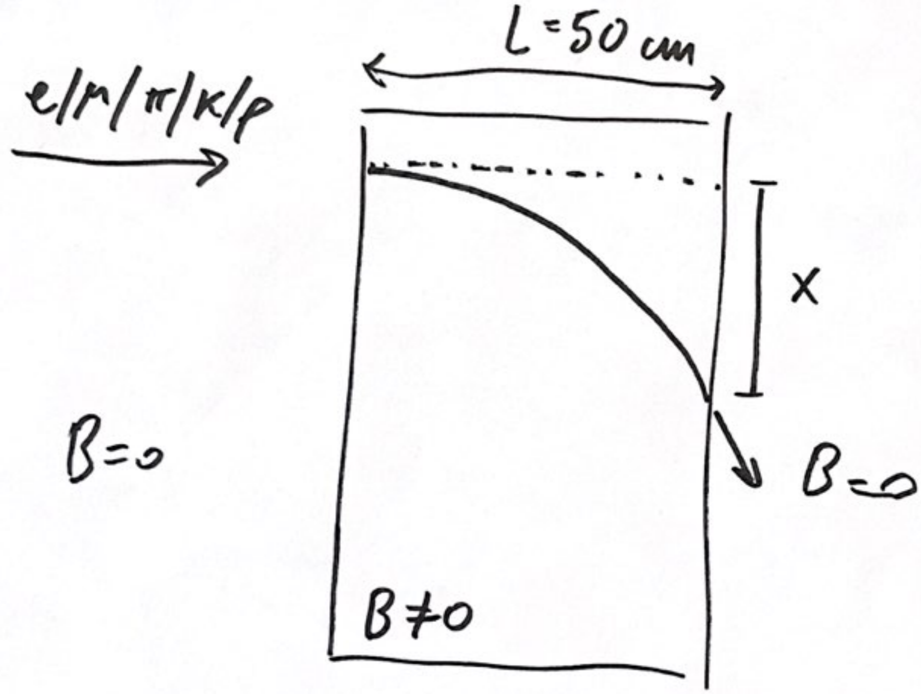
$\boxed{\text{Ex}}$ $e^+ \ \mu^+ \ \pi^+ \ K^+ \ p$ $p = 2\ \text{GeV}$

collineari $L = 50\ \text{cm}$

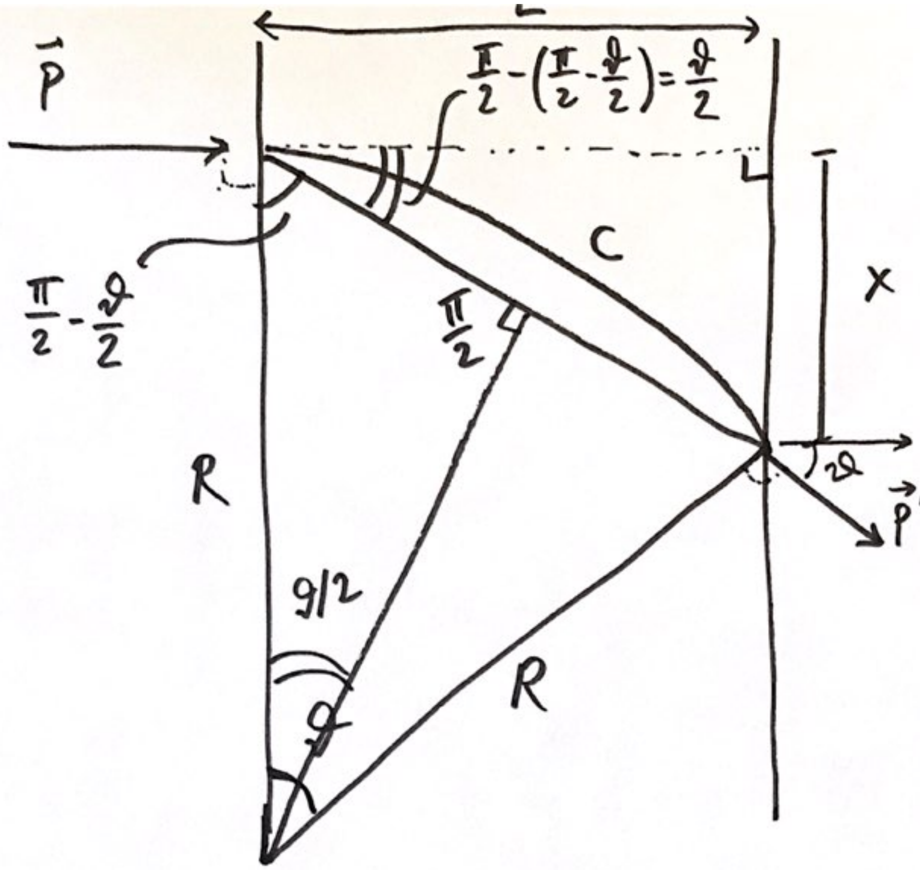
$B = 1.7\ \text{T}$

x rispetto alla linea di volo



$$\boxed{p = qRB}$$

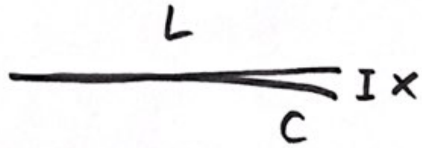
$$p[\text{GeV}] = 0.3\ R[\text{m}]\ B[\text{T}]$$



$$p = qRB$$

$$\begin{cases} \dfrac{x}{L} = \tan\dfrac{\vartheta}{2} \\[2ex] \dfrac{C}{R} = \vartheta \end{cases}$$

$\vartheta \ll 1$



$C \sim L$

$\tan\vartheta/2 \sim \vartheta/2$

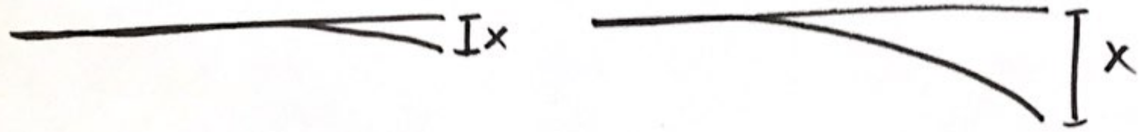
$$\Rightarrow \begin{cases} \dfrac{x}{L} = \tan\dfrac{\vartheta}{2} \sim \dfrac{\vartheta}{2} \\[2ex] \dfrac{C}{R} \sim \dfrac{L}{R} = \vartheta \end{cases}$$

$$p = qRB$$

$$\Rightarrow R = \frac{p}{qB}$$

$$x = \frac{L\vartheta}{2} = \frac{L^2}{2R}$$

$$x = \frac{qBL^2}{2p}$$

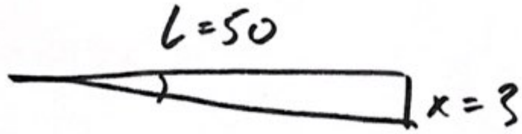


$$x \propto \frac{BL^2}{p}$$

KATRIN

GeV, T, m (!!)

$$x = \frac{0.3\,BL^2}{2p} = \frac{0.3\;\; 1.7 \cdot (0.5)^2}{2 \cdot 2} = 0.0319\,\mathrm{m}$$
$$= 3.19\,\mathrm{cm}$$

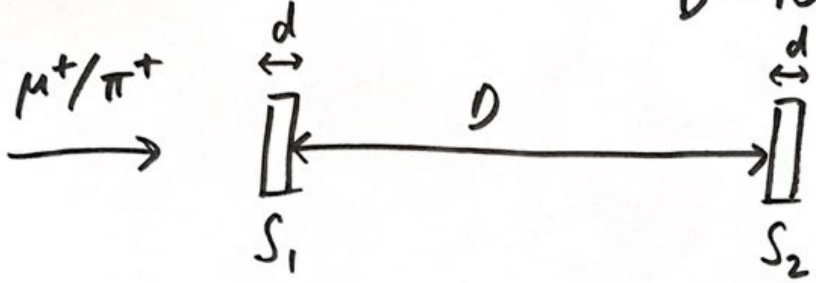


$$\frac{\sigma(p)}{p} \sim \frac{\sigma(x)}{x}$$

$\boxed{\text{Ex}}$ fascio con μ^+/π^+ $\quad p = 500$ MeV

NaI(Tl) $\quad d = 2$ cm

$D = 10$ m



$m(\mu) = 106$ MeV

$m(\pi) = 140$ MeV

NaI(Tl): $\rho = 3.67\ g/cm^3$

$I = 452$ eV

$X_0 = 2.59$ cm

$Z/A = 0.45$

ⓐ energia depositata nel primo scint. da π^+/μ^+

$$-\frac{dE}{dx} = C \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln\left(\frac{2 m_e \beta^2 \gamma^2}{\langle I \rangle}\right) - \beta^2 - \cancel{\frac{\delta}{2}} \right]$$

solo $e^{\pm}$

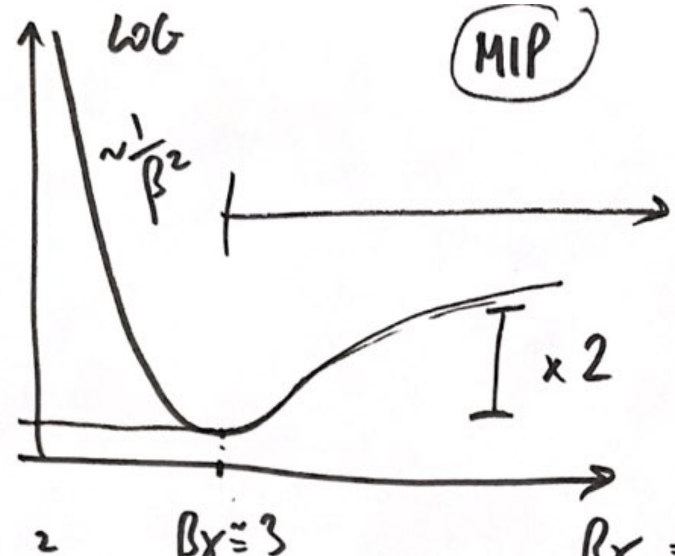
$C = 0.307\ MeV\ g^{-1}\ cm^2$ $\quad \left(\frac{g}{cm^3} = \frac{kg}{dm^3}\right)$

PARTICELLA: β, γ, z

MEZZO: $\rho, \frac{Z}{A}, I$ $\quad I \sim (10\ eV) \cdot Z$

~ 0.5

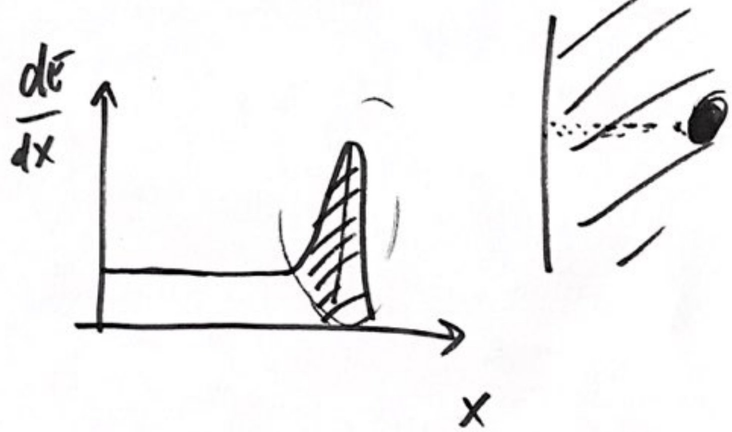
$$-\frac{1}{\rho}\frac{dE}{dx}$$



$\sim$ 1-2 MeV $g^{-1}cm^2$

$$\beta\gamma = \frac{p}{E}\frac{E}{m} = \frac{p}{m}$$

$$\left[\frac{dE}{dx}\right] = \frac{MeV}{cm}$$



$p < 500$ MeV

μ: $\quad m = 106$ MeV $\Rightarrow E = \sqrt{p^2 + m^2} = 511$ MeV

$$\beta = \frac{p}{E} = \frac{500}{511} = 0.978$$

$$\Rightarrow \beta\gamma = 4.71$$

$$\gamma = \frac{E}{m} = \frac{511}{106} = 4.82$$

$$-\left(\frac{dE}{dx}\right)_\mu = C\rho\,\frac{Z}{A}\,\frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e\beta^2\gamma^2}{I}\right)-\beta^2\right] =$$

$$= 0.307\cdot 3.67\cdot 0.45\cdot\frac{1}{0.978^2}\left[\ln\left(\frac{2\cdot 0.511\cdot 10^6\cdot 4.71^2}{452}\right)+\right.$$

$$\left(\frac{\text{MeV}}{\frac{\text{g}}{\text{cm}}}\right) \qquad\qquad \left.-\,0.978^2\right]$$

$$= 5.23\ \text{MeV/cm}$$

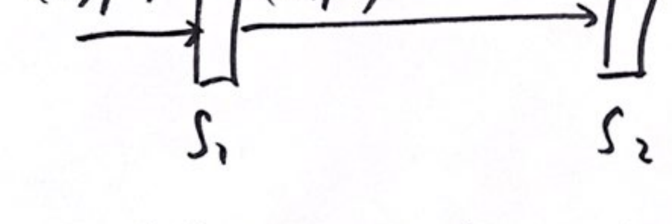
$$\Delta E_\mu = \left(\frac{dE}{dx}\right)_\mu\cdot d = 5.23\cdot 2 = \underline{10.5\ \text{MeV}}$$

π: $\quad p = 500\ \text{MeV} \quad m = 140\ \text{MeV} \Rightarrow E = 519\ \text{MeV}$

$$\beta = 0.963 \qquad \gamma = 3.70$$

$$\Rightarrow \beta\gamma = 3.57$$

$$\Rightarrow \left(\frac{dE}{dx}\right)_\pi = 5.11\ \text{MeV/cm} \qquad \Rightarrow \Delta E = d\cdot\frac{dE}{dx} = \underline{10.2\ \text{MeV}}$$



Calcolare il tempo di volo

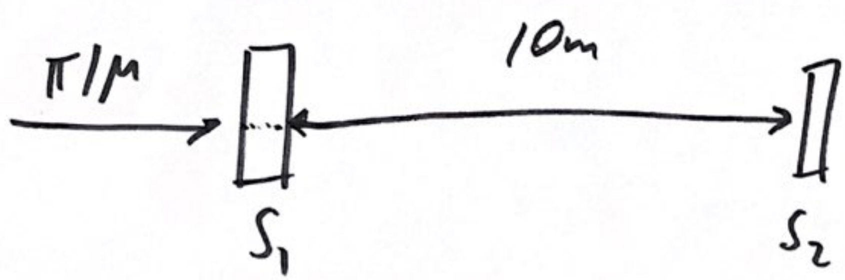
$$\begin{cases} E_\mu' = E_\mu - \Delta E_\mu = 511 - 10.5 = 500.5 \text{ MeV} \\ E_\pi' = E_\pi - \Delta E_\pi = 519 - 10.2 = 508.8 \text{ MeV} \end{cases}$$

$$\begin{cases} p_\mu' = \sqrt{E_\mu'^2 - m_\mu^2} = 489.1 \text{ MeV} \\ p_\pi' = \sqrt{E_\pi'^2 - m_\pi^2} = 489.2 \text{ MeV} \end{cases}$$

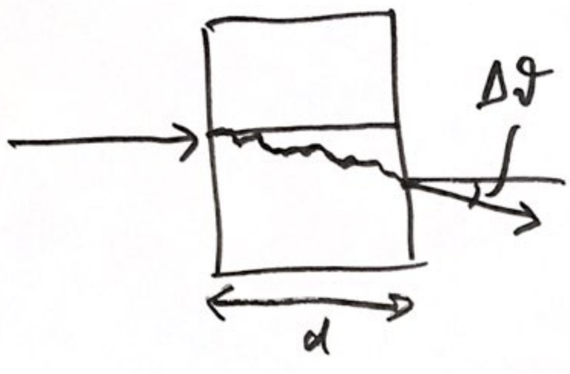
$$\Delta t = \frac{D}{v} = \frac{D}{\beta c}$$

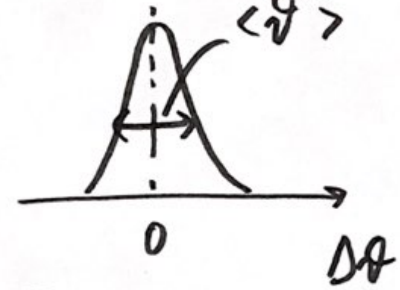
~~解~~ $$\begin{cases} \beta_\mu = \dfrac{p_\mu'}{E_\mu'} = \dfrac{489.1}{500.5} = 0.978 \\ \beta_\pi = \dfrac{489.2}{508.8} = 0.961 \end{cases}$$

$$\begin{cases} \Delta t_\mu = \dfrac{D}{\beta_\mu c} = \dfrac{10}{0.978 \cdot 3 \cdot 10^8} = 34.1 \text{ ns} \\ \Delta t_\pi = \dfrac{D}{\beta_\pi c} = \dfrac{10}{0.961 \cdot 3 \cdot 10^8} = 34.6 \text{ ns} \end{cases}$$



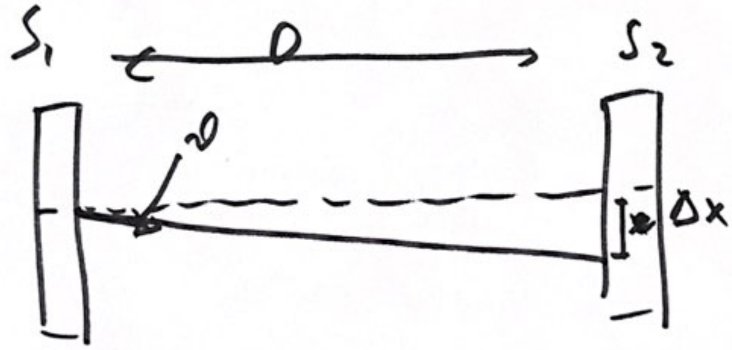
(c) deviazione rispetto alla direzione originale dovuta a scattering Coulombiano multiplo





$$\langle \vartheta^2 \rangle \simeq (21\ \text{MeV}) \cdot \frac{z}{\beta p} \cdot \sqrt{\frac{d}{X_0}}$$

$$\langle \vartheta^2 \rangle_\mu = 21\ \text{MeV} \cdot \frac{1}{0.978 \cdot 500} \cdot \sqrt{\frac{2}{2.59}} \sim 0.038\ \text{rad}$$



$$\Delta x = D \cdot \langle \vartheta^2 \rangle = 38\ \text{cm}$$

$\boxed{EX}$ elettroni $E = 25$ MeV

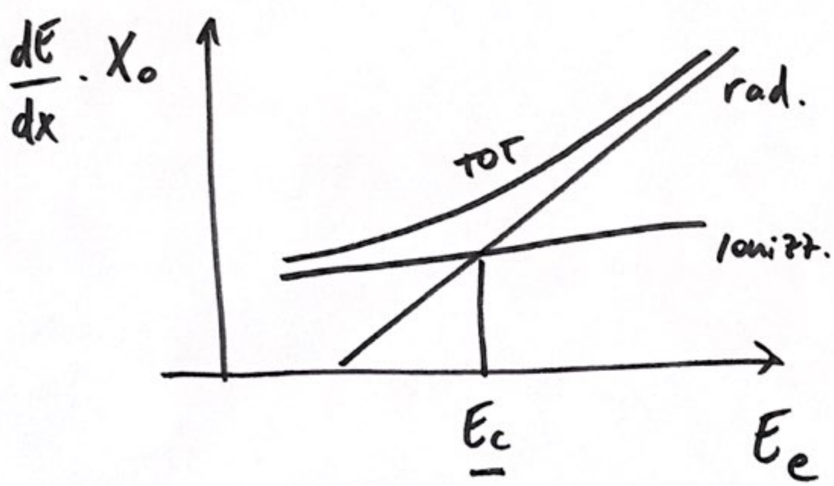
(a) energia persa 1 mm tessuto umano ~ H_2O

ACQUA: $\rho = 1.0 \ \frac{g}{cm^3}$ $\quad I = 80$ eV

$E_c = 78$ MeV $\quad X_0 = 36.1$ cm

$\frac{Z}{A} = 0.56 \quad \frac{\delta}{2} = 4.5$

$$-\frac{dE}{dx} = C\rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e\beta^2\gamma^2}{\langle I\rangle}\right) - \beta^2 - \frac{\delta}{2}\right]$$



$$\Delta E_e = \underbrace{\Delta E_{ion}} + \Delta E_{rad}$$

$$\Delta E_{ion} = \frac{dE}{dx} \cdot d$$

$$-\frac{dE}{dx} = 0.307 \cdot 1 \cdot 0.56 \cdot \frac{1}{\beta^2}\left[\ln \frac{2 \cdot 0.511 \cdot 10^6 \cdot \beta^2\gamma^2}{80} - \beta^2 - 4.5\right]$$

$E = 25$ MeV

$m = 0.511$ MeV

$$\Rightarrow p = \sqrt{E^2 - m^2} = 24.995 \text{ MeV}$$

$$\Rightarrow \beta = \frac{p}{E} = 0.9998$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{E}{m} = 50$$

$$\Rightarrow -\frac{dE}{dx} = 2.0 \text{ MeV/cm}$$

$$\Rightarrow \Delta E_{ion} = \frac{dE}{dx} \cdot d = 2.0 \cdot 0.1 = \underline{0.2 \text{ MeV}}$$

$$\underline{E(x) = E_0 e^{-x/X_0}}$$

$(x = X_0)$ $\quad E' = E_0 e^{-1} = \frac{E_0}{e}$

$$\Delta E(x) = E_0 - E(x) = E_0 - E_0 e^{-x/X_0}$$

$$= E_0 \left(1 - e^{-x/X_0}\right)$$

$$\Delta E_{rad} = 25 \text{ MeV} \left(1 - e^{-0.1/36.1}\right)$$

$$= 0.069 \text{ MeV}$$

$$\Delta E_{TOT} = \Delta E_{ion} + \Delta E_{rad} = 0.20 + 0.069 =$$

$$= 0.27 \text{ MeV}$$

$$\sigma \sim \frac{1}{M^2}$$

$$m_e = 0.511 \text{ MeV}$$

$$m_\mu = 106 \text{ MeV}$$

$$\sigma_\mu \sim \sigma_e \left(\frac{m_e}{m_\mu}\right)^2$$