

CKM Matrix and CP Violation in Standard Model

B physics at B factories
Lecture 15

DIPARTIMENTO DI FISICA



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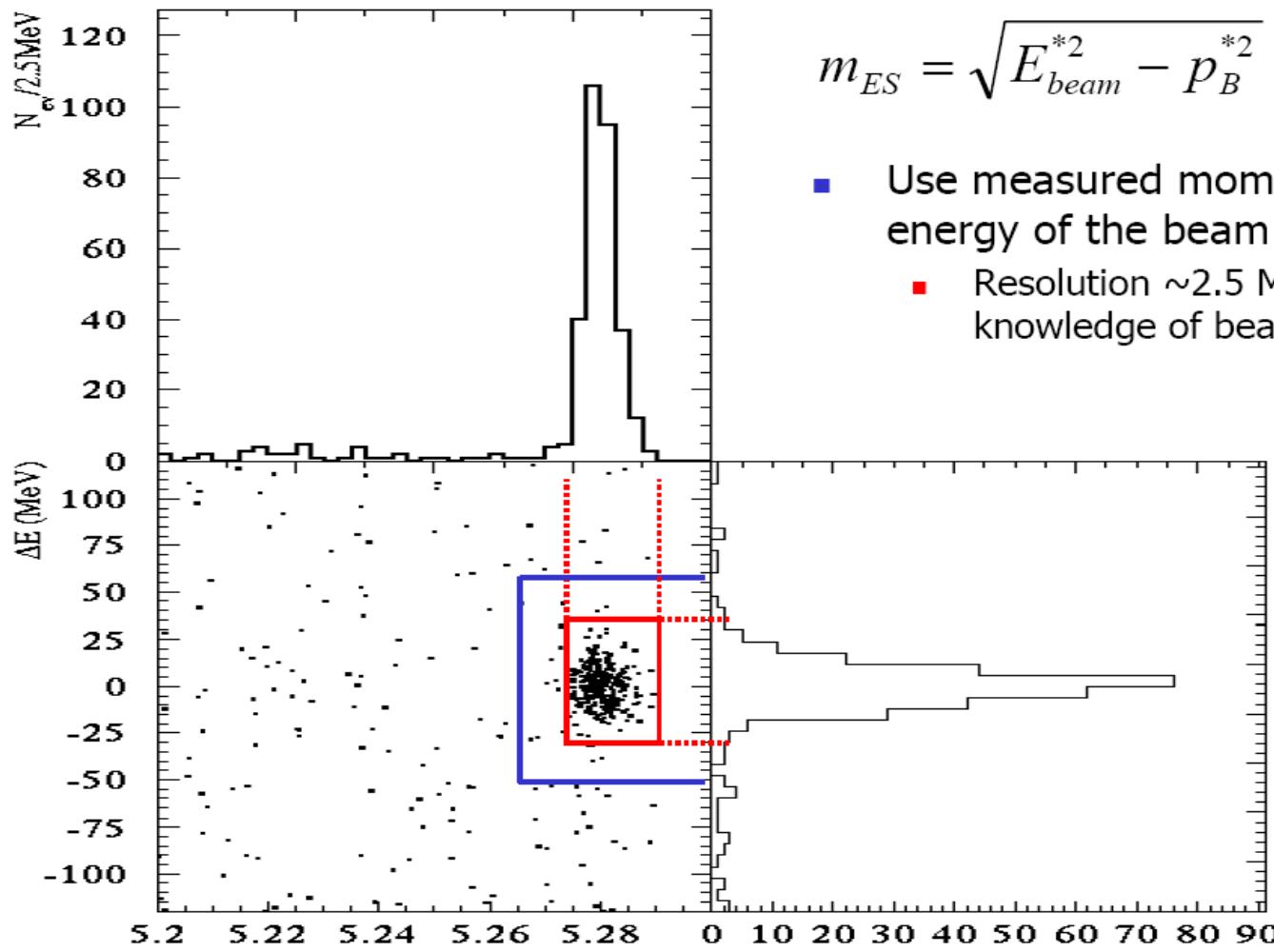
Shahram Rahatlou

Fisica delle Particelle Elementari, Anno Accademico 2015-16

<http://www.roma1.infn.it/people/rahatlou/particelle/>

Ingredients of B Reconstruction

- Take advantage of clean environment in $e^+e^- \rightarrow \gamma(4S) \rightarrow B\bar{B}$
 - Energy of each B meson is known in the center of mass



$$m_{ES} = \sqrt{E_{beam}^{*2} - p_B^{*2}}$$

Energy-substituted mass

- Use measured momenta of B daughters and energy of the beam
 - Resolution $\sim 2.5 \text{ MeV}/c^2$ dominated by knowledge of beam energy

Energy difference

$$\Delta E = E_B^* - E_{beam}^*$$

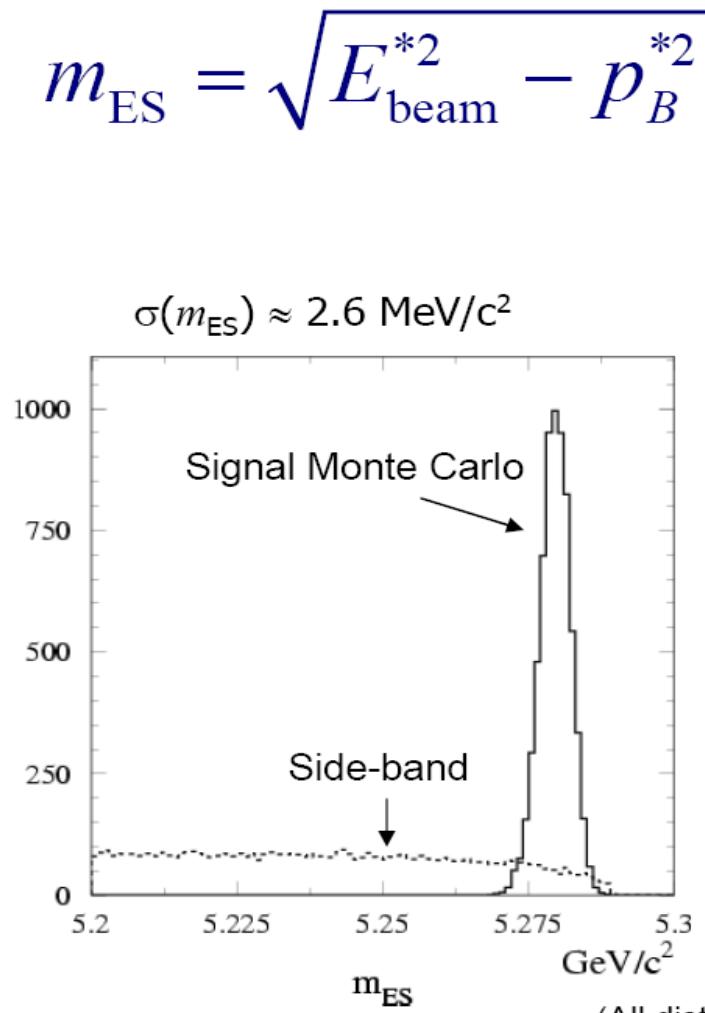
- Difference between total reconstructed and expected energy
 - Dominated by detector energy resolution
 - Resolution depends on particles in final states

Kinematic Variables

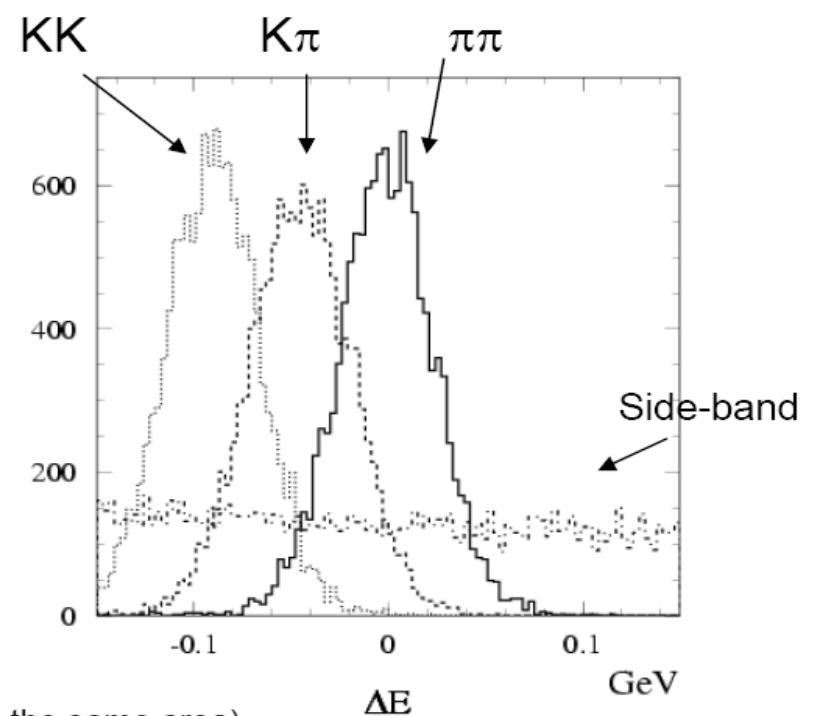
$$\Delta E = E_B^* - E_{\text{beam}}$$

- Dominated by tracking resolution
- Assume π mass for tracks
- Momentum dependent shift for $K\pi$ and KK

$$\sigma(\Delta E) \approx 26 \text{ MeV}$$



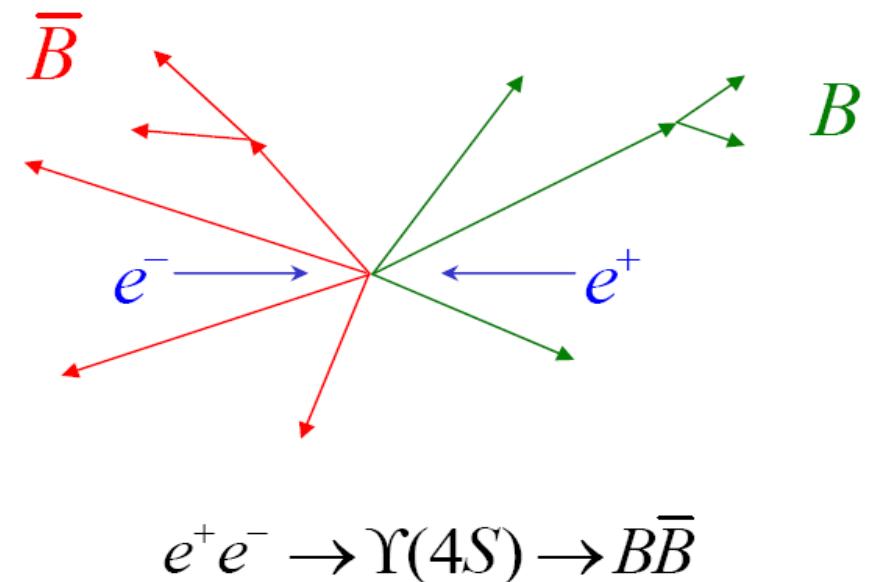
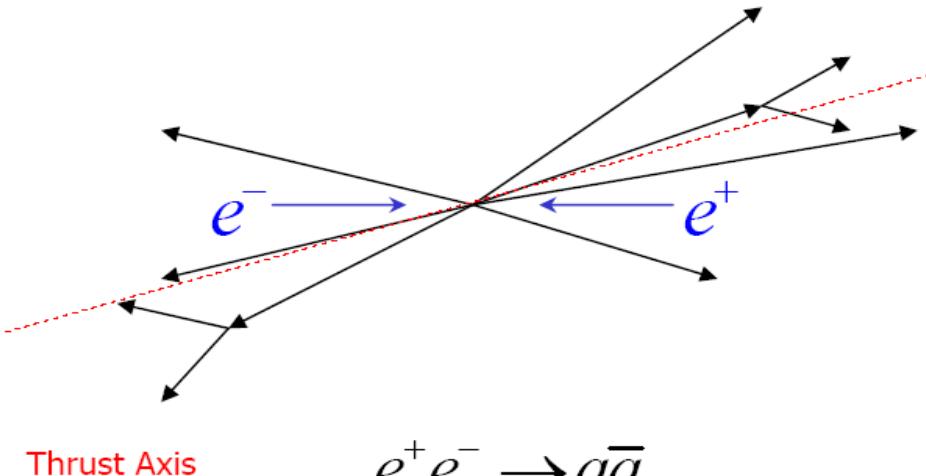
(All distributions are normalized to the same area)



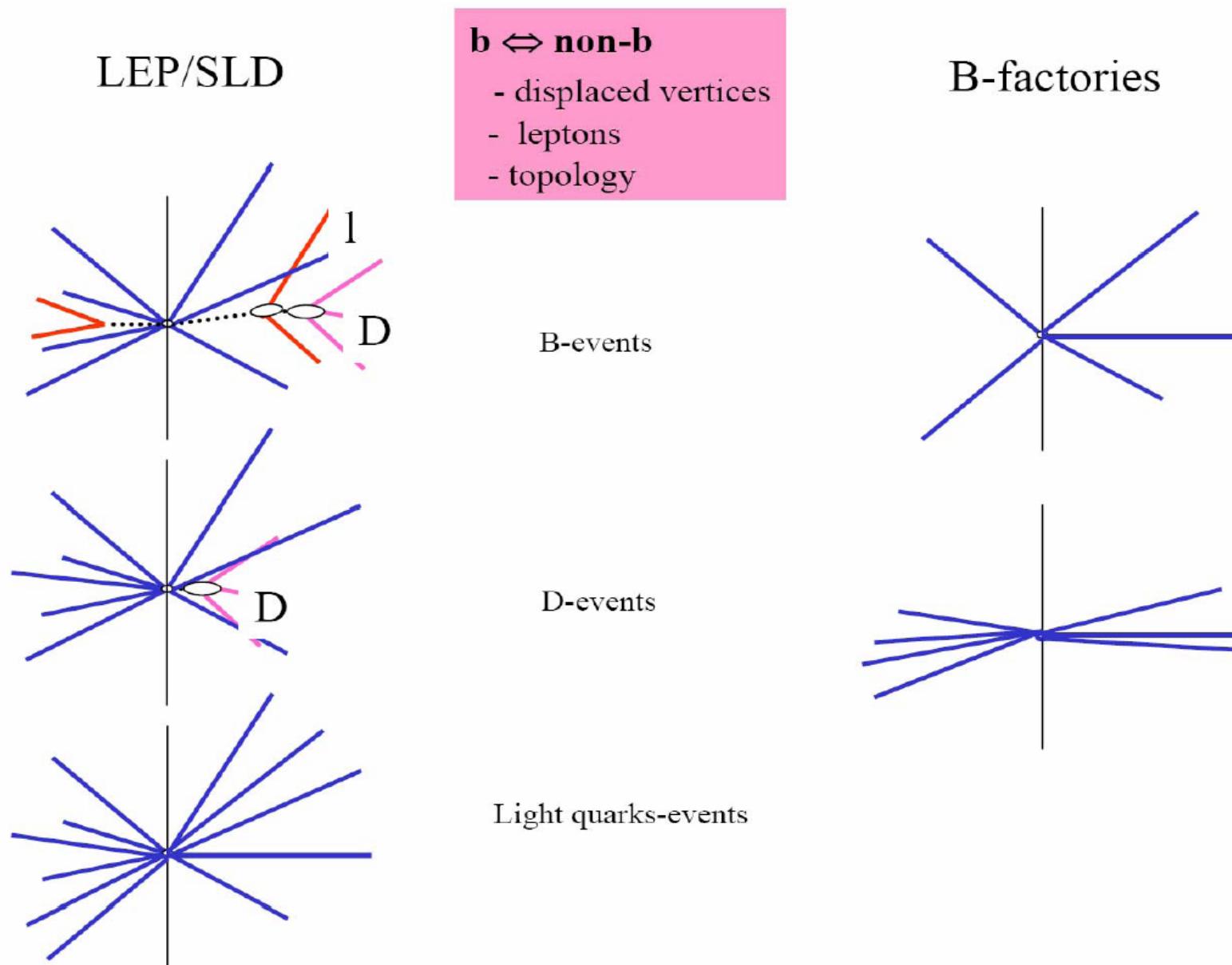
Continuum Background Rejection

- Main source of background: continuum $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$)
 - Branching fraction of interesting B decays $\leq 10^{-4}$
 - Branching fraction of D decays: $\simeq 10^{-2}$
- Overall branching fraction $< 10^{-6}$

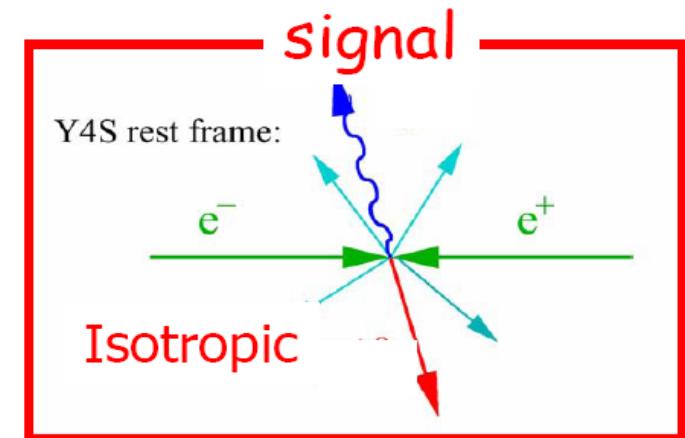
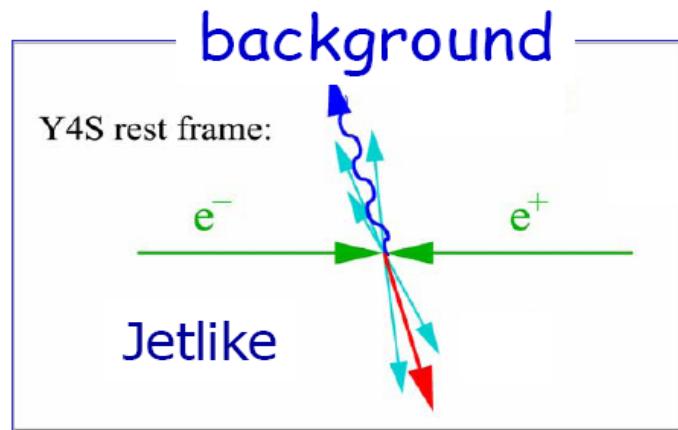
- Distinguish signal and background based on event topology
 - Neutral networks
 - Fisher discriminant



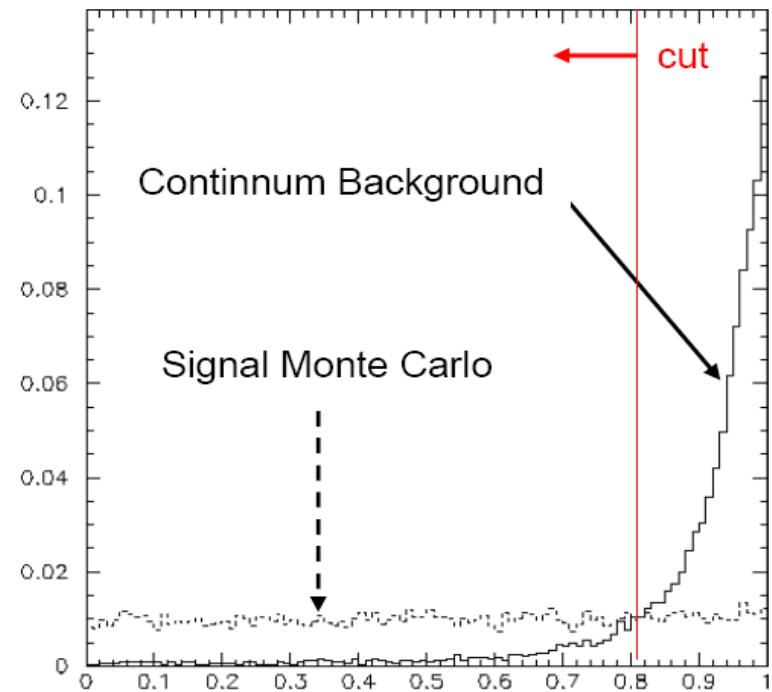
Event Topology at LEP/SLD vs. B Factories



Background Fighting: Sphericity Angle

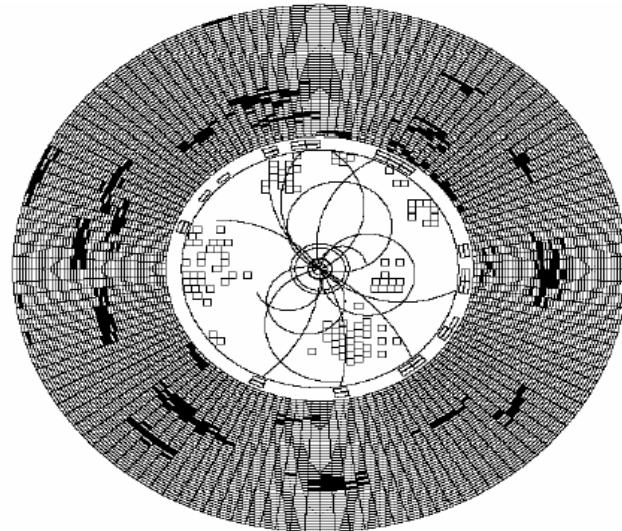


- $\cos\theta_s$: Angle between the B candidate and the rest of event

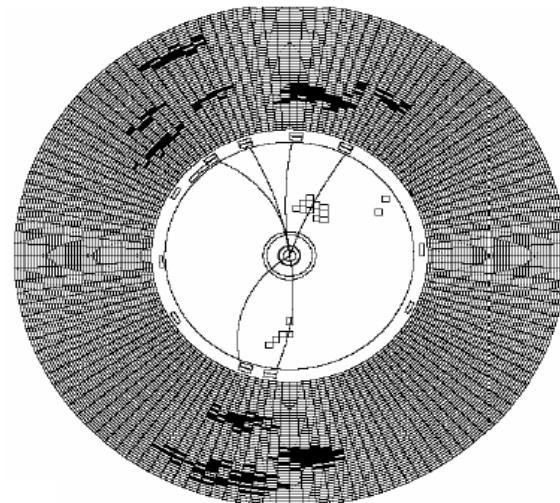


2nd Fox-Wolfram Moment

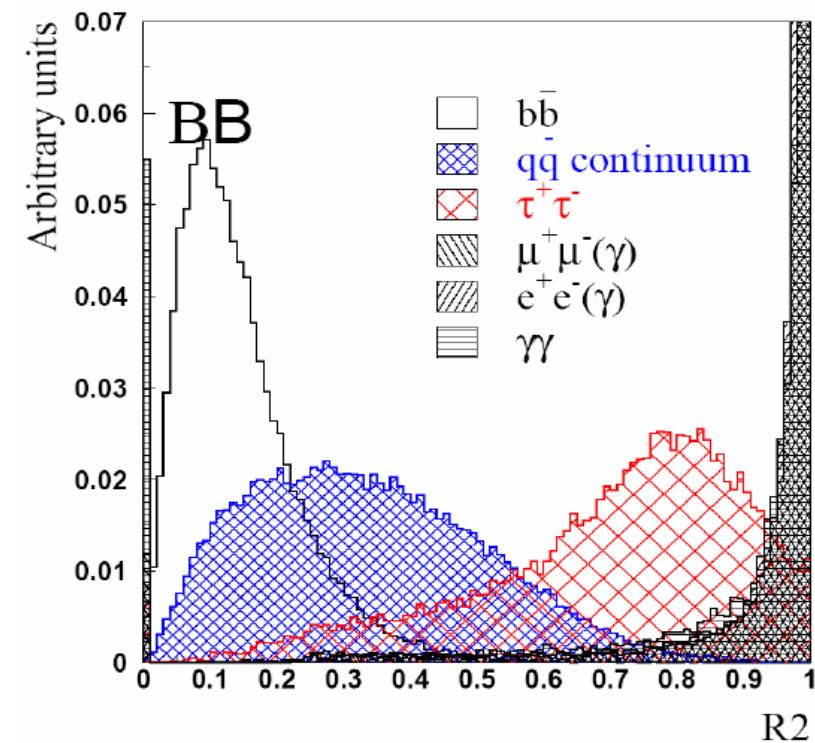
$\Upsilon(4S) \rightarrow B\bar{B}$ Decay



$e^+e^- \rightarrow u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}$ decays

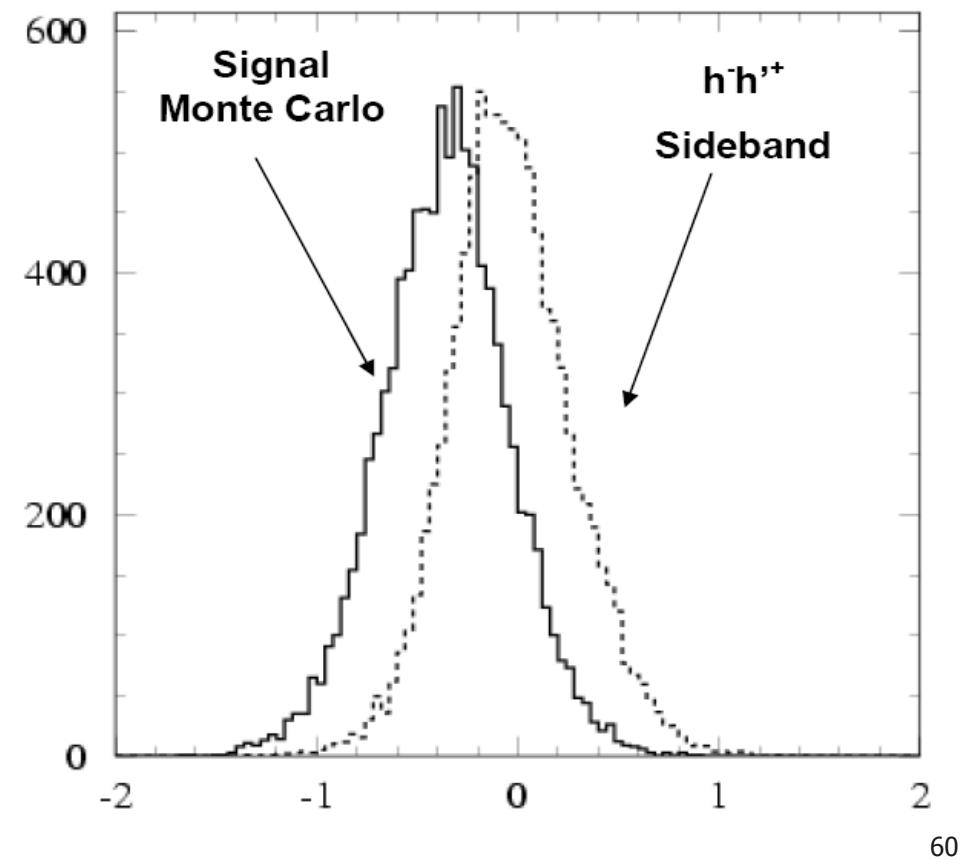
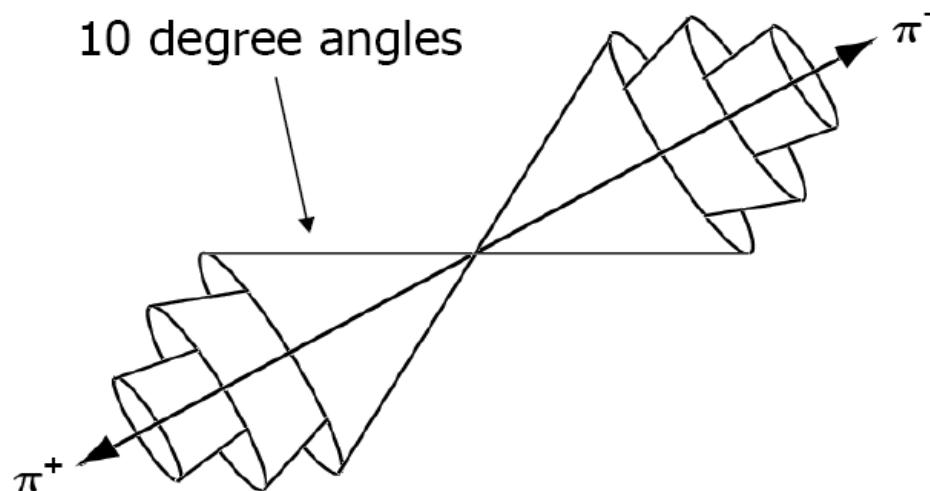


Differences in the event topology (Isotropic B Vs jet-like Continuum) and Energy flow structure in these events used to construct continuum background suppression tools.



Background Fighting: Fisher Discriminant

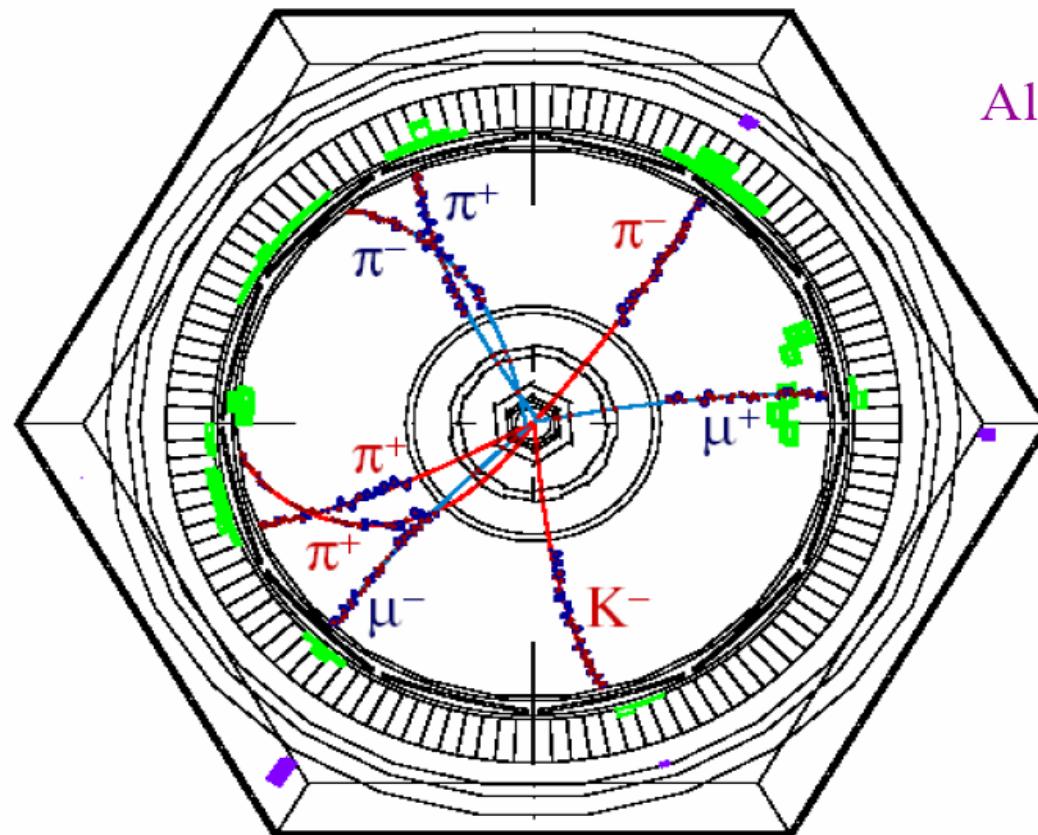
- Optimized linear combination of energy flow into cones about candidates
- Sensitive only to the rest of event
- Studied and calibrated on data: $B \rightarrow D^0\pi^-$, h^-h^+ sideband
- Validated with Monte Carlo



A Completely Reconstructed $\Upsilon(4S)$ event at BaBar

$$\bar{B}^0 \rightarrow D^{*+} \pi^-$$

$$B^0 \rightarrow \psi(2S) K_s^0$$



All particles accounted for
Nothing Missing !

$$D^{*+} \rightarrow D^0 \pi^+$$

$$\psi(2S) \rightarrow \mu^+ \mu^-$$

$$D^0 \rightarrow K^- \pi^+$$

$$K_s^0 \rightarrow \pi^+ \pi^-$$

Measurement of $|V_{ub}|$ and $|V_{cb}|$

$$\mathbf{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & \boxed{V_{ub}} \\ V_{cd} & V_{cs} & \boxed{V_{cb}} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \mathbf{v}_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \boxed{A\lambda^3(\rho-i\eta)} \\ -\lambda & 1-\lambda^2/2 & \boxed{A\lambda^2} \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$|V_{cb}|$ Exclusive

$|V_{cb}|$ from $B^0 \rightarrow D^{(*)} l \bar{\nu}$ Decays

Differential measurement of $B(B^0 \rightarrow D^* l \bar{\nu})$ allows for the extraction of $|V_{cb}|$ through the expression:

$$\frac{d\Gamma}{dw} \propto |V_{cb}|^2 F^2(w) G(w)$$

Form factor of $B \rightarrow D^*$ transition

$$w = v_B \cdot v_D = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}$$

Known kinematic factor

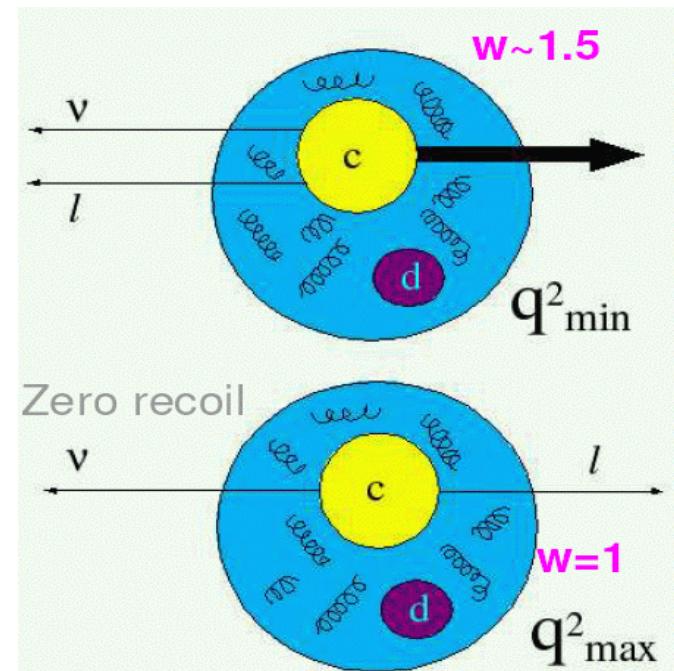
HQET and LQCD provide calculation at zero recoil

In reality the formula is slightly more complicated:

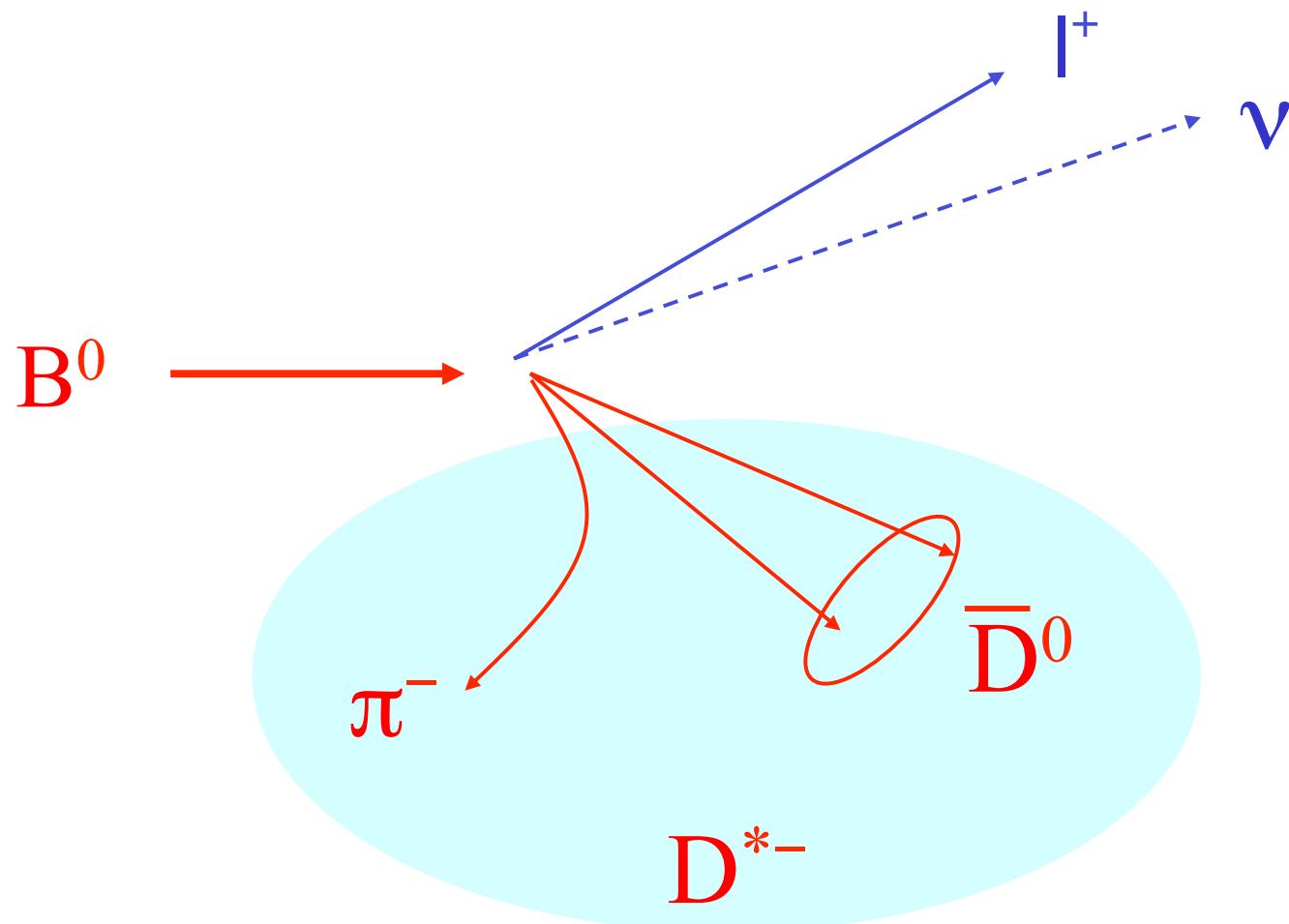
$$F^2(w)G(w) = h_{A_1}^2(w)\sqrt{w-1}(w+1)^2 \left\{ 2 \left[\frac{1-2wr+r^2}{(1-r)^2} \right] \times \left(1 + R_I(w)^2 \frac{w-1}{w+1} \right) + \left[1 + (1-R_2(w)) \frac{w-1}{1-r} \right]^2 \right\}$$

$$\text{where } r = \frac{M_{D^*}}{M_{B^0}}$$

$F \rightarrow 1$ for $w \rightarrow 1$

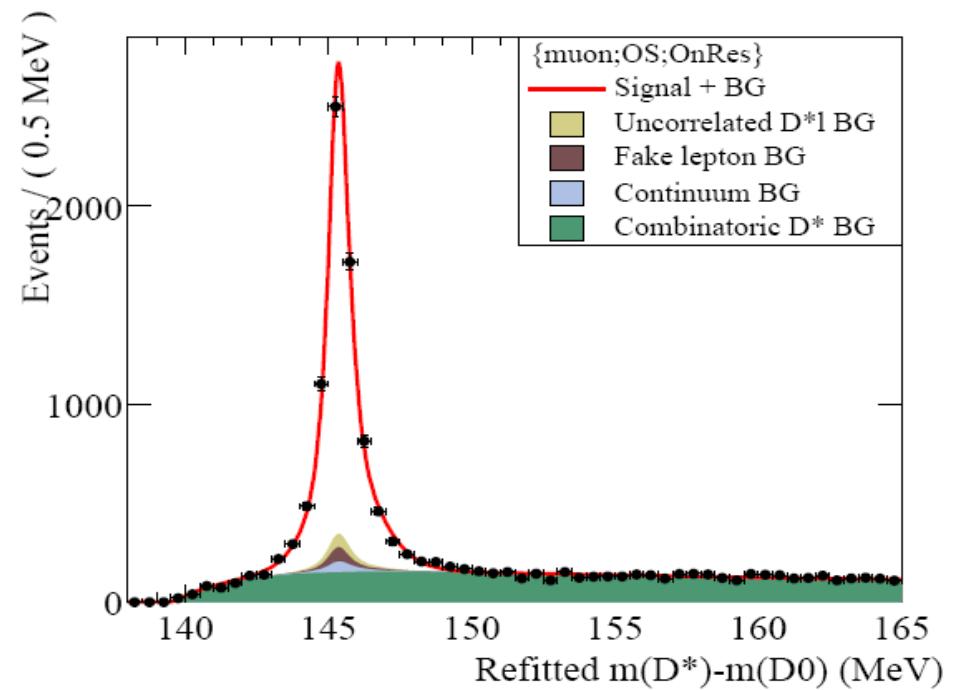
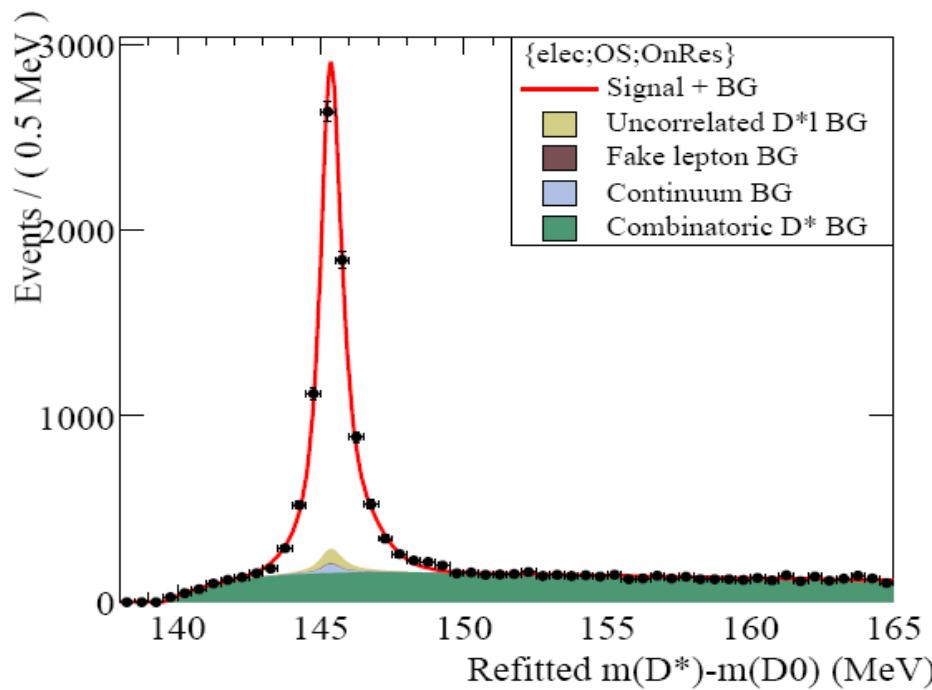


$B^0 \rightarrow D^{(*)} l \nu$ Decays



$D^{*+}-D^0$ mass difference

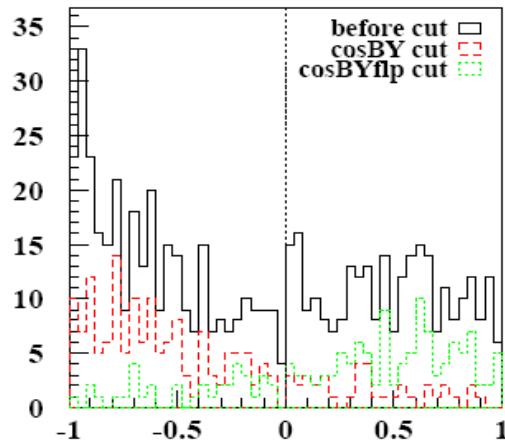
- D^0 mass: 1864 MeV
- D^* mass: 2010 MeV
- Fixed momentum for soft pion
 - Only experimental resolution



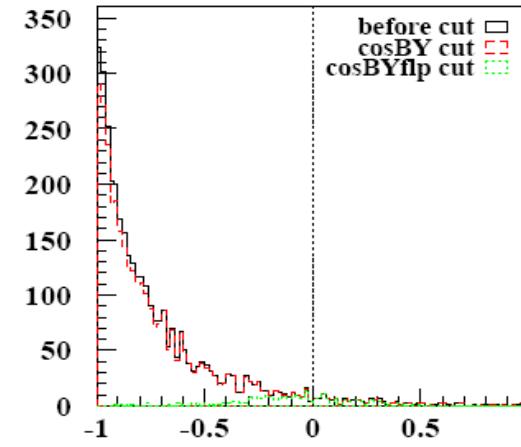
Angular Variables for $B^0 \rightarrow D^{(*)} l \nu$ Selection

- Angle between D^* and lepton

Background

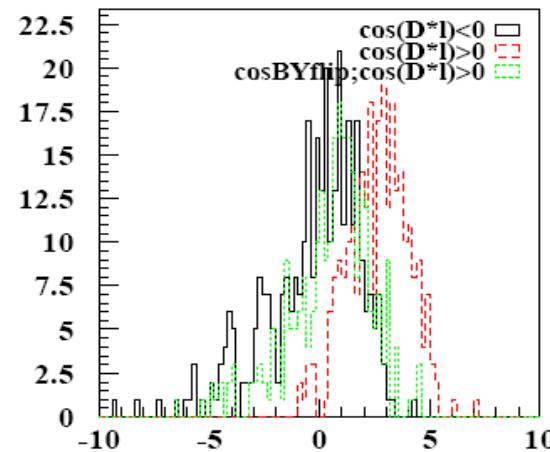


Signal

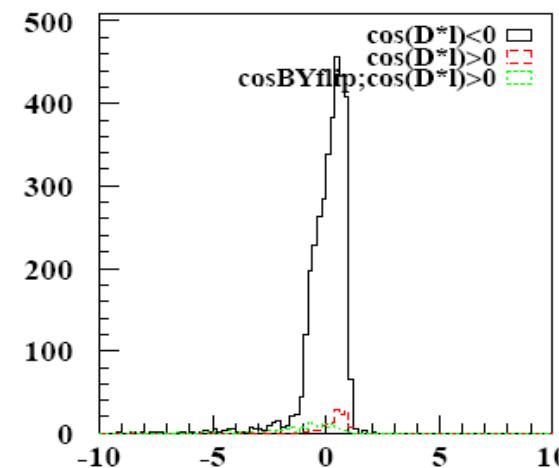


- Angle between B^0 and the $D^* l$ system
 - From kinematic quantities since B direction not measured

Background



Signal



Extraction of $|V_{cb}|$

Measurement:

- Determine number of $B^0 \rightarrow D^{*-} l^+ \nu$ candidates as function of w
- Obtain $h_{A_1}(w)|V_{cb}|$ distribution
- Fit differential spectrum and extrapolate to $w=1$

In BaBar:

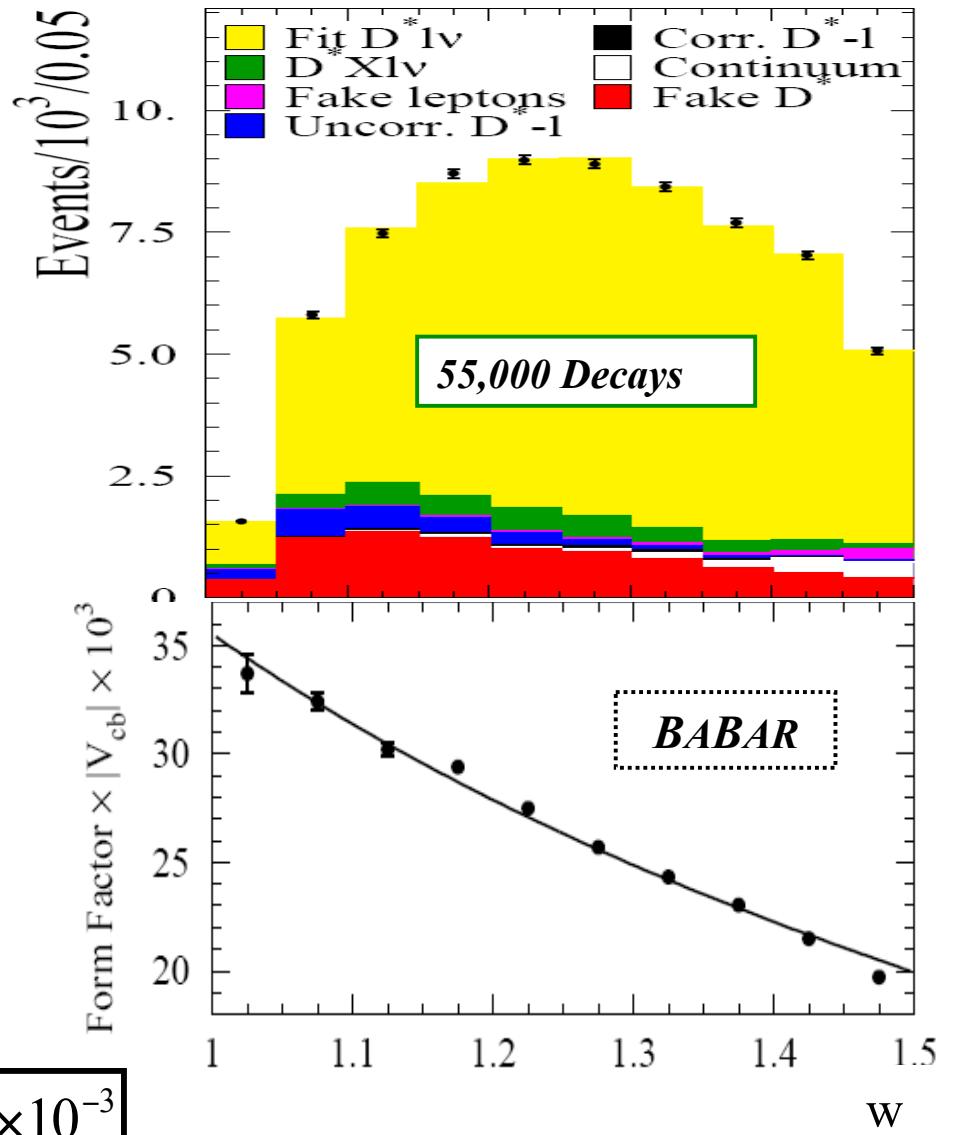
$$h_{A_1}(1)|V_{cb}| = (35.5 \pm 0.3_{\text{stat}} \pm 1.6_{\text{syst}})$$

and using

$$h_{A_1}(w=1) = 0.919^{+0.030}_{-0.035}$$

Hashimoto et al.
PRD 66, 014503
(LQCD)

$$|V_{cb}| = (38.7 \pm 0.3_{\text{stat}} \pm 1.7_{\text{syst}} \pm 1.5_{A_1}) \times 10^{-3}$$

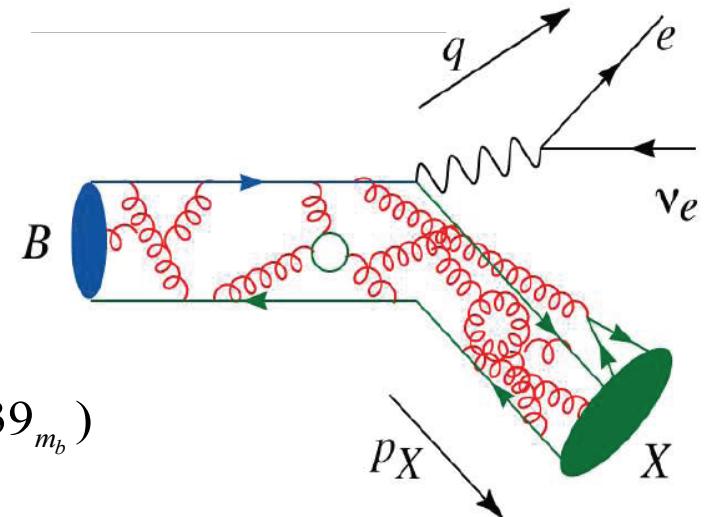


$|V_{ub}|$ Inclusive

$|V_{ub}|$ from Inclusive Semileptonic B Decays

Inclusive charmless semileptonic B decays,
 $B \rightarrow X_u l \bar{\nu}$, allow for the measurement of $|V_{ub}|$

$$|V_{ub}| = 0.00424 \sqrt{\frac{B(B \rightarrow X_u l \bar{\nu})}{0.002} \frac{1.61 \text{ ps}}{\tau_B}} \times (1 \pm 0.028_{\text{OPE}} \pm 0.039_{m_b})$$



Theoretically relatively simple, but

- hadronization effects and Fermi motion of b quark
- non-perturbative parametrizations (Shape Function, SF) affected by large uncertainties

Experimentally challenging, because

- $B \rightarrow X_c l \bar{\nu}$ background (60 times higher) separated in limited region of phase space
- extrapolation to full phase space introduces uncertainties

Lots of Theory Needed as Input for Interpretation

Traditional method:

- * Measure event yield in restricted phase space
- * Extrapolate to full BF (usually DeFazio & Neubert)
- * Full BF $\Rightarrow |V_{ub}|$

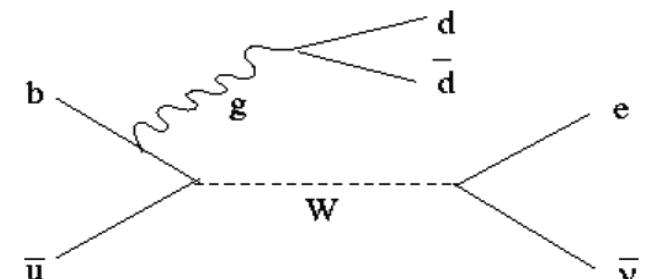
State-of-the-art:

- * Partial BF in restricted phase space ('unfolded')
Not depending on theoretical calculation on the market
- * Direct translation: Partial BF $\Rightarrow |V_{ub}|$ using existing calculations
- * Today: Bosch, Lange, Neubert, Paz (Used for recent results)
Bauer, Ligeti, Luke (Used for M_X - q^2 extraction)
Aglietti, Ricciardi, Ferrera (new; not used yet)

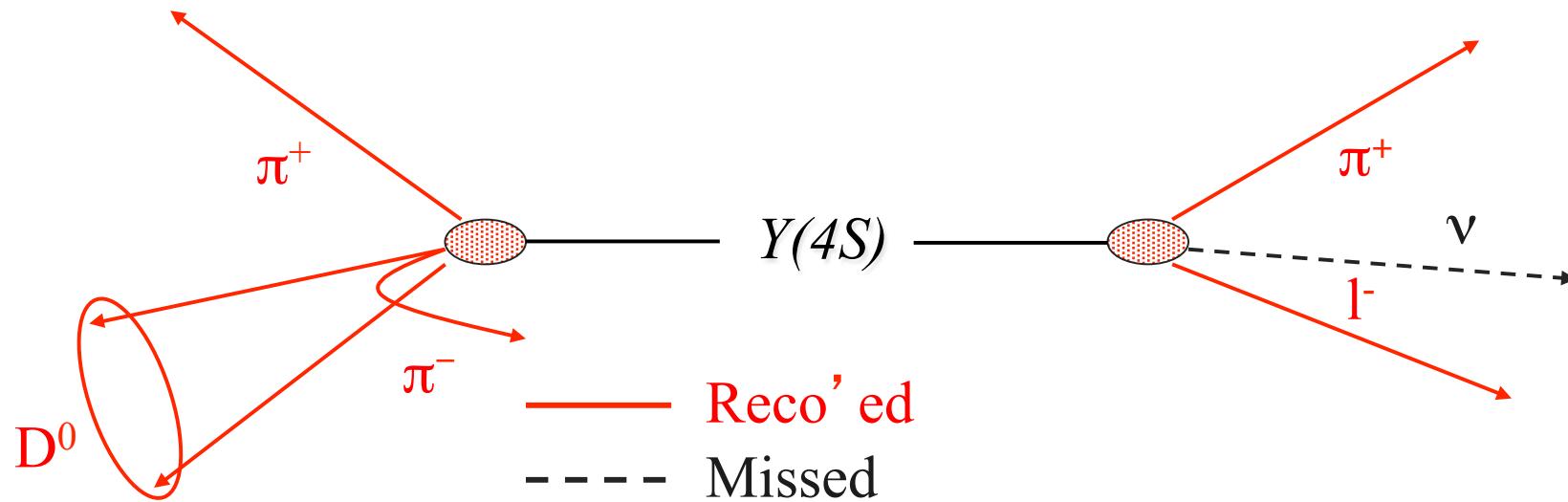
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Additional Uncertainties ('estimated'):

- * Subleading Shape Functions
(contributing differently in $B \rightarrow X_s \gamma$)
- * Weak Annihilation (important @ high q^2)



$|V_{ub}|$ inclusive from m_X and recoil

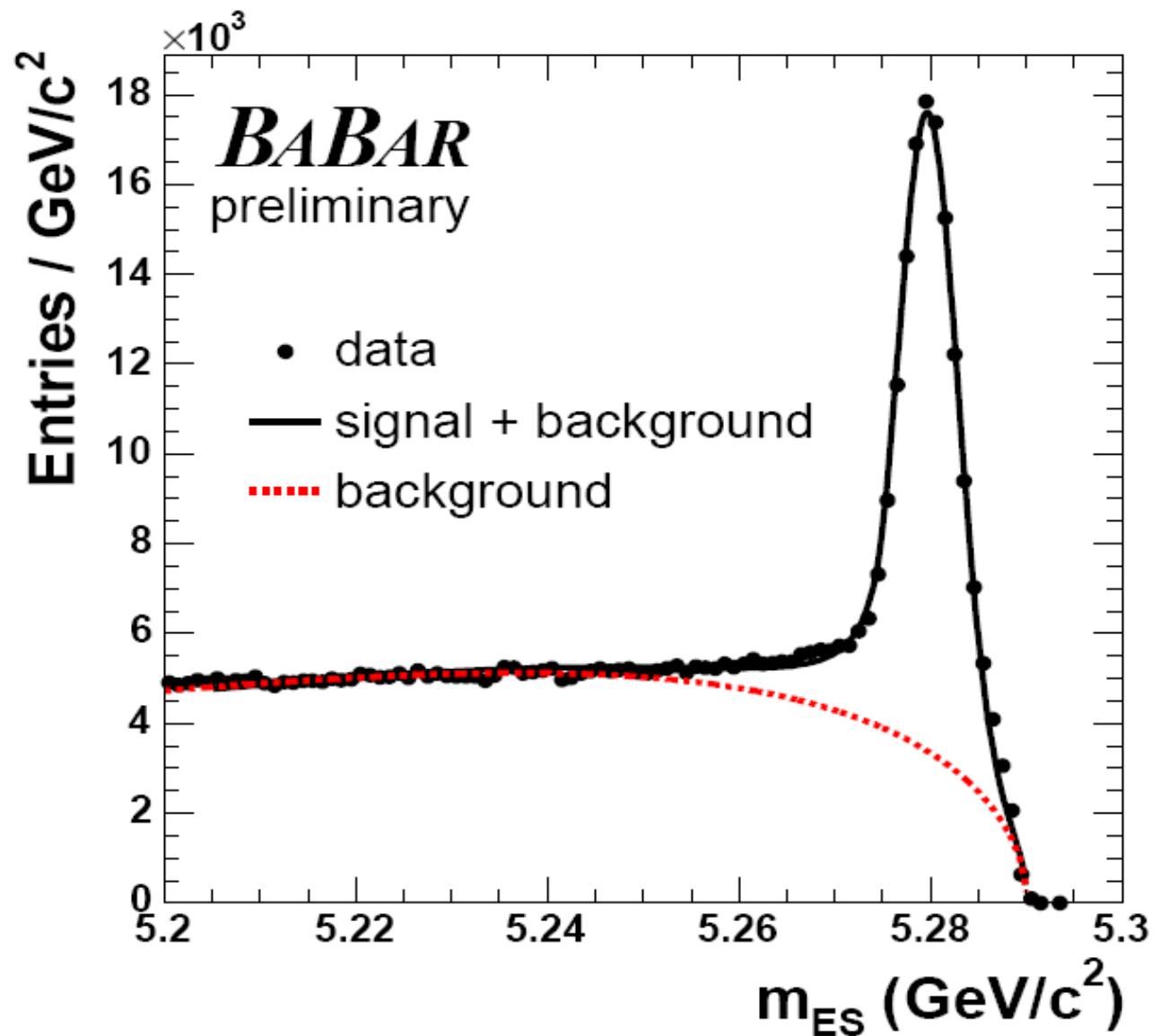


- Background $B \rightarrow X_c l \bar{\nu}$ separated using invariant mass of the X system (M_X)
- Full reconstruction of one B meson to improve signal/background ratio
- Look for semileptonic decay on the recoil side

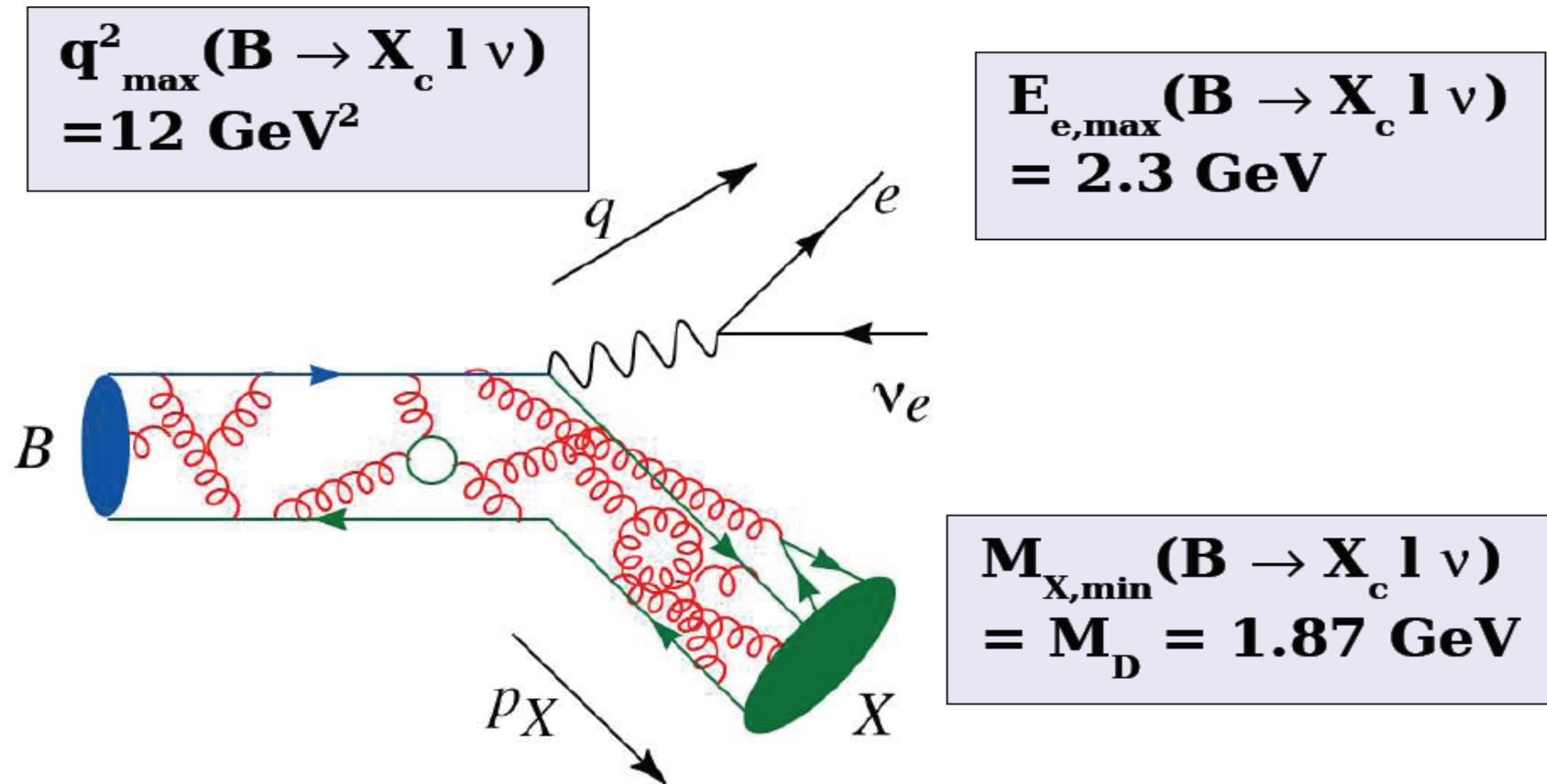
Fully Reconstructed B Mesons with Leptons in Recoil

Fully reconstruct as many decay modes as possible on one side

On the other side use lepton identification to clean up the sample:
 $p^*>0.5$ GeV for electrons
 $p^*>0.8$ GeV for muons

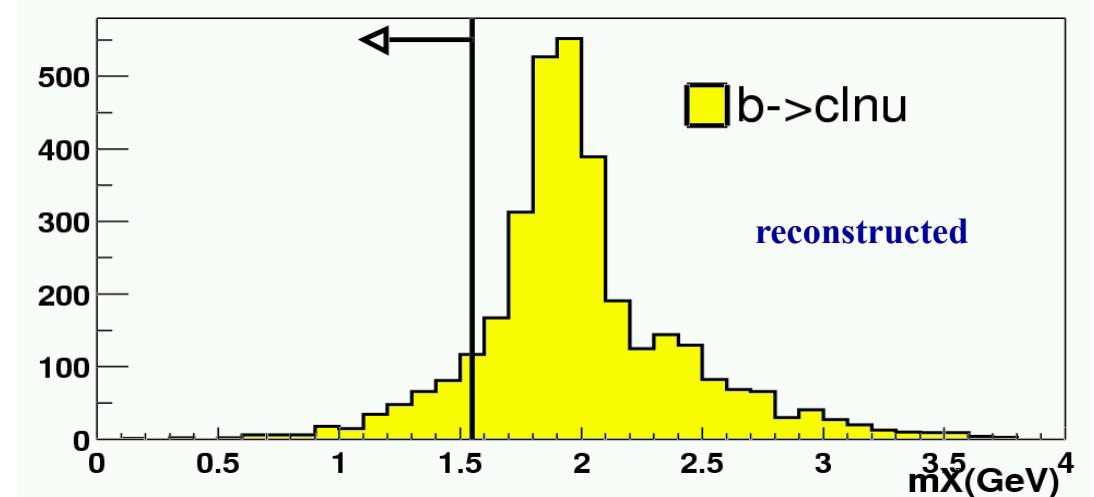


Kinematic Variables Useful for $|V_{ub}|$

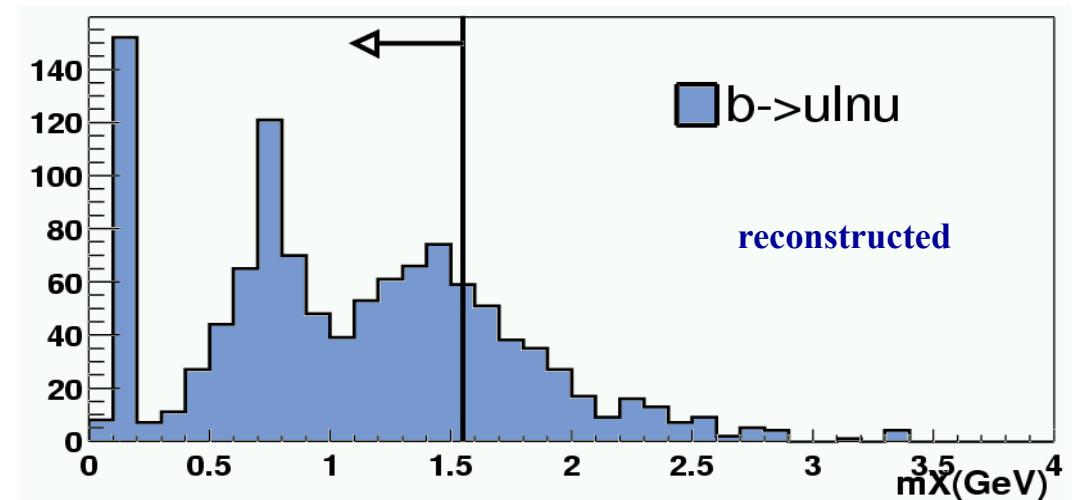


M_X to separate $b \rightarrow c l \bar{\nu}$ from $b \rightarrow u l \bar{\nu}$

$B \rightarrow D^* l \bar{\nu}$
 $B \rightarrow D^* l \bar{\nu}$
 $B \rightarrow D^{**} l \bar{\nu}$

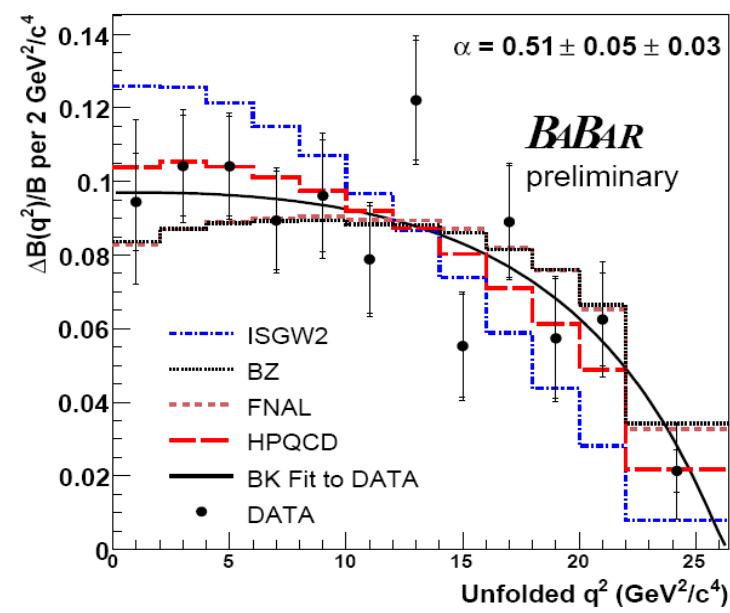
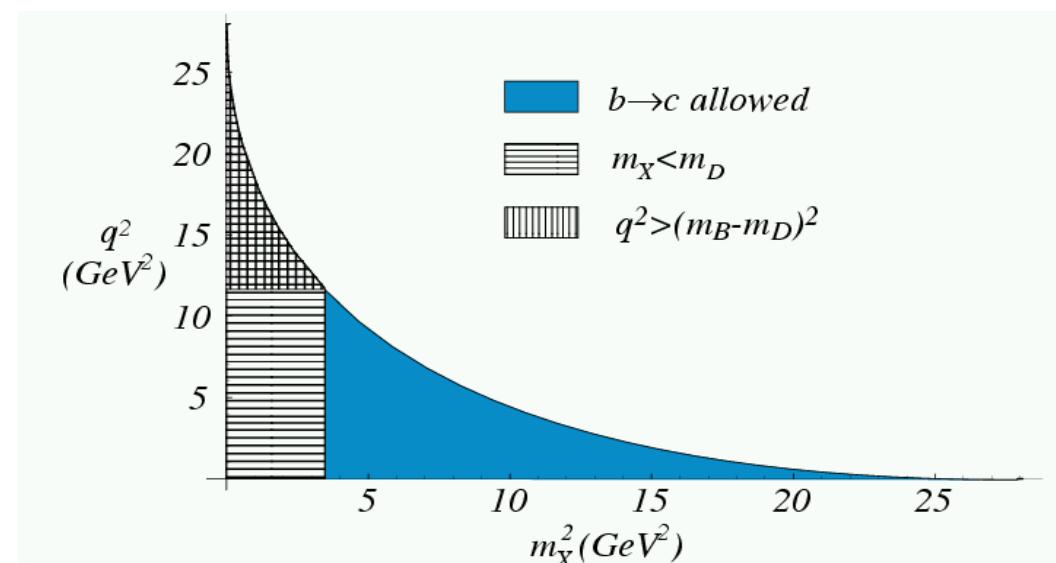
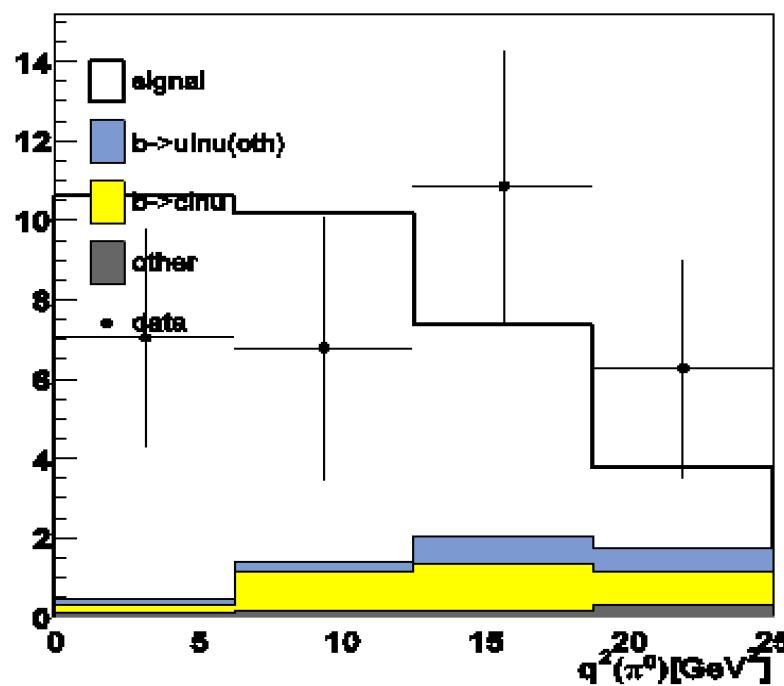
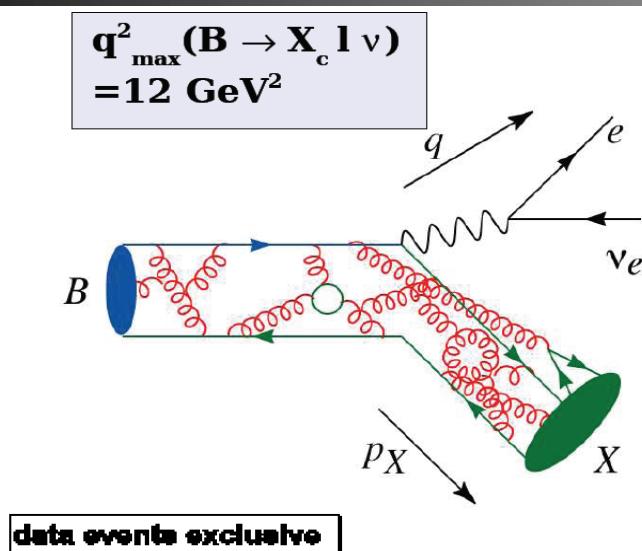


$B \rightarrow \pi l \bar{\nu}$
 $B \rightarrow \rho l \bar{\nu}$
 $B \rightarrow \eta l \bar{\nu}$



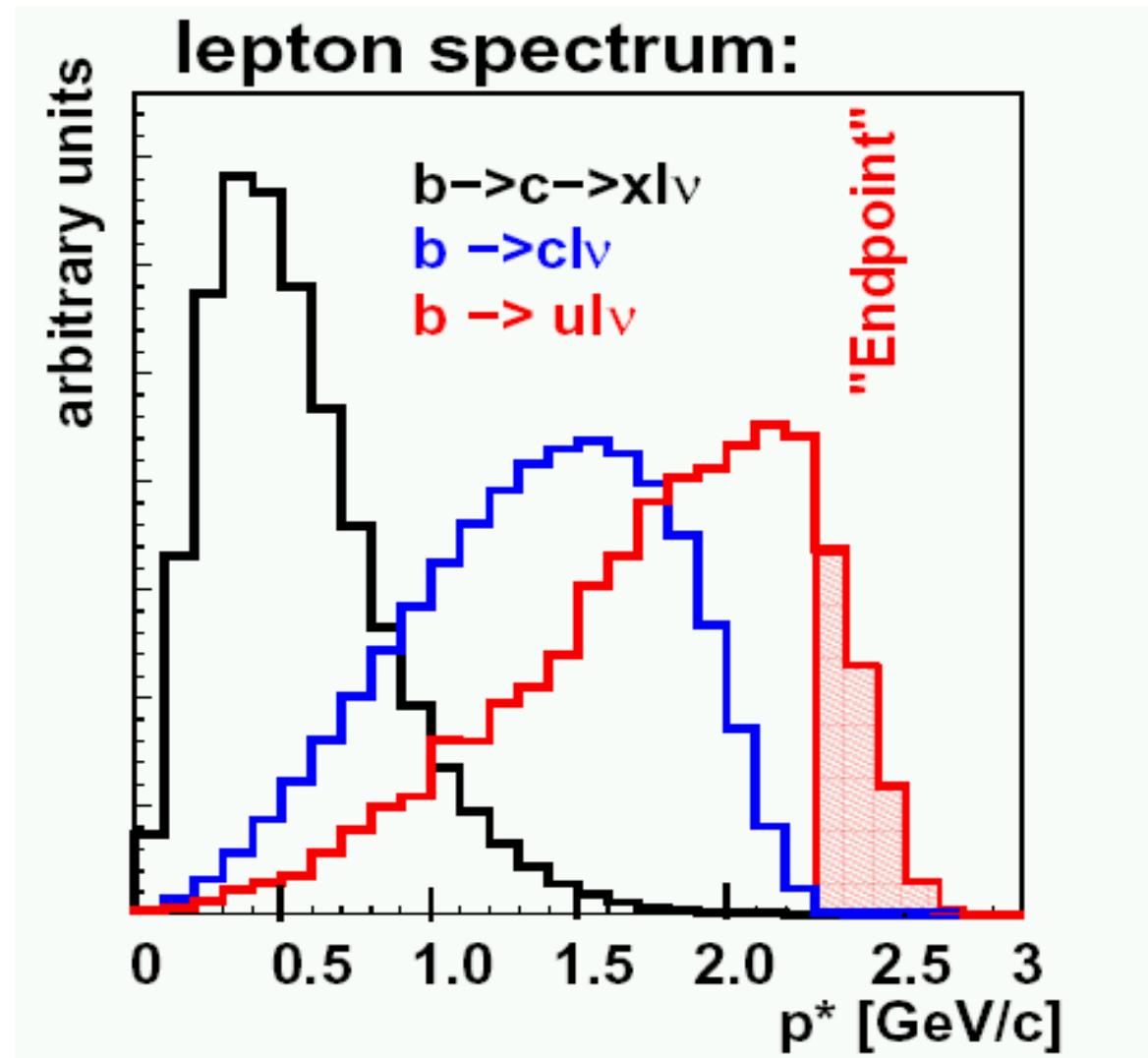
About 60-80% of signal for $M_X < M_D$

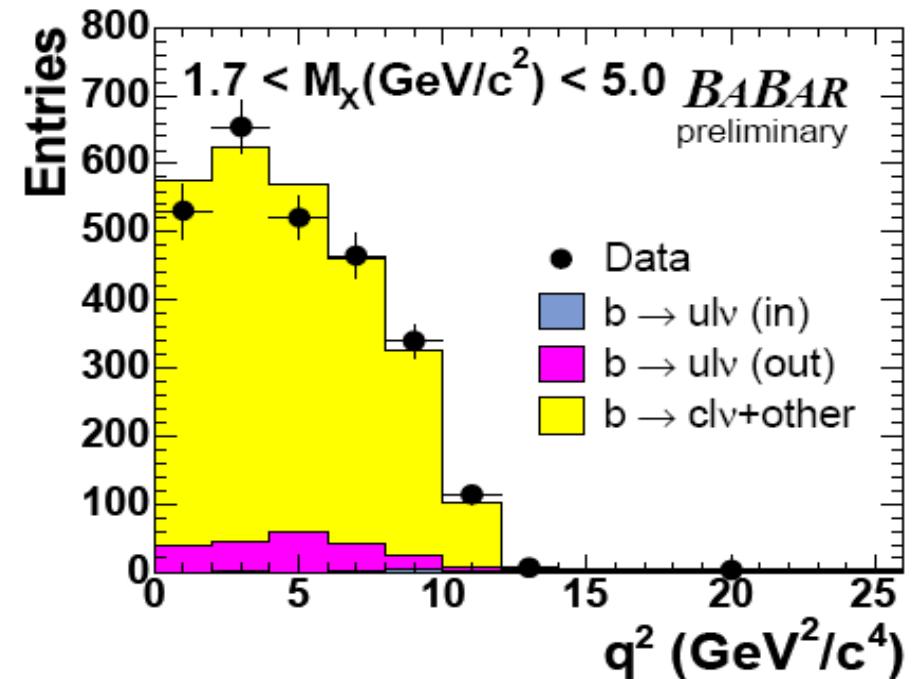
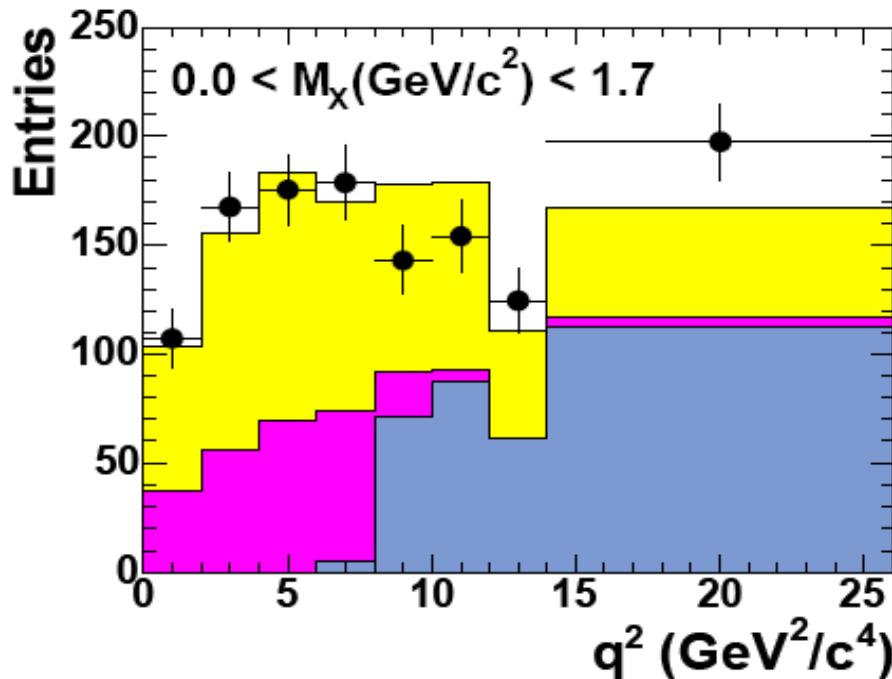
q^2 Spectrum



Lepton spectrum in Semileptonic Decays

- Cascade (secondary) leptons have lower momentum
- Requirement on momentum of leptons provides good separation between signal and cascade leptons
- Higher end of the spectrum mostly from decays mediated by V_{ub}
 - Lighter hadrons allows more momentum for leptons



$|V_{ub}|$ inclusive from m_X vs q^2 

$$|V_{ub}|^{\text{BLL}} = (4.82 \pm 0.26_{\text{stat}} \pm 0.25_{\text{syst}} \pm 0.46_{\text{th+SF}}) \times 10^{-3}$$

$$|V_{ub}|^{\text{BLNP}}_{\text{Belle } B \rightarrow X_s \gamma} = (5.00 \pm 0.27_{\text{stat}} \pm 0.26_{\text{syst}} \pm 0.46_{\text{SF}} \pm 0.28_{\text{th}}) \times 10^{-3}$$

$$|V_{ub}|^{\text{BLNP}}_{\text{BABAR } b \rightarrow c \ell \bar{\nu}} = (4.65 \pm 0.24_{\text{stat}} \pm 0.24_{\text{syst}}^{+0.46}_{-0.38\text{SF}} \pm 0.23_{\text{th}}) \times 10^{-3}$$

- Results vary slightly depending on particular calculation or experimental input to shape functions
- Uncertainty dominated by theory!

CKM Matrix and CP Violation in Standard Model

CP Violation in Standard Model
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Fisica delle Particelle Elementari, Anno Accademico 2014-15

<http://www.roma1.infn.it/people/rahatlou/particelle/>

Asymmetric Universe of Matter

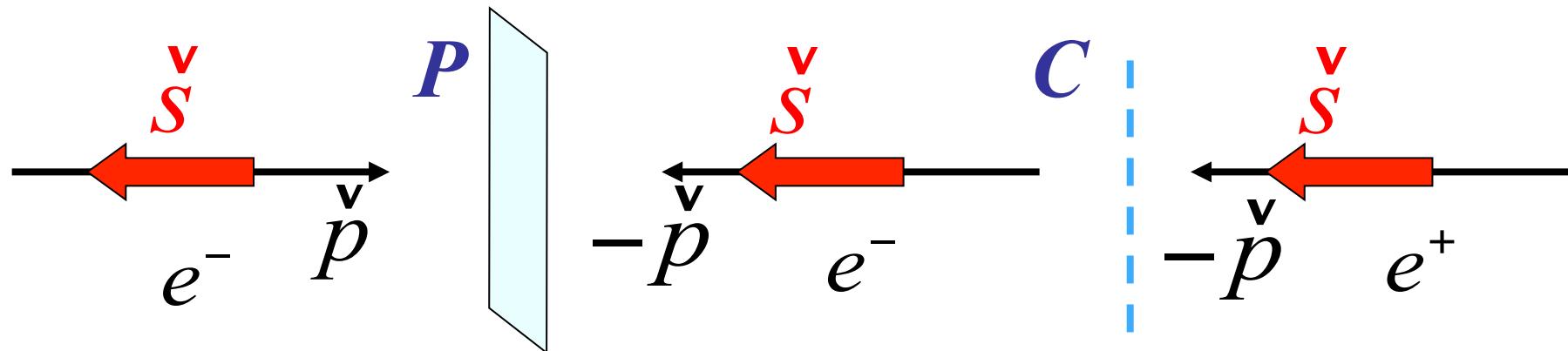
- Universe is very empty but in a biased way

$$\frac{n_{baryon}}{n_{photons}} \approx 10^{-18}$$

$$\frac{N(\text{anti-baryon})}{N(\text{baryon})} \leq 10^{-4} - 10^{-6}$$

- Absence of anti-nuclei amongst cosmic rays in our galaxy
- Absence of intense γ -ray emission due to annihilation of distant galaxies in collision with antimatter galaxies
- The early universe believed to have equal amount of matter and anti-matter
 - What happened to the anti-matter?
- **CP Violation** is one of the three ingredients required to generate such an asymmetry after the Big Bang (A. Sakharov, 1967)
 - Baryon-number violating processes
 - Non-equilibrium state during expansion
 - C and **CP Violation**

C and P Symmetries and Fundamental Interactions



- Parity, P
 - Parity reflects a system through the origin. Converts right-handed coordinate systems to left-handed ones.
 - Vectors change sign but axial vectors remain unchanged
 - $\mathbf{x} \rightarrow -\mathbf{x}$, $\mathbf{L} \rightarrow \mathbf{L}$
- Charge Conjugation, C
 - Charge conjugation turns a particle into its anti-particle
 - $\mathbf{e}^+ \rightarrow \mathbf{e}^-$, $\mathbf{K}^- \rightarrow \mathbf{K}^+$, $\gamma \rightarrow \gamma$

CP Symmetry, particles and anti-particles

- CP symmetry transforms a particle in its anti-particle



- CP is violated IF particles and anti-particles behave differently!

Weak Interactions and Symmetry Violation

- P and C are good symmetries of the strong and electromagnetic interactions
- Parity violation observed in 1957
 - Asymmetry in β decays of ${}^{60}\text{Co} \rightarrow {}^{60}\text{Ni} + e^- + \nu$
 - Electrons produced mostly in one hemisphere
- Charge-conjugation violation 1958
 - Only left-handed neutrinos and right-handed anti-neutrinos
- CP believed to be a good symmetry, but ...

A Shocker : Weak Interaction Violates Parity !

1956

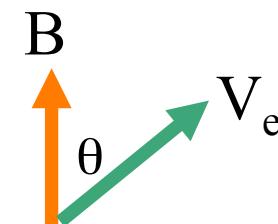
Observation of a spatial asymmetry in
the β -decay electrons from ${}^{60}\text{Co} \rightarrow {}^{60}\text{Ni} + e^- + \nu$



C.S.Wu

- Cold ${}^{60}\text{Co}$ inside a Solenoidal B Field
- ${}^{60}\text{Co}$ nuclei spin aligned with B field direction
- ${}^{60}\text{Co}$ undergoes β decayelectron emitted
- Measure electron intensity w.r.t B field dir.
- Result: Electrons preferentially emitted in opposite spin direction

$$I(\theta) = 1 - \frac{V_e}{c} \cos \theta$$



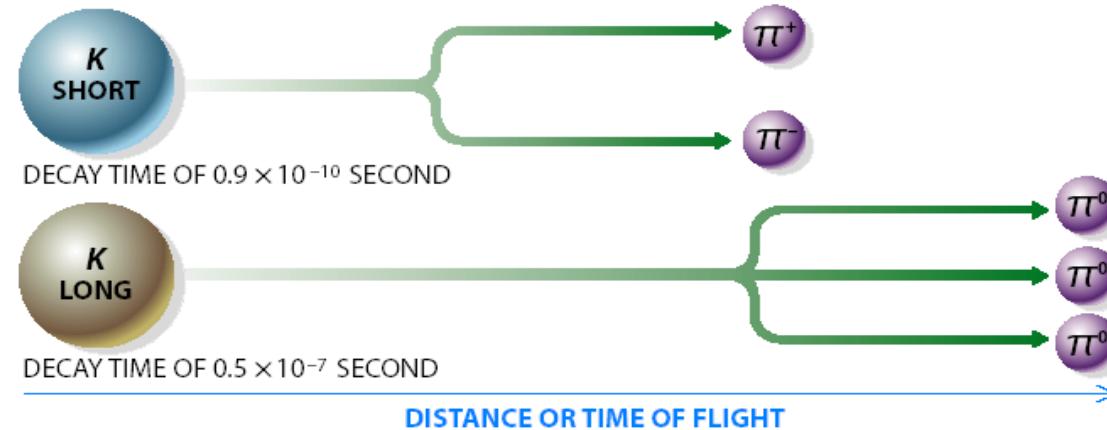
asymmetry of intensity \rightarrow Weak interaction violated Parity

CP Violation in Kaons

- CP conservation implies

$$\text{CP} = +1$$

$$\text{CP} = -1$$



- CP violation in kaons observed in 1964

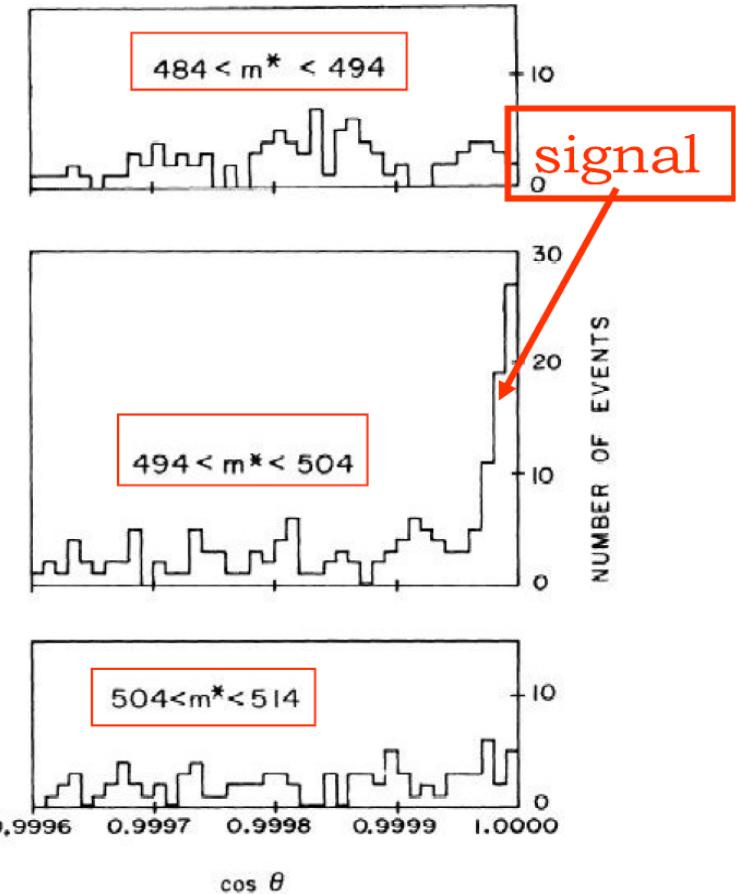
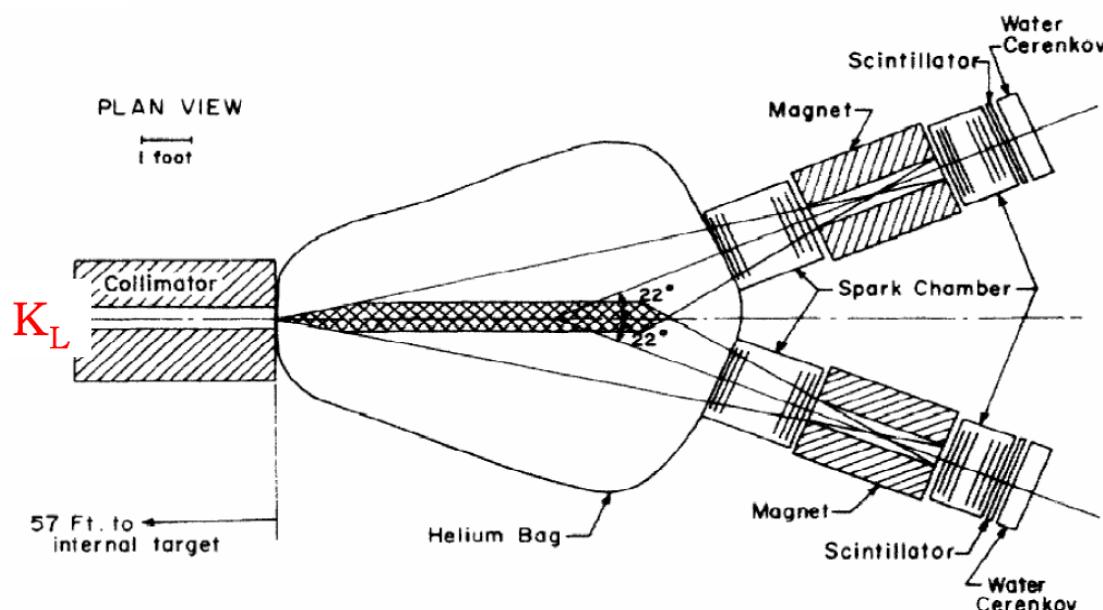
0.2% of
the time!



- No theoretical explanation!

Observation of CP Violation in Kaons

$$\frac{A(|K_L^0\rangle \rightarrow 2\pi)}{A(|K_s^0\rangle \rightarrow 2\pi)} = (2.27 \pm 0.02) 10^{-3}$$



2-body decay : the two π
are back-to-back: $|\cos\theta|=1$

Complex Coupling Constants and CP Violation

| Fermion bilinear | Boson field F | $\mathbf{P} F \mathbf{P}^\dagger$ | $\mathbf{C} F \mathbf{C}^\dagger$ | $\mathbf{CP} F \mathbf{CP}^\dagger$ |
|------------------------------------|--------------------------------|-----------------------------------|-----------------------------------|-------------------------------------|
| $\bar{\psi}\psi$ | Scalar $S^+(t, \vec{x})$ | $S^+(t, -\vec{x})$ | $S^-(t, \vec{x})$ | $S^-(t, -\vec{x})$ |
| $\bar{\psi}\gamma^5\psi$ | Pseudoscalar $P^+(t, \vec{x})$ | $-P^+(t, -\vec{x})$ | $P^-(t, \vec{x})$ | $-P^-(t, -\vec{x})$ |
| $\bar{\psi}\gamma_\mu\psi$ | Vector $V_\mu^+(t, \vec{x})$ | $V_\mu^+(t, -\vec{x})$ | $-V_\mu^-(t, \vec{x})$ | $-V_\mu^-(t, -\vec{x})$ |
| $\bar{\psi}\gamma_\mu\gamma^5\psi$ | Axial $A_\mu^+(t, \vec{x})$ | $-A_\mu^+(t, -\vec{x})$ | $A_\mu^-(t, \vec{x})$ | $-A_\mu^-(t, -\vec{x})$ |

Table 2.1: Properties of charged boson fields and corresponding fermion bilinear terms under \mathbf{P} , \mathbf{C} , and \mathbf{CP} . γ^5 and γ^μ are the Dirac matrices.

Generic interaction lagrangian with vector and axial fields

$$\mathcal{L} = a V_\mu^+(t, \vec{x}) V^{\mu-}(t, \vec{x}) + b A_\mu^+(t, \vec{x}) A^{\mu-}(t, \vec{x}) + c V_\mu^+(t, \vec{x}) A^{\mu-}(t, \vec{x}) + c^* A_\mu^+(t, \vec{x}) V^{\mu-}(t, \vec{x})$$

a, b: real constants
c: complex constant

Lagrangian after CP transformation

$$\mathbf{CP} \mathcal{L} \mathbf{CP}^\dagger = a V_\mu^-(t, -\vec{x}) V^{\mu+}(t, -\vec{x}) + b A_\mu^-(t, -\vec{x}) A^{\mu+}(t, -\vec{x}) + c V_\mu^-(t, -\vec{x}) A^{\mu+}(t, -\vec{x}) + c^* A_\mu^-(t, -\vec{x}) V^{\mu+}(t, -\vec{x}) .$$

Lagrangian invariant under CP **IF AND ONLY IF** $c=c^*$! c must be real

Reminder Kobayashi-Maskawa Mechanism of CP Violation

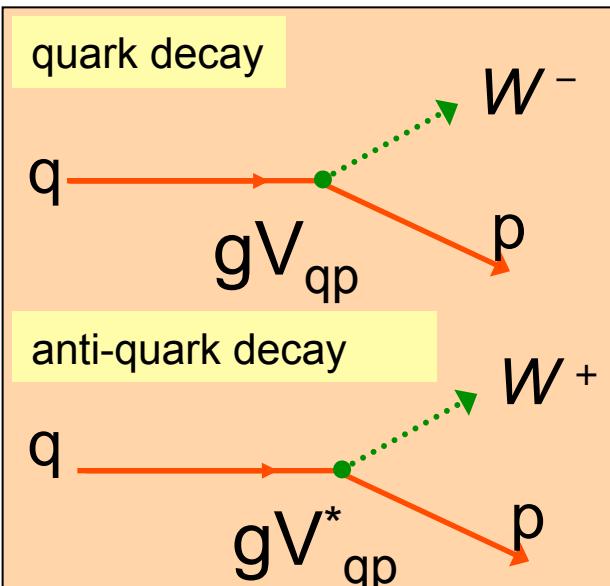
1972



Two Young
Postdocs at that
time !

- Proposed a daring explanation for CP violation in K decay:
- CP violation appears only in the charged current weak interaction of quarks
- There is a single source of CP Violation \Rightarrow **Complex Quantum Mechanical Phase** δ_{KM} in inter-quark coupling matrix
- Need at least **3 Generation of Quarks** (then not known) to facilitate this
- **CP is NOT an approximate symmetry**, $\delta_{KM} \approx 1$, it is **MAXIMALLY violated** !

CKM Matrix Revisited



$$\mathbf{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \mathbf{v}_{CKM} \equiv \begin{pmatrix} 1 & 1 & e^{-i\gamma} \\ 1 & 1 & 1 \\ e^{-i\beta} & 1 & 1 \end{pmatrix}$$

$$\mathbf{V}_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

| Quark families | # Angles | # Phases | # Irreducible Phases |
|----------------|------------|------------|------------------------------------|
| n | $n(n-1)/2$ | $n(n+1)/2$ | $n(n-1)/2 - (2n-1) = (n-1)(n-2)/2$ |
| 2 | 1 | 3 | 0 |
| 3 | 3 | 6 | 1 |
| 4 | 6 | 10 | 3 |

Only Source of
CP Violation
in SM

CP Violation built in the Standard Model through Kobayashi-Maskawa Mechanism!

Only one complex phase! All CP violating effects in SM related to each other
B and K decays CP Violating phenomena are caused by the same complex phase

Unitarity of CKM Matrix

$$V^\dagger V = VV^\dagger = 1$$

- All rows and columns must be orthonormal
 - 3 conditions for diagonal elements
 - 6 conditions for off-diagonal elements

$$\begin{aligned} |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 &= 1 \\ |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 &= 1 \\ |V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 &= 1 \end{aligned}$$

Magnitude of each term

$$\begin{aligned} V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} &= 0 & \lambda \lambda \lambda^5 \\ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} &= 0 & \lambda^3 \lambda^3 \lambda^3 \\ V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} &= 0 & \lambda^4 \lambda^2 \lambda^2 \\ V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} &= 0 & \lambda^3 \lambda^3 \lambda^3 \\ V_{td}^* V_{cd} + V_{ts}^* V_{cs} + V_{tb}^* V_{cb} &= 0 & \lambda^4 \lambda^2 \lambda^2 \\ V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} &= 0 & \lambda \lambda \lambda^5 \end{aligned}$$

Only condition with comparable size of all pieces and involving b decays

Unitarity Triangles

Unitarity condition of CKM Matrix → orthonormality of rows & columns

$$\sum_{(i=u,c,t)} V_{ij} V_{ik}^* = \delta_{jk} ; \quad \sum_{(i=d,s,b)} V_{ij} V_{kj}^* = \delta_{ik}$$

⇒ three conditions are interesting for understanding SM predictions for CP violation

$$V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0,$$

$$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0,$$

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0.$$

Each relation requires sum of three complex quantities to vanish

→ can be represented in the complex plane as a triangle

→ known as Unitarity Triangles

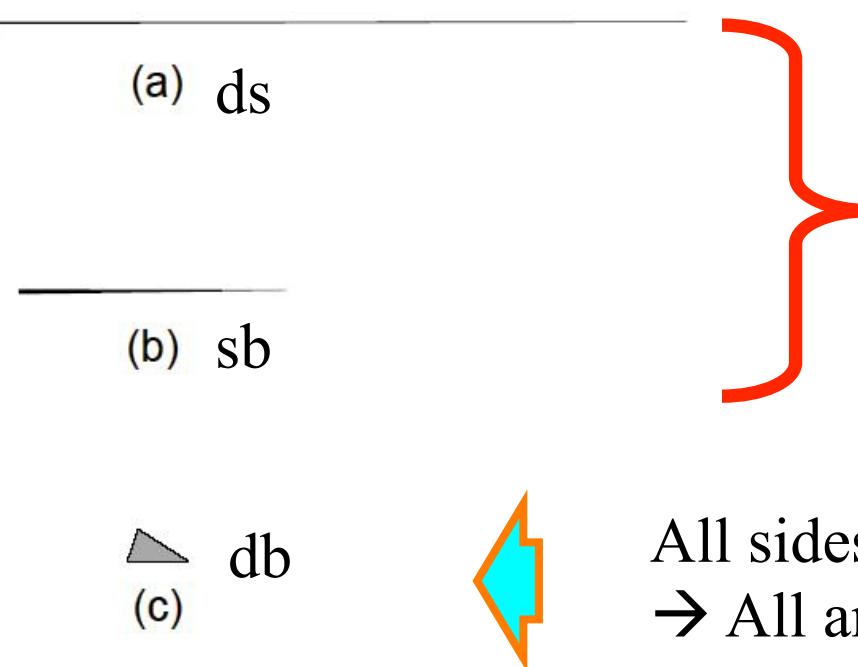
With the knowledge of $|V_{ij}|$ magnitudes, its instructive to draw the triangles

Three Unitarity Triangles Drawn to Common Scale

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0,$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0,$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.$$



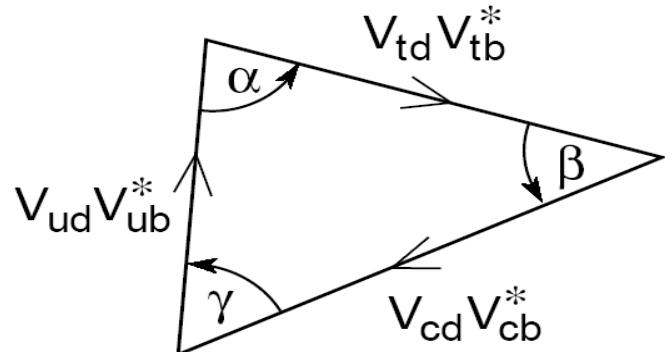
One side is much shorter than the other two → triangle collapses on a line

All sides of comparable length (λ^3)
→ All angles are large

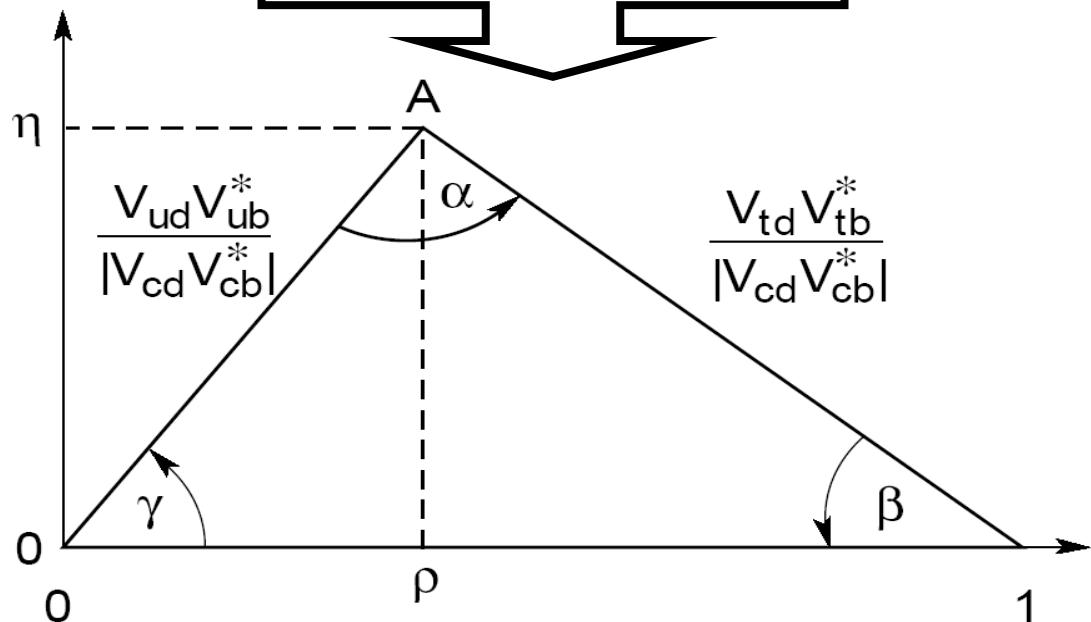
Experimentally ⇒ hard to measure small numbers
easier to measure larger numbers as in (c)

CKM Unitarity Triangle in B Decays

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



Rescaling, aligning



Angles of Unitarity Triangle

$$\alpha = \phi_2 \equiv \arg \left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right],$$

$$\beta = \phi_1 \equiv \arg \left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right],$$

$$\gamma = \phi_3 \equiv \arg \left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right]$$

All lengths involve b decays
Large CP Asymmetries predicted , \propto UT angles