CKM Matrix and CP Violation in Standard Model

Types of CP Violation. B⁰ oscillation Lecture 16

DIPARTIMENTO DI FISICA



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Measuring Complex Phase of CKM Matrix

- Branching fractions and lifetimes sensitive to magnitude of CKM elements
 - Decay probabilities usually include |V_{ii}|²
 - We looked for decays involving only one CKM element to make interpretation of experimental result possible
- Complex phase of CKM is a relative phase between matrix elements
- We need processes with interference of two different CKM elements

$$A_1 = Ae^{i\alpha}$$
$$A_2 = Be^{i\beta}$$

$$A_2 = Be^{i\beta}$$

$$A_{tot} = A_1 + A_2$$

Sensitive to phase difference!

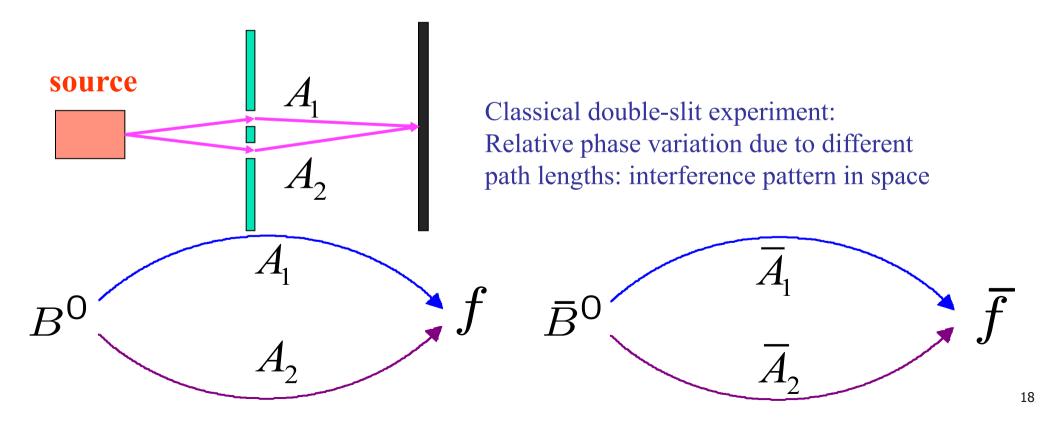
$$|A_{tot}|^2 = |A|^2 + |B|^2 + ABe^{i(\alpha-\beta)} + ABe^{-i(\alpha-\beta)}$$

CP Violation

 CP violation can be observed by comparing decay rates of particles and antiparticles

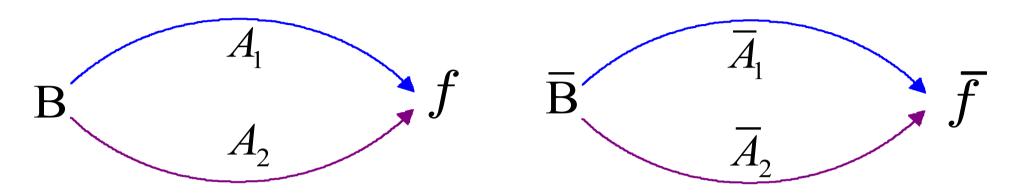
$$\Gamma(a \rightarrow f) \neq \Gamma(\overline{a} \rightarrow \overline{f}) \Rightarrow \text{CP Violation}$$

The difference in decay rates arises from a different interference term for the matter vs. antimatter process. Analogy to double-slit experiment:



CP Violation in B Meson System

Identify B final states which are arrived at by two paths



In B⁰ system, B⁰ \rightleftharpoons \overline{B}^0 oscillation provides one path other path(s) come from weak decay of B hadron

In B[±]system \Rightarrow no oscillation possible,

2 (or more) amplitudes must come from different weak decay

2 (or more) amplitudes must come from different weak decay of B

B Meson is heavy ⇒ many final states, multiple "paths."

2 classes of B decays come into play: "Tree" ⇒ spectator decay like

"Penguin" ⇒ FCNC loop diagrams with u,c,t

CP Violation Is a Quantum Phenomenon

- CPV is due to Quantum interference between two or more amplitudes
- Phase of QM amplitudes is the key
- Need to consider two types of phases
 - CP-conserving phases: don't change sign under CP
 - Sometimes called strong phases since they can arise from strong, final-state interactions
 - CP-violating phases: these do change sign under CP transformation
 - originate in the Weak interaction sector

$$A = Ae^{i\varphi}e^{i\delta}$$

$$\overline{A} = Ae^{-i\varphi}e^{i\delta}$$

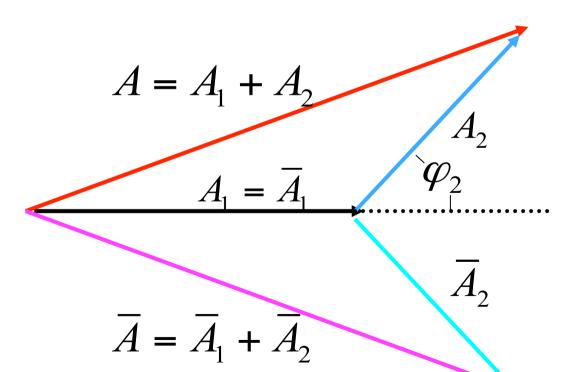
How can CP asymmetries arise?

- Suppose a decay can occur through two different processes, with amplitudes A_1 and A_2
- First, consider the case in which there is a (relative) CP-violating phase between A_1 and A_2 only

$$A = A_1 + a_2 e^{i\varphi_2}$$
$$\overline{A} = A_1 + a_2 e^{-i\varphi_2}$$

- Different decay rate for particle and anti-particle
 - Since new term added
- But no direct CP asymmetry

$$A = \bar{A}$$



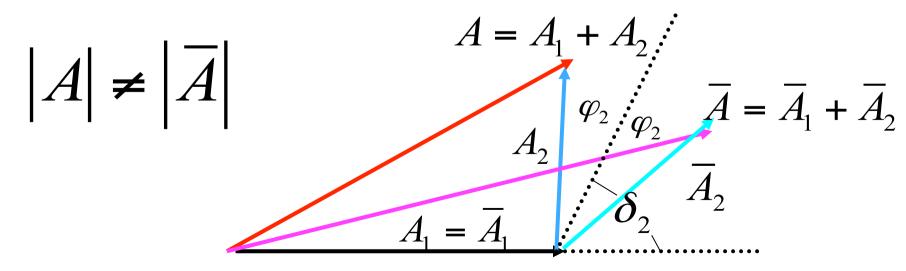
How can CP asymmetries arise?

 Next, introduce a relative CP-conserving phase in addition to the relative CP-violating phase

$$A = A_1 + a_2 e^{i(\varphi_2 + \delta_2)}$$

$$\bar{A} = A_1 + a_2 e^{i(-\varphi_2 + \delta_2)}$$

Now have a Direct CP Violation



Definition of CP Asymmetry

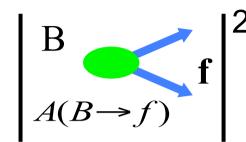
$$Asymmetry = \frac{\left|\overline{A}\right|^{2} - \left|A\right|^{2}}{\left|\overline{A}\right|^{2} + \left|A\right|^{2}} = \frac{2\left|A_{1}\right|\left|A_{2}\right|\sin(\delta_{1} - \delta_{2})\sin(\phi_{1} - \phi_{2})}{\left|A_{1}\right|^{2} + \left|A_{2}\right|^{2} + \left|A_{1}\right|\left|A_{2}\right|\cos(\delta_{1} - \delta_{2})\cos(\phi_{1} - \phi_{2})}$$

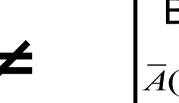
To extract the CP-violating phase from an observed CP asymmetry, we need to know the value of the CP-conserving phase difference

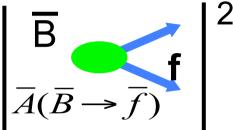
B system: extraordinary laboratory for quantum interference experiments: many final states, multiple "paths"→ Lots of channels for CP Violation

Overview of CP Violating Processes

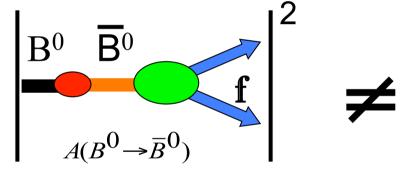
CP Violation in Decay a.k.a. Direct CPV

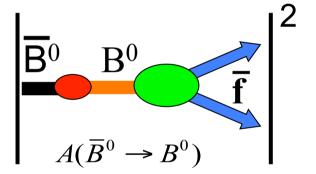




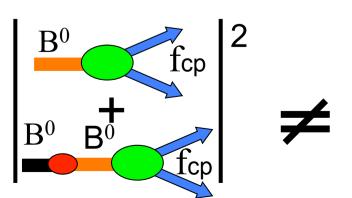


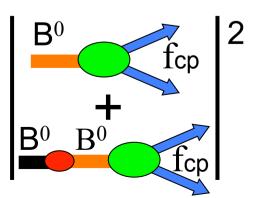
CP Violation in Mixing



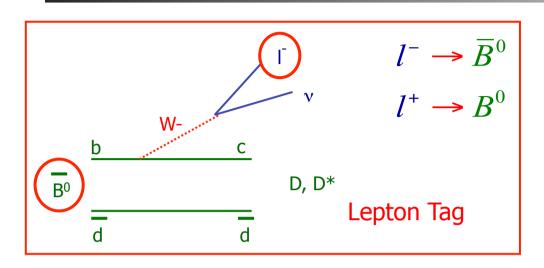


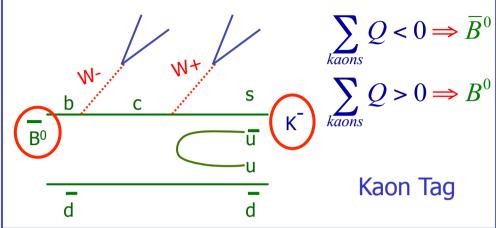
CP Violation in interference between Mixing and Decay

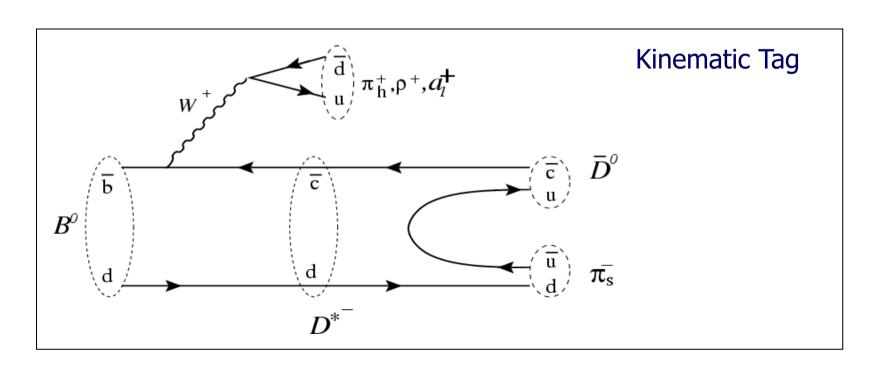




Separating B^0 and \overline{B}^0 mesons



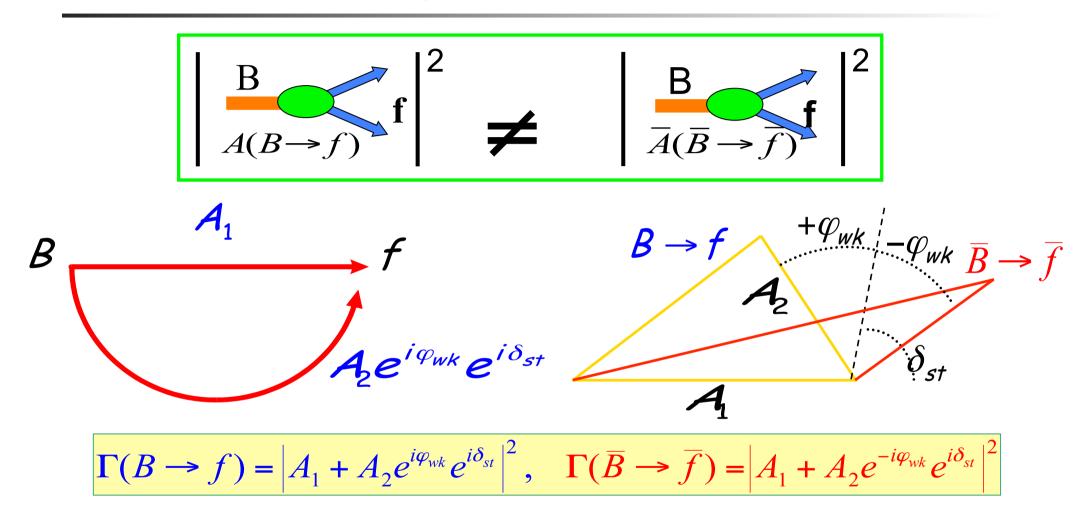




Direct CP Violation

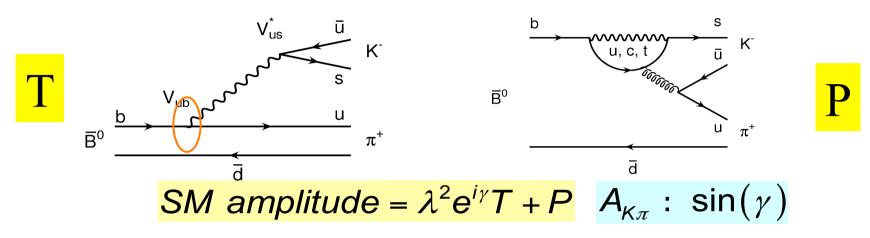


CPV in Decay a.k.a. Direct CP Violation

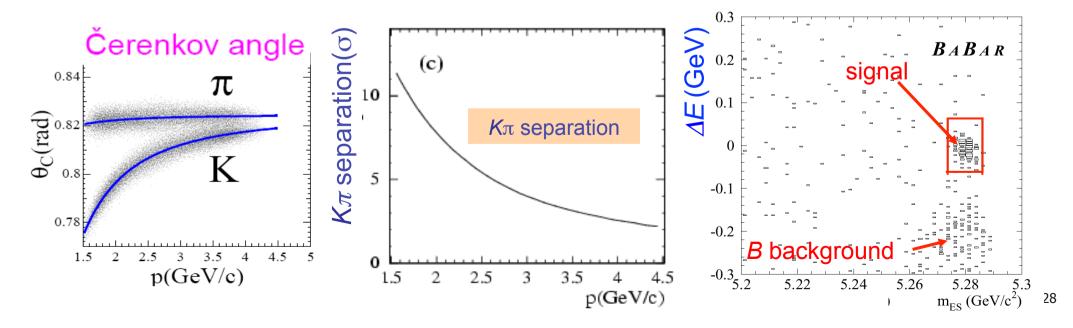


$$A_{CP} = \frac{Br(\overline{B} \to \overline{f}) - Br(B \to f)}{Br(\overline{B} \to \overline{f}) + Br(B \to f)} \equiv \frac{\left|\overline{A_f}\right|^2 - \left|A_f\right|^2}{\left|\overline{A_f}\right|^2 + \left|A_f\right|^2} \neq 0 \to \text{Direct } CPV$$

Direct CP Violation in $B^0 \rightarrow K^- \pi^+$



- Loop diagrams from New Physics (e.g. SUSY) can modify SM asymmetry via P
- Clean mode with "large" rate: 2 x 10⁻⁵
- Measure <u>charge</u> asymmetry, reject large $B \rightarrow \pi\pi$ background with Particle ID

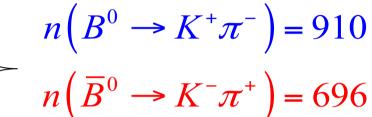


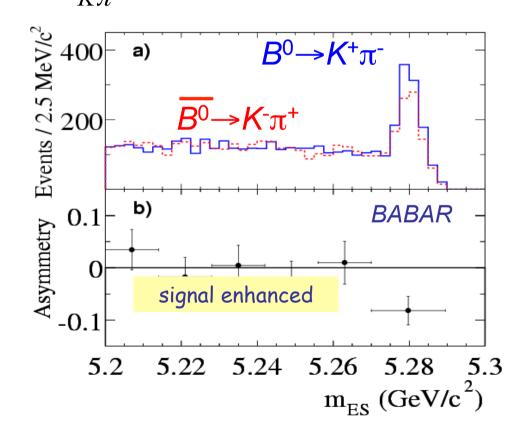
Observation of Direct CPV in B0 $\rightarrow K^-\pi^+$

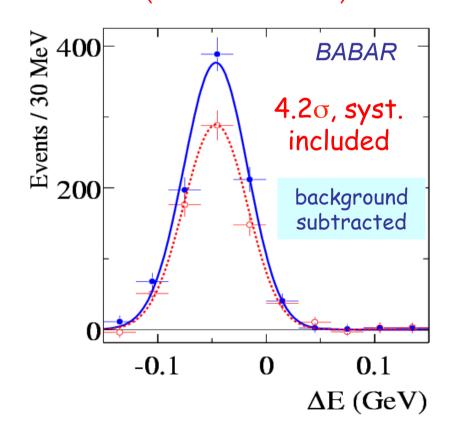
$$A_{K^-\pi^+} \equiv \frac{\Gamma(\overline{B} \to K^-\pi^+) - \Gamma(B \to K^+\pi^-)}{\Gamma(\overline{B} \to K^-\pi^+) + \Gamma(B \to K^+\pi^-)}$$

$$n_{K\pi} = 1606 \pm 51$$

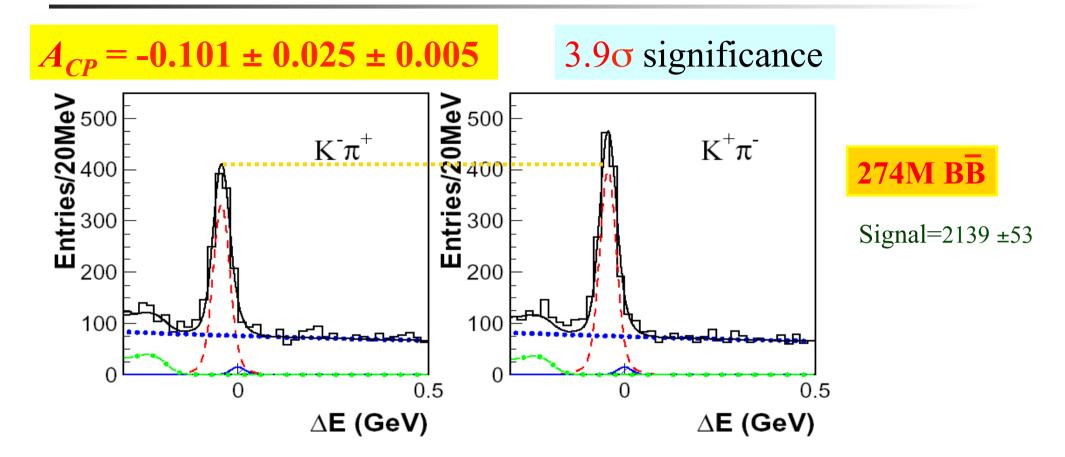
$$A_{K\pi} = -0.133 \pm 0.030 \pm 0.009$$







Confirmation of Direct CPV by Belle

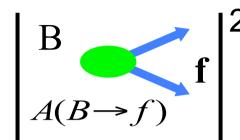


Non-Perturbative QCD uncertainties large, Standard Model CP Violation not precisely predictable

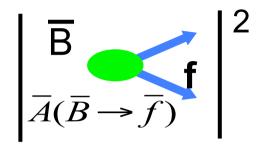
⇒ insufficient to prove or rule out contribution from New Physics

Overview of CP Violating Processes

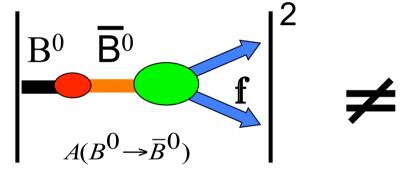
CP Violation in Decay a.k.a. Direct CPV

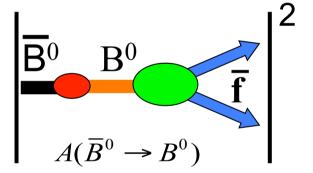




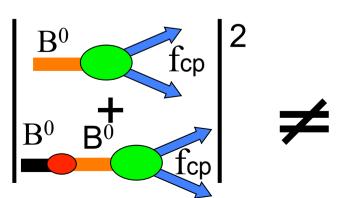


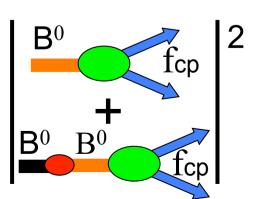
CP Violation in Mixing





CP Violation in interference between Mixing and Decay





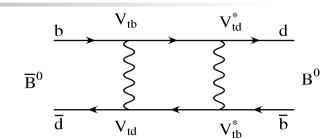
Snapshot of BB Event at BaBar

Inclusive Reconstruction: Look at some of decay products carrying information about their mother μ $\beta \gamma_{Y(4S)} = 0.55$ tag Y(4s)**Coherent BB pair** Exclusive Reconstruction: All particles $B^0 \rightarrow D^{*-}\pi^+$ in final state are identified

B⁰ – B Oscillation

B_d-B_d Oscillation and CP Violation

Necessary ingredient for two types of CP Violation



- Oscillation is an example of superposition principle in a two-state quantum system
 - Oscillation occurs because mass and flavor eigenstates are different
 - Flavor eigenstates $|B^0\rangle$ and $|\overline{B}^0\rangle$: physical states with definite quark structure and are produced as a consequence of the quark-level strong interactions.
 - CP eigenstates $|B_{CP=1}\rangle$ and $|B_{CP=-1}\rangle$: eigenstates of the the CP operation

$$CP|B_{CP=1}\rangle = +|B_{CP=1}\rangle$$

 $CP|B_{CP=-1}\rangle = -|B_{CP=-1}\rangle$

• Mass eigenstates $|B_L\rangle$ and $|B_H\rangle$: eigenstates of the full Hamiltonian and, hence, with definite mass M and decay width $\Gamma \equiv 1/\tau$. These states evolve in time in a definite fashion according to

$$|B_L, t\rangle = e^{-\Gamma_L t} e^{-iM_L t} |B_L, t = 0\rangle$$
 (2.28)

$$|B_H, t\rangle = e^{-\Gamma_H t} e^{-iM_H t} |B_H, t = 0\rangle.$$
 (2.29)

Phenomenology of B⁰ Time Development

- An initially B⁰ or B $^{\overline{0}}$ system evolves with time as a mixture of flavor eigenstates $|\psi(t)=a|B^0\rangle+b|\overline{B}^0\rangle$
- Evolution regulated by time-dependent Schrödinger equation

Wigner-Weisskopf Approximation
$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = \mathbf{H}\begin{pmatrix} a \\ b \end{pmatrix} \equiv (\mathbf{M} - \frac{i}{2}\mathbf{\Gamma})\begin{pmatrix} a \\ b \end{pmatrix}$$

• M and Γ computed to 2^{na} order of perturbation theory

$$M_{ij} = m_B \delta_{ij} + \langle i | H_W^{\Delta B=2} | j \rangle + P \sum_n \frac{1}{m_B - E_n} \langle i | H_W^{\Delta B=1} | n \rangle \langle n | H_W^{\Delta B=1} | j \rangle$$

$$\Gamma_{ij} = 2\pi \sum_n \delta(E_n - m_B) \langle i | H_W^{\Delta B=1} | n \rangle \langle n | H_W^{\Delta B=1} | j \rangle .$$

- Virtual intermediate states contribute to M
- Γ receives contributions from physical states to which B⁰ or B⁰ can decay

Mass Eigenstates of Effective Hamiltonian

Solving the Schroedinger equation

$$H|\psi\rangle = \lambda |\psi\rangle$$

Two complex eigenvalues

$$\lambda_{\pm} = M - \frac{i}{2}\Gamma - \sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12})}$$

Mass eigenstates

$$|B_L, t\rangle = e^{-\Gamma_L t} e^{-iM_L t} |B_L, t = 0\rangle$$

$$|B_H, t\rangle = e^{-\Gamma_H t} e^{-iM_H t} |B_H, t = 0\rangle$$

$$\Delta m_d \equiv m_H - m_L \equiv \mathcal{R}e(\lambda_+ - \lambda_-) \qquad \qquad \Gamma = 1/\tau_{B^0} = \frac{1}{2}(\Gamma_H + \Gamma_L) \qquad \qquad M = \frac{1}{2}(M_H + M_L)$$

$$\Delta \Gamma \equiv \Gamma_H - \Gamma_L \equiv 2\mathcal{I}m(\lambda_+ - \lambda_-) \qquad \qquad \Delta \Gamma = \Gamma_H - \Gamma_L \qquad \qquad \Delta m_d = M_H - M_L$$

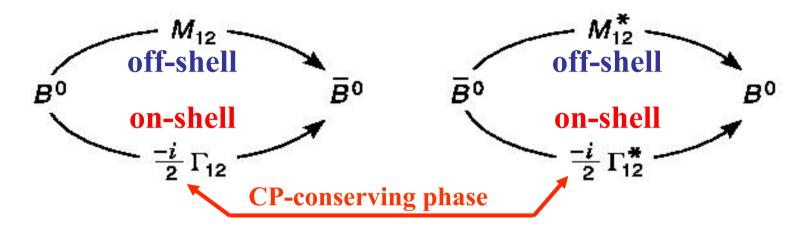
Interpretation of Effective Hamiltonian

 The effective Hamiltonian for the two-state system is not Hermitian since mesons decay

$$M_{12} = (V_{tb}V_{td}^*)^2 \frac{G_F^2}{8\pi^2} \frac{M_W^2}{m_B} S\left(\frac{m_t^2}{M_W^2}\right) \eta_B b_B(\mu) \langle B^0 | Q(\mu) | \overline{B}^0 \rangle$$

what we are after calculable perturbatively nonperturbative

Driving $B^0 \leftrightarrow B^0$ Oscillation



In B° meson system, final states that both B° and \overline{B}° can decay into have very small rates Decays like $b \rightarrow c \ \overline{c}d$ or $b \rightarrow u \ \overline{u}d$ are suppressed due to associated CKM elements in W decay

$$\left|\frac{\Gamma_{12}}{M_{12}}\right| = O(\frac{m_b^2}{m_t^2}) = 1$$

B Oscillation is driven by M_{12} , which is dominated by Top quark in the loop

Differences between K and B Mesons

- Formalism for time evolution can be applied to both K and B mesons
- B mesons
 - Very few common states accessible by both B⁰ and B⁰
 - Comparable lifetime and oscillation frequency

$$\Delta\Gamma/\Gamma \lesssim \mathcal{O}(10^{-2})$$
 $x_d \equiv \Delta m_d/\Gamma = 0.73 \pm 0.05$

Mass eigenstates have very similar lifetimes but different masses

$$\Delta\Gamma \ll \Delta m_d$$

- Kaons
 - Mass eigenstates with similar masses
 - Very different lifetimes

$$\Delta\Gamma_K = \Gamma_K - \Gamma_L \cong \Gamma_K + \Gamma_L \cong \Gamma_K$$

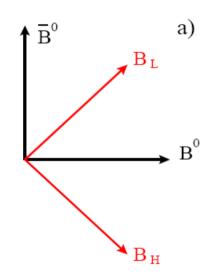
Relation Between Mass and Flavor states

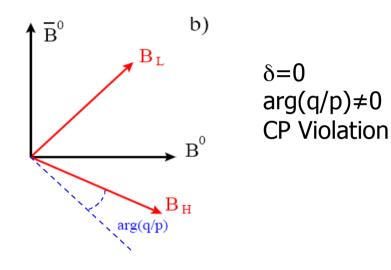
$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle, \qquad |B^0\rangle = \frac{1}{2p}(|B_L\rangle + |B_H\rangle)$$

 $|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle \qquad |\bar{B}^0\rangle = \frac{1}{2q}(|B_L\rangle - |B_H\rangle)$

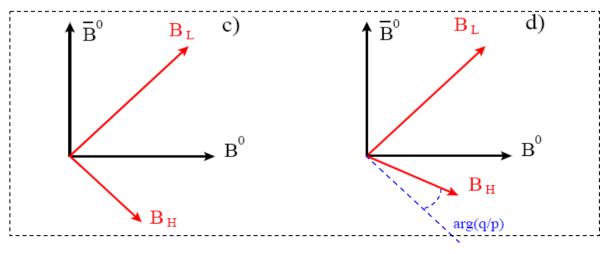
$$\delta \equiv \langle B_L | B_H \rangle \equiv |p|^2 - |q|^2$$

 δ =0 arg(q/p)=0 No CP Violation





 $\delta \neq 0$ regardless arg(q/p) CP Violation



CPV in Mixing

Time Development of Physical States

Evolution of a pure B⁰ or B⁰ state at t=0

$$\begin{aligned} \left| B_{phys}^{0}(t) \right\rangle &= \frac{1}{2p} \left(e^{-\Gamma_{L}t} e^{-iM_{L}t} \left(p \left| B^{0} \right\rangle + q \left| \overline{B}^{0} \right\rangle \right) + e^{-\Gamma_{H}t} e^{-iM_{H}t} \left(p \left| B^{0} \right\rangle - q \left| \overline{B}^{0} \right\rangle \right) \right) \\ \left| \overline{B}_{phys}^{0}(t) \right\rangle &= \frac{1}{2q} \left(e^{-\Gamma_{L}t} e^{-iM_{L}t} \left(p \left| B^{0} \right\rangle + q \left| \overline{B}^{0} \right\rangle \right) - e^{-\Gamma_{H}t} e^{-iM_{H}t} \left(p \left| B^{0} \right\rangle - q \left| \overline{B}^{0} \right\rangle \right) \right) \end{aligned}$$

After some math

$$\Gamma = 1/\tau_{B^0} = \frac{1}{2}(\Gamma_H + \Gamma_L)$$

$$\Delta\Gamma = \Gamma_H - \Gamma_L$$

$$M = \frac{1}{2}(M_H + M_L)$$

$$\Delta m_d = M_H - M_L$$

$$\begin{vmatrix} B_{phys}^{0}(t) \rangle = g_{+}(t) | B^{0} \rangle + (q/p) g_{-}(t) | \overline{B}^{0} \rangle$$

$$\begin{vmatrix} \overline{B}_{phys}^{0}(t) \rangle = (p/q) g_{-}(t) | B^{0} \rangle + g_{+}(t) | \overline{B}^{0} \rangle$$

$$g_{+}(t) = e^{-iMt} e^{-\Gamma t/2} \cos(\Delta m_{d} t/2)$$

$$g_{-}(t) = e^{-iMt} e^{-\Gamma t/2} i \sin(\Delta m_{d} t/2)$$

Prob of $B^0 \rightarrow \overline{B}^0$ oscillates as function of time!

Time evolution of B^0 and $\overline{B^0}$ mesons

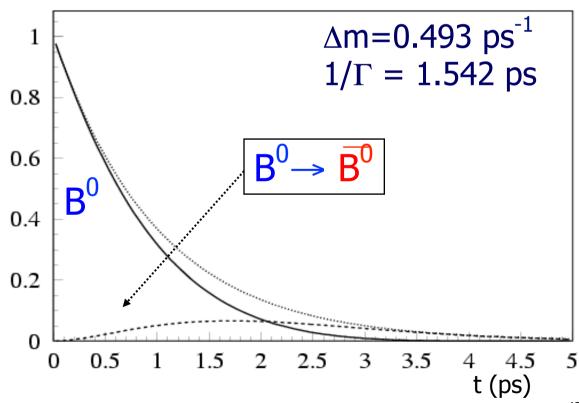
$$|B^{0}(t)\rangle = e^{-iMt}e^{-\Gamma t}\left(\cos\frac{\Delta m\ t}{2} \qquad |B^{0}\rangle + i\,\sin\frac{\Delta m\ t}{2}\cdot\frac{q}{p}\ |\overline{B^{0}}\rangle\right)$$

$$|\overline{B^0}(t)\rangle = e^{-iMt}e^{-\Gamma t}\left(i\,\sin\frac{\Delta m\ t}{2}\cdot\frac{p}{q}\ |B^0\rangle + \cos\frac{\Delta m\ t}{2} \quad |\overline{B^0}\rangle\right)$$

$$\frac{q}{p} = \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} = e^{-i2\beta}$$

$$P(B^{0} \to \overline{B}^{0}) \propto e^{-\Gamma t} \left(1 - \cos(\Delta m \ t) \right)$$

Slow oscillation compared to the lifetime



Quantum Entanglement in $\Upsilon(4S) \rightarrow B^0B^0$ Decays

$$\Upsilon(4s) \to B^0 \bar{B}^0 \qquad \text{With $L=1$}$$
 Spin = 1 0 0

- Strong interaction: CP is and flavor beauty number are conserved
 - Must have one b and one anti-b quarks in final state

$$|B_{\rm phys}^0 \overline{B}_{\rm phys}^0\rangle = \frac{a}{\sqrt{2}} |B_L B_H\rangle + \frac{b}{\sqrt{2}} |B_H B_L\rangle$$

Time evolution given by mass eigenstates

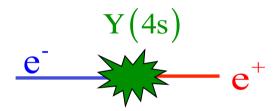
$$|B_{\text{phys}}^0 \overline{B}_{\text{phys}}^0; t_1, t_2\rangle = a e^{i\lambda_+ t_1} e^{i\lambda_- t_2} |B_L B_H\rangle + b e^{i\lambda_- t_1} e^{i\lambda_+ t_2} |B_H B_L\rangle$$

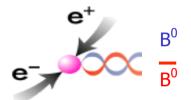
• Bose-Einstein Statistics requires wave function $|\Psi>$ to be symmetric at all times

$$|\Psi\rangle = |\Psi_{\rm flavor}\rangle |\Psi_{\rm space}\rangle$$

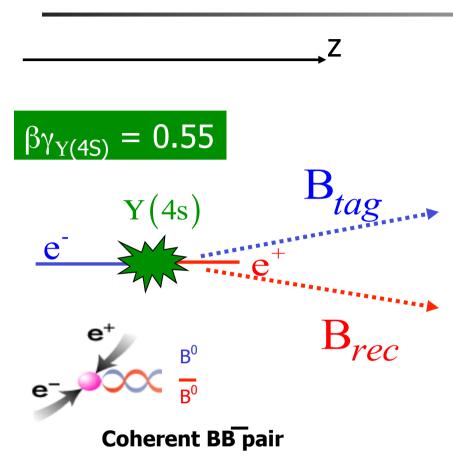
- L=-1 implies asymmetric spatial wave function
- We need a=-b which means a B⁰ and a B⁰ meson at all times until one of them decays!
 - Example of Einstein-Podolsky-Rosen Paradox

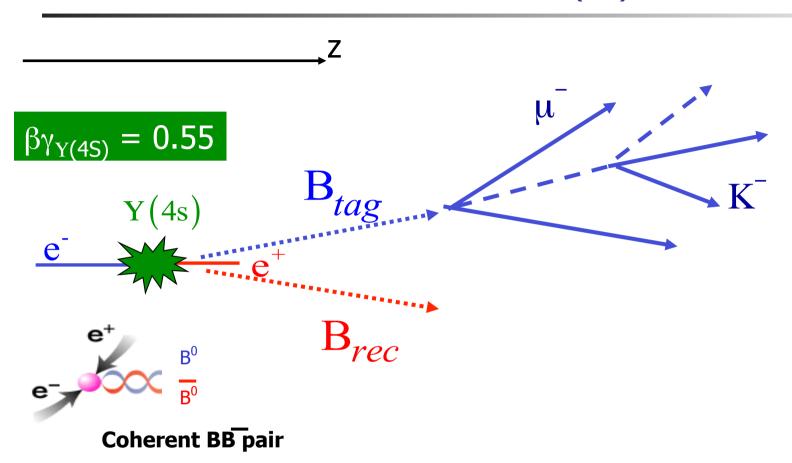
$\beta \gamma_{Y(4S)} = 0.55$

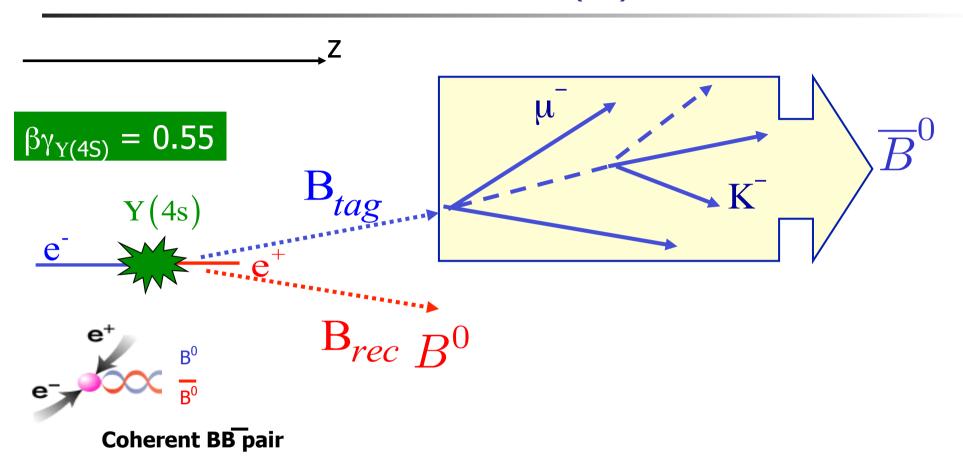


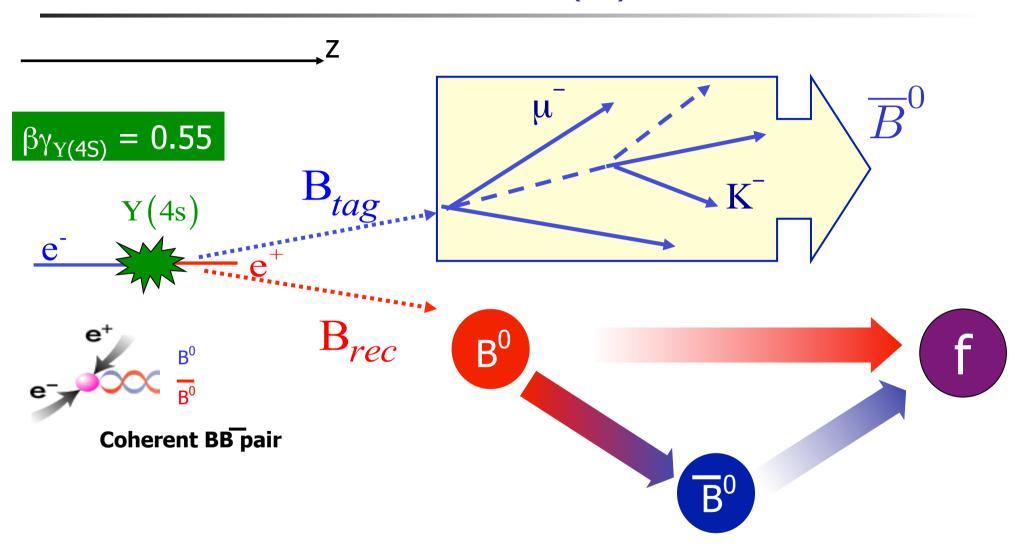


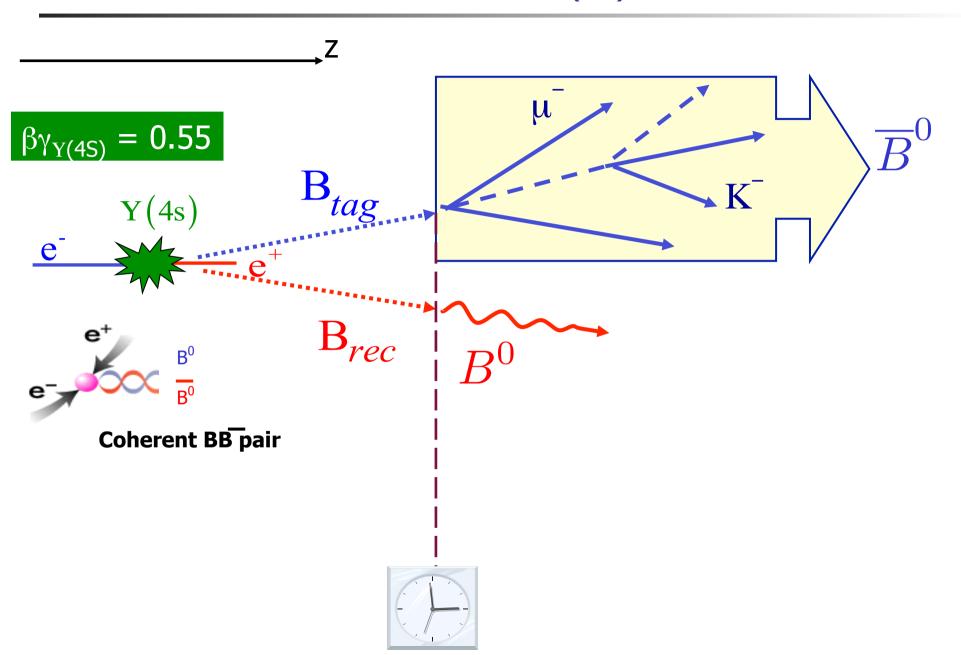
Coherent BB pair

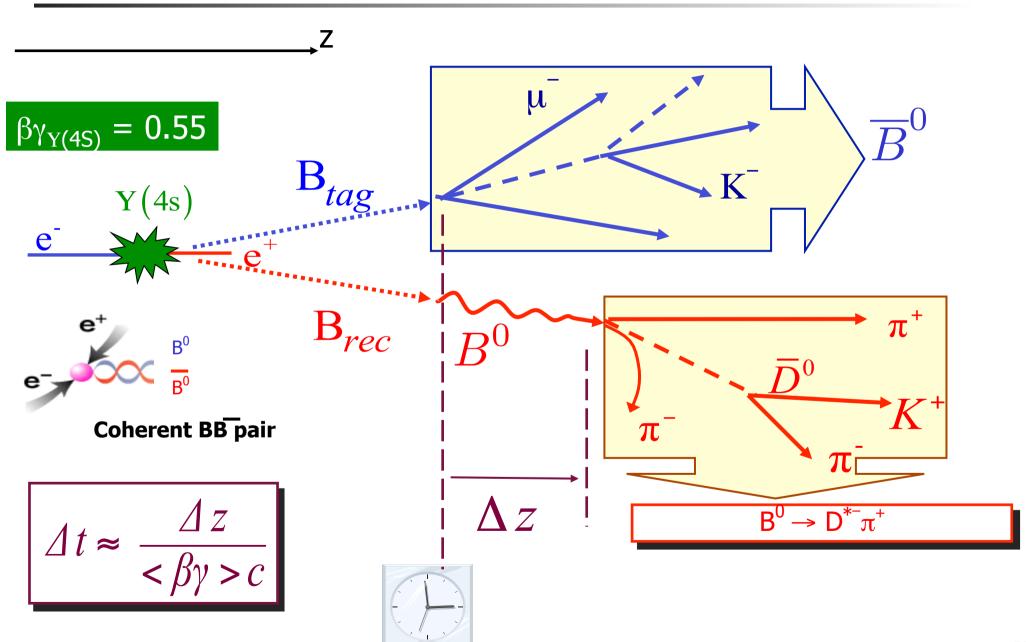




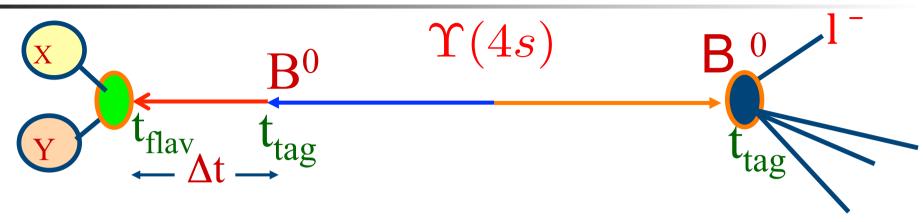








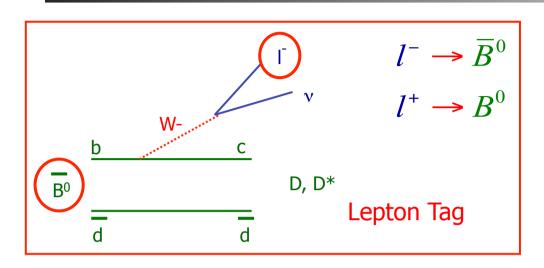
Quantum Correlation at $\Upsilon(4S)$

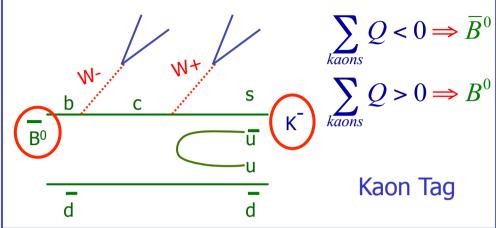


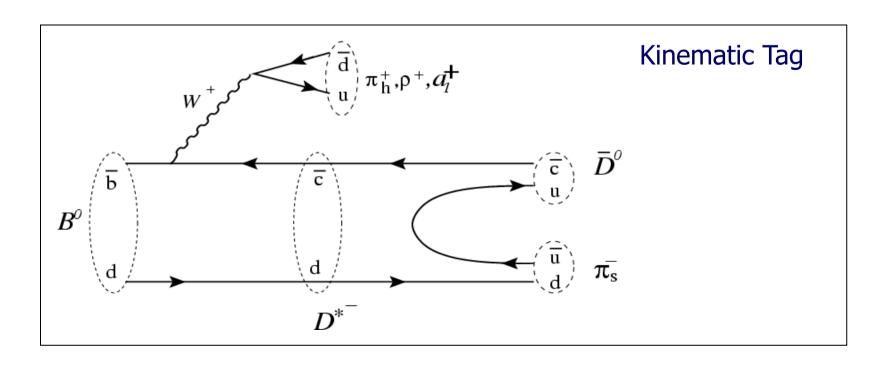
- Decay of first B (B⁰) at time t_{taq} ensures the other B is B⁰
 - End of Quantum entanglement! Defines a ref. time (clock)
- At $t > t_{tag}$, B^0 has some probability to oscillate into B^0 before it decays at time t_{flav} into a flavor specific state
- Two possibilities in the $\Upsilon(4S)$ event depending on whether the 2^{nd} B oscillated or not:

no oscillation/mixing
$$\Rightarrow$$
 B^0 \bar{B}^0 in final state oscillation/mixing \Rightarrow \bar{B}^0 \bar{B}^0 in final state

Separating B^0 and \overline{B}^0 mesons







Flavor Tagging Performance

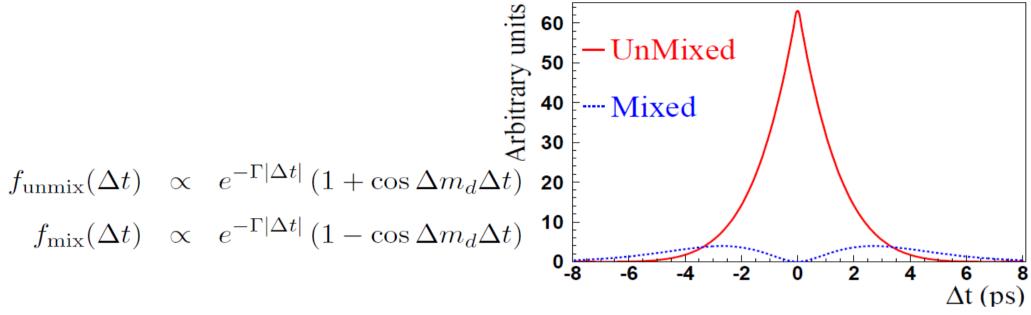
The large sample of fully reconstructed events provides the precise determination of the tagging parameters required in the CP fit

Tagging category	Fraction of tagged events ε (%)	Wrong tag fraction w (%)	$Q = \epsilon (1-2w)^2 (\%)$
Lepton	11.1 ± 0.2	8.6 ± 0.9	7.6 ± 0.4
Kaon	34.7 ± 0.4	18.1 ± 0.7	14.1 ± 0.6
NT1	7.7 ± 0.2	22.0 ± 1.5	2.4 ± 0.3
NT2	14.0 ± 0.3	37.3 ± 1.3	0.9 ± 0.2
ALL	67.5 ± 0.5		25.1 ± 0.8

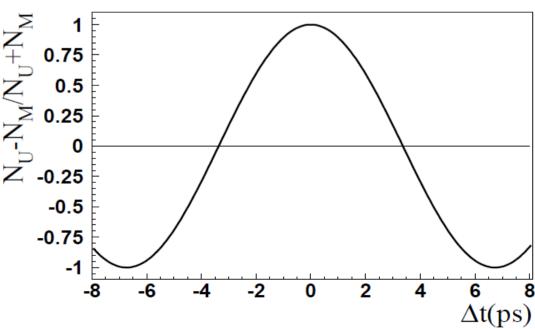
Highest "efficiency"

Smallest mistag fraction

Time Dependent B Oscillation (Or Mixing) at $\Upsilon(4S)$



$$\mathcal{A}_{ ext{mix}}(\Delta t) = rac{f_{ ext{unmix}} - f_{ ext{mix}}}{f_{ ext{unmix}} + f_{ ext{mix}}} egin{array}{ccc} egin{array}{cccc} egin{array}{cccc} egin{array}{ccc} egin{array}{ccc} egin{array}{cccc} egin{array}{cccc} egin{array}{cccc} egin{array}{ccc} egin{array}{cccc} egin{array}{cccc} egin{array}{cccc} egin{array}{ccc} egin{array}{cccc} egi$$

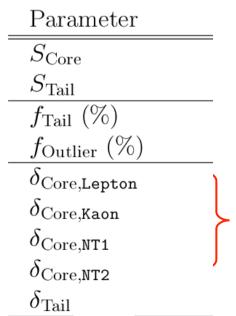


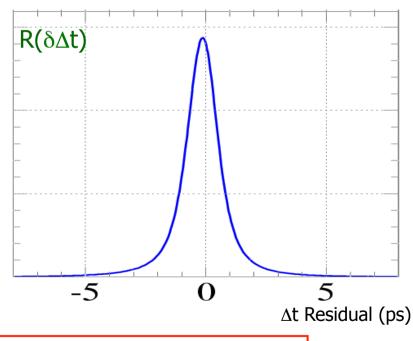
∆t Resolution Function

$$R\left(\delta\Delta t\right) = \left(1 - f_{tail} - f_{outl}\right) \cdot G_{core}\left(\delta\Delta t, S_{core}, \delta_{core,i}\right) \longleftarrow Core$$

$$+ f_{tail} \cdot G_{tail}\left(\delta\Delta t, S_{tail}, \delta_{tail}\right) \longleftarrow Tail$$

Outlier
$$+ f_{outl} \cdot G_{outl} \left(\delta \Delta t, \sigma_{outl} = 0 ps, \delta_{outl} = 8 ps \right)$$



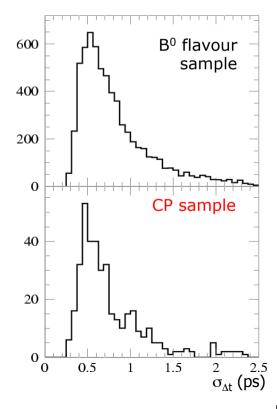


Different bias scale factor For each tagging category

$$\sigma_{core} = S_{core} \cdot \sigma_{\Delta t}^{evt}$$

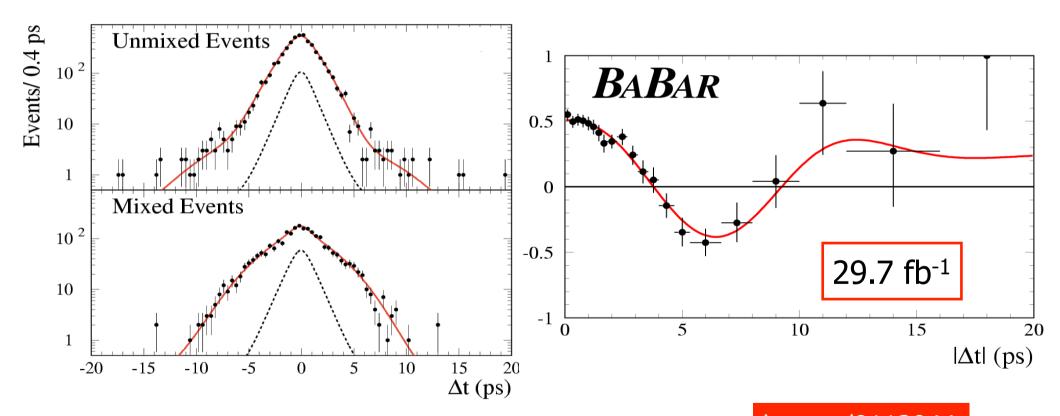
$$\sigma_{tail} = S_{tail} \cdot \sigma_{\Delta t}^{evt}$$

Use the event-by-event uncertainty on ∆t



B⁰B⁰ Mixing Fit Result

$$Asym\left(\Delta t\right) = \frac{N(unmixed) - N(mixed)}{N(unmixed) + N(mixed)} \sim (1 - 2\langle w \rangle) \times \cos\left(\Delta m_d \Delta t\right)$$

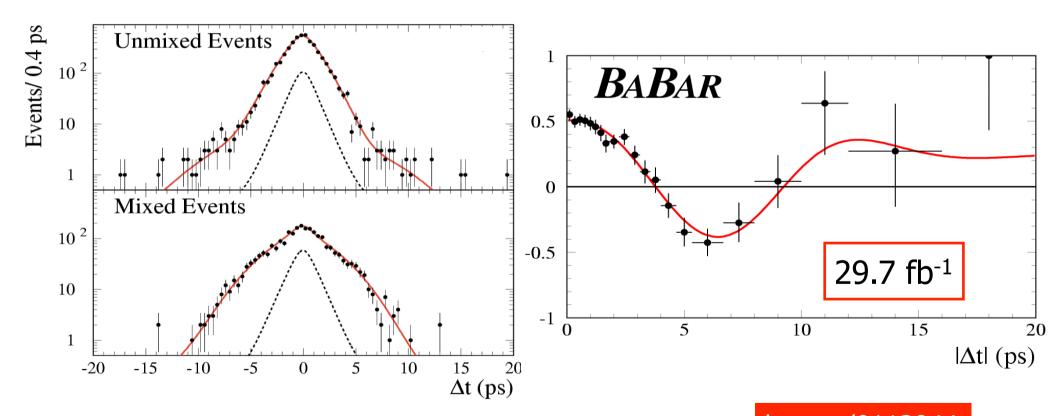


 $\Delta m_d = 0.516 \pm 0.016 \text{ (stat)} \pm 0.010 \text{ (syst) ps}^{-1}$

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