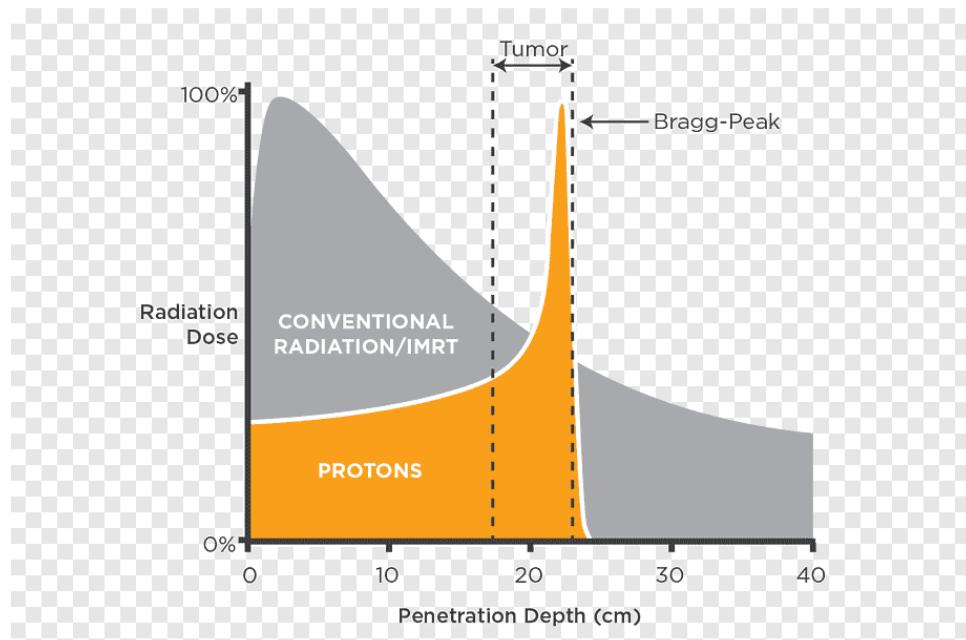
Bethe-Bloch.

Br piccoli: Perde tutta energ.

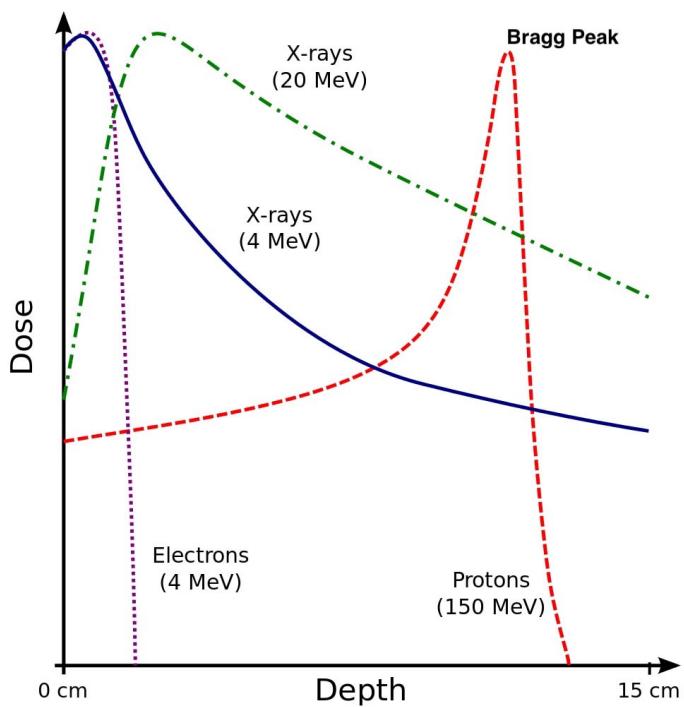




Dose : en. rivelate nel tessuto.



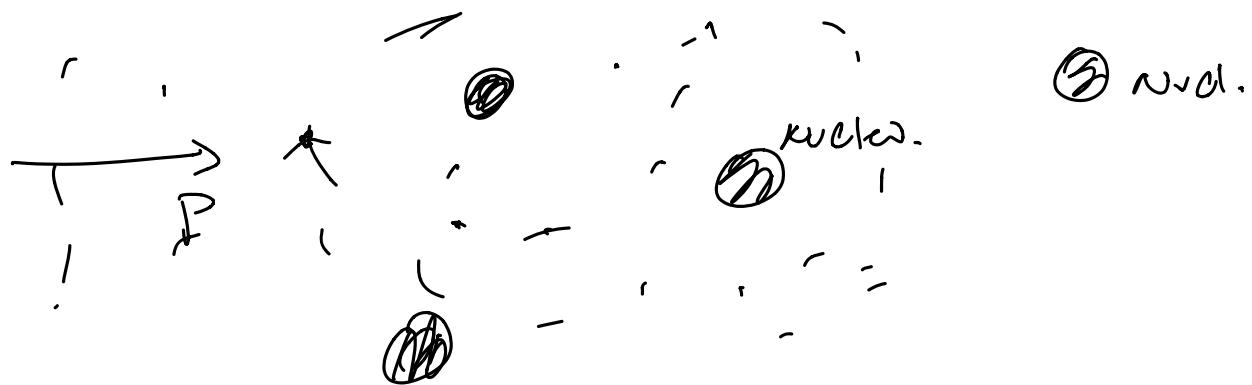
Adroterapie: $P, {}^{12}C$



adattropia: rilascia grande e localmente
di energie vicine al termine.

Perdita di energia per ionizzazione dovuta:

interazione fra proiettile ed elettroni:

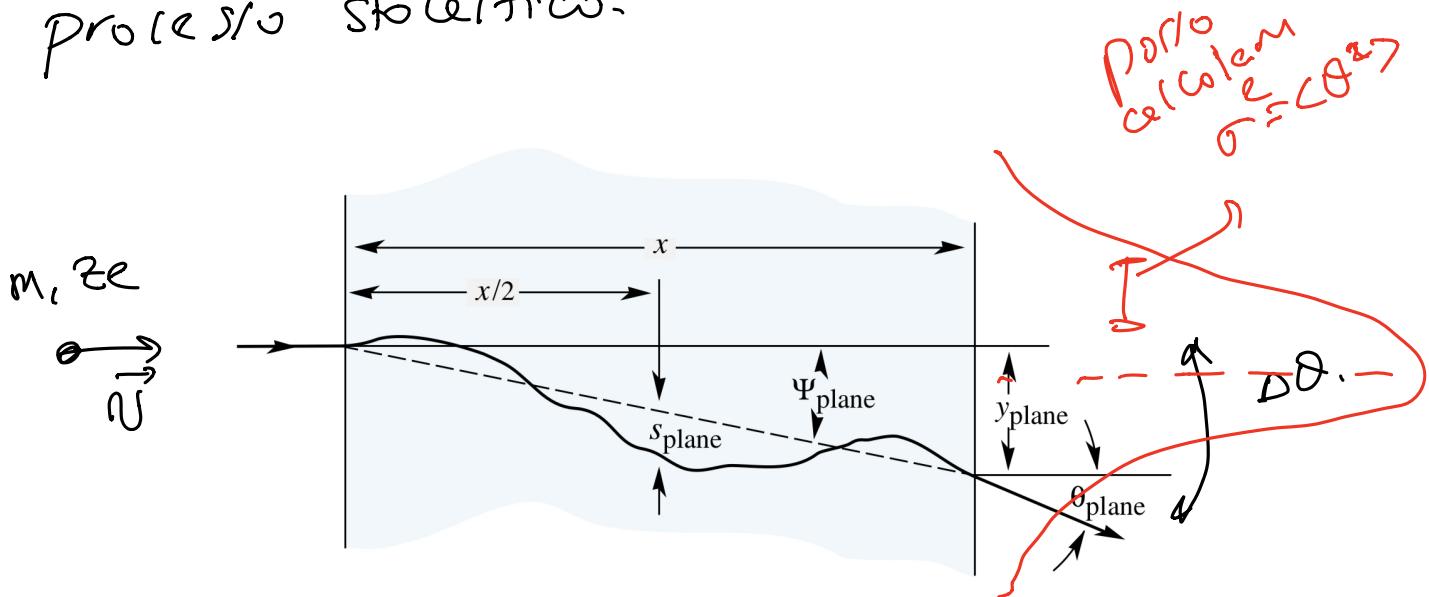


$\frac{dE}{dx}$ per urti con nucleo: trascurabile
elastici

urti contro nucleo $\frac{dE}{dx}$ trascurabile.
 $\frac{dp}{dx}$ grande

nell'urto Rutherford con nuclei $\Rightarrow \Delta\theta$ grande.
 \Rightarrow deviazione eufolare.

processo stocastico.



Se spessore grande \rightarrow teorema di limite centrale.

$$f(\Delta\theta) \propto G(\langle\theta\rangle=0, \sigma=\sqrt{\langle\theta^2\rangle})$$

$$\langle\theta^n\rangle = \frac{\int \theta^n \left(\frac{d\sigma}{d\Omega} \right) d\Omega}{\int \frac{d\sigma}{d\Omega} d\Omega}$$

set. d'urto
di Rutherford $\frac{1}{\sin^2 \frac{\theta}{2}}$

$$f(\Delta\theta) \propto \frac{1}{\sqrt{\theta^2}} e^{-\frac{\Delta\theta^2}{\theta^2}}$$

$$d\Omega = \sin\theta d\theta d\varphi = 2\pi \sin\theta d\theta$$

$$\frac{d\sigma}{d\Omega} \propto \frac{d\sigma}{d\theta}$$

Prob. di avere urto
in $[\theta, \theta + d\theta]$

$$\langle \Theta \rangle = \emptyset$$

Diffusione Coulombica multiple
(doppia
a: nucleo)

$$\langle \Theta^2 \rangle = \text{E1 MeV} \frac{Z}{\beta c P} \sqrt{\frac{X}{X_0}}$$

β, P, Z : del proiettile.

X : spessore percorso in cm. (fiscale)

X_0 : lunghezza di radiazione Caratteristica
del mezzo.

$$[X_0] = \frac{\text{cm}}{\text{g/cm}^3} = \frac{1}{\text{g cm}^{-2}}$$

$$\frac{1}{X_0} = 4 \pi e^2 \alpha \rho \frac{N_A}{A} Z^2 \ln(183 Z^{-1/3})$$

Il significato sarà chiaro quando tratteremo
passaggio di e^- , γ nel mezzo