Isospin

introdotto da Heisenberg per spiegare inter forte / nucleare per neutroni e protoni (nucleoni)

Isospin simile al momento angolare

$$\text{nucleone} = \begin{pmatrix} p \\ n \end{pmatrix} \qquad \begin{aligned} p &= |\tfrac{1}{2}, +\tfrac{1}{2}\rangle \quad I = 1/2 \quad I_3 = +1/2 \\ n &= |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \end{aligned}$$

Mesoni π : $\pi^{\pm}$, π^0

$M_{\pi^\pm} \sim 140$ MeV simili

$M_{\pi^0} \sim 135$ Me

$|\pi^+\rangle = |I = 1, I_3 = +1\rangle \quad |\pi^0\rangle = |1, 0\rangle, \quad |\pi^-\rangle = |1, -1\rangle$

utilizzare l'isospin

1) classificazione di adroni scoperti (o da scoprire)
2) stimare sezioni d'urto di processi forti
3) prevedere esistenza di alcuni stati ancora non osservati

Isospin si conserva nelle interazioni forti
$\Rightarrow$ Trasf. di Isospin è una simm. della H_F.

$$\Delta I = 0 \qquad \underset{I, I_3}{|i\rangle} \xrightarrow[H_F]{} \underset{I, I_3}{|f\rangle}. \qquad I_3 = I_f$$

Deuterio

stato legato np ${}^2_1H$

$n = |\tfrac{1}{2}, -1/2\rangle \qquad p = |1/2, +1/2\rangle$.

$np \quad I = ? \quad I_3 = ?$

np : $I_3 = \frac{1}{2} - \frac{1}{2} = 0$ $\qquad I = 0, 1.$

t r i p l e t t o

pp := $|\frac{1}{2}\frac{1}{2}\rangle|\frac{1}{2}1/2\rangle = |1, +1\rangle \qquad I_3 = +1, I = 1.$

pn : $\frac{1}{\sqrt{2}}|\frac{1}{2}\frac{1}{2}\rangle|\frac{1}{2}\frac{-1}{2}\rangle + \frac{1}{\sqrt{2}}|\frac{1}{2}-\frac{1}{2}\rangle|\frac{1}{2},-1/2\rangle \qquad I = 1, I_3 = 0.$

nn := $|\frac{1}{2},-\frac{1}{2}\rangle|\frac{1}{2},-1/2\rangle = |1,-1\rangle \qquad I = 1.$

pn: $\frac{1}{\sqrt{2}}\left(|\frac{1}{2}\frac{1}{2}\rangle|\frac{1}{2},-\frac{1}{2}\rangle - |\frac{1}{2}\frac{1}{2}\rangle|\frac{1}{2},-1/2\rangle\right)$

$I = 0, I_3 = 0.$

singoletto di isospin.

se isospin è una simm. della forte nucleare.

se np è $|I=1, I_3=0\rangle \Rightarrow$ esistere pp nn

ma sperimentalmente non esistono pp, nn.

$\Rightarrow$ np è $I=0, I_3=0$.

singoletto di isospin.

deutone (nucleo del deuterio) $I=0$

Isospin e sezioni d'urto.

(a) $$p + p \longrightarrow d + \pi^+$$

deutone (nucleo deuterio)

	p	p	d	π^+
Q	$+1$	$+1$	$+1$	$+1$
I_3	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+1$

$|f\rangle = |I=1, I_3=+1\rangle$

$|i\rangle = |I=1, I_3=+1\rangle$

(b) $$p + n \longrightarrow d + \pi^0$$

	p	n	d	π^0
Q	$+1$	0	$+1$	0
I_3	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0
			$\lvert 0,0\rangle$	$\lvert 1,0\rangle$

$|f\rangle = |I=1, I_3=0\rangle$

$p+n = \frac{1}{\sqrt{2}}\left(|1,0\rangle + |0,0\rangle\right)$

i: $I_3 = 0$. $I = 0, 1$

$|i\rangle = \frac{1}{\sqrt{2}}\left(|1,0\rangle + |0,0\rangle\right) \longrightarrow |f\rangle = |1,0\rangle$

(c) $n+n \longrightarrow d + \pi^-$

	n	n	d	π^-
I_3	$-\frac{1}{2}$	$-\frac{1}{2}$	0	-1
Q	0	0	$+$	-1

$|f\rangle = |1,-1\rangle$

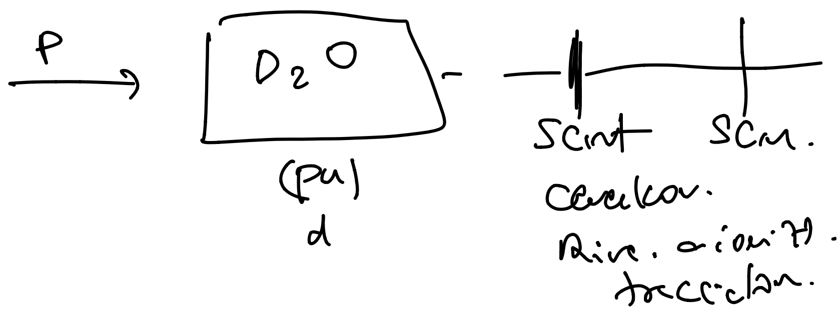
$|1,-1\rangle$

$M_a : M_b : M_c = 1 : \frac{1}{\sqrt{2}} : 1$

$\sigma \propto |M|^2$ (regola di Fermi).

$\sigma_a : \sigma_b : \sigma_c \doteq 1 : \frac{1}{2} : 1$

$N(p+p \to d+\pi^+) : N(p+n \to d+\pi^0) \doteq 2 : 1$



Si è misurato sperim. $\frac{\sigma_a}{\sigma_b} \simeq 2$

Diffusione pione-nucleone:

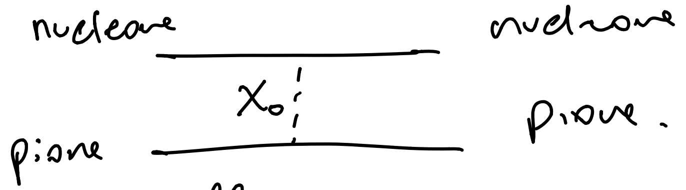
π + nucleone $\longrightarrow$ pione + nucleone.

$\begin{pmatrix} \pi^+ \\ \pi^- \\ \pi^0 \end{pmatrix}$ $I=1$ $\begin{pmatrix} p \\ n \end{pmatrix}$ $I=1/2$ $\begin{pmatrix} \pi^+ \\ \pi^- \\ \pi^0 \end{pmatrix}$ $I=1$ $\begin{pmatrix} p \\ n \end{pmatrix}$ $I=\frac{1}{2}$

$\pi^+ + p \rightarrow \pi^+ + p$

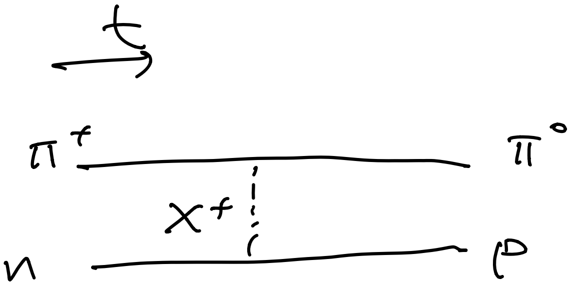
$\pi^0 + p \rightarrow \pi^0 + p$

$\vdots$



diffusione elastica

processi con scambio di carica



$\pi^+ + n \rightarrow \pi^0 + p$

$\pi^0 + p \rightarrow \pi^+ + n$

$\pi^0 + n \rightarrow \pi^- + p$

$\pi^- + p \rightarrow \pi^0 + n$

pione + nucleone

$$|I=1, a\rangle + |I=\tfrac{1}{2}, b\rangle = \alpha\,|I=\tfrac{3}{2}, I_3 = a+b\rangle + \beta\,|I=\tfrac{1}{2}, I_3 = a+b\rangle .$$

Table 3.3. *Clebsch–Gordan coefficients in pion–nucleon scattering*

		$I=\frac{3}{2}$				$I=\frac{1}{2}$	
Pion	Nucleon	$I_3=\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
π^+	p	1					
π^+	n		$\sqrt{\frac{1}{3}}$			$\sqrt{\frac{2}{3}}$	
π^0	p		$\sqrt{\frac{2}{3}}$			$-\sqrt{\frac{1}{3}}$	
π^0	n			$\sqrt{\frac{2}{3}}$			$\sqrt{\frac{1}{3}}$
π^-	p			$\sqrt{\frac{1}{3}}$			$-\sqrt{\frac{2}{3}}$
π^-	n				1		

pione + nucleone $\xrightarrow{H_F}$ pione + nucleone.

$|i\rangle = \alpha|I=3/2\rangle + \beta|I=1/2\rangle$ $\qquad$ $|f\rangle = \gamma|I=3/2\rangle + \delta|I=1/2\rangle$

$$\langle f|H_F|i\rangle = A\,\overbrace{\langle f, I=3/2|\,H_3\,|i, I=3/2\rangle}^{M_3} + B\,\overbrace{\langle f, I=1/2|\,H_1\,|i, I=1/2\rangle}^{M_1}.$$

$H_F = H_3 + H_1$ $\qquad$ se inter forte conserva Isospin.

$\Delta I = 0$

$I_f = I_i$

$\sigma \propto |M_{fi}|^2$

$M_{fi} = \langle f|H_F|i\rangle = a M_3 + b M_1$

$N(\text{event.}) \propto \sigma \propto |M_{fi}|^2 \propto |a|^2|M_3|^2 + |b|^2 \cancel{} |M_1|^2 + ab\, M_1 M_3$

sperimentalmente 3 reazioni facili da osservare.

(a) $\pi^+ + p \to \pi^+ + p$ $\qquad M_a = M_3$

$\sigma_a \propto |M_3|^2$

(f) $\pi^- + n \to \pi^- + n$ $\qquad M_f = M_3$

$\sigma_f \propto |M_3|^2$

Mi aspetto $N(\pi^+ + p \to \pi^+ + p) = N(\pi^- + n \to \pi^- + n)$

(c) $\pi^- + p \to \pi^- + p$ $\qquad M_c = \frac{1}{3} M_3 + \frac{2}{3} M_1$

(j) $\pi^- + p \to \pi^0 + n$ $\qquad M_j = \frac{\sqrt{2}}{3}(M_3 - M_1)$

(a) $\pi^+ + p \to \pi^+ + p$ $\quad N_a \propto \sigma_a \propto |M_3|^2$

(c) $\pi^- + p \to \pi^- + p$ $\quad N_c \propto \sigma_c \propto \left(\frac{1}{3}M_3 + \frac{2}{3}M_1\right)^2$

$$= \frac{1}{9}(M_3 + 2M_1)^2$$

(j) $\pi^- + p \to \pi^0 + n$ $\quad N_j \propto \sigma_j \propto \frac{2}{9}|M_3 - M_1)^2$

2 scenari:

$M_1 \gg M_3$

$$\sigma_a \simeq 0$$
$$\sigma_c \simeq \frac{4}{9}\,|M_1|^2$$
$$\sigma_j \simeq \frac{2}{9}|M_1|^2$$

N eventi: $\quad 0 : 2 : 1$

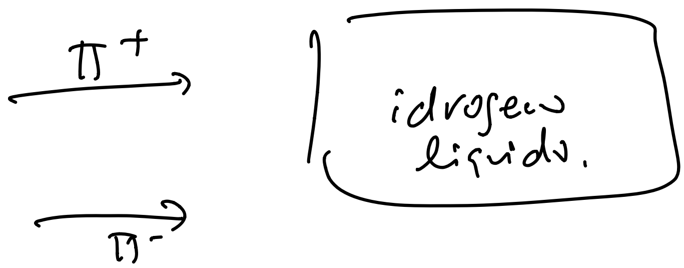
$M_3 \gg M_1$

(a) $\sigma_a \simeq |M_3|^2$ $\qquad \pi^+ + p \to \pi^+ + p.$

(c) $\sigma_c \simeq \frac{1}{9}|M_3|^2$ $\qquad \pi^- + p \to \pi^- + p.$

(j) $\sigma_j \simeq \frac{2}{9}|M_3|^2$ $\qquad \pi^- + p \to \pi^0 + n$

$$\frac{N(\pi^+ + p)}{N(\pi^- + p)} = \frac{\sigma_a}{\sigma_j + \sigma_c} = \frac{9}{3} = 3$$



conto.

conto

$$\frac{\sigma(\pi^+ + p)}{\sigma(\pi^- + p)} = \frac{N(\pi^+ + p)}{N(\pi^- + p)}$$

$$\frac{N(\pi^+ + p)}{N(\pi^- + p)} \sim 3$$

Conferma di isospin

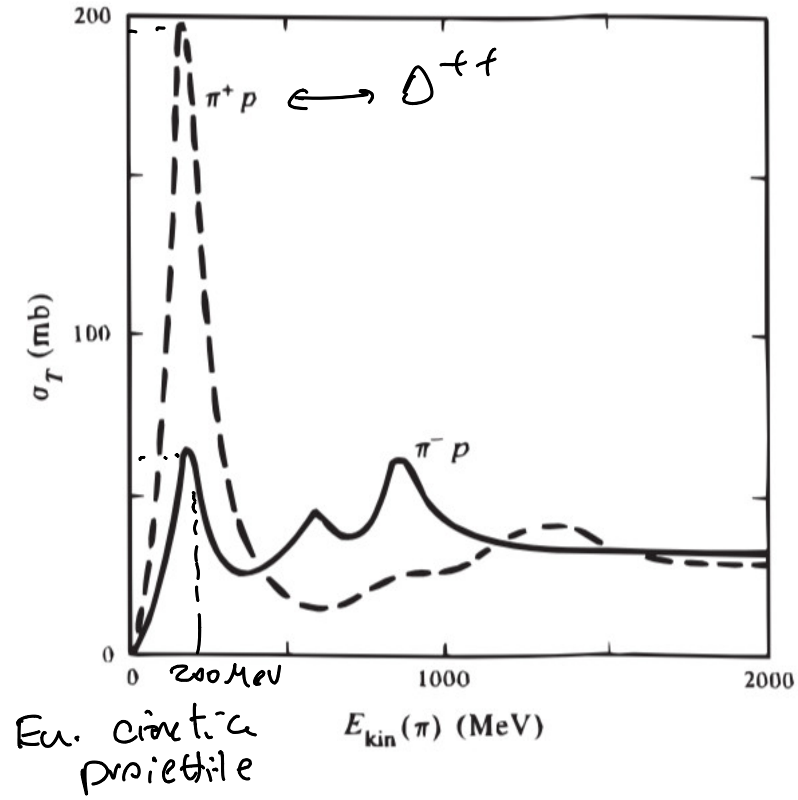


Figure 5.35: Total cross section as a function of pion kinetic energy for the scattering of positive and negative pions from protons. (1 mb = 1 millibarn = 10^{-27} cm^2.)

Relazione tra l'isospin e la carica

Nucleo $A = Z + N$.

l'isospin del nucleo $I_3 = Z \times \frac{1}{2} + N\left(-\frac{1}{2}\right)$

$$= \frac{Z}{2} - (A - Z)\frac{1}{2}$$

$$\Rightarrow \quad Z = I_3 + \frac{A}{2}$$

Z: carica in unità di e.

A: # barioni

Gell-Mann – Nishijima hanno proposto.

$$Q = I_3 + \frac{B}{2} \qquad B\text{: numero barionico.}$$

$$p: \quad Q = \frac{1}{2} + \frac{1}{2} = 1.$$

$$n = -\frac{1}{2} + \frac{1}{2} = \emptyset.$$

funziona anche per i mesoni: $\pi : I = 1$, $I_3 \in (-1, 0, +1)$.

$$\pi^+ : \quad Q = +1 + \frac{0}{2} = +1.$$

Formule rotte dalla scoperta delle particelle strane:

$$\begin{pmatrix} K^+ \\ K^0 \end{pmatrix} : \quad +1 = Q.$$

$$Q = I_3 + \frac{B}{2} = +\frac{1}{2} + \frac{0}{2} = +\frac{1}{2}. \quad \begin{matrix} 0 \\ 0. \end{matrix}$$

Modifiche delle formule:

Ipercarica: $Y = B + S$ → stranezza.

$$Q = I_3 + \frac{B+S}{2}$$

1974 scoperta del Charm. (quark).

1977 beaty / bottom.

1995 top.

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

→ $I = \frac{1}{2}$ $\quad u = |\frac{1}{2}, +\frac{1}{2}\rangle$

$d = |\frac{1}{2}, -\frac{1}{2}\rangle$

isospin:

$|c\rangle = |s\rangle = |t\rangle = |b\rangle = |0, 0\rangle.$

nuovi num. quantici.

- s: stranezza.
- c: charm.
- b: beaty.
- t: top.

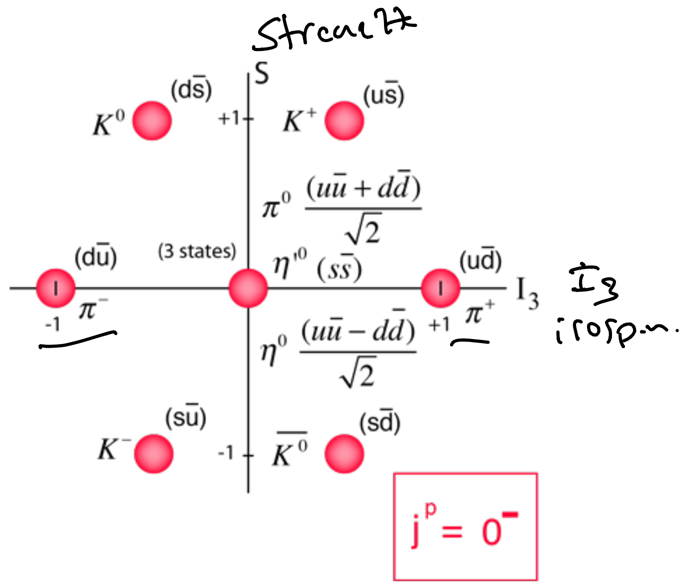
$$Q = I_3 + \frac{B+S+C+b+t}{2}$$

→ # quantici.

→ Ipercarica Y

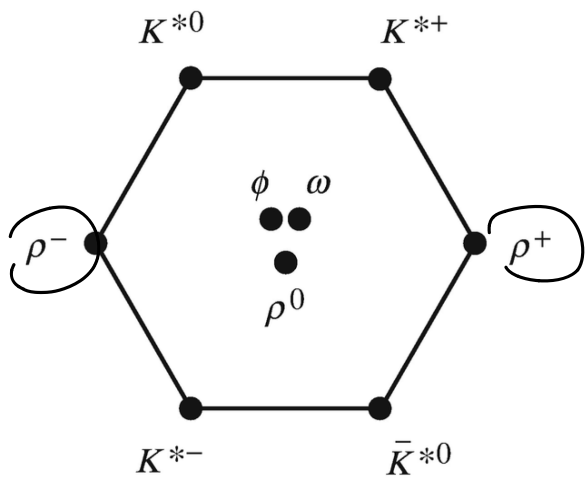
quark.	num. quantico.		
$\bar{s}$	Stranezza $= +1.$	s	$S=-1.$
$\bar{b}$	beauty $= +1$	b	$b=-1.$
c	$C=+1$	$\bar{c}$	charm # quantico. $C=-1$
t	$t=+1$	$\bar{t}$	$t=-1$

Mesoni pseudoscalari.



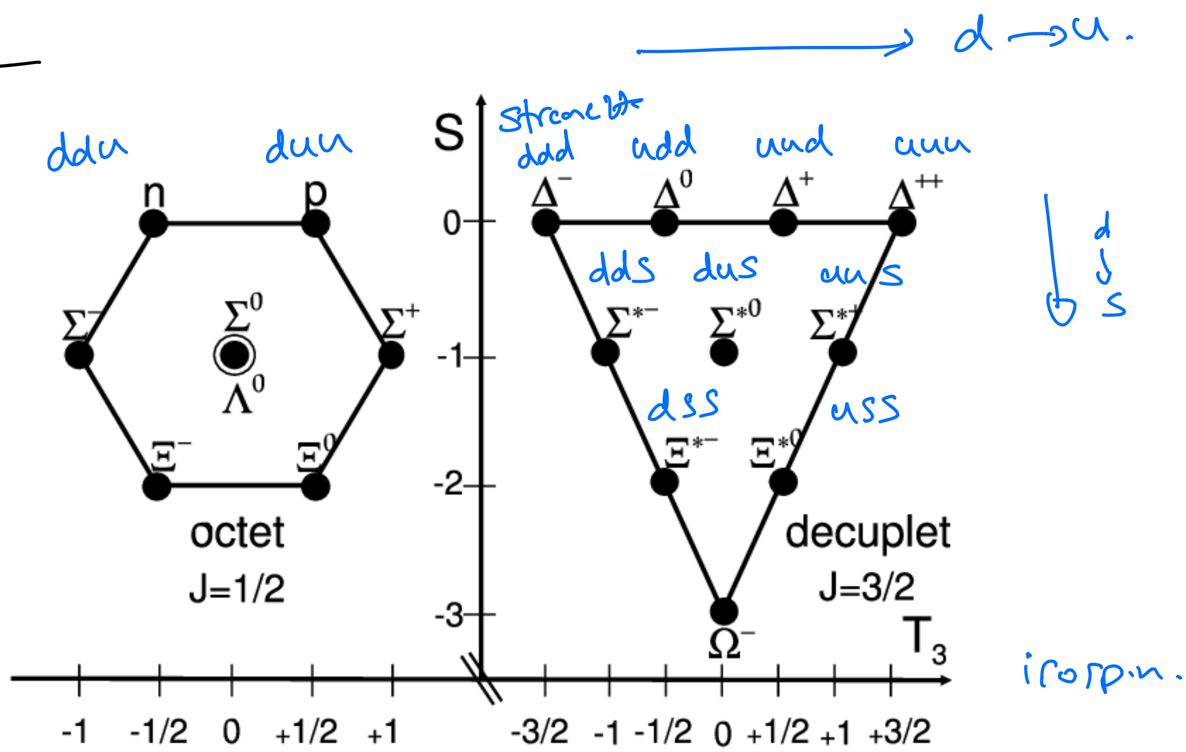
8 + 1

Mesoni vettori



8+1

Barioni:

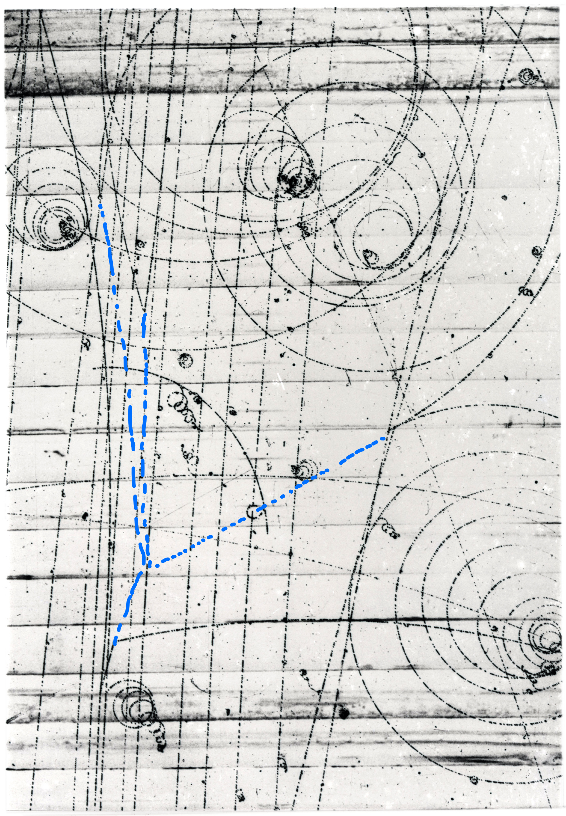


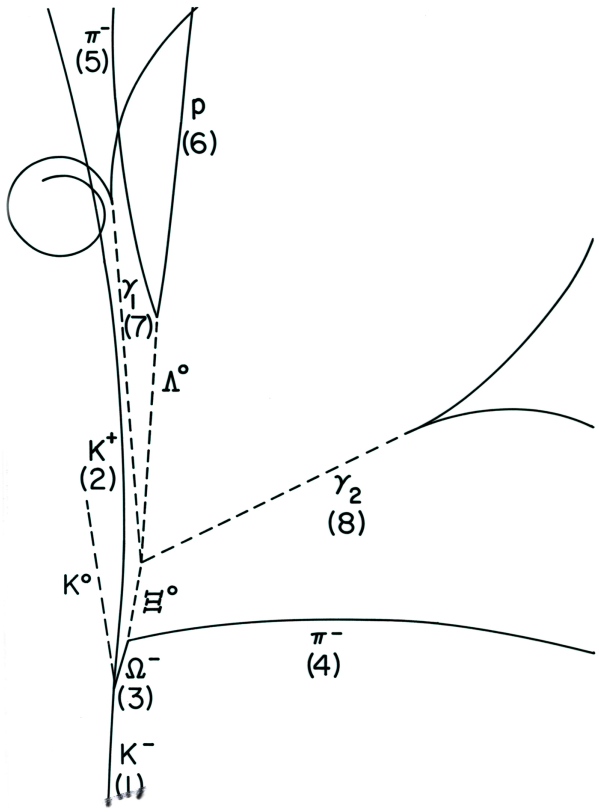
Ω^- : sss $S = -3$ barione   predice carica.

$$Q = I_3 + \frac{B+S}{2} = 0 + \frac{1-3}{2} = -1$$

Gell-Mann ha calcolato $M_\Omega = 1680$ MeV (1961)

1964 Camere a bolle. $M_\Omega = 167$





	K^- +	p →	Ω^- +	K^+ +	K^0
S	-1	0	-3	+1	+1
B	0	+1	1	0	0
Q	-1	+1	-1	+1	0

← curvatura traccia