NEUTRINO PHYSICS

Neutrino Oscillation Theory

DIPARTIMENTO DI FISICA

SAPIENZA
UNIVERSITÀ DI ROMA

Fisica delle Particelle Elementari, Anno Accademico 2015-16

http://www.roma1.infn.it/people/rahatlou/particelle

REFERENCES

- Lectures:
  - **E.Lisi "*Neutrinos:theory and phenomenology*":** http://www.lnf.infn.it/conference/lnfss/14/pageparts/schedule/scheduleMON.php
  - E.Lisi *"Physics of Massive neutrinos"*: http://www.pd.infn.it/~laveder/unbound/scuole/2006/bari-2006/dottorato/
  - L. Ludovici "*Lezioni sulle oscillazioni di neutrino*": http://www.roma1.infn.it/people/longo/corso.html
  - International Neutrino School 2014: https://indico.cern.ch/event/300715/contributions

- Review:
  - PDG review 2014: http://pdg.lbl.gov/2014/reviews/rpp2014-rev-neutrino-mixing.pdf

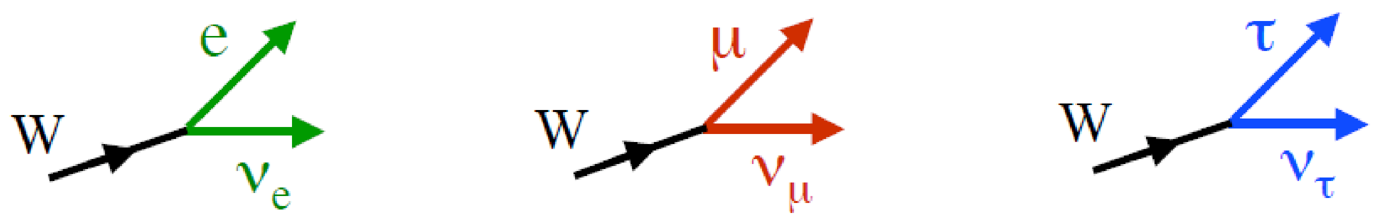
- Conference:
  - Neutrino 2014: https://indico.fnal.gov/conferenceOtherViews.py?view=standard&confId=8022

- Web page:
  - Several articles and recent results: http://www.nu.to.infn.it/

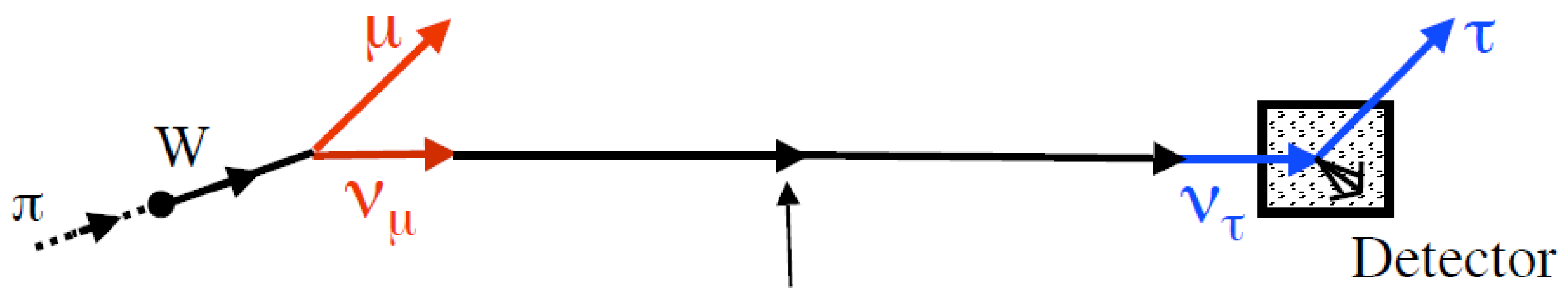
- Book:
  - D. Giunti, C.W. Kim: *"Fundamentals of Neutrino Physics and Astrophysics"*, Oxford University Pres

WHY NEUTRINO PHYSICS?

- Neutrino flavour:



- Experimental observation (1998-today):



- Implications:

Neutrino flavour change

  - Neutrinos mix: lepton flavour not conserved
  - Neutrinos have non zero mass: there must be some ν mass spectrum

- Mixing angles?
- Neutrino masses?
- Dirac or Majorana ?
- CP violation in leptonic sector? Leptogenesis?

NEUTRINOS IN THE STANDARD MODEL

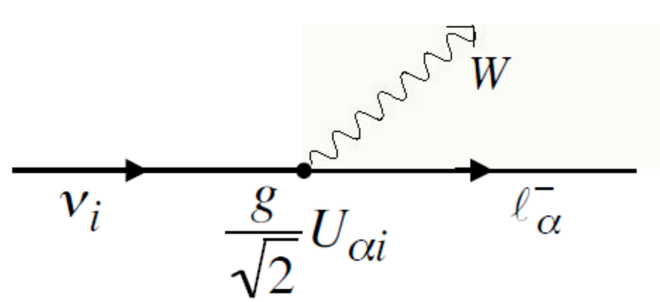
- Fermions described by a Dirac field Ψ
- Standard Model(MS): chiral theory $SU(2)_L \times U(1)_Y$
  - Chirality projector $P_L^{(R)}=(1\pm\gamma_5)/2$, $P_L^{(R)}\Psi=\Psi_L^{(R)}$
  - Ψ_L and Ψ_R have different properties under $SU(2)_L$
    - Ψ_R SU(2) singlet $\Rightarrow$ doesn't couple with W,Z bosons
    - Ψ_L SU(2) doublet
    - Mass term (after EW symmetry breaking): $m\overline{\Psi}_L \Psi_R$ + h.c. $\Rightarrow$mix Ψ_L and Ψ_R
- Minimal Standard Model(MMS):
  - neutrinos are massless!
  - there are only 3 neutrinos lighter than $M_Z/2$
- Neutrino interaction:
  - Charged current: $$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\sum_{\alpha=e,\mu,\tau}(\bar{\ell}_{L\alpha}\gamma^\lambda\nu_{L\alpha}W_\lambda^- + \overline{\nu}_{L\alpha}\gamma^\lambda\ell_{L\alpha}W_\lambda^+)$$
  - Neutral current: $$\mathcal{L}_Z = -\frac{g}{cos\theta_W}\sum_{\alpha=e,\mu,\tau}\overline{\nu}_{L\alpha}\gamma^\lambda\nu_{L\alpha}Z_\lambda$$

NEUTRINO MIXING

- **Higgs mechanism** (like in the quark sector):
  - Introduce Dirac Mass Term and diagonalize the mass matrix
  - Unitary matrix appears in Interaction Lagrangian
  - Neutral current not affected: GIM mechanism

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau \, i=1,2,3} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \bar{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

- $A(W^+ \rightarrow l_\alpha + \nu_i) = g/\sqrt{2} U_{\alpha i}^*$



- Orthogonality: 3 flavors ⇒ at least 3 mass eigenstates

$$|\nu_\alpha\rangle = \sum_{i=1,2,3} U_{\alpha i}^{*} |\nu_i\rangle \qquad |\nu_i\rangle = \sum_{\alpha=e,\mu,\tau} U_{\alpha i} |\nu_\alpha\rangle$$

- Flavour fraction of $|\nu_i\rangle = |U_{\alpha i}|^2$

> ν field: creates $\bar{\nu}$ and destroys ν
> This is why U* appear when ket |ν⟩ used

NOTATION FOR MIXING

- Three flavor states $\nu_e\ \nu_\mu\ \nu_\tau$ mixed with mass states $\nu_1\ \nu_2\ \nu_3$:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad \nu_\alpha = U_{\alpha i}\,\nu_i$$

- If these are the only ν states in nature, the matrix U is unitary :
  $UU^{+} = I$ (although $U \neq U^{*}$ in general)

- For antineutrinos, $U \to U^{*}$ (see also next exercise)

- As for quarks, the unitary mixing matrix U can be expressed in terms of four independent physical parameters :
  3 mixing angles + 1 CP-violating phase

from E.Lisi "Neutrinos: theory and phenomenology"

NOTATION FOR MIXING

- The Particle Data Group notation is currently "universal"

$$U = O_{23}\,\Gamma_\delta\, O_{13}\,\Gamma_\delta^\dagger\, O_{12}$$

← with $\Gamma_\delta =$ diag$(1, 1, e^{i\delta})$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

← with $c_{ij} = \cos\theta_{ij}$
$s_{ij} = \sin\theta_{ij}$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$$

- U is often called "Pontecorvo-Maki-Nakagawa-Sakata" (PMNS) matrix.

from E.Lisi "Neutrinos: theory and phenomenology"

THE MIXING MATRIX

- Experimentally, we now know that:

$$U = (23)\quad(13)\quad(12)$$

$\sin^2\theta_{23} \sim 0.5$ ~maximal

$\sin^2\theta_{13} \sim 0.02$ small ($\delta = ?$)

$\sin^2\theta_{12} \sim 0.3$ large

- The presence of two small parameters, $s^2_{13} \sim 0.02$ and $\delta m^2/\Delta m^2 \sim 1/30$, makes 3ν mixing approximately reducible to "effective 2ν mixing" in several cases of phenomenological interest.

- Goal of many current and future experiments is to find evidence of "genuine 3ν effects" beyond 2ν approximations.

from E.Lisi "Neutrinos: theory and phenomenology"

COMPARE WITH QUARKS

- PMNS matrix

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} = \begin{bmatrix} 0.82 \pm 0.01 & 0.54 \pm 0.02 & 0.15 \pm 0.03 \\ 0.35 \pm 0.06 & 0.70 \pm 0.06 & 0.62 \pm 0.06 \\ 0.44 \pm 0.06 & 0.45 \pm 0.06 & 0.77 \pm 0.06 \end{bmatrix}$$

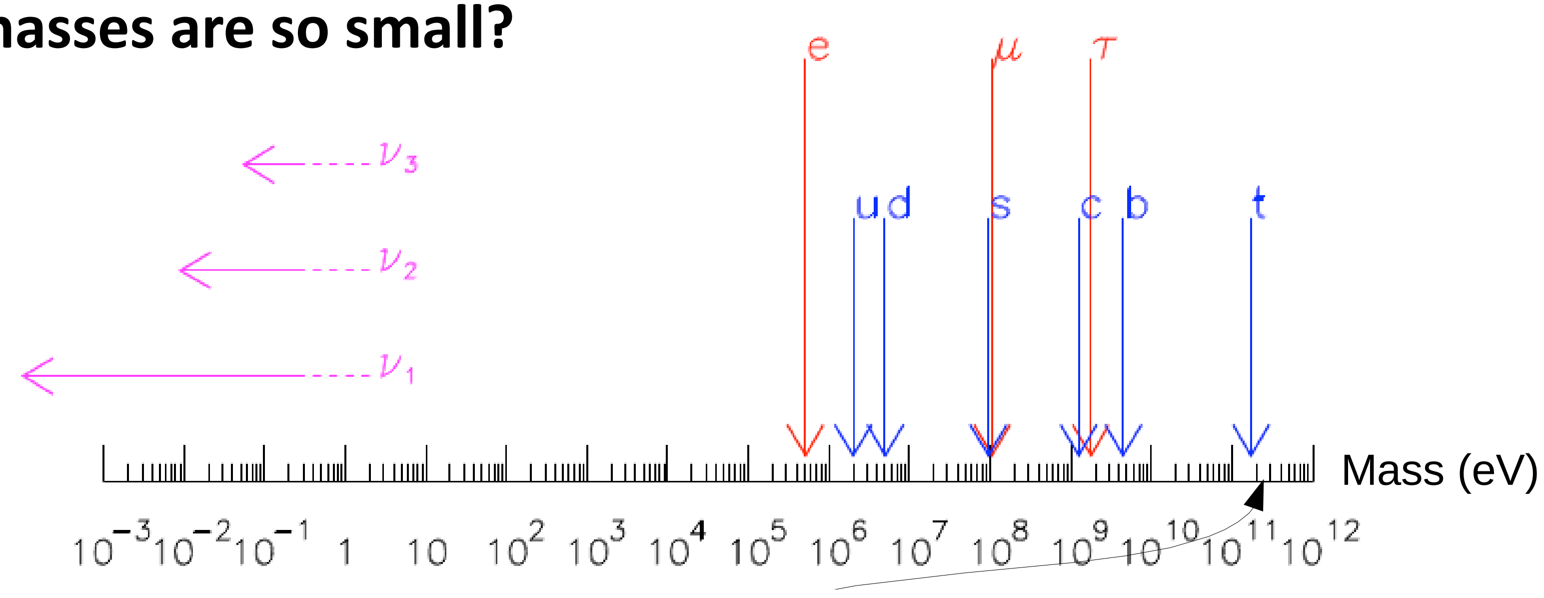
- Compare with the CKM Matrix:

phase and
signs neglected

$$V_{CKM} = \begin{bmatrix} 0.97 & 0.22 & 0.003 \\ 0.22 & 0.97 & 0.04 \\ 0.009 & 0.04 & 0.99 \end{bmatrix}$$

Different than other leptons

- **Why neutrino masses are so small?**



- **Quarks** and **charged leptons** are produced in ordinary processes as **mass eigenstates**
  - One change of basis:mass↔flavour; decay rates $\propto U_{ij}$
- **Neutrinos** are produced and detected as **flavour eigenstates**
  - Two changes of basis:flavour↔mass↔flavour: MQ interference
  - Neutrinos mass difference very tiny→Interference on Macroscopic scale
- **We can produce a ν_μ but not a ν_3**
  - For this reason **unitary triangles,** useful because experiments can measure both sides and angles, **have no practical use in lepton flavour mixing**

⇒ Neutrino Oscillation theory

Neutrino flavour oscillation

$m_i \ll E$ in almost all cases of phenomenological interest. Then:

- set $\beta = v/c \simeq 1$, $x \simeq t$, $\partial_x \simeq \partial_t$
- Chirality flips (LH → RH) of $\mathcal{O}(m_i/E)$ can be ignored

$$\boxed{i \frac{d}{dx} |\nu\rangle = \hat{H} |\nu\rangle}$$

with formal solution $|\nu(x)\rangle = \hat{S}(x,0)\,|\nu(0)\rangle$ where $\hat{S}$ is the evolution operator from 0 to x.

from E.Lisi "Neutrinos: theory and phenomenology"

FLAVOUR EVOLUTION IN VACUUM

- For a ν beam of momentum p traveling in vacuum, in the mass eigenstate basis the $\hat{H}$ matrix reads:

$$E = \sqrt{m_i^2 + p^2} \simeq p + m_i^2/2p$$

$$H_{mass} = \begin{pmatrix} E_1 & & \\ & E_2 & \\ & & E_3 \end{pmatrix} \simeq p \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{1}{2E} \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}$$

$\leftarrow$ diagonal

- However, in flavor basis:

$H_{flavor} = U H_{mass} U^{+}$ $\leftarrow$ non diagonal : flavor <u>not</u> conserved

- We shall work out several consequences of this simple hamiltonian, and then add corrections for propagation in matter. Main output: flavor oscillation probabilities,

$$P(\nu_\alpha \to \nu_\beta) = |S_{\beta\alpha}|^2 \begin{cases} \alpha = \beta : & \text{survival probability} \\ \alpha \neq \beta : & \text{transition probability} \end{cases}$$

from E.Lisi "Neutrinos: theory and phenomenology"

2 NEUTRINOS CASE

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = U \begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$ (components) with $U = U^* = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$ for 2ν. $(U^+ = U^T)$

Mass basis: $$H_{mass} = p\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \frac{1}{2E}\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix} = \frac{1}{2E}\begin{pmatrix}-\frac{\delta m^2}{2} & 0\\ 0 & +\frac{\delta m^2}{2}\end{pmatrix} + \cancel{\text{const} \times \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}}$$

where $\delta m^2 = m_2^2 - m_1^2$

The evolution operator $\hat{S}(x,0)$ formally solves the equation $\hat{H}|\nu\rangle = i\frac{d}{dt}|\nu\rangle$ with initial condition $|\nu\rangle = |\nu(0)\rangle$:

$$|\nu(x)\rangle = \hat{S}(x,0)\,|\nu(0)\rangle \qquad \hat{S} = e^{-i\hat{H}x}$$

Flavor basis: $$S_{flavor} = U\, S_{mass}\, U^T$$
$$= \cos\left(\frac{\delta m^2 x}{4E}\right)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - i\sin\left(\frac{\delta m^2 x}{4E}\right)\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}$$

$$S_{e\mu} = -i\sin 2\theta\, \sin\left(\frac{\delta m^2 x}{4E}\right)$$

$$P_{e\mu} = |S_{e\mu}|^2 = \sin^2 2\theta\, \sin^2\left(\frac{\delta m^2 x}{4E}\right)$$

from E.Lisi "Neutrinos: theory and phenomenology"

Mass Splittings

- Three mass states $\nu_1\ \nu_2\ \nu_3$ with masses $m_1\ m_2\ m_3$
- For ultrarelativistic ν in vacuum : $E = \sqrt{m_i^2 + p^2} \simeq p + m_i^2/2p$
- Neutrino oscillations probe $\Delta E \propto \Delta m_{ij}^2$
- 3 neutrinos $\rightarrow$ 2 independent Δm_{ij}^2, say, δm^2 and Δm^2
- Experimentally : very different values, $\delta m^2/\Delta m^2 \sim 1/30$

$$\boxed{\begin{array}{l} \delta m^2 \simeq 7.5 \times 10^{-5}\ \mathrm{eV}^2 \\ \Delta m^2 \simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2 \end{array}}$$

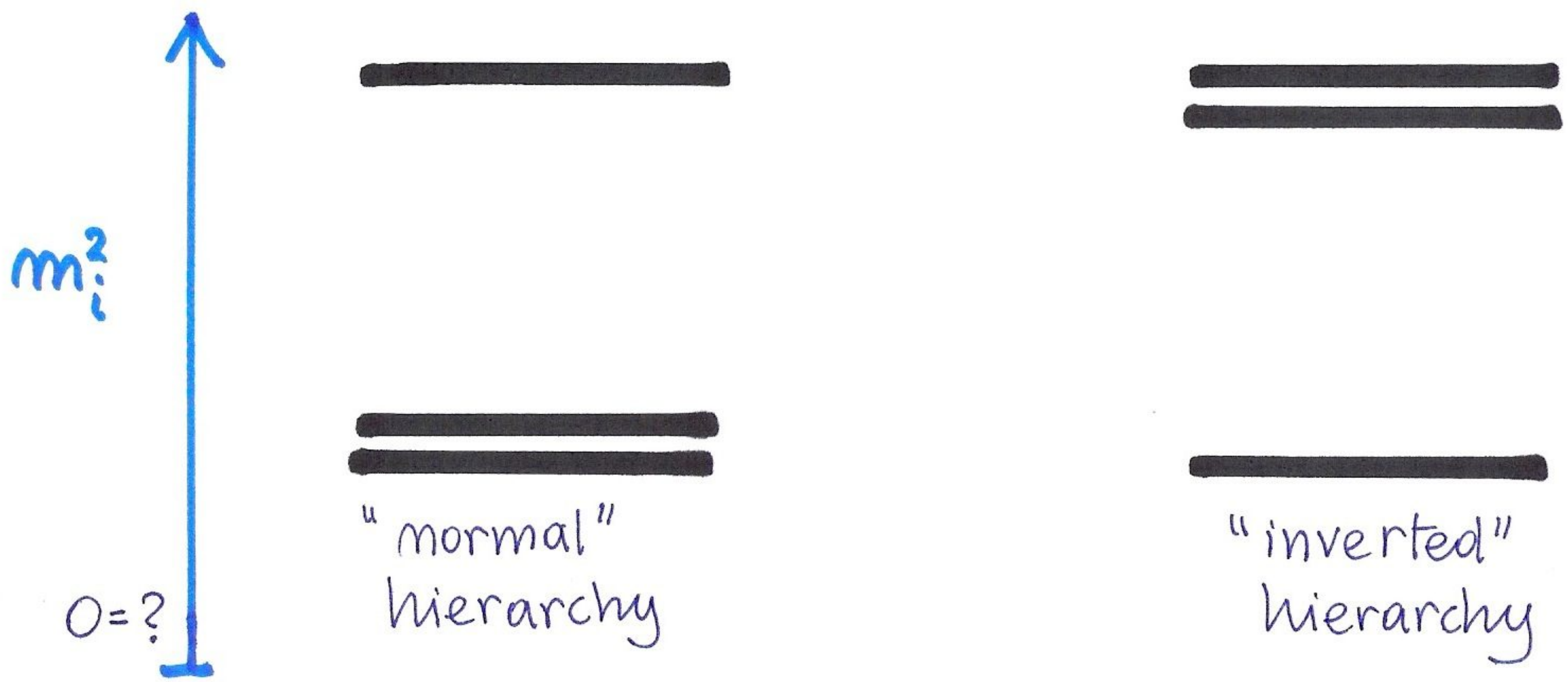
$\leftarrow$ "small" or "solar" splitting

$\leftarrow$ "large" or "atmospheric" splitting

from E.Lisi "Neutrinos: theory and phenomenology"

Mass Hierarchy

- Very difficult to probe both splittings in the same experiment!
- Two possible arrangements ("hierarchies") for such splittings:



- Absolute ν mass scale unknown; lightest m_i could be zero
- However, upper limits exist: $m_i \lesssim \mathcal{O}(eV)$

from E.Lisi "Neutrinos: theory and phenomenology"

δM^2 AND ΔM^2

- In both hierarchies, there is a "doublet" of close mass states and a "lone" mass state. Universal convention:

(ν_1, ν_2) is the doublet, with ν_1 being the lightest : $m_1 < m_2$

ν_3 is the lone state, with $m_3 \gtrless m_{1,2}$

- Splittings : $\delta m^2 = m_2^2 - m_1^2 > 0$ (>0 by definition)

$\Delta m^2 \simeq m_3^2 - m_{1,2}^2 \gtrless 0$ (has a physical sign)

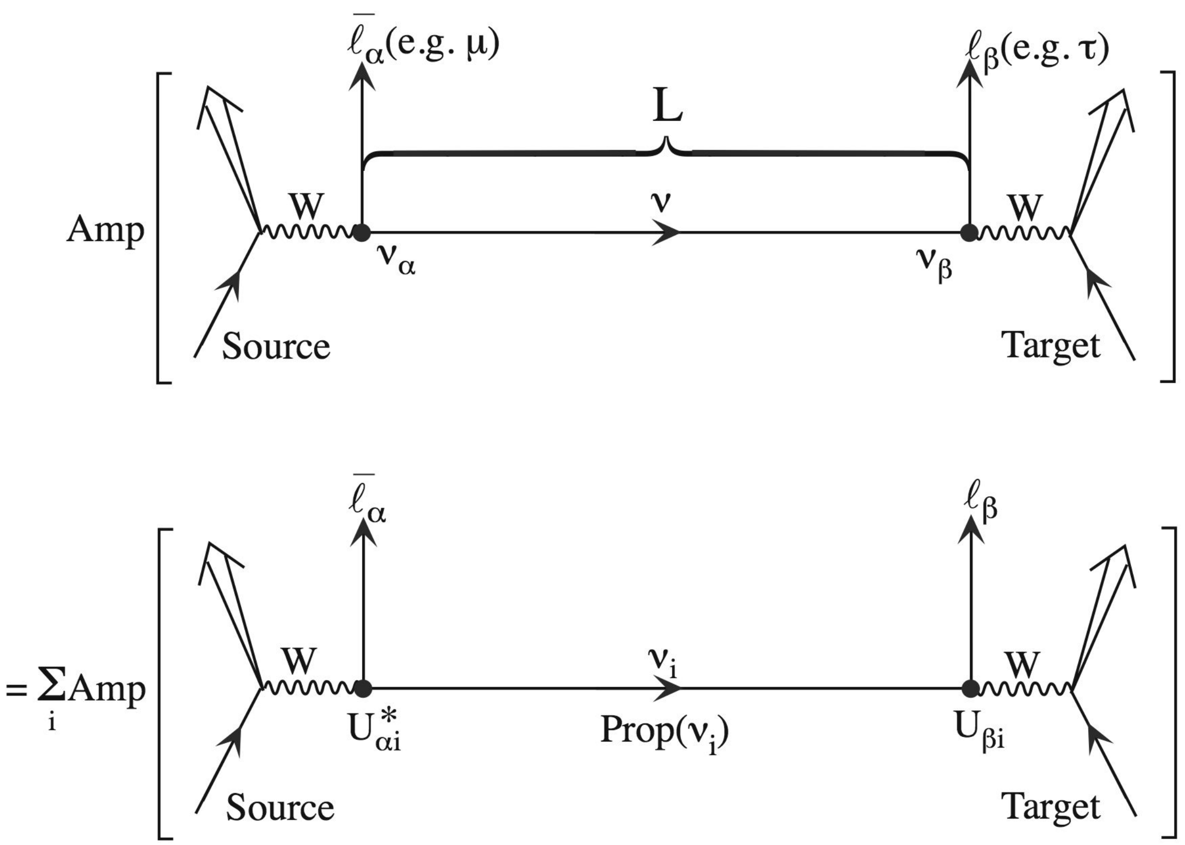
- We use : $\Delta m^2 = \frac{1}{2}(\Delta m^2_{31} + \Delta m^2_{32})$ for the sake of precision

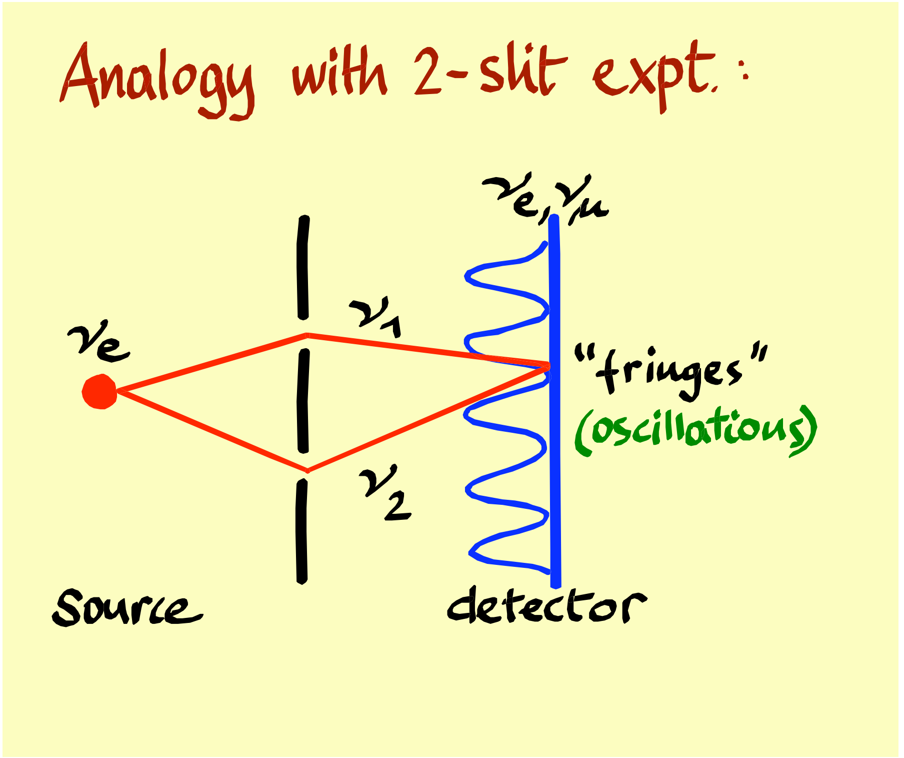
(others prefer to use either $\Delta m^2 = \Delta m^2_{31}$ or $\Delta m^2 = \Delta m^2_{32}$)

from E.Lisi "Neutrinos: theory and phenomenology"

2 SLIT ANALOGY

$$P[\nu_\alpha \rightarrow \nu_\beta] = sin^2(2\theta)sin^2(\Delta m^2 L/4E)$$





- Length scales
  - L:Baseline
  - Oscillation length: $\lambda=4\pi E/\Delta m^2$
- Fringes not visibile if $\lambda<L$ or large experimental smearing $<sin^2(\Delta m^2 L/4E)>\approx 0.5$

from E.Lisi "Physics of Massive neutrinos"

OSCILLATION WITH 2 NEUTRINOS

- Two flavour eigenstates and two mass eigenstates

$$U = \begin{array}{c c} & \begin{array}{cc} \nu_1 & \nu_2 \end{array} \\ \begin{array}{c} \nu_e \\ \nu_\mu \end{array} & \left[\begin{array}{cc} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{array} \right] \end{array}$$

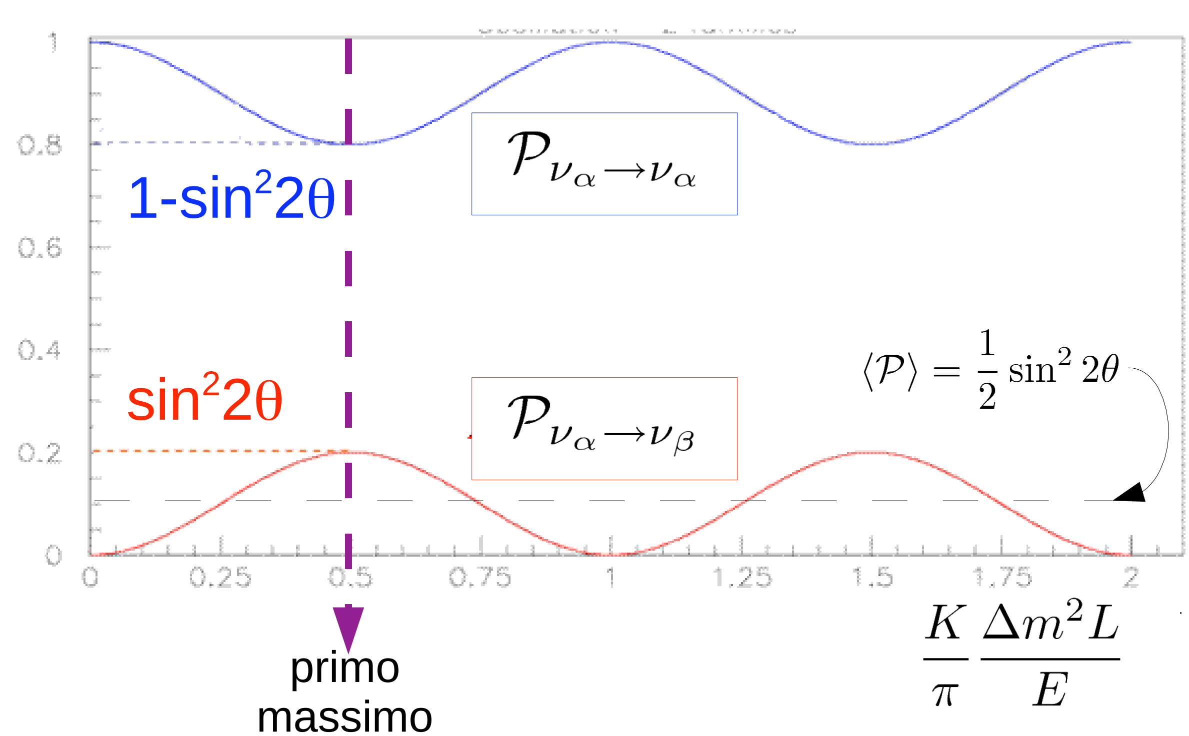
$\nu_e = \nu_1 cos\vartheta + \nu_2 sin\vartheta$

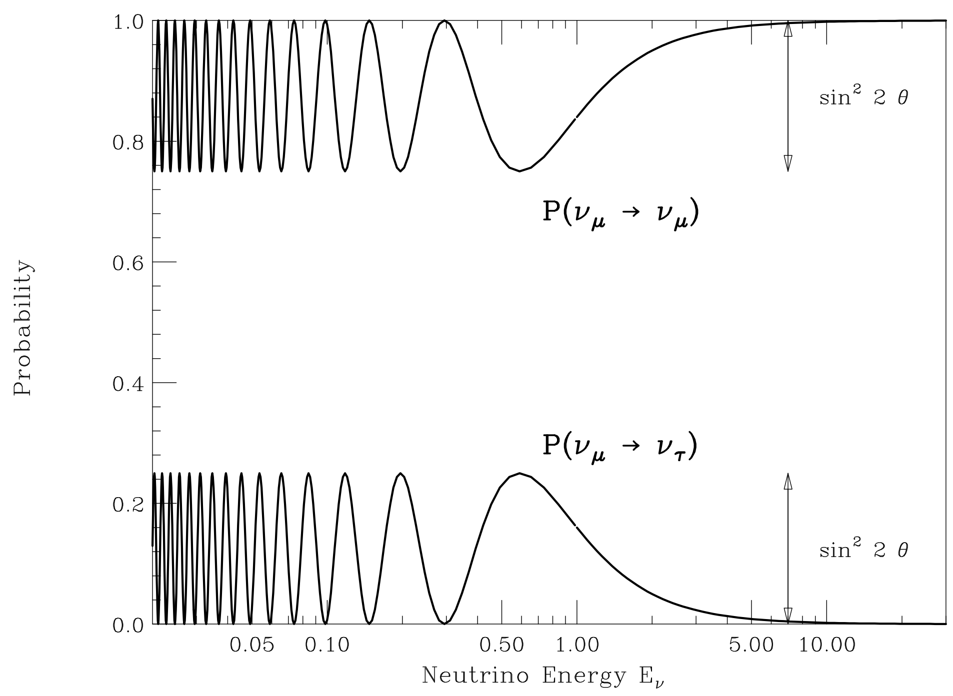
$\nu_\mu = - \nu_1 sin\vartheta + \nu_2 cos\vartheta$

$$P[\nu_\alpha \rightarrow \nu_\beta] = sin^2(2\theta) sin^2(\Delta m^2 L/4E)$$

$$P[\nu_\alpha \rightarrow \nu_\alpha] = 1 - sin^2(2\theta) sin^2(\Delta m^2 L/4E)$$

- one angle θ , one splitting Δm^2
- no distinction between $\theta \Leftrightarrow \pi/2-\theta$, $\Delta m^2 \Leftrightarrow -\Delta m^2$





SENSITIVITY TO OSCILLATION

- In real life: production and detection region are not point, neutrinos not monochromatic, experimental energy resolution not perfect
  - loss of coherence⇒ oscillation washed out

$$P[\nu_\alpha \rightarrow \nu_\beta] = sin^2(2\theta) sin^2(\Delta m^2 L/4E)$$

$\Delta m^2 L/4\pi E$<<1 (short baseline)

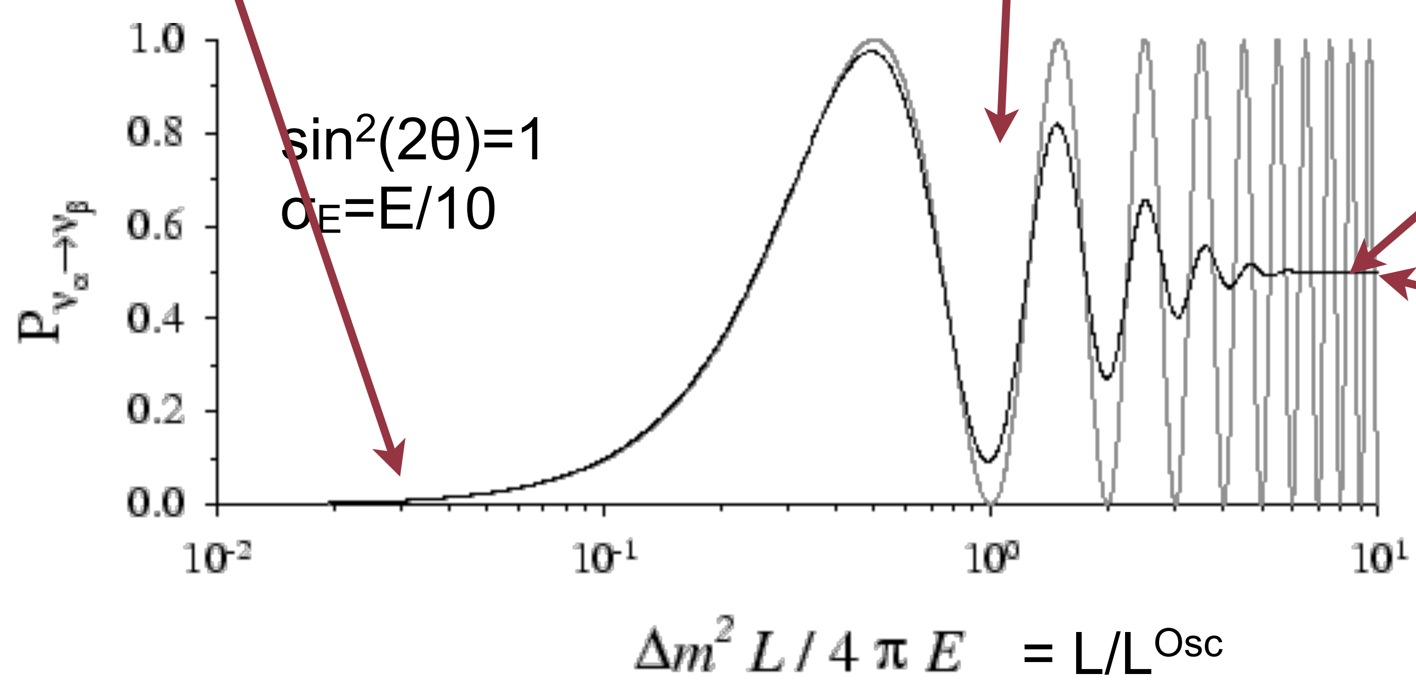
oscillation doesn't evolve

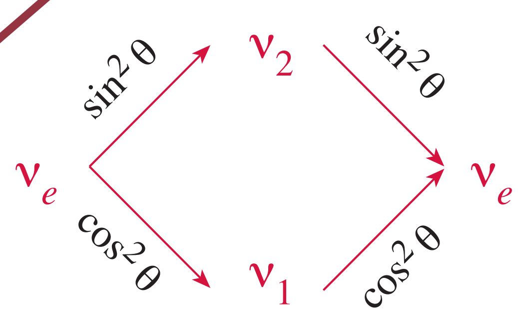
$\Delta m^2 L/4\pi E$~1 max sensitivity

can observe n~$\Delta E/E$ oscillations

$\Delta m^2 L/4\pi E$>>1 (averaged regime)

phase lost due to poor L/E resolution





Σ Probabilities rather than Amplitudes

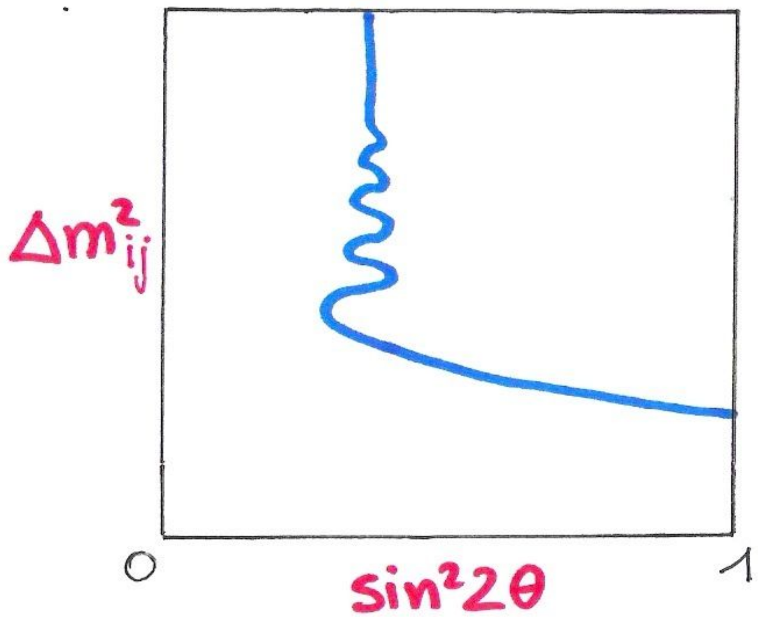
$P(\nu_\alpha \Rightarrow \nu_\alpha)=sin^4(\theta)+cos^4(\theta)=1-0.5\ sin^2(2\theta)$

$P(\nu_\alpha \Rightarrow \nu_\beta)=0.5\ sin^2(2\theta)$

SENSITIVITY PLOT

- Experiments measure some "averaged" $P_{\alpha\beta} \simeq \sin^2 2\theta \left\langle \sin^2\left(\frac{\Delta m^2_{ij}\, x}{4E}\right)\right\rangle$

- Curve of iso-$P_{\alpha\beta}$:

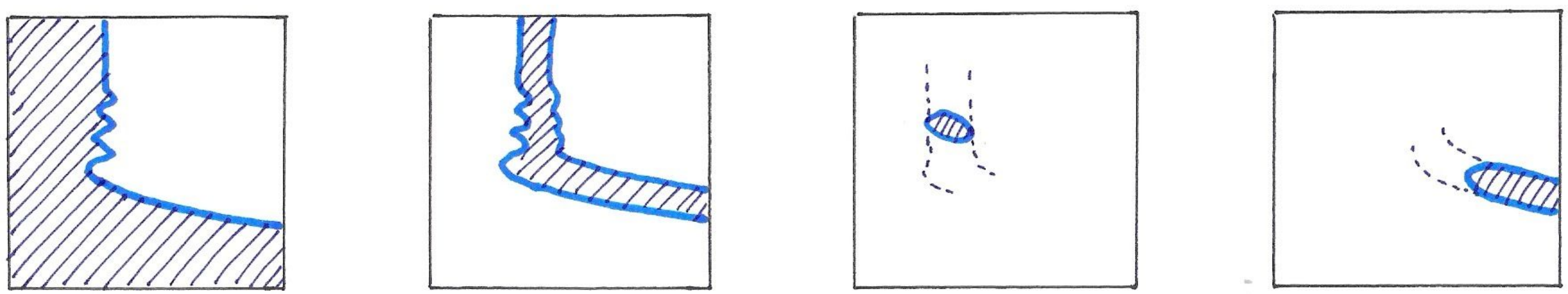


← $\frac{\Delta m^2_{ij}\, x}{4E} \gg 1$, $\langle \cdots \rangle \sim \frac{1}{2}$, fast oscillations

← $\frac{\Delta m^2_{ij}\, x}{4E} \sim \mathcal{O}(1)$

← $\frac{\Delta m^2_{ij}\, x}{4E} \ll 1$, vanishing oscillations

- Possible expt. constraints:



No signal | Signal | Precise signal at small mixing | Precise signal at large mixing

(need ≥ 2 expts or spectral data in 1 expt)

from E.Lisi "Neutrinos: theory and phenomenology"

INTERPRETATION OF SENSITIVITY PLOT

Appearance experiment: search for ν_β in a flux Φ of ν_α $\quad P[\nu_\alpha \rightarrow \nu_\beta] = sin^2(2\theta) sin^2(\Delta m^2 L/4E)$

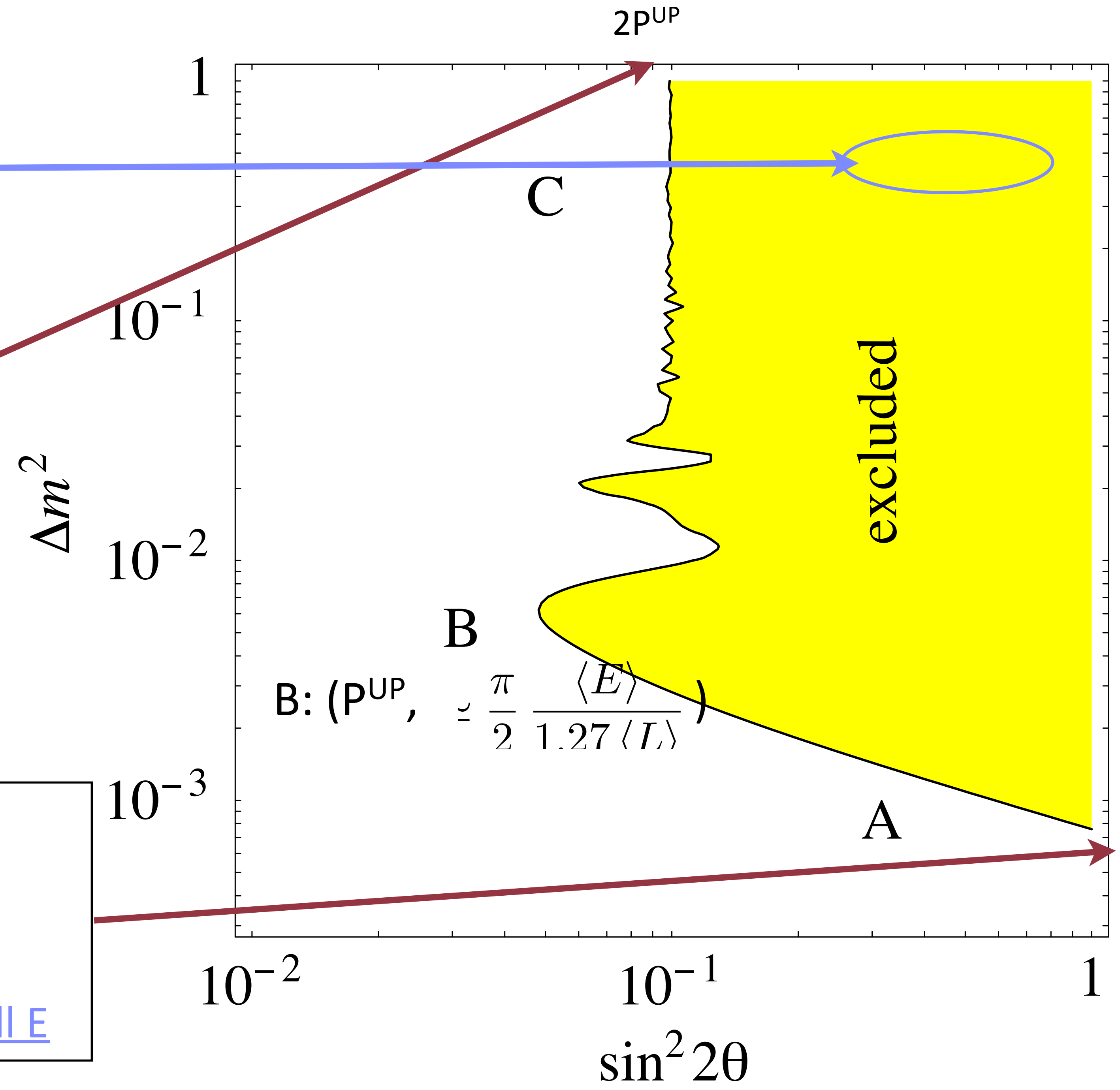
Null result: $P[\nu_\alpha \rightarrow \nu_\beta] < P^{UP} \propto \Phi(\nu_\alpha)^{-0.5}$

Positive results: inclusion curves

C: $\Delta m^2 L/4\pi E >> 1$;

$P(\nu_\alpha \Rightarrow \nu_\beta) = 0.5\, sin^2(2\theta) < P^{UP} \propto \Phi^{-0.5}$

big Flux to have small angle



A: $\Delta m^2 L/4\pi E << 1$

$P(\nu_\alpha \Rightarrow \nu_\beta) = sin^2(2\theta)(\Delta m^2 L/4E)^2 < P^{UP} \propto \Phi^{-0.5}$

$$\Delta m^2_{min} \simeq \frac{\sqrt{P^{UP}}}{1.27\sqrt{\langle L^2 \rangle \langle E^{-2} \rangle}}$$

big L (small Φ), small E

OSCILLATION WITH 3 NEUTRINOS

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i<j} \mathrm{Re}\, J_{\alpha\beta}^{ij} \sin^2\left(\frac{\Delta m^2_{ij}\, x}{4E}\right) - 2 \sum_{i<j} \mathrm{Im}\, J_{\alpha\beta}^{ij} \sin\left(\frac{\Delta m^2_{ij}\, x}{2E}\right)$$

$$\Delta m^2_{ij} = m^2_i - m^2_j$$

$$J_{\alpha\beta}^{ij} = U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}$$

$$\frac{\Delta m^2_{ij}}{4E} = 1.267 \left(\frac{\Delta m^2_{ij}}{eV^2}\right)\left(\frac{x}{m}\right)\left(\frac{MeV}{E}\right)$$

- Comments:
  - If no mixing $P(\nu_\alpha \Rightarrow \nu_\beta)=\delta_{\alpha\beta}$; **flavour change ⇒ mixing**
  - If degenerate masses $P(\nu_\alpha \Rightarrow \nu_\beta)=\delta_{\alpha\beta}$; **flavour change ⇒ no degenerate $m_\nu \neq 0$**
  - $P(\nu_\alpha \Rightarrow \nu_\beta)$ depends only on **squared-mass splitting** **⇒ no absolute mass value**
  - Neutrino flavour change does not change the total flux (if no sterile νs): $\sum_\beta P(\nu_\alpha \Rightarrow \nu_\beta)=1$

APPEARANCE AND DISAPPEARANCE

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i<j} \mathrm{Re}\, J^{ij}_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{ij}\, x}{4E}\right) - 2 \sum_{i<j} \mathrm{Im}\, J^{ij}_{\alpha\beta} \sin\left(\frac{\Delta m^2_{ij}\, x}{2E}\right)$$

$$\Delta m^2_{ij} = m^2_i - m^2_j$$

$$J^{ij}_{\alpha\beta} = U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}$$

$$\frac{\Delta m^2_{ij}}{4E} = 1.267 \left(\frac{\Delta m^2_{ij}}{eV^2}\right)\left(\frac{x}{m}\right)\left(\frac{MeV}{E}\right)$$

- Comments:

– **Flavour change in vacuum oscillates with L/E:** macroscopic quantum interference phenomena

– **$\sin^2(\Delta m^2_{ij} L/E)$** appreciable when its argument is **O(1)**: an experiment L/E is sensitive to **$\Delta m^2_{ij} \cong L/E$**

➡ **$E \approx 1$GeV, $L \approx 10^4$ km, $\Delta m^2_{ij} \approx 10^{-4}$ eV^2**

➡ **$E \approx 1$MeV, $L \approx 100$ km, $\Delta m^2_{ij} \approx 10^{-5}$ eV^2**
$P(\nu_\alpha \Rightarrow \nu_\beta)$ depends only on squared-mass splitting $\Rightarrow$ no absolute mass value

– Flavour change can be detected in two modes:
- **Appearance:** see $\nu_{\beta\neq\alpha}$ in a ν_α beam
- **Disappearance:** see some of known ν_α disappear in ν_α beam

ANTI-NEUTRINOS VACUUM OSCILLATION

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i<j} \mathrm{Re}\, J_{\alpha\beta}^{ij} \sin^2\left(\frac{\Delta m^2_{ij}\, x}{4E}\right) - 2 \sum_{i<j} \mathrm{Im}\, J_{\alpha\beta}^{ij} \sin\left(\frac{\Delta m^2_{ij}\, x}{2E}\right)$$

$$\Delta m^2_{ij} = m^2_i - m^2_j$$

$$J_{\alpha\beta}^{ij} = U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}$$

$$\frac{\Delta m^2_{ij}}{4E} = 1.267 \left(\frac{\Delta m^2_{ij}}{eV^2}\right)\left(\frac{x}{m}\right)\left(\frac{MeV}{E}\right)$$

- If CPT holds: $P[\bar{\nu}_\alpha \to \bar{\nu}_\beta] = P[\nu_\beta \to \nu_\alpha]$

$$P[\nu_\beta \to \nu_\alpha; U] = P[\nu_\alpha \to \nu_\beta; U^*] \quad \Rightarrow \quad P[\bar{\nu}_\alpha \to \bar{\nu}_\beta, U] = P[\nu_\alpha \to \nu_\beta; U^*]$$

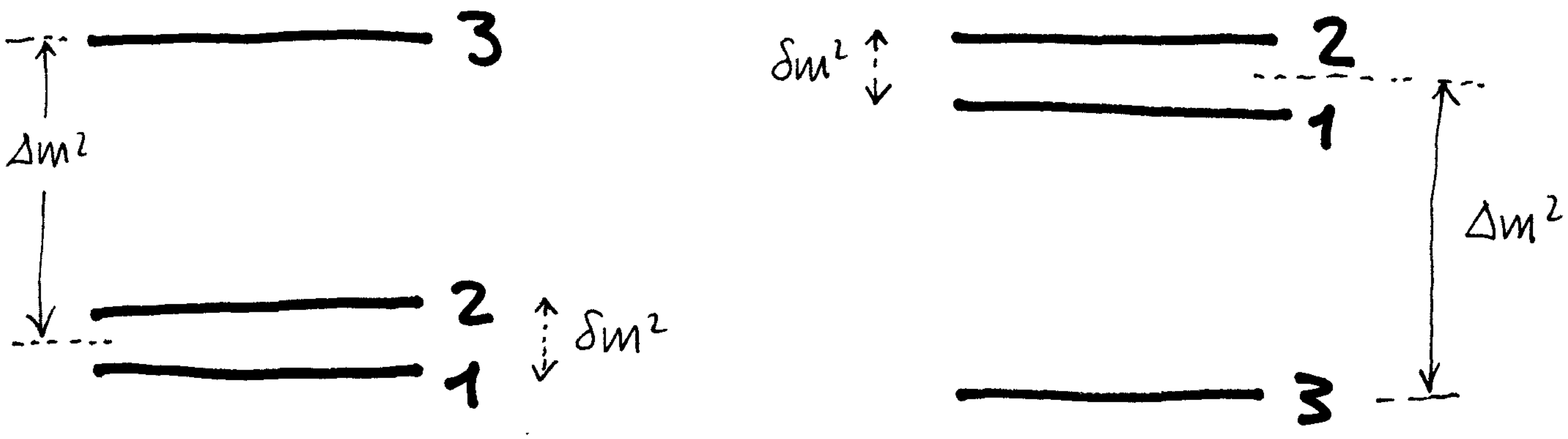
- **Imaginary term** changes sign for anti-neutrinos: **complex U ⇒ CP violation**

- **No CP violation in disappearance experiment**
  - $\Delta_{\alpha\beta}=P(\nu_\alpha \Rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \Rightarrow \bar{\nu}_\beta)$
  - CPT: $P(\nu_\alpha \Rightarrow \nu_\beta)= P(\bar{\nu}_\beta \Rightarrow \bar{\nu}_\alpha) \Rightarrow \Delta_{\alpha\beta} = -\Delta_{\beta\alpha}$
  - $\boldsymbol{\Delta_{\alpha\alpha} = 0}$

- **Need at least 3 flavours**
  - If only two flavours αβ: $P(\nu_\alpha \Rightarrow \nu_\beta) = 1 - P(\nu_\alpha \Rightarrow \nu_\alpha) = \mathrm{CPT} = 1 - P(\bar{\nu}_\alpha \Rightarrow \bar{\nu}_\alpha) = P(\bar{\nu}_\alpha \Rightarrow \bar{\nu}_\beta)$

OSCILLATION SEARCHES SENSITIVE TO ΔM^2

Mass Splitting δM² and ΔM²



normal hierarchy

inverted hierarchy

Our convention:

$$\begin{cases} \delta m^2 \stackrel{\text{def}}{=} m_2^2 - m_1^2 \geq 0 \quad \text{always} \\ \Delta m^2 \stackrel{\text{def}}{=} \left| m_3^2 - \dfrac{m_1^2+m_2^2}{2} \right| = \left| \dfrac{\Delta m_{31}^2 + \Delta m_{32}^2}{2} \right| \end{cases}$$

Squared mass matrix $\mathcal{M}^2 = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$

$$\mathcal{M}^2 = \frac{m_1+m_2}{2}\mathbb{1} + \mathrm{diag}\left(-\frac{\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm\Delta m^2\right)$$

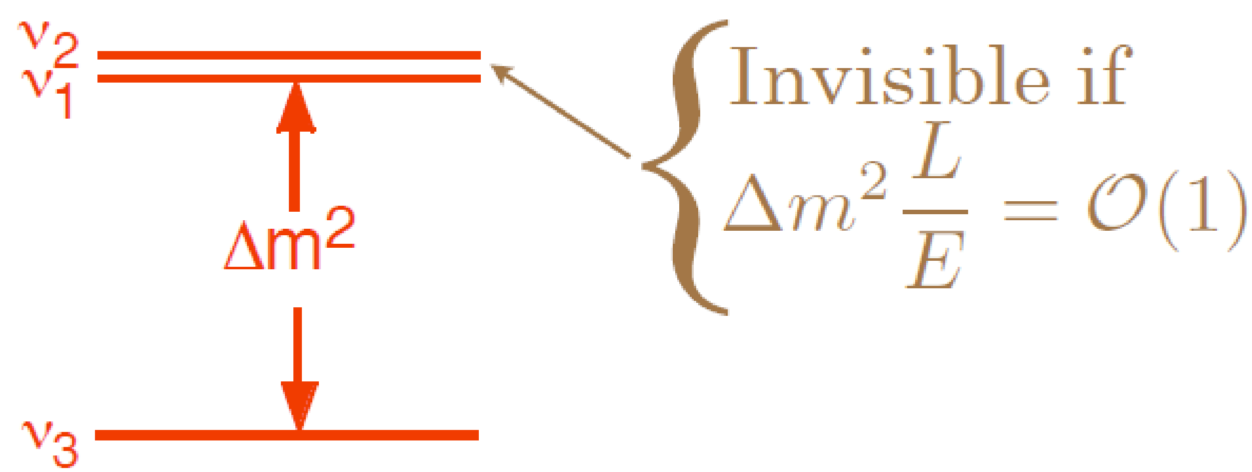
$+$: normal

$-$: inverted

from E.Lisi "Physics of Massive neutrinos"

ONE MASS-SCALE DOMINANCE

- 3 flavours ⇒ 3 angles, 1 phase, 2 independent Δm^2
- 3 flavours + one mass scale dominance



$$P_{\alpha\alpha} = P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha) \simeq 1 - 4|U_{\alpha 3}|^2(1-|U_{\alpha 3}|^2)\sin^2\left(\frac{\Delta m^2 x}{4E}\right)$$

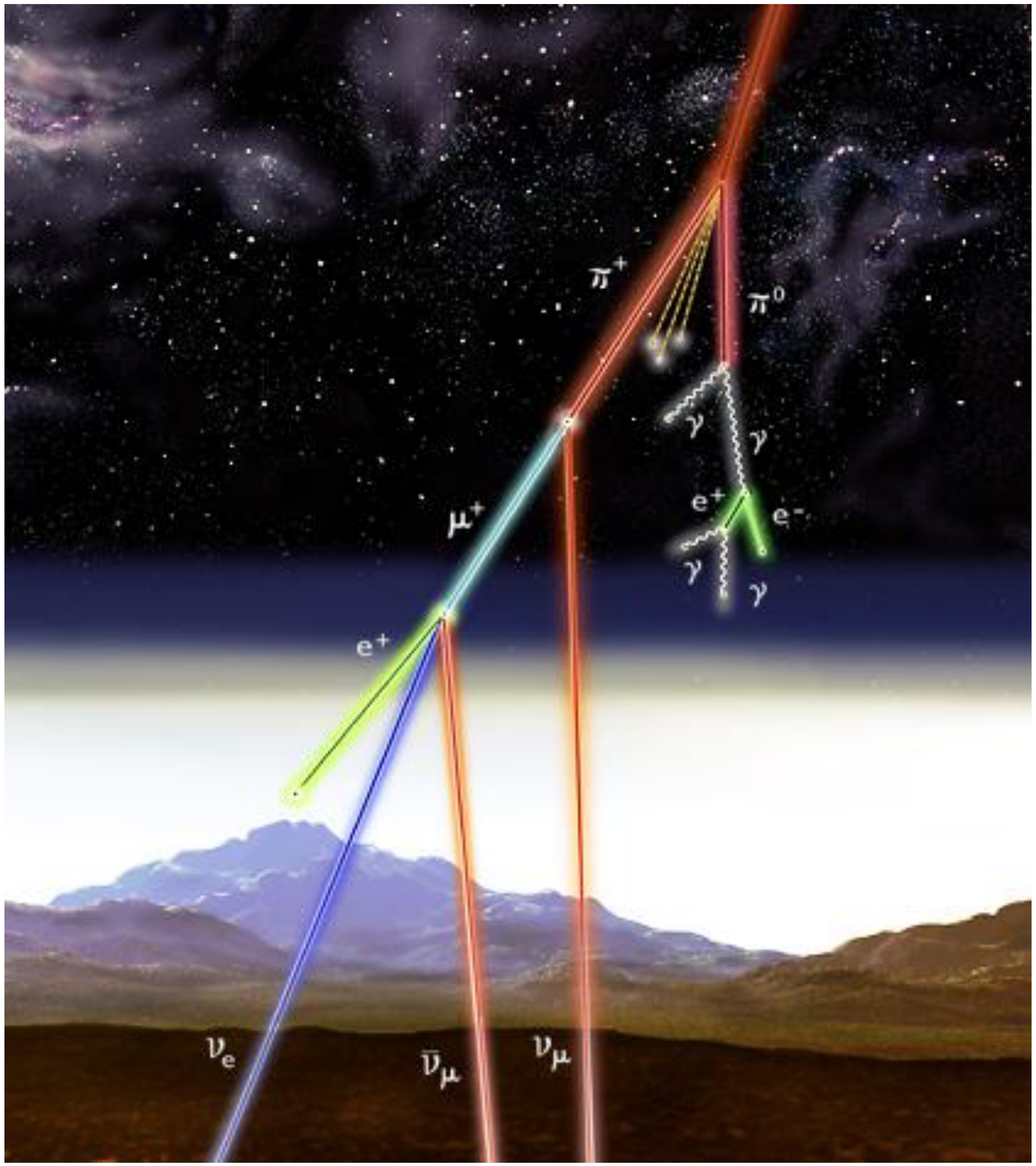
$$P_{\alpha\beta} = P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) \simeq 4|U_{\alpha 3}|^2|U_{\beta 3}|^2\sin^2\left(\frac{\Delta m^2 x}{4E}\right) \qquad \alpha \neq \beta$$

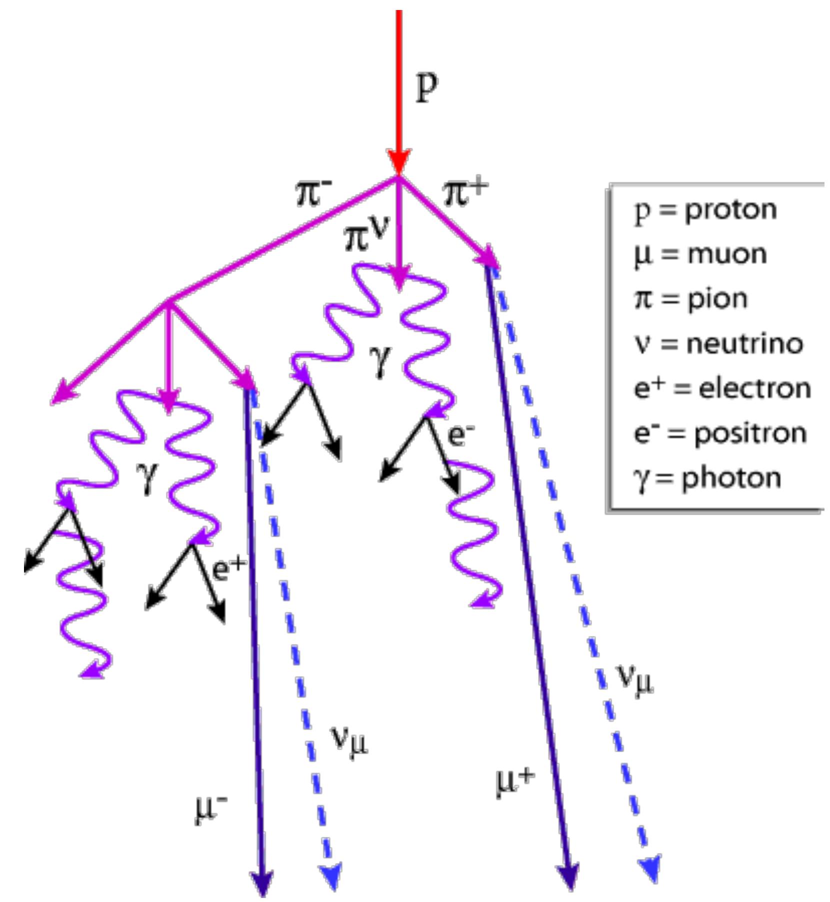
$$|U| = \begin{pmatrix} \cdot & \cdot & |U_{e3}| \\ \cdot & \cdot & |U_{\mu 3}| \\ \cdot & \cdot & |U_{\tau 3}| \end{pmatrix}$$

  - Δm^2,mixing with ν_3 ϑ_{23}, ϑ_{13}, $|U_{e3}|=\sin(\vartheta_{13})$, $|U_{\mu 3}|=\cos(\vartheta_{13})\sin(\vartheta_{23})$, $|U_{\tau 3}|=\cos(\vartheta_{13})\cos(\vartheta_{23})$
  - No sensitivity to hierarchy ($\Delta m^2 \leftrightarrow -\Delta m^2$), CP($U \leftrightarrow U^*$), δm^2, ϑ_{12}, $\bar{\nu}/\nu$

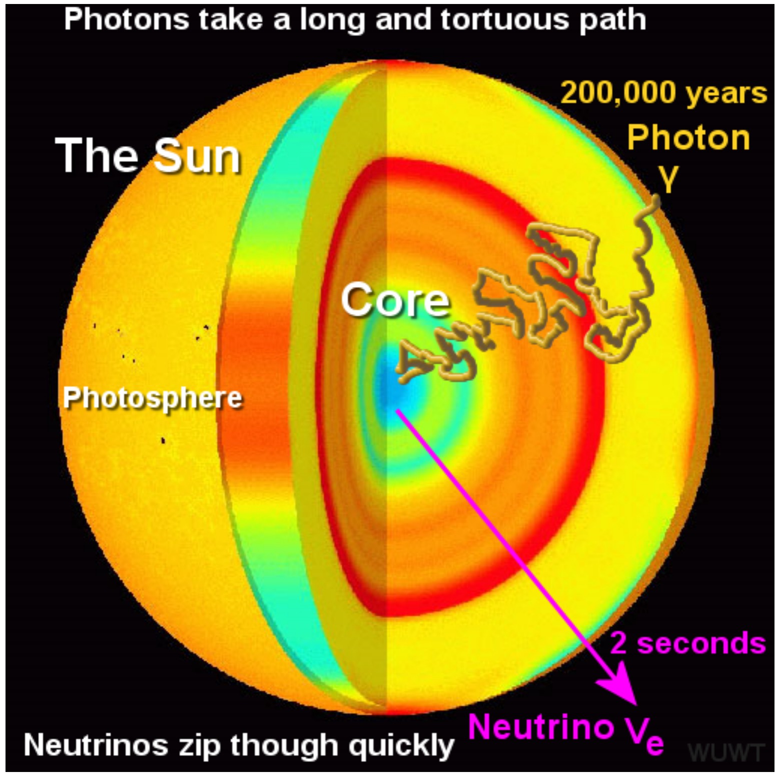
Sources of Neutrinos

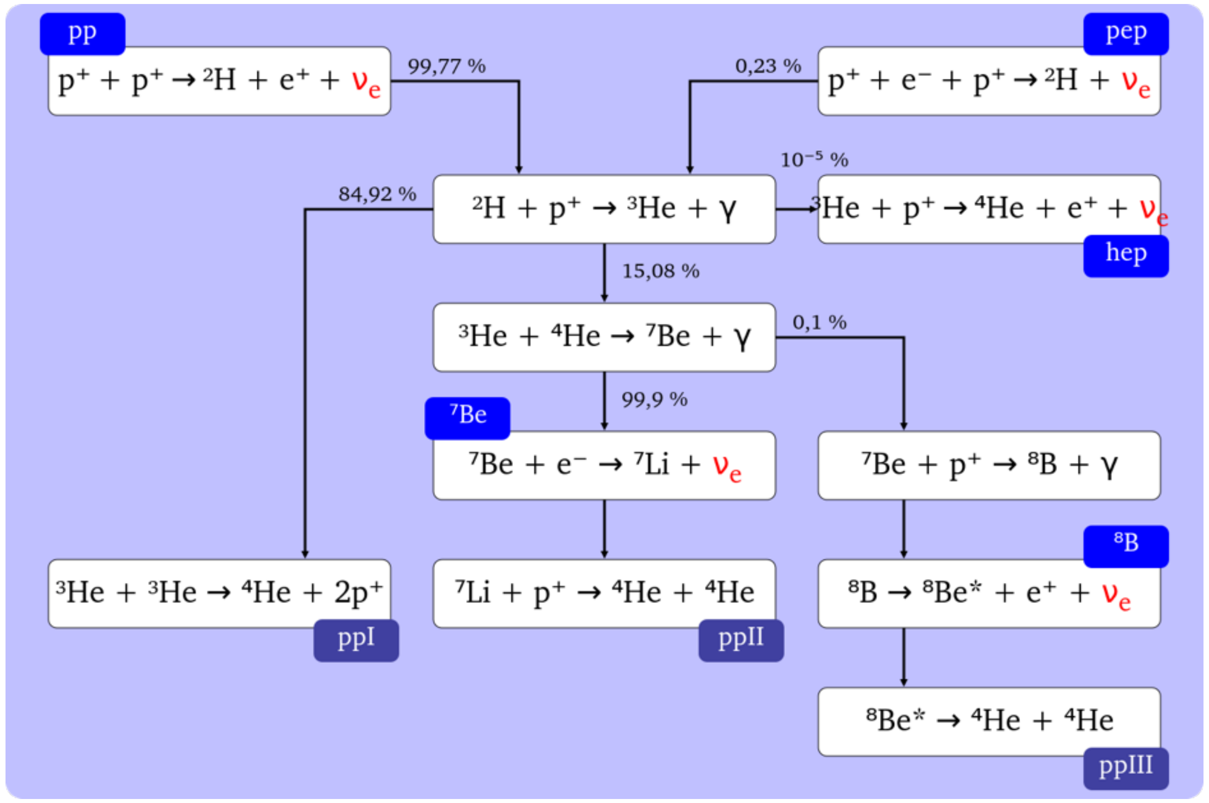
Atmosphere Neutrinos

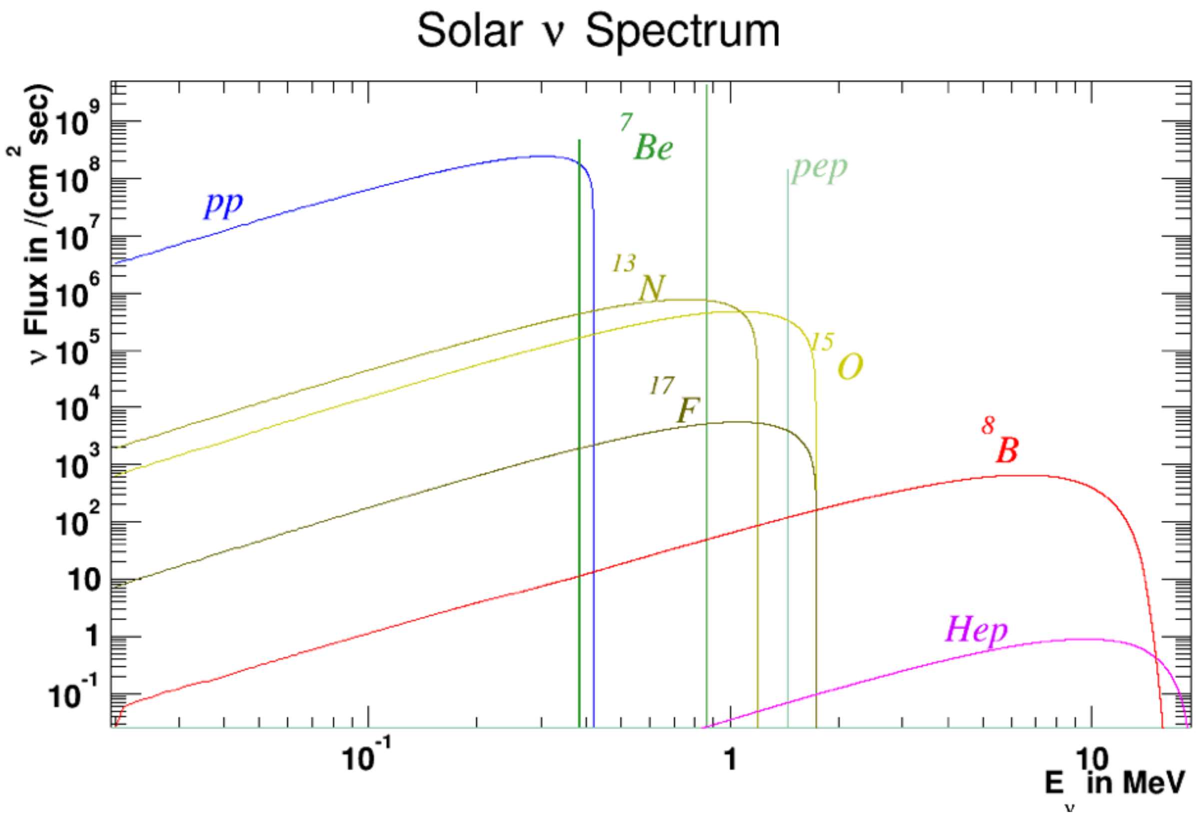




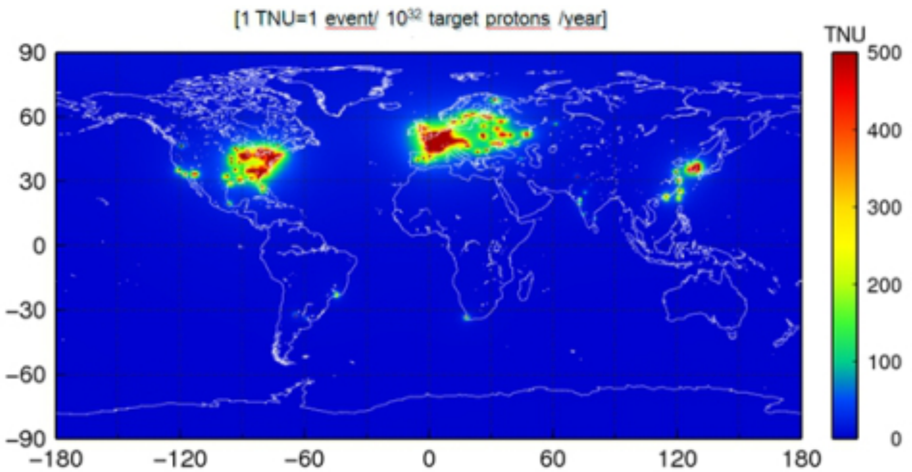
SOLAR NEUTRINOS

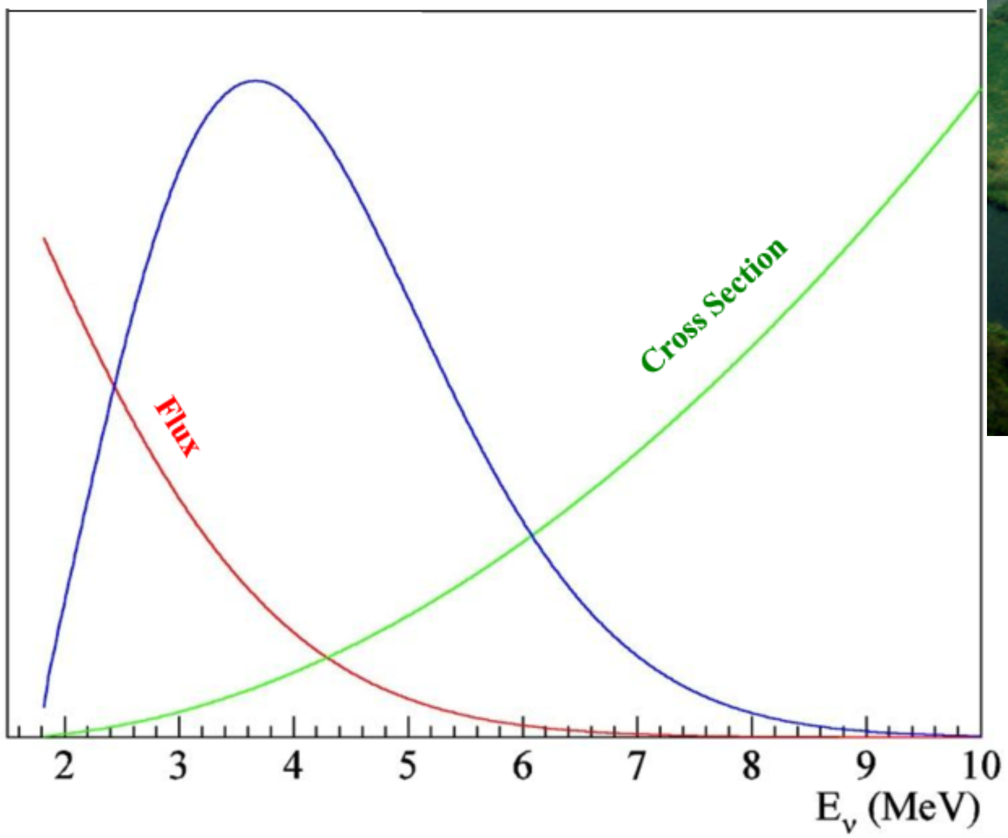




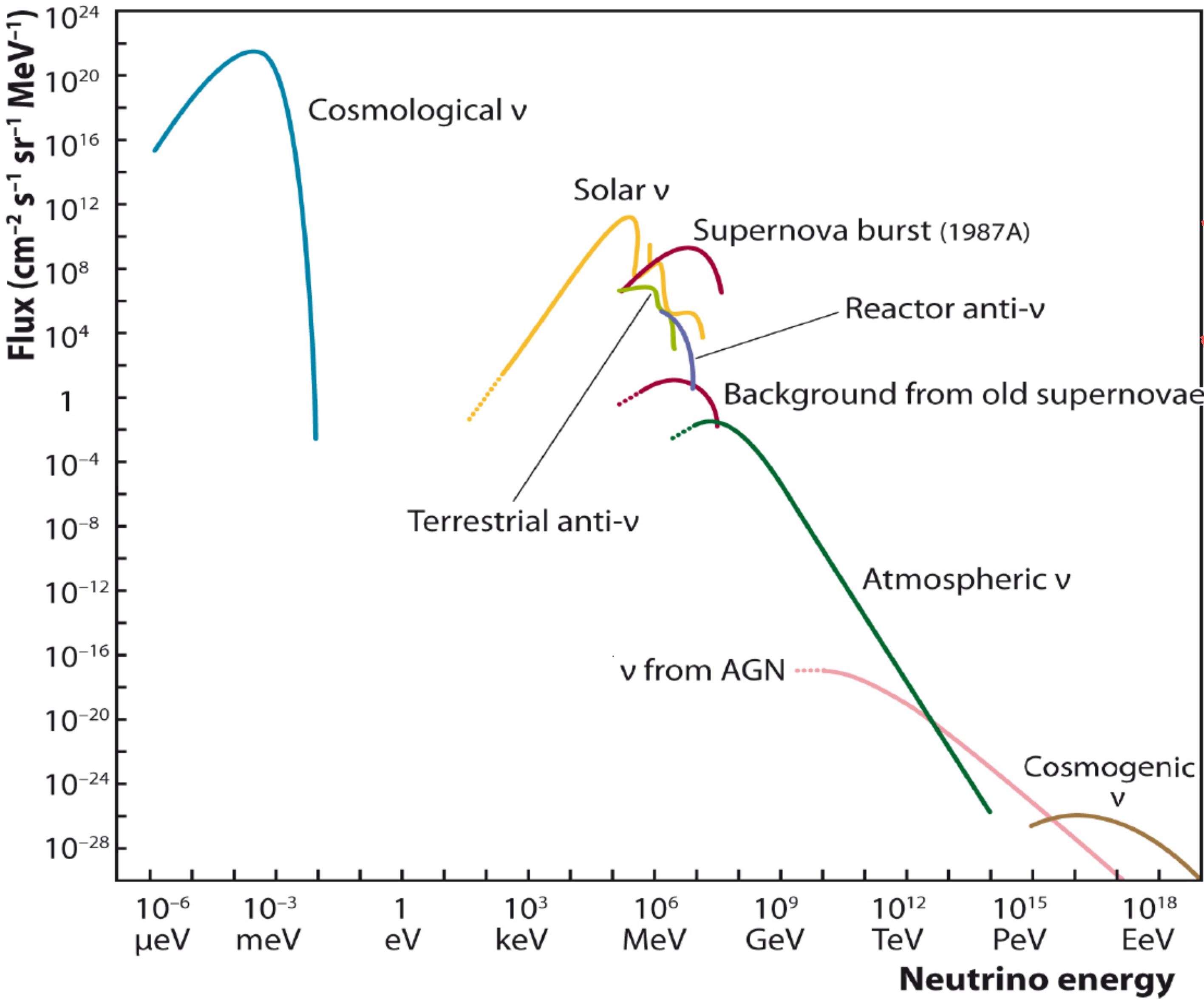


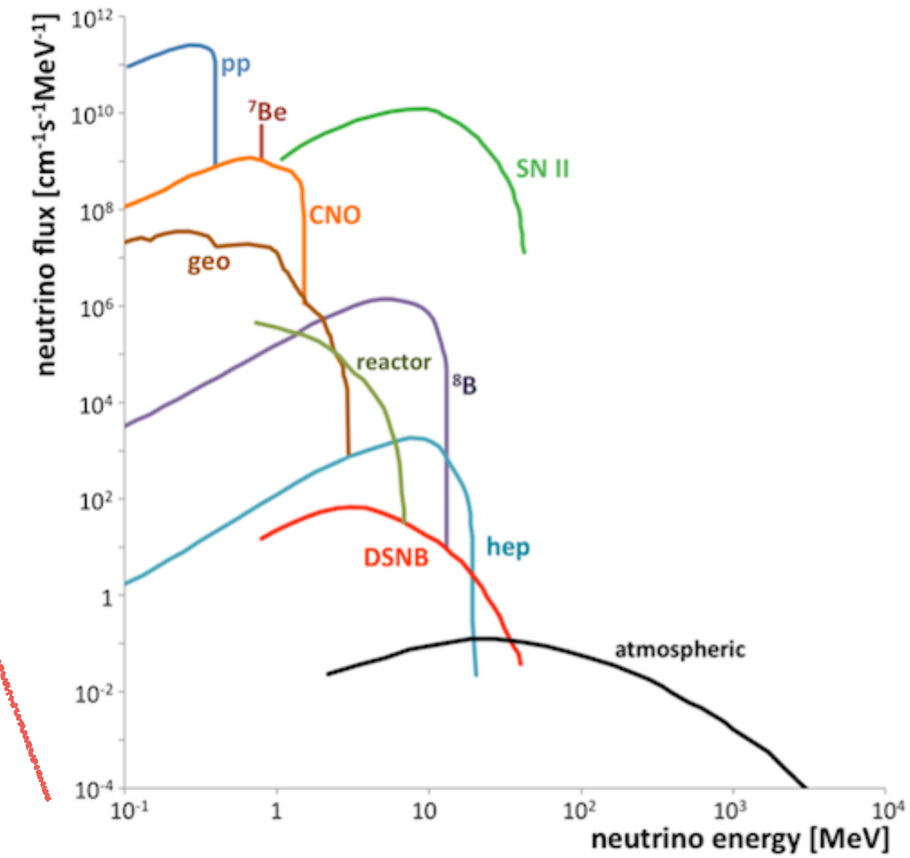
Reactor Neutrinos





NEUTRINO ENERGY SPECTRUM





EXPERIMENTAL OBSERVABLES

- atmospheric ν experiments (ATM) Super-Kamiokande ...
- long-baseline accelerator expts. (LBL) K2K, MINOS, T2K, OPERA...
- short-baseline reactor expts. (SBR) CHOOZ, Daya Bay, RENO, D-Chooz...

OPERA (LBL) : $P(\nu_\mu \to \nu_\tau) \simeq \cos^4\theta_{13} \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$ (*)

ATM + LBL : $P(\nu_\mu \to \nu_\mu) \simeq 1 - 4 c_{13}^2 s_{23}^2 (1 - c_{13}^2 s_{23}^2) \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$ (*)

ATM + LBL : $P(\nu_\mu \to \nu_e) \simeq s_{23}^2 \cdot \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$ (**)

SBR : $P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$ (**)

(*) : reduces to 2ν form for $\theta_{13} \to 0$ (pure $\nu_\mu \leftrightarrow \nu_\tau$ oscillations)

(**) : vanishes for $\theta_{13} \to 0$

from E.Lisi "Neutrinos: theory and phenomenology"

SHORT BASELINE REACTOR

Short-baseline reactor experiments look for $\bar{\nu}_e$ oscillations at $x = L \sim O(1\,\mathrm{Km})$ and $E \sim$ few MeV. At these energies, CC reactions in the final state can produce e^+ but not μ^+ or τ^+; therefore, only $P(\bar{\nu}_e \to \bar{\nu}_e)$ is observable (disappearance) but not $P(\bar{\nu}_e \to \bar{\nu}_\mu)$ or $P(\bar{\nu}_e \to \bar{\nu}_\tau)$ (appearance). Moreover, it is $\delta m^2 L/4E \ll 1$, while $\Delta m^2 L/4E \sim \mathcal{O}(1)$.

$$P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

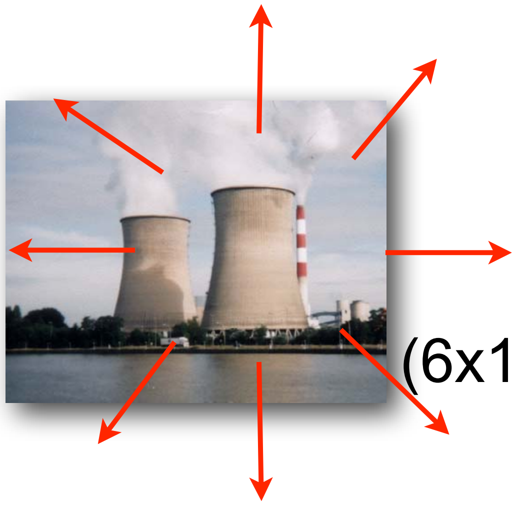
Intuitively: two of the three mixing rotations have ~no effect

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{\cancel{(2\,3)}}_{\substack{\text{mixes}\\ \text{unobservable}\\ \text{flavors } (\nu_\mu, \nu_\tau)}} (1\,3)\, \underbrace{\cancel{(1\,2)}}_{\substack{\text{mixes}\\ \sim\text{degenerate}\\ \text{states } (\nu_1, \nu_2)}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad \to \text{only } (1\,3) \text{ physical}$$

Note that, in this case, δ is unobservable, as well as sign$(\pm\Delta m^2)$, and $P(\bar{\nu}_e \to \bar{\nu}_e) = P(\nu_e \to \nu_e)$.

from E.Lisi "Neutrinos: theory and phenomenology"

REACTOR ANTI DISAPPEARANCE EXPERIMENT



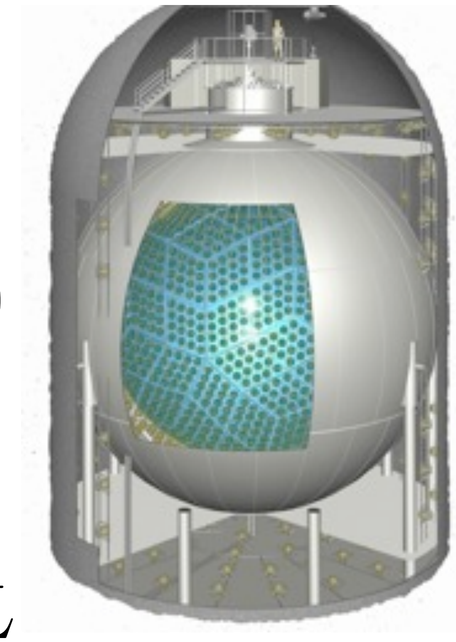
$\overline{\nu}_e$

($6\text{x}10^{20}$ $\overline{\nu}_e$/s/reactor)

($\overline{\nu}_e$ p⇒e⁺n) N_{e^+}

$N_{\mu^+} = 0$

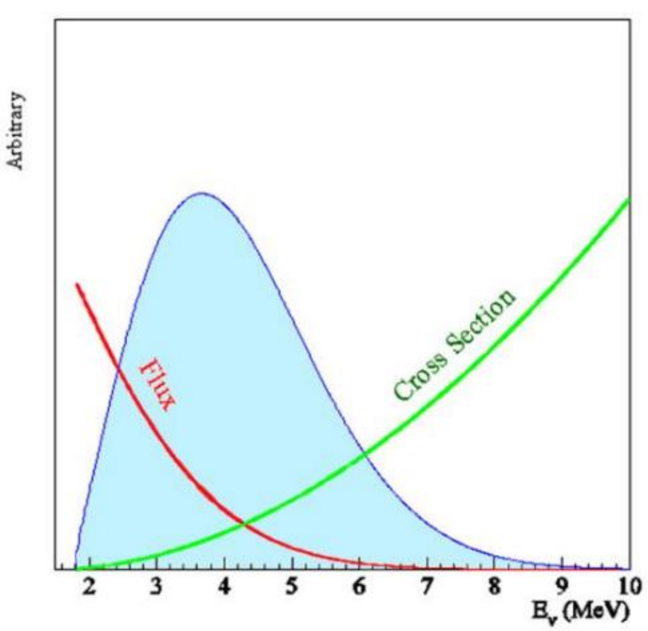
$N_{\tau^+} = 0$

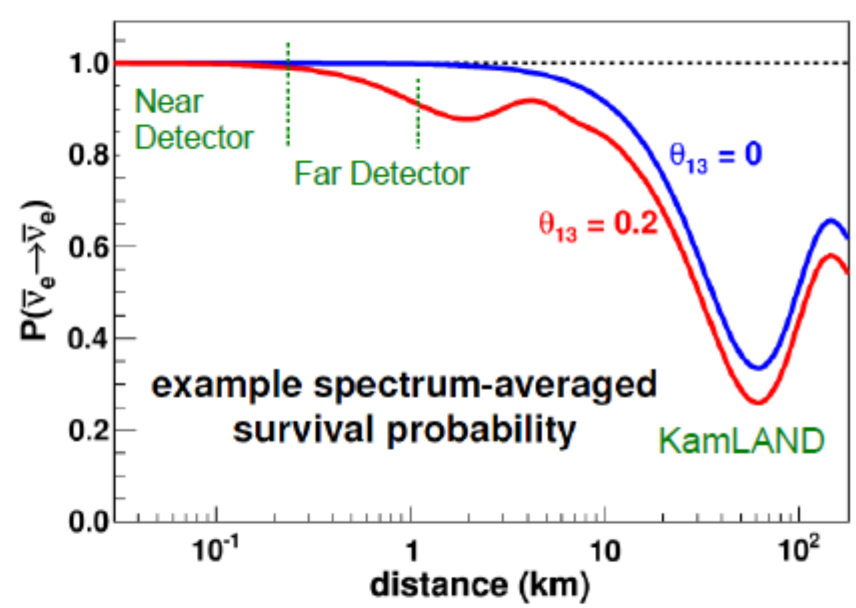


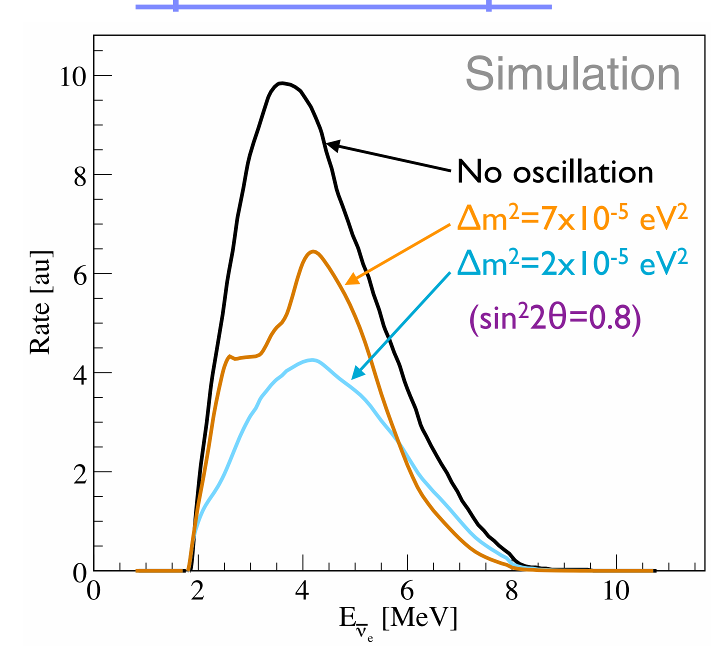
$$P(\overline{\nu}_e \to \overline{\nu}_e) = 1 - \sin^2 2\theta \, \sin^2 \frac{1.27 \Delta m^2 L}{E}$$

L

Change in normalization and spectral shape

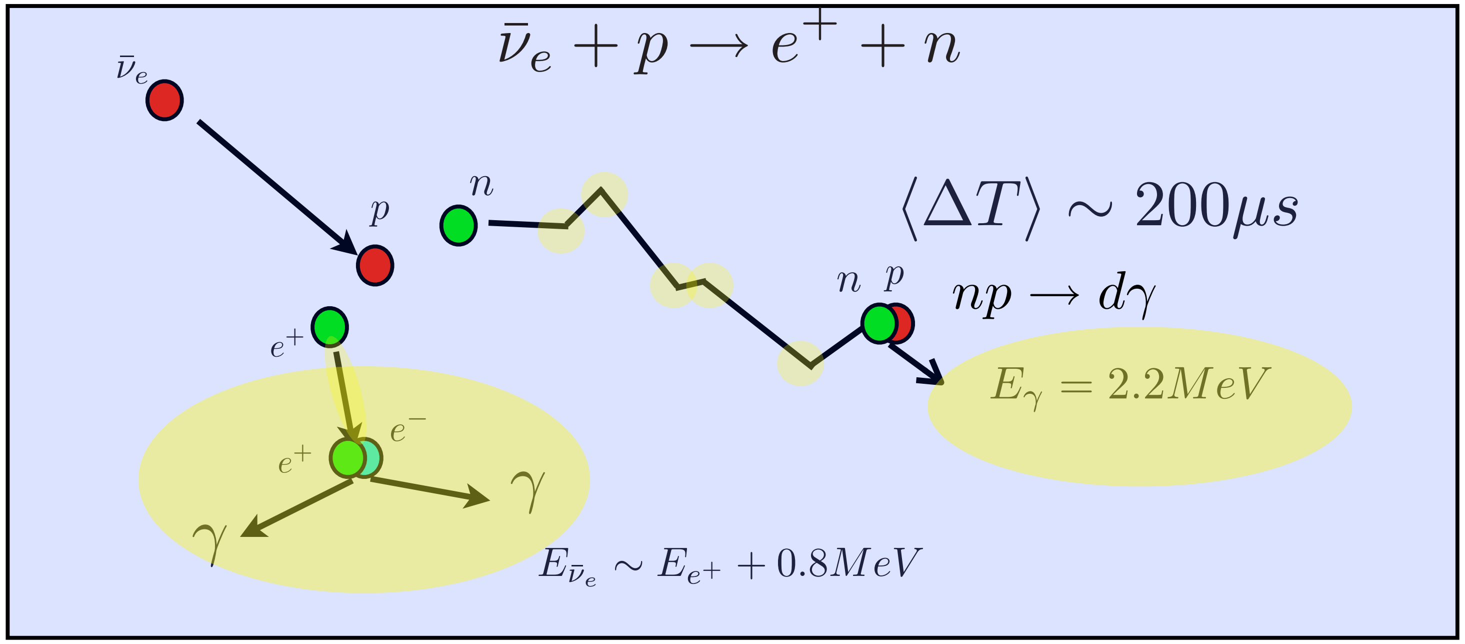






DETECTION

- Electron scattering:
  - no energy threshold, $\sigma_0=0.4\cdot 10^{-44}cm^2\,E/MeV$
- Inverse Beta Decay $\bar{\nu}_e\, p \Rightarrow e^+ n$
  - energy threshold E_{th}=1.8 MeV, $\sigma=9.3\cdot 10^{-43}cm^2\,E_\nu/MeV$

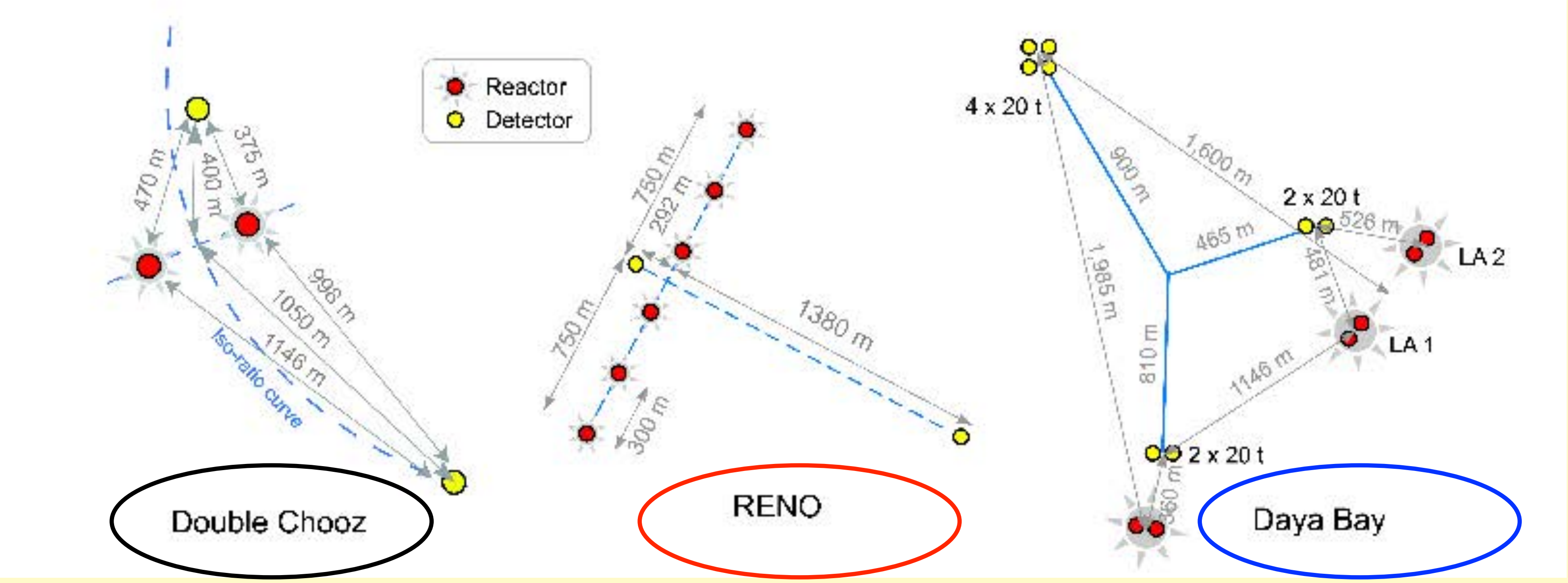


free proton rich material(scintillator)
\+
Energy
and
Time
measurement

  - Prompt positron: $E_\nu \simeq E_{prompt} + \langle E_n \rangle + 0.8$ MeV
    - $E_{prompt} = E_\gamma + e^+$ kinetic energy
    - $\langle E_n \rangle \simeq$ average neutron recoil O(10keV)
  - Delayed (≃207 μs) neutron capture: $E_\gamma \simeq 2.2$ MeV

THREE EXPERIMENTS

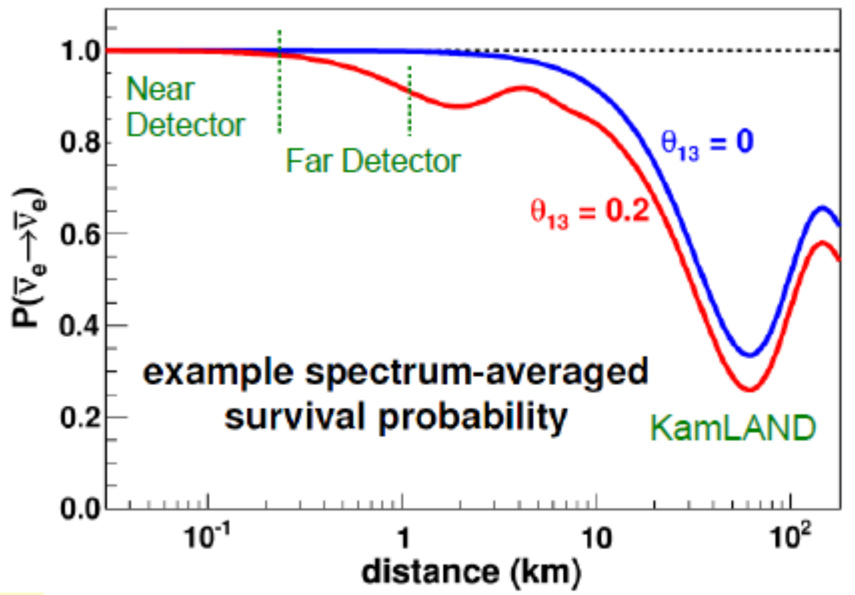
Need near and far detector to reduce systematics on ν flux composition/spectrum

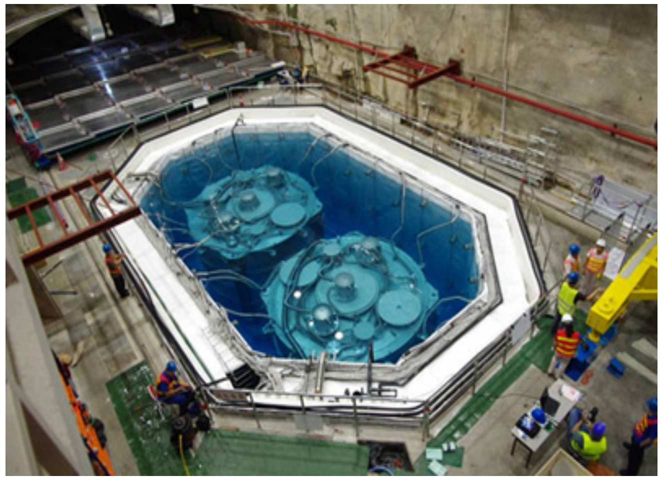


Running with FD;
ND this year!

Running with
ND & FD

Running with
ND & FD

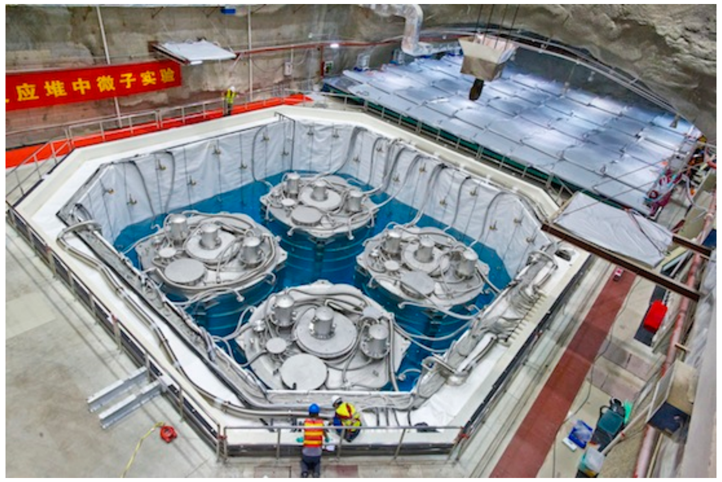




E.g, for
Daya Bay:

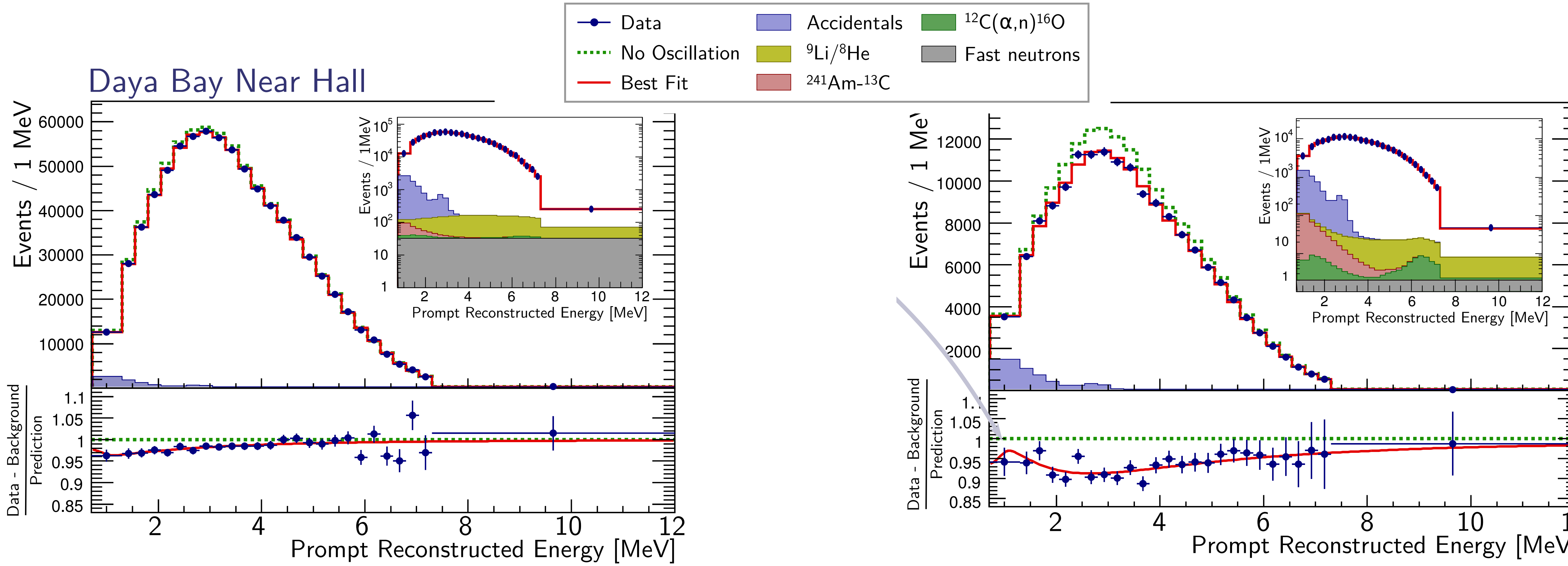
← ND

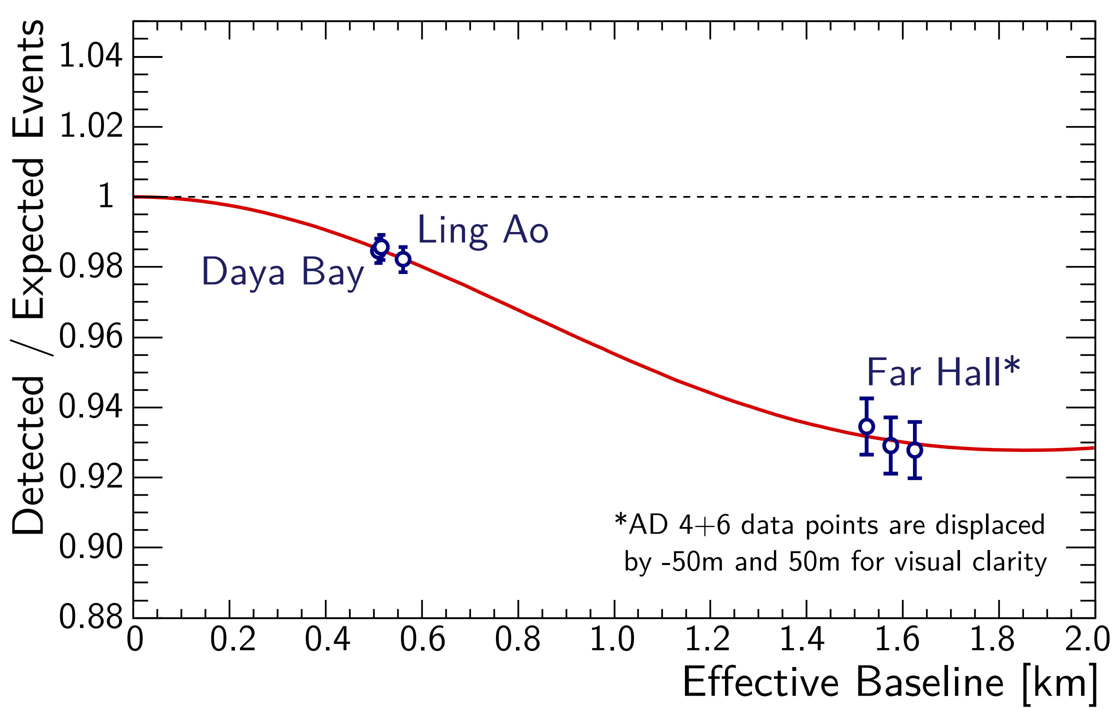
FD →

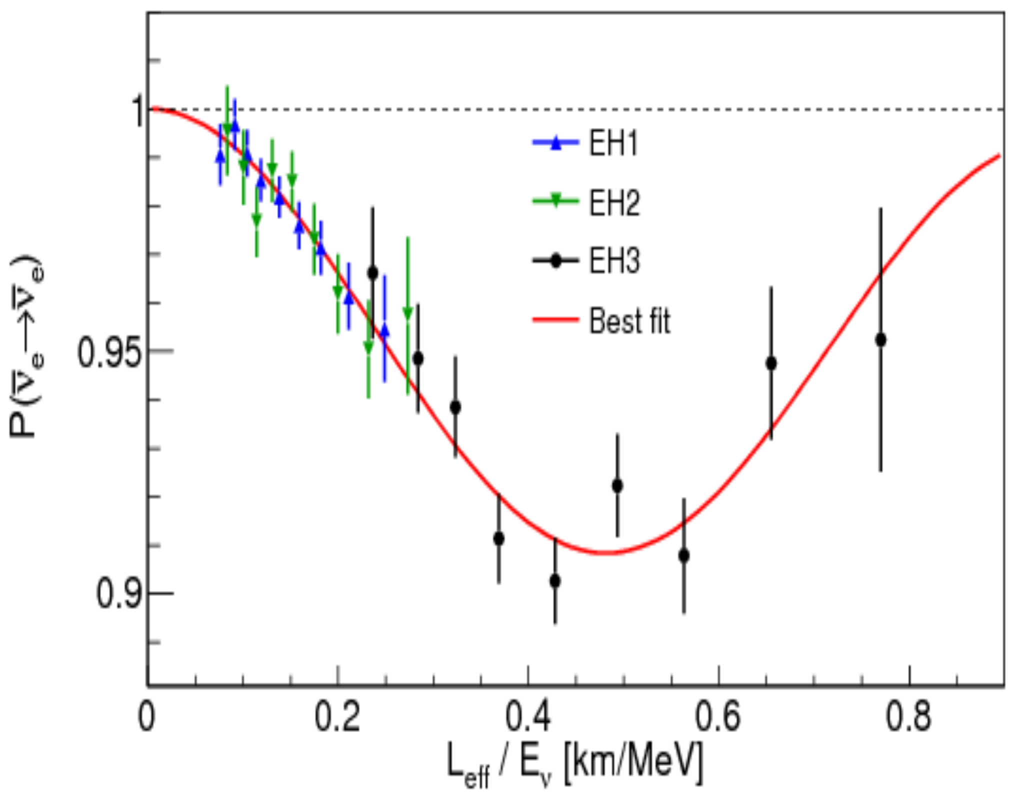


from E.Lisi "Neutrinos: theory and phenomenology"

RESULTS ($\sin^2(2\theta_{13})=0.090^{+0.008}_{-0.007}$)







MEASURING Θ13: APPEARANCE

- In vacuum: $P[\nu_\mu \to \nu_e] \simeq P_{atm} + 2\sqrt{P_{atm}P_{sol}}cos(\Delta_{32}+\delta) + P_{sol}$

$$\sqrt{P}_{atm} = sin\theta_{23}sin2\theta_{13}sin\Delta_{31} \qquad \sqrt{P}_{sol} = cos\theta_{23}cos\theta_{13}sin2\theta_{12}sin\Delta_{21}$$

- Matter effects complicates the picture
- T2K:Off-Axis beam, L/E tuned at first maximum: 28 events observed , 5 expected if θ_{13}=0

$\sin^2(2\theta_{13})=0.15^{+0.04}_{-0.03}$