$$\underline{P}_{in}^2 = \underline{P}_f^2 \Rightarrow |\underline{P}_{pi}|^2 + |\underline{P}_{x_{in}}|^2 + 2\,\underline{P}_{p_i}\cdot\underline{P}_{x_i}$$

$$= |\underline{P}_{pf}|^2 + |\underline{P}_{xf}|^2 + 2\,\underline{P}_{pf}\cdot\underline{P}_{xf}$$

$$E_p^i \cdot m_X = E_p^f E_x^f - \vec{P}_p^f \cdot \vec{P}_x^f$$

1) $\theta = 0 \Rightarrow \vec{P}_p^f \cdot \vec{P}_x^f = p_p^f\, p_x^f$

$$E_p^i\, m_X = E_p^f E_x^f - p_p^f\, p_x^f$$

2) $p_p^i = p_p^f + p_x^f \Rightarrow p_p^f = p_p^i - p_x^f$

3) $E_p^i + m_X = E_p^f + E_x = E_p^f + m_X + T$

$$E_p^i = E_p^f + T \Rightarrow E_p^f = E_p^i - T$$

$E_x = E_x^f$

$p_x = p_x^f$

4) $E_x = T + m_X$

$$E_p^i\, m_X = E_p^f E_x - p_p^f\, p_x$$

$$E_p^f = E_p^i - T$$

$$E_p^i\, m_X = (E_p^i - T) E_x - p_p^f\, p_x$$

$E_x = T + m_X$

$$= (E_p^i - T)(m_X + T) - p_p^f\, p_x$$

$$= E_p^i m_X + E_p^i T - m_X T - T^2 - p_p^f\, p_x$$

$$E_p^i T - m_x T - T^2 - p_p^f p_x = 0$$

$$p_p^f = p_p^i - p_x$$

$$p_p^f p_x = (p_p^i - p_x) p_x = p_p^i p_x - p_x^2$$

$$p_x^2 = E_x^2 - m_x^2$$

$$p_p^f p_x = p_p^i p_x - E_x^2 + m_x^2$$

$$(E_p^i - m_x) T - T^2 - p_p^i p_x + E_x^2 - m_x^2 = 0$$

$$(E_p^i - m_x) T - T^2 + E_x^2 - m_x^2 = p_p^i p_x$$

$$E_x = m_x + T$$

$$(E_p^i - m_x) T - T^2 + T^2 + m_x^2 + 2 m_x T - m_x^2 = p_p^i p_x$$

$$\frac{(E_p^i + m_x) T}{p_p^i} = \frac{p_p^i p_x}{p_p^i}$$

$$p_x = \frac{E_p^i + m_x}{p_p^i} T$$

$$E_p^i = \gamma m_p$$

$$p_p^i = \gamma \beta m_p$$

Faccio il quadrato.

$$p_x^2 = \left(\frac{E_p^i + m_x}{p_p^i}\right)^2 T^2$$

$$p_x^2 = E_x^2 - m_x^2 = (T + m_x)^2 - m_x^2 = T^2 + 2m_x T + m_x^2 - m_x^2$$

$$p_x^2 = T^2 + 2m_x T$$

$$T^2 + 2m_x T = \left(\frac{E_p^i + m_x}{p_p^i}\right)^2 T^2$$

$$\Rightarrow \quad 2m_x = \left[\left(\frac{E_p^i + m_x}{p_p^i}\right)^2 - 1\right] T$$

$$\frac{E_p^{i\,2} + 2m_x E_p^i + m_x^2 - p_p^{i\,2}}{(p_p^i)^2} = \frac{m_p^2 + 2m_x E_p^i + m_x^2}{(p_p^i)^2}$$

$$\Rightarrow \quad T = \frac{2m_x}{\dfrac{m_p^2 + m_x^2 + 2m_x E_p^i}{(p_p^i)^2}}$$

$$T = \frac{2m_x (p_p^i)^2}{m_p^2 + 2m_x E_p^i + m_x^2}$$

$$p_p^i = \gamma\beta\, m_p$$

$$E_p^i = \gamma m_p$$

$$T = \frac{2m_x \gamma^2\beta^2 m_p^2}{m_p^2 + 2m_x \cdot \gamma m_p + m_x^2}$$

$$T = \frac{2m_x \gamma^2\beta^2}{1 + 2\gamma\left(\frac{m_x}{m_p}\right) + \left(\frac{m_x}{m_p}\right)^2}$$