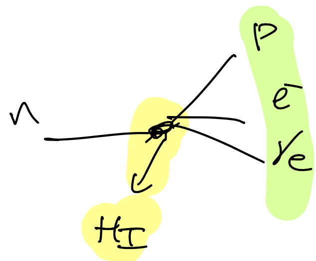
Decadimento β $\qquad n \rightarrow p + e^- + \bar{\nu}_e$

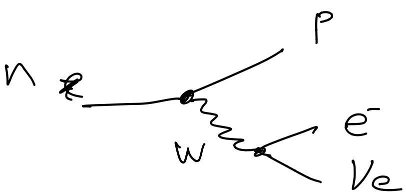
$$\Gamma(n \rightarrow p e^- \bar{\nu}_e) = 2\pi \, |M_{fi}|^2 \, \rho(E)\Big|_{E_f = E_i}$$

Regola d'oro di Fermi

$$M_{fi} = i \int d^3r \, \psi_f^* \, H_I \, \psi_i \qquad \rho(E) = \delta(E_f - E_i) \frac{dn}{dE}$$



$$H_I = G_F$$

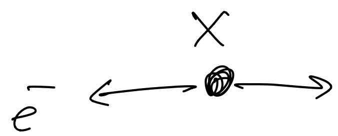


$$|i\rangle = |n\rangle \qquad |f\rangle = |p e^- \bar{\nu}_e\rangle$$

$$M_{fi} = -i \int d^3r \, \psi_p^* \psi_e^* \psi_\nu^* \, G_F \, \psi_n = -i \, G_F \int d^3r \, \psi_p^* \psi_n \, \psi_e^* \psi_{\nu_e}^*$$

$$\psi_e^* = e^{i \vec{p} \cdot \vec{r}} \qquad \psi_\nu^* = e^{i \vec{q} \cdot \vec{r}}$$

$$\vec{p} \equiv \vec{p}_e \qquad \vec{q} \equiv \vec{p}_\nu$$



$$K_Y \simeq 0$$

$$\psi_e^* \psi_\nu^* = e^{i(\vec{p} + \vec{q}) \cdot \vec{r}}$$

$$|\vec{p} + \vec{q}| \qquad E_f = E_i$$

$$E_i = M_X = M_Y + \underbrace{\cancel{K_Y}}_{\simeq 0} + E_e + E_\nu \equiv E_f$$

$$E_T = M_X - M_Y = E_e + E_\nu \qquad n \rightarrow p + e^- + \bar{\nu}_e$$

energia trasferita $\qquad m_n - m_p \simeq 1\,\text{MeV}$

$$|\vec{r}| \simeq 1\,\text{fm}$$

$|\vec{p}+\vec{q}| \cdot |\vec{r}| \simeq 1 \text{ MeV} \times 1 \text{ fm} \simeq 1 \text{ MeV} \frac{1}{200 \text{ MeV}}$

$\simeq 0.005$

$e^{i(\vec{p}+\vec{q})\cdot\vec{r}} \simeq 1 + O(|\vec{p}+\vec{q}\cdot\vec{r}|) + - -$

$$M_{fi} = -i\frac{G_F}{V} \underbrace{\int_{Vol.} d^3r\, \varphi_p^* \varphi_n}_{\text{Calcolo teorico}} = -i\frac{G_F N}{V}$$

$|N| \simeq 1$

$$\psi_e \simeq e^{i\vec{p}\cdot\vec{r}}$$

$$\int d^3r\, |\psi_e|^2 = 1$$

$$\psi_e \propto \frac{1}{\sqrt{V}} \qquad \psi_\nu \propto \frac{1}{\sqrt{V}}$$

$$\psi_e = \frac{1}{\sqrt{V}} e^{i\vec{p}\cdot\vec{r}}$$

$M_{fi} = -i\frac{G_F}{V} N \qquad |M_{fi}|^2 = M_{fi}^* M_{fi} = \frac{G_F^2}{V^2}|N|^2$

$\rho(E)$: densità degli stati

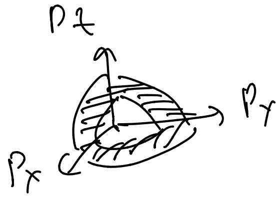
$\vec{0} = \vec{P}_y + \vec{P}_e + \vec{P}_\nu$ Conserv. dell'impulso.

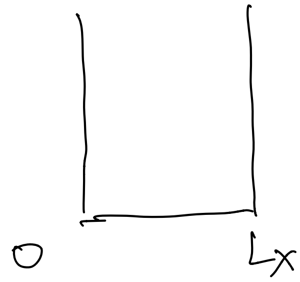
$\vec{P}_y = -(\vec{P}_e + \vec{P}_\nu)$ Rinculo $\Rightarrow x$ non ha gradi di lib.

$x \to y + e + \nu.$

$0 \quad \vec{P}_y + \vec{P}_e + \vec{P}_\nu$

$|\vec{P}_y| \simeq 0 \Rightarrow \vec{P}_e \simeq -\vec{P}_{\nu e}$ per il fatto che $M_x - M_y \simeq 1$ MeV





$$f \simeq \sin(Kx) = \sin\left(\frac{p}{\hbar}x\right)$$

$$= \sin(px)$$

$$p \cdot L_x = n\pi$$

$$\Rightarrow \quad p_x = n_x \frac{\pi}{L_x} \qquad p_y = n_y \frac{\pi}{L_y} \qquad p_z = n_z \frac{\pi}{L_z}$$

$$n^2 = n_x^2 + n_y^2 + n_z^2 \qquad \vec{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$$

$$d^3p = dp_x dp_y dp_z = \frac{\pi^3}{L_x L_y L_z} dn_x dn_y dn_z = \frac{\pi^3}{V} dn$$

$$d^3p = 4\pi^2 p^2 dp$$

Volume guscio con $p_x, p_y, p_z > 0$

$$\frac{1}{8} 4\pi p^2 dp = \frac{\pi^3}{V} dn.$$

$$\Rightarrow \quad dn = \frac{V}{(2\pi)^3} 4\pi p^2 dp$$

$$\delta(E_f - E_i) dn = \delta(E_f - E_i) \underbrace{\frac{V}{(2\pi)^3} 4\pi p^2 dp}_{dn_e} \underbrace{\frac{V}{(2\pi)^3} 4\pi q^2 dq}_{dn_\nu}$$

$$m_\nu = 0 \quad \Rightarrow \quad q = E_\nu \qquad q^2 dq = E_\nu dE_\nu$$

$$\delta(E_f - E_i) \quad \Rightarrow \quad M_X = M_Y + E_\nu + E_e \quad \Rightarrow \quad E_\nu + E_e = M_X - M_Y = E_T$$

$$E_\nu = E_T - E_e$$

$$\delta(\sim) \quad q^2 dq \quad p^2 dp \longrightarrow \frac{V^2}{(2\pi)^6}(4\pi)^2 (E_T - E_e)^2 \, p^2 dP$$

$$\rho(E) = \frac{V^2}{(2\pi)^6}(4\pi)^2 \int (E_T - E_e)^2 \, p^2 dP$$

$$E^2 = p^2 + m^2 \Rightarrow \cancel{2}E\,dE = \cancel{2}P\,dP.$$

$$\rho(E) \propto \int_{m_e}^{E_T} (E_T - E_e)^2 \sqrt{E_e^2 - m_e^2}\, E_e \, dE_e$$

$X \to Y$

$\nu_e \longleftarrow \bullet \longrightarrow e^-$ $\qquad \min(E_e) = m_e$

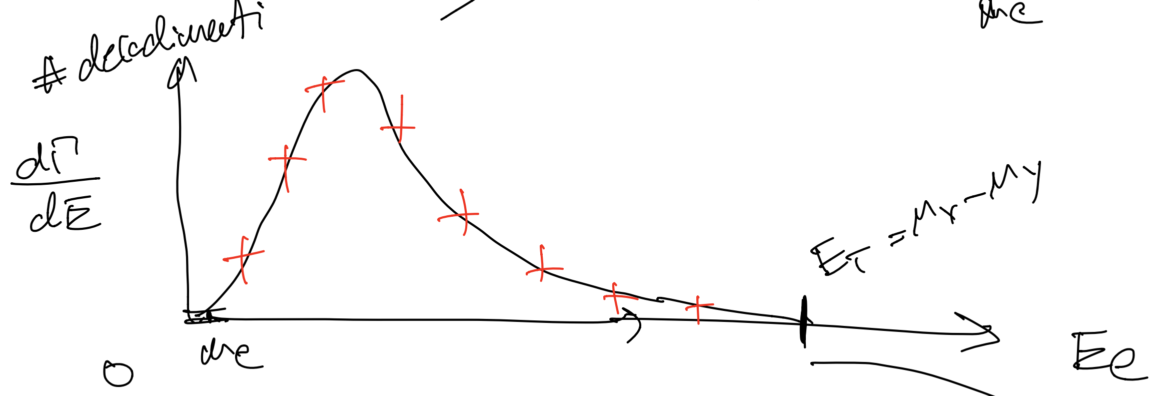
$\max(E_e) =$

$E_\nu + E_e = M_X - M_Y.$

$\hookrightarrow \cancel{m_\nu} + K_\nu + E_e = M_X - M_Y$

$$\max(E_e) = M_X - M_Y = E_T$$

$$\Gamma(n \to p e \bar{\nu}_e) = 2\pi \frac{G_F^2}{\cancel{V^2}} |N|^2 \frac{\cancel{V^2}}{(2\pi)^6}(4\pi)^2 \int_{m_e}^{M_X - M_Y} (E_T - E_e)^2 \sqrt{E_e^2 - m_e^2}\, E_e \, dE_e$$



$$\frac{d\Gamma}{dE} \propto G_F^2 (E_T - E_e)^2 \sqrt{E_e^2 - m_e^2}\, E_e \qquad |N|^2$$

conto teorico.

$$E_e \longrightarrow E_T = M_X - M_Y$$

prendere caso $E_T = M_X - M_Y \gtrsim 1$ abb. grande.

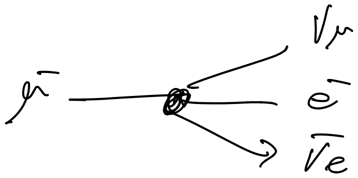
$$E_e \simeq P_e \gg m_e$$

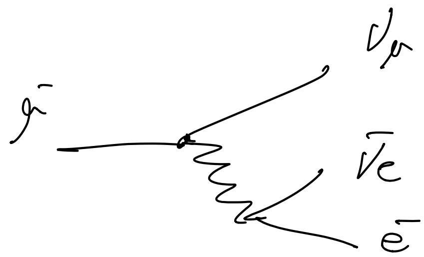
$$\int_0^{E_T} (E_T - E_e)^2 \, E_e \, E_e \, dE_e \quad \propto \quad (E_T)^5 = (M_X - M_Y)^5$$

$$\Gamma(X \to Y e^- \bar{\nu}_e) \propto G_F^2 \, (M_X - M_Y)^5$$

Regola di Sargent

$$\mu^- \to e^- \, \bar{\nu}_e \, \nu_\mu$$





$$\frac{1}{\tau} = \Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \propto G_F^2 \, (m_\mu)^5$$

$$E_T = m_\mu - m_\nu = m_\mu$$

$$\tau \propto G_F^{-2} \, m_\mu^{-5}$$

M $\quad m_\nu \neq 0 \Rightarrow E_T = M_X - M_Y - m_\nu$

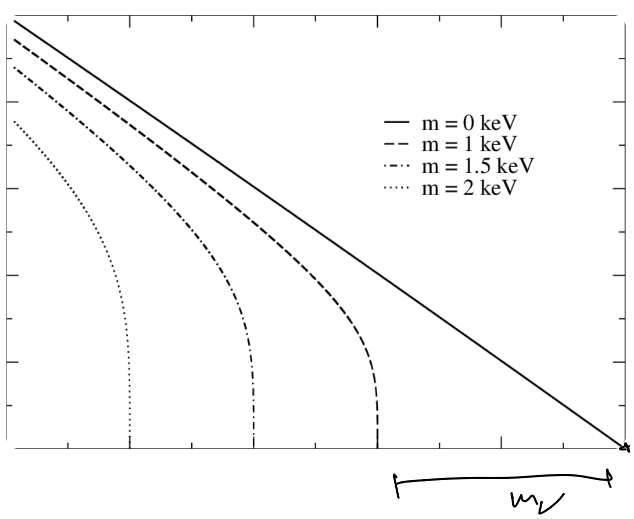


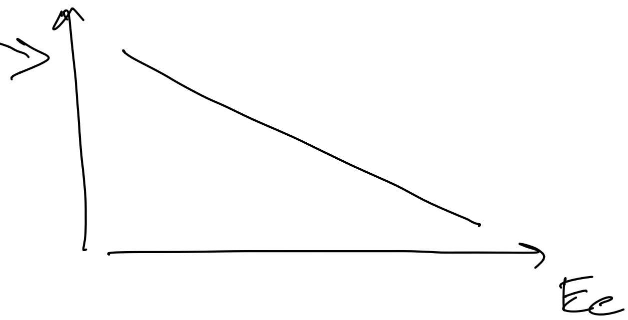
Grafico di Kurie

endpoint

$$\sqrt{\frac{1}{p\,E_e}\frac{d\Gamma}{dE}} \propto (E_T - E_e)$$

$$\int$$

$$\frac{\#\,\text{decadiv}}{E_e}$$



$$G_F \simeq 1 \times 10^{-5}\ \text{GeV}^{-2}$$

$$[\Gamma] = [E] \qquad [G_F]^2$$

$$n \to p + e^- + \bar{\nu}_e$$

$$\bar{\nu}_e + p \to n + e^+$$