

NEUTRINO PHYSICS

Atmospheric and Solar Evidence

DIPARTIMENTO DI FISICA



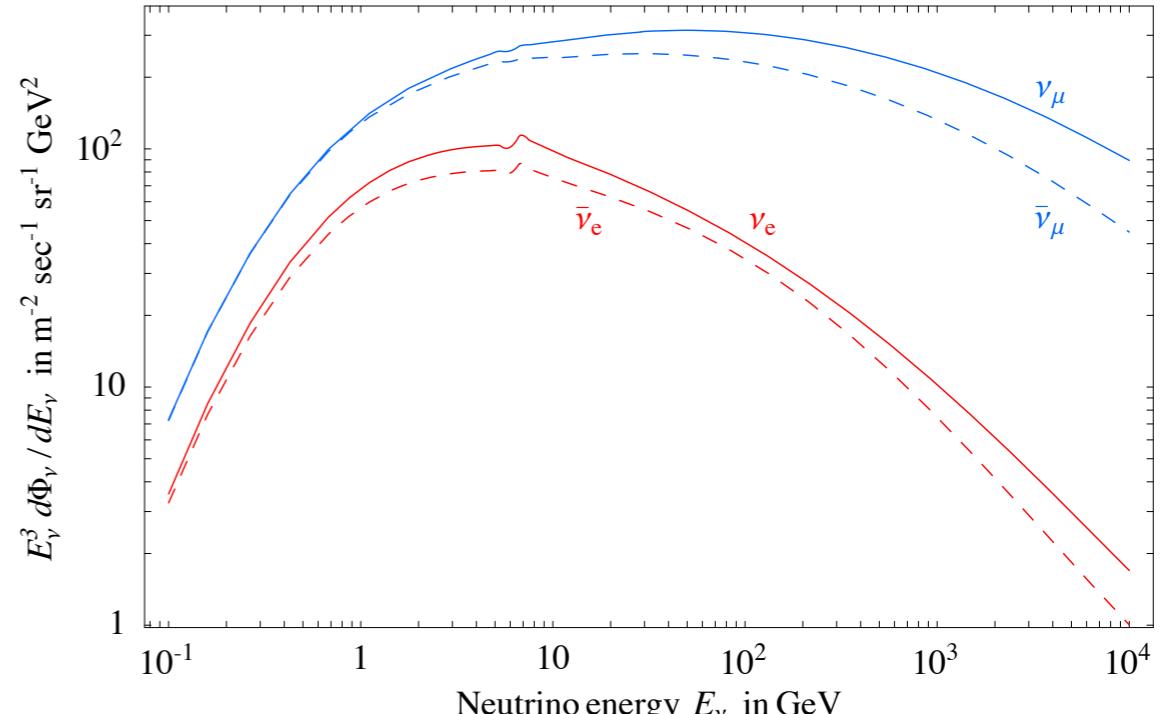
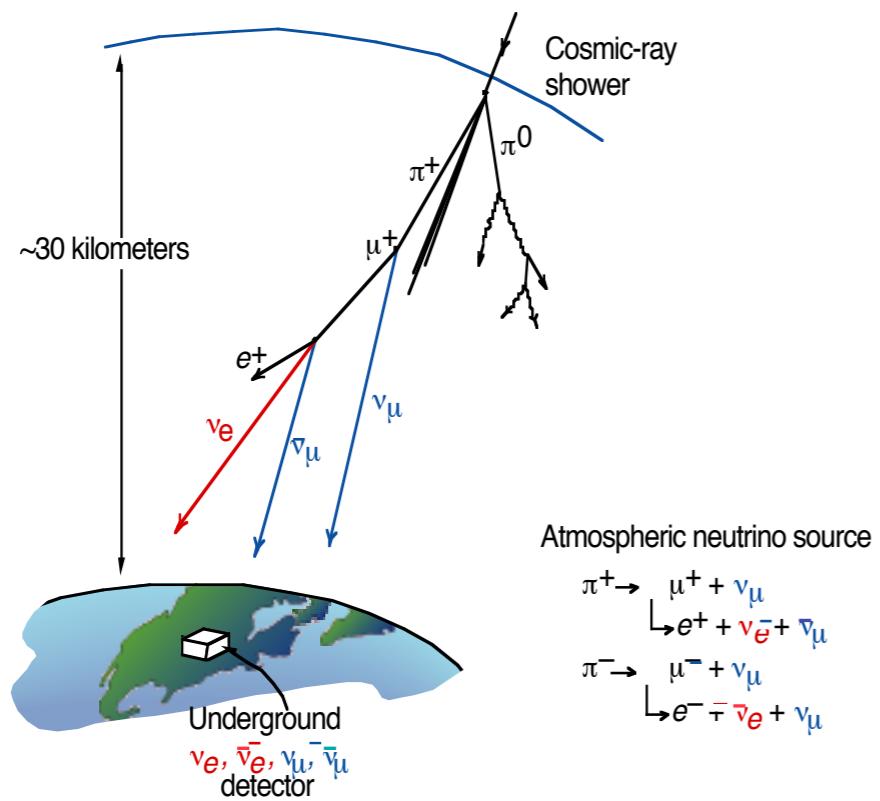
SAPIENZA
UNIVERSITÀ DI ROMA

Fisica delle Particelle Elementari, Anno Accademico 2015-16
<http://www.roma1.infn.it/people/rahatlou/particelle>

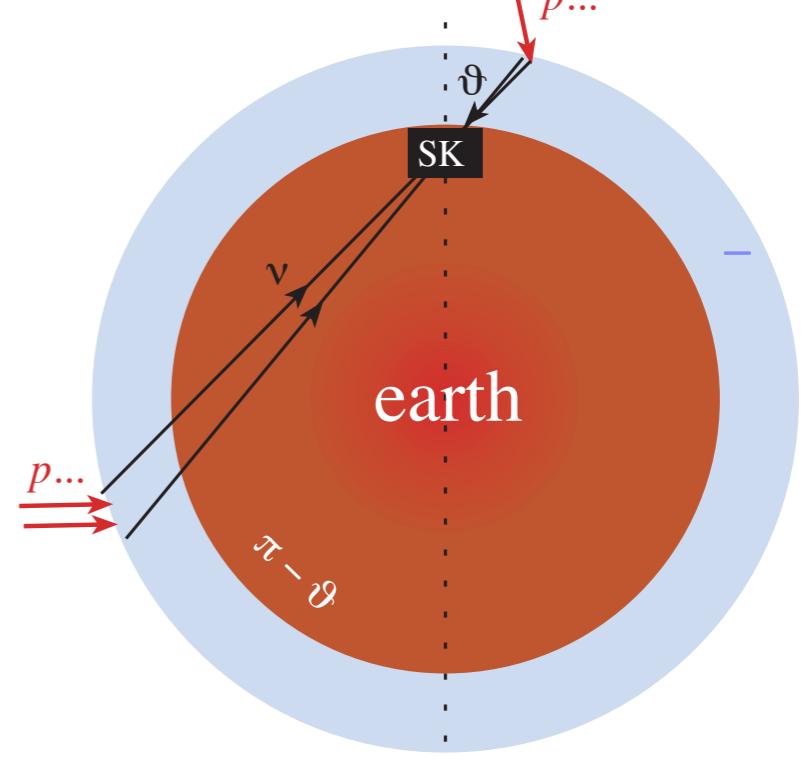
ATMOSPHERIC EVIDENCE

THE ATMOSPHERIC NEUTRINOS

- Primary cosmics produce π in upper atmosphere: $\pi \rightarrow \mu\nu_\mu$, $\mu \rightarrow e\nu_\mu\nu_e$ $\varphi(\nu_\mu) = 2\varphi(\nu_e)$ at few % level

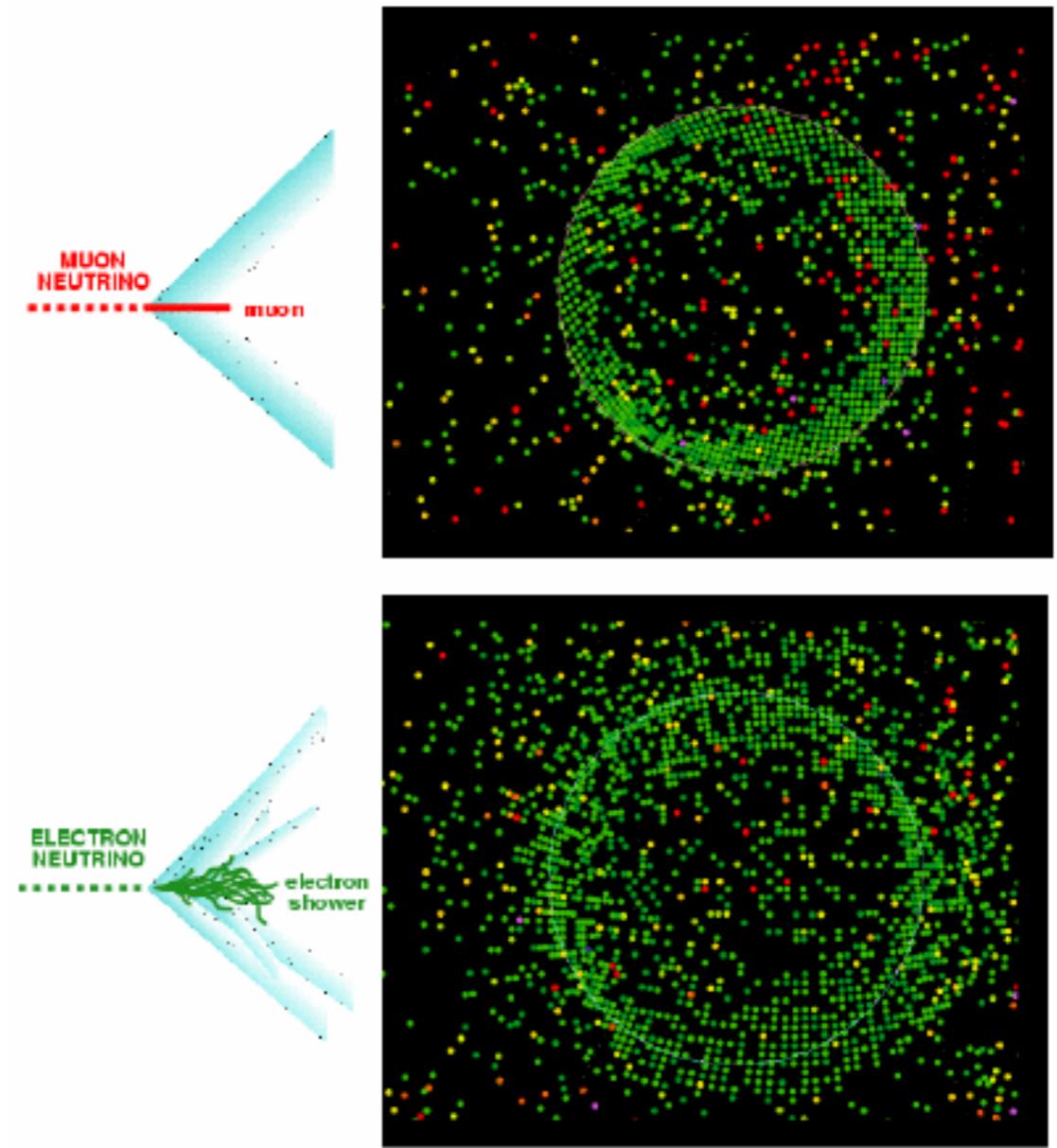


- Isotropy of the >2GeV cosmic rays flux +Gauss' law :
- ν rate up/down symmetric
- no need knowledge flux



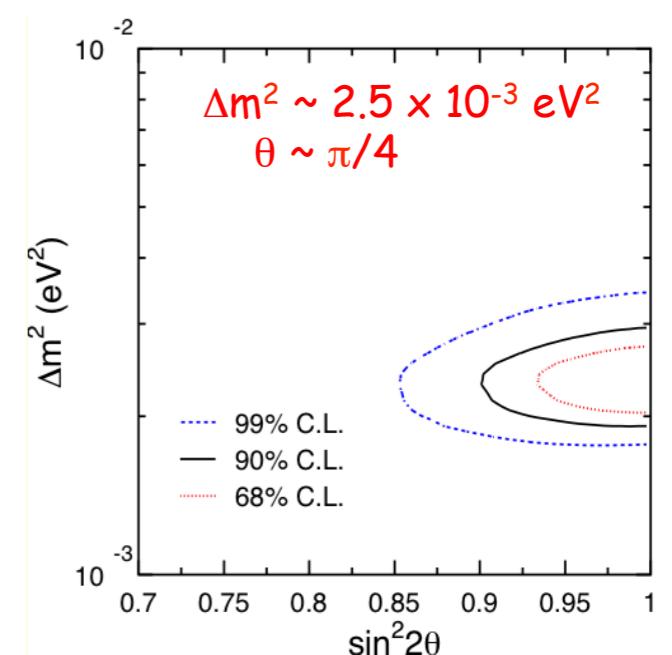
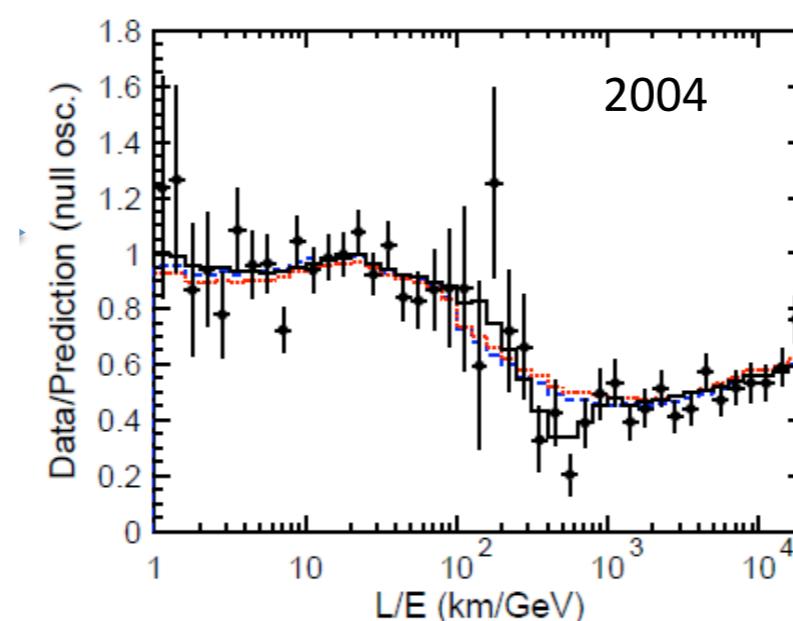
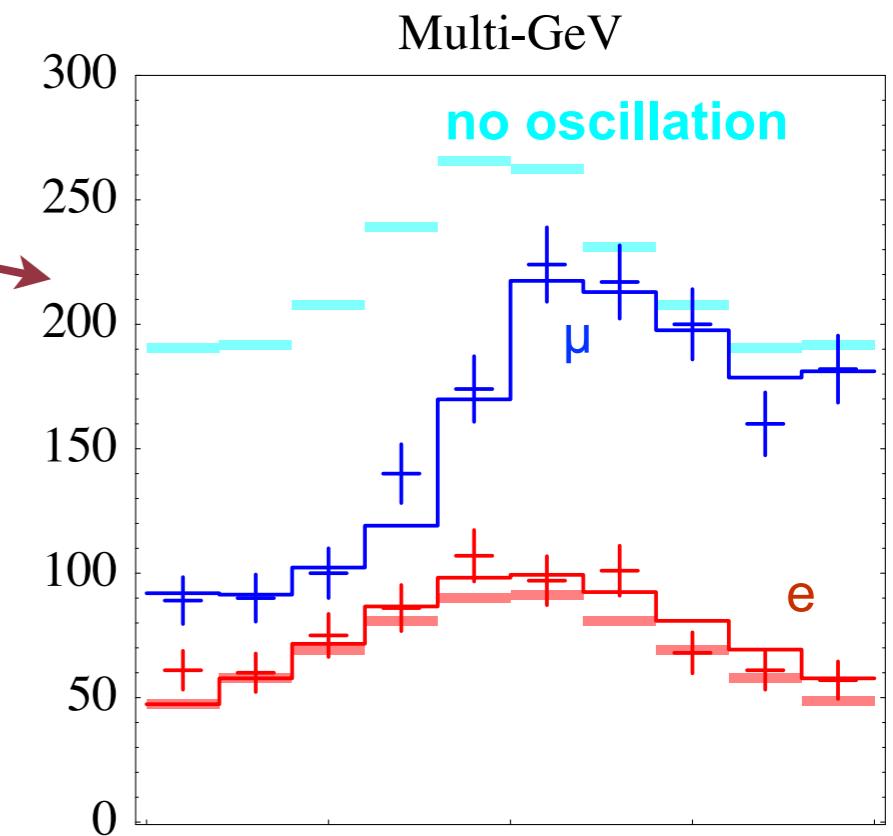
SUPERKAMIOKANDE

- Water Cherenkov $H_2O + PMT$
 - Unsegmented \Rightarrow requires low rate and low multiplicity \Rightarrow underground $\Phi = 2/cm^2 s \Rightarrow \approx 1\text{kevents/year}$
 - CC interaction on nucleons: $v_l N \rightarrow l N'$, $\sigma(v) \approx 3\sigma(\bar{v})$
 - e/ μ separation **not** $e(\mu)^+, e(\mu)^-$
 - v from everywhere: **no E_v reconstruction**
 - wide energy range: **GeV-TeV**
 - $E_l \geq 1.5\text{GeV} \Rightarrow \theta_{vl} \approx 15^\circ \Rightarrow$ direction \Rightarrow measure L



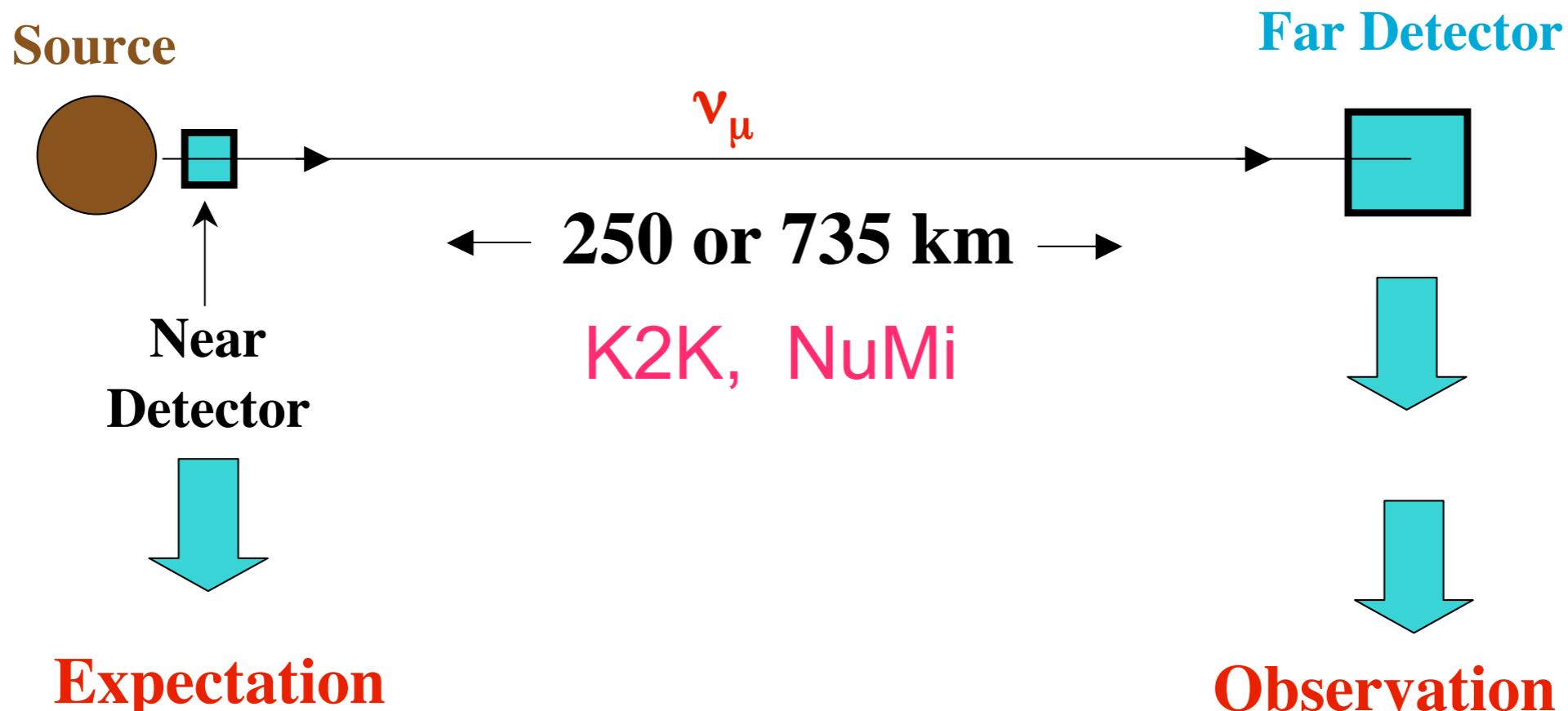
ATMOSPHERIC ANOMALY

- $[\varphi(v_\mu)/\varphi(v_e)]/[\varphi(v_\mu)/\varphi(v_e)]_{MC} \sim 0.65$
- Anomaly without relying on knowledge of neutrino fluxes
 - v_e as expected, v_μ deficit from below
 - $v_\mu \Rightarrow v_e$? not at leading order
 - $v_\mu \Rightarrow v_\tau$? yes
- $P_{\mu\mu} = 1 - \sin^2(2\vartheta_{23}) \sin^2(\Delta m^2 L / 4E_\nu)$
 - Zenith no oscillation, Nadir average oscillation
 - ▶ $\Rightarrow P_{\mu\mu} = 1 - 0.5 \cdot \sin^2(2\vartheta_{23}) = N^\uparrow / N^\downarrow \Rightarrow \vartheta = 45^\circ$
 - Oscillations start horizontal:
 - ▶ $E_\nu \sim \text{GeV}$, $L \sim 1000 \text{ km}$, $\Rightarrow \Delta m^2 \sim E_\nu / L \sim 3 \cdot 10^{-3} \text{ eV}^2$
- Restricting to cleanest events



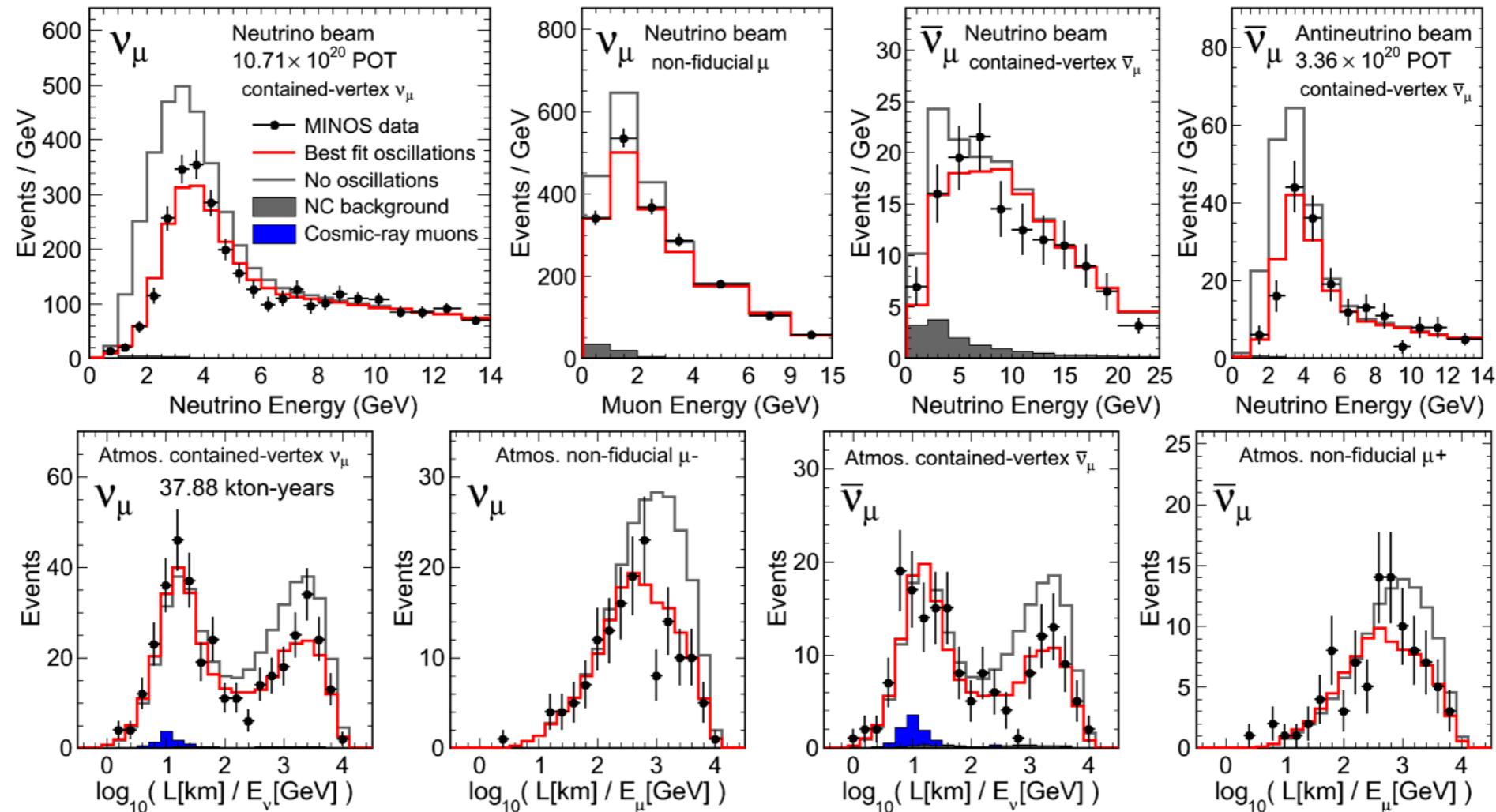
TEST OF $P_{\mu\mu}$ ANOMALY

- $\Delta m^2_{\text{Atm}} \sim E_\nu / L \sim 3 \cdot 10^{-3} \text{ eV}^2$
- Build an experiment and ν_μ beam with $\Delta m^2_{\text{Atm}} L / E_\nu \sim 1$,
 - $E_\nu \approx \text{few GeV} \approx m_p$, $L \approx \text{O}(100 \text{ km})$, $\theta_{\mu\nu} \approx 1$ (if $\nu_\mu n \Rightarrow \mu^- p$ dominant reaction)



Neutrino beam: p on target $\rightarrow K\pi(\mu)$ focus and decay \rightarrow shielding \rightarrow $90\% \nu_\mu, 8\% \bar{\nu}_\mu, 1\% \nu_e, 1\% \bar{\nu}_e$
 $\pi^+ \rightarrow \mu^+ \nu_\mu$ (99.99%), $\pi^+ \rightarrow e^+ \nu_e$ ($10^{-2}\%$), $K^+ \rightarrow \mu^+ \nu_\mu$ (63%), $K^+ \rightarrow \pi^0 e^\pm (\mu^\pm) \bar{\nu} (\bar{e})$ (4%), $K^0 \rightarrow \pi^\pm e^\pm (\mu^\pm) \bar{\nu} (\bar{e})$ ($\approx 30\%$)

TEST OF $P_{\mu\mu}$ ANOMALY AND $|\Delta m_{\text{ATM}}|$



exp: 3564
seen 2894

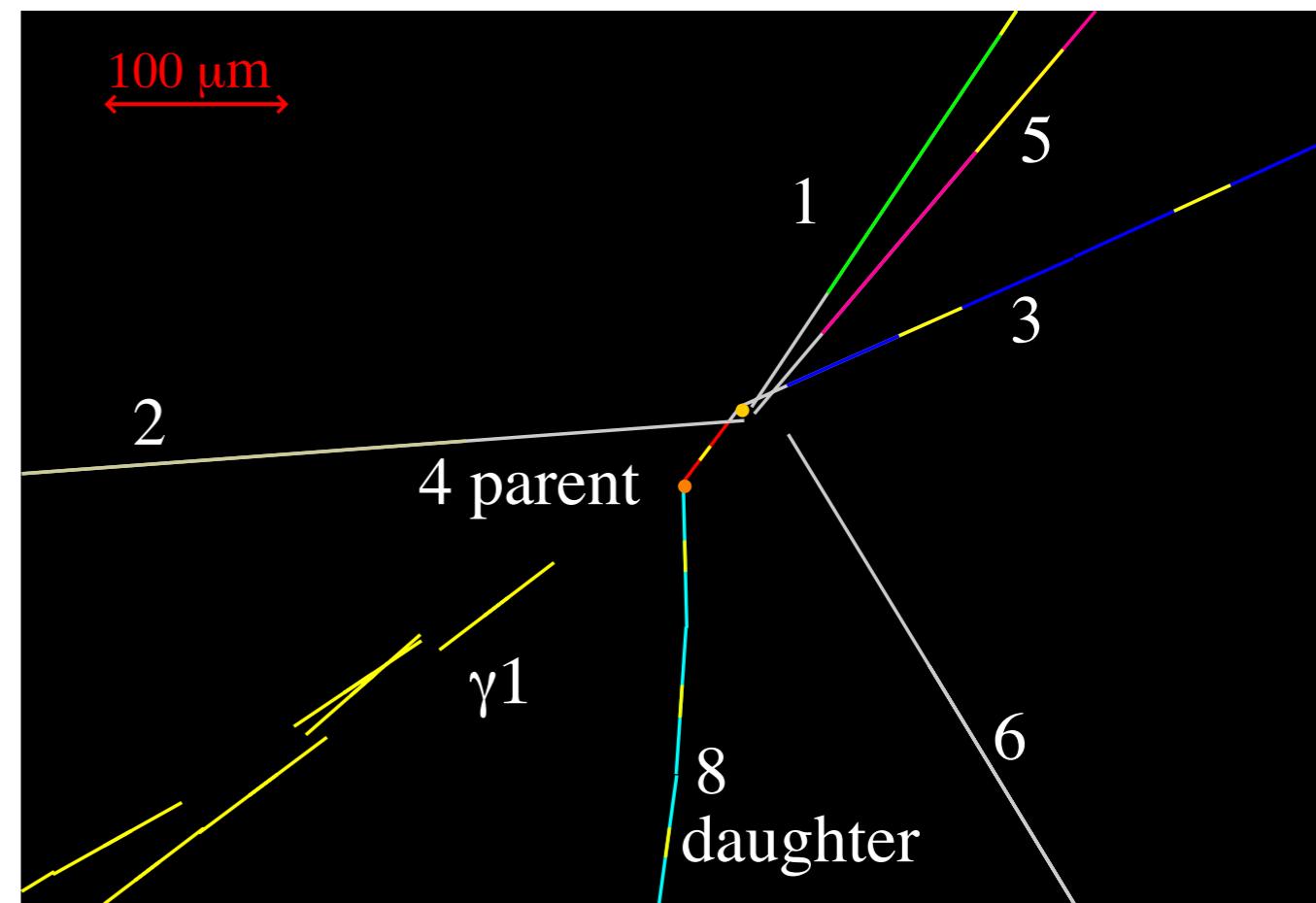
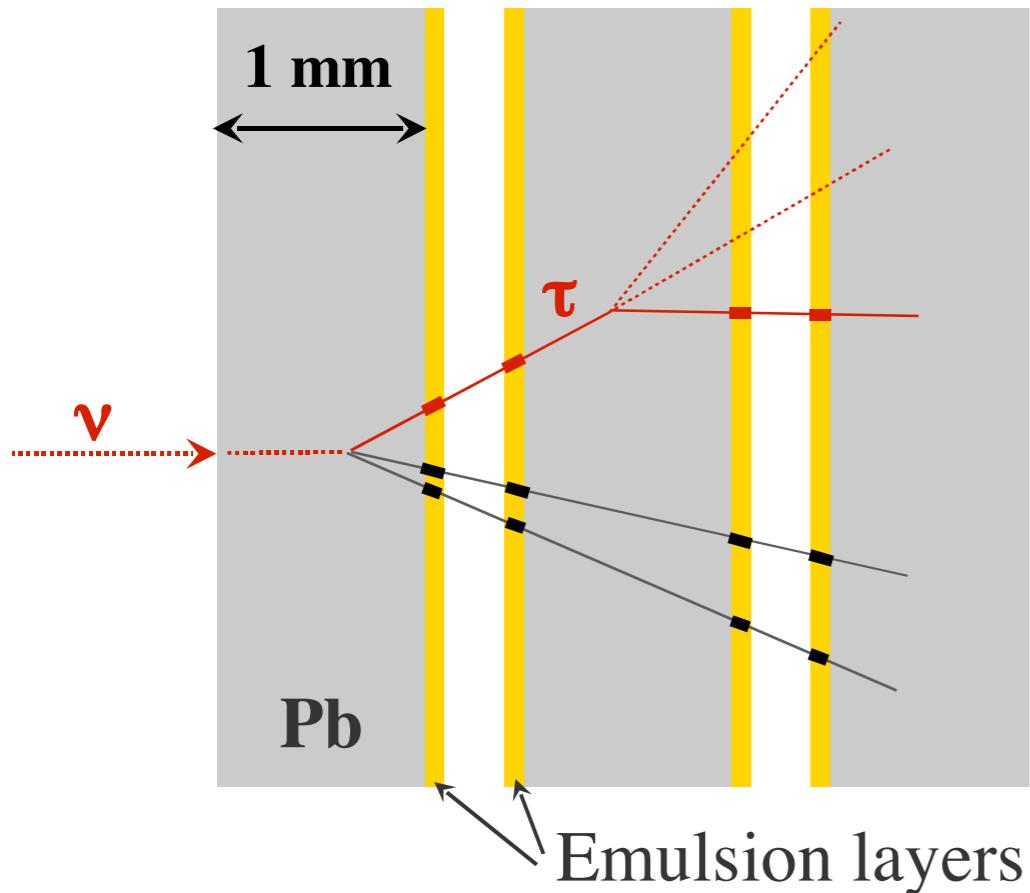
- Pulsed ν_μ beam
 - Kamiokande: WC, pulsed beam used to discriminate against atmospheric ν
 - MINOS: magnetized calorimeter
- $\nu_\mu n \rightarrow \mu^- p$ dominant reaction
 - E_ν reconstructed from E_μ and $\theta_{\mu\nu} \approx 1$ since ν source known
 - observe energy-dependent ν_μ deficit $|\Delta m_{\text{atm}}^2| \approx (2.38 \pm 0.15) \cdot 10^{-3} \text{ eV}^2$

$$\theta_{\text{atm}} \approx \theta_{23} \approx 45^\circ,$$

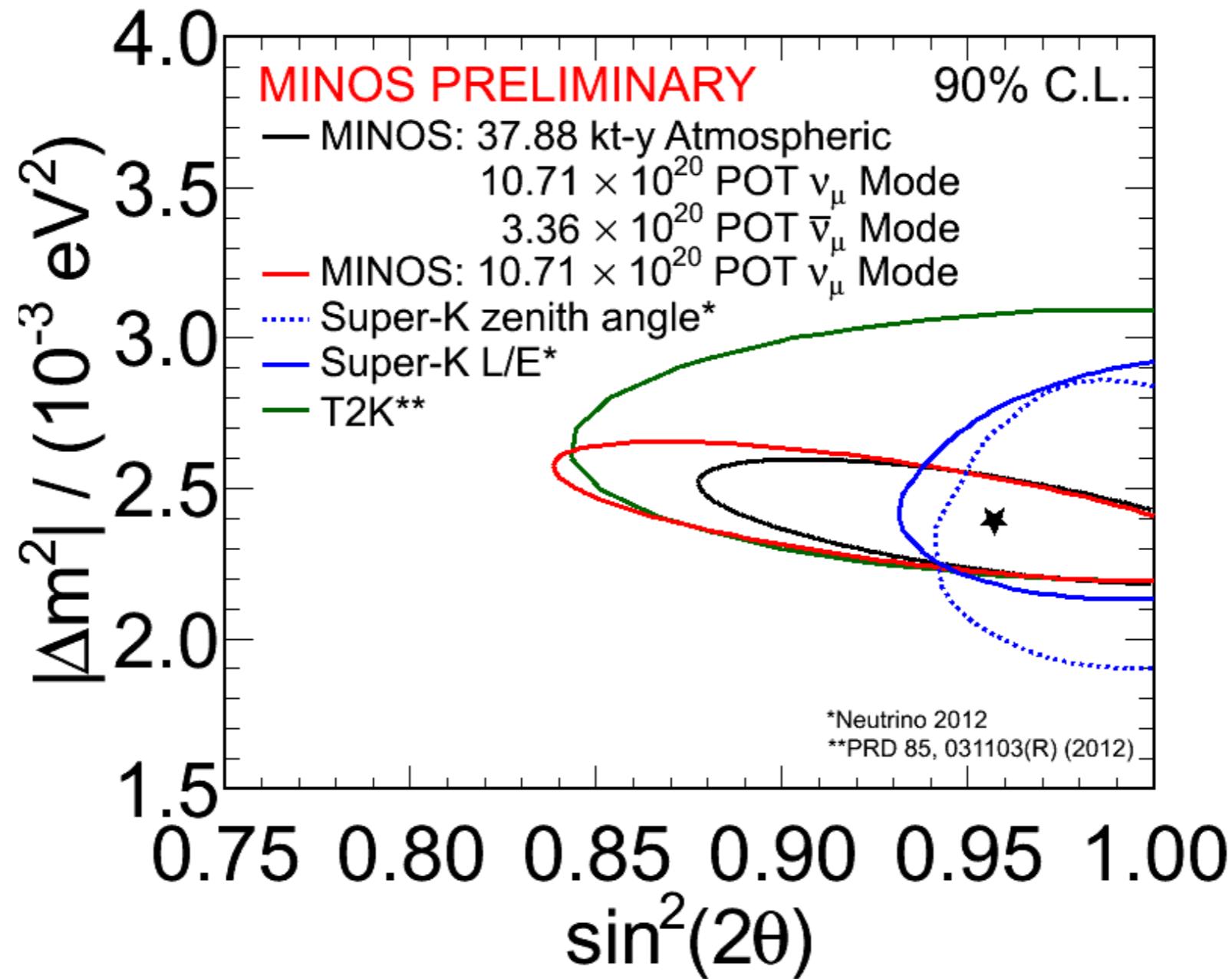
$$|\Delta m_{13}^2| \approx |\Delta m_{23}^2| \approx |\Delta m_{\text{atm}}^2| \approx (2.38 \pm 0.15) \cdot 10^{-3} \text{ eV}^2$$

OPERA: $\nu_\mu \rightarrow \nu_\tau$

- Appearance of ν_τ $\nu_\tau N \rightarrow \tau^- N'$
 - Need $O(kTon)$ target mass + high granularity detector ($\sigma \sim \mu\text{m}$) for τ detection ($c\tau \sim 87 \mu\text{m}$) and bkgd rejection (mainly $\nu_\mu N \rightarrow c\mu X$)
 - Lead-nuclear emulsion
- ν_μ beam: CERN-LNGS: $L \approx 750 \text{ km}$, $\langle E_\nu \rangle \approx 17 \text{ GeV}$



ATMOSPHERIC NEUTRINO SUMMARY



- Large Δm and large mixing angle observed

Oscillation Searches Sensitive to δM^2

EXPERIMENTS SENSITIVE TO δm^2 IN $\Delta m^2 \rightarrow \infty$

Previously we have considered experiments with sensitivity to Δm^2 in the limit $\Delta m^2 \rightarrow 0$. At the other end of the spectrum, there are expts. with leading sensitivity to δm^2 , for which one can take $\Delta m^2 \rightarrow \infty$:

$$\frac{\delta m^2 x}{4E} \sim \mathcal{O}(1) \quad \text{and} \quad \frac{\Delta m^2 x}{4E} \gg 1$$

This is the case, for instance, of long-baseline reactor experiments (KamLAND) with large x and relatively low E . At low $E \sim \text{few MeV}$, the main observable is the disappearance probability P_{ee} . Prove that:

$$P_{ee} \simeq \cos^4 \theta_{13} \left[1 - \sin^2 2\theta_{12} \sin^2 \left(\frac{\delta m^2 x}{4E} \right) \right] + \sin^4 \theta_{13}$$

(which does not depend on hierarchy, $\nu/\bar{\nu}$, ϕ)

Namely, the 3ν probability (for $\theta_{13} \neq 0$) is related to the 2ν probability (at $\theta_{13} = 0$) by the relation:

$$P_{ee}^{3\nu} = \cos^4 \theta_{13} P_{ee}^{2\nu} + \sin^4 \theta_{13}$$

from E.Lisi "Neutrinos: theory and phenomenology"

EXPERIMENTS SENSITIVE TO δm^2 IN $\Delta M^2 \rightarrow \infty$

Important note: The Δm^2 -averaged form for P_{ee} ,

$$P_{ee}^{3\nu} = C_{13}^4 P_{ee}^{2\nu}(\delta m^2, \theta_{12}) + S_{13}^4$$

holds not only for KamLAND, but also for solar neutrinos (proof omitted) where, however, $P_{ee}^{2\nu}$ takes a very different form due to matter effects in the Sun.

Therefore, via $P_{ee}^{3\nu}$, solar+KamLAND experiments allow to set constraints on δm^2 and on the 1st-row elements of the PMNS matrix (in absolute value) :

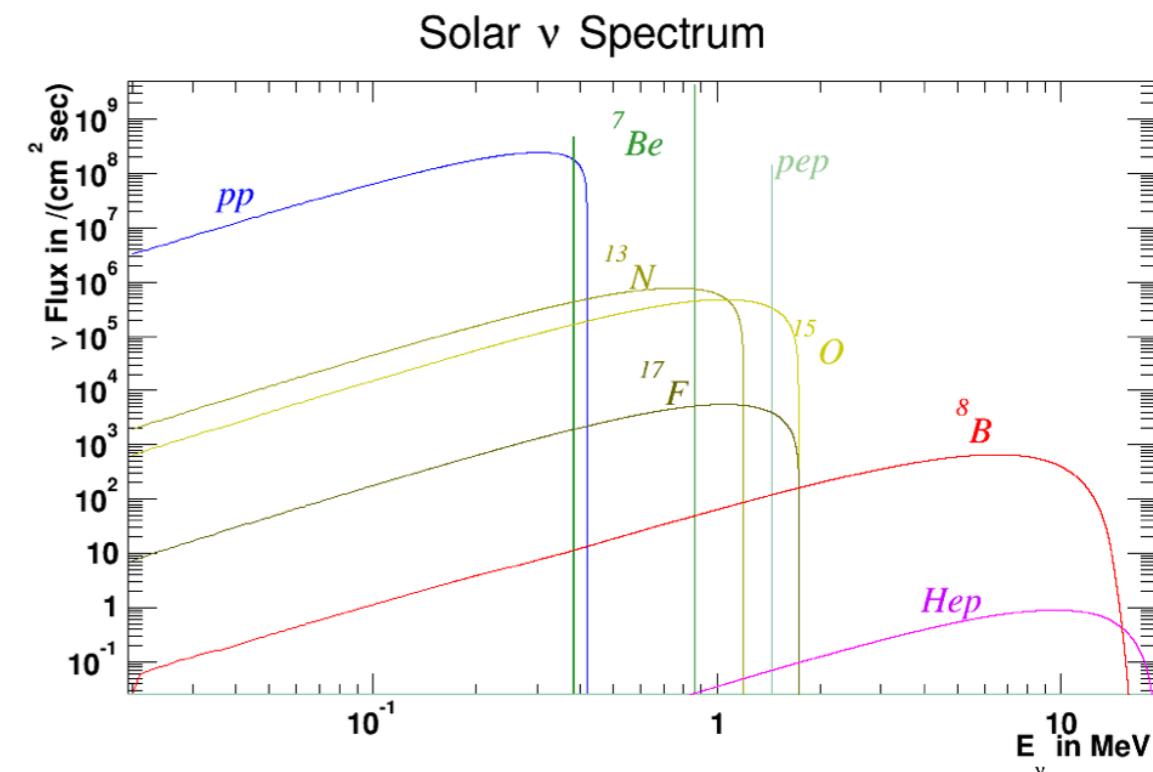
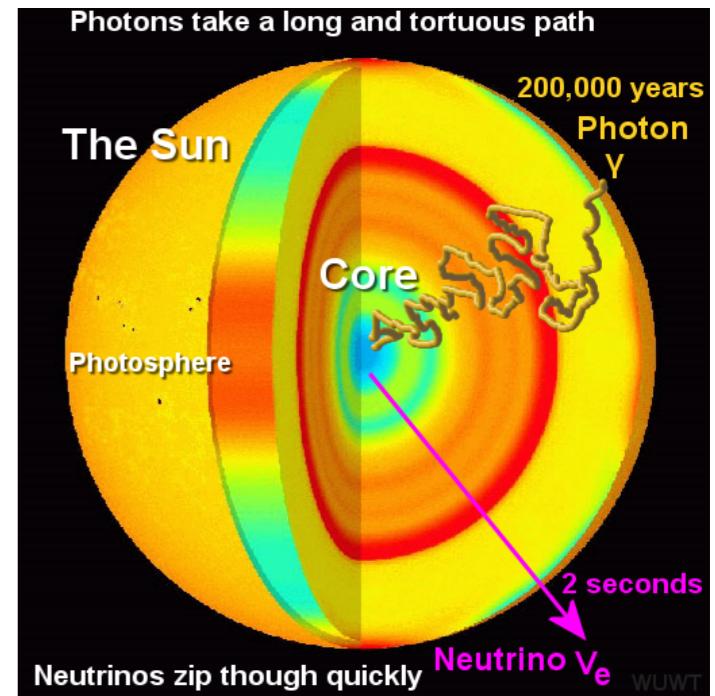
$$|U| = \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{pmatrix} \leftarrow \text{functions of } \theta_{12}, \theta_{13}$$

Recap	Solar + KamLAND	θ_{12}	θ_{13}	δm^2
of leading sensitivity :	ATM + LBL accel.	θ_{23}	θ_{13}	$ \Delta m^2 $
	SBL reactors		θ_{13}	$ \Delta m^2 $

SOLAR EVIDENCE

SOLAR NEUTRINOS

- Must account for neutrino propagation through matter
 - neutrinos produced in Sun core
 - electron neutrinos interact with matter
 - ▶ charged current t channel
- Mikheyev-Smirnov-Wolfenstein (MSW) effects
 - account for interaction with atoms in the Sun
 - electron neutrino continuously regenerated
 - neutrino cross section increases with neutrino energy in t-channel
 - ▶ energy $\ll M_w$ in the propagator
 - matter effects increase with neutrino energy
- Matter effects small or negligible for very low energy neutrinos
- Matter effects significant for higher energy neutrinos from Sun
- Direct impact on observed oscillation amplitude and frequency

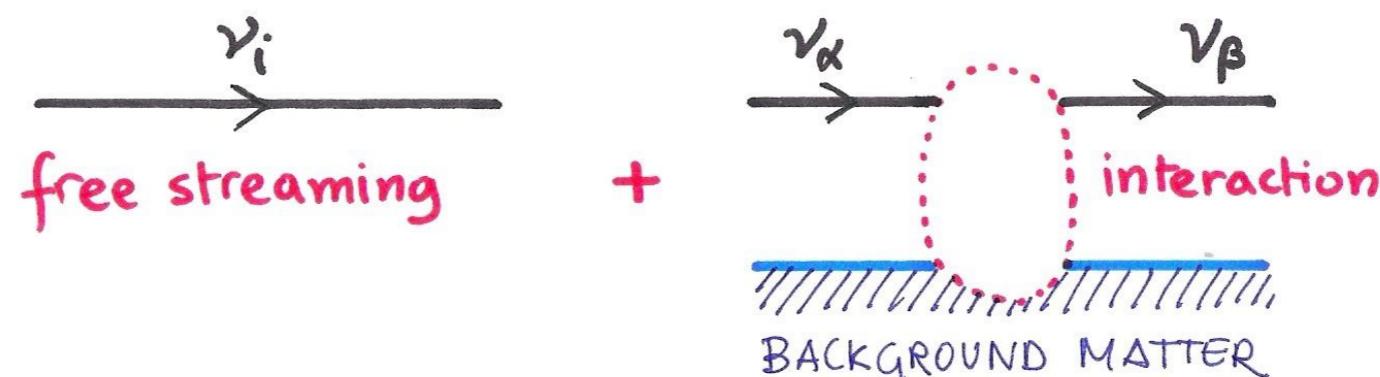


NEUTRINO OSCILLATION IN MATTER

- Matter effect analogous to light in medium (air, water): even if absorption negligible, matter effect can reduce $v=c/n$. In some materials (birefringence) n changed in a polarized dependent way
- Coherent forward scattering \Rightarrow flavour dependent refraction index $n=1+\mathcal{V}/E_\nu$
- Flavour basis: $H=H_{\text{VAC}} + V$
 - $H_{\text{VAC}} = U H_{\text{Mass}} U^\dagger$
 - V = dynamical contribution to the forward scattering

$$H_{\text{flavor}} = \frac{1}{2E} \square \begin{pmatrix} m_1^2 & m_2^2 & m_3^2 \\ m_2^2 & m_3^2 & m_1^2 \\ m_3^2 & m_1^2 & m_2^2 \end{pmatrix} \square^\dagger + \begin{pmatrix} v_{ee} & v_{e\mu} & v_{e\tau} \\ v_{\mu e} & v_{\mu\mu} & v_{\mu\tau} \\ v_{\tau e} & v_{\tau\mu} & v_{\tau\tau} \end{pmatrix}$$

VACUUM (KINEMATICS) MATTER (DYNAMICS)



from E.Lisi "Physics of Massive neutrinos"

NEUTRINO OSCILLATION IN MATTER

Within the Standard Model, and in ordinary matter:

$$V_{\alpha\beta} = \left(\begin{array}{ccc} \nu_e & \nu_e & 0 \\ 0 & \nu_\mu & \nu_\mu \\ p, n, e & p, n, e & 0 \end{array} \right)_{NC} + \left(\begin{array}{ccc} \nu_e & \nu_e & 0 \\ 0 & 0 & 0 \\ e & e & 0 \end{array} \right)_{CC}$$

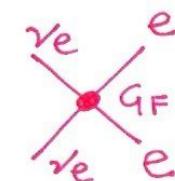
+
 NC CC

↑
proportional to unity
and unobservable

↑
observable in ν_e
oscillations

Relevant term is the $\nu_e - \nu_{\mu,\tau}$ energy difference: $V_{CC} \approx$

(No analogous for μ, τ , which are absent in ordinary matter)



from E.Lisi "Physics of Massive neutrinos"

NEUTRINO OSCILLATION IN MATTER

- It turns out that the V_{cc}^{ee} interaction energy is (see next page):

$$V = \sqrt{2} G_F N_e$$

where N_e = electron number density, and $V \rightarrow -V$ for $\nu \rightarrow \bar{\nu}$

- Then, the Hamiltonian of ν propagation in matter reads:

$$H_{\text{flavor}} = \frac{1}{2E} \left[\begin{matrix} m_1^2 & m_2^2 & m_3^2 \\ m_2^2 & m_1^2 & 0 \\ m_3^2 & 0 & m_1^2 \end{matrix} \right] + \frac{A}{2E} \left(\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \right)$$

where

$$A = 2\sqrt{2} G_F N_e E$$

- The relative size of matter/vacuum terms is given by $A/\Delta m_{ij}^2$.
Roughly speaking, one may expect sizable effects for $A/\Delta m_{ij}^2 \sim \mathcal{O}(1)$.
- The dependence $A = A(x)$ makes the evolution nontrivial in many cases.

$$\frac{A}{\Delta m_{ij}^2} = 1.526 \times 10^{-7} \left(\frac{N_e}{\text{mol/cm}^3} \right) \left(\frac{E}{\text{MeV}} \right) \left(\frac{eV^2}{\Delta m_{ij}^2} \right)$$

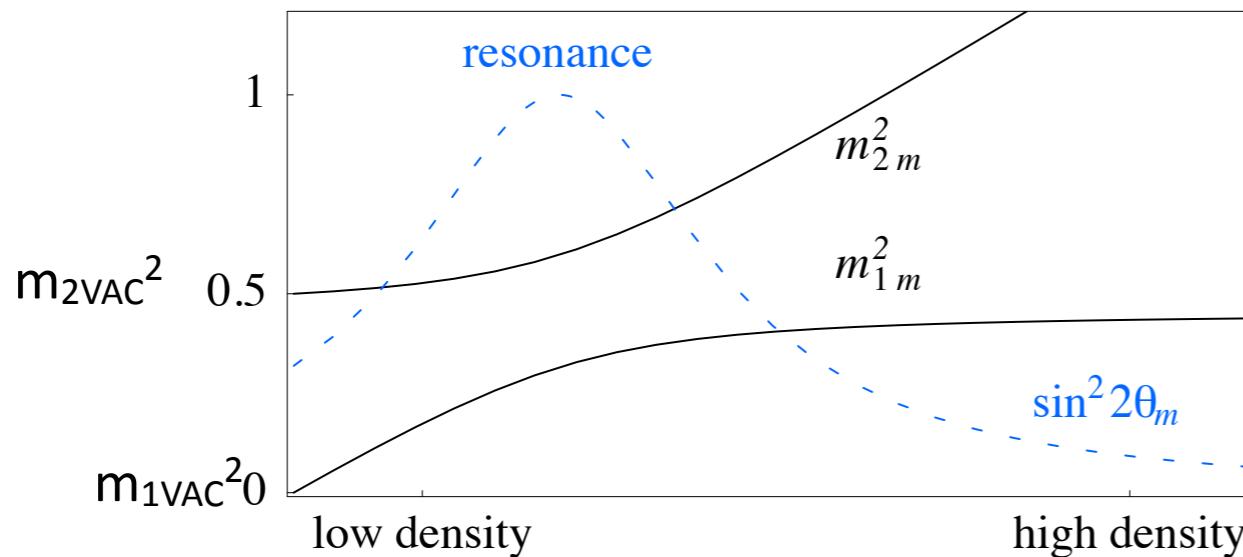
from E.Lisi "Neutrinos: theory and phenomenology"

2 V OSCILLATION IN CONSTANT MATTER

$$P_{e\mu} = \sin^2 2\tilde{\theta} \sin^2 \left(\frac{\Delta\tilde{m}^2 L}{4E} \right) \quad (\text{tutorial})$$

$$\sin 2\tilde{\theta} = \frac{\sin 2\theta}{\sqrt{(\cos 2\theta - \frac{A}{\Delta m^2})^2 + \sin^2 2\theta}} \quad \frac{\Delta\tilde{m}^2}{\Delta m^2} = \frac{\sin 2\theta}{\sin 2\tilde{\theta}}$$

↑
“Breit-Wigner” resonance form



- $A(v) = -A(\bar{v}) \Rightarrow$ asymmetry between neutrinos and antineutrinos: no resonance
- Matter effect distinguishes between $\theta \Leftrightarrow \pi/2 - \theta$, $\Delta m^2 \Leftrightarrow -\Delta m^2$ and change energy dependence
- Oscillation suppressed when matter dominates: independent $\lambda_{\text{MAT}} = \pi/\sqrt{2}(G_F N_e) \approx 3000$ km in the earth mantle
- Matter effect important when $\lambda_{\text{MAT}} \approx \lambda_{\text{VAC}}$

Can get a MSW resonant behavior for $c_{2\theta} \sim A/\Delta m^2$

$$\rightarrow \Delta m^2 c_{2\theta} = 2\sqrt{2} G_F N_e E$$

$$\rightarrow \sin^2 2\tilde{\theta} \sim 1 \text{ (enhanc.)}$$

$$\rightarrow \Delta\tilde{m}^2 \text{ minimized}$$

Can get suppression for $A \gg \Delta m^2 \rightarrow \sin^2 2\tilde{\theta} \sim 0$

Limiting cases:

$$A/\Delta m^2 \ll 1: (\Delta\tilde{m}^2, \tilde{\theta}) \approx (\Delta m^2, \theta) \quad \leftarrow \text{vacuum-like}$$

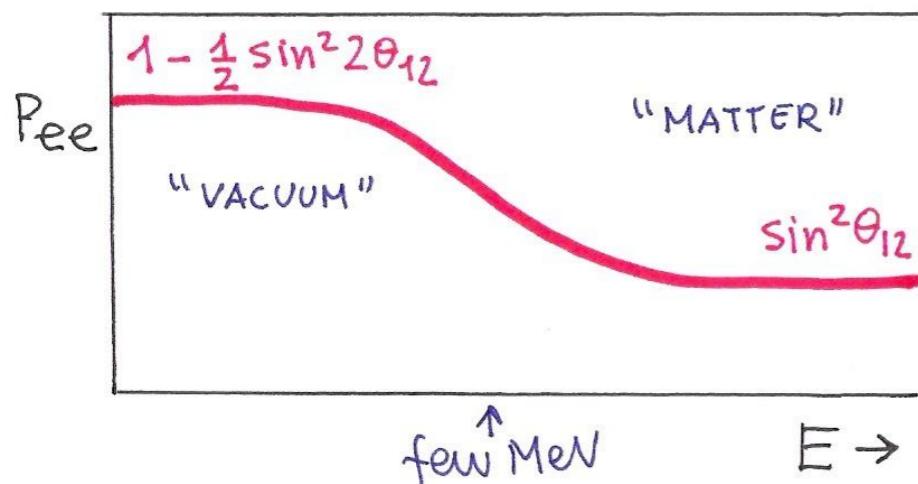
$$A/\Delta m^2 \approx \cos 2\theta: (\Delta\tilde{m}^2, \tilde{\theta}) \approx (\Delta m^2 \sin 2\theta, \pi/4) \quad \leftarrow \text{reson.}$$

$$A/\Delta m^2 \gg 1: (\Delta\tilde{m}^2, \tilde{\theta}) \approx (A, \pi/2) \quad \leftarrow \text{matter dominance}$$

MATTER EFFECT IN SLOW VARYING DENSITY

It turns out that, for the $(\delta m^2, \theta_{12})$ values chosen by nature, the adiabatic approximation can be applied to solar ν_e . In this case, $\tilde{\theta}_{12}(x_f) = \theta_{12}$ (vacuum value at the exit from the Sun), while $\tilde{\theta}_{12}(x_i)$ must be evaluated at the production point x_i . Limiting cases:

- $E \lesssim \text{few MeV}$ (vacuum dominance): $A/\delta m^2 \lesssim 1$ and $\tilde{\theta}_{12}(x_i) \approx \theta_{12}$
- $$Pee \approx C_{12}^4 + S_{12}^4 = 1 - \frac{1}{2} \sin^2 2\theta_{12}$$
- This is the averaged vacuum probability, octant symmetric.
- $E \gtrsim \text{few MeV}$ (matter dominance): $A/\delta m^2 \gtrsim 1$ and $\tilde{\theta}_{12}(x_i) \approx \pi/2$
- $$Pee \approx \sin^2 \theta_{12}$$
- This is the matter-dominated probability, octant-asymmetric



The Pee transition from "low" to "high" E is a signature of matter effects in the Sun.

Thanks to matter effects we can determine the octant of the mixing angle θ_{12} .

NEUTRINOS FROM THE SUN

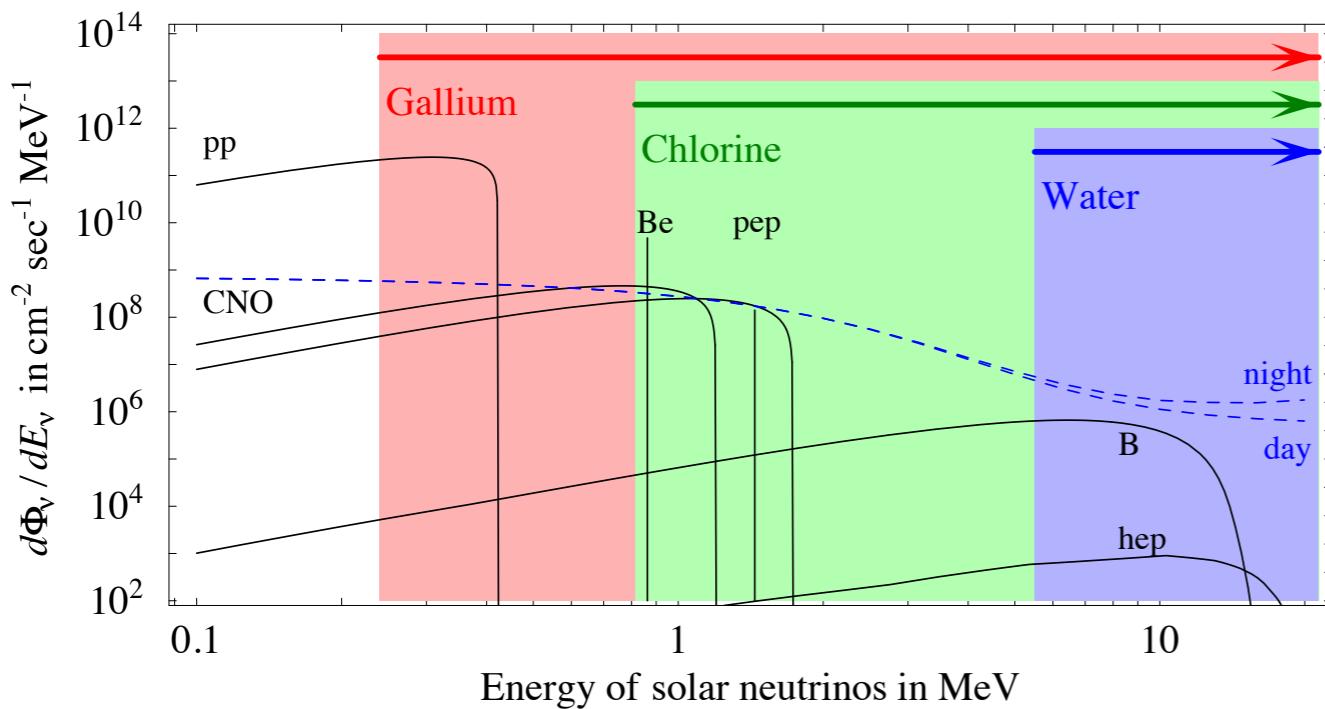
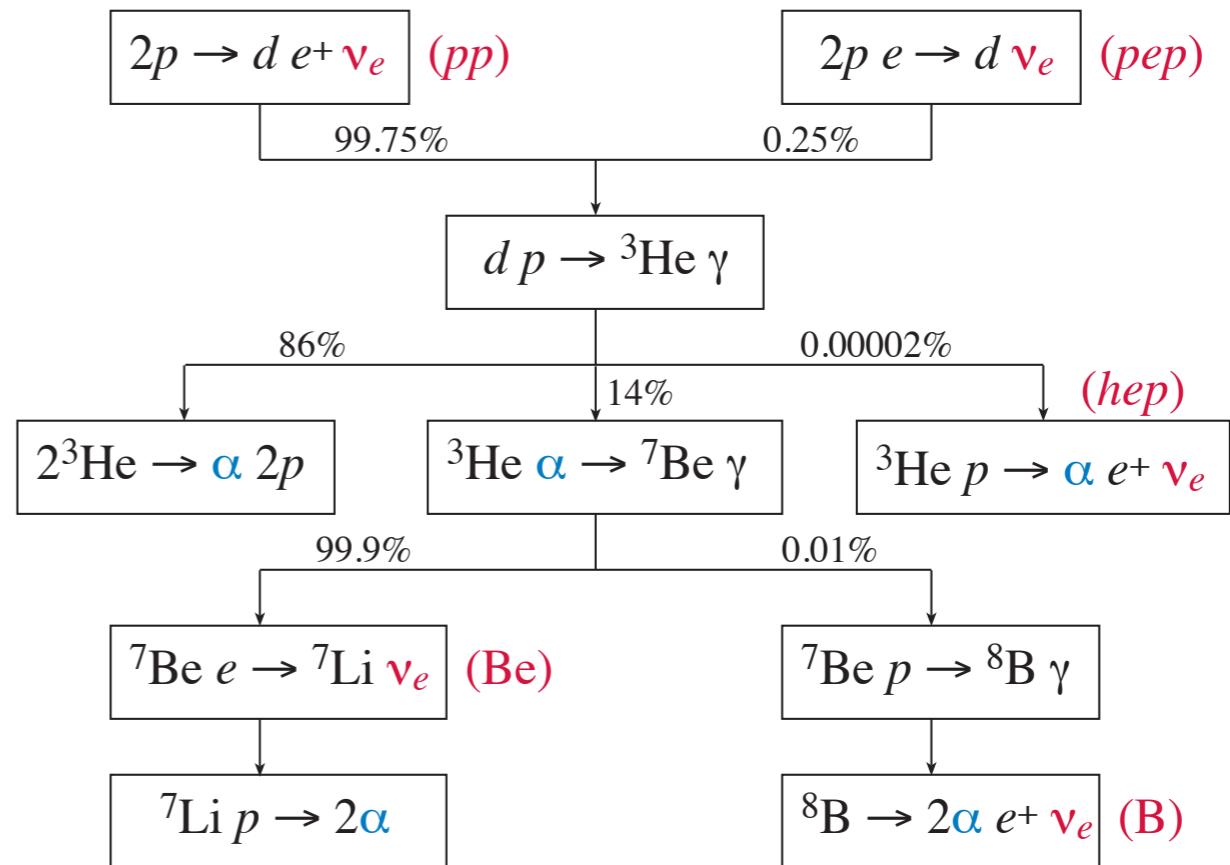
In the center of the sun



$$Q = 26.73 \text{ MeV}$$

$$\langle E_\nu \rangle \approx 0.3 \text{ MeV}$$

$$\text{Solar } \Phi(\nu) \approx 6 \cdot 10^{10} \text{ v/cm}^2\text{s}$$



Radio Chemical exp:

-Homestake: $\nu^{71}\text{Ga} \rightarrow e^- {}^{71}\text{Ge}$

-Gallex: $\nu^{37}\text{Cl} \rightarrow e^- {}^{37}\text{Ar}$

Real Time Water Čerenkov exp:

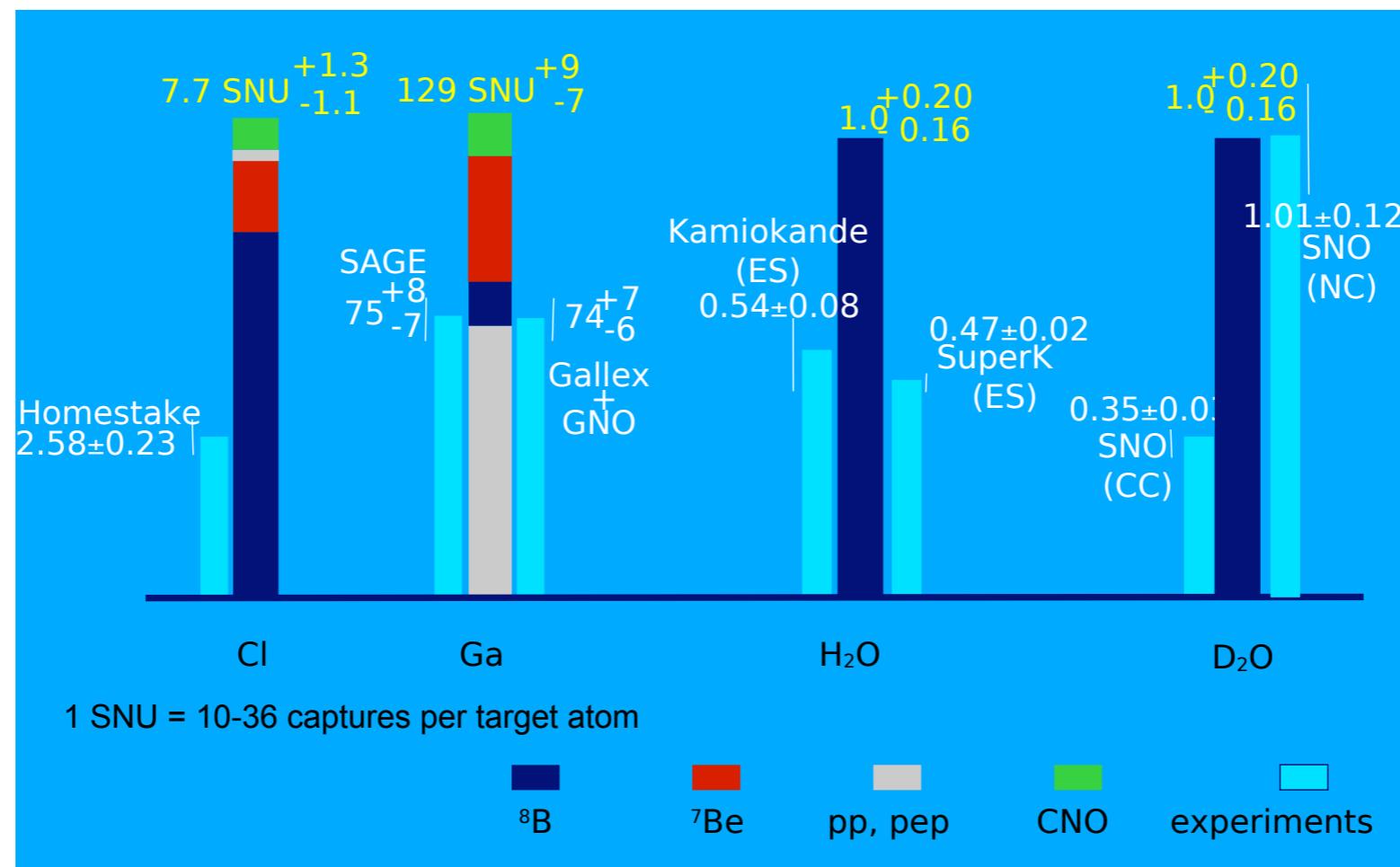
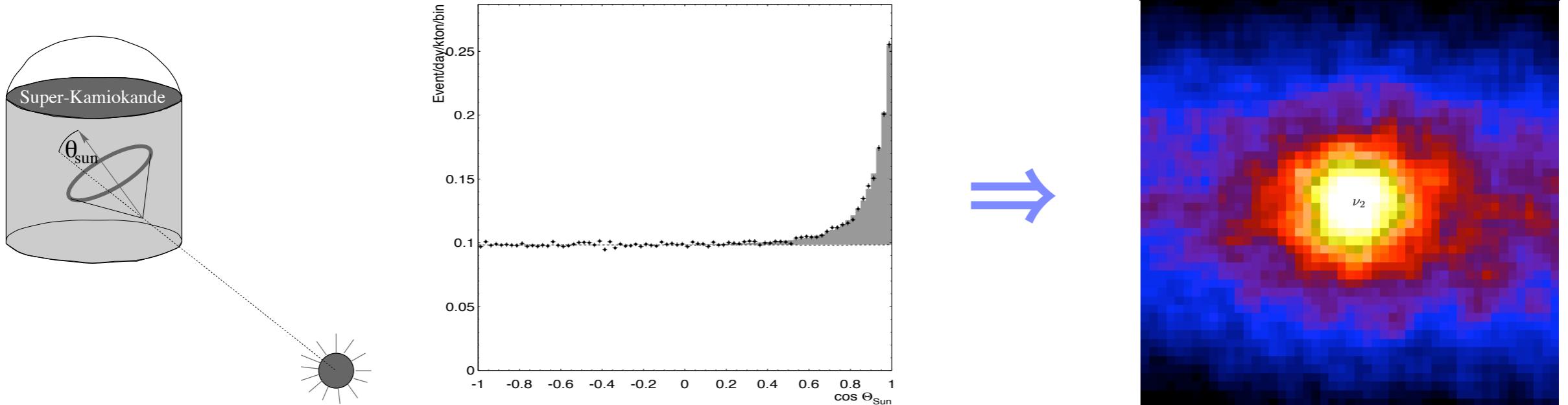
-SuperKamiokande: ES $\nu_{e,\mu,\tau} e \rightarrow \nu_{e,\mu,\tau} e$

-SNO: ES+CC+NC

Real Time scintillator exp:

Borexino: ES $\nu_{e,\mu,\tau} e \rightarrow \nu_{e,\mu,\tau} e$

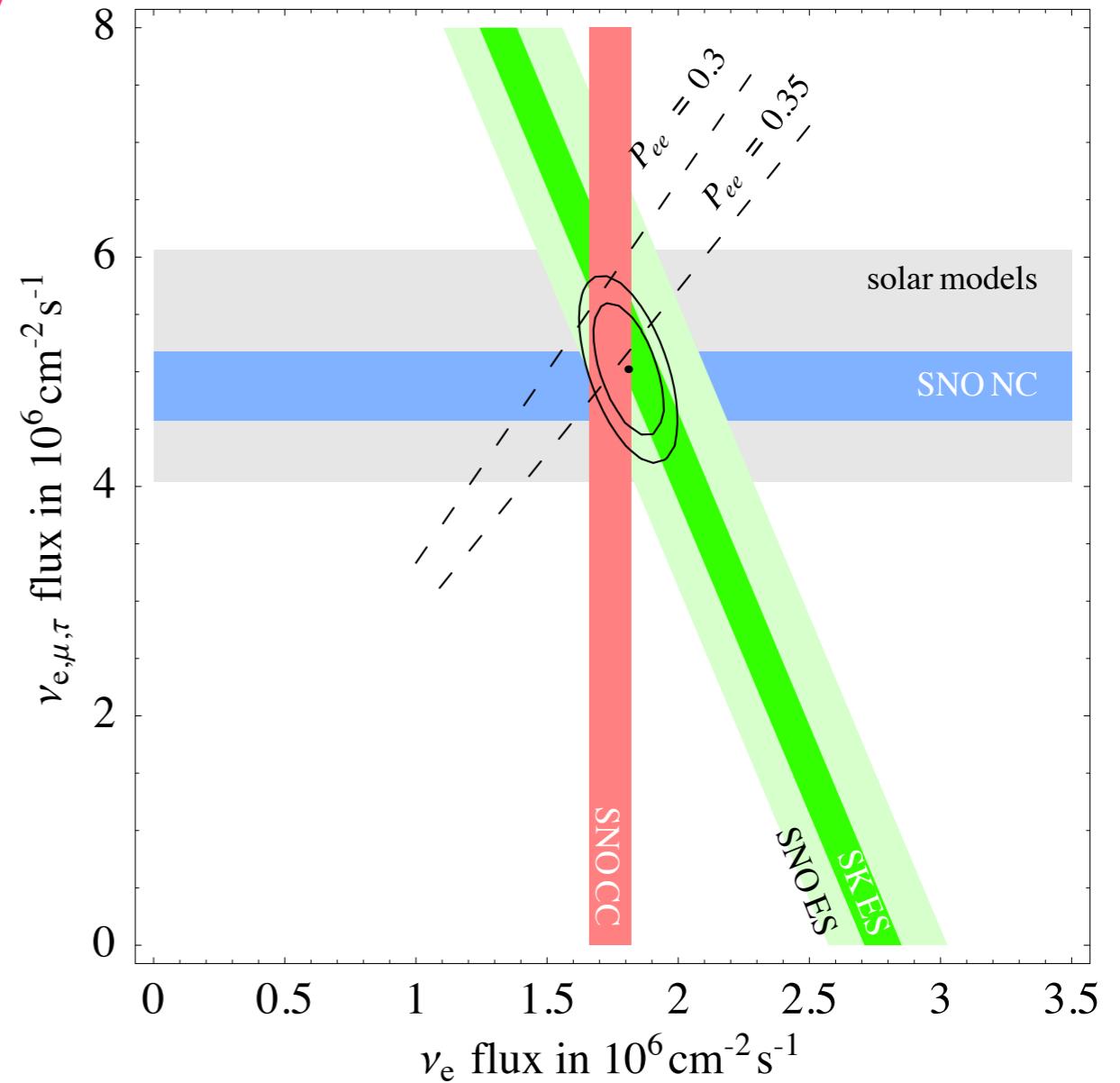
MEASURED NEUTRINO FLUX



SNO:TOTAL FLUX MEASUREMENT

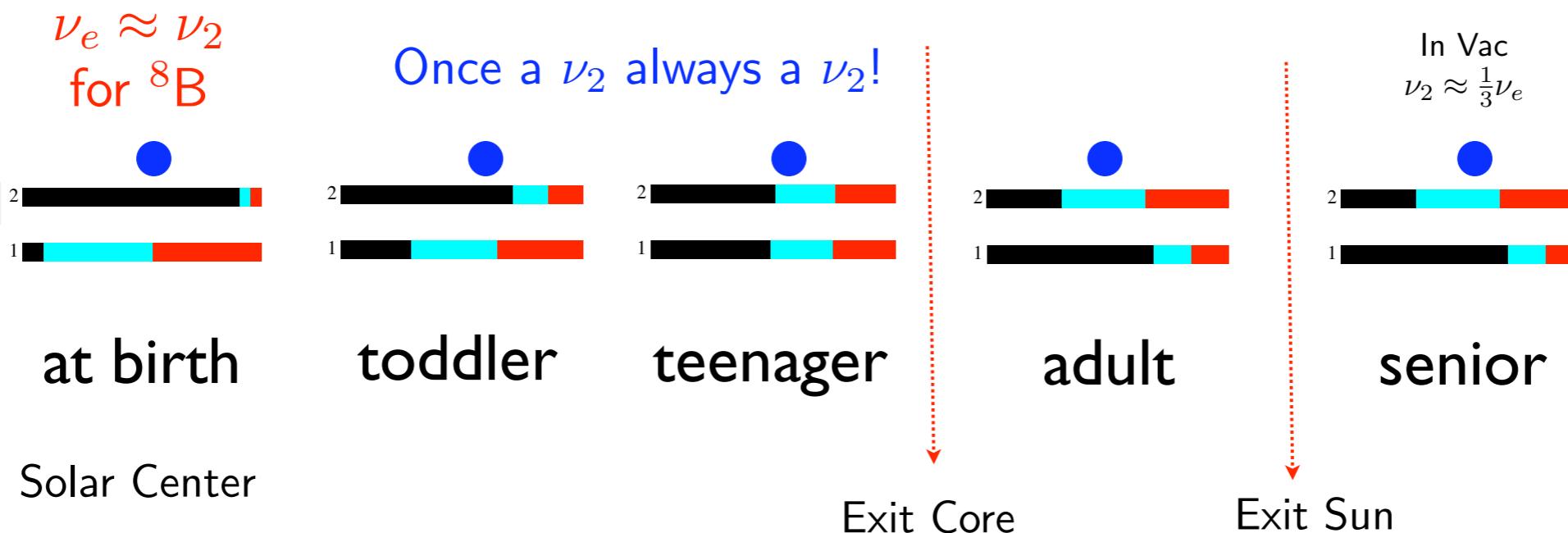
- D₂O Čerenkov detector:
 - ES: $\nu_{e,\mu,\tau} + e^- \rightarrow \nu_{e,\mu,\tau} + e^- \Rightarrow \Phi(\nu_e) + 0.155 \Phi(\nu_{\mu,\tau})$
 - CC: $\nu_e + D \rightarrow e^- + p + p \Rightarrow \Phi(\nu_e)$
 - NC: $\nu + D \rightarrow \nu + p + n \Rightarrow \Phi(\nu_{e,\mu,\tau})$
- NC rate as expected from Solar Model
- ES rate consistent with SK
- CC rate:

$$\frac{\phi(\nu_e)}{\phi(\nu_e) + \phi(\nu_{\mu,\tau})} = 0.357 \pm 0.030$$
- $P_{ee} < 0.5$ can't be vacuum oscillation!
 - Averaged oscillations $|\Delta m^2_{12}| \approx 7.58 \cdot 10^{-5} \text{ eV}^2$, $L \approx 150 \cdot 10^6 \text{ km}$
 - ▶ $P_{ee} = 1 - 0.5 \cdot \sin^2(2\theta_{\text{sun}}) > 0.5$



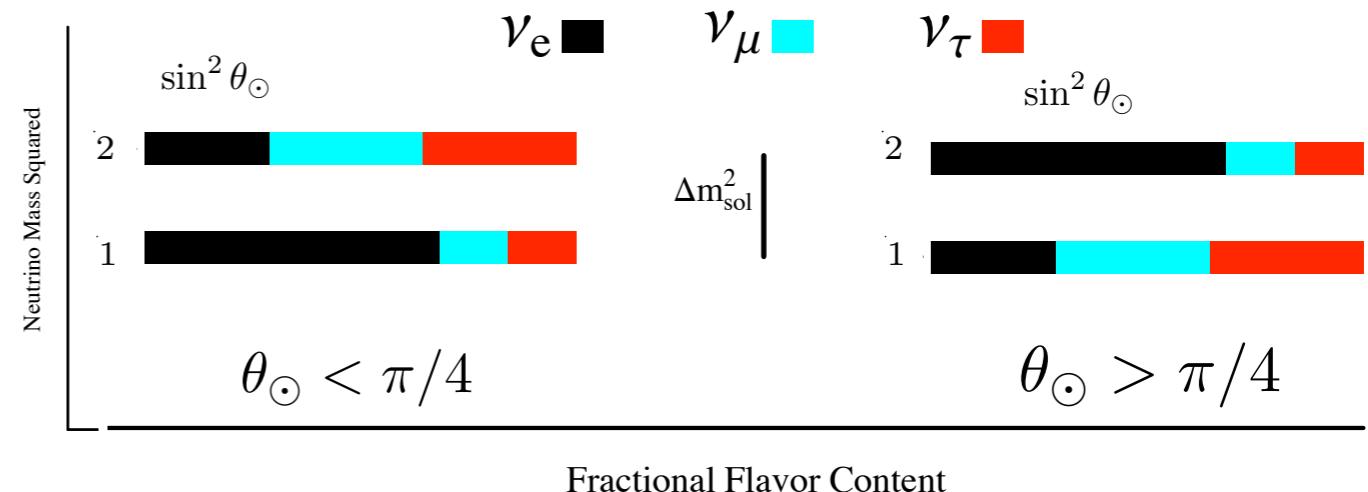
MATTER EFFECT

- At the center of the sun $V_W = 0.75 \cdot 10^{-5} \text{ eV}^2/\text{MeV} \Rightarrow A/\Delta m^2 = E/3\text{MeV}$
 - For pp ν_e 's ($E \approx 0.2 \text{ MeV}$) H_{vac} dominates \Rightarrow Average osc. $P_{ee} = 1 - 0.5 \cdot \sin^2(2\theta_{\text{sun}})$ \Rightarrow no Δm sensitivity
 - For ${}^8\text{B} \nu_e$'s ($E \approx 8 \text{ MeV}$) H_M dominates $\Rightarrow H_{\text{Tot}}$ diagonal $\Rightarrow \nu_e$ heavier mass eigenstate $\nu_2 (V > 0)$
 - ▶ ${}^8\text{B} \nu_e$'s propagate outward adiabatically
- At the exit of the core: $\nu_2 = \nu_e \sin(\theta_{\text{sun}}) + \nu_x \cos(\theta_{\text{sun}}) \Rightarrow P_{ee} = \sin^2(\theta_{\text{sun}})$

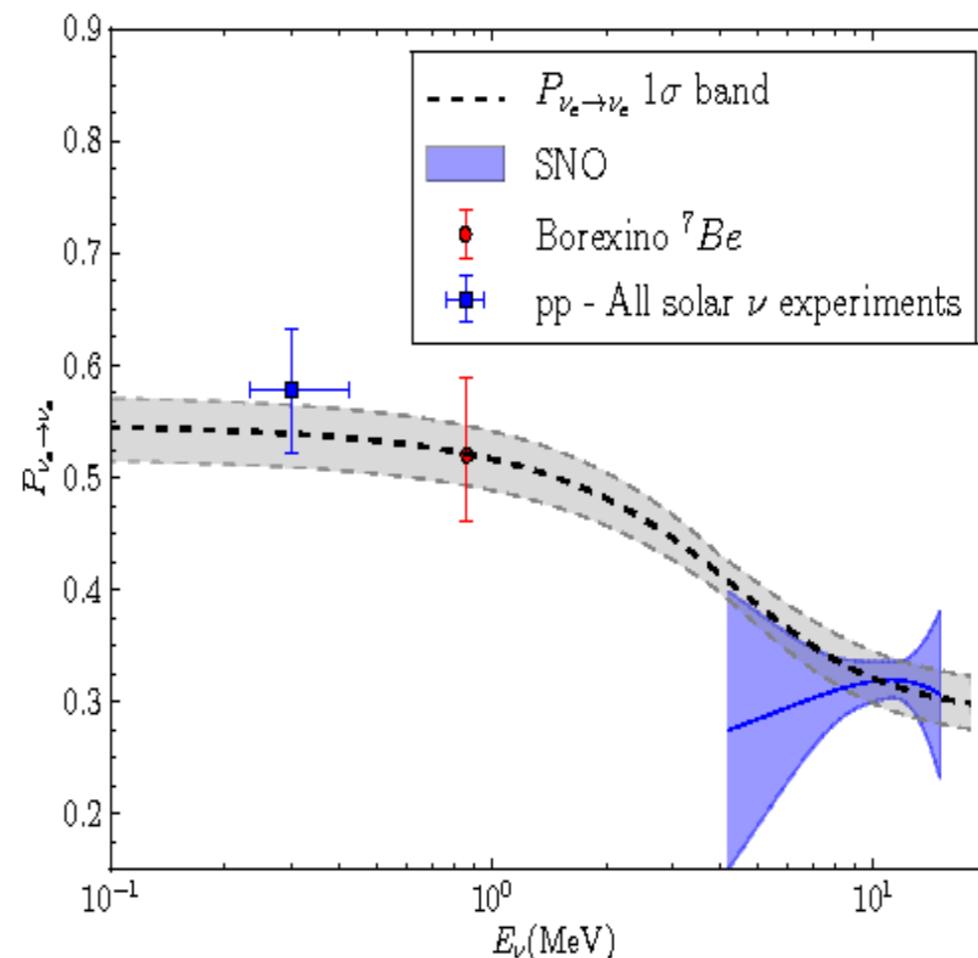


LARGE MIXING ANGLE + MSW

- Theory: $P_{ee} = 1.15 \cdot \sin^2(\theta_{\text{sun}})$
- Result: $P_{ee} = 0.357 \pm 0.030$
 $\Rightarrow \tan^2(\theta_{\text{sun}}) = 0.45 \pm 0.05, \theta_{\text{sun}} \approx 30^\circ$
- Octant and mass hierarchy resolved



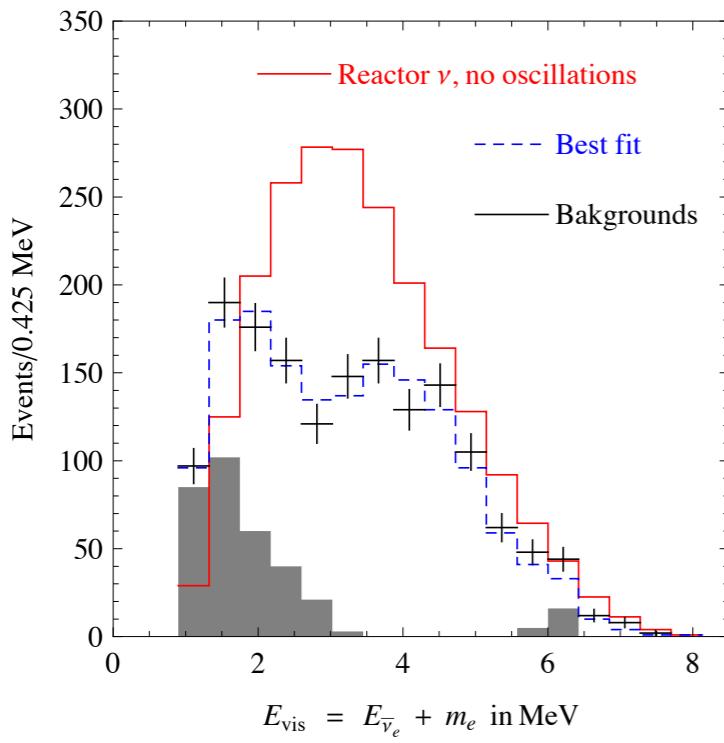
The overall picture



REACTOR EVIDENCE

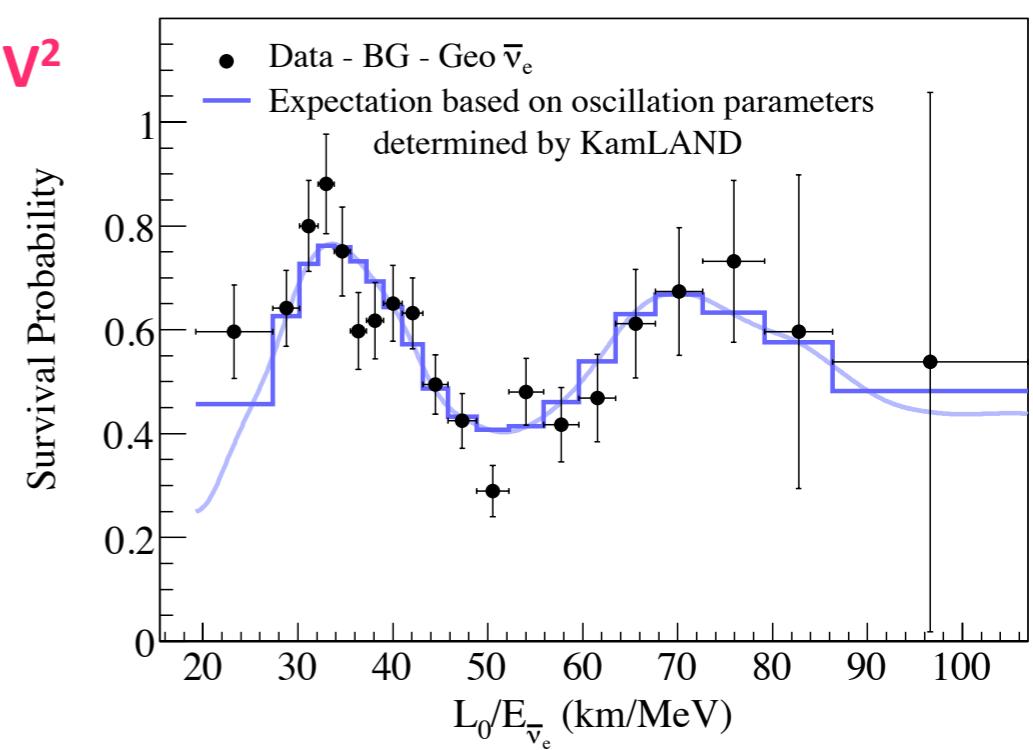
- Solar ν_e disappearance: first signal of neutrino anomaly
 - $\vartheta_{13} \sim 9^\circ \Rightarrow \nu_3$ couples feebly to electrons: solar neutrino born ν_e , they are a mixture of ν_1 and ν_2
 - Solar neutrino flavour change is $\nu_e \Rightarrow \nu_X$ where ν_X combination of ν_μ and ν_τ (look same below CC Eth)
 - 2 neutrino system: $\theta_{\text{sun}} \approx \theta_{12}$, $\Delta m_{\text{sun}}^2 \approx \Delta m_{12}^2$
- Reactors: $E_\nu \approx \text{MeV}$, $L \approx 180\text{km}$
 - Atmospheric and matter effect negligible ($|x| \sim E/10\text{GeV}$)

$$P[\bar{\nu}_e \rightarrow \bar{\nu}_e] = \cos^4(\theta_{13})(1 - \sin^2(2\theta_{12})\sin^2(\Delta m_{12}^2 L/4E))$$



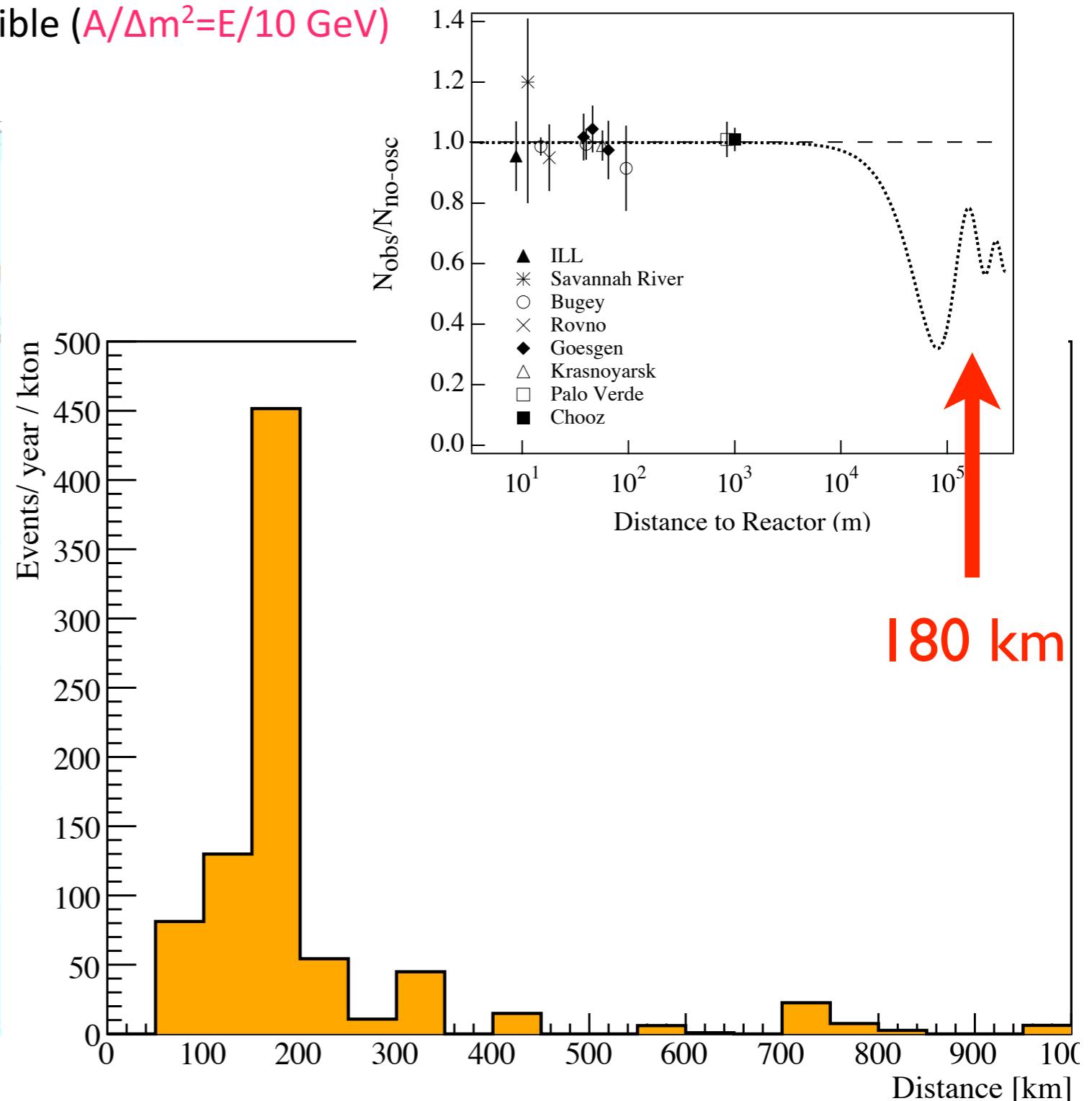
$$|\Delta m_{12}^2| \approx (7.58 \pm 0.21) \cdot 10^{-5} \text{ eV}^2$$

No distinction between
 $\theta \Leftrightarrow \pi/2 - \theta$, $\Delta m^2 \Leftrightarrow -\Delta m^2$



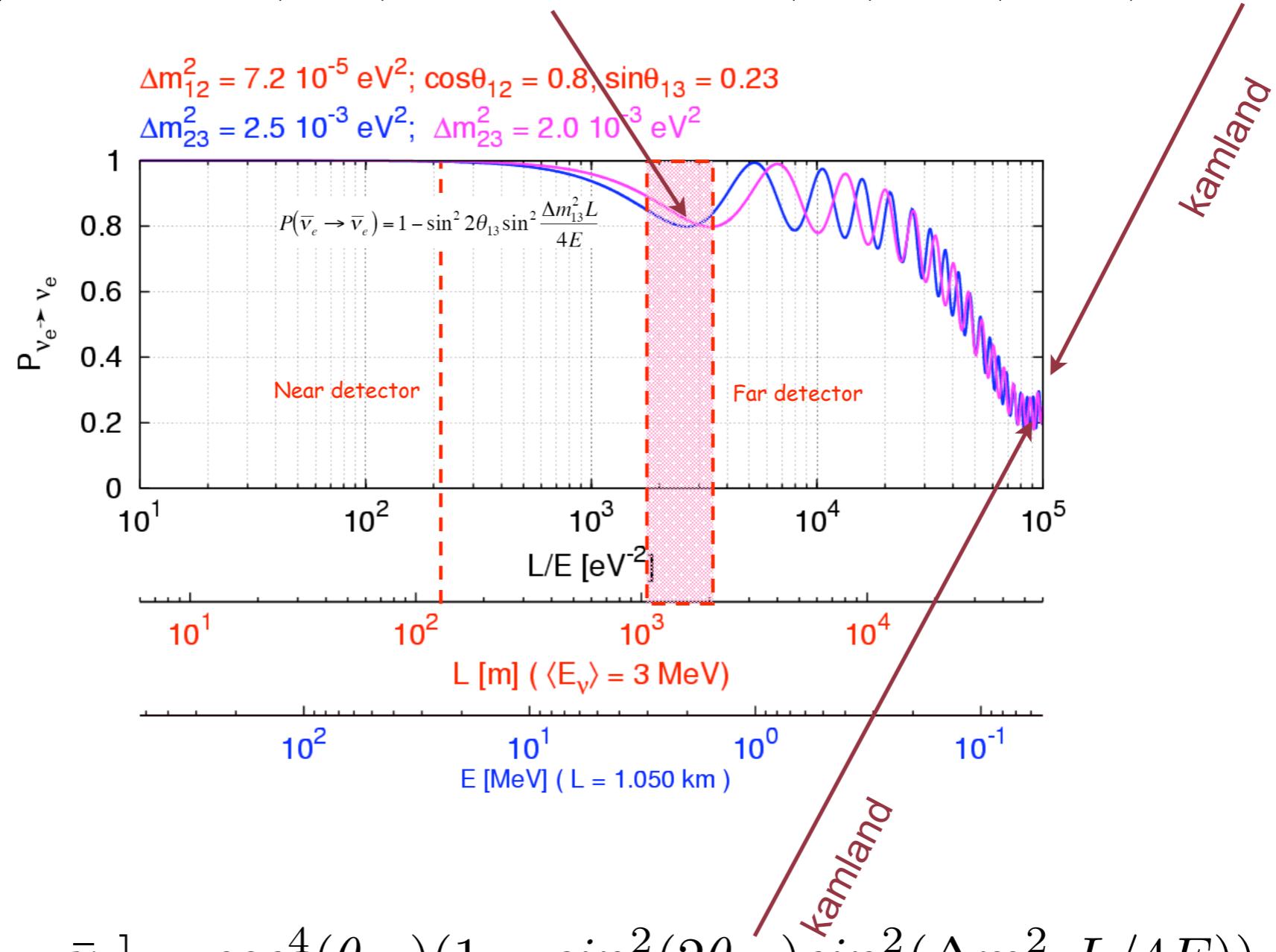
REACTOR CONFIRMATION: KAMLAND

- Reactors: $E_v \approx \text{MeV}$, $L \approx 180\text{ km}$
 - Atmospheric and matter effect negligible ($A/\Delta m^2 = E/10 \text{ GeV}$)



REMINDER

$$P[\nu_e \rightarrow \nu_e] = 1 - \sin^2(2\theta_{13})\sin^2\Delta_{atm} - \cos^4(\theta_{13})\sin^2(2\theta_{sun})\sin^2\Delta_{sun}$$



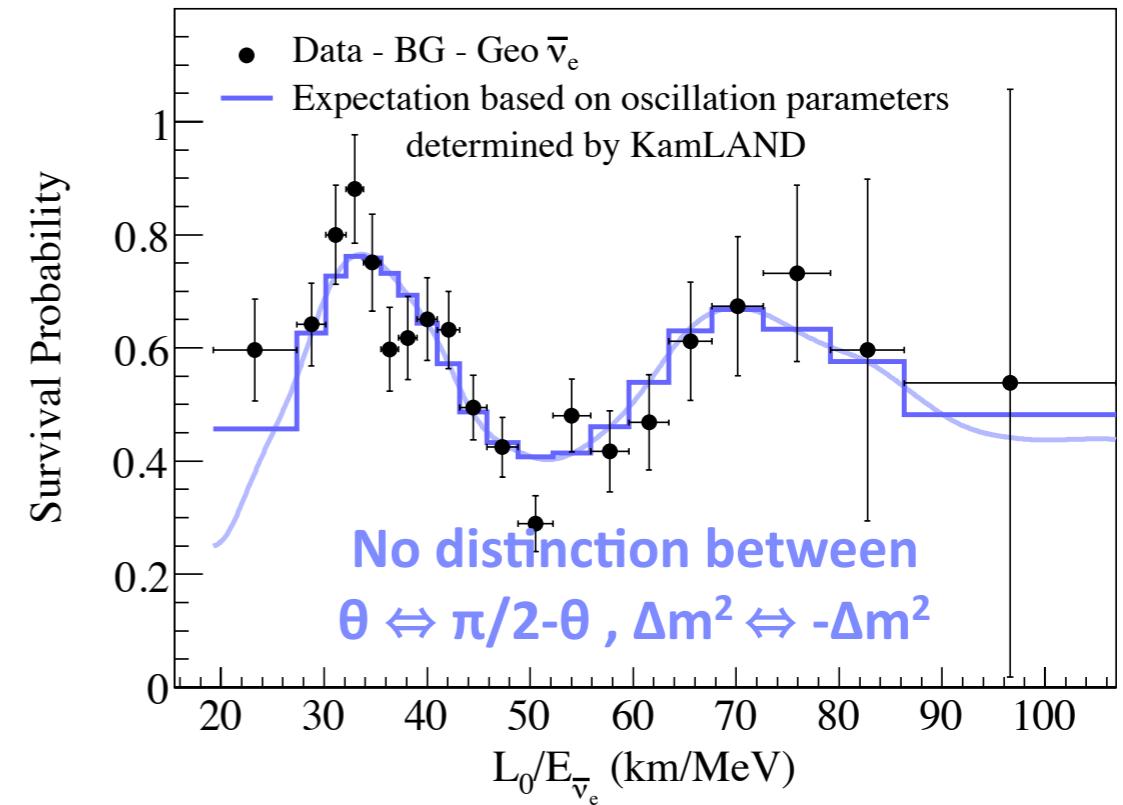
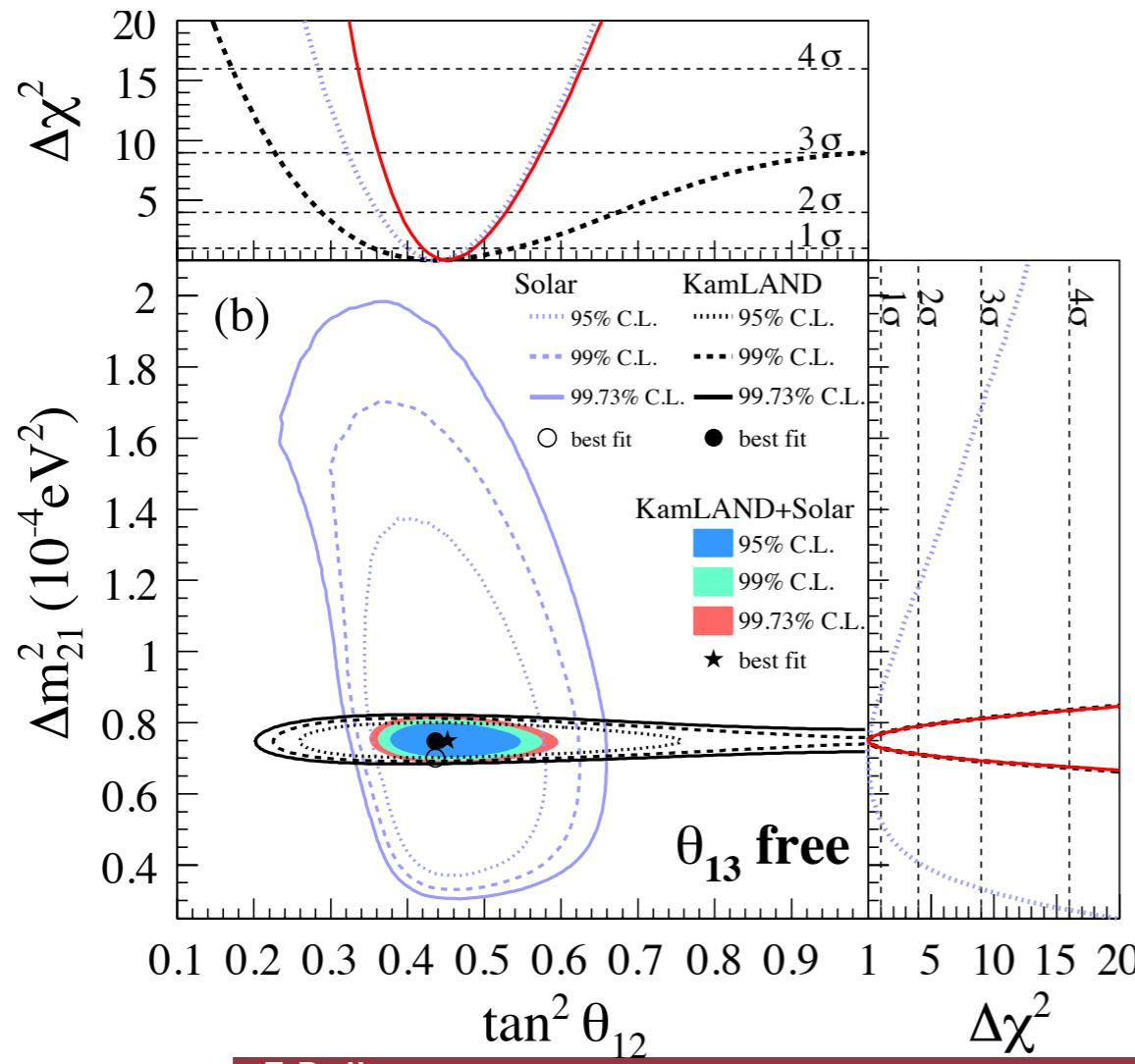
$$P[\bar{\nu}_e \rightarrow \bar{\nu}_e] = \cos^4(\theta_{13})(1 - \sin^2(2\theta_{12})\sin^2(\Delta m_{12}^2 L/4E))$$

Big effect: ν source knowledge not critical

RESULTS

One period of oscillation observed

$$P[\bar{\nu}_e \rightarrow \bar{\nu}_e] = \cos^4(\theta_{13})(1 - \sin^2(2\theta_{12})\sin^2(\Delta m_{12}^2 L/4E))$$

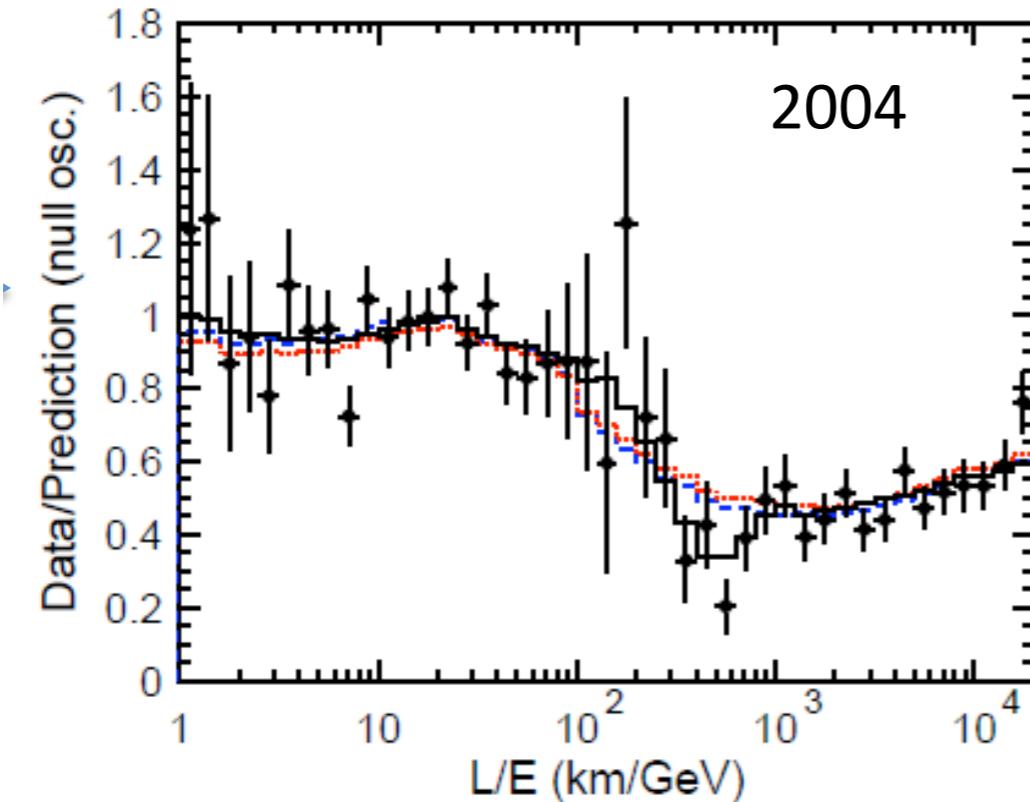


In the two neutrino framework
complementary of solar/reactor exp.

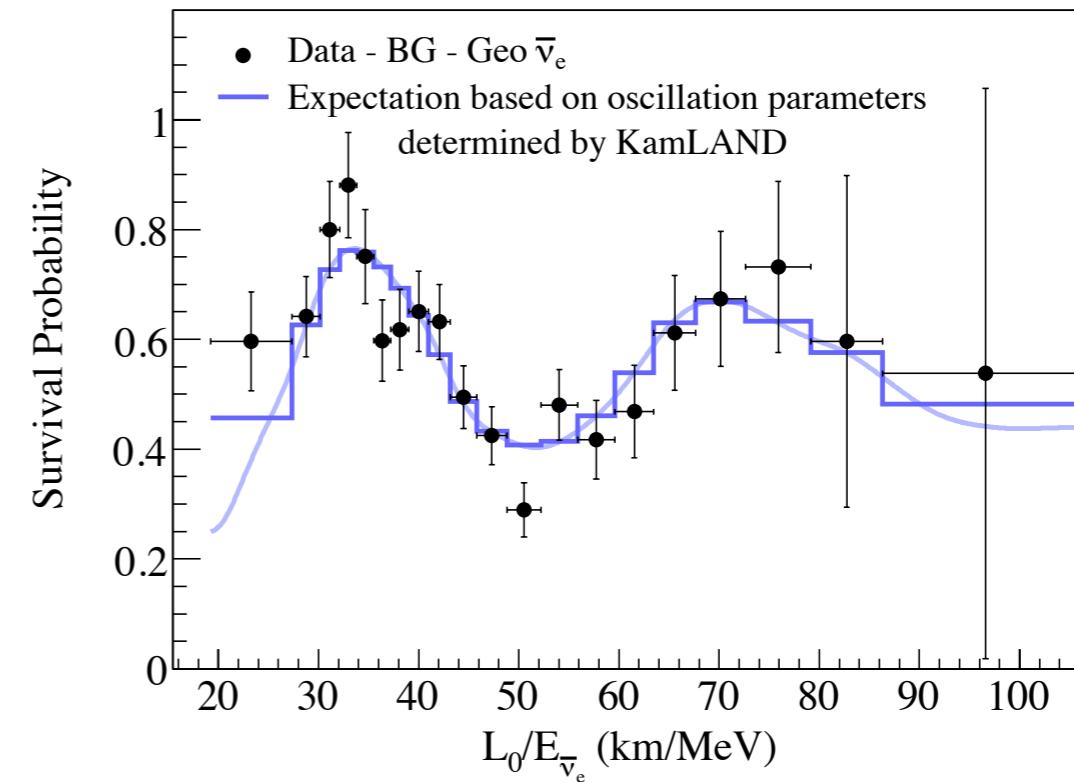
$$|\Delta m_{12}^2| \approx (7.58 \pm 0.21) \cdot 10^{-5} \text{ eV}^2$$

SUMMARY: THE TURNING POINT

Atmospheric: $\Delta m^2 \sim 2.4 \cdot 10^{-3} \text{ eV}^2$



Solar: $\delta m^2 \sim 7.6 \cdot 10^{-5} \text{ eV}^2$



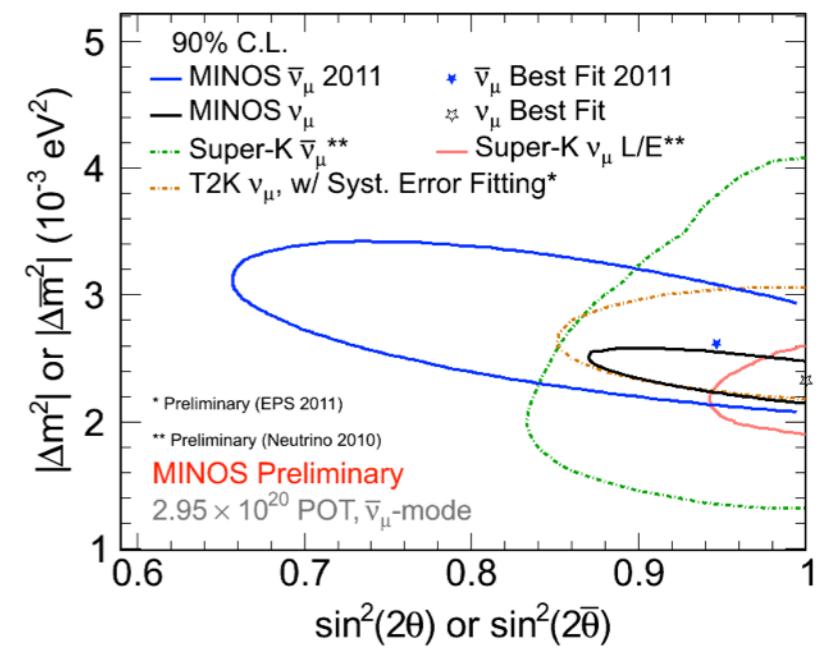
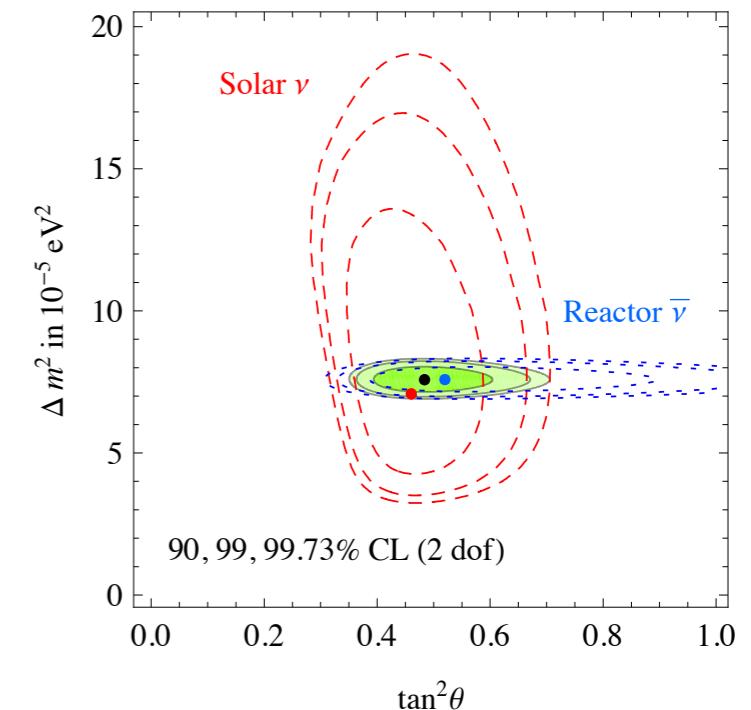
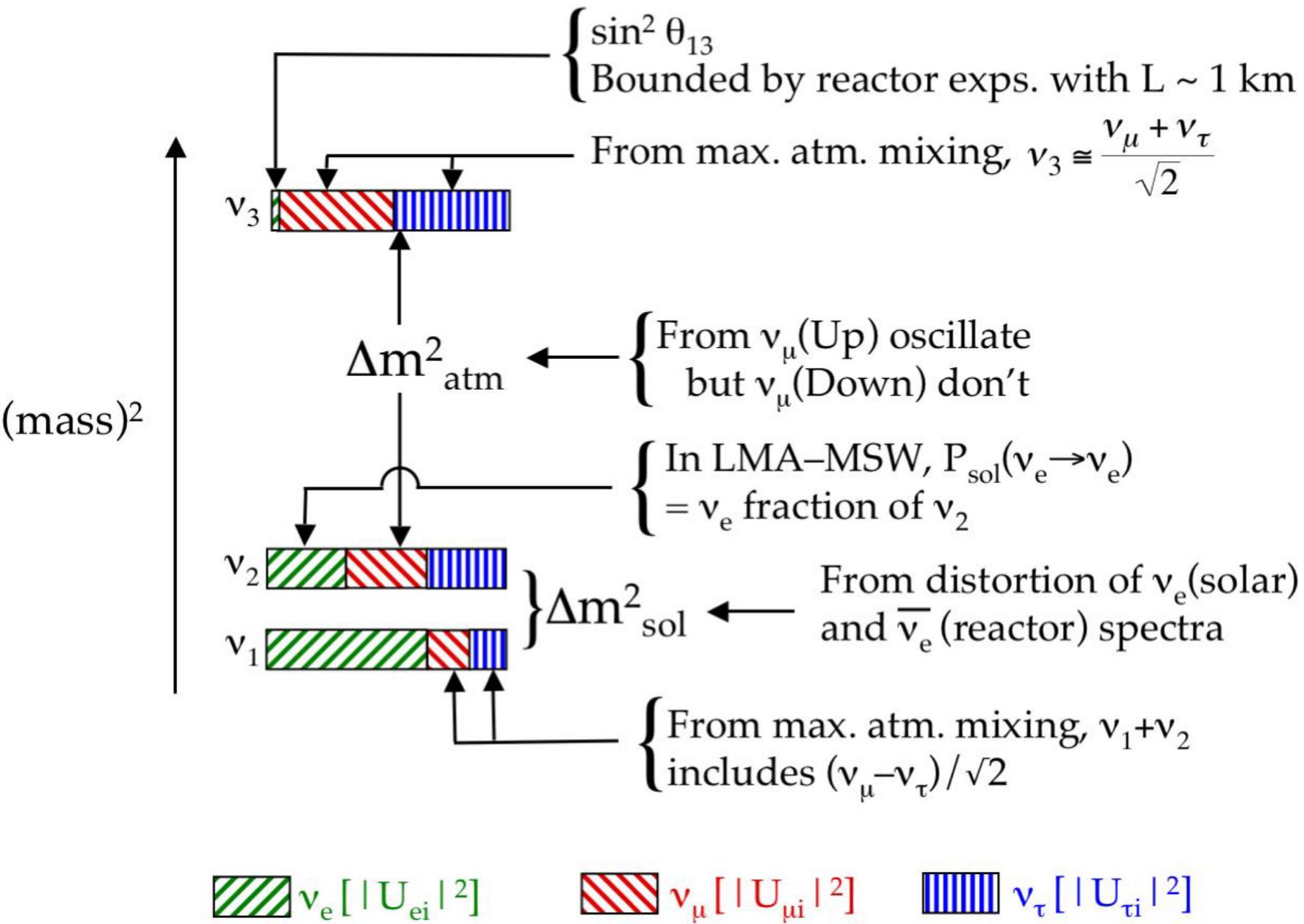
- One Mass scale dominance:
 - Only 3 parameters: $\Delta m^2, \vartheta_{23}, \vartheta_{13}$, $|U_{e3}| = \sin(\vartheta_{13})$, $|U_{\mu 3}| = \cos(\vartheta_{13}) \sin(\vartheta_{23})$
 - Experiment with $\Delta m^2 L/E = O(1)$ sensitive $|U_{e3}|$ (flavour content of ν_3)
- At solar energies(MeV) μ, τ under production threshold:
 - mixing depends only on first raw: $\delta m^2, \vartheta_{13}, \vartheta_{12}$, $|U_{e3}| = \sin(\vartheta_{13})$, $|U_{e2}| = \cos(\vartheta_{13}) \sin(\vartheta_{12})$
- ϑ_{13} only link between oscillations: $\vartheta_{13} \sim 9^\circ \Rightarrow$ decoupled

$$\begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

MASS HIERARCHY

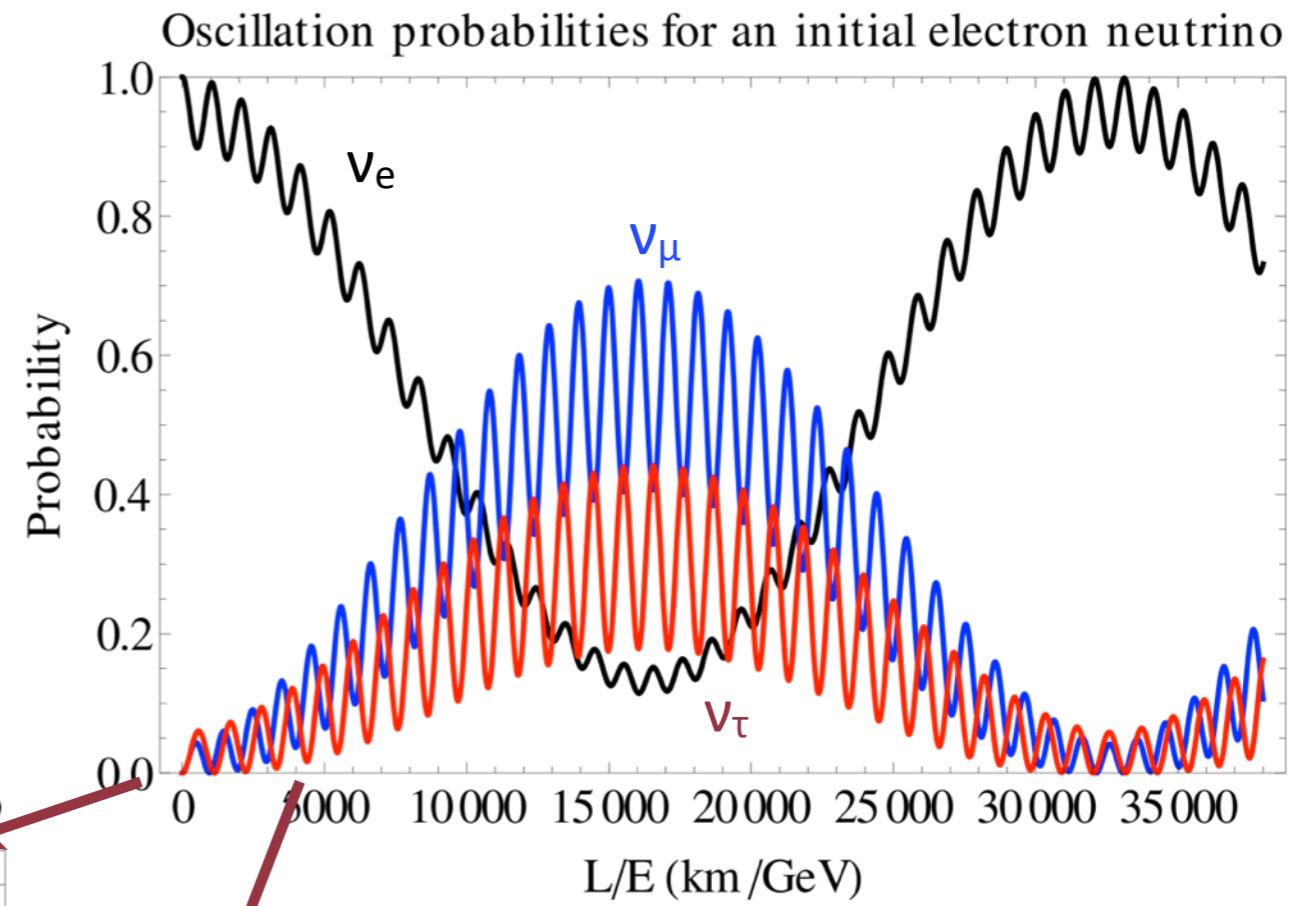
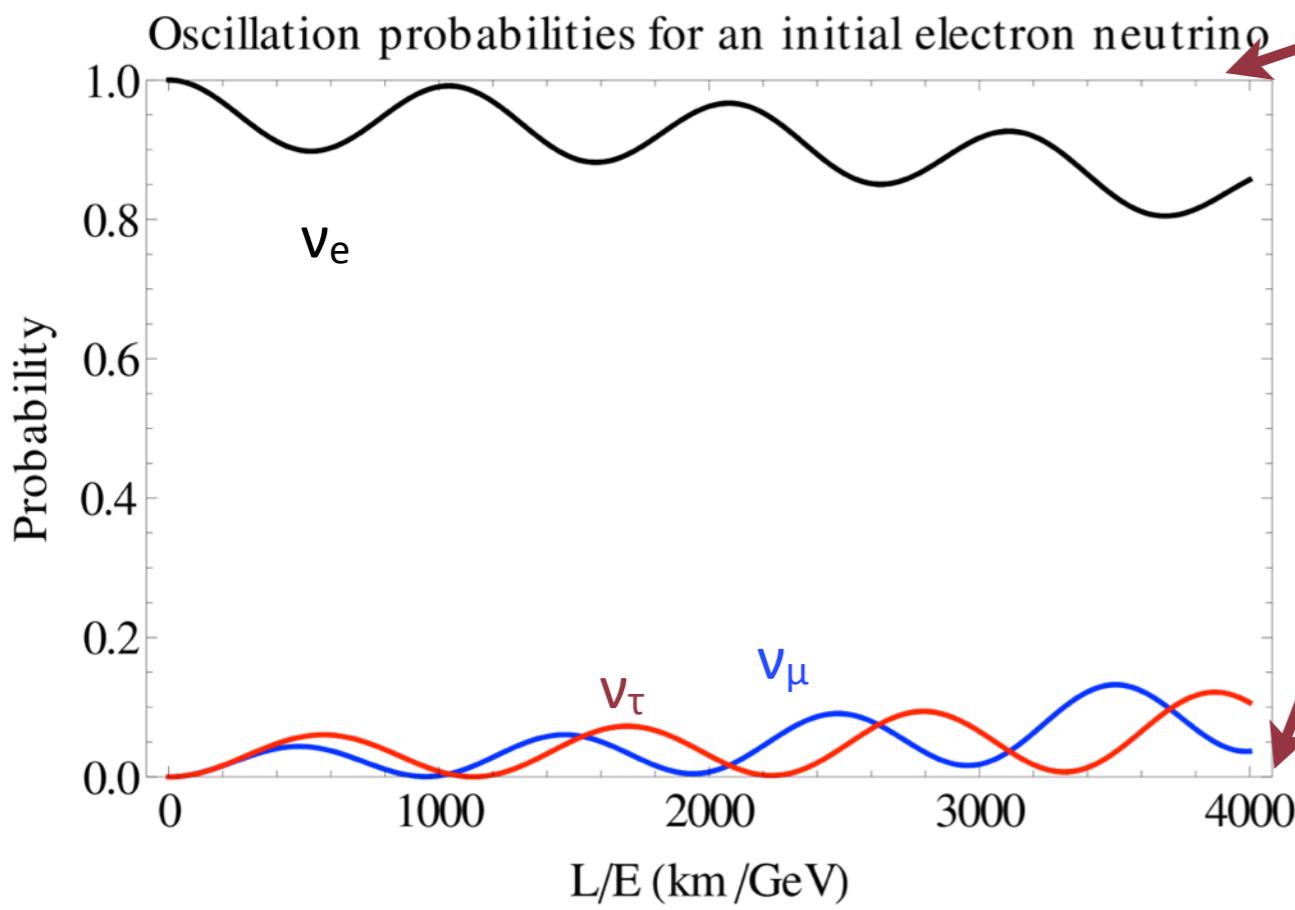
$$\theta_{\text{atm}} \approx \theta_{23} \approx 45^\circ, \theta_{\text{sun}} \approx \theta_{12} \approx 30^\circ, \theta_{13} \approx 9^\circ$$

$$|\Delta m^2_{13}| \approx |\Delta m^2_{23}| \approx |\Delta m^2_{\text{atm}}| \approx (2.40 \pm 0.15) \cdot 10^{-3} \text{ eV}^2 \quad \Delta m^2_{12} \approx \Delta m^2_{\text{sol}} \approx (7.58 \pm 0.21) \cdot 10^{-5} \text{ eV}^2$$



ν_e OSCILLATION

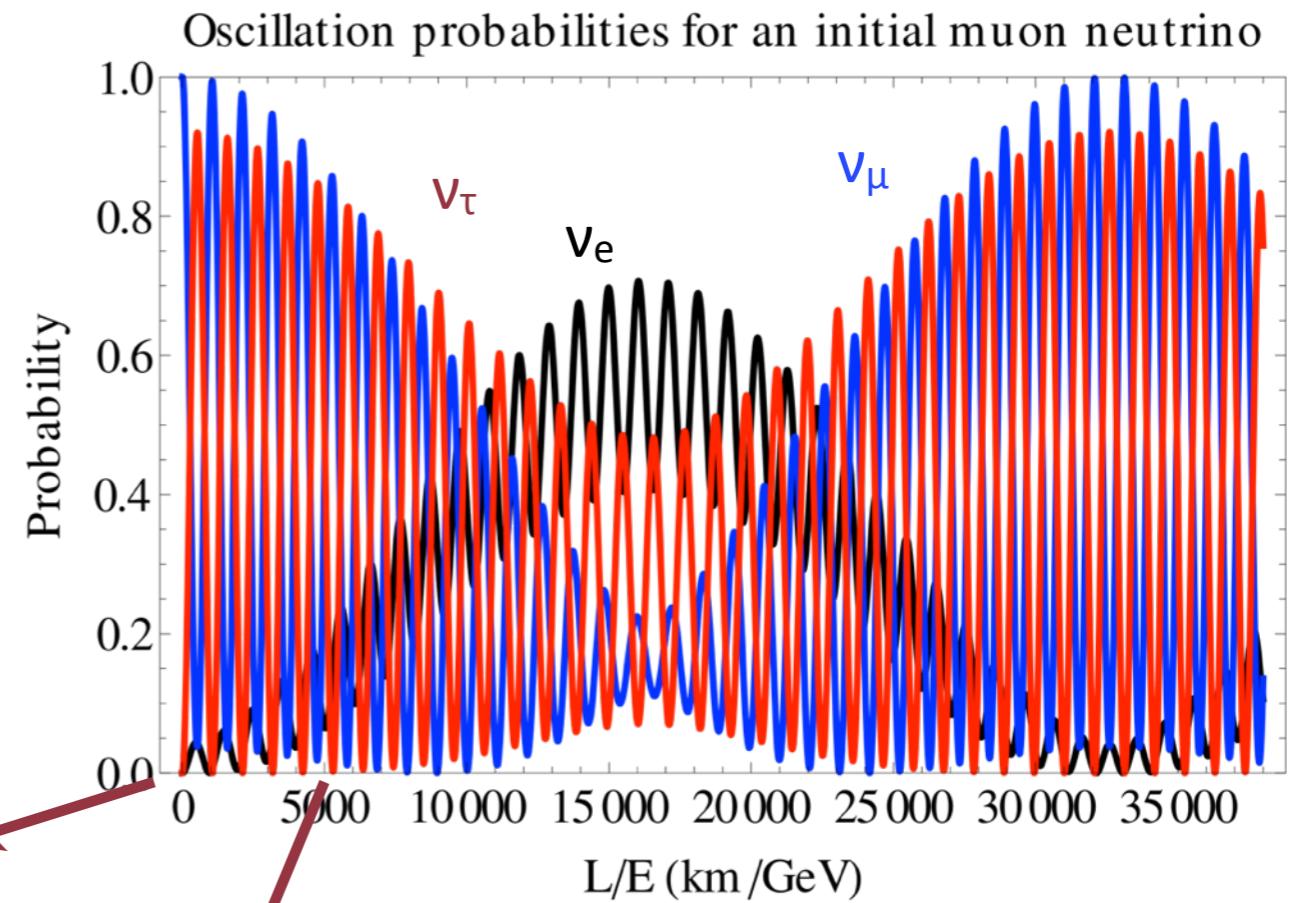
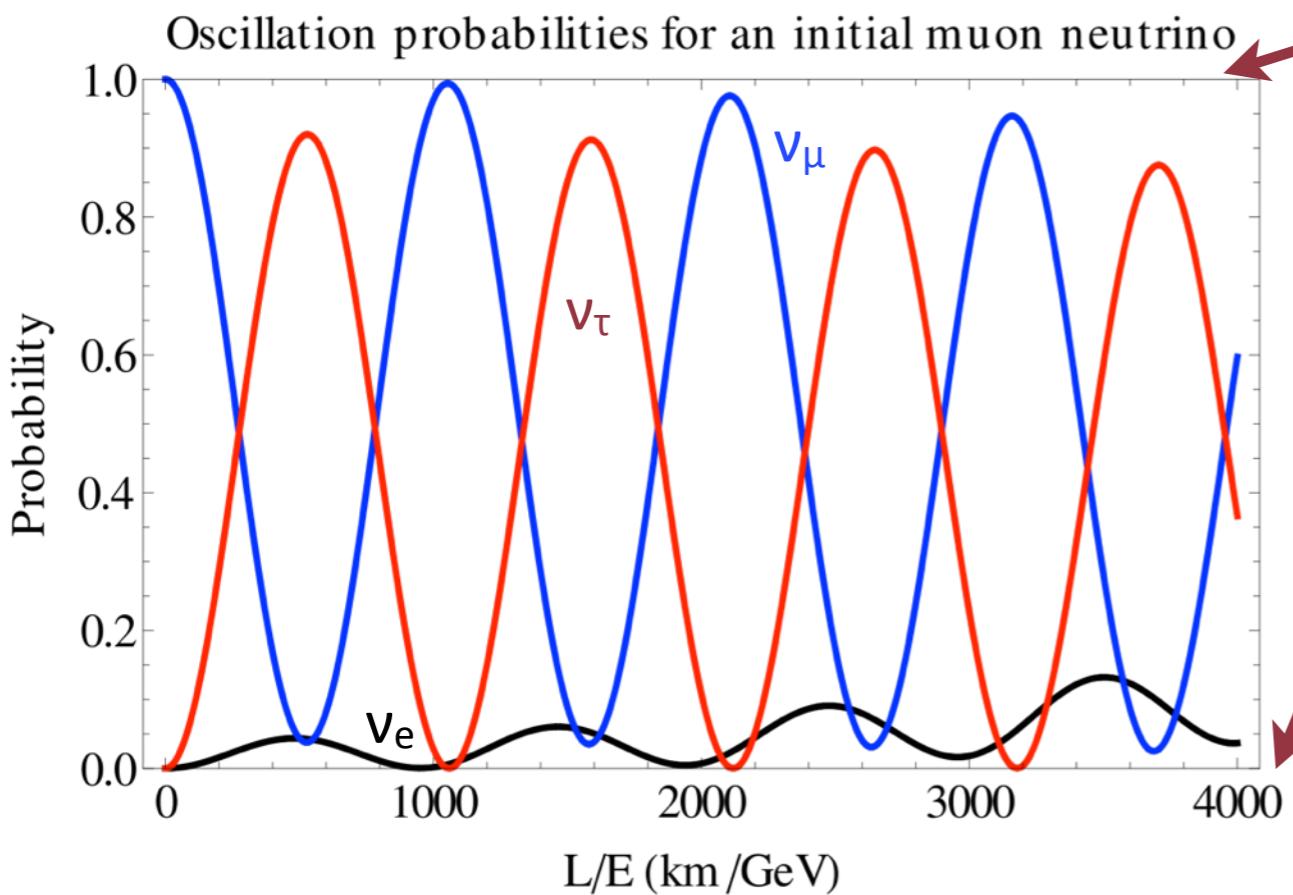
Short Baseline



Long Baseline

ν_μ OSCILLATION

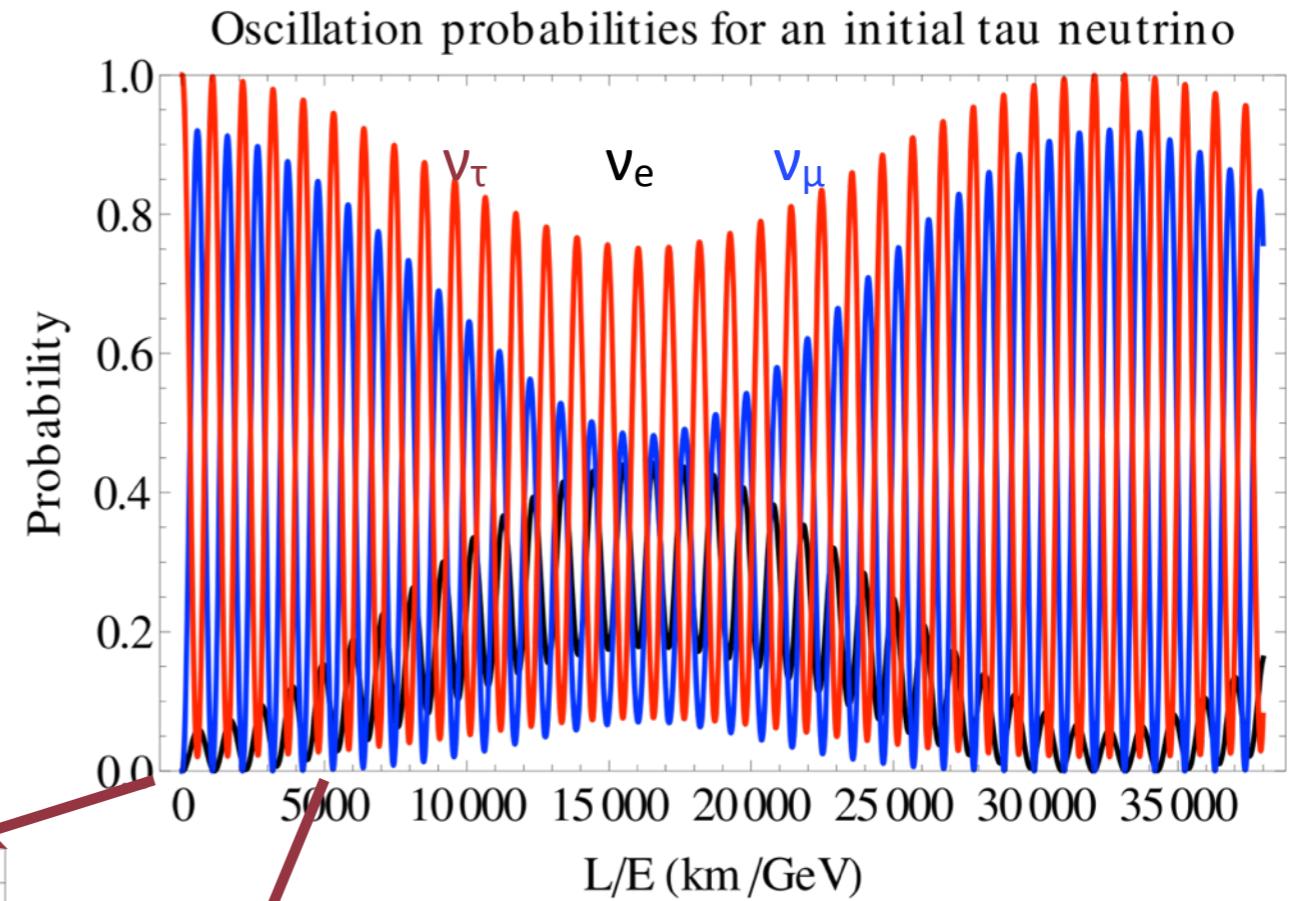
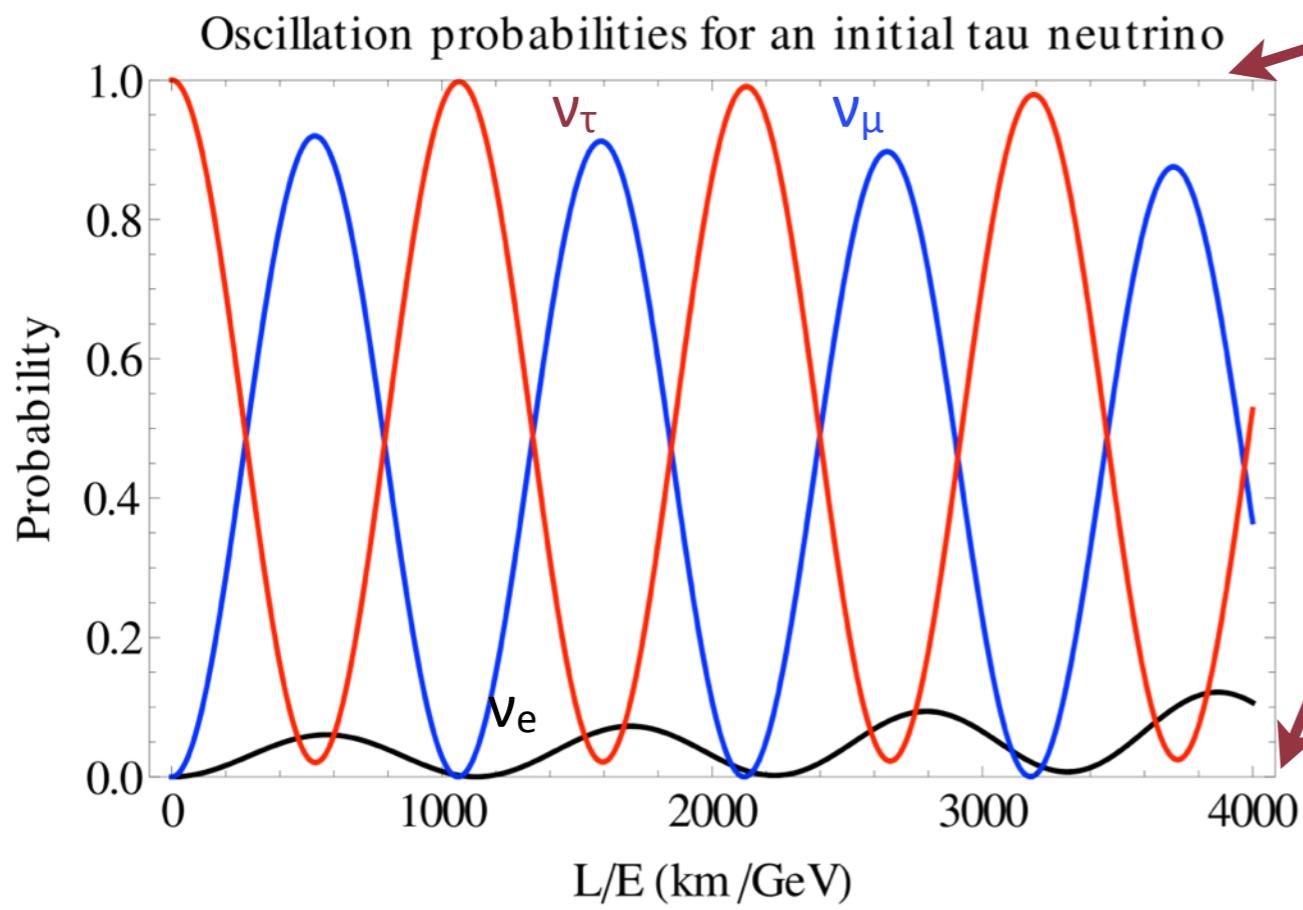
Short Baseline



Long Baseline

ν_τ OSCILLATION

Short Baseline



Long Baseline