Decadimento

$\lambda \equiv \Gamma$ prob. di decad. per unità di tempo

$$N(t) = N(0)\, e^{-\Gamma t} = e^{-t/\tau} = e^{-\lambda t}$$

$$\tau = \langle t \rangle = \frac{1}{\lambda} \quad \text{vita media prop.}$$

nel rif. solidale con le particelle.

In LAB

$$\tau_{LAB} = \gamma \tau \qquad N(t) = N(0)\, e^{-\frac{t_{LAB}}{\gamma\tau}}$$

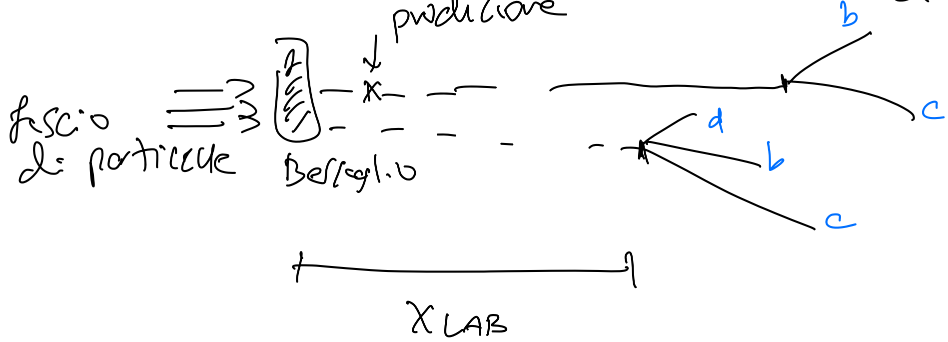
$$t_{LAB} = \frac{x_{LAB}}{v_{LAB}} = \frac{x_{LAB}}{\beta c}$$

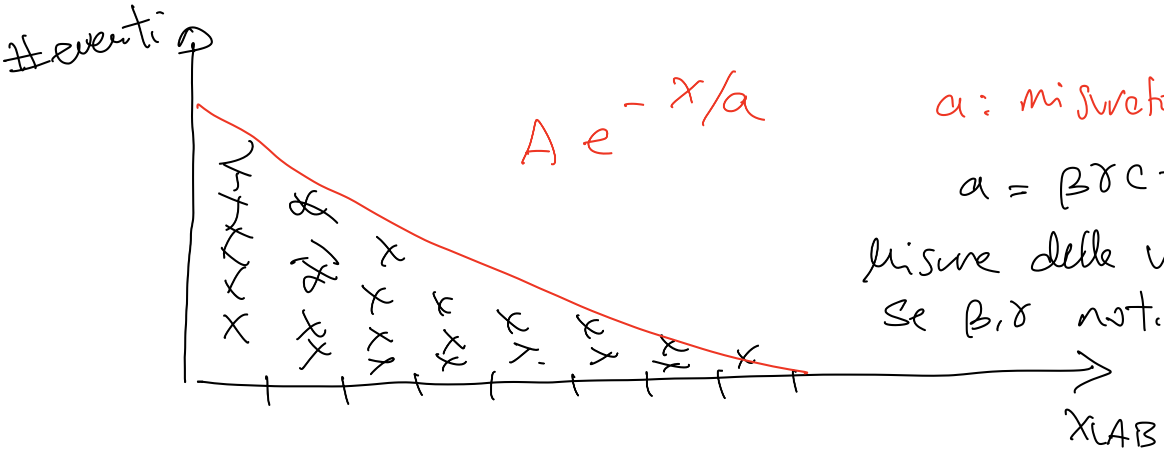
$$N(t) = N(0)\, e^{-\frac{x_{LAB}}{\beta\gamma c\tau}}$$

x_{LAB}: distanza percorsa dalla produzione al decadimento.

$$1\ \text{fm} = \frac{1}{200\ \text{MeV}}$$

$a \to b + c$
$b + c + d$ } prodotti di decadimento





a: misurato dai dati

$$a = \beta\gamma c\tau$$

Misura della vita media se β, γ noti

Perché decadimento:

particelle libere con H_0

$$H \psi_j(x,t) = E_j(x,t)$$

$$i \frac{d}{dt} \psi(x,t) = H \psi(x,t)$$

E_j: autovalori di energia

$$\psi_j(x,t) = \underbrace{\psi_j(\vec{x}, t=0)}_{\text{spaziale}} \; e^{-iE_jt}$$

$$|\psi_j(x,t)|^2 = |\psi_j(\vec{x})|^2$$

Introduciamo l'interazione come perturb. dipendente dal tempo.

$$\cancel{H} = H_0 + V$$

$$E_j = \underset{\substack{\downarrow \\ H_0}}{E_j^0} + \langle j|V|j\rangle + \sum_{j\neq K} \frac{|\langle j|V|K\rangle|^2}{E_j - E_K} - i\pi \sum_{j\neq K} |\langle j|V|K\rangle|^2 \delta(E_j - E_K)$$

j, K : su tutti autovalori di energia del sistema.

$$\Gamma = 2\pi \sum_{j\neq K} |\langle j|V|K\rangle|^2 \delta(E_j - E_K)$$

larghezza di decadimento

$$\psi_j(\vec{x},t) = \psi_j(\vec{x})\, e^{-iE_jt} = \underbrace{\psi_j(\vec{x})\, e^{-iE_j^0t}}_{\text{come prima con } H_0} \; \underbrace{e^{-\frac{\Gamma}{2}t}}_{\text{Dovuta a interazione}}$$

$$|\psi_j(x,t)|^2 = |\psi_j(\vec{x})|^2 \, e^{-\Gamma t}$$

Decadimento in 2 Corpi

$$a \rightarrow b + c$$

$$\underline{P_a} = \underline{P_b} + \underline{P_c} \qquad \text{conservazione 4-impulso.}$$

$$E_a = E_b + E_c$$

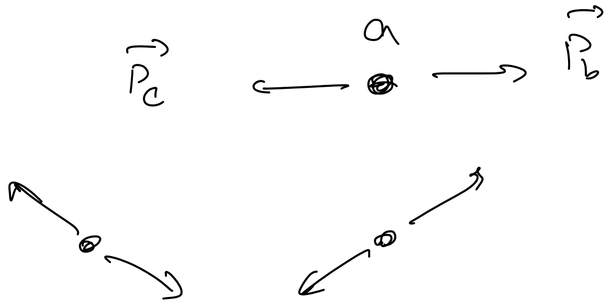
$$\vec{P_a} = \vec{P_b} + \vec{P_c}$$

Rif. Solidale con a

$$P_a = 0 \Rightarrow E_a = \sqrt{\cancel{P_a^2} + m_a^2} = m_a \Rightarrow m_a = E_b + E_c$$

$$\Rightarrow \vec{0} = \vec{P_b} + \vec{P_c} \Rightarrow \vec{P_b} = -\vec{P_c}$$

conservazione di impulso.



$|\vec{p}_b| = |\vec{p}_c| = p^*$

impulso di una figlia nel rf di a

$$M_a = \sqrt{p^{*2}+m_b^2} + \sqrt{p^{*2}+m_c^2}$$

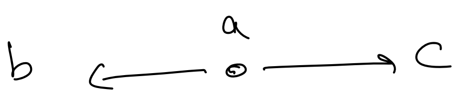
$$\left(M_a - \sqrt{p^{*2}+m_b^2}\right)^2 = p^{*2}+m_c^2$$

$$M_a^2 + \cancel{p^{*2}} + m_b^2 - 2M_a\sqrt{p^{*2}+m_b^2} = \cancel{p^{*2}} + m_c^2$$

$$2M_a\sqrt{p_*^2+m_b^2} = M_a^2 + m_b^2 - m_c^2$$

quadrato ai due lati:

$$p^* = \frac{m_a^2 - m_b^2 - m_c^2}{2m_a}$$



$$M_a = E_b + E_c = m_b + K_b + m_c + K_c$$

$$= m_b + m_c + K_b + K_c$$

$$E = \sqrt{m^2+p^2} = m + K$$

$K \geq 0$

Limite cinematico è

$K_b = K_c = 0.$

$$Q = M_a - m_b - m_c = K_b + K_c \geq 0.$$

Q-value del decadimento

$Q \geq 0$ in tutti decadimenti permessi

Decadimenti radiattivi: $\alpha \equiv {}^{4}_{2}He$

Z: protoni

A-Z: neutroni

$${}^{A}_{Z}N \longrightarrow {}^{A-4}_{Z-2}N + \alpha$$

Decadimento α del nucleo

$$Q = M(A,Z) - M(A-4, Z-4) - M_\alpha \geq 0$$

Decadimento β:

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

$m_n = 936$ MeV

$m_p = 935$ MeV β^-

$m_e = 0.511$ MeV

$m_{\bar{\nu}} \simeq 0$ MeV

$$p \longrightarrow n + e^+ + \nu_e$$

$m_{e^+} = m_{e^-} = 0.511$ MeV. β^+

$Q = m_p - m_N - m_{e^+} < 0$ Non avviene.

$${}^{A}_{Z}N \longrightarrow {}^{A}_{Z+1}N + e^- + \bar{\nu}_e \quad \beta^-$$

$${}^{A}_{Z}N \longrightarrow {}^{A}_{Z-1}N + e^+ + \nu_e \quad \beta^+$$

$a \longrightarrow 1 + \dots N$ decadimento a N particelle

$$Q = m_a - m_1 - \dots - m_N \geq 0$$

ν_e : neutrino

$\bar{\nu}_e$: anti neutrino.

p : protone

$\bar{p}$: anti-protone

$$\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$$

	Mode		Fraction (Γ_i/Γ)	Confidence level
Γ_1	$e^-\bar{\nu}_e\nu_\mu$		$\approx 100\%$	
Γ_2	$e^-\bar{\nu}_e\nu_\mu\gamma$		[a] $(6.0\pm0.5)\times10^{-8}$	
Γ_3	$e^-\bar{\nu}_e\nu_\mu e^+e^-$		[b] $(3.4\pm0.4)\times10^{-5}$	
	Lepton Family number (*LF*) violating modes			
Γ_4	$e^-\nu_e\bar{\nu}_\mu$	*LF*	[c] < 1.2 %	90%
Γ_5	$e^-\gamma$	*LF*	$< 4.2 \times10^{-13}$	90%
Γ_6	$e^-e^+e^-$	*LF*	$< 1.0 \times10^{-12}$	90%
Γ_7	$e^-2\gamma$	*LF*	$< 7.2 \times10^{-11}$	90%

$\mu^- \longrightarrow e^- + \gamma$

$M_\mu = 106$ MeV

$M_e = 0.5$ MeV

$M_\gamma = 0$ MeV

$Q = 106 - 0.5 = 105.5$ MeV

violazione del numero leptonico

$\pi^\pm$: $(u\bar{d}), (\bar{u}d)$ mesoni $M_\pi = 140$ MeV

$M_\pi - M_e / M_\mu > 0$

$\pi^+ \longrightarrow \mu^+ \nu_\mu$

$e^+ \nu_e$

	Mode		Fraction (Γ_i/Γ)	Confidence level
Γ_1	$\mu^+\nu_\mu$	[a]	(99.98770±0.00004) %	
Γ_2	$\mu^+\nu_\mu\gamma$	[b]	(2.00 ±0.25) $\times 10^{-4}$	
Γ_3	$e^+\nu_e$	[a]	(1.230 ±0.004) $\times 10^{-4}$	
Γ_4	$e^+\nu_e\gamma$	[b]	(7.39 ±0.05) $\times 10^{-7}$	
Γ_5	$e^+\nu_e\pi^0$		(1.036 ±0.006) $\times 10^{-8}$	
Γ_6	$e^+\nu_e e^+ e^-$		(3.2 ±0.5) $\times 10^{-9}$	
Γ_7	$\mu^+\nu_\mu\nu\bar{\nu}$		< 9 $\times 10^{-6}$	90%
Γ_8	$e^+\nu_e\nu\bar{\nu}$		< 1.6 $\times 10^{-7}$	90%

$Q_\mu = M_\pi - M_\mu = 140 - 106$ MeV $= 34$ MeV.

$Q_e = M_\pi - M_e = 139.5$ MeV

$Q_e >> Q_\mu \Rightarrow \pi^+ \longrightarrow e^+$ dovrebbe essere più abbondante

$M_H = 125$ GeV

H DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Confidence level
Γ_1	WW^*	(25.7 ±2.5) %	
Γ_2	ZZ^*	(2.80±0.30) %	
Γ_3	$\gamma\gamma$	(2.50±0.20) $\times 10^{-3}$	
Γ_4	$b\bar{b}$	(53 ±8) %	
Γ_5	e^+e^-	< 3.0 $\times 10^{-4}$	95%
Γ_6	$\mu^+\mu^-$	(2.6 ±1.3) $\times 10^{-4}$	
Γ_7	$\tau^+\tau^-$	(6.0 $^{+0.8}_{-0.7}$) %	
Γ_8	$Z\gamma$	(3.4 ±1.1) $\times 10^{-3}$	
Γ_9	$Z\rho(770)$	< 1.21 %	95%
Γ_{10}	$Z\phi(1020)$	< 3.6 $\times 10^{-3}$	95%
Γ_{11}	$Z\eta_c$		
Γ_{12}	ZJ/ψ	< 1.9 $\times 10^{-3}$	95%
Γ_{13}	$Z\psi(2S)$	< 6.6 $\times 10^{-3}$	95%
Γ_{14}	$J/\psi\gamma$	< 2.0 $\times 10^{-4}$	95%
Γ_{15}	$J/\psi J/\psi$	< 3.8 $\times 10^{-4}$	95%

$H \rightarrow W^+ W^{-*}$ $\quad M_W = 80$ GeV

$\rightarrow Z^0 Z^{0*}$ $\quad M_Z = 90$ GeV

$Q = M_H - 2M_W < 0$

1 dei due W, Z virtuale (off-shell)

$H \rightarrow \gamma\gamma$

$\gamma \longleftarrow \overset{H}{\bullet} \longrightarrow \gamma$

$H \rightarrow b\bar{b}$

$H \longrightarrow$ Γ_1 WW

Γ_2 ZZ

Γ_3 $\gamma\gamma$

$\Gamma = \Sigma_i \Gamma_i = \Gamma_{tot}$ di decadimento.

$$\frac{\Gamma_i}{\Gamma} = \text{Branching Fraction / Ratio}$$

$$0 < \frac{\Gamma_i}{\Gamma} < 1$$ percentuale di decadimento in un canale specifico.

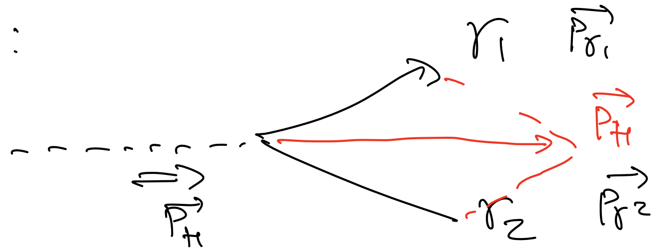
$$\tau = \frac{1}{\Gamma} = \frac{1}{\Sigma_i \Gamma_i}$$

$$H \to \gamma\gamma \qquad \underline{P}_H = \underline{P}_{\gamma_1} + \underline{P}_{\gamma_2}$$

Nel rif. dell'Higgs.

$$M_H = E_{\gamma_1} + E_{\gamma_2}.$$

LAB:



$$\underline{P}_H^{LAB} = \underline{P}_{\gamma_1}^{LAB} + \underline{P}_{\gamma_2}^{LAB}.$$

$$|\underline{P}_H|^2 = m_H^2$$ invariante di Lorentz

misurate sperimentalmente (E_1^γ, p_1^γ) $(E_{\gamma_2}, \vec{p}_2^\gamma)$

$$E_{\gamma_1} \equiv P_{\gamma_1} \qquad E_{\gamma_2} \equiv P_{\gamma_2}$$

Misurare $E_{\gamma_1}, E_{\gamma_2}$.

$$(E_{\gamma_1}\ \theta_{\gamma_1}, \varphi_{\gamma_1}) \equiv \vec{P}_1$$

$$(E_{\gamma_2}, \theta_{\gamma_2}, \varphi_{\gamma_2}) \equiv \vec{P}_2$$

$$\underline{P}_{\gamma_1} = (P_{\gamma_1}, \vec{P}_{\gamma_1})$$

$$\underline{P}_{\gamma_2} = (P_{\gamma_2}, \vec{P}_{\gamma_2})$$

$$(\underline{P}_{\gamma_1} + \underline{P}_{\gamma_2})^2 = m_H^2$$

$\Rightarrow$ misura di M_H da energie, θ, φ dei fotoni.

$a \rightarrow b + c$

$$(\underline{P}_b + \underline{P}_c)^2 = m^2_{inv}$$ masse invariante.

$M_{inv} \simeq M_a$

$$a: \quad \psi(\vec{x},t) = \psi(\vec{x})\, e^{-iEt}\, e^{-\frac{\Gamma}{2}t}$$

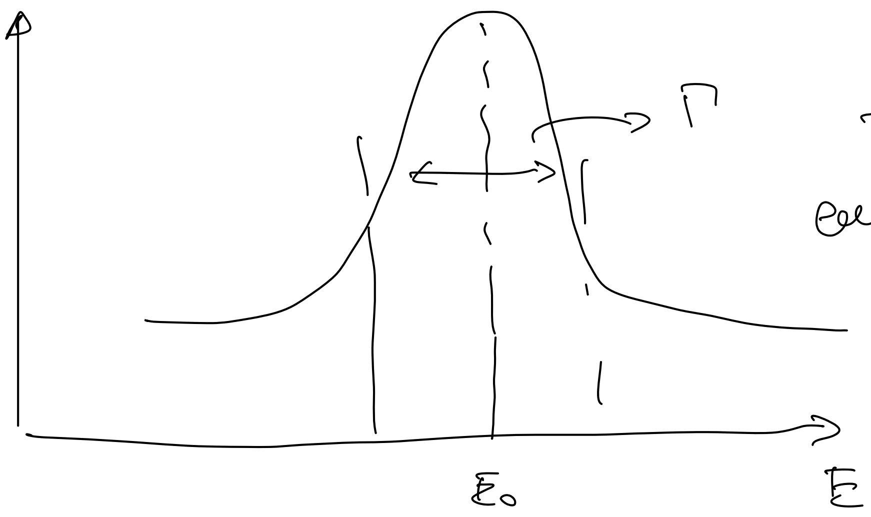
$$\chi(E) = C\int_0^\infty dt'\, e^{iEt'}\, \psi(x)\, e^{-iE_0 t}\, e^{-\frac{\Gamma}{2}t}$$

$$C\,\psi(x)\int dt'\, e^{i(E-E_0)t'}\, e^{-\frac{\Gamma}{2}t'}$$

$$= C\,\frac{}{i(E-E_0)-\frac{\Gamma}{2}}$$

Lorentz
Cauchy
Breit-Wigner.

$$|\chi(E)|^2 = C\,|\psi(x)|^2\,\frac{1}{(E-E_0)^2+\frac{\Gamma^2}{4}}$$



70% area entro Γ

$$E = m + \underline{K} = \pm\sqrt{m^2 + p^2} = m\sqrt{1 + \frac{p^2}{m^2}} = m\left(1 + \frac{1}{2}\frac{p^2}{m^2}\right)$$

$$K = E - m \qquad = \underbrace{m}_{} + \underbrace{\frac{1}{2}\frac{p^2}{m}}_{} \qquad \frac{p}{m} \ll 1$$

$$e^{iEt} \qquad\qquad e^{i(-E)(-t)}$$

$$-\sqrt{p^2 + m^2}$$

$$t < 0$$

$$+\sqrt{p^2 + m^2}$$

$$t > 0$$