$\alpha + N \rightarrow \alpha + N.$

$|i\rangle = |\alpha N\rangle \qquad |f\rangle = |\alpha N\rangle$

assumiamo N fermo; urto elastico contro il muro.

$\Rightarrow$ solo $\vec{p}_\alpha$ può variare

$$\Gamma(i \to f) = 2\pi |M_{fi}|^2 \rho(E).$$

$$M_{fi} = -i \int_V d^3V \, \psi_f^* H_I \psi_I$$

Approssimazione di Born.

$$\psi_i = \frac{e^{i\vec{p}\cdot\vec{r}}}{\sqrt{2\pi}} \quad \text{onde libere}$$

p

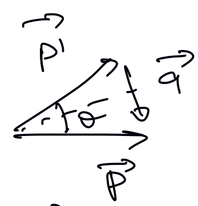
$$\psi_i = \frac{e^{i\vec{p}\cdot\vec{r}}}{\sqrt{2\pi}} \qquad \psi_f = \frac{e^{i\vec{p}\,'\cdot\vec{r}}}{\sqrt{2\pi}}$$

$$H_I = \frac{e^2}{4\pi\varepsilon_0} \frac{Z_\alpha \cdot Z_N}{r} = \frac{\alpha Z_\alpha Z_N}{r} = \frac{A}{r}$$

$$M_{fi} = -i \frac{1}{2\pi} \int d^3r \, e^{-i\vec{p}\,'\cdot\vec{r}} \frac{A}{r} e^{+i\vec{p}\cdot\vec{r}} = -i\frac{A}{2\pi}\int d^3r \, \frac{e^{-i(\vec{p}\,'-\vec{p})\cdot\vec{r}}}{r}$$

$q = 2p \sin\frac{\theta}{2}$

$|\vec{p}| = |\vec{p}\,'|$



$-p' + \vec{p} = \vec{q} \Rightarrow \vec{p} = p' + \vec{q}$

$$M_{fi} = -i\frac{A}{2\pi}\int d^3r \, \frac{e^{i\vec{q}\cdot\vec{r}}}{r} \qquad d^3r = \sin\theta \, d\theta \, d\varphi \, r^2 dr$$

$$\vec{q}\cdot\vec{r} = qr\cos\theta \qquad \int_0^\pi \sin\theta \, d\theta = -\int_1^{-1} d\cos\theta = \int_{-1}^{1} d\cos\theta.$$

$$\int_{-1}^{1} d\cos\theta \, e^{iqr\cos\theta} = \frac{1}{iqr}\left[e^{iqr} - e^{-iqr}\right].$$

$$M_{fi} = -i\frac{A}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} r^2 dr \, \frac{1}{r}\, \frac{1}{iqr}\left[e^{iqr} - e^{-iqr}\right]$$

$$= -i\frac{A}{2\pi}(2\pi)\frac{1}{iq}\int_0^{\infty}\left[e^{iqr} - e^{-iqr}\right] dr$$

anzichè usare $V(r) = \frac{A}{r}$ usiamo $V(r) = A\frac{e^{-\lambda r}}{r}$

e poi alla fine $\lim_{\lambda \to 0}$

$$M_{fi} = -\frac{iA}{\cancel{2\pi}}\frac{(\cancel{2\pi})}{iq}\lim_{\lambda\to 0}\int_0^{\infty} e^{-\lambda r}\left(e^{iqr} - e^{-iqr}\right) dr$$

$\alpha = \lambda - iq$

$$\int_0^{\infty} e^{-\lambda r} e^{iqr} dr = \frac{1}{\lambda - iq}\int_0^{\infty} \alpha\, e^{-\alpha r} dr$$

$$= \frac{1}{\lambda - iq}\int_0^{\infty} e^{-z} dz = \frac{1}{\lambda - iq}[0 - 1]$$

analogamente

$$\int_0^{\infty} e^{-\lambda r} e^{-iqr} dr = \frac{1}{\lambda + iq}[0-1].$$

$$\int_0^{\infty} e^{-\lambda r}\left(e^{iqr} - e^{-iqr}\right) dr = \frac{1}{\lambda - iq} - \frac{1}{\lambda + iq} = \frac{\lambda + iq - \lambda + iq}{\lambda^2 - (-q^2)}$$

$$= \frac{2iq}{\lambda^2 + q^2}$$

$$M_{fi} = \lim_{\lambda \to 0}\left[-i\frac{A}{i\cancel{q}}\frac{2i\cancel{q}}{\lambda^2 + q^2}\right] = -2iA\frac{1}{q^2}$$

$$\Rightarrow |M_{fi}|^2 = \frac{4A^2}{q^4}$$

$$\rho(E) = \int dn\, \delta(E_f - E_i)$$

$$dn = \frac{1}{8} \frac{(2\pi)^3}{V} p^2 dp \qquad p: \text{impulso delle particelle } \alpha$$

particelle α non relativistica.

$$E = \frac{p^2}{2m} \Rightarrow p^2 = 2mE \Rightarrow \cancel{2} p\,dp = \cancel{2} m\,dE$$

$$p\,dp = m\,dE$$

$$p = \sqrt{2mE} \qquad \Rightarrow \qquad p^2 dp = p\, m\, dE$$

$$\rho(E) = \int dn\, \delta(E_f - E_i) = \frac{(2\pi)^3}{8V} \int p^2 dp\, \delta(E_f - E_i)$$

$$= \frac{(2\pi)^3}{8V} \int p\, m\, \delta(E_f - E_i)\, dE = \frac{(2\pi)^3}{8V} m \underbrace{\sqrt{2mE_i}}_{= p_i}$$

p del proiettile.

Ricordiamo di nuovo che

$$\Gamma(i \to f) = \frac{dN_r}{dt} \frac{1}{N_B} \frac{1}{N_P} = \sigma \cdot \frac{v_P}{V}$$

velocità proiettile

$$= 2\pi |M_{fi}|^2 \rho(E).$$

$$\Rightarrow (2\pi) \frac{4A^2}{q^4} \frac{(2\pi)^3}{8\cancel{V}} m \sqrt{\cancel{2}m\cancel{E_i}} = \sigma \frac{v_P}{\cancel{V}} = \sigma \sqrt{\frac{\cancel{2}}{m}} \cancel{\sqrt{E_i}}$$

$$E = \frac{p^2}{2m} = \frac{1}{2} m v_P^2 \Rightarrow v_P = \sqrt{\frac{2E_i}{m}}$$

$$\Rightarrow \quad \sigma = \frac{4A^2}{q^4} \frac{(2\pi)^4}{8} m^2$$

$$q = 2p \sin\frac{\theta}{2} \Rightarrow q^4 = 16\, p^4 \sin^4 \frac{\theta}{2}$$

$$E = \frac{p^2}{2m} \Rightarrow p^4 = 4m^2E^2$$

$$\Rightarrow \quad \Gamma = \frac{\cancel{4} A^2}{16 \times \cancel{4}\, \cancel{m^2}\, E^2 \sin^4\frac{\theta}{2}} \; \frac{(2\pi)^4}{8} \cancel{m^2}$$

$$= \frac{(2\pi)^4}{8} \left(\frac{A}{4E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}} \qquad A = \alpha Z_\alpha Z_N$$

Controllore